Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/199-7.4.2-Exponentials-of-inverse-hyperbolic-cotangent-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [935]. This is test number [199].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (935)	0.00 (0)
Mathematica	97.75 (914)	2.25 (21)
Fricas	89.95 (841)	10.05 (94)
Maple	83.85 (784)	16.15 (151)
Mupad	55.40 (518)	44.60 (417)
Maxima	54.01 (505)	45.99 (430)
Giac	47.49 (444)	52.51 (491)
Sympy	22.03 (206)	77.97 (729)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

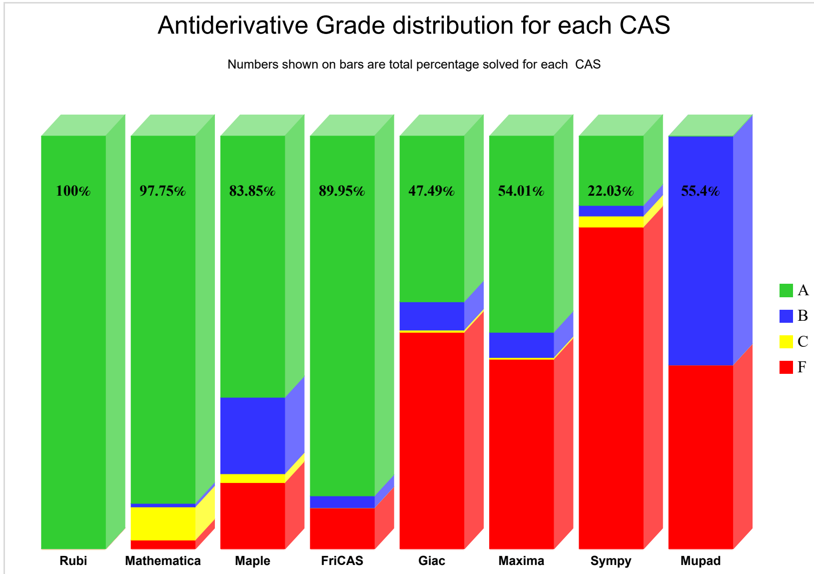
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

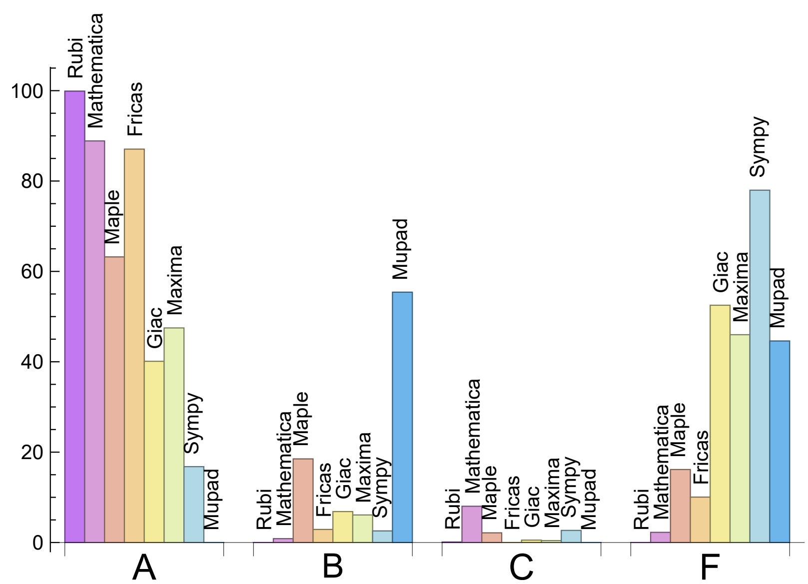
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.89	0.00	0.11	0.00
Mathematica	88.88	0.86	8.02	2.25
Fricas	87.06	2.89	0.00	10.05
Maple	63.21	18.50	2.14	16.15
Maxima	47.49	6.10	0.43	45.99
Giac	40.11	6.84	0.53	52.51
Sympy	16.79	2.57	2.67	77.97
Mupad	N/A	55.40	0.00	44.60

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	21	80.95 %	19.05 %	0.00 %
Maple	151	100.00 %	0.00 %	0.00 %
Fricas	94	100.00 %	0.00 %	0.00 %
Giac	491	66.80 %	0.00 %	33.20 %
Maxima	430	100.00 %	0.00 %	0.00 %
Sympy	729	68.86 %	23.87 %	7.27 %
Mupad	417	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

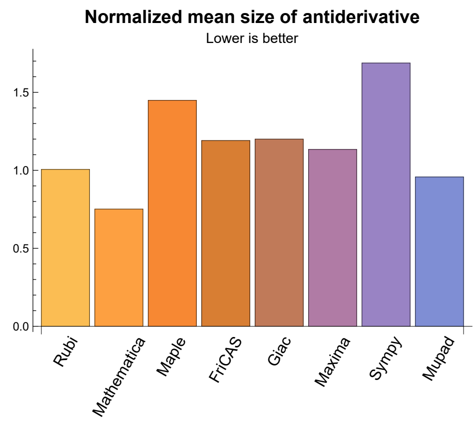
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

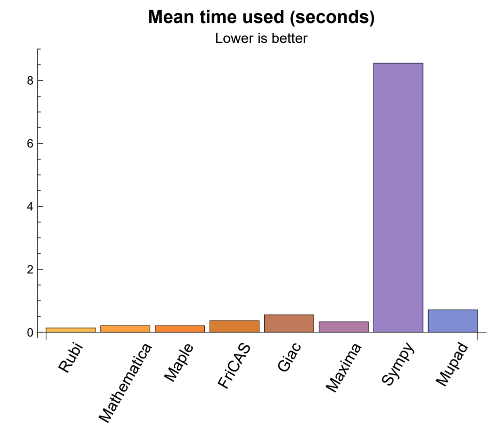
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	134.16	1.01	111.00	1.00
Mathematica	0.21	81.97	0.75	70.50	0.67
Maple	0.21	164.43	1.45	96.00	0.92
Maxima	0.33	115.90	1.13	97.00	1.03
Fricas	0.37	152.98	1.19	99.00	1.08
Sympy	8.55	146.48	1.69	62.00	1.00
Giac	0.55	123.96	1.20	91.00	1.01
Mupad	0.71	97.81	0.96	81.00	0.90

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 61, 70, 79, 88, 97, 106, 113, 120, 126, 133, 135, 136, 138, 158, 159, 160, 161, 162, 178, 179, 180, 181, 182, 198, 199, 200, 201, 217, 218, 219, 220, 257, 279, 280, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 331, 332, 334, 355, 356, 357, 358, 379, 380, 381, 382, 383, 396, 397, 398, 399, 400, 413, 414, 415, 416, 429, 430, 431, 432, 433, 442, 480, 481, 482, 483, 490, 491, 492, 493, 507, 508, 509, 511, 512, 513, 514, 515, 517, 518, 520, 533, 534, 535, 537, 556, 557, 558, 559, 561, 562, 563, 573, 574, 575, 576, 578, 579, 580, 590, 591, 592, 593, 595, 596, 597, 606, 607, 608, 609, 611, 612, 613, 623, 624, 625, 626, 627, 628, 629, 636, 651, 652, 653, 654, 655, 662, 676, 715, 731, 734, 744, 746, 756, 761, 762, 766, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 796, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 829, 832, 848, 857, 873, 885, 888, 909, 912, 931}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

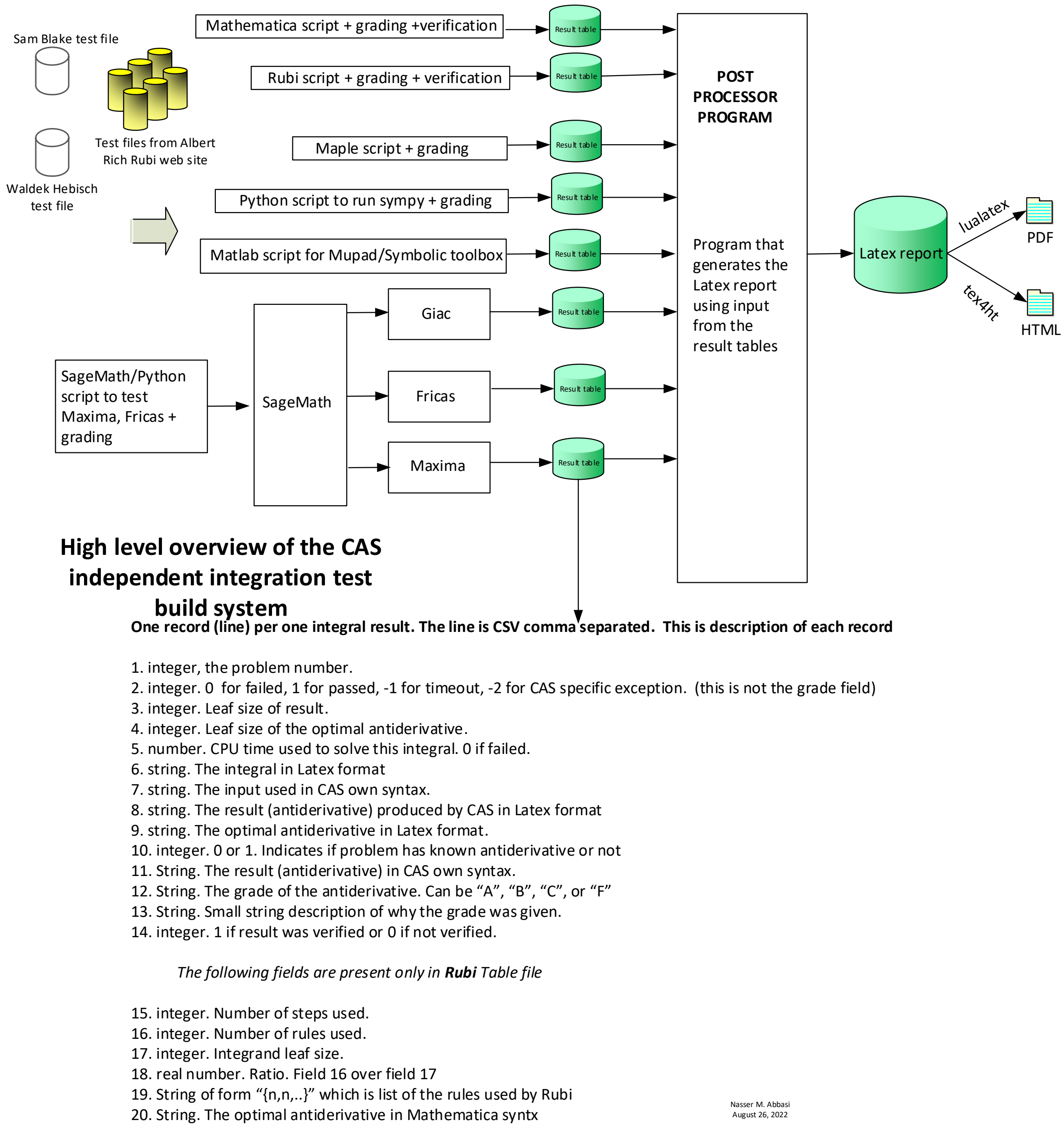
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

B grade: { }

C grade: { 172 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 65, 66, 68, 69, 71, 77, 78, 80, 81, 83, 84, 86, 87, 89, 95, 96, 98, 99, 101, 102, 104, 105, 107, 114, 121, 122, 124, 127, 132, 134, 137, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 550, 552, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 732, 733, 735, 738, 739, 740, 741, 742, 743, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade: { 365, 381, 398, 426, 584, 736, 737, 744 }

C grade: { 61, 64, 67, 70, 72, 73, 74, 75, 76, 79, 82, 85, 88, 90, 91, 92, 93, 94, 97, 100, 103, 106, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 123, 125, 126, 128, 129, 130, 131, 133, 135, 136, 138, 172, 267, 268, 269, 429, 430, 431, 432, 451, 452, 453, 454, 458, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 498, 499, 569, 602, 731, 734 }

F grade: { 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 545, 546, 547, 548, 549, 551, 553, 554, 933, 934, 935 }

2.1.3 Maple

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 404, 405, 406, 407, 408, 409, 410, 411, 412, 417, 421, 422, 423, 424, 425, 426, 427, 428, 435, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 475, 476, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 653, 654, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 676, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 713, 715, 716, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 161, 162, 174, 178, 181, 182, 199, 200, 201, 212, 217, 218, 219, 220, 289, 290, 291, 292, 293, 379, 380, 381, 382, 383, 384, 385, 386, 396, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 418, 419, 420, 429, 430, 431, 432, 433, 434, 436, 437, 450, 451, 452, 453, 454, 471, 472, 473, 474, 477, 478, 479, 500, 501, 502, 526, 527, 528, 529, 530, 531, 532, 586, 623, 624, 625, 626, 629, 630, 631, 632, 651, 652, 655, 656, 657, 658, 673, 675, 677, 678, 679, 680, 681, 712, 714, 717, 718, 719, 720, 776, 777, 778, 779, 792, 793, 794, 795, 796, 810, 811, 812, 813, 825, 826, 827, 828, 829, 838, 839, 840, 843, 844, 845, 863, 864, 865, 868, 869, 870, 890, 891, 892, 893, 894, 914, 915, 916, 917, 918 }

C grade: { 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 132, 134, 137, 503, 504, 505, 506 }

F grade: { 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 126, 127, 128, 129, 130, 131, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.4 Maxima

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 195, 196, 202, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 283, 284, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 323, 324, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 359, 360, 361, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 673, 674, 675, 676, 677, 712, 713, 714, 715, 716, 732, 733, 740, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 880, 904 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 53, 54, 158, 159, 160, 161, 178, 179, 180, 181, 187, 194, 198, 199, 200, 201, 212, 222, 281, 282, 285, 286, 379, 380, 381, 382, 396, 397, 398, 399, 414, 416, 417, 435, 584, 587, 636, 662 }

C grade: { 321, 322, 325, 326 }

F grade: { 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 277, 278, 295, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 354, 355, 356, 357, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 200, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 369, 370, 371, 379, 380, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 491, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 585, 586, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927 }

B grade: { 5, 6, 21, 37, 38, 129, 130, 131, 183, 194, 201, 242, 290, 381, 442, 466, 467, 492, 493, 519, 520, 572, 577, 584, 587, 588, 605 }

C grade: { }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 295, 335, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 489, 516, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.6 Sympy

A grade: { 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 169, 170, 171, 172, 173, 174, 175, 188, 189, 190, 191, 192, 193, 195, 196, 207, 208, 209, 210, 211, 212, 213, 214, 215, 237, 238, 239, 240, 241, 242, 261, 262, 263, 264, 265, 266, 267, 268, 269, 288, 292, 296, 297, 301, 302, 303, 304, 305, 321, 324, 325, 333, 334, 336, 337, 341, 342, 343, 344, 345, 387, 388, 389, 390, 391, 392, 393, 394, 395, 404, 405, 406, 407, 408, 409, 410, 411, 412, 421, 422, 423, 424, 425, 427, 428, 502, 528, 564, 565, 566, 567, 568, 569, 570, 571, 572, 585, 588, 589, 598, 599, 600, 601, 602, 603, 604, 605, 780, 781, 782, 783, 784, 785, 786, 787, 788, 797, 798, 799, 800, 801, 802, 803, 804, 805, 814, 815, 816, 817, 818, 819, 820, 821 }

B grade: { 134, 167, 168, 176, 187, 194, 235, 236, 246, 306, 307, 308, 309, 346, 347, 348, 349, 426, 581, 582, 583, 584, 586, 587 }

C grade: { 137, 319, 323, 327, 328, 369, 370, 371, 447, 552, 623, 624, 625, 626, 651, 652, 740, 767, 770, 838, 839, 840, 863, 864, 865 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 19, 20, 21, 22, 23, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 291, 293, 294, 295, 298, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 322, 326, 329, 330, 331, 332, 335, 338, 339, 340, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 396, 397, 398, 399, 400, 401, 402, 403, 413, 414, 415, 416, 417, 418, 419, 420, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 573, 574, 575, 576, 577, 578, 579, 580, 590, 591, 592, 593, 594, 595, 596, 597, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 768, 769, 771, 772, 773, 774, 775, 776, 777, 778, 779, 789, 790, 791, 792, 793, 794, 795, 796, 806, 807, 808, 809, 810, 811, 812, 813, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.7 Giac

A grade: { 1, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 25, 26, 27, 28, 30, 31, 32, 34, 35, 36, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 158, 159, 160, 161, 163, 164, 165, 166, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 180, 181, 182, 185, 186, 188, 189, 190, 192, 193, 195, 196, 198, 199, 200, 201, 203, 204, 205, 207, 208, 209, 210, 211, 213, 214, 215, 222, 226, 228, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 245, 248, 249, 250, 251, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 311, 333, 334, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 382, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 400, 401, 403, 409, 410, 411, 412, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 435, 498, 499, 500, 556, 557, 558, 559, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 581, 582, 585, 587, 588, 589, 590, 591, 592, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 611, 623, 624, 625, 626, 627, 651, 652, 653, 674, 675, 676, 677, 678, 713, 714, 715, 716, 717, 772, 773, 774, 775, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 797, 798, 801, 802, 803, 804, 805, 806, 807, 808, 809, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 838, 839, 840, 863, 864, 865, 886, 887, 888, 890, 892, 910, 911, 912, 914, 916 }

B grade: { 5, 6, 7, 8, 9, 29, 37, 39, 41, 174, 183, 184, 191, 194, 212, 235, 236, 237, 294, 301, 302, 379, 380, 381, 396, 398, 399, 404, 405, 406, 407, 408, 413, 414, 450, 451, 452, 453, 454, 501, 530, 531, 532, 577, 583, 584, 586, 610, 629, 655, 679, 680, 681, 718, 719, 720, 799, 800, 891, 893, 894, 915, 917, 918 }

C grade: { 321, 322, 325, 326, 328 }

F grade: { 2, 18, 19, 20, 21, 22, 23, 24, 33, 38, 40, 50, 51, 52, 53, 54, 55, 56, 57, 58, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 167, 177, 187, 197, 202, 206, 216, 217, 218, 219, 220, 221, 223, 224, 225, 227, 229, 244, 246, 247, 252, 253, 254, 255, 257, 259, 270, 271, 272, 273, 274, 277, 295, 296, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 327, 329, 330, 331, 332, 335, 336, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 383, 384, 386, 402, 418, 419, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 560, 561, 562, 563, 579, 580, 594, 595, 596, 597, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 628, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 654, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 776, 777, 778, 779, 793, 795, 796, 810, 811, 812, 813, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 889, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 913, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 252, 253, 254, 255, 256, 257, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 336, 337, 338, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 359, 360, 361, 369, 370, 371, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 494, 495, 496, 497, 503, 504, 505, 506, 521, 522, 523, 538, 539, 540, 541, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 629, 630, 631, 632, 639, 655, 656, 657, 658, 665, 732, 733, 740, 741, 742, 743, 747, 748, 749, 750, 753, 754, 757, 758, 759, 760, 763, 764, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 881, 882, 885, 905, 906, 909 }

C grade: { }

F grade: { 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 177, 197, 206, 216, 231, 232, 233, 234, 247, 248, 249, 250, 251, 258, 259, 260, 277, 278, 295, 299, 300, 310, 311, 312, 313, 314, 315, 316, 317, 318, 329, 330, 331, 332, 333, 334, 335, 339, 340, 354, 355, 356, 357, 358, 362, 363, 364, 365, 366, 367, 368, 372, 373, 374, 375, 376, 377, 378, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 498, 499, 500, 501, 502, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 633, 634, 635, 636, 637, 638, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 734, 735, 736, 737, 738, 739, 744, 745, 746, 751, 752, 755, 756, 761, 762, 765, 766, 767, 768, 769, 770, 771, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845,

846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 907, 908, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935
}

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	68	193	203	92	0	111	171
N.S.	1	1.00	0.60	1.69	1.78	0.81	0.00	0.97	1.50
time (sec)	N/A	0.084	0.049	0.109	0.265	0.349	0.000	0.415	1.257

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	60	173	166	84	0	0	133
N.S.	1	1.00	0.67	1.92	1.84	0.93	0.00	0.00	1.48
time (sec)	N/A	0.063	0.035	0.081	0.261	0.331	0.000	0.000	0.063

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	152	128	73	0	77	98
N.S.	1	1.00	0.78	2.41	2.03	1.16	0.00	1.22	1.56
time (sec)	N/A	0.042	0.025	0.079	0.256	0.345	0.000	0.410	0.063

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	97	90	64	0	57	58
N.S.	1	1.00	1.14	2.69	2.50	1.78	0.00	1.58	1.61
time (sec)	N/A	0.024	0.017	0.075	0.256	0.321	0.000	0.411	0.044

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	36	131	69	57	0	63	37
N.S.	1	1.00	1.64	5.95	3.14	2.59	0.00	2.86	1.68
time (sec)	N/A	0.029	0.011	0.074	0.467	0.345	0.000	0.405	1.180

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	27	220	53	46	0	66	55
N.S.	1	1.00	1.12	9.17	2.21	1.92	0.00	2.75	2.29
time (sec)	N/A	0.016	0.015	0.092	0.465	0.352	0.000	0.398	0.055

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	260	91	60	0	143	81
N.S.	1	1.00	1.11	6.84	2.39	1.58	0.00	3.76	2.13
time (sec)	N/A	0.022	0.031	0.095	0.457	0.341	0.000	0.398	1.203

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	284	136	68	0	148	105
N.S.	1	1.00	0.68	3.79	1.81	0.91	0.00	1.97	1.40
time (sec)	N/A	0.042	0.054	0.098	0.476	0.377	0.000	0.415	0.064

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	59	308	172	76	0	226	129
N.S.	1	1.00	0.67	3.50	1.95	0.86	0.00	2.57	1.47
time (sec)	N/A	0.057	0.064	0.100	0.469	0.347	0.000	0.413	0.085

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	42	43	42	37	47	38
N.S.	1	1.00	1.00	0.98	1.00	0.98	0.86	1.09	0.88
time (sec)	N/A	0.038	0.014	0.102	0.259	0.375	0.047	0.409	0.036

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	34	33	27	38	30
N.S.	1	1.00	1.00	1.03	1.03	1.00	0.82	1.15	0.91
time (sec)	N/A	0.033	0.011	0.081	0.253	0.345	0.047	0.412	0.036

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	25	20	30	23
N.S.	1	1.00	1.00	1.04	1.00	0.96	0.77	1.15	0.88
time (sec)	N/A	0.022	0.009	0.081	0.248	0.323	0.035	0.411	0.041

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	16	10	14	13
N.S.	1	1.00	1.00	1.00	0.93	1.14	0.71	1.00	0.93
time (sec)	N/A	0.009	0.010	0.093	0.250	0.348	0.028	0.397	1.171

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	13	13	10	15	14
N.S.	1	1.00	1.00	1.00	0.93	0.93	0.71	1.07	1.00
time (sec)	N/A	0.026	0.006	0.079	0.252	0.367	0.064	0.423	0.043

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	22	15	20	14
N.S.	1	1.00	1.00	1.00	0.95	1.16	0.79	1.05	0.74
time (sec)	N/A	0.030	0.008	0.101	0.260	0.335	0.067	0.418	1.190

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	30	35	26	32	23
N.S.	1	1.00	1.00	0.94	0.91	1.06	0.79	0.97	0.70
time (sec)	N/A	0.030	0.009	0.114	0.256	0.334	0.094	0.406	0.044

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	38	38	43	34	40	30
N.S.	1	1.00	1.00	0.95	0.95	1.08	0.85	1.00	0.75
time (sec)	N/A	0.034	0.011	0.111	0.263	0.349	0.091	0.391	0.042

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	75	471	182	112	0	0	154
N.S.	1	1.00	0.64	3.99	1.54	0.95	0.00	0.00	1.31
time (sec)	N/A	0.738	0.055	0.097	0.251	0.337	0.000	0.000	1.257

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	421	145	103	0	0	117
N.S.	1	1.00	0.72	4.58	1.58	1.12	0.00	0.00	1.27
time (sec)	N/A	0.645	0.048	0.095	0.255	0.343	0.000	0.000	0.065

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	248	110	92	0	0	59
N.S.	1	1.00	0.87	4.00	1.77	1.48	0.00	0.00	0.95
time (sec)	N/A	0.585	0.034	0.093	0.253	0.367	0.000	0.000	1.293

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	372	90	104	0	0	54
N.S.	1	1.00	1.15	8.09	1.96	2.26	0.00	0.00	1.17
time (sec)	N/A	0.563	0.042	0.084	0.459	0.353	0.000	0.000	0.037

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	593	72	74	0	0	57
N.S.	1	1.00	0.80	11.63	1.41	1.45	0.00	0.00	1.12
time (sec)	N/A	0.048	0.061	0.100	0.457	0.358	0.000	0.000	0.049

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	642	110	88	0	0	83
N.S.	1	1.00	0.62	7.05	1.21	0.97	0.00	0.00	0.91
time (sec)	N/A	0.294	0.068	0.100	0.479	0.365	0.000	0.000	0.076

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	66	666	154	96	0	0	152
N.S.	1	1.00	0.71	7.16	1.66	1.03	0.00	0.00	1.63
time (sec)	N/A	0.484	0.084	0.102	0.460	0.355	0.000	0.000	1.240

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	55	58	66	49	78	57
N.S.	1	1.00	1.00	0.96	1.02	1.16	0.86	1.37	1.00
time (sec)	N/A	0.044	0.033	0.127	0.258	0.336	0.089	0.402	0.044

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	49	57	39	69	49
N.S.	1	1.00	1.00	1.00	1.04	1.21	0.83	1.47	1.04
time (sec)	N/A	0.039	0.027	0.131	0.259	0.357	0.077	0.399	1.167

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	39	41	49	31	64	38
N.S.	1	1.00	1.00	1.00	1.05	1.26	0.79	1.64	0.97
time (sec)	N/A	0.026	0.022	0.125	0.261	0.348	0.064	0.397	0.042

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	26	38	19	46	25
N.S.	1	1.00	0.96	0.96	0.96	1.41	0.70	1.70	0.93
time (sec)	N/A	0.012	0.015	0.123	0.263	0.348	0.060	0.410	0.036

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	18	8	57	12
N.S.	1	1.00	1.00	1.00	0.92	1.38	0.62	4.38	0.92
time (sec)	N/A	0.028	0.008	0.131	0.258	0.337	0.093	0.403	0.034

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	55	26	40	28
N.S.	1	1.00	1.00	0.97	1.06	1.72	0.81	1.25	0.88
time (sec)	N/A	0.032	0.019	0.162	0.260	0.334	0.119	0.401	0.052

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	43	48	73	41	62	41
N.S.	1	1.00	1.00	0.93	1.04	1.59	0.89	1.35	0.89
time (sec)	N/A	0.036	0.023	0.181	0.255	0.334	0.139	0.398	1.196

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	56	81	49	74	49
N.S.	1	1.00	1.00	0.94	1.04	1.50	0.91	1.37	0.91
time (sec)	N/A	0.039	0.033	0.187	0.250	0.346	0.170	0.396	0.062

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	68	193	203	91	0	0	172
N.S.	1	1.00	0.60	1.69	1.78	0.80	0.00	0.00	1.51
time (sec)	N/A	0.088	0.046	0.089	0.255	0.343	0.000	0.000	1.213

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	60	173	166	83	0	86	134
N.S.	1	1.00	0.67	1.92	1.84	0.92	0.00	0.96	1.49
time (sec)	N/A	0.068	0.037	0.087	0.252	0.339	0.000	0.418	0.054

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	49	152	130	73	0	71	97
N.S.	1	1.00	0.77	2.38	2.03	1.14	0.00	1.11	1.52
time (sec)	N/A	0.047	0.026	0.080	0.259	0.339	0.000	0.420	0.057

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	98	90	64	0	52	58
N.S.	1	1.00	1.14	2.65	2.43	1.73	0.00	1.41	1.57
time (sec)	N/A	0.026	0.018	0.076	0.256	0.360	0.000	0.413	1.189

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	133	70	57	0	59	37
N.S.	1	1.00	1.70	6.65	3.50	2.85	0.00	2.95	1.85
time (sec)	N/A	0.032	0.011	0.069	0.463	0.347	0.000	0.409	0.031

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	220	55	47	0	0	55
N.S.	1	1.00	1.04	8.80	2.20	1.88	0.00	0.00	2.20
time (sec)	N/A	0.018	0.015	0.083	0.468	0.340	0.000	0.000	1.198

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	260	93	60	0	157	82
N.S.	1	1.00	1.02	6.50	2.32	1.50	0.00	3.92	2.05
time (sec)	N/A	0.024	0.034	0.089	0.458	0.344	0.000	0.407	1.201

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	52	284	137	68	0	0	105
N.S.	1	1.00	0.68	3.74	1.80	0.89	0.00	0.00	1.38
time (sec)	N/A	0.046	0.057	0.089	0.464	0.341	0.000	0.000	1.204

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	59	308	173	77	0	258	129
N.S.	1	1.00	0.67	3.50	1.97	0.88	0.00	2.93	1.47
time (sec)	N/A	0.061	0.066	0.095	0.476	0.337	0.000	0.415	1.221

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	43	42	37	47	38
N.S.	1	1.00	1.00	1.00	1.02	1.00	0.88	1.12	0.90
time (sec)	N/A	0.039	0.021	0.095	0.249	0.330	0.044	0.410	1.171

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	34	33	27	38	31
N.S.	1	1.00	1.00	1.06	1.03	1.00	0.82	1.15	0.94
time (sec)	N/A	0.033	0.021	0.081	0.248	0.343	0.035	0.402	0.037

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	26	25	20	30	23
N.S.	1	1.00	1.00	1.08	1.04	1.00	0.80	1.20	0.92
time (sec)	N/A	0.022	0.019	0.083	0.261	0.320	0.033	0.406	0.039

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	16	10	14	13
N.S.	1	1.00	1.00	1.08	1.00	1.23	0.77	1.08	1.00
time (sec)	N/A	0.009	0.020	0.078	0.249	0.325	0.034	0.412	0.029

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	15	14
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.15	1.08
time (sec)	N/A	0.025	0.012	0.089	0.249	0.340	0.058	0.408	0.043

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	23	15	20	14
N.S.	1	1.00	1.00	1.06	1.00	1.28	0.83	1.11	0.78
time (sec)	N/A	0.028	0.010	0.119	0.252	0.333	0.075	0.405	1.196

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	30	35	26	32	24
N.S.	1	1.00	1.00	0.97	0.94	1.09	0.81	1.00	0.75
time (sec)	N/A	0.030	0.012	0.120	0.260	0.352	0.077	0.411	0.044

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	38	43	34	40	31
N.S.	1	1.00	1.00	0.98	0.95	1.08	0.85	1.00	0.78
time (sec)	N/A	0.031	0.020	0.121	0.252	0.373	0.089	0.408	1.211

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	83	539	223	92	0	0	192
N.S.	1	1.00	0.61	3.96	1.64	0.68	0.00	0.00	1.41
time (sec)	N/A	0.717	0.107	0.106	0.254	0.349	0.000	0.000	0.076

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	75	471	186	84	0	0	156
N.S.	1	1.00	0.65	4.06	1.60	0.72	0.00	0.00	1.34
time (sec)	N/A	0.611	0.135	0.105	0.258	0.336	0.000	0.000	0.058

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	66	421	151	75	0	0	120
N.S.	1	1.00	0.73	4.68	1.68	0.83	0.00	0.00	1.33
time (sec)	N/A	0.590	0.044	0.100	0.260	0.334	0.000	0.000	1.223

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	248	111	66	0	0	78
N.S.	1	1.00	0.90	4.13	1.85	1.10	0.00	0.00	1.30
time (sec)	N/A	0.552	0.035	0.091	0.255	0.333	0.000	0.000	0.040

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	369	89	74	0	0	54
N.S.	1	1.00	1.20	8.02	1.93	1.61	0.00	0.00	1.17
time (sec)	N/A	0.529	0.060	0.083	0.452	0.389	0.000	0.000	0.033

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	594	72	49	0	0	59
N.S.	1	1.00	0.77	11.21	1.36	0.92	0.00	0.00	1.11
time (sec)	N/A	0.052	0.099	0.105	0.457	0.372	0.000	0.000	0.048

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	641	112	61	0	0	118
N.S.	1	1.00	0.64	7.37	1.29	0.70	0.00	0.00	1.36
time (sec)	N/A	0.285	0.188	0.109	0.461	0.335	0.000	0.000	0.059

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	666	157	69	0	0	153
N.S.	1	1.00	0.69	6.94	1.64	0.72	0.00	0.00	1.59
time (sec)	N/A	0.480	0.167	0.112	0.470	0.351	0.000	0.000	1.232

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	75	690	193	77	0	0	190
N.S.	1	1.00	0.56	5.19	1.45	0.58	0.00	0.00	1.43
time (sec)	N/A	0.541	0.036	0.125	0.469	0.331	0.000	0.000	1.236

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.101	5.146	0.006	0.458	0.340	0.000	0.468	0.107

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	192
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.081	5.123	0.005	0.456	0.361	0.000	0.456	1.242

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	399	0	187	103	0	172	157
N.S.	1	1.00	2.23	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.062	4.342	0.004	0.460	0.357	0.000	0.446	0.081

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	149	93	0	139	120
N.S.	1	1.00	0.46	0.00	1.05	0.65	0.00	0.98	0.85
time (sec)	N/A	0.041	0.112	0.003	0.477	0.414	0.000	0.437	0.077

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	51	0	111	84	0	108	78
N.S.	1	1.00	0.53	0.00	1.16	0.88	0.00	1.12	0.81
time (sec)	N/A	0.025	0.061	0.003	0.465	0.370	0.000	0.431	0.065

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	291	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.172	0.031	0.014	0.474	0.366	0.000	0.424	0.083

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	148	0	186	396	0	186	87
N.S.	1	1.00	0.55	0.00	0.70	1.48	0.00	0.70	0.33
time (sec)	N/A	0.158	0.165	0.003	0.465	0.399	0.000	0.447	1.195

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	173	0	226	413	0	223	132
N.S.	1	1.00	0.54	0.00	0.71	1.29	0.00	0.70	0.41
time (sec)	N/A	0.177	0.153	0.002	0.469	0.352	0.000	0.436	0.075

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	270	427	0	271	168
N.S.	1	1.00	0.26	0.00	0.76	1.20	0.00	0.76	0.47
time (sec)	N/A	0.193	0.078	0.004	0.466	0.417	0.000	0.446	1.211

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.093	5.159	0.006	0.468	0.344	0.000	0.528	0.095

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	192
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.082	5.129	0.004	0.463	0.368	0.000	0.511	1.215

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	399	0	187	103	0	172	157
N.S.	1	1.00	2.23	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.061	7.009	0.003	0.472	0.394	0.000	0.479	0.085

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	152	95	0	141	120
N.S.	1	1.00	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.038	0.122	0.004	0.473	0.352	0.000	0.481	1.206

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	56	0	112	86	0	109	79
N.S.	1	1.00	0.57	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.025	0.045	0.003	0.461	0.377	0.000	0.447	1.186

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	291	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.166	0.035	0.013	0.460	0.366	0.000	0.448	0.052

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	46	0	187	376	0	187	88
N.S.	1	1.00	0.17	0.00	0.70	1.40	0.00	0.70	0.33
time (sec)	N/A	0.150	0.051	0.001	0.480	0.367	0.000	0.426	0.078

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	76	0	229	405	0	225	132
N.S.	1	1.00	0.24	0.00	0.72	1.27	0.00	0.71	0.41
time (sec)	N/A	0.173	0.058	0.002	0.456	0.358	0.000	0.451	0.089

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	277	413	0	271	168
N.S.	1	1.00	0.26	0.00	0.78	1.16	0.00	0.76	0.47
time (sec)	N/A	0.193	0.088	0.003	0.456	0.379	0.000	0.463	1.212

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	275	152	0	254	248
N.S.	1	1.00	0.69	0.00	0.96	0.53	0.00	0.89	0.86
time (sec)	N/A	0.110	5.168	0.004	0.467	0.383	0.000	0.478	1.264

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	238	144	0	223	211
N.S.	1	1.00	0.64	0.00	0.95	0.58	0.00	0.89	0.84
time (sec)	N/A	0.094	5.155	0.005	0.458	0.345	0.000	0.456	0.143

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	441	0	203	136	0	192	176
N.S.	1	1.00	2.07	0.00	0.95	0.64	0.00	0.90	0.83
time (sec)	N/A	0.076	6.626	0.004	0.464	0.362	0.000	0.448	0.092

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	80	0	166	128	0	161	139
N.S.	1	1.00	0.45	0.00	0.94	0.73	0.00	0.91	0.79
time (sec)	N/A	0.046	0.152	0.003	0.464	0.366	0.000	0.471	1.210

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	67	0	131	117	0	141	98
N.S.	1	1.00	0.52	0.00	1.01	0.90	0.00	1.08	0.75
time (sec)	N/A	0.031	0.089	0.003	0.471	0.371	0.000	0.454	0.064

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	30	0	244	358	0	252	118
N.S.	1	1.00	0.09	0.00	0.76	1.12	0.00	0.79	0.37
time (sec)	N/A	0.203	0.050	0.012	0.467	0.360	0.000	0.450	1.180

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	173	0	204	451	0	217	107
N.S.	1	1.00	0.58	0.00	0.68	1.51	0.00	0.73	0.36
time (sec)	N/A	0.175	0.467	0.003	0.468	0.388	0.000	0.421	1.217

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	186	0	244	469	0	243	152
N.S.	1	1.00	0.53	0.00	0.70	1.34	0.00	0.69	0.43
time (sec)	N/A	0.203	0.339	0.004	0.459	0.373	0.000	0.459	0.084

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	288	477	0	291	188
N.S.	1	1.00	0.27	0.00	0.75	1.24	0.00	0.76	0.49
time (sec)	N/A	0.214	0.206	0.003	0.462	0.378	0.000	0.452	1.264

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.095	5.215	0.003	0.466	0.351	0.000	0.440	0.080

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	193
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.082	5.159	0.004	0.466	0.347	0.000	0.455	1.210

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	389	0	187	102	0	172	157
N.S.	1	1.00	2.17	0.00	1.04	0.57	0.00	0.96	0.88
time (sec)	N/A	0.063	5.718	0.003	0.461	0.346	0.000	0.440	1.202

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	66	0	151	93	0	140	121
N.S.	1	1.00	0.46	0.00	1.06	0.65	0.00	0.99	0.85
time (sec)	N/A	0.041	0.115	0.003	0.464	0.332	0.000	0.440	0.061

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	33	0	111	84	0	108	79
N.S.	1	1.00	0.34	0.00	1.14	0.87	0.00	1.11	0.81
time (sec)	N/A	0.026	0.034	0.002	0.463	0.373	0.000	0.425	1.181

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	30	0	224	291	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.165	0.035	0.012	0.467	0.338	0.000	0.427	1.176

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	33	0	186	368	0	186	88
N.S.	1	1.00	0.12	0.00	0.69	1.37	0.00	0.69	0.33
time (sec)	N/A	0.153	0.035	0.002	0.460	0.351	0.000	0.418	1.186

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	56	0	227	396	0	223	132
N.S.	1	1.00	0.18	0.00	0.71	1.24	0.00	0.70	0.41
time (sec)	N/A	0.173	0.050	0.003	0.459	0.382	0.000	0.439	0.065

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	277	412	0	271	169
N.S.	1	1.00	0.26	0.00	0.78	1.16	0.00	0.76	0.47
time (sec)	N/A	0.194	0.086	0.003	0.465	0.382	0.000	0.438	0.069

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	173	0	259	119	0	234	229
N.S.	1	1.00	0.68	0.00	1.02	0.47	0.00	0.92	0.91
time (sec)	N/A	0.097	5.212	0.006	0.467	0.407	0.000	0.480	1.214

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	149	0	224	111	0	203	193
N.S.	1	1.00	0.69	0.00	1.04	0.51	0.00	0.94	0.89
time (sec)	N/A	0.078	5.174	0.004	0.478	0.406	0.000	0.489	1.180

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	389	0	187	103	0	172	157
N.S.	1	1.00	2.17	0.00	1.04	0.58	0.00	0.96	0.88
time (sec)	N/A	0.060	6.133	0.004	0.464	0.398	0.000	0.458	0.057

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	70	0	152	95	0	141	121
N.S.	1	1.00	0.49	0.00	1.07	0.67	0.00	0.99	0.85
time (sec)	N/A	0.040	0.128	0.002	0.464	0.367	0.000	0.462	1.186

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	55	0	112	86	0	109	79
N.S.	1	1.00	0.56	0.00	1.14	0.88	0.00	1.11	0.81
time (sec)	N/A	0.024	0.078	0.003	0.463	0.345	0.000	0.425	1.183

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	28	0	224	291	0	232	101
N.S.	1	1.00	0.10	0.00	0.77	1.00	0.00	0.80	0.35
time (sec)	N/A	0.159	0.049	0.013	0.471	0.419	0.000	0.431	1.181

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	149	0	187	402	0	187	88
N.S.	1	1.00	0.55	0.00	0.70	1.49	0.00	0.70	0.33
time (sec)	N/A	0.153	0.189	0.004	0.473	0.494	0.000	0.429	0.052

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	174	0	228	419	0	225	132
N.S.	1	1.00	0.55	0.00	0.71	1.31	0.00	0.71	0.41
time (sec)	N/A	0.173	0.135	0.001	0.467	0.368	0.000	0.434	1.182

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	93	0	270	427	0	271	169
N.S.	1	1.00	0.26	0.00	0.76	1.20	0.00	0.76	0.47
time (sec)	N/A	0.196	0.095	0.004	0.472	0.366	0.000	0.454	0.064

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	198	0	279	119	0	254	253
N.S.	1	1.00	0.69	0.00	0.97	0.41	0.00	0.89	0.88
time (sec)	N/A	0.110	5.239	0.004	0.466	0.467	0.000	0.439	0.081

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	161	0	244	111	0	223	217
N.S.	1	1.00	0.64	0.00	0.98	0.44	0.00	0.89	0.87
time (sec)	N/A	0.102	5.202	0.004	0.461	0.393	0.000	0.452	0.080

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	389	0	207	103	0	192	181
N.S.	1	1.00	1.83	0.00	0.97	0.48	0.00	0.90	0.85
time (sec)	N/A	0.075	5.660	0.004	0.468	0.360	0.000	0.454	0.074

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	113	0	172	95	0	161	145
N.S.	1	1.00	0.64	0.00	0.98	0.54	0.00	0.91	0.82
time (sec)	N/A	0.050	0.193	0.003	0.458	0.375	0.000	0.437	1.198

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	31	0	132	86	0	129	103
N.S.	1	1.00	0.24	0.00	1.02	0.66	0.00	0.99	0.79
time (sec)	N/A	0.033	0.050	0.003	0.461	0.355	0.000	0.421	1.188

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	28	0	244	308	0	252	118
N.S.	1	1.00	0.09	0.00	0.76	0.96	0.00	0.79	0.37
time (sec)	N/A	0.203	0.067	0.013	0.479	0.351	0.000	0.435	1.144

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	31	0	204	377	0	204	106
N.S.	1	1.00	0.10	0.00	0.68	1.26	0.00	0.68	0.35
time (sec)	N/A	0.177	0.056	0.003	0.469	0.363	0.000	0.409	1.180

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	94	0	247	405	0	243	153
N.S.	1	1.00	0.27	0.00	0.70	1.15	0.00	0.69	0.44
time (sec)	N/A	0.198	0.124	0.005	0.473	0.366	0.000	0.413	0.070

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	104	0	297	413	0	291	188
N.S.	1	1.00	0.27	0.00	0.77	1.07	0.00	0.76	0.49
time (sec)	N/A	0.219	0.116	0.002	0.467	0.356	0.000	0.435	0.073

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	340	705	220	173	0	215	168
N.S.	1	1.00	1.19	2.47	0.77	0.61	0.00	0.75	0.59
time (sec)	N/A	0.170	5.438	4.751	0.471	0.345	0.000	0.436	0.132

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	167	1158	194	168	0	191	142
N.S.	1	1.00	0.65	4.49	0.75	0.65	0.00	0.74	0.55
time (sec)	N/A	0.146	0.233	5.448	0.467	0.341	0.000	0.432	1.221

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	35	987	167	160	0	168	115
N.S.	1	1.00	0.16	4.43	0.75	0.72	0.00	0.75	0.52
time (sec)	N/A	0.124	0.032	4.770	0.460	0.343	0.000	0.419	0.098

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	26	2714	0	340	0	261	167
N.S.	1	1.00	0.06	6.75	0.00	0.85	0.00	0.65	0.42
time (sec)	N/A	0.370	0.026	7.816	0.000	0.367	0.000	0.409	1.267

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	39	1487	152	223	0	152	109
N.S.	1	1.00	0.17	6.38	0.65	0.96	0.00	0.65	0.47
time (sec)	N/A	0.255	0.036	8.952	0.489	0.360	0.000	0.410	1.230

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	124	896	178	240	0	175	136
N.S.	1	1.00	0.48	3.45	0.68	0.92	0.00	0.67	0.52
time (sec)	N/A	0.276	0.200	13.773	0.475	0.366	0.000	0.427	0.106

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	133	1498	205	246	0	199	161
N.S.	1	1.00	0.46	5.22	0.71	0.86	0.00	0.69	0.56
time (sec)	N/A	0.280	0.139	15.273	0.475	0.373	0.000	0.443	1.255

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	340	613	149	100	0	144	171
N.S.	1	1.00	2.17	3.90	0.95	0.64	0.00	0.92	1.09
time (sec)	N/A	0.044	4.899	0.949	0.491	0.337	0.000	0.418	1.191

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	165	403	123	95	0	120	145
N.S.	1	1.00	1.27	3.10	0.95	0.73	0.00	0.92	1.12
time (sec)	N/A	0.031	0.234	0.951	0.474	0.335	0.000	0.417	0.049

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	85	397	96	87	0	97	118
N.S.	1	1.00	0.89	4.14	1.00	0.91	0.00	1.01	1.23
time (sec)	N/A	0.019	0.113	0.968	0.473	0.352	0.000	0.416	0.046

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	26	1038	140	86	0	79	82
N.S.	1	1.00	0.17	6.70	0.90	0.55	0.00	0.51	0.53
time (sec)	N/A	0.035	0.029	0.878	0.483	0.381	0.000	0.418	1.416

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	87	407	98	97	0	99	118
N.S.	1	1.00	0.88	4.11	0.99	0.98	0.00	1.00	1.19
time (sec)	N/A	0.028	0.096	1.149	0.472	0.334	0.000	0.401	0.025

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	134	507	124	111	0	122	145
N.S.	1	1.00	1.03	3.90	0.95	0.85	0.00	0.94	1.12
time (sec)	N/A	0.034	0.168	1.162	0.484	0.326	0.000	0.420	0.027

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	399	0	341	457	0	308	227
N.S.	1	1.00	0.93	0.00	0.79	1.07	0.00	0.72	0.53
time (sec)	N/A	0.238	6.109	0.006	0.466	0.374	0.000	0.454	1.307

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	319	0	304	448	0	288	190
N.S.	1	1.00	0.81	0.00	0.78	1.14	0.00	0.73	0.48
time (sec)	N/A	0.164	0.554	0.006	0.483	0.360	0.000	0.448	1.275

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	56	0	265	423	0	0	149
N.S.	1	1.00	0.16	0.00	0.75	1.20	0.00	0.00	0.42
time (sec)	N/A	0.131	0.036	0.004	0.465	0.411	0.000	0.000	1.264

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	919	919	30	0	0	2289	0	661	648
N.S.	1	1.00	0.03	0.00	0.00	2.49	0.00	0.72	0.71
time (sec)	N/A	0.650	0.030	0.020	0.000	0.424	0.000	1.097	1.398

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	676	46	0	0	2874	0	432	162
N.S.	1	1.00	0.07	0.00	0.00	4.25	0.00	0.64	0.24
time (sec)	N/A	0.432	0.075	0.006	0.000	0.435	0.000	0.527	1.249

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	72	0	0	2931	0	461	210
N.S.	1	1.00	0.10	0.00	0.00	4.01	0.00	0.63	0.29
time (sec)	N/A	0.464	0.060	0.005	0.000	0.434	0.000	0.553	1.263

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	201	0	0	0	0	-1
N.S.	1	1.00	1.04	4.47	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.018	0.224	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	228	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.926	0.225	0.030	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	106	0	0	100	0	-1
N.S.	1	1.00	0.74	3.03	0.00	0.00	2.86	0.00	-0.03
time (sec)	N/A	0.027	0.009	0.135	0.000	0.000	2.205	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	128	0	0	0	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.280	0.013	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	115	0	0	0	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.173	0.014	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	93	0	0	119	0	-1
N.S.	1	1.00	0.75	2.58	0.00	0.00	3.31	0.00	-0.03
time (sec)	N/A	0.028	0.009	0.088	0.000	0.000	2.099	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	192	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.804	0.167	0.014	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.424	0.013	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.388	0.012	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.348	0.013	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.392	0.011	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.381	0.014	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.447	0.016	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.560	0.013	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.546	0.010	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.440	0.011	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.246	0.016	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	118	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.564	0.015	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	98	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.214	0.016	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.129	0.013	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	142	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.112	0.016	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.027	0.014	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	107	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.463	0.016	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	132	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.428	0.014	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	148	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.348	0.014	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	131	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.061	0.058	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	80	183	259	125	0	138	214
N.S.	1	1.00	0.61	1.39	1.96	0.95	0.00	1.05	1.62
time (sec)	N/A	0.201	0.079	0.090	0.260	0.392	0.000	0.426	0.126

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	73	141	221	115	0	118	177
N.S.	1	1.00	0.70	1.34	2.10	1.10	0.00	1.12	1.69
time (sec)	N/A	0.146	0.095	0.085	0.271	0.429	0.000	0.415	0.079

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	121	181	103	0	98	138
N.S.	1	1.00	0.82	1.55	2.32	1.32	0.00	1.26	1.77
time (sec)	N/A	0.092	0.107	0.086	0.266	0.341	0.000	0.429	1.210

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	51	93	132	77	0	58	94
N.S.	1	1.00	1.09	1.98	2.81	1.64	0.00	1.23	2.00
time (sec)	N/A	0.045	0.059	0.041	0.273	0.384	0.000	0.408	1.198

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	249	78	87	0	0	48
N.S.	1	1.00	1.18	4.88	1.53	1.71	0.00	0.00	0.94
time (sec)	N/A	0.131	0.078	0.128	0.271	0.390	0.000	0.000	0.073

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	36	23	57	0	49	23
N.S.	1	1.00	1.03	1.09	0.70	1.73	0.00	1.48	0.70
time (sec)	N/A	0.067	0.072	0.119	0.264	0.358	0.000	0.420	1.181

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	42	41	39	77	0	85	39
N.S.	1	1.00	0.63	0.61	0.58	1.15	0.00	1.27	0.58
time (sec)	N/A	0.084	0.080	0.135	0.259	0.366	0.000	0.422	1.170

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	51	50	55	96	0	105	56
N.S.	1	1.00	0.51	0.50	0.55	0.96	0.00	1.05	0.56
time (sec)	N/A	0.157	0.045	0.128	0.265	0.344	0.000	0.445	0.043

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	59	58	71	116	0	125	72
N.S.	1	1.00	0.44	0.44	0.53	0.87	0.00	0.94	0.54
time (sec)	N/A	0.230	0.047	0.129	0.270	0.328	0.000	0.466	1.174

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	29	49	28	124	0	28
N.S.	1	1.00	0.67	0.69	1.17	0.67	2.95	0.00	0.67
time (sec)	N/A	0.044	0.016	0.117	0.278	0.340	0.586	0.000	1.214

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	47	60	60	66	60	60
N.S.	1	1.00	0.62	1.27	1.62	1.62	1.78	1.62	1.62
time (sec)	N/A	0.040	0.014	0.147	0.267	0.324	0.032	0.405	0.035

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	30	37	37	36	37	37
N.S.	1	1.00	0.81	0.81	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.037	0.012	0.124	0.262	0.330	0.022	0.398	0.049

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	31	38	38	37	38	38
N.S.	1	1.00	0.81	0.84	1.03	1.03	1.00	1.03	1.03
time (sec)	N/A	0.041	0.011	0.128	0.261	0.328	0.019	0.398	0.046

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	17	18	18	15	18	15
N.S.	1	1.00	0.85	0.85	0.90	0.90	0.75	0.90	0.75
time (sec)	N/A	0.031	0.006	0.131	0.255	0.323	0.022	0.414	0.029

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	26	26	13	12	12	12	12	9
N.S.	1	1.86	1.86	0.93	0.86	0.86	0.86	0.86	0.64
time (sec)	N/A	0.008	0.007	0.086	0.273	0.349	0.012	0.395	0.023

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	30	29	20	31	29
N.S.	1	1.00	0.94	0.91	0.94	0.91	0.62	0.97	0.91
time (sec)	N/A	0.042	0.011	0.119	0.256	0.328	0.064	0.409	1.195

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	25	30	26	26	24	34	13
N.S.	1	1.00	1.79	2.14	1.86	1.86	1.71	2.43	0.93
time (sec)	N/A	0.033	0.007	0.120	0.259	0.343	0.086	0.406	1.191

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	47	47	49	21	46
N.S.	1	1.00	0.62	0.81	1.27	1.27	1.32	0.57	1.24
time (sec)	N/A	0.041	0.011	0.123	0.255	0.347	0.119	0.392	1.201

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	30	57	57	60	21	56
N.S.	1	1.00	0.62	0.81	1.54	1.54	1.62	0.57	1.51
time (sec)	N/A	0.042	0.012	0.118	0.253	0.337	0.183	0.407	0.087

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	155	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.094	0.059	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	192	259	126	0	138	214
N.S.	1	1.00	0.76	1.83	2.47	1.20	0.00	1.31	2.04
time (sec)	N/A	0.110	0.079	0.085	0.257	0.342	0.000	0.437	0.092

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	124	221	109	0	84	176
N.S.	1	1.00	0.82	1.59	2.83	1.40	0.00	1.08	2.26
time (sec)	N/A	0.092	0.117	0.090	0.268	0.347	0.000	0.428	1.202

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	130	181	103	0	98	139
N.S.	1	1.00	0.82	1.67	2.32	1.32	0.00	1.26	1.78
time (sec)	N/A	0.113	0.108	0.083	0.259	0.358	0.000	0.415	1.201

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	162	135	81	0	74	97
N.S.	1	1.00	0.82	2.49	2.08	1.25	0.00	1.14	1.49
time (sec)	N/A	0.123	0.087	0.085	0.261	0.339	0.000	0.417	1.204

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	345	95	120	0	35	63
N.S.	1	1.00	0.79	4.31	1.19	1.50	0.00	0.44	0.79
time (sec)	N/A	0.189	0.069	0.157	0.259	0.355	0.000	0.413	0.066

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	36	36	23	77	0	69	23
N.S.	1	1.00	1.09	1.09	0.70	2.33	0.00	2.09	0.70
time (sec)	N/A	0.073	0.042	0.137	0.274	0.323	0.000	0.442	0.037

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	41	41	39	95	0	125	39
N.S.	1	1.00	0.61	0.61	0.58	1.42	0.00	1.87	0.58
time (sec)	N/A	0.091	0.045	0.141	0.253	0.355	0.000	0.467	1.176

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	50	50	55	116	0	145	56
N.S.	1	1.00	0.53	0.53	0.59	1.23	0.00	1.54	0.60
time (sec)	N/A	0.182	0.045	0.143	0.259	0.347	0.000	0.482	1.186

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	58	58	71	134	0	165	72
N.S.	1	1.00	0.46	0.46	0.57	1.07	0.00	1.32	0.58
time (sec)	N/A	0.255	0.049	0.134	0.255	0.356	0.000	0.525	0.045

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	74	153	81	530	0	57
N.S.	1	1.00	0.76	1.12	2.32	1.23	8.03	0.00	0.86
time (sec)	N/A	0.056	0.062	0.158	0.278	0.361	0.556	0.000	1.354

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	45	59	59	63	42	59
N.S.	1	1.00	0.58	0.85	1.11	1.11	1.19	0.79	1.11
time (sec)	N/A	0.046	0.016	0.179	0.253	0.349	0.032	0.409	1.190

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	23	28	28	29	42	24
N.S.	1	1.00	1.00	0.72	0.88	0.88	0.91	1.31	0.75
time (sec)	N/A	0.037	0.021	0.175	0.250	0.319	0.024	0.392	0.044

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	29	37	37	37	42	37
N.S.	1	1.00	0.86	0.83	1.06	1.06	1.06	1.20	1.06
time (sec)	N/A	0.038	0.014	0.172	0.255	0.343	0.025	0.398	0.046

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	25	25	24	40	19
N.S.	1	1.00	1.59	0.94	1.47	1.47	1.41	2.35	1.12
time (sec)	N/A	0.031	0.017	0.158	0.249	0.363	0.020	0.401	0.031

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	24	28	26	50	26
N.S.	1	1.00	0.96	0.89	0.89	1.04	0.96	1.85	0.96
time (sec)	N/A	0.024	0.009	0.218	0.260	0.348	0.057	0.408	1.181

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	41	44	49	37	57	42
N.S.	1	1.00	0.75	0.85	0.92	1.02	0.77	1.19	0.88
time (sec)	N/A	0.046	0.017	0.169	0.252	0.350	0.119	0.388	0.058

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	42	51	51	51	50	25
N.S.	1	1.00	1.00	1.68	2.04	2.04	2.04	2.00	1.00
time (sec)	N/A	0.035	0.008	0.153	0.261	0.367	0.141	0.415	1.192

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	41	65	65	70	42	29
N.S.	1	1.00	0.60	0.79	1.25	1.25	1.35	0.81	0.56
time (sec)	N/A	0.049	0.015	0.151	0.257	0.342	0.187	0.400	1.230

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	42	77	77	80	42	29
N.S.	1	1.00	0.58	0.79	1.45	1.45	1.51	0.79	0.55
time (sec)	N/A	0.045	0.014	0.158	0.264	0.326	0.205	0.412	1.238

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	76	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.032	0.060	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	72	196	221	114	0	109	176
N.S.	1	1.00	0.57	1.54	1.74	0.90	0.00	0.86	1.39
time (sec)	N/A	0.234	0.090	0.129	0.264	0.396	0.000	0.404	1.228

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	176	181	104	0	90	140
N.S.	1	1.00	0.64	1.76	1.81	1.04	0.00	0.90	1.40
time (sec)	N/A	0.188	0.066	0.125	0.259	0.380	0.000	0.416	1.211

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	153	135	81	0	68	96
N.S.	1	1.00	0.82	2.35	2.08	1.25	0.00	1.05	1.48
time (sec)	N/A	0.116	0.040	0.095	0.269	0.359	0.000	0.421	0.064

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	34	76	55	47	0	33	24
N.S.	1	1.00	1.48	3.30	2.39	2.04	0.00	1.43	1.04
time (sec)	N/A	0.069	0.036	0.148	0.263	0.329	0.000	0.411	0.062

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	36	23	39	0	0	23
N.S.	1	1.00	0.96	1.29	0.82	1.39	0.00	0.00	0.82
time (sec)	N/A	0.071	0.073	0.162	0.258	0.347	0.000	0.000	0.035

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	34	41	39	57	0	45	38
N.S.	1	1.00	0.55	0.66	0.63	0.92	0.00	0.73	0.61
time (sec)	N/A	0.086	0.079	0.162	0.255	0.399	0.000	0.431	0.035

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	43	50	55	77	0	65	55
N.S.	1	1.00	0.45	0.53	0.58	0.81	0.00	0.68	0.58
time (sec)	N/A	0.160	0.084	0.171	0.253	0.435	0.000	0.432	0.040

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	51	58	71	95	0	85	71
N.S.	1	1.00	0.40	0.45	0.55	0.74	0.00	0.66	0.55
time (sec)	N/A	0.176	0.102	0.172	0.252	0.373	0.000	0.475	1.192

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.023	0.120	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	50	63	68	68	75	63
N.S.	1	1.00	0.62	0.55	0.69	0.75	0.75	0.82	0.69
time (sec)	N/A	0.050	0.019	0.138	0.253	0.346	0.072	0.400	1.168

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	42	52	57	56	64	52
N.S.	1	1.00	0.66	0.58	0.71	0.78	0.77	0.88	0.71
time (sec)	N/A	0.045	0.026	0.158	0.254	0.576	0.059	0.399	0.038

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	39	34	41	45	41	52	41
N.S.	1	1.00	0.71	0.62	0.75	0.82	0.75	0.95	0.75
time (sec)	N/A	0.042	0.015	0.125	0.259	0.377	0.057	0.403	0.044

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	24	35	26
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.92	1.35	1.00
time (sec)	N/A	0.025	0.011	0.089	0.256	0.532	0.042	0.395	0.039

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	15	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.07	1.00
time (sec)	N/A	0.035	0.011	0.132	0.255	0.403	0.015	0.394	0.039

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	28	29	23	20	25	12
N.S.	1	1.00	1.00	2.33	2.42	1.92	1.67	2.08	1.00
time (sec)	N/A	0.035	0.011	0.131	0.258	0.408	0.067	0.395	0.055

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	40	48	46	39	46	31
N.S.	1	1.00	0.97	1.21	1.45	1.39	1.18	1.39	0.94
time (sec)	N/A	0.044	0.020	0.127	0.260	0.327	0.117	0.387	0.066

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	52	63	76	54	51	46
N.S.	1	1.00	0.69	1.02	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.047	0.026	0.134	0.256	0.333	0.148	0.408	0.074

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	44	64	84	113	78	89	65
N.S.	1	1.00	0.64	0.93	1.22	1.64	1.13	1.29	0.94
time (sec)	N/A	0.055	0.041	0.128	0.264	0.337	0.215	0.409	1.195

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.083	0.059	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	86	542	244	114	0	0	199
N.S.	1	1.00	0.57	3.57	1.61	0.75	0.00	0.00	1.31
time (sec)	N/A	0.288	0.112	0.135	0.256	0.345	0.000	0.000	0.087

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	78	474	204	104	0	0	163
N.S.	1	1.00	0.60	3.67	1.58	0.81	0.00	0.00	1.26
time (sec)	N/A	0.239	0.086	0.140	0.262	0.355	0.000	0.000	1.194

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	68	422	156	81	0	0	117
N.S.	1	1.00	0.74	4.59	1.70	0.88	0.00	0.00	1.27
time (sec)	N/A	0.164	0.088	0.103	0.249	0.335	0.000	0.000	0.067

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	248	78	63	0	0	48
N.S.	1	1.00	1.02	4.68	1.47	1.19	0.00	0.00	0.91
time (sec)	N/A	0.134	0.051	0.124	0.262	0.414	0.000	0.000	1.167

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	22	22	0	0	22
N.S.	1	1.00	0.93	1.25	0.79	0.79	0.00	0.00	0.79
time (sec)	N/A	0.077	0.039	0.159	0.261	0.473	0.000	0.000	0.029

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	33	33	48	31	0	22	38
N.S.	1	1.00	1.57	1.57	2.29	1.48	0.00	1.05	1.81
time (sec)	N/A	0.068	0.045	0.164	0.265	0.386	0.000	0.412	1.168

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	45	65	58	0	0	50
N.S.	1	1.00	0.82	0.74	1.07	0.95	0.00	0.00	0.82
time (sec)	N/A	0.089	0.056	0.171	0.264	0.366	0.000	0.000	1.232

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	57	61	82	77	0	0	51
N.S.	1	1.00	0.61	0.65	0.87	0.82	0.00	0.00	0.54
time (sec)	N/A	0.215	0.051	0.168	0.254	0.358	0.000	0.000	1.207

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	66	69	97	96	0	0	60
N.S.	1	1.00	0.53	0.55	0.78	0.77	0.00	0.00	0.48
time (sec)	N/A	0.276	0.058	0.181	0.267	0.347	0.000	0.000	0.066

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	83	69	99	105	0	147	76
N.S.	1	1.00	0.33	0.27	0.39	0.41	0.00	0.58	0.30
time (sec)	N/A	0.149	0.038	0.126	0.264	0.342	0.000	0.425	1.456

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	75	61	83	94	0	0	102
N.S.	1	1.00	0.38	0.31	0.42	0.48	0.00	0.00	0.52
time (sec)	N/A	0.133	0.031	0.126	0.270	0.331	0.000	0.000	1.433

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	137	67	53	67	83	0	97	60
N.S.	1	1.19	0.58	0.46	0.58	0.72	0.00	0.84	0.52
time (sec)	N/A	0.116	0.026	0.127	0.261	0.358	0.000	0.417	1.408

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	89	57	43	45	64	0	0	50
N.S.	1	1.16	0.74	0.56	0.58	0.83	0.00	0.00	0.65
time (sec)	N/A	0.104	0.021	0.119	0.262	0.376	0.000	0.000	1.372

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	36	26	50	0	49	43
N.S.	1	1.00	1.48	1.24	0.90	1.72	0.00	1.69	1.48
time (sec)	N/A	0.021	0.015	0.076	0.274	0.377	0.000	0.422	1.310

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	83	0	239	0	94	-1
N.S.	1	1.00	0.84	0.70	0.00	2.03	0.00	0.80	-0.01
time (sec)	N/A	0.106	0.043	0.180	0.000	0.359	0.000	0.423	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	116	118	0	281	0	60	-1
N.S.	1	1.00	0.91	0.92	0.00	2.20	0.00	0.47	-0.01
time (sec)	N/A	0.117	0.075	0.161	0.000	0.392	0.000	0.445	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	123	167	0	337	0	78	-1
N.S.	1	1.00	0.64	0.87	0.00	1.75	0.00	0.40	-0.01
time (sec)	N/A	0.135	0.106	0.155	0.000	0.442	0.000	0.435	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	139	219	0	393	0	105	-1
N.S.	1	1.00	0.56	0.88	0.00	1.57	0.00	0.42	-0.00
time (sec)	N/A	0.153	0.111	0.157	0.000	0.389	0.000	0.457	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	33	32	60	170	205	32
N.S.	1	1.00	0.85	0.82	0.80	1.50	4.25	5.12	0.80
time (sec)	N/A	0.057	0.032	0.164	0.263	0.348	7.035	0.409	0.039

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	33	32	49	80	141	32
N.S.	1	1.00	0.85	0.82	0.80	1.22	2.00	3.52	0.80
time (sec)	N/A	0.060	0.028	0.161	0.250	0.320	4.686	0.397	0.034

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	33	32	32	61	71	32
N.S.	1	1.00	0.75	0.82	0.80	0.80	1.52	1.78	0.80
time (sec)	N/A	0.059	0.022	0.165	0.257	0.333	3.847	0.401	0.033

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	33	30	19	31	44	32
N.S.	1	1.00	0.61	0.87	0.79	0.50	0.82	1.16	0.84
time (sec)	N/A	0.053	0.018	0.125	0.249	0.349	1.574	0.407	0.029

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	31	30	29	49	32	19
N.S.	1	1.00	0.58	0.86	0.83	0.81	1.36	0.89	0.53
time (sec)	N/A	0.056	0.018	0.140	0.254	0.344	7.733	0.412	1.207

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	33	26	44	29	36	20
N.S.	1	1.00	0.89	0.87	0.68	1.16	0.76	0.95	0.53
time (sec)	N/A	0.059	0.033	0.128	0.251	0.409	16.768	0.392	0.030

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	33	24	56	31	34	20
N.S.	1	1.00	0.85	0.82	0.60	1.40	0.78	0.85	0.50
time (sec)	N/A	0.059	0.043	0.130	0.253	0.339	12.615	0.415	1.209

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	33	26	66	31	36	20
N.S.	1	1.00	0.85	0.82	0.65	1.65	0.78	0.90	0.50
time (sec)	N/A	0.060	0.041	0.140	0.251	0.346	23.285	0.412	1.190

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	77	66	106	105	0	130	110
N.S.	1	1.00	0.39	0.34	0.54	0.53	0.00	0.66	0.56
time (sec)	N/A	0.133	0.032	0.138	0.274	0.357	0.000	0.444	1.445

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	69	58	90	94	0	0	102
N.S.	1	1.00	0.50	0.42	0.66	0.69	0.00	0.00	0.74
time (sec)	N/A	0.123	0.027	0.122	0.268	0.347	0.000	0.000	1.437

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	59	50	74	83	0	80	93
N.S.	1	1.00	0.66	0.56	0.83	0.93	0.00	0.90	1.04
time (sec)	N/A	0.109	0.030	0.118	0.271	0.328	0.000	0.421	1.386

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	43	42	41	61	129	0	81
N.S.	1	1.00	1.39	1.35	1.32	1.97	4.16	0.00	2.61
time (sec)	N/A	0.025	0.023	0.079	0.268	0.336	149.575	0.000	1.385

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	105	107	0	250	0	0	-1
N.S.	1	1.00	0.64	0.66	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.045	0.155	0.000	0.407	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	116	135	0	288	0	75	-1
N.S.	1	1.00	0.66	0.76	0.00	1.63	0.00	0.42	-0.01
time (sec)	N/A	0.123	0.099	0.159	0.000	0.363	0.000	0.426	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	125	174	0	341	0	78	-1
N.S.	1	1.00	0.67	0.93	0.00	1.82	0.00	0.42	-0.01
time (sec)	N/A	0.135	0.104	0.155	0.000	0.359	0.000	0.443	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	142	226	0	393	0	105	-1
N.S.	1	1.00	0.57	0.90	0.00	1.57	0.00	0.42	-0.00
time (sec)	N/A	0.157	0.099	0.148	0.000	0.351	0.000	0.460	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	147	278	0	449	0	129	-1
N.S.	1	1.00	0.48	0.91	0.00	1.46	0.00	0.42	-0.00
time (sec)	N/A	0.163	0.131	0.152	0.000	0.404	0.000	0.460	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	311	86	84	128	105	0	0	110
N.S.	1	1.60	0.44	0.43	0.66	0.54	0.00	0.00	0.57
time (sec)	N/A	0.164	0.035	0.083	0.284	0.364	0.000	0.000	1.408

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	254	78	76	112	94	0	0	102
N.S.	1	1.58	0.48	0.47	0.70	0.58	0.00	0.00	0.63
time (sec)	N/A	0.145	0.032	0.084	0.271	0.338	0.000	0.000	1.363

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	197	70	68	96	83	0	0	94
N.S.	1	1.54	0.55	0.53	0.75	0.65	0.00	0.00	0.73
time (sec)	N/A	0.133	0.028	0.083	0.276	0.320	0.000	0.000	1.371

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	137	60	58	72	64	0	0	82
N.S.	1	1.44	0.63	0.61	0.76	0.67	0.00	0.00	0.86
time (sec)	N/A	0.119	0.025	0.084	0.271	0.341	0.000	0.000	1.342

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	89	50	48	54	50	0	43	71
N.S.	1	1.44	0.81	0.77	0.87	0.81	0.00	0.69	1.15
time (sec)	N/A	0.098	0.019	0.085	0.267	0.336	0.000	0.414	1.276

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	46	29	44	0	0	34
N.S.	1	1.00	0.97	1.59	1.00	1.52	0.00	0.00	1.17
time (sec)	N/A	0.024	0.016	0.080	0.265	0.341	0.000	0.000	1.250

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	78	0	141	0	64	-1
N.S.	1	1.00	1.00	1.03	0.00	1.86	0.00	0.84	-0.01
time (sec)	N/A	0.115	0.034	0.154	0.000	0.346	0.000	0.432	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	116	123	0	281	0	0	-1
N.S.	1	1.00	0.85	0.90	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.083	0.152	0.000	0.348	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	125	172	0	341	0	88	-1
N.S.	1	1.00	0.65	0.89	0.00	1.77	0.00	0.46	-0.01
time (sec)	N/A	0.138	0.114	0.152	0.000	0.405	0.000	0.435	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	88	101	123	204	129	161	112
N.S.	1	1.00	0.64	0.74	0.90	1.49	0.94	1.18	0.82
time (sec)	N/A	0.108	0.052	0.205	0.488	0.340	48.222	0.401	0.082

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	87	109	182	110	134	95
N.S.	1	1.00	0.69	0.75	0.94	1.57	0.95	1.16	0.82
time (sec)	N/A	0.091	0.040	0.201	0.465	0.333	32.158	0.397	0.065

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	71	73	95	146	92	107	78
N.S.	1	1.00	0.75	0.77	1.00	1.54	0.97	1.13	0.82
time (sec)	N/A	0.088	0.030	0.203	0.460	0.344	20.422	0.400	1.248

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	79	119	73	77	61
N.S.	1	1.00	0.80	0.78	1.04	1.57	0.96	1.01	0.80
time (sec)	N/A	0.070	0.023	0.198	0.462	0.353	2.280	0.414	1.238

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	45	68	118	60	51	47
N.S.	1	1.00	1.00	0.78	1.17	2.03	1.03	0.88	0.81
time (sec)	N/A	0.062	0.019	0.191	0.458	0.328	9.973	0.399	0.074

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	29	52	88	41	36	28
N.S.	1	1.00	1.00	0.78	1.41	2.38	1.11	0.97	0.76
time (sec)	N/A	0.064	0.013	0.179	0.465	0.343	8.757	0.402	0.083

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	37	50	71	146	61	54	47
N.S.	1	1.00	0.65	0.88	1.25	2.56	1.07	0.95	0.82
time (sec)	N/A	0.069	0.017	0.190	0.463	0.386	8.757	0.419	1.234

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	39	64	81	196	82	73	65
N.S.	1	1.00	0.47	0.77	0.98	2.36	0.99	0.88	0.78
time (sec)	N/A	0.077	0.027	0.188	0.467	0.370	16.029	0.407	0.095

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	39	78	101	252	100	93	79
N.S.	1	1.00	0.38	0.75	0.97	2.42	0.96	0.89	0.76
time (sec)	N/A	0.085	0.023	0.193	0.467	0.345	12.737	0.410	0.094

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	84	92	152	105	0	0	110
N.S.	1	1.00	0.23	0.25	0.41	0.29	0.00	0.00	0.30
time (sec)	N/A	0.173	0.037	0.102	0.278	0.354	0.000	0.000	1.453

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	76	84	136	94	0	0	102
N.S.	1	1.00	0.24	0.27	0.44	0.30	0.00	0.00	0.33
time (sec)	N/A	0.165	0.031	0.100	0.285	0.357	0.000	0.000	1.410

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	68	76	120	83	0	0	94
N.S.	1	1.00	0.27	0.30	0.47	0.33	0.00	0.00	0.37
time (sec)	N/A	0.145	0.028	0.105	0.283	0.347	0.000	0.000	1.378

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	57	65	93	63	0	0	81
N.S.	1	1.00	0.29	0.33	0.48	0.32	0.00	0.00	0.42
time (sec)	N/A	0.142	0.024	0.102	0.272	0.334	0.000	0.000	1.356

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	48	56	75	50	0	0	71
N.S.	1	1.00	0.35	0.41	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.107	0.019	0.099	0.287	0.326	0.000	0.000	1.300

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	48	51	48	44	0	42	34
N.S.	1	1.00	0.56	0.60	0.56	0.52	0.00	0.49	0.40
time (sec)	N/A	0.098	0.020	0.096	0.270	0.332	0.000	0.396	1.398

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	41	46	45	43	0	41	32
N.S.	1	1.00	1.41	1.59	1.55	1.48	0.00	1.41	1.10
time (sec)	N/A	0.026	0.021	0.089	0.281	0.359	0.000	0.427	1.353

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	122	85	0	235	0	0	-1
N.S.	1	1.00	1.02	0.71	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.053	0.164	0.000	0.384	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	140	129	0	285	0	90	-1
N.S.	1	1.00	0.76	0.70	0.00	1.55	0.00	0.49	-0.01
time (sec)	N/A	0.137	0.103	0.164	0.000	0.352	0.000	0.433	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	41	67	110	57	0	59	94
N.S.	1	1.00	0.41	0.68	1.11	0.58	0.00	0.60	0.95
time (sec)	N/A	0.050	0.024	0.099	0.260	0.356	0.000	0.409	1.226

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	40	57	87	54	0	48	68
N.S.	1	1.00	0.51	0.72	1.10	0.68	0.00	0.61	0.86
time (sec)	N/A	0.031	0.016	0.070	0.253	0.358	0.000	0.416	0.044

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	22	50	24	0	15	18
N.S.	1	1.00	1.17	1.22	2.78	1.33	0.00	0.83	1.00
time (sec)	N/A	0.037	0.012	0.027	0.267	0.378	0.000	0.427	1.207

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	39	48	83	51	0	38	63
N.S.	1	1.00	1.11	1.37	2.37	1.46	0.00	1.09	1.80
time (sec)	N/A	0.033	0.012	0.068	0.256	0.331	0.000	0.401	0.035

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	52	79	138	66	0	71	118
N.S.	1	1.00	0.39	0.59	1.04	0.50	0.00	0.53	0.89
time (sec)	N/A	0.073	0.025	0.109	0.265	0.328	0.000	0.405	0.048

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	47	69	112	61	0	60	94
N.S.	1	1.00	0.44	0.65	1.06	0.58	0.00	0.57	0.89
time (sec)	N/A	0.058	0.020	0.112	0.252	0.416	0.000	0.414	0.046

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	52	70	138	66	0	72	118
N.S.	1	1.00	0.73	0.99	1.94	0.93	0.00	1.01	1.66
time (sec)	N/A	0.077	0.022	0.120	0.261	0.343	0.000	0.425	1.187

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	60	112	61	0	60	90
N.S.	1	1.00	0.89	1.13	2.11	1.15	0.00	1.13	1.70
time (sec)	N/A	0.059	0.017	0.112	0.256	0.330	0.000	0.402	1.183

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	15	32	26	15	0	14	26
N.S.	1	1.00	0.68	1.45	1.18	0.68	0.00	0.64	1.18
time (sec)	N/A	0.036	0.014	0.081	0.253	0.330	0.000	0.396	0.055

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	35	31	31	29	21	14
N.S.	1	1.00	0.82	1.59	1.41	1.41	1.32	0.95	0.64
time (sec)	N/A	0.042	0.008	0.105	0.258	0.331	2.337	0.410	0.029

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	106	74	66	0	65	43
N.S.	1	1.00	0.87	2.26	1.57	1.40	0.00	1.38	0.91
time (sec)	N/A	0.090	0.031	0.123	0.259	0.399	0.000	0.413	1.190

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	38	106	44	61	0	49	28
N.S.	1	1.00	1.15	3.21	1.33	1.85	0.00	1.48	0.85
time (sec)	N/A	0.091	0.018	0.115	0.257	0.376	0.000	0.408	1.193

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	36	110	44	44	0	44	28
N.S.	1	1.00	0.80	2.44	0.98	0.98	0.00	0.98	0.62
time (sec)	N/A	0.056	0.025	0.116	0.260	0.334	0.000	0.415	0.028

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	37	11	11	8	22	11
N.S.	1	1.00	0.86	1.76	0.52	0.52	0.38	1.05	0.52
time (sec)	N/A	0.046	0.011	0.085	0.258	0.341	3.459	0.397	0.175

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	43	146	56	84	0	79	40
N.S.	1	1.00	0.78	2.65	1.02	1.53	0.00	1.44	0.73
time (sec)	N/A	0.109	0.034	0.123	0.259	0.345	0.000	0.410	0.044

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	35	13	31	0	46	13
N.S.	1	1.00	1.00	1.46	0.54	1.29	0.00	1.92	0.54
time (sec)	N/A	0.050	0.013	0.087	0.265	0.362	0.000	0.411	0.021

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.131	0.023	0.059	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	64	50	55	69	184	0	57
N.S.	1	1.00	0.46	0.36	0.39	0.49	1.31	0.00	0.41
time (sec)	N/A	0.142	0.023	0.121	0.264	0.323	36.319	0.000	1.379

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	56	42	41	61	136	81	49
N.S.	1	1.00	0.61	0.46	0.45	0.66	1.48	0.88	0.53
time (sec)	N/A	0.110	0.018	0.122	0.266	0.348	20.072	0.417	1.367

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	43	36	26	50	0	49	43
N.S.	1	1.00	1.48	1.24	0.90	1.72	0.00	1.69	1.48
time (sec)	N/A	0.021	0.014	0.072	0.278	0.362	0.000	0.408	0.002

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	70	0	207	0	88	-1
N.S.	1	1.00	0.80	0.74	0.00	2.20	0.00	0.94	-0.01
time (sec)	N/A	0.124	0.037	0.150	0.000	0.346	0.000	0.417	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	76	78	0	229	0	101	-1
N.S.	1	1.00	0.78	0.80	0.00	2.36	0.00	1.04	-0.01
time (sec)	N/A	0.127	0.027	0.160	0.000	0.351	0.000	0.426	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	48	75	74	44	83	189	83
N.S.	1	1.00	0.48	0.74	0.73	0.44	0.82	1.87	0.82
time (sec)	N/A	0.153	0.048	0.182	0.270	0.326	1.934	0.401	0.040

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	40	61	60	36	68	142	66
N.S.	1	1.00	0.50	0.76	0.75	0.45	0.85	1.78	0.82
time (sec)	N/A	0.142	0.037	0.178	0.255	0.328	1.802	0.410	0.055

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	31	47	44	27	48	92	46
N.S.	1	1.00	0.54	0.82	0.77	0.47	0.84	1.61	0.81
time (sec)	N/A	0.098	0.031	0.177	0.251	0.343	2.180	0.402	0.050

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	33	30	19	31	44	32
N.S.	1	1.00	0.61	0.87	0.79	0.50	0.82	1.16	0.84
time (sec)	N/A	0.053	0.018	0.128	0.255	0.321	1.559	0.400	0.002

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	49	82	39	40	31
N.S.	1	1.00	1.00	0.82	1.26	2.10	1.00	1.03	0.79
time (sec)	N/A	0.124	0.020	0.192	0.455	0.363	2.213	0.402	1.191

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	45	62	97	119	48	34
N.S.	1	1.00	1.00	1.07	1.48	2.31	2.83	1.14	0.81
time (sec)	N/A	0.130	0.021	0.200	0.471	0.375	4.181	0.405	0.060

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	65	103	117	270	76	54
N.S.	1	1.00	0.81	0.96	1.51	1.72	3.97	1.12	0.79
time (sec)	N/A	0.135	0.035	0.224	0.460	0.368	8.775	0.411	1.232

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	80	134	133	439	104	74
N.S.	1	1.00	0.71	0.90	1.51	1.49	4.93	1.17	0.83
time (sec)	N/A	0.140	0.040	0.198	0.479	0.353	9.164	0.418	1.202

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	71	93	163	149	639	131	91
N.S.	1	1.00	0.65	0.85	1.48	1.35	5.81	1.19	0.83
time (sec)	N/A	0.148	0.051	0.212	0.466	0.375	13.765	0.418	0.066

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	130	161	0	304	0	0	-1
N.S.	1	1.00	0.42	0.52	0.00	0.98	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.080	0.161	0.000	0.349	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	122	143	0	288	0	145	-1
N.S.	1	1.00	0.47	0.55	0.00	1.10	0.00	0.56	-0.00
time (sec)	N/A	0.199	0.058	0.161	0.000	0.366	0.000	0.418	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	114	125	0	272	0	0	-1
N.S.	1	1.00	0.54	0.59	0.00	1.29	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.050	0.158	0.000	0.364	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	105	106	0	250	0	0	-1
N.S.	1	1.00	0.64	0.65	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.041	0.149	0.000	0.380	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	120	107	0	352	0	0	-1
N.S.	1	1.00	0.71	0.63	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.044	0.152	0.000	0.362	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	120	117	0	390	0	0	-1
N.S.	1	1.00	0.70	0.68	0.00	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.055	0.175	0.000	0.347	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	132	144	0	428	0	0	-1
N.S.	1	1.00	0.59	0.64	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.107	0.165	0.000	0.365	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	140	165	0	444	0	0	-1
N.S.	1	1.00	0.51	0.60	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.143	0.165	0.000	0.350	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	148	186	0	460	0	0	-1
N.S.	1	1.00	0.46	0.58	0.00	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.177	0.168	0.000	0.345	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	46	37	27	33	194	0	48
N.S.	1	1.00	0.32	0.26	0.19	0.23	1.35	0.00	0.33
time (sec)	N/A	0.088	0.016	0.111	0.259	0.332	71.446	0.000	1.325

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	41	32	22	28	0	0	38
N.S.	1	1.00	0.38	0.30	0.21	0.26	0.00	0.00	0.36
time (sec)	N/A	0.071	0.012	0.073	0.259	0.326	0.000	0.000	1.259

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	46	34	22	45	107	63	35
N.S.	1	1.00	0.44	0.33	0.21	0.43	1.03	0.61	0.34
time (sec)	N/A	0.083	0.015	0.119	0.260	0.325	175.101	0.404	1.309

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	29	17	40	0	49	30
N.S.	1	1.00	0.60	0.43	0.25	0.59	0.00	0.72	0.44
time (sec)	N/A	0.069	0.012	0.105	0.257	0.331	0.000	0.418	1.250

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	39	30	20	26	129	0	38
N.S.	1	1.00	0.36	0.28	0.19	0.24	1.21	0.00	0.36
time (sec)	N/A	0.071	0.011	0.106	0.263	0.330	5.965	0.000	1.252

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	34	25	15	21	56	0	21
N.S.	1	1.00	0.49	0.36	0.21	0.30	0.80	0.00	0.30
time (sec)	N/A	0.058	0.011	0.071	0.260	0.374	3.305	0.000	1.235

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	41	29	17	40	80	49	30
N.S.	1	1.00	0.58	0.41	0.24	0.56	1.13	0.69	0.42
time (sec)	N/A	0.070	0.011	0.105	0.263	0.341	12.493	0.396	1.267

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	34	24	12	33	0	33	25
N.S.	1	1.00	1.70	1.20	0.60	1.65	0.00	1.65	1.25
time (sec)	N/A	0.013	0.010	0.080	0.262	0.341	0.000	0.406	1.272

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	26	25	13	21	48	0	21
N.S.	1	1.00	0.36	0.34	0.18	0.29	0.66	0.00	0.29
time (sec)	N/A	0.067	0.011	0.072	0.255	0.322	6.278	0.000	1.242

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	22	7	18	19	23	18
N.S.	1	1.00	0.64	0.67	0.21	0.55	0.58	0.70	0.55
time (sec)	N/A	0.051	0.007	0.072	0.258	0.333	6.506	0.406	1.224

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	69	66	0	72	0	0	-1
N.S.	1	1.00	0.55	0.52	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.033	0.094	0.000	0.334	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	63	55	0	66	0	0	-1
N.S.	1	1.00	0.70	0.61	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.020	0.092	0.000	0.336	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	65	47	0	46	0	0	-1
N.S.	1	1.00	0.70	0.51	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.026	0.090	0.000	0.360	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	37	0	26	0	0	-1
N.S.	1	1.00	0.71	0.64	0.00	0.45	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.016	0.080	0.000	0.351	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	75	90	0	84	126	41	-1
N.S.	1	1.00	0.58	0.69	0.00	0.65	0.97	0.32	-0.01
time (sec)	N/A	0.090	0.052	0.096	0.000	0.340	79.461	0.434	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	58	79	0	76	70	32	-1
N.S.	1	1.00	0.64	0.88	0.00	0.84	0.78	0.36	-0.01
time (sec)	N/A	0.076	0.063	0.099	0.000	0.334	76.938	0.382	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	102	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.083	0.058	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	185	67	65	83	69	230	0	88
N.S.	1	1.30	0.47	0.46	0.58	0.49	1.62	0.00	0.62
time (sec)	N/A	0.159	0.025	0.141	0.279	0.349	106.241	0.000	1.286

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	137	58	56	69	60	182	69	57
N.S.	1	1.32	0.56	0.54	0.66	0.58	1.75	0.66	0.55
time (sec)	N/A	0.126	0.021	0.136	0.283	0.347	51.330	0.400	1.287

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	89	50	48	54	50	0	43	71
N.S.	1	1.44	0.81	0.77	0.87	0.81	0.00	0.69	1.15
time (sec)	N/A	0.095	0.018	0.078	0.271	0.343	0.000	0.414	0.002

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	74	80	0	206	0	40	-1
N.S.	1	1.00	0.79	0.85	0.00	2.19	0.00	0.43	-0.01
time (sec)	N/A	0.136	0.031	0.146	0.000	0.363	0.000	0.408	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	78	90	0	232	0	48	-1
N.S.	1	1.00	0.81	0.94	0.00	2.42	0.00	0.50	-0.01
time (sec)	N/A	0.138	0.027	0.159	0.000	0.395	0.000	0.418	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	85	101	123	168	126	159	114
N.S.	1	1.00	0.61	0.73	0.88	1.21	0.91	1.14	0.82
time (sec)	N/A	0.178	0.083	0.216	0.459	0.337	5.146	0.418	0.084

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	75	97	153	95	105	80
N.S.	1	1.00	0.80	0.77	1.00	1.58	0.98	1.08	0.82
time (sec)	N/A	0.168	0.063	0.216	0.462	0.351	3.634	0.414	1.258

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	70	73	95	136	92	105	80
N.S.	1	1.00	0.72	0.75	0.98	1.40	0.95	1.08	0.82
time (sec)	N/A	0.115	0.058	0.218	0.467	0.345	3.102	0.396	0.092

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	59	79	119	73	77	61
N.S.	1	1.00	0.80	0.78	1.04	1.57	0.96	1.01	0.80
time (sec)	N/A	0.066	0.023	0.192	0.475	0.362	2.163	0.432	0.002

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	58	98	157	80	67	57
N.S.	1	1.00	1.00	0.78	1.32	2.12	1.08	0.91	0.77
time (sec)	N/A	0.148	0.022	0.202	0.478	0.349	3.181	0.406	0.084

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	71	111	176	162	71	61
N.S.	1	1.00	1.00	0.91	1.42	2.26	2.08	0.91	0.78
time (sec)	N/A	0.148	0.028	0.213	0.482	0.402	4.505	0.415	1.244

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	93	94	152	204	352	106	88
N.S.	1	1.00	0.88	0.89	1.43	1.92	3.32	1.00	0.83
time (sec)	N/A	0.173	0.052	0.221	0.475	0.384	8.537	0.410	1.235

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	101	108	183	220	614	133	105
N.S.	1	1.00	0.80	0.85	1.44	1.73	4.83	1.05	0.83
time (sec)	N/A	0.184	0.061	0.223	0.474	0.421	9.380	0.409	0.128

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	109	122	212	236	991	160	122
N.S.	1	1.00	0.74	0.82	1.43	1.59	6.70	1.08	0.82
time (sec)	N/A	0.196	0.076	0.230	0.475	0.374	16.560	0.415	1.262

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	73	81	117	77	0	0	74
N.S.	1	1.00	0.26	0.29	0.42	0.27	0.00	0.00	0.26
time (sec)	N/A	0.183	0.034	0.100	0.303	0.345	0.000	0.000	1.415

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	65	73	104	69	0	0	88
N.S.	1	1.00	0.28	0.32	0.45	0.30	0.00	0.00	0.38
time (sec)	N/A	0.170	0.026	0.099	0.286	0.332	0.000	0.000	1.340

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	57	65	91	61	0	0	58
N.S.	1	1.00	0.31	0.36	0.50	0.34	0.00	0.00	0.32
time (sec)	N/A	0.137	0.022	0.098	0.298	0.332	0.000	0.000	1.331

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	48	56	75	50	0	0	71
N.S.	1	1.00	0.35	0.41	0.55	0.36	0.00	0.00	0.52
time (sec)	N/A	0.106	0.020	0.089	0.289	0.350	0.000	0.000	0.002

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	78	80	0	207	0	0	-1
N.S.	1	1.00	0.56	0.57	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.081	0.149	0.000	0.351	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	79	86	0	233	0	0	-1
N.S.	1	1.00	0.56	0.61	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.056	0.167	0.000	0.358	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	90	103	0	262	0	0	-1
N.S.	1	1.00	0.47	0.54	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.059	0.178	0.000	0.384	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	98	114	0	278	0	0	-1
N.S.	1	1.00	0.41	0.48	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.063	0.173	0.000	0.337	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	106	125	0	294	0	0	-1
N.S.	1	1.00	0.37	0.44	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.064	0.183	0.000	0.368	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	116	104	122	185	0	0	223
N.S.	1	1.00	0.42	0.37	0.44	0.67	0.00	0.00	0.80
time (sec)	N/A	0.183	0.052	0.125	0.268	0.341	0.000	0.000	1.791

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	78	61	68	93	0	0	140
N.S.	1	1.00	0.61	0.48	0.54	0.73	0.00	0.00	1.10
time (sec)	N/A	0.107	0.034	0.100	0.279	0.349	0.000	0.000	1.339

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	58	34	37	44	0	0	55
N.S.	1	1.00	1.61	0.94	1.03	1.22	0.00	0.00	1.53
time (sec)	N/A	0.019	0.013	0.094	0.275	0.346	0.000	0.000	1.263

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.019	0.059	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.027	0.059	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.034	0.060	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	1.832	0.055	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	144	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.727	0.054	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.353	0.016	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	87	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.133	0.062	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	33	0	58	187	0	32
N.S.	1	1.00	0.69	0.69	0.00	1.21	3.90	0.00	0.67
time (sec)	N/A	0.075	0.101	0.083	0.000	0.367	11.679	0.000	1.527

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	64	46	0	128	1112	0	113
N.S.	1	1.00	0.62	0.44	0.00	1.23	10.69	0.00	1.09
time (sec)	N/A	0.100	0.163	0.083	0.000	0.352	52.641	0.000	1.647

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	83	68	0	228	3534	0	180
N.S.	1	1.00	0.37	0.30	0.00	1.02	15.78	0.00	0.80
time (sec)	N/A	0.177	0.204	0.083	0.000	0.353	171.540	0.000	1.806

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	103	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.066	0.077	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	101	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.047	0.072	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.041	0.072	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.036	0.066	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	94	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.027	0.063	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	117	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.073	0.065	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	138	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.107	0.066	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	175	224	224	156	0	248	183
N.S.	1	1.00	1.54	1.96	1.96	1.37	0.00	2.18	1.61
time (sec)	N/A	0.178	0.122	0.067	0.485	0.351	0.000	0.423	1.317

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	167	200	201	146	0	221	163
N.S.	1	1.00	1.90	2.27	2.28	1.66	0.00	2.51	1.85
time (sec)	N/A	0.127	0.097	0.062	0.504	0.360	0.000	0.420	0.110

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	158	168	125	119	0	137	90
N.S.	1	1.00	2.55	2.71	2.02	1.92	0.00	2.21	1.45
time (sec)	N/A	0.064	0.108	0.058	0.489	0.357	0.000	0.408	1.223

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	63	66	48	0	42	60
N.S.	1	1.00	1.15	2.33	2.44	1.78	0.00	1.56	2.22
time (sec)	N/A	0.022	0.026	0.033	0.490	0.352	0.000	0.403	0.063

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	250	116	94	0	0	62
N.S.	1	1.00	0.90	3.57	1.66	1.34	0.00	0.00	0.89
time (sec)	N/A	0.134	0.055	0.102	0.273	0.338	0.000	0.000	1.222

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	94	339	137	134	0	0	104
N.S.	1	1.00	0.90	3.23	1.30	1.28	0.00	0.00	0.99
time (sec)	N/A	0.196	0.042	0.108	0.268	0.350	0.000	0.000	0.100

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	431	153	170	0	63	121
N.S.	1	1.00	0.75	3.12	1.11	1.23	0.00	0.46	0.88
time (sec)	N/A	0.265	0.050	0.115	0.260	0.343	0.000	0.418	1.216

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	112	523	169	204	0	0	137
N.S.	1	1.00	0.65	3.06	0.99	1.19	0.00	0.00	0.80
time (sec)	N/A	0.348	0.059	0.122	0.260	0.368	0.000	0.000	0.109

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	47	57	67	63	58	51
N.S.	1	1.00	1.03	0.77	0.93	1.10	1.03	0.95	0.84
time (sec)	N/A	0.095	0.060	0.175	0.255	0.345	0.135	0.406	0.074

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	32	37	43	39	38	35
N.S.	1	1.00	1.05	0.80	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.085	0.046	0.174	0.270	0.336	0.090	0.428	0.050

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	31	37	45	37	38	35
N.S.	1	1.00	1.05	0.79	0.95	1.15	0.95	0.97	0.90
time (sec)	N/A	0.085	0.042	0.138	0.258	0.345	0.078	0.412	1.180

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	21	15	16	19
N.S.	1	1.00	1.00	1.06	1.00	1.31	0.94	1.00	1.19
time (sec)	N/A	0.081	0.037	0.128	0.264	0.324	0.034	0.403	0.034

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	1.00
time (sec)	N/A	0.050	0.019	0.108	0.265	0.332	0.029	0.422	1.169

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	35	35	40	26	36	34
N.S.	1	1.00	0.81	0.95	0.95	1.08	0.70	0.97	0.92
time (sec)	N/A	0.082	0.020	0.088	0.265	0.344	0.074	0.414	0.060

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	49	55	70	49	42	54
N.S.	1	1.00	0.96	0.92	1.04	1.32	0.92	0.79	1.02
time (sec)	N/A	0.102	0.047	0.097	0.265	0.342	0.144	0.408	1.214

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	61	75	100	73	50	71
N.S.	1	1.00	0.86	0.84	1.03	1.37	1.00	0.68	0.97
time (sec)	N/A	0.111	0.050	0.092	0.269	0.392	0.192	0.406	0.078

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	73	93	126	94	58	90
N.S.	1	1.00	0.82	0.84	1.07	1.45	1.08	0.67	1.03
time (sec)	N/A	0.119	0.058	0.090	0.274	0.346	0.248	0.409	0.089

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	175	233	223	156	0	248	183
N.S.	1	1.00	1.70	2.26	2.17	1.51	0.00	2.41	1.78
time (sec)	N/A	0.094	0.149	0.064	0.474	0.352	0.000	0.424	0.130

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	51	105	151	85	0	75	119
N.S.	1	1.00	0.84	1.72	2.48	1.39	0.00	1.23	1.95
time (sec)	N/A	0.037	0.048	0.058	0.472	0.329	0.000	0.396	1.227

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	154	174	125	114	0	138	90
N.S.	1	1.00	2.44	2.76	1.98	1.81	0.00	2.19	1.43
time (sec)	N/A	0.089	0.104	0.059	0.469	0.353	0.000	0.420	1.213

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	73	145	114	88	0	91	82
N.S.	1	1.00	1.49	2.96	2.33	1.80	0.00	1.86	1.67
time (sec)	N/A	0.116	0.073	0.084	0.472	0.341	0.000	0.430	0.095

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	346	133	128	0	63	100
N.S.	1	1.00	0.67	3.30	1.27	1.22	0.00	0.60	0.95
time (sec)	N/A	0.204	0.072	0.103	0.269	0.336	0.000	0.415	1.239

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	438	153	170	0	63	120
N.S.	1	1.00	0.75	3.17	1.11	1.23	0.00	0.46	0.87
time (sec)	N/A	0.271	0.051	0.118	0.260	0.343	0.000	0.434	0.091

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	112	530	169	204	0	0	137
N.S.	1	1.00	0.68	3.21	1.02	1.24	0.00	0.00	0.83
time (sec)	N/A	0.345	0.058	0.122	0.259	0.334	0.000	0.000	0.110

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	120	622	185	240	0	63	153
N.S.	1	1.00	0.59	3.05	0.91	1.18	0.00	0.31	0.75
time (sec)	N/A	0.440	0.064	0.122	0.272	0.336	0.000	0.486	1.245

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	48	59	67	63	123	51
N.S.	1	1.00	1.03	0.75	0.92	1.05	0.98	1.92	0.80
time (sec)	N/A	0.092	0.056	0.250	0.298	0.329	0.143	0.408	0.064

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	31	36	31	59	27
N.S.	1	1.00	1.00	0.90	1.03	1.20	1.03	1.97	0.90
time (sec)	N/A	0.083	0.044	0.210	0.256	0.327	0.068	0.404	0.047

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	30	34	43	37	98	31
N.S.	1	1.00	1.05	0.79	0.89	1.13	0.97	2.58	0.82
time (sec)	N/A	0.087	0.042	0.194	0.262	0.382	0.085	0.434	1.184

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	24	27	32	26	94	25
N.S.	1	1.00	1.07	0.89	1.00	1.19	0.96	3.48	0.93
time (sec)	N/A	0.081	0.038	0.162	0.296	0.335	0.049	0.403	1.181

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	24	23	17	55	24
N.S.	1	1.00	1.00	0.88	0.96	0.92	0.68	2.20	0.96
time (sec)	N/A	0.054	0.022	0.154	0.275	0.349	0.121	0.395	0.069

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	47	49	64	41	74	48
N.S.	1	1.00	0.96	0.89	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.089	0.024	0.132	0.266	0.333	0.135	0.388	0.062

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	61	75	100	73	94	71
N.S.	1	1.00	0.89	0.86	1.06	1.41	1.03	1.32	1.00
time (sec)	N/A	0.108	0.050	0.135	0.267	0.335	0.192	0.407	0.075

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	93	126	94	109	90
N.S.	1	1.00	0.80	0.82	1.04	1.42	1.06	1.22	1.01
time (sec)	N/A	0.117	0.052	0.135	0.263	0.396	0.257	0.401	1.225

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	79	85	113	154	114	124	109
N.S.	1	1.00	0.75	0.81	1.08	1.47	1.09	1.18	1.04
time (sec)	N/A	0.129	0.058	0.129	0.278	0.326	0.311	0.399	1.252

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	175	290	223	156	0	265	185
N.S.	1	1.00	1.30	2.15	1.65	1.16	0.00	1.96	1.37
time (sec)	N/A	0.302	0.098	0.108	0.466	0.356	0.000	0.428	0.121

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	167	266	201	143	0	232	163
N.S.	1	1.00	1.58	2.51	1.90	1.35	0.00	2.19	1.54
time (sec)	N/A	0.224	0.083	0.102	0.461	0.361	0.000	0.420	1.255

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	55	227	126	113	0	130	90
N.S.	1	1.00	0.71	2.95	1.64	1.47	0.00	1.69	1.17
time (sec)	N/A	0.163	0.131	0.099	0.458	0.334	0.000	0.412	1.224

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	73	136	114	88	0	85	82
N.S.	1	1.00	1.49	2.78	2.33	1.80	0.00	1.73	1.67
time (sec)	N/A	0.100	0.072	0.081	0.469	0.345	0.000	0.418	0.071

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	28	44	27	0	24	39
N.S.	1	1.00	1.00	1.47	2.32	1.42	0.00	1.26	2.05
time (sec)	N/A	0.028	0.032	0.112	0.262	0.327	0.000	0.412	0.053

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	255	120	97	0	0	62
N.S.	1	1.00	0.95	3.49	1.64	1.33	0.00	0.00	0.85
time (sec)	N/A	0.079	0.025	0.148	0.253	0.337	0.000	0.000	1.227

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	94	344	137	134	0	0	105
N.S.	1	1.00	0.90	3.28	1.30	1.28	0.00	0.00	1.00
time (sec)	N/A	0.199	0.044	0.140	0.261	0.347	0.000	0.000	1.246

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	436	153	170	0	59	121
N.S.	1	1.00	0.75	3.16	1.11	1.23	0.00	0.43	0.88
time (sec)	N/A	0.266	0.055	0.144	0.256	0.341	0.000	0.449	1.238

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	51	60	71	56	62	61
N.S.	1	1.00	1.03	0.78	0.92	1.09	0.86	0.95	0.94
time (sec)	N/A	0.098	0.058	0.177	0.260	0.357	0.232	0.407	0.095

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	43	51	62	42	53	51
N.S.	1	1.00	1.04	0.80	0.94	1.15	0.78	0.98	0.94
time (sec)	N/A	0.093	0.044	0.155	0.260	0.353	0.188	0.401	0.078

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	31	40	43	31	42	40
N.S.	1	1.00	1.05	0.78	1.00	1.08	0.78	1.05	1.00
time (sec)	N/A	0.089	0.040	0.130	0.261	0.369	0.143	0.397	1.223

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	23	22	17	25	23
N.S.	1	1.00	1.00	0.87	1.00	0.96	0.74	1.09	1.00
time (sec)	N/A	0.057	0.020	0.112	0.249	0.356	0.106	0.408	0.064

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	23	20	19	17	21	19
N.S.	1	1.00	1.10	1.15	1.00	0.95	0.85	1.05	0.95
time (sec)	N/A	0.077	0.013	0.092	0.261	0.343	0.039	0.405	1.201

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	39	36	34	27	34	36	17
N.S.	1	1.00	2.17	2.00	1.89	1.50	1.89	2.00	0.94
time (sec)	N/A	0.092	0.047	0.092	0.252	0.351	0.074	0.409	1.238

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	48	53	59	56	51	52
N.S.	1	1.00	0.98	0.84	0.93	1.04	0.98	0.89	0.91
time (sec)	N/A	0.104	0.049	0.095	0.261	0.333	0.168	0.404	0.090

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	60	69	93	73	57	68
N.S.	1	1.00	0.97	0.80	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.111	0.060	0.098	0.257	0.326	0.220	0.408	0.096

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	567	672	246	157	0	0	211
N.S.	1	1.00	3.46	4.10	1.50	0.96	0.00	0.00	1.29
time (sec)	N/A	0.371	0.725	0.121	0.483	0.336	0.000	0.000	0.137

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	663	450	225	146	0	0	190
N.S.	1	1.00	4.91	3.33	1.67	1.08	0.00	0.00	1.41
time (sec)	N/A	0.295	0.294	0.115	0.482	0.355	0.000	0.000	1.273

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	424	600	149	120	0	0	117
N.S.	1	1.00	4.04	5.71	1.42	1.14	0.00	0.00	1.11
time (sec)	N/A	0.222	0.261	0.115	0.478	0.346	0.000	0.000	0.102

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	234	376	135	92	0	0	107
N.S.	1	1.00	3.12	5.01	1.80	1.23	0.00	0.00	1.43
time (sec)	N/A	0.155	0.321	0.085	0.486	0.349	0.000	0.000	0.093

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	250	120	69	0	0	87
N.S.	1	1.00	0.85	3.47	1.67	0.96	0.00	0.00	1.21
time (sec)	N/A	0.120	0.056	0.100	0.260	0.345	0.000	0.000	0.056

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	69	250	125	67	0	0	90
N.S.	1	1.00	0.93	3.38	1.69	0.91	0.00	0.00	1.22
time (sec)	N/A	0.071	0.026	0.095	0.249	0.353	0.000	0.000	1.188

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	50	92	42	0	42	41
N.S.	1	1.00	0.73	1.11	2.04	0.93	0.00	0.93	0.91
time (sec)	N/A	0.035	0.016	0.137	0.271	0.351	0.000	0.400	0.069

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	94	523	160	134	0	0	128
N.S.	1	1.00	0.85	4.71	1.44	1.21	0.00	0.00	1.15
time (sec)	N/A	0.103	0.046	0.149	0.262	0.334	0.000	0.000	1.235

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	615	176	170	0	0	144
N.S.	1	1.00	0.75	4.46	1.28	1.23	0.00	0.00	1.04
time (sec)	N/A	0.268	0.057	0.157	0.270	0.343	0.000	0.000	0.087

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	279	109	166	0	437	0	0	-1
N.S.	1	1.19	0.46	0.71	0.00	1.86	0.00	0.00	-0.00
time (sec)	N/A	0.117	0.078	0.082	0.000	0.384	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	221	101	149	0	415	0	0	-1
N.S.	1	1.13	0.52	0.76	0.00	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.054	0.056	0.000	0.382	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	189	89	132	0	381	0	0	-1
N.S.	1	1.20	0.57	0.84	0.00	2.43	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.046	0.056	0.000	0.392	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	70	106	0	313	0	0	-1
N.S.	1	1.00	0.60	0.91	0.00	2.68	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.035	0.050	0.000	0.399	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	295	0	0	-1
N.S.	1	1.00	0.85	1.12	0.00	3.78	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.030	0.044	0.000	0.410	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	93	151	0	517	0	0	-1
N.S.	1	1.00	0.61	0.99	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.045	0.142	0.000	0.426	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	122	259	0	594	0	0	-1
N.S.	1	1.00	0.57	1.20	0.00	2.76	0.00	0.00	-0.00
time (sec)	N/A	0.103	0.064	0.158	0.000	0.442	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	135	366	0	668	0	0	-1
N.S.	1	1.00	0.49	1.32	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.119	0.080	0.161	0.000	0.447	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	91	163	0	234	0	0	-1
N.S.	1	1.00	0.64	1.14	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.102	0.119	0.000	0.388	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	83	144	0	212	731	0	-1
N.S.	1	1.00	0.70	1.22	0.00	1.80	6.19	0.00	-0.01
time (sec)	N/A	0.155	0.079	0.106	0.000	0.359	8.100	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	108	0	182	0	0	-1
N.S.	1	1.00	0.79	1.14	0.00	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.045	0.104	0.000	0.349	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	55	103	0	137	0	0	-1
N.S.	1	1.00	0.79	1.47	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.034	0.099	0.000	0.352	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	120	0	124	0	96	-1
N.S.	1	1.00	1.00	2.40	0.00	2.48	0.00	1.92	-0.02
time (sec)	N/A	0.109	0.022	0.138	0.000	0.351	0.000	0.427	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	194	0	176	0	167	-1
N.S.	1	1.00	0.61	2.77	0.00	2.51	0.00	2.39	-0.01
time (sec)	N/A	0.120	0.023	0.146	0.000	0.329	0.000	0.475	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	55	260	0	238	0	243	-1
N.S.	1	1.00	0.58	2.74	0.00	2.51	0.00	2.56	-0.01
time (sec)	N/A	0.141	0.022	0.159	0.000	0.340	0.000	0.517	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	58	328	0	294	0	316	-1
N.S.	1	1.00	0.49	2.78	0.00	2.49	0.00	2.68	-0.01
time (sec)	N/A	0.148	0.023	0.168	0.000	0.342	0.000	0.571	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	46	396	0	346	0	389	-1
N.S.	1	1.00	0.32	2.73	0.00	2.39	0.00	2.68	-0.01
time (sec)	N/A	0.163	0.029	0.179	0.000	0.360	0.000	0.681	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	109	178	0	437	0	0	-1
N.S.	1	1.00	0.41	0.66	0.00	1.63	0.00	0.00	-0.00
time (sec)	N/A	0.113	0.065	0.058	0.000	0.382	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	101	161	0	415	0	0	-1
N.S.	1	1.00	0.43	0.68	0.00	1.75	0.00	0.00	-0.00
time (sec)	N/A	0.102	0.057	0.053	0.000	0.372	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	89	144	0	381	0	0	-1
N.S.	1	1.00	0.57	0.92	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.045	0.051	0.000	0.383	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	66	118	0	315	0	0	-1
N.S.	1	1.00	0.56	1.00	0.00	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.031	0.048	0.000	0.395	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	93	160	0	512	0	0	-1
N.S.	1	1.00	0.61	1.05	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.041	0.147	0.000	0.441	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	115	259	0	581	0	0	-1
N.S.	1	1.00	0.53	1.20	0.00	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.107	0.062	0.152	0.000	0.435	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	135	373	0	668	0	0	-1
N.S.	1	1.00	0.49	1.36	0.00	2.43	0.00	0.00	-0.00
time (sec)	N/A	0.126	0.080	0.158	0.000	0.423	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	143	480	0	736	0	0	-1
N.S.	1	1.00	0.43	1.43	0.00	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.098	0.158	0.000	0.457	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	101	161	0	415	0	0	-1
N.S.	1	1.00	0.46	0.73	0.00	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.101	0.068	0.105	0.000	0.365	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	89	144	0	381	0	0	-1
N.S.	1	1.00	0.55	0.89	0.00	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.045	0.105	0.000	0.366	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	70	118	0	315	0	0	-1
N.S.	1	1.00	0.50	0.84	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.035	0.095	0.000	0.383	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	297	0	0	-1
N.S.	1	1.00	0.82	1.28	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.027	0.094	0.000	0.381	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	102	0	299	0	0	-1
N.S.	1	1.00	0.85	1.31	0.00	3.83	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.029	0.099	0.000	0.459	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	91	162	0	522	0	0	-1
N.S.	1	1.00	0.60	1.07	0.00	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.044	0.180	0.000	0.399	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	123	264	0	596	0	0	-1
N.S.	1	1.00	0.56	1.21	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.113	0.067	0.182	0.000	0.419	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	135	371	0	668	0	0	-1
N.S.	1	1.00	0.49	1.34	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.088	0.200	0.000	0.430	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	125	281	0	323	0	0	-1
N.S.	1	1.00	0.77	1.72	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.171	0.202	0.000	0.340	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	116	257	0	285	0	0	-1
N.S.	1	1.00	0.84	1.86	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.068	0.208	0.000	0.354	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	229	0	235	0	0	-1
N.S.	1	1.00	0.84	2.03	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.056	0.193	0.000	0.339	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	190	0	219	0	0	-1
N.S.	1	1.00	1.00	2.07	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.034	0.190	0.000	0.379	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	136	0	234	0	0	-1
N.S.	1	1.00	1.00	1.43	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.033	0.184	0.000	0.369	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	136	0	231	0	0	-1
N.S.	1	1.00	1.00	1.45	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.035	0.230	0.000	0.380	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	70	368	0	287	0	0	-1
N.S.	1	1.00	0.60	3.17	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.037	0.242	0.000	0.362	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	79	497	0	359	0	0	-1
N.S.	1	1.00	0.54	3.38	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.068	0.237	0.000	0.356	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	82	626	0	431	0	0	-1
N.S.	1	1.00	0.48	3.64	0.00	2.51	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.045	0.248	0.000	0.409	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	140	229	0	437	0	0	-1
N.S.	1	1.00	0.42	0.68	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.076	0.117	0.000	0.409	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	132	212	0	415	0	0	-1
N.S.	1	1.00	0.48	0.77	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.120	0.074	0.109	0.000	0.383	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	124	195	0	381	0	0	-1
N.S.	1	1.00	0.57	0.89	0.00	1.74	0.00	0.00	-0.00
time (sec)	N/A	0.103	0.069	0.107	0.000	0.398	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	71	169	0	315	0	0	-1
N.S.	1	1.00	0.45	1.07	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.031	0.103	0.000	0.373	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	299	0	0	-1
N.S.	1	1.00	0.48	1.04	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.034	0.094	0.000	0.378	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	69	149	0	303	0	0	-1
N.S.	1	1.00	0.58	1.26	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.029	0.095	0.000	0.382	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	64	149	0	311	0	0	-1
N.S.	1	1.00	0.55	1.27	0.00	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.031	0.095	0.000	0.361	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	90	264	0	524	0	0	-1
N.S.	1	1.00	0.45	1.33	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.044	0.186	0.000	0.471	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	121	290	0	594	0	0	-1
N.S.	1	1.00	0.45	1.09	0.00	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.051	0.194	0.000	0.404	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.157	0.022	0.020	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	147	121	0	337	0	0	-1
N.S.	1	1.00	0.90	0.74	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.270	0.048	0.000	0.425	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	148	102	0	317	0	0	-1
N.S.	1	1.00	1.19	0.82	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.159	0.045	0.000	0.410	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	87	0	295	0	0	-1
N.S.	1	1.00	0.85	1.12	0.00	3.78	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.032	0.039	0.000	0.387	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	132	88	0	275	0	0	-1
N.S.	1	1.00	1.74	1.16	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.163	0.045	0.000	0.432	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	41	0	58	0	0	47
N.S.	1	1.00	1.22	1.11	0.00	1.57	0.00	0.00	1.27
time (sec)	N/A	0.105	0.064	0.032	0.000	0.355	0.000	0.000	1.367

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	47	0	68	0	0	53
N.S.	1	1.00	0.75	0.61	0.00	0.88	0.00	0.00	0.69
time (sec)	N/A	0.129	0.061	0.032	0.000	0.359	0.000	0.000	1.394

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	66	55	0	77	0	0	100
N.S.	1	1.00	0.56	0.47	0.00	0.66	0.00	0.00	0.85
time (sec)	N/A	0.164	0.063	0.033	0.000	0.337	0.000	0.000	1.415

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	74	63	0	84	0	0	108
N.S.	1	1.00	0.47	0.40	0.00	0.53	0.00	0.00	0.68
time (sec)	N/A	0.200	0.071	0.037	0.000	0.375	0.000	0.000	1.441

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	50	172	0	179	0	142	-1
N.S.	1	1.00	0.38	1.32	0.00	1.38	0.00	1.09	-0.01
time (sec)	N/A	0.263	0.027	0.138	0.000	0.359	0.000	0.440	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	50	155	0	163	0	127	-1
N.S.	1	1.00	0.48	1.48	0.00	1.55	0.00	1.21	-0.01
time (sec)	N/A	0.253	0.025	0.135	0.000	0.370	0.000	0.439	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	139	0	147	0	112	-1
N.S.	1	1.00	0.96	1.74	0.00	1.84	0.00	1.40	-0.01
time (sec)	N/A	0.164	0.046	0.134	0.000	0.452	0.000	0.423	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	120	0	124	0	96	-1
N.S.	1	1.00	1.00	2.40	0.00	2.48	0.00	1.92	-0.02
time (sec)	N/A	0.103	0.024	0.131	0.000	0.375	0.000	0.436	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	99	0	111	39	0	-1
N.S.	1	1.00	1.00	2.11	0.00	2.36	0.83	0.00	-0.02
time (sec)	N/A	0.223	0.022	0.133	0.000	0.372	6.087	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	173	0	28	0	0	24
N.S.	1	1.00	0.67	4.12	0.00	0.67	0.00	0.00	0.57
time (sec)	N/A	0.220	0.025	0.123	0.000	0.325	0.000	0.000	1.262

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	36	192	0	36	0	0	32
N.S.	1	1.00	0.52	2.78	0.00	0.52	0.00	0.00	0.46
time (sec)	N/A	0.229	0.030	0.125	0.000	0.338	0.000	0.000	1.286

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	44	211	0	44	0	0	77
N.S.	1	1.00	0.46	2.20	0.00	0.46	0.00	0.00	0.80
time (sec)	N/A	0.237	0.035	0.127	0.000	0.375	0.000	0.000	1.278

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	52	230	0	52	0	0	98
N.S.	1	1.00	0.43	1.90	0.00	0.43	0.00	0.00	0.81
time (sec)	N/A	0.239	0.041	0.132	0.000	0.373	0.000	0.000	1.370

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	252	224	0	568	0	0	-1
N.S.	1	1.00	0.81	0.72	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.527	0.135	0.000	0.449	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	244	202	0	552	0	0	-1
N.S.	1	1.00	0.93	0.77	0.00	2.11	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.392	0.143	0.000	0.458	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	236	180	0	536	0	0	-1
N.S.	1	1.00	1.13	0.86	0.00	2.56	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.311	0.135	0.000	0.442	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	93	160	0	512	0	0	-1
N.S.	1	1.00	0.61	1.05	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.054	0.129	0.000	0.438	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	218	159	0	490	0	0	-1
N.S.	1	1.00	1.49	1.09	0.00	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.283	0.140	0.000	0.454	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	155	140	0	353	0	0	-1
N.S.	1	1.00	1.24	1.12	0.00	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.186	0.142	0.000	0.383	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	162	165	0	381	0	0	-1
N.S.	1	1.00	0.95	0.97	0.00	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.183	0.145	0.000	0.390	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	170	187	0	397	0	0	-1
N.S.	1	1.00	0.81	0.89	0.00	1.90	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.195	0.145	0.000	0.420	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	178	209	0	413	0	0	-1
N.S.	1	1.00	0.59	0.69	0.00	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.200	0.148	0.000	0.388	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	93	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.045	0.015	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	147	133	0	337	0	0	-1
N.S.	1	1.00	0.90	0.81	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.272	0.090	0.000	0.360	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	139	116	0	321	0	0	-1
N.S.	1	1.00	1.12	0.94	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.229	0.087	0.000	0.377	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	65	101	0	297	0	0	-1
N.S.	1	1.00	0.82	1.28	0.00	3.76	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.032	0.087	0.000	0.360	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	132	100	0	275	0	0	-1
N.S.	1	1.00	1.74	1.32	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.170	0.089	0.000	0.415	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	54	0	59	0	0	54
N.S.	1	1.00	0.66	0.77	0.00	0.84	0.00	0.00	0.77
time (sec)	N/A	0.128	0.066	0.082	0.000	0.393	0.000	0.000	1.331

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	58	62	0	69	0	0	62
N.S.	1	1.00	0.51	0.55	0.00	0.61	0.00	0.00	0.55
time (sec)	N/A	0.162	0.064	0.081	0.000	0.340	0.000	0.000	1.353

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	66	70	0	77	0	0	100
N.S.	1	1.00	0.44	0.47	0.00	0.52	0.00	0.00	0.67
time (sec)	N/A	0.216	0.068	0.085	0.000	0.345	0.000	0.000	1.408

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	116	259	0	271	0	0	-1
N.S.	1	1.00	0.67	1.51	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.099	0.187	0.000	0.350	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	108	237	0	259	0	0	-1
N.S.	1	1.00	0.73	1.61	0.00	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.315	0.070	0.184	0.000	0.345	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	215	0	239	0	0	-1
N.S.	1	1.00	0.82	1.76	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.056	0.187	0.000	0.350	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	189	0	219	0	0	-1
N.S.	1	1.00	1.00	2.05	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.059	0.181	0.000	0.397	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	228	0	203	80	0	-1
N.S.	1	1.00	1.00	2.65	0.00	2.36	0.93	0.00	-0.01
time (sec)	N/A	0.252	0.045	0.185	0.000	0.344	6.974	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	254	0	161	0	0	-1
N.S.	1	1.00	0.84	3.10	0.00	1.96	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.088	0.204	0.000	0.371	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	79	278	0	181	0	278	-1
N.S.	1	1.00	0.70	2.46	0.00	1.60	0.00	2.46	-0.01
time (sec)	N/A	0.270	0.108	0.197	0.000	0.397	0.000	0.750	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	87	302	0	201	0	356	-1
N.S.	1	1.00	0.77	2.67	0.00	1.78	0.00	3.15	-0.01
time (sec)	N/A	0.283	0.153	0.194	0.000	0.355	0.000	0.802	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	95	326	0	213	0	434	-1
N.S.	1	1.00	0.58	2.00	0.00	1.31	0.00	2.66	-0.01
time (sec)	N/A	0.310	0.184	0.199	0.000	0.354	0.000	0.950	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	167	197	0	353	0	0	-1
N.S.	1	1.00	0.55	0.65	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.548	0.100	0.000	0.385	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	159	180	0	337	0	0	-1
N.S.	1	1.00	0.63	0.72	0.00	1.34	0.00	0.00	-0.00
time (sec)	N/A	0.202	0.302	0.102	0.000	0.379	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	151	163	0	321	0	0	-1
N.S.	1	1.00	0.76	0.82	0.00	1.61	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.263	0.102	0.000	0.403	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	67	146	0	299	0	0	-1
N.S.	1	1.00	0.48	1.04	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.035	0.092	0.000	0.367	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	131	151	0	277	0	0	-1
N.S.	1	1.00	0.98	1.13	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.273	0.105	0.000	0.369	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	58	62	0	59	0	0	54
N.S.	1	1.00	0.53	0.57	0.00	0.54	0.00	0.00	0.50
time (sec)	N/A	0.152	0.068	0.096	0.000	0.341	0.000	0.000	1.361

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	70	70	0	69	0	0	62
N.S.	1	1.00	0.47	0.47	0.00	0.46	0.00	0.00	0.41
time (sec)	N/A	0.183	0.070	0.102	0.000	0.363	0.000	0.000	1.442

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	78	78	0	77	0	0	100
N.S.	1	1.00	0.41	0.41	0.00	0.41	0.00	0.00	0.53
time (sec)	N/A	0.288	0.074	0.107	0.000	0.336	0.000	0.000	1.415

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	86	86	0	85	0	0	108
N.S.	1	1.00	0.30	0.30	0.00	0.29	0.00	0.00	0.37
time (sec)	N/A	0.205	0.080	0.105	0.000	0.351	0.000	0.000	1.410

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	155	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.253	0.016	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.250	0.015	0.000	0.000	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	113	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.051	0.017	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	180.004	0.016	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	180.003	0.017	0.000	0.000	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	180.004	0.016	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	180.007	0.014	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	1.012	0.014	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.021	0.013	0.000	0.000	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.427	0.015	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	272	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	4.77	0.00	-0.02
time (sec)	N/A	0.074	0.018	0.064	0.000	0.000	5.101	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.839	0.013	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.177	0.013	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.035	0.067	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	279	415	169	0	218	362
N.S.	1	1.00	0.28	0.71	1.06	0.43	0.00	0.55	0.92
time (sec)	N/A	0.227	0.091	0.106	0.265	0.363	0.000	0.429	1.367

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	231	337	148	0	178	289
N.S.	1	1.00	0.30	0.74	1.08	0.47	0.00	0.57	0.92
time (sec)	N/A	0.172	0.099	0.100	0.261	0.336	0.000	0.450	1.308

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	183	259	125	0	137	214
N.S.	1	1.00	0.34	0.79	1.11	0.54	0.00	0.59	0.92
time (sec)	N/A	0.133	0.057	0.095	0.257	0.353	0.000	0.438	1.261

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	119	171	91	0	90	131
N.S.	1	1.00	0.42	0.82	1.18	0.63	0.00	0.62	0.90
time (sec)	N/A	0.081	0.056	0.046	0.255	0.347	0.000	0.406	0.070

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	23	22	34	0	0	22
N.S.	1	1.00	1.00	1.77	1.69	2.62	0.00	0.00	1.69
time (sec)	N/A	0.020	0.034	0.188	0.261	0.327	0.000	0.000	1.225

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	52	65	58	0	0	50
N.S.	1	1.00	0.98	1.02	1.27	1.14	0.00	0.00	0.98
time (sec)	N/A	0.044	0.047	0.185	0.261	0.333	0.000	0.000	0.071

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	68	99	86	0	0	60
N.S.	1	1.00	0.78	0.80	1.16	1.01	0.00	0.00	0.71
time (sec)	N/A	0.065	0.063	0.192	0.260	0.326	0.000	0.000	0.073

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	82	84	132	134	0	0	142
N.S.	1	1.00	0.69	0.71	1.11	1.13	0.00	0.00	1.19
time (sec)	N/A	0.090	0.090	0.194	0.252	0.380	0.000	0.000	0.062

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	47	85	113	113	119	113	113
N.S.	1	1.00	0.56	1.01	1.35	1.35	1.42	1.35	1.35
time (sec)	N/A	0.065	0.026	0.219	0.262	0.337	0.037	0.422	1.251

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	63	82	82	87	82	82
N.S.	1	1.00	0.57	0.91	1.19	1.19	1.26	1.19	1.19
time (sec)	N/A	0.057	0.020	0.200	0.263	0.323	0.030	0.402	0.042

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	54	70	70	70	70	70
N.S.	1	1.00	0.60	1.04	1.35	1.35	1.35	1.35	1.35
time (sec)	N/A	0.053	0.017	0.194	0.257	0.346	0.025	0.407	0.035

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	31	38	38	36	38	38
N.S.	1	1.00	0.66	0.89	1.09	1.09	1.03	1.09	1.09
time (sec)	N/A	0.043	0.012	0.179	0.257	0.313	0.019	0.404	0.053

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	23	14	21	21	20	21	17
N.S.	1	1.00	1.53	0.93	1.40	1.40	1.33	1.40	1.13
time (sec)	N/A	0.024	0.011	0.099	0.256	0.346	0.015	0.411	0.035

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	13	13	10	14	14
N.S.	1	1.00	1.12	0.94	0.81	0.81	0.62	0.88	0.88
time (sec)	N/A	0.043	0.012	0.141	0.258	0.333	0.051	0.404	0.035

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	52	63	76	54	51	46
N.S.	1	1.00	0.69	1.02	1.24	1.49	1.06	1.00	0.90
time (sec)	N/A	0.058	0.019	0.141	0.257	0.349	0.146	0.430	1.263

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	63	76	91	121	85	74	73
N.S.	1	1.00	0.73	0.88	1.06	1.41	0.99	0.86	0.85
time (sec)	N/A	0.071	0.027	0.151	0.270	0.341	0.220	0.416	0.091

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	82	100	140	217	141	91	121
N.S.	1	1.00	0.68	0.83	1.16	1.79	1.17	0.75	1.00
time (sec)	N/A	0.085	0.045	0.161	0.281	0.339	0.332	0.411	1.302

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	288	415	170	0	216	362
N.S.	1	1.00	0.28	0.73	1.06	0.43	0.00	0.55	0.92
time (sec)	N/A	0.222	0.087	0.112	0.273	0.340	0.000	0.414	0.215

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	240	337	147	0	177	289
N.S.	1	1.00	0.30	0.77	1.08	0.47	0.00	0.57	0.92
time (sec)	N/A	0.176	0.072	0.096	0.271	0.361	0.000	0.417	0.123

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	192	259	126	0	138	214
N.S.	1	1.00	0.34	0.82	1.11	0.54	0.00	0.59	0.92
time (sec)	N/A	0.139	0.059	0.102	0.272	0.357	0.000	0.441	1.246

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	183	171	91	0	90	133
N.S.	1	1.00	0.42	1.26	1.18	0.63	0.00	0.62	0.92
time (sec)	N/A	0.080	0.043	0.089	0.277	0.351	0.000	0.414	0.070

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	51	0	49	23
N.S.	1	1.00	1.00	1.33	1.28	2.83	0.00	2.72	1.28
time (sec)	N/A	0.021	0.035	0.130	0.279	0.354	0.000	0.418	0.032

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	43	52	55	77	0	65	55
N.S.	1	1.00	0.78	0.95	1.00	1.40	0.00	1.18	1.00
time (sec)	N/A	0.045	0.049	0.189	0.271	0.378	0.000	0.437	0.040

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	68	97	96	0	0	60
N.S.	1	1.00	0.73	0.75	1.07	1.05	0.00	0.00	0.66
time (sec)	N/A	0.071	0.067	0.187	0.266	0.344	0.000	0.000	1.264

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	84	131	124	0	0	76
N.S.	1	1.00	0.65	0.66	1.03	0.98	0.00	0.00	0.60
time (sec)	N/A	0.097	0.085	0.193	0.264	0.380	0.000	0.000	1.260

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	39	75	101	101	109	102	101
N.S.	1	1.00	0.59	1.14	1.53	1.53	1.65	1.55	1.53
time (sec)	N/A	0.059	0.025	0.276	0.262	0.352	0.041	0.410	1.243

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	69	92	92	100	90	92
N.S.	1	1.00	0.60	1.33	1.77	1.77	1.92	1.73	1.77
time (sec)	N/A	0.055	0.020	0.235	0.272	0.334	0.034	0.414	0.051

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	45	59	59	63	78	59
N.S.	1	1.00	0.66	1.29	1.69	1.69	1.80	2.23	1.69
time (sec)	N/A	0.044	0.015	0.229	0.261	0.355	0.029	0.409	0.034

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	49	16	47	47	48	64	47
N.S.	1	1.00	2.88	0.94	2.76	2.76	2.82	3.76	2.76
time (sec)	N/A	0.038	0.014	0.186	0.260	0.347	0.026	0.405	0.032

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	36	32	33	37	36	60	33
N.S.	1	1.00	0.78	0.70	0.72	0.80	0.78	1.30	0.72
time (sec)	N/A	0.032	0.010	0.146	0.265	0.381	0.058	0.422	0.051

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	28	19	19	17	27	12
N.S.	1	1.00	1.92	2.15	1.46	1.46	1.31	2.08	0.92
time (sec)	N/A	0.044	0.008	0.181	0.261	0.330	0.087	0.394	0.050

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	16	41	41	42	15	40
N.S.	1	1.00	0.94	0.89	2.28	2.28	2.33	0.83	2.22
time (sec)	N/A	0.042	0.014	0.177	0.265	0.341	0.108	0.411	0.062

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	76	102	147	99	91	83
N.S.	1	1.00	0.60	0.87	1.17	1.69	1.14	1.05	0.95
time (sec)	N/A	0.067	0.027	0.184	0.287	0.372	0.244	0.408	0.092

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	80	100	130	191	129	127	111
N.S.	1	1.00	0.66	0.82	1.07	1.57	1.06	1.04	0.91
time (sec)	N/A	0.080	0.039	0.198	0.270	0.377	0.328	0.410	1.298

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	279	415	170	0	196	362
N.S.	1	1.00	0.28	0.71	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.219	0.093	0.102	0.264	0.369	0.000	0.425	1.343

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	231	337	147	0	161	289
N.S.	1	1.00	0.30	0.74	1.08	0.47	0.00	0.51	0.92
time (sec)	N/A	0.178	0.103	0.102	0.270	0.348	0.000	0.411	0.113

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	183	259	126	0	126	214
N.S.	1	1.00	0.34	0.79	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.138	0.058	0.095	0.275	0.376	0.000	0.418	0.079

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	119	171	91	0	82	132
N.S.	1	1.00	0.42	0.82	1.18	0.63	0.00	0.57	0.91
time (sec)	N/A	0.086	0.054	0.049	0.268	0.355	0.000	0.419	1.243

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	23	23	0	0	23
N.S.	1	1.00	1.00	1.50	1.44	1.44	0.00	0.00	1.44
time (sec)	N/A	0.021	0.036	0.175	0.273	0.344	0.000	0.000	0.031

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	52	67	50	0	0	55
N.S.	1	1.00	0.87	0.95	1.22	0.91	0.00	0.00	1.00
time (sec)	N/A	0.043	0.046	0.203	0.269	0.356	0.000	0.000	0.051

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	64	68	102	76	0	0	109
N.S.	1	1.00	0.70	0.75	1.12	0.84	0.00	0.00	1.20
time (sec)	N/A	0.070	0.066	0.213	0.267	0.348	0.000	0.000	1.242

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	80	84	135	104	0	0	148
N.S.	1	1.00	0.63	0.66	1.06	0.82	0.00	0.00	1.17
time (sec)	N/A	0.095	0.088	0.225	0.262	0.354	0.000	0.000	0.038

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	39	61	80	80	87	80	80
N.S.	1	1.00	0.53	0.84	1.10	1.10	1.19	1.10	1.10
time (sec)	N/A	0.056	0.022	0.240	0.260	0.322	0.038	0.407	0.047

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	31	54	70	70	70	70	70
N.S.	1	1.00	0.56	0.98	1.27	1.27	1.27	1.27	1.27
time (sec)	N/A	0.052	0.019	0.191	0.258	0.326	0.025	0.395	0.039

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	30	37	37	36	37	37
N.S.	1	1.00	0.81	0.81	1.00	1.00	0.97	1.00	1.00
time (sec)	N/A	0.045	0.013	0.176	0.270	0.329	0.029	0.401	0.051

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	22	14	20	20	19	20	17
N.S.	1	1.00	1.38	0.88	1.25	1.25	1.19	1.25	1.06
time (sec)	N/A	0.023	0.011	0.102	0.266	0.340	0.014	0.403	0.035

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	12	12	10	14	12
N.S.	1	1.00	1.29	1.07	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.044	0.013	0.137	0.255	0.343	0.048	0.408	0.050

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	52	63	76	54	51	46
N.S.	1	1.00	0.67	1.06	1.29	1.55	1.10	1.04	0.94
time (sec)	N/A	0.056	0.022	0.139	0.258	0.343	0.143	0.408	0.065

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	76	91	121	85	74	73
N.S.	1	1.00	0.73	0.90	1.08	1.44	1.01	0.88	0.87
time (sec)	N/A	0.072	0.031	0.142	0.277	0.369	0.218	0.409	1.261

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	80	100	140	217	141	91	121
N.S.	1	1.00	0.67	0.84	1.18	1.82	1.18	0.76	1.02
time (sec)	N/A	0.085	0.043	0.157	0.280	0.367	0.324	0.404	1.284

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	111	288	415	169	0	198	362
N.S.	1	1.00	0.28	0.73	1.06	0.43	0.00	0.50	0.92
time (sec)	N/A	0.221	0.094	0.105	0.273	0.390	0.000	0.435	0.171

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	95	240	337	148	0	162	289
N.S.	1	1.00	0.30	0.77	1.08	0.47	0.00	0.52	0.92
time (sec)	N/A	0.179	0.074	0.100	0.264	0.336	0.000	0.402	0.130

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	79	192	259	125	0	126	214
N.S.	1	1.00	0.34	0.82	1.11	0.54	0.00	0.54	0.92
time (sec)	N/A	0.140	0.057	0.098	0.278	0.347	0.000	0.419	1.269

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	61	183	171	92	0	82	133
N.S.	1	1.00	0.42	1.26	1.18	0.63	0.00	0.57	0.92
time (sec)	N/A	0.082	0.061	0.095	0.262	0.364	0.000	0.410	0.066

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	23	34	0	49	23
N.S.	1	1.00	1.00	1.33	1.28	1.89	0.00	2.72	1.28
time (sec)	N/A	0.022	0.037	0.133	0.263	0.335	0.000	0.429	1.209

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	43	52	60	58	0	65	60
N.S.	1	1.00	0.78	0.95	1.09	1.05	0.00	1.18	1.09
time (sec)	N/A	0.045	0.046	0.175	0.264	0.348	0.000	0.441	0.049

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	68	103	86	0	0	116
N.S.	1	1.00	0.73	0.75	1.13	0.95	0.00	0.00	1.27
time (sec)	N/A	0.069	0.064	0.191	0.266	0.361	0.000	0.000	1.206

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	84	136	134	0	0	155
N.S.	1	1.00	0.65	0.66	1.07	1.06	0.00	0.00	1.22
time (sec)	N/A	0.098	0.088	0.200	0.266	0.348	0.000	0.000	0.039

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	79	113	0	117	0	0	-1
N.S.	1	1.00	0.34	0.49	0.00	0.51	0.00	0.00	-0.00
time (sec)	N/A	0.138	0.043	0.127	0.000	0.354	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	71	97	0	95	0	0	-1
N.S.	1	1.00	0.39	0.53	0.00	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.035	0.110	0.000	0.346	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	63	81	0	73	0	0	-1
N.S.	1	1.00	0.46	0.60	0.00	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.030	0.115	0.000	0.362	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	53	63	0	43	0	0	-1
N.S.	1	1.00	0.57	0.68	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.023	0.108	0.000	0.336	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	45	0	22	0	0	-1
N.S.	1	1.00	0.60	0.66	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.014	0.105	0.000	0.351	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	0	22	0	0	-1
N.S.	1	1.00	1.00	1.34	0.00	0.58	0.00	0.00	-0.03
time (sec)	N/A	0.099	0.014	0.092	0.000	0.355	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	86	0	0	-1
N.S.	1	1.00	0.62	0.92	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.026	0.096	0.000	0.397	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	136	0	0	-1
N.S.	1	1.00	0.45	0.92	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.042	0.094	0.000	0.367	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	101	241	0	191	0	0	-1
N.S.	1	1.00	0.36	0.87	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.056	0.098	0.000	0.362	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	167	454	192	329	1340	164	-1
N.S.	1	1.00	0.95	2.58	1.09	1.87	7.61	0.93	-0.01
time (sec)	N/A	0.121	0.103	0.210	0.514	0.411	150.001	0.434	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	151	375	173	286	1090	141	-1
N.S.	1	1.00	0.99	2.45	1.13	1.87	7.12	0.92	-0.01
time (sec)	N/A	0.107	0.138	0.203	0.477	0.377	33.013	0.420	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	296	154	241	476	116	-1
N.S.	1	1.00	1.04	2.28	1.18	1.85	3.66	0.89	-0.01
time (sec)	N/A	0.099	0.161	0.194	0.477	0.374	9.154	0.419	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	117	217	131	180	340	85	-1
N.S.	1	1.00	1.09	2.03	1.22	1.68	3.18	0.79	-0.01
time (sec)	N/A	0.088	0.065	0.190	0.480	0.377	4.912	0.410	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	136	47	134	0	62	-1
N.S.	1	1.00	0.88	1.58	0.55	1.56	0.00	0.72	-0.01
time (sec)	N/A	0.078	0.036	0.190	0.470	0.378	0.000	0.412	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	82	79	40	153	0	0	-1
N.S.	1	1.00	1.39	1.34	0.68	2.59	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.032	0.137	0.471	0.374	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	64	127	61	47	0	148	33
N.S.	1	1.00	1.25	2.49	1.20	0.92	0.00	2.90	0.65
time (sec)	N/A	0.073	0.025	0.177	0.262	0.385	0.000	0.430	1.294

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	53	206	80	75	0	0	56
N.S.	1	1.00	0.72	2.78	1.08	1.01	0.00	0.00	0.76
time (sec)	N/A	0.079	0.027	0.197	0.292	0.403	0.000	0.000	1.381

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	292	99	124	0	0	134
N.S.	1	1.00	0.99	3.01	1.02	1.28	0.00	0.00	1.38
time (sec)	N/A	0.087	0.036	0.188	0.266	0.523	0.000	0.000	1.453

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	112	378	118	152	0	0	177
N.S.	1	1.00	0.93	3.15	0.98	1.27	0.00	0.00	1.48
time (sec)	N/A	0.095	0.044	0.196	0.272	0.662	0.000	0.000	1.512

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	71	102	204	95	0	0	-1
N.S.	1	1.00	0.38	0.55	1.10	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.042	0.112	0.297	0.362	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	63	102	172	95	0	0	-1
N.S.	1	1.00	0.45	0.73	1.24	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.036	0.113	0.294	0.359	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	55	86	140	73	0	0	-1
N.S.	1	1.00	0.59	0.92	1.51	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.029	0.105	0.280	0.349	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	69	48	97	42	0	0	-1
N.S.	1	1.00	1.50	1.04	2.11	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.115	0.037	0.095	0.287	0.361	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	57	67	0	33	0	0	-1
N.S.	1	1.00	0.50	0.59	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.019	0.095	0.000	0.351	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	53	64	0	39	0	0	-1
N.S.	1	1.00	0.67	0.81	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.024	0.099	0.000	0.381	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	51	56	0	39	0	0	90
N.S.	1	1.00	1.09	1.19	0.00	0.83	0.00	0.00	1.91
time (sec)	N/A	0.119	0.033	0.094	0.000	0.339	0.000	0.000	1.555

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	71	169	0	139	0	0	-1
N.S.	1	1.00	0.38	0.91	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.039	0.092	0.000	0.349	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	99	241	0	190	0	0	-1
N.S.	1	1.00	0.36	0.87	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.157	0.056	0.097	0.000	0.366	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	79	113	0	117	0	0	-1
N.S.	1	1.00	0.34	0.48	0.00	0.50	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.041	0.114	0.000	0.334	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	71	97	0	95	0	0	-1
N.S.	1	1.00	0.38	0.52	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.036	0.108	0.000	0.327	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	63	81	0	73	0	0	-1
N.S.	1	1.00	0.45	0.58	0.00	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.030	0.110	0.000	0.336	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	53	63	0	43	0	0	-1
N.S.	1	1.00	0.56	0.66	0.00	0.45	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.023	0.115	0.000	0.329	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	45	0	22	0	0	-1
N.S.	1	1.00	0.59	0.65	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.014	0.092	0.000	0.319	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	51	0	22	0	0	-1
N.S.	1	1.00	1.00	1.38	0.00	0.59	0.00	0.00	-0.03
time (sec)	N/A	0.103	0.015	0.090	0.000	0.359	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	54	84	0	83	0	0	-1
N.S.	1	1.00	0.60	0.93	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.037	0.092	0.000	0.355	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	81	169	0	137	0	0	-1
N.S.	1	1.00	0.44	0.92	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.046	0.096	0.000	0.364	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	99	241	0	186	0	0	-1
N.S.	1	1.00	0.36	0.87	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.062	0.095	0.000	0.363	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	136	275	154	241	476	117	-1
N.S.	1	1.00	1.04	2.10	1.18	1.84	3.63	0.89	-0.01
time (sec)	N/A	0.099	0.093	0.201	0.465	0.374	8.343	0.442	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	117	202	130	180	338	85	-1
N.S.	1	1.00	1.08	1.87	1.20	1.67	3.13	0.79	-0.01
time (sec)	N/A	0.090	0.063	0.202	0.494	0.381	4.930	0.423	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	127	47	134	0	62	-1
N.S.	1	1.00	1.15	1.46	0.54	1.54	0.00	0.71	-0.01
time (sec)	N/A	0.081	0.038	0.187	0.464	0.351	0.000	0.416	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	100	73	39	151	0	0	-1
N.S.	1	1.00	1.67	1.22	0.65	2.52	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.044	0.138	0.466	0.343	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	63	115	60	47	0	148	33
N.S.	1	1.00	1.21	2.21	1.15	0.90	0.00	2.85	0.63
time (sec)	N/A	0.073	0.026	0.191	0.256	0.351	0.000	0.425	1.278

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	188	79	75	0	0	56
N.S.	1	1.00	1.05	2.51	1.05	1.00	0.00	0.00	0.75
time (sec)	N/A	0.079	0.034	0.186	0.258	0.392	0.000	0.000	1.400

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	268	98	124	0	0	134
N.S.	1	1.00	0.98	2.73	1.00	1.27	0.00	0.00	1.37
time (sec)	N/A	0.083	0.044	0.200	0.262	0.497	0.000	0.000	1.453

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	112	348	117	152	0	0	177
N.S.	1	1.00	0.93	2.88	0.97	1.26	0.00	0.00	1.46
time (sec)	N/A	0.093	0.051	0.195	0.274	0.645	0.000	0.000	1.494

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	71	102	204	95	0	0	-1
N.S.	1	1.00	0.38	0.54	1.08	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.040	0.110	0.286	0.338	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	63	102	172	95	0	0	-1
N.S.	1	1.00	0.44	0.72	1.21	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.034	0.112	0.281	0.337	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	55	86	140	73	0	0	-1
N.S.	1	1.00	0.58	0.91	1.47	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.028	0.112	0.278	0.354	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	69	48	97	42	0	0	-1
N.S.	1	1.00	1.47	1.02	2.06	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.038	0.094	0.288	0.342	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	56	67	0	33	0	0	-1
N.S.	1	1.00	0.50	0.60	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.020	0.097	0.000	0.337	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	52	62	0	38	0	0	-1
N.S.	1	1.00	0.68	0.81	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.025	0.095	0.000	0.340	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	56	0	39	0	0	58
N.S.	1	1.00	1.11	1.22	0.00	0.85	0.00	0.00	1.26
time (sec)	N/A	0.114	0.035	0.099	0.000	0.329	0.000	0.000	1.472

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	71	169	0	136	0	0	-1
N.S.	1	1.00	0.39	0.93	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.046	0.093	0.000	0.375	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	99	241	0	193	0	0	-1
N.S.	1	1.00	0.36	0.88	0.00	0.70	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.068	0.090	0.000	0.349	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	48	0	25	0	0	-1
N.S.	1	1.00	0.59	0.63	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.018	0.095	0.000	0.354	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	43	48	0	25	0	0	-1
N.S.	1	1.00	0.58	0.65	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.016	0.104	0.000	0.349	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	45	0	22	0	0	-1
N.S.	1	1.00	0.60	0.66	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.013	0.094	0.000	0.351	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	42	44	0	18	0	0	-1
N.S.	1	1.00	0.61	0.64	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.016	0.096	0.000	0.347	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	43	48	0	22	0	0	-1
N.S.	1	1.00	0.59	0.66	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.018	0.095	0.000	0.357	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	96	270	117	184	0	0	-1
N.S.	1	1.00	0.70	1.97	0.85	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.147	0.210	0.465	0.343	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	185	93	168	0	84	-1
N.S.	1	1.00	0.79	1.65	0.83	1.50	0.00	0.75	-0.01
time (sec)	N/A	0.258	0.123	0.211	0.478	0.355	0.000	0.401	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	163	70	150	0	72	-1
N.S.	1	1.00	0.93	1.92	0.82	1.76	0.00	0.85	-0.01
time (sec)	N/A	0.165	0.062	0.197	0.465	0.359	0.000	0.405	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	76	136	47	134	0	62	-1
N.S.	1	1.00	0.88	1.58	0.55	1.56	0.00	0.72	-0.01
time (sec)	N/A	0.077	0.039	0.195	0.475	0.390	0.000	0.404	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	97	129	90	191	0	95	-1
N.S.	1	1.00	1.29	1.72	1.20	2.55	0.00	1.27	-0.01
time (sec)	N/A	0.224	0.055	0.189	0.471	0.349	0.000	0.420	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	104	209	0	209	0	134	-1
N.S.	1	1.00	1.27	2.55	0.00	2.55	0.00	1.63	-0.01
time (sec)	N/A	0.239	0.060	0.214	0.000	0.378	0.000	0.428	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	76	239	0	148	0	200	-1
N.S.	1	1.00	0.97	3.06	0.00	1.90	0.00	2.56	-0.01
time (sec)	N/A	0.223	0.075	0.197	0.000	0.349	0.000	0.409	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	317	0	164	0	250	-1
N.S.	1	1.00	0.83	3.20	0.00	1.66	0.00	2.53	-0.01
time (sec)	N/A	0.245	0.076	0.211	0.000	0.370	0.000	0.411	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	95	341	0	180	0	324	-1
N.S.	1	1.00	0.73	2.62	0.00	1.38	0.00	2.49	-0.01
time (sec)	N/A	0.271	0.088	0.213	0.000	0.381	0.000	0.414	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	88	92	0	58	0	0	-1
N.S.	1	1.00	0.39	0.40	0.00	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.041	0.106	0.000	0.355	0.000	0.000	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	74	83	0	49	0	0	-1
N.S.	1	1.00	0.40	0.45	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.033	0.114	0.000	0.366	0.000	0.000	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	66	76	0	42	0	0	-1
N.S.	1	1.00	0.43	0.50	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.028	0.096	0.000	0.366	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	57	67	0	33	0	0	-1
N.S.	1	1.00	0.50	0.59	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.018	0.109	0.000	0.350	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	53	59	0	28	0	0	-1
N.S.	1	1.00	0.46	0.52	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.021	0.098	0.000	0.356	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	55	65	0	33	0	0	-1
N.S.	1	1.00	0.48	0.57	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.023	0.102	0.000	0.397	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	69	77	0	90	0	0	-1
N.S.	1	1.00	0.45	0.50	0.00	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.030	0.095	0.000	0.338	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	79	85	0	98	0	0	-1
N.S.	1	1.00	0.41	0.44	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.030	0.098	0.000	0.356	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	84	93	0	106	0	0	-1
N.S.	1	1.00	0.37	0.41	0.00	0.46	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.039	0.107	0.000	0.368	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	77	106	0	76	0	0	-1
N.S.	1	1.00	0.36	0.50	0.00	0.36	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.059	0.096	0.000	0.347	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	71	98	0	69	0	0	-1
N.S.	1	1.00	0.41	0.57	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.040	0.095	0.000	0.358	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	75	86	0	56	0	0	-1
N.S.	1	1.00	0.58	0.66	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.033	0.095	0.000	0.350	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	59	84	0	86	0	0	-1
N.S.	1	1.00	0.68	0.97	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.029	0.097	0.000	0.340	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	84	0	86	0	0	-1
N.S.	1	1.00	0.62	0.92	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.025	0.097	0.000	0.365	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	68	93	0	63	0	0	-1
N.S.	1	1.00	0.38	0.53	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.043	0.097	0.000	0.364	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	79	118	0	92	0	0	-1
N.S.	1	1.00	0.37	0.55	0.00	0.43	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.046	0.095	0.000	0.373	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	94	138	0	113	0	0	-1
N.S.	1	1.00	0.37	0.55	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	0.178	0.055	0.095	0.000	0.368	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	89	185	0	138	0	0	-1
N.S.	1	1.00	0.34	0.71	0.00	0.53	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.078	0.099	0.000	0.361	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	84	169	0	122	0	0	-1
N.S.	1	1.00	0.39	0.78	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.072	0.100	0.000	0.349	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	86	169	0	136	0	0	-1
N.S.	1	1.00	0.49	0.96	0.00	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.059	0.093	0.000	0.338	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	85	164	0	134	0	0	-1
N.S.	1	1.00	0.46	0.89	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.072	0.096	0.000	0.348	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	60	164	0	134	0	0	-1
N.S.	1	1.00	0.44	1.20	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.098	0.093	0.000	0.339	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	83	169	0	136	0	0	-1
N.S.	1	1.00	0.45	0.92	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.073	0.096	0.000	0.372	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	88	196	0	145	0	0	-1
N.S.	1	1.00	0.32	0.72	0.00	0.54	0.00	0.00	-0.00
time (sec)	N/A	0.185	0.069	0.097	0.000	0.371	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	99	225	0	174	0	0	-1
N.S.	1	1.00	0.32	0.73	0.00	0.57	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.066	0.095	0.000	0.368	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	48	0	25	0	0	-1
N.S.	1	1.00	0.59	0.63	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.021	0.102	0.000	0.342	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	43	48	0	25	0	0	-1
N.S.	1	1.00	0.58	0.65	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.018	0.096	0.000	0.345	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	45	0	22	0	0	-1
N.S.	1	1.00	0.59	0.65	0.00	0.32	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.015	0.093	0.000	0.346	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	46	0	20	0	0	-1
N.S.	1	1.00	0.63	0.66	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.014	0.099	0.000	0.382	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	48	0	22	0	0	-1
N.S.	1	1.00	0.61	0.67	0.00	0.31	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.018	0.110	0.000	0.381	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	96	261	117	184	0	0	-1
N.S.	1	1.00	0.70	1.91	0.85	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.091	0.200	0.475	0.359	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	176	93	168	0	84	-1
N.S.	1	1.00	0.79	1.57	0.83	1.50	0.00	0.75	-0.01
time (sec)	N/A	0.266	0.068	0.196	0.461	0.356	0.000	0.432	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	154	70	150	0	73	-1
N.S.	1	1.00	0.94	1.83	0.83	1.79	0.00	0.87	-0.01
time (sec)	N/A	0.171	0.057	0.189	0.457	0.352	0.000	0.411	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	127	47	134	0	62	-1
N.S.	1	1.00	1.15	1.46	0.54	1.54	0.00	0.71	-0.01
time (sec)	N/A	0.077	0.040	0.190	0.473	0.399	0.000	0.413	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	97	121	86	191	0	95	-1
N.S.	1	1.00	1.29	1.61	1.15	2.55	0.00	1.27	-0.01
time (sec)	N/A	0.233	0.058	0.192	0.485	0.343	0.000	0.418	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	104	200	0	210	0	134	-1
N.S.	1	1.00	1.27	2.44	0.00	2.56	0.00	1.63	-0.01
time (sec)	N/A	0.234	0.064	0.230	0.000	0.372	0.000	0.415	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	76	230	0	149	0	200	-1
N.S.	1	1.00	0.97	2.95	0.00	1.91	0.00	2.56	-0.01
time (sec)	N/A	0.227	0.080	0.217	0.000	0.400	0.000	0.425	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	308	0	165	0	250	-1
N.S.	1	1.00	0.81	3.05	0.00	1.63	0.00	2.48	-0.01
time (sec)	N/A	0.245	0.083	0.211	0.000	0.353	0.000	0.418	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	95	332	0	181	0	324	-1
N.S.	1	1.00	0.73	2.55	0.00	1.39	0.00	2.49	-0.01
time (sec)	N/A	0.271	0.094	0.209	0.000	0.361	0.000	0.416	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	87	92	0	58	0	0	-1
N.S.	1	1.00	0.38	0.41	0.00	0.26	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.044	0.093	0.000	0.366	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	72	83	0	49	0	0	-1
N.S.	1	1.00	0.39	0.45	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.031	0.114	0.000	0.350	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	65	76	0	42	0	0	-1
N.S.	1	1.00	0.43	0.50	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.043	0.096	0.000	0.340	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	56	67	0	33	0	0	-1
N.S.	1	1.00	0.50	0.60	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.033	0.094	0.000	0.333	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	50	61	0	26	0	0	-1
N.S.	1	1.00	0.45	0.54	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.025	0.103	0.000	0.420	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	56	64	0	33	0	0	-1
N.S.	1	1.00	0.49	0.56	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.023	0.096	0.000	0.389	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	68	77	0	88	0	0	-1
N.S.	1	1.00	0.45	0.51	0.00	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.027	0.092	0.000	0.351	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	78	85	0	98	0	0	-1
N.S.	1	1.00	0.40	0.44	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.030	0.099	0.000	0.361	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	83	93	0	104	0	0	-1
N.S.	1	1.00	0.37	0.41	0.00	0.46	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.037	0.099	0.000	0.335	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	74	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.038	0.068	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	129	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.147	0.113	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	62	54	74	0	0	93
N.S.	1	1.00	0.68	0.76	0.66	0.90	0.00	0.00	1.13
time (sec)	N/A	0.142	0.028	0.102	0.277	0.383	0.000	0.000	1.593

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	64	57	75	0	0	94
N.S.	1	1.00	0.70	0.77	0.69	0.90	0.00	0.00	1.13
time (sec)	N/A	0.151	0.024	0.097	0.276	0.372	0.000	0.000	1.452

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	110	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.203	0.112	0.000	0.000	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	75	0	0	0	0	0	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.040	0.063	0.000	0.000	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	267	0	0	0	0	0	-1
N.S.	1	1.00	3.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	1.522	0.061	0.000	0.000	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	-1
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	1.502	0.058	0.000	0.000	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	111	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.497	0.017	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	82	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.025	0.009	0.000	0.000	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	30	27	49	0	39
N.S.	1	1.00	1.00	1.00	1.67	1.50	2.72	0.00	2.17
time (sec)	N/A	0.023	0.038	0.092	0.273	0.351	0.625	0.000	1.449

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	55	0	79	0	0	106
N.S.	1	1.00	0.76	0.76	0.00	1.10	0.00	0.00	1.47
time (sec)	N/A	0.053	0.052	0.100	0.000	0.348	0.000	0.000	1.586

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	97	101	0	174	0	0	192
N.S.	1	1.00	0.76	0.80	0.00	1.37	0.00	0.00	1.51
time (sec)	N/A	0.088	0.076	0.102	0.000	0.342	0.000	0.000	1.729

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	152	167	0	309	0	0	314
N.S.	1	1.00	0.77	0.85	0.00	1.57	0.00	0.00	1.59
time (sec)	N/A	0.126	0.106	0.103	0.000	0.368	0.000	0.000	1.759

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	280	0	0	0	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	1.546	0.063	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	101	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.399	0.065	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	81	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.128	0.066	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.70
time (sec)	N/A	0.036	0.131	0.096	0.000	0.363	0.000	0.000	1.492

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	1.70
time (sec)	N/A	0.075	0.425	0.096	0.000	0.367	0.000	0.000	1.597

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	299	140	0	291	0	0	289
N.S.	1	1.00	1.80	0.84	0.00	1.75	0.00	0.00	1.74
time (sec)	N/A	0.120	1.031	0.093	0.000	0.366	0.000	0.000	1.737

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	260	218	0	453	0	0	441
N.S.	1	1.00	1.09	0.91	0.00	1.90	0.00	0.00	1.85
time (sec)	N/A	0.170	1.450	0.097	0.000	0.363	0.000	0.000	1.984

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	133	0	0	0	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.405	0.069	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	127	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.273	0.066	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	82	0	0	81
N.S.	1	1.00	0.93	1.07	0.00	1.78	0.00	0.00	1.76
time (sec)	N/A	0.061	0.127	0.093	0.000	0.337	0.000	0.000	1.435

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	49	0	80	0	0	78
N.S.	1	1.00	0.93	1.07	0.00	1.74	0.00	0.00	1.70
time (sec)	N/A	0.034	0.127	0.103	0.000	0.354	0.000	0.000	0.002

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	127	0	0	0	0	0	-1
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.589	0.063	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	201	0	0	0	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	1.430	0.065	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	110	93	0	175	0	0	175
N.S.	1	1.00	0.33	0.28	0.00	0.53	0.00	0.00	0.53
time (sec)	N/A	0.237	0.491	0.101	0.000	0.347	0.000	0.000	1.556

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	96	0	180	0	0	175
N.S.	1	1.00	1.07	0.94	0.00	1.76	0.00	0.00	1.72
time (sec)	N/A	0.133	0.511	0.093	0.000	0.347	0.000	0.000	1.556

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	108	86	0	171	0	0	176
N.S.	1	1.00	1.11	0.89	0.00	1.76	0.00	0.00	1.81
time (sec)	N/A	0.101	0.449	0.096	0.000	0.346	0.000	0.000	1.551

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	110	84	0	165	0	0	173
N.S.	1	1.00	1.08	0.82	0.00	1.62	0.00	0.00	1.70
time (sec)	N/A	0.072	0.154	0.097	0.000	0.350	0.000	0.000	0.002

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	944	944	220	0	0	0	0	0	-1
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	1.643	0.067	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	83	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.129	0.063	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	38	34	42	0	0	59
N.S.	1	1.00	0.71	0.75	0.67	0.82	0.00	0.00	1.16
time (sec)	N/A	0.078	0.053	0.115	0.274	0.366	0.000	0.000	1.336

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	40	36	44	0	0	59
N.S.	1	1.00	0.69	0.77	0.69	0.85	0.00	0.00	1.13
time (sec)	N/A	0.078	0.044	0.115	0.261	0.341	0.000	0.000	1.300

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.016	0.162	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.175	0.062	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	67	0	0	0	651	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	12.06	0.00	-0.02
time (sec)	N/A	0.062	0.014	0.115	0.000	0.000	13.826	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.155	0.068	0.000	0.000	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.184	0.059	0.000	0.000	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	73	0	0	0	651	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	11.84	0.00	-0.02
time (sec)	N/A	0.059	0.019	0.118	0.000	0.000	11.927	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	119	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.194	0.062	0.000	0.000	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	120	320	380	201	0	461	332
N.S.	1	1.00	0.35	0.94	1.11	0.59	0.00	1.35	0.97
time (sec)	N/A	0.170	0.165	0.094	0.469	0.369	0.000	0.437	1.419

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	104	272	302	179	0	355	258
N.S.	1	1.00	0.39	1.01	1.13	0.67	0.00	1.32	0.96
time (sec)	N/A	0.129	0.128	0.080	0.470	0.342	0.000	0.426	1.319

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	94	224	223	157	0	249	183
N.S.	1	1.00	0.48	1.15	1.15	0.81	0.00	1.28	0.94
time (sec)	N/A	0.089	0.082	0.069	0.462	0.381	0.000	0.437	1.334

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	53	163	117	104	0	130	84
N.S.	1	1.00	0.50	1.52	1.09	0.97	0.00	1.21	0.79
time (sec)	N/A	0.050	0.048	0.061	0.463	0.358	0.000	0.405	0.074

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	56	250	116	93	0	0	62
N.S.	1	1.00	0.54	2.40	1.12	0.89	0.00	0.00	0.60
time (sec)	N/A	0.051	0.081	0.135	0.268	0.335	0.000	0.000	0.078

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	83	530	160	134	0	0	128
N.S.	1	1.00	0.46	2.94	0.89	0.74	0.00	0.00	0.71
time (sec)	N/A	0.078	0.085	0.152	0.266	0.375	0.000	0.000	0.068

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	99	714	194	178	0	0	171
N.S.	1	1.00	0.39	2.81	0.76	0.70	0.00	0.00	0.67
time (sec)	N/A	0.113	0.090	0.177	0.257	0.374	0.000	0.000	0.097

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	115	898	230	274	0	0	210
N.S.	1	1.00	0.35	2.74	0.70	0.84	0.00	0.00	0.64
time (sec)	N/A	0.158	0.116	0.184	0.274	0.380	0.000	0.000	0.069

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	88	114	122	124	115	89
N.S.	1	1.00	1.00	0.69	0.90	0.96	0.98	0.91	0.70
time (sec)	N/A	0.118	0.064	0.336	0.258	0.349	0.409	0.399	0.103

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	64	81	89	88	82	65
N.S.	1	1.00	1.00	0.71	0.90	0.99	0.98	0.91	0.72
time (sec)	N/A	0.109	0.039	0.266	0.259	0.350	0.261	0.408	1.244

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	55	70	78	76	71	56
N.S.	1	1.00	1.00	0.72	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.105	0.042	0.230	0.262	0.339	0.185	0.383	1.218

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	31	35	43	39	36	32
N.S.	1	1.00	1.00	0.79	0.90	1.10	1.00	0.92	0.82
time (sec)	N/A	0.094	0.018	0.183	0.255	0.364	0.105	0.408	0.051

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	23
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.10
time (sec)	N/A	0.058	0.013	0.144	0.263	0.335	0.047	0.405	0.041

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	37	35	40	36	36	33
N.S.	1	1.00	0.78	1.03	0.97	1.11	1.00	1.00	0.92
time (sec)	N/A	0.104	0.022	0.104	0.257	0.338	0.070	0.392	0.049

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	60	69	93	73	57	68
N.S.	1	1.00	1.00	0.80	0.92	1.24	0.97	0.76	0.91
time (sec)	N/A	0.121	0.034	0.112	0.272	0.332	0.222	0.390	0.094

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	84	97	137	102	80	94
N.S.	1	1.00	0.75	0.76	0.88	1.25	0.93	0.73	0.85
time (sec)	N/A	0.134	0.051	0.122	0.259	0.351	0.545	0.398	1.281

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	98	108	145	233	156	96	142
N.S.	1	1.00	0.68	0.74	1.00	1.61	1.08	0.66	0.98
time (sec)	N/A	0.150	0.072	0.121	0.270	0.374	0.732	0.413	1.445

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	126	329	380	201	0	461	332
N.S.	1	1.00	0.37	0.96	1.11	0.59	0.00	1.34	0.97
time (sec)	N/A	0.164	0.133	0.092	0.466	0.374	0.000	0.438	1.378

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	110	281	302	179	0	355	258
N.S.	1	1.00	0.41	1.04	1.12	0.67	0.00	1.32	0.96
time (sec)	N/A	0.127	0.107	0.076	0.470	0.354	0.000	0.425	1.412

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	94	233	223	156	0	248	183
N.S.	1	1.00	0.48	1.19	1.14	0.80	0.00	1.27	0.94
time (sec)	N/A	0.090	0.083	0.065	0.462	0.359	0.000	0.428	0.137

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	235	118	106	0	130	84
N.S.	1	1.00	0.75	3.09	1.55	1.39	0.00	1.71	1.11
time (sec)	N/A	0.039	0.074	0.103	0.470	0.347	0.000	0.428	0.088

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	69	346	133	128	0	0	100
N.S.	1	1.00	0.48	2.40	0.92	0.89	0.00	0.00	0.69
time (sec)	N/A	0.067	0.198	0.112	0.252	0.338	0.000	0.000	1.345

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	78	438	153	170	0	63	121
N.S.	1	1.00	0.43	2.42	0.85	0.94	0.00	0.35	0.67
time (sec)	N/A	0.086	0.086	0.157	0.262	0.352	0.000	0.430	0.091

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	101	714	192	204	0	0	160
N.S.	1	1.00	0.40	2.80	0.75	0.80	0.00	0.00	0.63
time (sec)	N/A	0.121	0.233	0.170	0.263	0.347	0.000	0.000	1.406

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	117	766	226	248	0	0	203
N.S.	1	1.00	0.36	2.33	0.69	0.75	0.00	0.00	0.62
time (sec)	N/A	0.153	0.118	0.197	0.274	0.345	0.000	0.000	0.110

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	80	103	111	112	184	81
N.S.	1	1.00	1.00	0.69	0.89	0.96	0.97	1.59	0.70
time (sec)	N/A	0.111	0.032	0.415	0.278	0.377	0.381	0.402	0.086

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	71	92	100	100	160	72
N.S.	1	1.00	1.00	0.71	0.92	1.00	1.00	1.60	0.72
time (sec)	N/A	0.112	0.025	0.321	0.271	0.349	0.308	0.412	1.290

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	47	59	67	65	136	48
N.S.	1	1.00	1.00	0.75	0.94	1.06	1.03	2.16	0.76
time (sec)	N/A	0.106	0.018	0.310	0.268	0.578	0.166	0.393	0.058

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	40	46	56	53	112	43
N.S.	1	1.00	1.00	0.78	0.90	1.10	1.04	2.20	0.84
time (sec)	N/A	0.096	0.016	0.240	0.275	0.614	0.107	0.414	0.060

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	32	35	26	66	32
N.S.	1	1.00	1.00	0.88	0.97	1.06	0.79	2.00	0.97
time (sec)	N/A	0.064	0.017	0.186	0.270	0.403	0.150	0.414	0.067

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	49	49	64	41	74	48
N.S.	1	1.00	1.00	0.92	0.92	1.21	0.77	1.40	0.91
time (sec)	N/A	0.112	0.026	0.195	0.268	0.425	0.128	0.410	1.256

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	61	75	100	83	93	71
N.S.	1	1.00	0.89	0.86	1.06	1.41	1.17	1.31	1.00
time (sec)	N/A	0.115	0.029	0.145	0.271	0.460	0.192	0.421	1.267

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	84	107	163	114	130	104
N.S.	1	1.00	0.80	0.76	0.96	1.47	1.03	1.17	0.94
time (sec)	N/A	0.136	0.049	0.154	0.278	0.713	0.372	0.393	0.110

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	98	108	135	207	144	170	131
N.S.	1	1.00	0.67	0.74	0.92	1.42	0.99	1.16	0.90
time (sec)	N/A	0.149	0.069	0.170	0.275	0.520	0.539	0.401	0.148

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	120	320	380	201	0	524	332
N.S.	1	1.00	0.35	0.93	1.11	0.59	0.00	1.53	0.97
time (sec)	N/A	0.166	0.161	0.090	0.480	0.401	0.000	0.425	1.368

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	110	272	302	179	0	394	258
N.S.	1	1.00	0.41	1.01	1.12	0.67	0.00	1.46	0.96
time (sec)	N/A	0.123	0.230	0.085	0.479	0.464	0.000	0.432	0.109

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	94	224	223	156	0	264	183
N.S.	1	1.00	0.48	1.15	1.14	0.80	0.00	1.35	0.94
time (sec)	N/A	0.089	0.157	0.064	0.482	0.437	0.000	0.437	1.299

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	166	117	107	0	121	84
N.S.	1	1.00	0.51	1.54	1.08	0.99	0.00	1.12	0.78
time (sec)	N/A	0.051	0.050	0.059	0.487	0.355	0.000	0.426	1.278

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	57	250	121	67	0	0	86
N.S.	1	1.00	0.54	2.38	1.15	0.64	0.00	0.00	0.82
time (sec)	N/A	0.052	0.087	0.141	0.266	0.370	0.000	0.000	0.054

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	85	530	163	119	0	0	137
N.S.	1	1.00	0.47	2.96	0.91	0.66	0.00	0.00	0.77
time (sec)	N/A	0.083	0.075	0.165	0.268	0.465	0.000	0.000	1.296

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	101	714	197	161	0	0	178
N.S.	1	1.00	0.40	2.80	0.77	0.63	0.00	0.00	0.70
time (sec)	N/A	0.114	0.092	0.194	0.273	0.356	0.000	0.000	0.062

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	117	898	231	205	0	0	217
N.S.	1	1.00	0.36	2.73	0.70	0.62	0.00	0.00	0.66
time (sec)	N/A	0.153	0.114	0.189	0.266	0.514	0.000	0.000	1.282

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	64	81	89	88	82	67
N.S.	1	1.00	1.00	0.71	0.90	0.99	0.98	0.91	0.74
time (sec)	N/A	0.105	0.025	0.313	0.267	0.373	0.249	0.403	1.318

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	55	70	78	76	71	56
N.S.	1	1.00	1.00	0.72	0.92	1.03	1.00	0.93	0.74
time (sec)	N/A	0.109	0.019	0.258	0.257	0.485	0.181	0.407	1.251

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	37	43	39	38	35
N.S.	1	1.00	1.00	0.80	0.92	1.08	0.98	0.95	0.88
time (sec)	N/A	0.095	0.015	0.220	0.281	0.482	0.083	0.391	0.050

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	20	22	25
N.S.	1	1.00	1.00	1.05	1.00	1.24	0.95	1.05	1.19
time (sec)	N/A	0.058	0.014	0.161	0.262	0.491	0.045	0.389	1.227

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	37	34	38	36	36	33
N.S.	1	1.00	0.80	1.06	0.97	1.09	1.03	1.03	0.94
time (sec)	N/A	0.107	0.023	0.116	0.269	0.350	0.072	0.400	0.052

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	60	69	92	75	57	68
N.S.	1	1.00	0.96	0.82	0.95	1.26	1.03	0.78	0.93
time (sec)	N/A	0.117	0.035	0.120	0.285	0.391	0.225	0.398	1.307

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	104	84	97	137	102	80	94
N.S.	1	1.00	0.96	0.78	0.90	1.27	0.94	0.74	0.87
time (sec)	N/A	0.133	0.050	0.121	0.263	0.345	0.363	0.417	0.120

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	124	108	145	233	156	96	142
N.S.	1	1.00	0.87	0.76	1.01	1.63	1.09	0.67	0.99
time (sec)	N/A	0.149	0.069	0.123	0.266	0.407	0.615	0.398	0.153

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	126	329	379	201	0	525	332
N.S.	1	1.00	0.37	0.96	1.10	0.59	0.00	1.53	0.97
time (sec)	N/A	0.166	0.137	0.082	0.484	0.374	0.000	0.437	0.173

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	110	281	301	179	0	395	258
N.S.	1	1.00	0.41	1.04	1.12	0.67	0.00	1.47	0.96
time (sec)	N/A	0.126	0.109	0.078	0.478	0.382	0.000	0.443	1.263

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	94	233	224	156	0	264	183
N.S.	1	1.00	0.48	1.19	1.15	0.80	0.00	1.35	0.94
time (sec)	N/A	0.089	0.091	0.068	0.465	0.384	0.000	0.426	0.093

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	57	234	118	103	0	122	84
N.S.	1	1.00	0.75	3.08	1.55	1.36	0.00	1.61	1.11
time (sec)	N/A	0.038	0.057	0.111	0.484	0.348	0.000	0.427	0.061

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	69	346	140	96	0	59	114
N.S.	1	1.00	0.48	2.40	0.97	0.67	0.00	0.41	0.79
time (sec)	N/A	0.067	0.173	0.113	0.265	0.408	0.000	0.411	0.069

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	78	438	161	135	0	59	141
N.S.	1	1.00	0.43	2.42	0.89	0.75	0.00	0.33	0.78
time (sec)	N/A	0.085	0.158	0.154	0.270	0.361	0.000	0.435	0.046

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	101	714	199	179	0	0	183
N.S.	1	1.00	0.40	2.82	0.79	0.71	0.00	0.00	0.72
time (sec)	N/A	0.115	0.091	0.161	0.266	0.434	0.000	0.000	0.054

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	117	766	231	275	0	0	224
N.S.	1	1.00	0.36	2.34	0.71	0.84	0.00	0.00	0.69
time (sec)	N/A	0.151	0.118	0.192	0.258	0.439	0.000	0.000	0.063

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	96	112	0	96	0	0	-1
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.104	0.052	0.064	0.000	0.379	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	75	96	0	74	0	0	-1
N.S.	1	1.00	0.32	0.41	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.099	0.040	0.036	0.000	0.353	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	64	80	0	42	0	0	-1
N.S.	1	1.00	0.44	0.55	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.033	0.037	0.000	0.366	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	50	0	17	0	0	-1
N.S.	1	1.00	0.58	0.75	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.016	0.030	0.000	0.336	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	57	0	24	0	0	-1
N.S.	1	1.00	0.61	0.79	0.00	0.33	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.023	0.033	0.000	0.359	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	68	102	0	68	0	0	-1
N.S.	1	1.00	0.39	0.59	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.051	0.036	0.000	0.405	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	86	175	0	137	0	0	-1
N.S.	1	1.00	0.33	0.67	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.115	0.079	0.041	0.000	0.362	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	104	247	0	205	0	0	-1
N.S.	1	1.00	0.29	0.69	0.00	0.57	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.111	0.043	0.000	0.406	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	150	795	0	438	1059	561	-1
N.S.	1	1.00	0.40	2.14	0.00	1.18	2.85	1.51	-0.00
time (sec)	N/A	0.358	0.115	0.165	0.000	0.407	25.015	24.146	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	134	625	0	394	500	416	-1
N.S.	1	1.00	0.46	2.13	0.00	1.34	1.70	1.41	-0.00
time (sec)	N/A	0.318	0.090	0.135	0.000	0.413	13.401	1.568	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	115	454	0	317	376	266	-1
N.S.	1	1.00	0.54	2.13	0.00	1.49	1.77	1.25	-0.00
time (sec)	N/A	0.293	0.070	0.125	0.000	0.414	8.556	0.521	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	-1
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.058	0.100	0.000	0.371	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	68	177	0	216	0	0	-1
N.S.	1	1.00	0.61	1.59	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.050	0.121	0.000	0.379	0.000	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	326	0	280	0	0	-1
N.S.	1	1.00	0.77	2.65	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.057	0.157	0.000	0.370	0.000	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	105	462	0	352	0	0	-1
N.S.	1	1.00	0.52	2.28	0.00	1.73	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.067	0.150	0.000	0.363	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	133	572	0	496	0	0	-1
N.S.	1	1.00	0.47	2.02	0.00	1.75	0.00	0.00	-0.00
time (sec)	N/A	0.325	0.084	0.185	0.000	0.442	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	96	0	0	-1
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.059	0.037	0.000	0.363	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	94	112	0	96	0	0	-1
N.S.	1	1.00	0.29	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.108	0.055	0.042	0.000	0.390	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	87	96	0	72	0	0	-1
N.S.	1	1.00	0.37	0.41	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.095	0.038	0.037	0.000	0.381	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	59	69	0	44	0	0	-1
N.S.	1	1.00	0.40	0.47	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.064	0.034	0.000	0.344	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	50	65	0	27	0	0	-1
N.S.	1	1.00	0.46	0.60	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.046	0.036	0.000	0.373	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	56	85	0	49	0	0	-1
N.S.	1	1.00	0.49	0.74	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.066	0.035	0.000	0.352	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	64	102	0	81	0	0	-1
N.S.	1	1.00	0.37	0.60	0.00	0.47	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.105	0.035	0.000	0.474	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	86	175	0	138	0	0	-1
N.S.	1	1.00	0.32	0.66	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.118	0.144	0.043	0.000	0.436	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	140	247	0	207	0	0	-1
N.S.	1	1.00	0.39	0.69	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.138	0.044	0.000	0.367	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	96	0	0	-1
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.112	0.051	0.039	0.000	0.474	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	77	96	0	74	0	0	-1
N.S.	1	1.00	0.32	0.40	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.100	0.039	0.036	0.000	0.446	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	65	80	0	44	0	0	-1
N.S.	1	1.00	0.44	0.54	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.032	0.034	0.000	0.347	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	52	0	19	0	0	-1
N.S.	1	1.00	0.60	0.76	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.018	0.031	0.000	0.341	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	45	59	0	26	0	0	-1
N.S.	1	1.00	0.62	0.82	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.024	0.030	0.000	0.376	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	65	103	0	66	0	0	-1
N.S.	1	1.00	0.38	0.60	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.045	0.038	0.000	0.354	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	85	175	0	135	0	0	-1
N.S.	1	1.00	0.32	0.67	0.00	0.51	0.00	0.00	-0.00
time (sec)	N/A	0.117	0.082	0.039	0.000	0.369	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	104	247	0	201	0	0	-1
N.S.	1	1.00	0.29	0.69	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.104	0.043	0.000	0.388	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	150	795	0	438	1059	561	-1
N.S.	1	1.00	0.40	2.12	0.00	1.17	2.82	1.50	-0.00
time (sec)	N/A	0.352	0.115	0.151	0.000	0.369	25.205	24.189	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	134	625	0	394	500	416	-1
N.S.	1	1.00	0.46	2.13	0.00	1.34	1.71	1.42	-0.00
time (sec)	N/A	0.324	0.090	0.136	0.000	0.362	12.952	1.687	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	115	455	0	316	376	266	-1
N.S.	1	1.00	0.54	2.14	0.00	1.48	1.77	1.25	-0.00
time (sec)	N/A	0.296	0.077	0.122	0.000	0.357	9.496	0.546	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	-1
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.241	0.057	0.102	0.000	0.366	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	68	179	0	212	0	0	-1
N.S.	1	1.00	0.61	1.60	0.00	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.049	0.123	0.000	0.364	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	95	326	0	279	0	0	-1
N.S.	1	1.00	0.77	2.63	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.057	0.136	0.000	0.350	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	105	462	0	351	0	0	-1
N.S.	1	1.00	0.54	2.37	0.00	1.80	0.00	0.00	-0.01
time (sec)	N/A	0.308	0.066	0.148	0.000	0.384	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	131	572	0	495	0	0	-1
N.S.	1	1.00	0.49	2.12	0.00	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.084	0.178	0.000	0.400	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	97	112	0	96	0	0	-1
N.S.	1	1.00	0.30	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.057	0.037	0.000	0.340	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	94	112	0	96	0	0	-1
N.S.	1	1.00	0.29	0.35	0.00	0.30	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.055	0.043	0.000	0.347	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	81	96	0	74	0	0	-1
N.S.	1	1.00	0.34	0.41	0.00	0.31	0.00	0.00	-0.00
time (sec)	N/A	0.100	0.045	0.036	0.000	0.345	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	59	69	0	42	0	0	-1
N.S.	1	1.00	0.40	0.47	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.064	0.034	0.000	0.343	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	47	67	0	25	0	0	-1
N.S.	1	1.00	0.44	0.63	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.048	0.033	0.000	0.411	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	54	87	0	47	0	0	-1
N.S.	1	1.00	0.48	0.77	0.00	0.42	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.064	0.034	0.000	0.392	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	63	102	0	81	0	0	-1
N.S.	1	1.00	0.38	0.61	0.00	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.054	0.035	0.000	0.378	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	85	175	0	137	0	0	-1
N.S.	1	1.00	0.32	0.66	0.00	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.073	0.039	0.000	0.364	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	105	247	0	208	0	0	-1
N.S.	1	1.00	0.29	0.69	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.119	0.043	0.000	0.362	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	63	44	72	0	0	-1
N.S.	1	1.00	0.66	0.79	0.55	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.026	0.026	0.303	0.354	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	24	0	0	46
N.S.	1	1.00	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.178	0.019	0.033	0.000	0.402	0.000	0.000	1.442

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	43	52	0	21	0	0	45
N.S.	1	1.00	0.61	0.73	0.00	0.30	0.00	0.00	0.63
time (sec)	N/A	0.121	0.018	0.028	0.000	0.358	0.000	0.000	1.400

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	50	0	17	0	0	-1
N.S.	1	1.00	0.58	0.75	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.016	0.030	0.000	0.331	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	50	0	21	0	0	-1
N.S.	1	1.00	0.61	0.71	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.018	0.033	0.000	0.357	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	53	0	21	0	0	63
N.S.	1	1.00	1.02	1.15	0.00	0.46	0.00	0.00	1.37
time (sec)	N/A	0.173	0.017	0.033	0.000	0.353	0.000	0.000	1.402

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	93	196	0	222	0	128	-1
N.S.	1	1.00	0.58	1.22	0.00	1.39	0.00	0.80	-0.01
time (sec)	N/A	0.380	0.064	0.116	0.000	0.373	0.000	0.424	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	84	174	0	204	0	116	-1
N.S.	1	1.00	0.68	1.41	0.00	1.66	0.00	0.94	-0.01
time (sec)	N/A	0.338	0.051	0.123	0.000	0.348	0.000	0.407	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	77	147	0	188	0	106	-1
N.S.	1	1.00	0.79	1.50	0.00	1.92	0.00	1.08	-0.01
time (sec)	N/A	0.226	0.043	0.114	0.000	0.361	0.000	0.414	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	-1
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.052	0.104	0.000	0.373	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	306	0	252	0	127	-1
N.S.	1	1.00	0.70	2.62	0.00	2.15	0.00	1.09	-0.01
time (sec)	N/A	0.381	0.059	0.120	0.000	0.360	0.000	0.493	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	78	347	0	177	0	194	-1
N.S.	1	1.00	0.70	3.13	0.00	1.59	0.00	1.75	-0.01
time (sec)	N/A	0.379	0.049	0.116	0.000	0.355	0.000	0.495	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	86	378	0	201	0	231	-1
N.S.	1	1.00	0.63	2.76	0.00	1.47	0.00	1.69	-0.01
time (sec)	N/A	0.372	0.061	0.119	0.000	0.358	0.000	0.729	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	217	0	316	-1
N.S.	1	1.00	0.60	2.63	0.00	1.39	0.00	2.03	-0.01
time (sec)	N/A	0.404	0.061	0.125	0.000	0.366	0.000	1.626	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	233	0	362	-1
N.S.	1	1.00	0.56	2.47	0.00	1.29	0.00	2.00	-0.01
time (sec)	N/A	0.417	0.073	0.130	0.000	0.369	0.000	1.972	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	71	89	0	48	0	0	-1
N.S.	1	1.00	0.38	0.48	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.036	0.064	0.000	0.351	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	63	82	0	41	0	0	-1
N.S.	1	1.00	0.41	0.54	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.031	0.033	0.000	0.368	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	54	73	0	32	0	0	-1
N.S.	1	1.00	0.48	0.65	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.030	0.035	0.000	0.345	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	50	65	0	27	0	0	-1
N.S.	1	1.00	0.46	0.60	0.00	0.25	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.043	0.034	0.000	0.361	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	52	67	0	32	0	0	-1
N.S.	1	1.00	0.48	0.62	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.057	0.037	0.000	0.372	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	66	82	0	85	0	0	-1
N.S.	1	1.00	0.45	0.56	0.00	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.068	0.036	0.000	0.336	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	76	90	0	93	0	0	-1
N.S.	1	1.00	0.40	0.48	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.073	0.037	0.000	0.347	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	81	98	0	101	0	0	-1
N.S.	1	1.00	0.36	0.44	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.084	0.039	0.000	0.341	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	90	106	0	109	0	0	-1
N.S.	1	1.00	0.34	0.40	0.00	0.41	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.101	0.038	0.000	0.357	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	50	65	46	73	0	0	-1
N.S.	1	1.00	0.62	0.80	0.57	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.081	0.023	0.299	0.333	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	53	0	24	0	0	46
N.S.	1	1.00	0.59	0.70	0.00	0.32	0.00	0.00	0.61
time (sec)	N/A	0.191	0.033	0.030	0.000	0.333	0.000	0.000	1.363

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	42	52	0	21	0	0	45
N.S.	1	1.00	0.58	0.72	0.00	0.29	0.00	0.00	0.62
time (sec)	N/A	0.128	0.019	0.033	0.000	0.345	0.000	0.000	1.319

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	41	52	0	19	0	0	-1
N.S.	1	1.00	0.60	0.76	0.00	0.28	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.016	0.028	0.000	0.346	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	41	50	0	21	0	0	-1
N.S.	1	1.00	0.59	0.72	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.018	0.033	0.000	0.342	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	53	0	21	0	0	63
N.S.	1	1.00	1.00	1.13	0.00	0.45	0.00	0.00	1.34
time (sec)	N/A	0.178	0.019	0.030	0.000	0.369	0.000	0.000	1.362

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	93	196	0	222	0	128	-1
N.S.	1	1.00	0.57	1.20	0.00	1.36	0.00	0.79	-0.01
time (sec)	N/A	0.384	0.064	0.112	0.000	0.354	0.000	0.421	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	84	173	0	204	0	117	-1
N.S.	1	1.00	0.68	1.40	0.00	1.65	0.00	0.94	-0.01
time (sec)	N/A	0.336	0.051	0.111	0.000	0.382	0.000	0.401	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	147	0	188	0	106	-1
N.S.	1	1.00	1.01	1.48	0.00	1.90	0.00	1.07	-0.01
time (sec)	N/A	0.220	0.042	0.112	0.000	0.424	0.000	0.417	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	197	0	267	0	0	-1
N.S.	1	1.00	0.69	1.70	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.055	0.096	0.000	0.347	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	306	0	252	0	127	-1
N.S.	1	1.00	0.70	2.62	0.00	2.15	0.00	1.09	-0.01
time (sec)	N/A	0.364	0.053	0.119	0.000	0.350	0.000	0.478	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	78	348	0	176	0	194	-1
N.S.	1	1.00	0.70	3.11	0.00	1.57	0.00	1.73	-0.01
time (sec)	N/A	0.374	0.045	0.115	0.000	0.382	0.000	0.485	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	86	378	0	200	0	231	-1
N.S.	1	1.00	0.61	2.70	0.00	1.43	0.00	1.65	-0.01
time (sec)	N/A	0.380	0.050	0.118	0.000	0.370	0.000	0.726	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	94	410	0	216	0	316	-1
N.S.	1	1.00	0.60	2.63	0.00	1.38	0.00	2.03	-0.01
time (sec)	N/A	0.386	0.058	0.118	0.000	0.346	0.000	1.581	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	447	0	232	0	362	-1
N.S.	1	1.00	0.56	2.47	0.00	1.28	0.00	2.00	-0.01
time (sec)	N/A	0.402	0.061	0.141	0.000	0.369	0.000	2.042	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	69	89	0	48	0	0	-1
N.S.	1	1.00	0.37	0.48	0.00	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.036	0.065	0.000	0.388	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	62	82	0	41	0	0	-1
N.S.	1	1.00	0.41	0.54	0.00	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.030	0.035	0.000	0.374	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	53	73	0	32	0	0	-1
N.S.	1	1.00	0.47	0.65	0.00	0.29	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.021	0.035	0.000	0.354	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	47	67	0	25	0	0	-1
N.S.	1	1.00	0.44	0.63	0.00	0.23	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.020	0.030	0.000	0.348	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	53	66	0	32	0	0	-1
N.S.	1	1.00	0.49	0.61	0.00	0.30	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.026	0.036	0.000	0.355	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	65	82	0	83	0	0	-1
N.S.	1	1.00	0.45	0.56	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.033	0.036	0.000	0.350	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	75	90	0	91	0	0	-1
N.S.	1	1.00	0.40	0.48	0.00	0.49	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.035	0.038	0.000	0.364	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	80	98	0	99	0	0	-1
N.S.	1	1.00	0.36	0.44	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.039	0.037	0.000	0.356	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	89	106	0	107	0	0	-1
N.S.	1	1.00	0.34	0.40	0.00	0.41	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.053	0.037	0.000	0.351	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	123	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.170	0.016	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	94	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.190	0.020	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	180	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	0.888	0.014	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	146	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.146	0.339	0.021	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	112	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.263	0.018	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.557	0.014	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.338	0.015	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.348	0.014	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [498] had the largest ratio of [27]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.00	10	0.600
2	A	7	6	1.00	10	0.600
3	A	6	6	1.00	8	0.750
4	A	5	5	1.00	6	0.833
5	A	6	6	1.00	10	0.600
6	A	3	3	1.00	10	0.300
7	A	3	3	1.00	10	0.300
8	A	7	5	1.00	10	0.500
9	A	5	4	1.00	10	0.400
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	10	0.300
13	A	4	3	1.00	8	0.375
14	A	4	3	1.00	12	0.250
15	A	4	3	1.00	12	0.250
16	A	4	3	1.00	12	0.250
17	A	4	3	1.00	12	0.250
18	A	14	9	1.00	12	0.750
19	A	12	8	1.00	10	0.800
20	A	8	7	1.00	8	0.875
21	A	8	7	1.00	12	0.583
22	A	5	5	1.00	12	0.417
23	A	9	7	1.00	12	0.583
24	A	10	9	1.00	12	0.750
25	A	4	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	12	0.250
27	A	4	3	1.00	10	0.300
28	A	4	3	1.00	8	0.375
29	A	4	3	1.00	12	0.250
30	A	4	3	1.00	12	0.250
31	A	4	3	1.00	12	0.250
32	A	4	3	1.00	12	0.250
33	A	8	6	1.00	12	0.500
34	A	7	6	1.00	12	0.500
35	A	6	6	1.00	10	0.600
36	A	5	5	1.00	8	0.625
37	A	6	6	1.00	12	0.500
38	A	3	3	1.00	12	0.250
39	A	3	3	1.00	12	0.250
40	A	7	5	1.00	12	0.417
41	A	5	4	1.00	12	0.333
42	A	4	3	1.00	12	0.250
43	A	4	3	1.00	12	0.250
44	A	4	3	1.00	10	0.300
45	A	4	3	1.00	8	0.375
46	A	4	3	1.00	12	0.250
47	A	4	3	1.00	12	0.250
48	A	4	3	1.00	12	0.250
49	A	4	3	1.00	12	0.250
50	A	19	9	1.00	12	0.750
51	A	14	9	1.00	12	0.750
52	A	12	8	1.00	10	0.800
53	A	8	7	1.00	8	0.875
54	A	8	7	1.00	12	0.583
55	A	5	5	1.00	12	0.417
56	A	9	7	1.00	12	0.583
57	A	10	9	1.00	12	0.750
58	A	14	11	1.00	12	0.917
59	A	11	8	1.00	14	0.571
60	A	10	8	1.00	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	9	8	1.00	14	0.571
62	A	7	7	1.00	12	0.583
63	A	6	6	1.00	10	0.600
64	A	17	14	1.00	14	1.000
65	A	13	10	1.00	14	0.714
66	A	14	11	1.00	14	0.786
67	A	15	12	1.00	14	0.857
68	A	11	8	1.00	14	0.571
69	A	10	8	1.00	14	0.571
70	A	9	8	1.00	14	0.571
71	A	7	7	1.00	12	0.583
72	A	6	6	1.00	10	0.600
73	A	17	14	1.00	14	1.000
74	A	13	10	1.00	14	0.714
75	A	14	11	1.00	14	0.786
76	A	15	12	1.00	14	0.857
77	A	12	9	1.00	14	0.643
78	A	11	9	1.00	14	0.643
79	A	10	9	1.00	14	0.643
80	A	8	7	1.00	12	0.583
81	A	7	6	1.00	10	0.600
82	A	19	16	1.00	14	1.143
83	A	14	11	1.00	14	0.786
84	A	15	11	1.00	14	0.786
85	A	16	12	1.00	14	0.857
86	A	11	8	1.00	14	0.571
87	A	10	8	1.00	14	0.571
88	A	9	8	1.00	14	0.571
89	A	7	7	1.00	12	0.583
90	A	6	6	1.00	10	0.600
91	A	17	14	1.00	14	1.000
92	A	13	10	1.00	14	0.714
93	A	14	11	1.00	14	0.786
94	A	15	12	1.00	14	0.857
95	A	11	8	1.00	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	10	8	1.00	14	0.571
97	A	9	8	1.00	14	0.571
98	A	7	7	1.00	12	0.583
99	A	6	6	1.00	10	0.600
100	A	17	14	1.00	14	1.000
101	A	13	10	1.00	14	0.714
102	A	14	11	1.00	14	0.786
103	A	15	12	1.00	14	0.857
104	A	12	9	1.00	14	0.643
105	A	11	9	1.00	14	0.643
106	A	10	9	1.00	14	0.643
107	A	8	7	1.00	12	0.583
108	A	7	6	1.00	10	0.600
109	A	19	16	1.00	14	1.143
110	A	14	11	1.00	14	0.786
111	A	15	11	1.00	14	0.786
112	A	16	12	1.00	14	0.857
113	A	16	11	1.00	12	0.917
114	A	14	10	1.00	10	1.000
115	A	13	9	1.00	8	1.125
116	A	25	13	1.00	12	1.083
117	A	14	10	1.00	12	0.833
118	A	15	11	1.00	12	0.917
119	A	16	12	1.00	12	1.000
120	A	6	5	1.00	12	0.417
121	A	4	4	1.00	10	0.400
122	A	3	3	1.00	8	0.375
123	A	4	4	1.00	12	0.333
124	A	3	3	1.00	12	0.250
125	A	4	4	1.00	12	0.333
126	A	19	15	1.00	14	1.071
127	A	17	14	1.00	12	1.167
128	A	16	13	1.00	10	1.300
129	A	39	20	1.00	14	1.429
130	A	25	11	1.00	14	0.786

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	26	12	1.00	14	0.857
132	A	5	5	1.00	12	0.417
133	A	9	5	1.00	12	0.417
134	A	4	4	1.00	12	0.333
135	A	4	3	1.00	10	0.300
136	A	4	3	1.00	12	0.250
137	A	4	4	1.00	12	0.333
138	A	9	5	1.00	12	0.417
139	A	2	2	1.00	14	0.143
140	A	2	2	1.00	14	0.143
141	A	2	2	1.00	14	0.143
142	A	2	2	1.00	14	0.143
143	A	2	2	1.00	14	0.143
144	A	2	2	1.00	14	0.143
145	A	2	2	1.00	12	0.167
146	A	2	2	1.00	12	0.167
147	A	2	2	1.00	14	0.143
148	A	2	2	1.00	12	0.167
149	A	5	5	1.00	12	0.417
150	A	3	3	1.00	10	0.300
151	A	2	2	1.00	8	0.250
152	A	4	4	1.00	12	0.333
153	A	2	2	1.00	12	0.167
154	A	3	3	1.00	12	0.250
155	A	4	4	1.00	12	0.333
156	A	4	4	1.00	12	0.333
157	A	4	4	1.00	16	0.250
158	A	9	8	1.00	16	0.500
159	A	8	8	1.00	16	0.500
160	A	7	7	1.00	16	0.438
161	A	6	6	1.00	14	0.429
162	A	7	7	1.00	16	0.438
163	A	3	3	1.00	16	0.188
164	A	4	4	1.00	16	0.250
165	A	6	6	1.00	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	8	6	1.00	16	0.375
167	A	5	4	1.00	18	0.222
168	A	4	3	1.00	18	0.167
169	A	4	3	1.00	18	0.167
170	A	4	3	1.00	18	0.167
171	A	4	3	1.00	18	0.167
172	C	1	1	1.86	16	0.062
173	A	4	3	1.00	18	0.167
174	A	3	3	1.00	18	0.167
175	A	4	3	1.00	18	0.167
176	A	4	3	1.00	18	0.167
177	A	5	4	1.00	18	0.222
178	A	8	7	1.00	18	0.389
179	A	7	6	1.00	18	0.333
180	A	8	8	1.00	18	0.444
181	A	8	8	1.00	16	0.500
182	A	9	8	1.00	18	0.444
183	A	3	3	1.00	18	0.167
184	A	4	4	1.00	18	0.222
185	A	6	6	1.00	18	0.333
186	A	7	6	1.00	18	0.333
187	A	5	4	1.00	18	0.222
188	A	4	3	1.00	18	0.167
189	A	5	4	1.00	18	0.222
190	A	4	3	1.00	18	0.167
191	A	3	3	1.00	18	0.167
192	A	4	3	1.00	16	0.188
193	A	4	3	1.00	18	0.167
194	A	3	3	1.00	18	0.167
195	A	4	3	1.00	18	0.167
196	A	4	3	1.00	18	0.167
197	A	3	3	1.00	18	0.167
198	A	9	7	1.00	18	0.389
199	A	8	7	1.00	18	0.389
200	A	7	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	5	1.00	18	0.278
202	A	3	3	1.00	18	0.167
203	A	4	4	1.00	18	0.222
204	A	6	6	1.00	18	0.333
205	A	7	6	1.00	18	0.333
206	A	4	4	1.00	18	0.222
207	A	4	3	1.00	18	0.167
208	A	4	3	1.00	18	0.167
209	A	4	3	1.00	18	0.167
210	A	4	3	1.00	16	0.188
211	A	3	3	1.00	18	0.167
212	A	4	4	1.00	18	0.222
213	A	5	4	1.00	18	0.222
214	A	5	4	1.00	18	0.222
215	A	5	4	1.00	18	0.222
216	A	3	3	1.00	18	0.167
217	A	10	8	1.00	18	0.444
218	A	9	8	1.00	18	0.444
219	A	8	8	1.00	16	0.500
220	A	6	6	1.00	18	0.333
221	A	3	3	1.00	18	0.167
222	A	3	3	1.00	18	0.167
223	A	5	5	1.00	18	0.278
224	A	6	5	1.00	18	0.278
225	A	7	5	1.00	18	0.278
226	A	7	6	1.00	18	0.333
227	A	6	6	1.00	18	0.333
228	A	5	5	1.19	18	0.278
229	A	4	4	1.16	18	0.222
230	A	1	1	1.00	18	0.056
231	A	5	5	1.00	18	0.278
232	A	5	5	1.00	18	0.278
233	A	6	5	1.00	18	0.278
234	A	7	5	1.00	18	0.278
235	A	5	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	5	4	1.00	20	0.200
237	A	5	4	1.00	20	0.200
238	A	5	4	1.00	20	0.200
239	A	5	4	1.00	20	0.200
240	A	5	4	1.00	20	0.200
241	A	5	4	1.00	20	0.200
242	A	5	4	1.00	20	0.200
243	A	6	6	1.00	20	0.300
244	A	5	5	1.00	20	0.250
245	A	4	4	1.00	20	0.200
246	A	1	1	1.00	20	0.050
247	A	6	5	1.00	20	0.250
248	A	6	5	1.00	20	0.250
249	A	6	5	1.00	20	0.250
250	A	7	5	1.00	20	0.250
251	A	8	5	1.00	20	0.250
252	A	8	6	1.60	20	0.300
253	A	7	6	1.58	20	0.300
254	A	6	6	1.54	20	0.300
255	A	5	5	1.44	20	0.250
256	A	4	4	1.44	20	0.200
257	A	1	1	1.00	20	0.050
258	A	4	4	1.00	20	0.200
259	A	5	5	1.00	20	0.250
260	A	6	5	1.00	20	0.250
261	A	10	6	1.00	20	0.300
262	A	9	6	1.00	20	0.300
263	A	8	6	1.00	20	0.300
264	A	7	6	1.00	20	0.300
265	A	6	6	1.00	20	0.300
266	A	5	5	1.00	20	0.250
267	A	6	6	1.00	20	0.300
268	A	7	6	1.00	20	0.300
269	A	8	6	1.00	20	0.300
270	A	9	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	8	6	1.00	20	0.300
272	A	7	6	1.00	20	0.300
273	A	6	6	1.00	20	0.300
274	A	5	5	1.00	20	0.250
275	A	4	4	1.00	20	0.200
276	A	1	1	1.00	20	0.050
277	A	5	5	1.00	20	0.250
278	A	6	5	1.00	20	0.250
279	A	7	6	1.00	9	0.667
280	A	6	5	1.00	8	0.625
281	A	3	3	1.00	11	0.273
282	A	6	6	1.00	10	0.600
283	A	8	6	1.00	11	0.546
284	A	7	5	1.00	10	0.500
285	A	8	8	1.00	13	0.615
286	A	7	7	1.00	12	0.583
287	A	3	3	1.00	11	0.273
288	A	4	4	1.00	10	0.400
289	A	8	8	1.00	13	0.615
290	A	7	7	1.00	12	0.583
291	A	5	5	1.00	11	0.454
292	A	3	3	1.00	10	0.300
293	A	9	9	1.00	13	0.692
294	A	3	3	1.00	12	0.250
295	A	3	3	1.00	21	0.143
296	A	5	4	1.00	21	0.190
297	A	4	4	1.00	19	0.210
298	A	1	1	1.00	18	0.056
299	A	5	5	1.00	21	0.238
300	A	5	5	1.00	21	0.238
301	A	5	4	1.00	23	0.174
302	A	5	4	1.00	23	0.174
303	A	5	4	1.00	21	0.190
304	A	5	4	1.00	20	0.200
305	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	6	6	1.00	23	0.261
307	A	7	7	1.00	23	0.304
308	A	8	7	1.00	23	0.304
309	A	9	7	1.00	23	0.304
310	A	10	7	1.00	23	0.304
311	A	9	7	1.00	23	0.304
312	A	7	6	1.00	21	0.286
313	A	6	5	1.00	20	0.250
314	A	8	8	1.00	23	0.348
315	A	8	8	1.00	23	0.348
316	A	9	9	1.00	23	0.391
317	A	10	9	1.00	23	0.391
318	A	11	9	1.00	23	0.391
319	A	6	6	1.00	13	0.462
320	A	5	5	1.00	12	0.417
321	A	5	5	1.00	15	0.333
322	A	4	4	1.00	14	0.286
323	A	5	5	1.00	13	0.385
324	A	4	4	1.00	12	0.333
325	A	4	4	1.00	15	0.267
326	A	1	1	1.00	14	0.071
327	A	4	4	1.00	13	0.308
328	A	3	3	1.00	12	0.250
329	A	6	6	1.00	15	0.400
330	A	5	5	1.00	14	0.357
331	A	5	5	1.00	13	0.385
332	A	4	4	1.00	12	0.333
333	A	6	6	1.00	15	0.400
334	A	5	5	1.00	14	0.357
335	A	4	4	1.00	23	0.174
336	A	6	5	1.30	23	0.217
337	A	5	5	1.32	21	0.238
338	A	4	4	1.44	20	0.200
339	A	5	5	1.00	23	0.217
340	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	9	7	1.00	23	0.304
342	A	9	7	1.00	23	0.304
343	A	8	7	1.00	21	0.333
344	A	7	6	1.00	20	0.300
345	A	9	8	1.00	23	0.348
346	A	9	8	1.00	23	0.348
347	A	10	9	1.00	23	0.391
348	A	11	9	1.00	23	0.391
349	A	12	9	1.00	23	0.391
350	A	8	6	1.00	23	0.261
351	A	7	6	1.00	23	0.261
352	A	6	6	1.00	21	0.286
353	A	5	5	1.00	20	0.250
354	A	6	6	1.00	23	0.261
355	A	6	6	1.00	23	0.261
356	A	7	7	1.00	23	0.304
357	A	8	7	1.00	23	0.304
358	A	9	7	1.00	23	0.304
359	A	6	6	1.00	24	0.250
360	A	4	4	1.00	24	0.167
361	A	1	1	1.00	22	0.045
362	A	3	3	1.00	24	0.125
363	A	3	3	1.00	24	0.125
364	A	3	3	1.00	18	0.167
365	A	3	3	1.00	18	0.167
366	A	3	3	1.00	18	0.167
367	A	3	3	1.00	16	0.188
368	A	3	3	1.00	18	0.167
369	A	3	3	1.00	18	0.167
370	A	4	4	1.00	18	0.222
371	A	6	6	1.00	18	0.333
372	A	3	3	1.00	20	0.150
373	A	3	3	1.00	20	0.150
374	A	3	3	1.00	20	0.150
375	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	3	3	1.00	20	0.150
377	A	4	4	1.00	20	0.200
378	A	5	4	1.00	20	0.200
379	A	9	9	1.00	20	0.450
380	A	8	8	1.00	20	0.400
381	A	7	7	1.00	20	0.350
382	A	3	3	1.00	18	0.167
383	A	7	7	1.00	20	0.350
384	A	8	7	1.00	20	0.350
385	A	9	7	1.00	20	0.350
386	A	10	7	1.00	20	0.350
387	A	5	4	1.00	22	0.182
388	A	5	4	1.00	22	0.182
389	A	5	4	1.00	22	0.182
390	A	6	5	1.00	22	0.227
391	A	5	4	1.00	20	0.200
392	A	5	4	1.00	22	0.182
393	A	5	4	1.00	22	0.182
394	A	5	4	1.00	22	0.182
395	A	5	4	1.00	22	0.182
396	A	8	8	1.00	22	0.364
397	A	4	4	1.00	22	0.182
398	A	8	8	1.00	22	0.364
399	A	8	8	1.00	20	0.400
400	A	8	7	1.00	22	0.318
401	A	9	7	1.00	22	0.318
402	A	10	7	1.00	22	0.318
403	A	11	7	1.00	22	0.318
404	A	5	4	1.00	22	0.182
405	A	6	5	1.00	22	0.227
406	A	5	4	1.00	22	0.182
407	A	5	4	1.00	22	0.182
408	A	5	4	1.00	20	0.200
409	A	5	4	1.00	22	0.182
410	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	5	4	1.00	22	0.182
412	A	5	4	1.00	22	0.182
413	A	10	8	1.00	22	0.364
414	A	9	8	1.00	22	0.364
415	A	8	8	1.00	22	0.364
416	A	7	7	1.00	20	0.350
417	A	2	2	1.00	22	0.091
418	A	6	6	1.00	22	0.273
419	A	8	7	1.00	22	0.318
420	A	9	7	1.00	22	0.318
421	A	5	4	1.00	22	0.182
422	A	5	4	1.00	22	0.182
423	A	5	4	1.00	22	0.182
424	A	5	4	1.00	20	0.200
425	A	5	4	1.00	22	0.182
426	A	6	5	1.00	22	0.227
427	A	5	4	1.00	22	0.182
428	A	5	4	1.00	22	0.182
429	A	11	9	1.00	22	0.409
430	A	10	9	1.00	22	0.409
431	A	9	9	1.00	22	0.409
432	A	8	8	1.00	20	0.400
433	A	6	6	1.00	22	0.273
434	A	6	6	1.00	22	0.273
435	A	3	3	1.00	22	0.136
436	A	7	7	1.00	22	0.318
437	A	9	7	1.00	22	0.318
438	A	8	7	1.19	22	0.318
439	A	7	7	1.13	22	0.318
440	A	7	7	1.20	22	0.318
441	A	5	5	1.00	22	0.227
442	A	4	4	1.00	22	0.182
443	A	8	7	1.00	22	0.318
444	A	9	8	1.00	22	0.364
445	A	10	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	12	9	1.00	24	0.375
447	A	11	9	1.00	24	0.375
448	A	10	9	1.00	24	0.375
449	A	9	9	1.00	24	0.375
450	A	8	8	1.00	24	0.333
451	A	9	9	1.00	24	0.375
452	A	10	9	1.00	24	0.375
453	A	11	9	1.00	24	0.375
454	A	12	9	1.00	24	0.375
455	A	8	8	1.00	24	0.333
456	A	8	7	1.00	24	0.292
457	A	6	5	1.00	24	0.208
458	A	5	5	1.00	24	0.208
459	A	8	7	1.00	24	0.292
460	A	9	8	1.00	24	0.333
461	A	10	8	1.00	24	0.333
462	A	11	8	1.00	24	0.333
463	A	7	7	1.00	24	0.292
464	A	6	6	1.00	24	0.250
465	A	6	6	1.00	24	0.250
466	A	4	4	1.00	24	0.167
467	A	4	4	1.00	24	0.167
468	A	9	8	1.00	24	0.333
469	A	10	9	1.00	24	0.375
470	A	11	10	1.00	24	0.417
471	A	14	10	1.00	24	0.417
472	A	13	10	1.00	24	0.417
473	A	12	10	1.00	24	0.417
474	A	11	9	1.00	24	0.375
475	A	11	9	1.00	24	0.375
476	A	12	10	1.00	24	0.417
477	A	12	10	1.00	24	0.417
478	A	13	10	1.00	24	0.417
479	A	14	10	1.00	24	0.417
480	A	9	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	8	8	1.00	24	0.333
482	A	7	7	1.00	24	0.292
483	A	6	6	1.00	24	0.250
484	A	6	6	1.00	24	0.250
485	A	5	5	1.00	24	0.208
486	A	5	5	1.00	24	0.208
487	A	9	8	1.00	24	0.333
488	A	10	9	1.00	24	0.375
489	A	3	3	1.00	25	0.120
490	A	6	5	1.00	25	0.200
491	A	5	5	1.00	23	0.217
492	A	4	4	1.00	22	0.182
493	A	4	4	1.00	25	0.160
494	A	2	2	1.00	25	0.080
495	A	3	3	1.00	25	0.120
496	A	4	4	1.00	25	0.160
497	A	5	4	1.00	25	0.160
498	A	11	9	1.00	27	0.333
499	A	10	9	1.00	27	0.333
500	A	9	9	1.00	25	0.360
501	A	8	8	1.00	24	0.333
502	A	8	8	1.00	27	0.296
503	A	7	6	1.00	27	0.222
504	A	7	6	1.00	27	0.222
505	A	7	6	1.00	27	0.222
506	A	7	6	1.00	27	0.222
507	A	11	8	1.00	27	0.296
508	A	10	8	1.00	27	0.296
509	A	9	8	1.00	25	0.320
510	A	8	7	1.00	24	0.292
511	A	8	7	1.00	27	0.259
512	A	5	4	1.00	27	0.148
513	A	6	5	1.00	27	0.185
514	A	7	6	1.00	27	0.222
515	A	8	6	1.00	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	4	4	1.00	27	0.148
517	A	6	5	1.00	27	0.185
518	A	5	5	1.00	25	0.200
519	A	4	4	1.00	24	0.167
520	A	4	4	1.00	27	0.148
521	A	3	3	1.00	27	0.111
522	A	4	4	1.00	27	0.148
523	A	5	5	1.00	27	0.185
524	A	14	10	1.00	27	0.370
525	A	13	10	1.00	27	0.370
526	A	12	10	1.00	25	0.400
527	A	11	9	1.00	24	0.375
528	A	11	9	1.00	27	0.333
529	A	9	8	1.00	27	0.296
530	A	10	9	1.00	27	0.333
531	A	11	9	1.00	27	0.333
532	A	11	9	1.00	27	0.333
533	A	9	8	1.00	27	0.296
534	A	8	8	1.00	27	0.296
535	A	7	7	1.00	25	0.280
536	A	6	6	1.00	24	0.250
537	A	6	5	1.00	27	0.185
538	A	4	3	1.00	27	0.111
539	A	5	4	1.00	27	0.148
540	A	6	5	1.00	27	0.185
541	A	4	3	1.00	27	0.111
542	A	5	5	1.00	20	0.250
543	A	3	3	1.00	22	0.136
544	A	5	5	1.00	22	0.227
545	A	3	3	1.00	24	0.125
546	A	3	3	1.00	24	0.125
547	A	3	3	1.00	24	0.125
548	A	3	3	1.00	24	0.125
549	A	3	3	1.00	22	0.136
550	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	3	3	1.00	23	0.130
552	A	7	7	1.00	22	0.318
553	A	3	3	1.00	20	0.150
554	A	3	3	1.00	22	0.136
555	A	9	9	1.00	22	0.409
556	A	13	5	1.00	20	0.250
557	A	11	5	1.00	20	0.250
558	A	9	5	1.00	20	0.250
559	A	7	5	1.00	18	0.278
560	A	1	1	1.00	20	0.050
561	A	2	2	1.00	20	0.100
562	A	3	2	1.00	20	0.100
563	A	4	2	1.00	20	0.100
564	A	4	3	1.00	22	0.136
565	A	4	3	1.00	22	0.136
566	A	4	3	1.00	22	0.136
567	A	4	3	1.00	22	0.136
568	A	3	3	1.00	20	0.150
569	A	3	3	1.00	22	0.136
570	A	5	4	1.00	22	0.182
571	A	5	4	1.00	22	0.182
572	A	5	4	1.00	22	0.182
573	A	13	5	1.00	22	0.227
574	A	11	5	1.00	22	0.227
575	A	9	5	1.00	22	0.227
576	A	7	5	1.00	20	0.250
577	A	1	1	1.00	22	0.045
578	A	2	2	1.00	22	0.091
579	A	3	2	1.00	22	0.091
580	A	4	2	1.00	22	0.091
581	A	4	3	1.00	22	0.136
582	A	4	3	1.00	22	0.136
583	A	4	3	1.00	22	0.136
584	A	3	3	1.00	22	0.136
585	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	3	3	1.00	22	0.136
587	A	3	3	1.00	22	0.136
588	A	5	4	1.00	22	0.182
589	A	5	4	1.00	22	0.182
590	A	13	5	1.00	22	0.227
591	A	11	5	1.00	22	0.227
592	A	9	5	1.00	22	0.227
593	A	7	5	1.00	20	0.250
594	A	1	1	1.00	22	0.045
595	A	2	2	1.00	22	0.091
596	A	3	2	1.00	22	0.091
597	A	4	2	1.00	22	0.091
598	A	4	3	1.00	22	0.136
599	A	4	3	1.00	22	0.136
600	A	4	3	1.00	22	0.136
601	A	3	3	1.00	20	0.150
602	A	3	3	1.00	22	0.136
603	A	5	4	1.00	22	0.182
604	A	5	4	1.00	22	0.182
605	A	5	4	1.00	22	0.182
606	A	13	5	1.00	22	0.227
607	A	11	5	1.00	22	0.227
608	A	9	5	1.00	22	0.227
609	A	7	5	1.00	20	0.250
610	A	1	1	1.00	22	0.045
611	A	2	2	1.00	22	0.091
612	A	3	2	1.00	22	0.091
613	A	4	2	1.00	22	0.091
614	A	4	3	1.00	22	0.136
615	A	4	3	1.00	22	0.136
616	A	4	3	1.00	22	0.136
617	A	4	3	1.00	22	0.136
618	A	3	2	1.00	22	0.091
619	A	3	3	1.00	22	0.136
620	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	5	4	1.00	22	0.182
622	A	5	4	1.00	22	0.182
623	A	10	7	1.00	24	0.292
624	A	9	7	1.00	24	0.292
625	A	8	7	1.00	24	0.292
626	A	7	7	1.00	24	0.292
627	A	6	6	1.00	24	0.250
628	A	5	5	1.00	24	0.208
629	A	4	4	1.00	24	0.167
630	A	5	5	1.00	24	0.208
631	A	6	5	1.00	24	0.208
632	A	7	5	1.00	24	0.208
633	A	4	3	1.00	24	0.125
634	A	4	3	1.00	24	0.125
635	A	4	3	1.00	24	0.125
636	A	3	3	1.00	24	0.125
637	A	4	3	1.00	24	0.125
638	A	4	3	1.00	24	0.125
639	A	3	3	1.00	24	0.125
640	A	5	4	1.00	24	0.167
641	A	5	4	1.00	24	0.167
642	A	4	3	1.00	24	0.125
643	A	4	3	1.00	24	0.125
644	A	4	3	1.00	24	0.125
645	A	4	3	1.00	24	0.125
646	A	3	2	1.00	24	0.083
647	A	3	3	1.00	24	0.125
648	A	5	4	1.00	24	0.167
649	A	5	4	1.00	24	0.167
650	A	5	4	1.00	24	0.167
651	A	8	7	1.00	24	0.292
652	A	7	7	1.00	24	0.292
653	A	6	6	1.00	24	0.250
654	A	5	5	1.00	24	0.208
655	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	5	5	1.00	24	0.208
657	A	6	5	1.00	24	0.208
658	A	7	5	1.00	24	0.208
659	A	4	3	1.00	24	0.125
660	A	4	3	1.00	24	0.125
661	A	4	3	1.00	24	0.125
662	A	3	3	1.00	24	0.125
663	A	4	3	1.00	24	0.125
664	A	4	3	1.00	24	0.125
665	A	3	3	1.00	24	0.125
666	A	5	4	1.00	24	0.167
667	A	5	4	1.00	24	0.167
668	A	4	3	1.00	25	0.120
669	A	4	3	1.00	23	0.130
670	A	3	2	1.00	22	0.091
671	A	4	3	1.00	25	0.120
672	A	4	3	1.00	25	0.120
673	A	8	7	1.00	27	0.259
674	A	7	7	1.00	27	0.259
675	A	6	6	1.00	25	0.240
676	A	6	6	1.00	24	0.250
677	A	9	9	1.00	27	0.333
678	A	9	9	1.00	27	0.333
679	A	7	7	1.00	27	0.259
680	A	8	8	1.00	27	0.296
681	A	9	8	1.00	27	0.296
682	A	4	3	1.00	27	0.111
683	A	4	3	1.00	27	0.111
684	A	4	3	1.00	25	0.120
685	A	4	3	1.00	24	0.125
686	A	4	3	1.00	27	0.111
687	A	4	3	1.00	27	0.111
688	A	4	3	1.00	27	0.111
689	A	4	3	1.00	27	0.111
690	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	4	3	1.00	25	0.120
692	A	4	3	1.00	25	0.120
693	A	4	3	1.00	25	0.120
694	A	5	4	1.00	23	0.174
695	A	5	4	1.00	22	0.182
696	A	4	3	1.00	25	0.120
697	A	4	3	1.00	25	0.120
698	A	4	3	1.00	25	0.120
699	A	4	3	1.00	25	0.120
700	A	4	3	1.00	25	0.120
701	A	5	4	1.00	25	0.160
702	A	5	4	1.00	25	0.160
703	A	5	4	1.00	23	0.174
704	A	5	4	1.00	22	0.182
705	A	4	3	1.00	25	0.120
706	A	4	3	1.00	25	0.120
707	A	4	3	1.00	27	0.111
708	A	4	3	1.00	25	0.120
709	A	3	2	1.00	24	0.083
710	A	4	3	1.00	27	0.111
711	A	4	3	1.00	27	0.111
712	A	8	7	1.00	27	0.259
713	A	7	7	1.00	27	0.259
714	A	6	6	1.00	25	0.240
715	A	6	6	1.00	24	0.250
716	A	9	9	1.00	27	0.333
717	A	9	9	1.00	27	0.333
718	A	7	7	1.00	27	0.259
719	A	8	8	1.00	27	0.296
720	A	9	8	1.00	27	0.296
721	A	4	3	1.00	27	0.111
722	A	4	3	1.00	27	0.111
723	A	4	3	1.00	25	0.120
724	A	4	3	1.00	24	0.125
725	A	4	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	4	3	1.00	27	0.111
727	A	4	3	1.00	27	0.111
728	A	4	3	1.00	27	0.111
729	A	4	3	1.00	27	0.111
730	A	5	4	1.00	27	0.148
731	A	8	6	1.00	27	0.222
732	A	4	3	1.00	25	0.120
733	A	4	3	1.00	27	0.111
734	A	8	6	1.00	27	0.222
735	A	5	4	1.00	27	0.148
736	A	3	3	1.00	22	0.136
737	A	3	3	1.00	22	0.136
738	A	3	3	1.00	20	0.150
739	A	2	2	1.00	8	0.250
740	A	1	1	1.00	22	0.045
741	A	2	2	1.00	22	0.091
742	A	3	2	1.00	22	0.091
743	A	4	2	1.00	22	0.091
744	A	3	3	1.00	24	0.125
745	A	3	3	1.00	24	0.125
746	A	3	3	1.00	24	0.125
747	A	1	1	1.00	24	0.042
748	A	2	2	1.00	24	0.083
749	A	3	2	1.00	24	0.083
750	A	4	2	1.00	24	0.083
751	A	7	6	1.00	27	0.222
752	A	4	4	1.00	27	0.148
753	A	1	1	1.00	25	0.040
754	A	1	1	1.00	24	0.042
755	A	5	5	1.00	27	0.185
756	A	8	6	1.00	27	0.222
757	A	6	4	1.00	27	0.148
758	A	2	2	1.00	27	0.074
759	A	2	2	1.00	25	0.080
760	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	15	6	1.00	27	0.222
762	A	3	3	1.00	22	0.136
763	A	3	3	1.00	23	0.130
764	A	3	3	1.00	23	0.130
765	A	4	4	1.00	22	0.182
766	A	3	3	1.00	22	0.136
767	A	4	4	1.00	22	0.182
768	A	3	3	1.00	20	0.150
769	A	3	3	1.00	22	0.136
770	A	4	4	1.00	22	0.182
771	A	3	3	1.00	22	0.136
772	A	14	8	1.00	20	0.400
773	A	12	8	1.00	20	0.400
774	A	10	8	1.00	20	0.400
775	A	9	9	1.00	18	0.500
776	A	6	6	1.00	20	0.300
777	A	8	6	1.00	20	0.300
778	A	10	6	1.00	20	0.300
779	A	12	6	1.00	20	0.300
780	A	5	4	1.00	22	0.182
781	A	5	4	1.00	22	0.182
782	A	5	4	1.00	22	0.182
783	A	5	4	1.00	22	0.182
784	A	5	4	1.00	20	0.200
785	A	5	4	1.00	22	0.182
786	A	5	4	1.00	22	0.182
787	A	5	4	1.00	22	0.182
788	A	5	4	1.00	22	0.182
789	A	14	8	1.00	22	0.364
790	A	12	8	1.00	22	0.364
791	A	10	8	1.00	22	0.364
792	A	8	8	1.00	20	0.400
793	A	7	6	1.00	22	0.273
794	A	9	8	1.00	22	0.364
795	A	10	6	1.00	22	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
796	A	12	6	1.00	22	0.273
797	A	5	4	1.00	22	0.182
798	A	5	4	1.00	22	0.182
799	A	5	4	1.00	22	0.182
800	A	5	4	1.00	22	0.182
801	A	5	4	1.00	20	0.200
802	A	5	4	1.00	22	0.182
803	A	5	4	1.00	22	0.182
804	A	5	4	1.00	22	0.182
805	A	5	4	1.00	22	0.182
806	A	14	8	1.00	22	0.364
807	A	12	8	1.00	22	0.364
808	A	10	8	1.00	22	0.364
809	A	9	9	1.00	20	0.450
810	A	6	6	1.00	22	0.273
811	A	8	6	1.00	22	0.273
812	A	10	6	1.00	22	0.273
813	A	12	6	1.00	22	0.273
814	A	5	4	1.00	22	0.182
815	A	5	4	1.00	22	0.182
816	A	5	4	1.00	22	0.182
817	A	5	4	1.00	20	0.200
818	A	5	4	1.00	22	0.182
819	A	5	4	1.00	22	0.182
820	A	5	4	1.00	22	0.182
821	A	5	4	1.00	22	0.182
822	A	14	8	1.00	22	0.364
823	A	12	8	1.00	22	0.364
824	A	10	8	1.00	22	0.364
825	A	8	8	1.00	20	0.400
826	A	7	6	1.00	22	0.273
827	A	9	8	1.00	22	0.364
828	A	10	6	1.00	22	0.273
829	A	12	6	1.00	22	0.273
830	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	4	3	1.00	22	0.136
832	A	4	3	1.00	22	0.136
833	A	4	3	1.00	22	0.136
834	A	4	3	1.00	22	0.136
835	A	4	3	1.00	22	0.136
836	A	4	3	1.00	22	0.136
837	A	4	3	1.00	22	0.136
838	A	15	11	1.00	24	0.458
839	A	13	11	1.00	24	0.458
840	A	11	11	1.00	24	0.458
841	A	9	9	1.00	24	0.375
842	A	7	7	1.00	24	0.292
843	A	7	7	1.00	24	0.292
844	A	9	8	1.00	24	0.333
845	A	11	8	1.00	24	0.333
846	A	4	3	1.00	24	0.125
847	A	4	3	1.00	24	0.125
848	A	4	3	1.00	24	0.125
849	A	4	3	1.00	24	0.125
850	A	4	3	1.00	24	0.125
851	A	4	3	1.00	24	0.125
852	A	4	3	1.00	24	0.125
853	A	4	3	1.00	24	0.125
854	A	4	3	1.00	24	0.125
855	A	4	3	1.00	24	0.125
856	A	4	3	1.00	24	0.125
857	A	4	3	1.00	24	0.125
858	A	4	3	1.00	24	0.125
859	A	4	3	1.00	24	0.125
860	A	4	3	1.00	24	0.125
861	A	4	3	1.00	24	0.125
862	A	4	3	1.00	24	0.125
863	A	15	11	1.00	24	0.458
864	A	13	11	1.00	24	0.458
865	A	11	11	1.00	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	9	9	1.00	24	0.375
867	A	7	7	1.00	24	0.292
868	A	7	7	1.00	24	0.292
869	A	9	8	1.00	24	0.333
870	A	11	8	1.00	24	0.333
871	A	4	3	1.00	24	0.125
872	A	4	3	1.00	24	0.125
873	A	4	3	1.00	24	0.125
874	A	4	3	1.00	24	0.125
875	A	4	3	1.00	24	0.125
876	A	4	3	1.00	24	0.125
877	A	4	3	1.00	24	0.125
878	A	4	3	1.00	24	0.125
879	A	4	3	1.00	24	0.125
880	A	4	3	1.00	25	0.120
881	A	4	3	1.00	25	0.120
882	A	3	2	1.00	23	0.087
883	A	4	3	1.00	22	0.136
884	A	4	3	1.00	25	0.120
885	A	3	3	1.00	25	0.120
886	A	9	8	1.00	27	0.296
887	A	8	7	1.00	27	0.259
888	A	7	6	1.00	25	0.240
889	A	9	9	1.00	24	0.375
890	A	9	9	1.00	27	0.333
891	A	7	6	1.00	27	0.222
892	A	8	7	1.00	27	0.259
893	A	10	8	1.00	27	0.296
894	A	11	8	1.00	27	0.296
895	A	4	3	1.00	27	0.111
896	A	4	3	1.00	27	0.111
897	A	4	3	1.00	25	0.120
898	A	4	3	1.00	24	0.125
899	A	4	3	1.00	27	0.111
900	A	4	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	4	3	1.00	27	0.111
902	A	4	3	1.00	27	0.111
903	A	4	3	1.00	27	0.111
904	A	4	3	1.00	27	0.111
905	A	4	3	1.00	27	0.111
906	A	3	2	1.00	25	0.080
907	A	4	3	1.00	24	0.125
908	A	4	3	1.00	27	0.111
909	A	3	3	1.00	27	0.111
910	A	9	8	1.00	27	0.296
911	A	8	7	1.00	27	0.259
912	A	7	6	1.00	25	0.240
913	A	9	9	1.00	24	0.375
914	A	9	9	1.00	27	0.333
915	A	7	6	1.00	27	0.222
916	A	8	7	1.00	27	0.259
917	A	10	8	1.00	27	0.296
918	A	11	8	1.00	27	0.296
919	A	4	3	1.00	27	0.111
920	A	4	3	1.00	27	0.111
921	A	4	3	1.00	25	0.120
922	A	4	3	1.00	24	0.125
923	A	4	3	1.00	27	0.111
924	A	4	3	1.00	27	0.111
925	A	4	3	1.00	27	0.111
926	A	4	3	1.00	27	0.111
927	A	4	3	1.00	27	0.111
928	A	4	4	1.00	20	0.200
929	A	5	5	1.00	22	0.227
930	A	7	5	1.00	22	0.227
931	A	6	6	1.00	24	0.250
932	A	4	4	1.00	24	0.167
933	A	3	3	1.00	22	0.136
934	A	3	3	1.00	23	0.130
935	A	3	3	1.00	23	0.130

Chapter 3

Listing of integrals

Local contents

3.1 $\int e^{\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] 3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4+2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2*
(1-1/a^2/x^2)^(1/2)/a^2+1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)
^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6304, 849, 821, 272, 65, 214}

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}+\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^3,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2)
 + (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{9}{a^2}+\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{24}\text{Subst}\left(\int \frac{-\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x\right.$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{3\text{Subst}\left(\int -\right.}{}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{3\text{Subst}\left(\int -\right.}{}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{3\text{Subst}\left(\int \right.}{}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{3\tanh^{-1}\left(\sqrt{\right.}}{8}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.60

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(16+9ax+8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^3,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
+ Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(94) = 188$.
time = 0.11, size = 193, normalized size = 1.69

method	result
risch	$\frac{(6a^3x^3+8a^2x^2+9ax+16)(ax-1)}{24a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax+1)(ax-1)}}{8a^3\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax-15\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-8((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}+15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right.\right.}{24\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(a*x-1)*(-6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-15*(a^2)^(1/2)*(a^2*x^2
-1)^(1/2)*a*x-8*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+15*ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-24*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(1/2))/(a^2)^(1/2))-24*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/((a*x-1)/
(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a^4/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(94) = 188$.
time = 0.27, size = 203, normalized size = 1.78

$$\frac{1}{24}a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-49\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-39\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/24*a*(2*(9*((a*x - 1)/(a*x + 1))^(7/2) - 49*((a*x - 1)/(a*x + 1))^(5/2) +
 31*((a*x - 1)/(a*x + 1))^(3/2) - 39*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1
)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 9*log(sqrt((a*x - 1)/(a*x + 1))
 + 1)/a^5 - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Fricas [A]
time = 0.35, size = 92, normalized size = 0.81

$$\frac{\left(6\,a^4x^4+14\,a^3x^3+17\,a^2x^2+25\,ax+16\right)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="fricas")
[Out] 1/24*((6*a^4*x^4 + 14*a^3*x^3 + 17*a^2*x^2 + 25*a*x + 16)*sqrt((a*x - 1)/(a
*x + 1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3,x)
[Out] Integral(x**3/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.41, size = 111, normalized size = 0.97

$$\frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2\,x\left(\frac{3\,x}{a\mathrm{sgn}\left(ax+1\right)}+\frac{4}{a^2\mathrm{sgn}\left(ax+1\right)}\right)+\frac{9}{a^3\mathrm{sgn}\left(ax+1\right)}\right)x+\frac{16}{a^4\mathrm{sgn}\left(ax+1\right)}\right)-\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,a^3|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3,x, algorithm="giac")
[Out] 1/24*sqrt(a^2*x^2 - 1)*((2*x*(3*x/(a*sgn(a*x + 1)) + 4/(a^2*sgn(a*x + 1)))
+ 9/(a^3*sgn(a*x + 1)))*x + 16/(a^4*sgn(a*x + 1))) - 3/8*log(abs(-x*abs(a)
+ sqrt(a^2*x^2 - 1)))/(a^3*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.26, size = 171, normalized size = 1.50

$$\frac{\frac{13\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12}+\frac{49\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{12}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}+\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((13*((a*x - 1)/(a*x + 1))^(1/2))/4 - (31*((a*x - 1)/(a*x + 1))^(3/2))/12 +
 (49*((a*x - 1)/(a*x + 1))^(5/2))/12 - (3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(
a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (
a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (3*atanh(((a*
x - 1)/(a*x + 1))^(1/2)))/(4*a^4)
```

3.2 $\int e^{\coth^{-1}(ax)} x^2 \, dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2+1/2*x^2*
(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6304, 849, 821, 272, 65, 214}

$$\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2,x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{4}{a^2}+\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.67

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(4+3ax+2a^2x^2)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^2,x]
```

[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + 3*a*x + 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(74) = 148$.
time = 0.08, size = 173, normalized size = 1.92

method	result
risch	$\frac{(2a^2x^2+3ax+4)(ax-1)}{6a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax+1)(ax-1)}}{2a^2\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}+3\sqrt{a^2}\sqrt{a^2x^2-1}\,ax+6\sqrt{a^2}\sqrt{(ax+1)(ax-1)}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)(ax-1)}\,a^3\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x,method=_RETURNVERBOSE)

[Out] 1/6*(a*x-1)*(2*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+6*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+6*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a^3/(a^2)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(74) = 148$.
time = 0.26, size = 166, normalized size = 1.84

$$-\frac{1}{6}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="maxima")

[Out] -1/6*a*(2*(3*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) + 9*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)

Fricas [A]

time = 0.33, size = 84, normalized size = 0.93

$$\frac{\left(2\,a^3x^3+5\,a^2x^2+7\,ax+4\right)\sqrt{\frac{ax-1}{ax+1}}+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 + 5*a^2*x^2 + 7*a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) + 3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2,x)
```

```
[Out] Integral(x**2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 0.06, size = 133, normalized size = 1.48

$$\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + ((a*x
- 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x -
 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x +
 1))^(1/2))/a^3
```

3.3 $\int e^{\coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=63

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\, x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\, x^2+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2+x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6304, 849, 821, 272, 65, 214}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]*x,x]

[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/a + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^2)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{2}\text{Subst}\left(\int \frac{-\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.78

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x,x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/
(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(53) = 106$.
time = 0.08, size = 152, normalized size = 2.41

method	result
risch	$\frac{(ax+2)(ax-1)}{2a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax+1)(ax-1)}}{2a\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(\sqrt{a^2}\sqrt{a^2x^2-1}\,ax+2\sqrt{a^2}\sqrt{(ax+1)(ax-1)}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a+2a\ln\left(\frac{a^2x+\sqrt{a^2}}{}\right.\right.}{2\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)(ax-1)}\,a^2\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x-1)*((a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+2*(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+2*a*ln((a
^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(
1/2)/((a*x+1)*(a*x-1))^(1/2)/a^2/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(53) = 106$.
time = 0.26, size = 128, normalized size = 2.03

$$\frac{1}{2}\,a\left(\frac{2\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(((a*x - 1)/(a*x + 1))^(3/2) - 3*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^3 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Fricas [A]
time = 0.35, size = 73, normalized size = 1.16

$$\frac{(a^2x^2+3\,ax+2)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 + 3*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(
a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x,x)
```

```
[Out] Integral(x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.41, size = 77, normalized size = 1.22

$$\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x}{a\mathrm{sgn}\left(ax+1\right)}+\frac{2}{a^2\mathrm{sgn}\left(ax+1\right)}\right)-\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*(x/(a*sgn(a*x + 1)) + 2/(a^2*sgn(a*x + 1))) - 1/2*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 0.06, size = 98, normalized size = 1.56

$$\frac{3\sqrt{\frac{a\,x-1}{a\,x+1}}-\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a^2+\frac{a^2\,\left(a\,x-1\right)^2}{\left(a\,x+1\right)^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(
a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(
a*x + 1))^(1/2))/a^2
```

3.4 $\int e^{\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=36

$$\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

[Out] arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6303, 821, 272, 65, 214}

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x],x]

[Out] Sqrt[1 - 1/(a^2*x^2)]*x + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{2a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x+a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.14

$$\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x],x]
```

```
[Out] Sqrt[1 - 1/(a^2*x^2)]*x + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(32) = 64$.
time = 0.08, size = 97, normalized size = 2.69

method	result	size
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	90
default	$\frac{(ax-1)\left(\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}+a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*((a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((a*x+
1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^
(1/2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.
time = 0.26, size = 90, normalized size = 2.50

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.32, size = 64, normalized size = 1.78

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.58

$$-\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{a\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x
^2 - 1)/(a*sgn(a*x + 1))
```

Mupad [B]
time = 0.04, size = 58, normalized size = 1.61

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.5 $\int \frac{e^{\coth^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=22

$$-\csc^{-1}(ax)+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

```
[Out] -arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6304, 858, 222, 272, 65, 214}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x,x]
```

```
[Out] -ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x}\,dx &= -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.64

$$-\text{ArcSin}\left(\frac{1}{ax}\right) + \log\left(x\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x,x]
```

```
[Out] -ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(20) = 40$.
time = 0.07, size = 131, normalized size = 5.95

method	result
default	$\frac{(ax-1)\left(\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}-\sqrt{a^2x^2-1}\,\sqrt{a^2}-\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}+a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(a}{}\right.\right.}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*((a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-
arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a
*x-1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)
/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(20) = 40$.
time = 0.47, size = 69, normalized size = 3.14

$$a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.
time = 0.35, size = 57, normalized size = 2.59

$$2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(1/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(20) = 40.
time = 0.41, size = 63, normalized size = 2.86

$$\frac{2\ \arctan\left(-x|a| + \sqrt{a^2x^2-1}\right)}{\operatorname{sgn}(ax+1)} - \frac{a\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - a*log(abs(-x*abs(a)
+ sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.18, size = 37, normalized size = 1.68

$$2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
```

3.6 $\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=24

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)$$

[Out] -a*arccsc(a*x)+a*(1-1/a^2/x^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6304, 655, 222}

$$a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]/x^2,x]

[Out] a*Sqrt[1 - 1/(a^2*x^2)] - a*ArcCsc[a*x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= a\sqrt{1-\frac{1}{a^2x^2}} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.12

$$a\left(\sqrt{1-\frac{1}{a^2x^2}} - \text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^2,x]
[Out] a*(Sqrt[1 - 1/(a^2*x^2)] - ArcSin[1/(a*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(22) = 44.
time = 0.09, size = 220, normalized size = 9.17

method	result
risch	$\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}} - \frac{a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax+1)(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,ax-(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-\ln\left(\right.\right.}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] (a*x-1)*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+((a*x+1)*(a*x-1))^(1/2)*(a^2
)^(1/2)*a*x-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x
```

```
-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-a*x*(a^2)^(1/2
)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)
)/(a^2)^(1/2))*a^2*x)/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/x/(a^
2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.
time = 0.46, size = 53, normalized size = 2.21

$$2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}+\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) + arctan(sqrt((a*x
 - 1)/(a*x + 1))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.
time = 0.35, size = 46, normalized size = 1.92

$$\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(1/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(22) = 44$.
time = 0.40, size = 66, normalized size = 2.75

$$\frac{2\,a\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{\operatorname{sgn}\left(ax+1\right)}+\frac{2\,|a|}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)\operatorname{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) + 2*abs(a)/(((x*abs(
a) - sqrt(a^2*x^2 - 1))^2 + 1)*sgn(a*x + 1))
```

Mupad [B]
time = 0.05, size = 55, normalized size = 2.29

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (2*a*((a*x - 1)/(a*x + 1))^(1/2))/(
(a*x - 1)/(a*x + 1) + 1)
```

3.7 $\int \frac{e^{\coth^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=38

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] -1/2*a^2*arccsc(a*x)+1/2*a*(2*a+1/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6304, 794, 222}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right)-\frac{1}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^3,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a + x^(-1)))/2 - (a^2*ArcCsc[a*x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right) - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{1}{x}\right) - \frac{1}{2}a^2\csc^{-1}(ax)$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.11

$$\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}\,(1+2ax) - ax\text{ArcSin}\left(\frac{1}{ax}\right)\right)}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^3,x]
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a*x) - a*x*ArcSin[1/(a*x)]))/(2*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(32) = 64.
time = 0.10, size = 260, normalized size = 6.84

method	result
risch	$\frac{(ax-1)(2ax+1)}{2x^2\sqrt{\frac{ax-1}{ax+1}}} - \frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax+1)(ax-1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)\left(2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2-2\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax-\sqrt{a^2x^2-1}\,\sqrt{a^2}\right.}{2\sqrt{\frac{ax-}{ax-}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x-1)*(2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+2*((a*x+1)*(a*x-1))^(1
/2)*(a^2)^(1/2)*a^2*x^2-2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-(a^2*x^2-1)^(1/
2)*(a^2)^(1/2)*a^2*x^2-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/
2))*a^3*x^2-a^2*x^2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+2*ln((a^2*x+(a^
2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2*x^2-1)^(3/2)*(a
^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/x^2/(a^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(32) = 64$.
time = 0.46, size = 91, normalized size = 2.39

$$\left(a \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + \frac{a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 3\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1} + \frac{(ax-1)^2}{(ax+1)^2} + 1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] (a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (a*((a*x - 1)/(a*x + 1))^(3/2) + 3*a
*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^
2 + 1))*a
```

Fricas [**A**]
time = 0.34, size = 60, normalized size = 1.58

$$\frac{2\,a^2x^2 \arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right) + (2\,a^2x^2 + 3\,ax + 1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (2*a^2*x^2 + 3*a*x + 1)*
sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(32) = 64$.
time = 0.40, size = 143, normalized size = 3.76

$$\frac{a^2 \arctan\left(-x|a| + \sqrt{a^2x^2 - 1}\right)}{\operatorname{sgn}(ax+1)} - \frac{\left(x|a| - \sqrt{a^2x^2 - 1}\right)^3 a^2 - 2\left(x|a| - \sqrt{a^2x^2 - 1}\right)^2 a|a| - \left(x|a| - \sqrt{a^2x^2 - 1}\right)a^2 - 2\,a|a|}{\left(\left(x|a| - \sqrt{a^2x^2 - 1}\right)^2 + 1\right)^2 \operatorname{sgn}(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - ((x*abs(a) - sqrt(
a^2*x^2 - 1))^3*a^2 - 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*abs(a) - (x*abs(
a) - sqrt(a^2*x^2 - 1))*a^2 - 2*a*abs(a))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^
2 + 1)^2*sgn(a*x + 1))
```

Mupad [B]
time = 1.20, size = 81, normalized size = 2.13

$$a^2\sqrt{\frac{a\,x-1}{a\,x+1}} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2} + a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right) + \frac{3\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] a^2*((a*x - 1)/(a*x + 1))^(1/2) + ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) + a^2
*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (3*a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x
)
```

3.8 $\int \frac{e^{\coth^{-1}(ax)}}{x^4} dx$

Optimal. Leaf size=75

$$a^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

```
[Out] -1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)+a^3*(1-1/a^2/x^2)^(1/2)+1/
2*a^2*(1-1/a^2/x^2)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6304, 811, 655, 201, 222}

$$-\frac{1}{2}a^3\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+a^3\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] a^3*Sqrt[1 - 1/(a^2*x^2)] - (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[1 -
 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x^4}\,dx &= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(a^2\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + a^2\text{Subst}\left(\int \left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx, x, \frac{1}{x}\right)\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} - a^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) + a^2\text{Subst}\Bigg(\int
\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - a^3\csc^{-1}(ax) + \frac{1}{2}a^2\text{Subst}\Bigg(\int
\\
&= a^3\sqrt{1-\frac{1}{a^2x^2}} - \frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x} - \frac{1}{2}a^3\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.68

$$\frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(2+3ax+4a^2x^2)}{x^2} - 3a^2\text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^4,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x + 4*a^2*x^2))/x^2 - 3*a^2*ArcSin[1/(a
*x)]))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(63) = 126$.
time = 0.10, size = 284, normalized size = 3.79

method	result
risch	$\frac{(ax-1)(4a^2x^2+3ax+2)}{6x^3\sqrt{\frac{ax-1}{ax+1}}} - \frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax+1)(ax-1)}}{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+6*(a^2*x^2-1)^(3/2)*
(a^2)^(1/2)*a^2*x^2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+6*ln((a^2*x+(a^
2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+3*a^3*x^3*(a^2)^(1/2)*arct
an(1/(a^2*x^2-1)^(1/2))-6*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-6*ln(
(a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+3*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*a*x+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1)
)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/x^3/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(63) = 126$.
time = 0.48, size = 136, normalized size = 1.81

$$\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{3\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (3*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 9*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

Fricas [A]
time = 0.38, size = 68, normalized size = 0.91

$$\frac{6\,a^{3}x^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(4\,a^{3}x^{3}+7\,a^{2}x^{2}+5\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (4*a^3*x^3 + 7*a^2*x^2 +
 5*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{4}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(63) = 126.
time = 0.42, size = 148, normalized size = 1.97

$$\frac{a^{3}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)}{\operatorname{sgn}\left(ax+1\right)}-\frac{3\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{5}a^{3}-12\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}a^{2}|a|-3\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)a^{3}-4\,a^{2}|a|}{3\left(\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}+1\right)^{3}\operatorname{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] a^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/3*(3*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^5*a^3 - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^2*abs(a)
- 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*a^3 - 4*a^2*abs(a))/(((x*abs(a) - sqrt(a
^2*x^2 - 1))^2 + 1)^3*sgn(a*x + 1))
```

Mupad [B]
time = 0.06, size = 105, normalized size = 1.40

$$\frac{2\,a^{3}\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^{3}}+a^{3}\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{7\,a^{2}\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x}+\frac{5\,a\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3)
 + a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (7*a^2*((a*x - 1)/(a*x + 1))^(1/
2))/(6*x) + (5*a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

3.9 $\int \frac{e^{\coth^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=88

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{3}{8}a^4\csc^{-1}(ax)$$

```
[Out] -3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a+9/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2
/x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6304, 847, 794, 222}

$$-\frac{3}{8}a^4\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a + 9/x))/24 + (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) - (3*a^4*ArcCsc[a*x])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
```

```
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^5}\,dx = -\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{1}{4}a^2\text{Subst}\left(\int \frac{x^2\left(-\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{12}a^4\text{Subst}\left(\int \frac{x\left(\frac{8}{a^2}+\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a+\frac{9}{x}\right) + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} + \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2} - \frac{3}{8}a^4\csc^{-1}(ax)$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 0.67

$$\frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(6+8ax+9a^2x^2+16a^3x^3\right)}{x^3} - 9a^3\text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/x^5,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(6 + 8*a*x + 9*a^2*x^2 + 16*a^3*x^3))/x^3 - 9*a^
3*ArcSin[1/(a*x)]))/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(74) = 148$.
time = 0.10, size = 308, normalized size = 3.50

method	result
risch	$\frac{(ax-1)(16a^3x^3+9a^2x^2+8ax+6)}{24x^4\sqrt{\frac{ax-1}{ax+1}}} - \frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{(ax+1)(ax-1)}}{8\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)\left(-24\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+9a^4x^4\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(a*x-1)*(-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^5*x^5+24*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*a^3*x^3+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+9*a^4*x^4*(a
^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+24*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^
(1/2))/(a^2)^(1/2))*a^5*x^4-24*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-
24*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+15*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+8*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+6*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1
/2)/x^4/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(74) = 148$.
time = 0.47, size = 172, normalized size = 1.95

$$\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{9\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+39\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4(ax-1)}{ax+1}+\frac{6(ax-1)^2}{(ax+1)^2}+\frac{4(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + (9*a^3*((a*x - 1)/(a*x + 1)
)^(7/2) + 49*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 31*a^3*((a*x - 1)/(a*x + 1))
^(3/2) + 39*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
```

```
- 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
1))*a
```

Fricas [A]
time = 0.35, size = 76, normalized size = 0.86

$$\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\left(16\,a^4x^4+25\,a^3x^3+17\,a^2x^2+14\,ax+6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) + (16*a^4*x^4 + 25*a^3*x
^3 + 17*a^2*x^2 + 14*a*x + 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^5\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

```
[Out] Integral(1/(x**5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(74) = 148.
time = 0.41, size = 226, normalized size = 2.57

$$\frac{3\,a^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,\mathrm{sgn}\left(ax+1\right)}-\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^7a^4+33\left(x|a|-\sqrt{a^2x^2-1}\right)^5a^4-48\left(x|a|-\sqrt{a^2x^2-1}\right)^4a^3|a|-33\left(x|a|-\sqrt{a^2x^2-1}\right)^3a^4-64\left(x|a|-\sqrt{a^2x^2-1}\right)^2a^3|a|-9\left(x|a|-\sqrt{a^2x^2-1}\right)a^4-16\,a^3|a|}{12\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^4\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/sgn(a*x + 1) - 1/12*(9*(x*abs
(a) - sqrt(a^2*x^2 - 1))^7*a^4 + 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*a^4 -
48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a) - 33*(x*abs(a) - sqrt(a^2*x^
2 - 1))^3*a^4 - 64*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a) - 9*(x*abs(a
) - sqrt(a^2*x^2 - 1))*a^4 - 16*a^3*abs(a))/(((x*abs(a) - sqrt(a^2*x^2 - 1)
)^2 + 1)^4*sgn(a*x + 1))
```

Mupad [B]
time = 0.08, size = 129, normalized size = 1.47

$$\frac{2\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4}+\frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}+\frac{17\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2}+\frac{25\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x}+\frac{7\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 + ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4)
 + (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 + (17*a^2*((a*x - 1)/(a*x +
1))^(1/2))/(24*x^2) + (25*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (7*a*((
a*x - 1)/(a*x + 1))^(1/2))/(12*x^3)
```

3.10 $\int e^{2\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=43

$$\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

```
[Out] 2*x/a^3+x^2/a^2+2/3*x^3/a+1/4*x^4+2*ln(-a*x+1)/a^4
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\frac{2\log(1-ax)}{a^4}+\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3,x]
```

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^3\,dx = -\int e^{2\tanh^{-1}(ax)}x^3\,dx$$
$$= -\int \frac{x^3(1+ax)}{1-ax}\,dx$$
$$= -\int \left(-\frac{2}{a^3}-\frac{2x}{a^2}-\frac{2x^2}{a}-x^3-\frac{2}{a^3(-1+ax)}\right)dx$$
$$= \frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$\frac{2x}{a^3}+\frac{x^2}{a^2}+\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1-ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3,x]
```

```
[Out] (2*x)/a^3 + x^2/a^2 + (2*x^3)/(3*a) + x^4/4 + (2*Log[1 - a*x])/a^4
```

Maple [A]
time = 0.10, size = 42, normalized size = 0.98

method	result	size
norman	$\frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2x}{a^3}+\frac{2x^3}{3a}+\frac{2\ln(ax-1)}{a^4}$	39
risch	$\frac{x^2}{a^2}+\frac{x^4}{4}+\frac{2x}{a^3}+\frac{2x^3}{3a}+\frac{2\ln(ax-1)}{a^4}$	39
default	$\frac{\frac{1}{4}a^3x^4+\frac{2}{3}a^2x^3+a\,x^2+2x}{a^3}+\frac{2\ln(ax-1)}{a^4}$	42
meijerg	$\frac{\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)}{a^4}-\frac{-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)}{a^4}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/4*a^3*x^4+2/3*a^2*x^3+a*x^2+2*x)+2/a^4*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 43, normalized size = 1.00

$$\frac{3\,a^3x^4+8\,a^2x^3+12\,ax^2+24\,x}{12\,a^3}+\frac{2\,\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^4 + 8*a^2*x^3 + 12*a*x^2 + 24*x)/a^3 + 2*log(a*x - 1)/a^4
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.98

$$\frac{3\,a^4x^4+8\,a^3x^3+12\,a^2x^2+24\,ax+24\,\log\left(ax-1\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x + 24*log(a*x - 1))/a^4
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.86

$$\frac{x^4}{4}+\frac{2x^3}{3a}+\frac{x^2}{a^2}+\frac{2x}{a^3}+\frac{2\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3,x)
```

```
[Out] x**4/4 + 2*x**3/(3*a) + x**2/a**2 + 2*x/a**3 + 2*log(a*x - 1)/a**4
```

Giac [A]
time = 0.41, size = 47, normalized size = 1.09

$$\frac{3\,a^4x^4+8\,a^3x^3+12\,a^2x^2+24\,ax}{12\,a^4}+\frac{2\,\log\left(|ax-1|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*x^4 + 8*a^3*x^3 + 12*a^2*x^2 + 24*a*x)/a^4 + 2*log(abs(a*x - 1)
)/a^4
```

Mupad [B]
time = 0.04, size = 38, normalized size = 0.88

$$\frac{2\,\ln\left(a\,x-1\right)}{a^4}+\frac{2\,x}{a^3}+\frac{x^4}{4}+\frac{2\,x^3}{3\,a}+\frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*log(a*x - 1))/a^4 + (2*x)/a^3 + x^4/4 + (2*x^3)/(3*a) + x^2/a^2
```

3.11 $\int e^{2\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=33

$$\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

```
[Out] 2*x/a^2+x^2/a+1/3*x^3+2*ln(-a*x+1)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\frac{2\log(1-ax)}{a^3}+\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^2\,dx = -\int e^{2\tanh^{-1}(ax)}x^2\,dx$$
$$= -\int \frac{x^2(1+ax)}{1-ax}\,dx$$
$$= -\int \left(-\frac{2}{a^2}-\frac{2x}{a}-x^2-\frac{2}{a^2(-1+ax)}\right)dx$$
$$= \frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{2x}{a^2}+\frac{x^2}{a}+\frac{x^3}{3}+\frac{2\log(1-ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2,x]
```

```
[Out] (2*x)/a^2 + x^2/a + x^3/3 + (2*Log[1 - a*x])/a^3
```

Maple [A]
time = 0.08, size = 34, normalized size = 1.03

method	result	size
norman	$\frac{x^2}{a}+\frac{x^3}{3}+\frac{2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	31
risch	$\frac{x^2}{a}+\frac{x^3}{3}+\frac{2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	31
default	$\frac{\frac{1}{3}a^2x^3+a\,x^2+2x}{a^2}+\frac{2\ln(ax-1)}{a^3}$	34
meijerg	$-\frac{-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)}{a^3}+\frac{\frac{ax(3ax+6)}{6}+\ln(-ax+1)}{a^3}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/3*a^2*x^3+a*x^2+2*x)+2/a^3*ln(a*x-1)
```

Maxima [A]
time = 0.25, size = 34, normalized size = 1.03

$$\frac{a^2x^3+3\,ax^2+6\,x}{3\,a^2}+\frac{2\,\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="maxima")
[Out] 1/3*(a^2*x^3 + 3*a*x^2 + 6*x)/a^2 + 2*log(a*x - 1)/a^3
```

Fricas [A]
time = 0.34, size = 33, normalized size = 1.00

$$\frac{a^3x^3 + 3\,a^2x^2 + 6\,ax + 6\,\log\left(ax - 1\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="fricas")
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x + 6*log(a*x - 1))/a^3
```

Sympy [A]
time = 0.05, size = 27, normalized size = 0.82

$$\frac{x^3}{3} + \frac{x^2}{a} + \frac{2x}{a^2} + \frac{2\log\left(ax - 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2,x)
[Out] x**3/3 + x**2/a + 2*x/a**2 + 2*log(a*x - 1)/a**3
```

Giac [A]
time = 0.41, size = 38, normalized size = 1.15

$$\frac{a^3x^3 + 3\,a^2x^2 + 6\,ax}{3\,a^3} + \frac{2\,\log\left(|ax - 1|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2,x, algorithm="giac")
[Out] 1/3*(a^3*x^3 + 3*a^2*x^2 + 6*a*x)/a^3 + 2*log(abs(a*x - 1))/a^3
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.91

$$\frac{2\,\ln\left(a\,x - 1\right)}{a^3} + \frac{2\,x}{a^2} + \frac{x^3}{3} + \frac{x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/(a*x - 1),x)
[Out] (2*log(a*x - 1))/a^3 + (2*x)/a^2 + x^3/3 + x^2/a
```

3.12 $\int e^{2\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=26

$$\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

```
[Out] 2*x/a+1/2*x^2+2*ln(-a*x+1)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6261, 78}

$$\frac{2\log(1-ax)}{a^2}+\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x,x]
```

```
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x\,dx = -\int e^{2\tanh^{-1}(ax)}x\,dx$$
$$= -\int \frac{x(1+ax)}{1-ax}\,dx$$
$$= -\int \left(-\frac{2}{a}-x-\frac{2}{a(-1+ax)}\right)dx$$
$$= \frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1-ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x,x]
[Out] (2*x)/a + x^2/2 + (2*Log[1 - a*x])/a^2
```

Maple [A]
time = 0.08, size = 27, normalized size = 1.04

method	result	size
norman	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
risch	$\frac{x^2}{2}+\frac{2x}{a}+\frac{2\ln(ax-1)}{a^2}$	24
default	$\frac{\frac{1}{2}a\,x^2+2x}{a}+\frac{2\ln(ax-1)}{a^2}$	27
meijerg	$\frac{\frac{ax(3ax+6)}{6}+\ln(-ax+1)}{a^2}-\frac{-ax-\ln(-ax+1)}{a^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x,x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*a*x^2+2*x)+2/a^2*ln(a*x-1)
```

Maxima [A]
time = 0.25, size = 26, normalized size = 1.00

$$\frac{ax^2+4\,x}{2\,a}+\frac{2\,\log\left(ax-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 + 4*x)/a + 2*log(a*x - 1)/a^2
```

Fricas [**A**]
time = 0.32, size = 25, normalized size = 0.96

$$\frac{a^2x^2 + 4\,ax + 4\,\log\left(ax - 1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2 + 4*a*x + 4*log(a*x - 1))/a^2
```

Sympy [**A**]
time = 0.03, size = 20, normalized size = 0.77

$$\frac{x^2}{2} + \frac{2x}{a} + \frac{2\log\left(ax - 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x)
```

```
[Out] x**2/2 + 2*x/a + 2*log(a*x - 1)/a**2
```

Giac [**A**]
time = 0.41, size = 30, normalized size = 1.15

$$\frac{a^2x^2 + 4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax - 1|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*x^2 + 4*a*x)/a^2 + 2*log(abs(a*x - 1))/a^2
```

Mupad [**B**]
time = 0.04, size = 23, normalized size = 0.88

$$\frac{2\,\ln\left(a\,x - 1\right)}{a^2} + \frac{2\,x}{a} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*log(a*x - 1))/a^2 + (2*x)/a + x^2/2
```

3.13 $\int e^{2\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=14

$$x+\frac{2\log(1-ax)}{a}$$

[Out] x+2*ln(-a*x+1)/a

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6260, 45}

$$\frac{2\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x]),x]

[Out] x + (2*Log[1 - a*x])/a

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6260

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\,dx &= -\int e^{2\tanh^{-1}(ax)}\,dx \\
&= -\int \frac{1+ax}{1-ax}\,dx \\
&= -\int \left(-1-\frac{2}{-1+ax}\right)dx \\
&= x+\frac{2\log(1-ax)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$x+\frac{2\log(1-ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x]),x]
```

```
[Out] x + (2*Log[1 - a*x])/a
```

Maple [A]
time = 0.09, size = 14, normalized size = 1.00

method	result	size
default	$x+\frac{2\ln(ax-1)}{a}$	14
norman	$x+\frac{2\ln(ax-1)}{a}$	14
risch	$x+\frac{2\ln(ax-1)}{a}$	14
meijerg	$-\frac{-ax-\ln(-ax+1)}{a}+\frac{\ln(-ax+1)}{a}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+2/a*ln(a*x-1)
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.93

$$x+\frac{2\,\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="maxima")
```

```
[Out] x + 2*log(a*x - 1)/a
```

Fricas [A]
time = 0.35, size = 16, normalized size = 1.14

$$\frac{ax + 2\log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="fricas")
```

```
[Out] (a*x + 2*log(a*x - 1))/a
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.71

$$x + \frac{2\log(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x)
```

```
[Out] x + 2*log(a*x - 1)/a
```

Giac [A]
time = 0.40, size = 14, normalized size = 1.00

$$x + \frac{2\log(|ax - 1|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1),x, algorithm="giac")
```

```
[Out] x + 2*log(abs(a*x - 1))/a
```

Mupad [B]
time = 1.17, size = 13, normalized size = 0.93

$$x + \frac{2\ln(a\,x - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(a*x - 1),x)
```

```
[Out] x + (2*log(a*x - 1))/a
```

3.14 $\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=14

$$-\log(x)+2\log(1-ax)$$

[Out] -ln(x)+2*ln(-a*x+1)

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$2\log(1-ax)-\log(x)$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/x,x]

[Out] -Log[x] + 2*Log[1 - a*x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x}\,dx \\
&= -\int \frac{1+ax}{x(1-ax)}\,dx \\
&= -\int \left(\frac{1}{x} - \frac{2a}{-1+ax}\right)dx \\
&= -\log(x) + 2\log(1-ax)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\log(x) + 2\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x,x]
```

```
[Out] -Log[x] + 2*Log[1 - a*x]
```

Maple [A]
time = 0.08, size = 14, normalized size = 1.00

method	result	size
default	$-\ln(x) + 2\ln(ax-1)$	14
norman	$-\ln(x) + 2\ln(ax-1)$	14
risch	$-\ln(x) + 2\ln(-ax+1)$	15
meijerg	$2\ln(-ax+1) - \ln(x) - \ln(-a)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+2*ln(a*x-1)
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.93

$$2\,\log(ax-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] 2*log(a*x - 1) - log(x)
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.93

$$2\log\left(ax - 1\right) - \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="fricas")
[Out] 2*log(a*x - 1) - log(x)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.71

$$-\log\left(x\right) + 2\log\left(x - \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x)
[Out] -log(x) + 2*log(x - 1/a)
```

Giac [A]
time = 0.42, size = 15, normalized size = 1.07

$$2\log\left(|ax - 1|\right) - \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x,x, algorithm="giac")
[Out] 2*log(abs(a*x - 1)) - log(abs(x))
```

Mupad [B]
time = 0.04, size = 14, normalized size = 1.00

$$2\ln\left(3 - 3\,a\,x\right) - \ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(a*x - 1)),x)
[Out] 2*log(3 - 3*a*x) - log(x)
```

3.15 $\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{x} - 2a\log(x) + 2a\log(1-ax)$$

[Out] 1/x-2*a*ln(x)+2*a*ln(-a*x+1)

Rubi [A]

time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$-2a\log(x) + 2a\log(1-ax) + \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/x^2,x]

[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{x^2}\,dx\\ &= -\int \frac{1+ax}{x^2(1-ax)}\,dx\\ &= -\int \left(\frac{1}{x^2}+\frac{2a}{x}-\frac{2a^2}{-1+ax}\right)dx\\ &= \frac{1}{x}-2a\log(x)+2a\log(1-ax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{x}-2a\log(x)+2a\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^2,x]
```

```
[Out] x^(-1) - 2*a*Log[x] + 2*a*Log[1 - a*x]
```

Maple [A]
time = 0.10, size = 19, normalized size = 1.00

method	result	size
default	$\frac{1}{x}-2a\ln(x)+2a\ln(ax-1)$	19
norman	$\frac{1}{x}-2a\ln(x)+2a\ln(ax-1)$	19
risch	$\frac{1}{x}-2a\ln(x)+2a\ln(-ax+1)$	20
meijerg	$-a(-\ln(-ax+1)+\ln(x)+\ln(-a))+a\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/x-2*a*ln(x)+2*a*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.95

$$2\,a\log(ax-1)-2\,a\log(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*log(a*x - 1) - 2*a*log(x) + 1/x
```

Fricas [A]
time = 0.33, size = 22, normalized size = 1.16

$$\frac{2\,ax\log\left(ax-1\right)-2\,ax\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*log(a*x - 1) - 2*a*x*log(x) + 1)/x
```

Sympy [A]
time = 0.07, size = 15, normalized size = 0.79

$$2a\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**2,x)
```

```
[Out] 2*a*(-log(x) + log(x - 1/a)) + 1/x
```

Giac [A]
time = 0.42, size = 20, normalized size = 1.05

$$2\,a\log\left(\left|ax-1\right|\right)-2\,a\log\left(\left|x\right|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*log(abs(a*x - 1)) - 2*a*log(abs(x)) + 1/x
```

Mupad [B]
time = 1.19, size = 14, normalized size = 0.74

$$\frac{1}{x}-4\,a\,\mathrm{atanh}(2\,a\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(a*x - 1)),x)
```

```
[Out] 1/x - 4*a*atanh(2*a*x - 1)
```

3.16 $\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=33

$$\frac{1}{2x^2} + \frac{2a}{x} - 2a^2\log(x) + 2a^2\log(1-ax)$$

```
[Out] 1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(-a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$-2a^2\log(x) + 2a^2\log(1-ax) + \frac{2a}{x} + \frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x^3,x]
```

```
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{x^3}\,dx$$
$$= -\int \frac{1+ax}{x^3(1-ax)}\,dx$$
$$= -\int \left(\frac{1}{x^3}+\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{-1+ax}\right)dx$$
$$= \frac{1}{2x^2}+\frac{2a}{x}-2a^2\log(x)+2a^2\log(1-ax)$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{2x^2}+\frac{2a}{x}-2a^2\log(x)+2a^2\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^3,x]
[Out] 1/(2*x^2) + (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 - a*x]
```

Maple [A]
time = 0.11, size = 31, normalized size = 0.94

method	result
norman	$\frac{\frac{1}{2}+2ax}{x^2}-2a^2\ln(x)+2a^2\ln(ax-1)$
default	$\frac{1}{2x^2}+\frac{2a}{x}-2a^2\ln(x)+2a^2\ln(ax-1)$
risch	$\frac{\frac{1}{2}+2ax}{x^2}-2a^2\ln(x)+2a^2\ln(-ax+1)$
meijerg	$a^2\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)-a^2\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2/x^2+2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 30, normalized size = 0.91

$$2\,a^2\log(ax-1)-2\,a^2\log(x)+\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 2*a^2*log(a*x - 1) - 2*a^2*log(x) + 1/2*(4*a*x + 1)/x^2
```

Fricas [A]
time = 0.33, size = 35, normalized size = 1.06

$$\frac{4\,a^2x^2\log\left(ax-1\right)-4\,a^2x^2\log\left(x\right)+4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2*log(a*x - 1) - 4*a^2*x^2*log(x) + 4*a*x + 1)/x^2
```

Sympy [A]
time = 0.09, size = 26, normalized size = 0.79

$$2a^2\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**3,x)
```

```
[Out] 2*a**2*(-log(x) + log(x - 1/a)) + (4*a*x + 1)/(2*x**2)
```

Giac [A]
time = 0.41, size = 32, normalized size = 0.97

$$2\,a^2\log\left(|ax-1|\right)-2\,a^2\log\left(|x|\right)+\frac{4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 2*a^2*log(abs(a*x - 1)) - 2*a^2*log(abs(x)) + 1/2*(4*a*x + 1)/x^2
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.70

$$\frac{2\,a\,x+\frac{1}{2}}{x^2}-4\,a^2\,\mathrm{atanh}(2\,a\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(a*x - 1)),x)
```

```
[Out] (2*a*x + 1/2)/x^2 - 4*a^2*atanh(2*a*x - 1)
```

3.17 $\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=40

$$\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

```
[Out] 1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(-a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$-2a^3\log(x)+2a^3\log(1-ax)+\frac{2a^2}{x}+\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/x^4,x]
```

```
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{x^4}\,dx$$
$$= -\int \frac{1+ax}{x^4(1-ax)}\,dx$$
$$= -\int \left(\frac{1}{x^4}+\frac{2a}{x^3}+\frac{2a^2}{x^2}+\frac{2a^3}{x}-\frac{2a^4}{-1+ax}\right)dx$$
$$= \frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\log(x)+2a^3\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/x^4,x]
[Out] 1/(3*x^3) + a/x^2 + (2*a^2)/x - 2*a^3*Log[x] + 2*a^3*Log[1 - a*x]
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.95

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}-2a^3\ln(x)+2a^3\ln(ax-1)$
default	$\frac{1}{3x^3}+\frac{a}{x^2}+\frac{2a^2}{x}-2a^3\ln(x)+2a^3\ln(ax-1)$
risch	$\frac{\frac{1}{3}+2a^2x^2+ax}{x^3}+2a^3\ln(-ax+1)-2a^3\ln(x)$
meijerg	$-a^3\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)+a^3\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{}{3x}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/3/x^3+a/x^2+2*a^2/x-2*a^3*ln(x)+2*a^3*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 38, normalized size = 0.95

$$2\,a^3\log(ax-1)-2\,a^3\log(x)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] 2*a^3*log(a*x - 1) - 2*a^3*log(x) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x^3
```

Fricas [A]
time = 0.35, size = 43, normalized size = 1.08

$$\frac{6\,a^3x^3\log\left(ax-1\right)-6\,a^3x^3\log\left(x\right)+6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(6*a^3*x^3*log(a*x - 1) - 6*a^3*x^3*log(x) + 6*a^2*x^2 + 3*a*x + 1)/x^3
```

Sympy [A]
time = 0.09, size = 34, normalized size = 0.85

$$2a^3\left(-\log\left(x\right)+\log\left(x-\frac{1}{a}\right)\right)+\frac{6a^2x^2+3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x**4,x)
```

```
[Out] 2*a**3*(-log(x) + log(x - 1/a)) + (6*a**2*x**2 + 3*a*x + 1)/(3*x**3)
```

Giac [A]
time = 0.39, size = 40, normalized size = 1.00

$$2\,a^3\log\left(|ax-1|\right)-2\,a^3\log\left(|x|\right)+\frac{6\,a^2x^2+3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*log(abs(a*x - 1)) - 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 + 3*a*x + 1)/x
^3
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.75

$$\frac{2\,a^2\,x^2+a\,x+\frac{1}{3}}{x^3}-4\,a^3\,\mathrm{atanh}(2\,a\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(a*x - 1)),x)
```

```
[Out] (a*x + 2*a^2*x^2 + 1/3)/x^3 - 4*a^3*atanh(2*a*x - 1)
```

3.18 $\int e^{3\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=118

$$-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3+\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] 11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3-4*(1-1/a^2/x^2)^(1/2)/a^2/(a-1/x)+14/
3*x*(1-1/a^2/x^2)^(1/2)/a^2+3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/
x^2)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6304, 6874, 665, 277, 270, 272, 44, 65, 214}

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2,x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a - x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x
)/(3*a^2) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^
3)/3 + (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^4\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a^3(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}} + \frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}}\right.\right.$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\text{Subst}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x,\right.}{a^3}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{3\text{Subst}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{4\tanh^{-}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a-\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{11\tanh}{}$$

Mathematica [A]
time = 0.06, size = 75, normalized size = 0.64

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-52+19ax+7a^2x^2+2a^3x^3)}{-1+ax} + 33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-52 + 19*a*x + 7*a^2*x^2 + 2*a^3*x^3))/(-1 + a
*x) + 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(100) = 200$.
time = 0.10, size = 471, normalized size = 3.99

method	result
risch	$\frac{(2a^2x^2+9ax+28)(ax-1)}{6a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a^2\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{a^4\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax+1)(ax-}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$9\sqrt{a^2x^2-1}\sqrt{a^2}\,a^3x^3+2\sqrt{a^2}\,((ax+1)(ax-1))^{\frac{3}{2}}a^2x^2+42\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2-18\sqrt{a^2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^3*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+2*(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(3/2)*a^2*x^2+42*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)
^(1/2))*a^3*x^2-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-9*ln((a^2*x+(a^2*x
^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-4*(a^2)^(1/2)*((a*x+1)*(a*x-1
))^(3/2)*a*x+42*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-84*ln((a^2*x+(a
^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+9*(a^2)^(1/2)*(a^2*x^
2-1)^(1/2)*a*x+18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2
*x-10*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-84*((a*x+1)*(a*x-1))^(1/2)*(a^2)^
(1/2)*a*x+42*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))-
9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+42*(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)
/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.25, size = 182, normalized size = 1.54

$$-\frac{1}{6}a\left(\frac{2\left(\frac{75(ax-1)}{ax+1}-\frac{88(ax-1)^2}{(ax+1)^2}+\frac{33(ax-1)^3}{(ax+1)^3}-12\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-3a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^4\sqrt{\frac{ax-1}{ax+1}}}-\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}+\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(75*(a*x - 1)/(a*x + 1) - 88*(a*x - 1)^2/(a*x + 1)^2 + 33*(a*x -
1)^3/(a*x + 1)^3 - 12)/(a^4*((a*x - 1)/(a*x + 1))^(7/2) - 3*a^4*((a*x - 1)/
(a*x + 1))^(5/2) + 3*a^4*((a*x - 1)/(a*x + 1))^(3/2) - a^4*sqrt((a*x - 1)/(
a*x + 1))) - 33*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 + 33*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^4)
```

Fricas [A]
time = 0.34, size = 112, normalized size = 0.95

$$\frac{33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-33\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^4x^4+9\,a^3x^3+26\,a^2x^2-33\,ax-52)\sqrt{\frac{ax-1}{ax+1}}}{6\,(a^4x-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(33*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 33*(a*x - 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (2*a^4*x^4 + 9*a^3*x^3 + 26*a^2*x^2 - 33*a*x
- 52)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.26, size = 154, normalized size = 1.31

$$\frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}-\frac{\frac{88\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{11\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{25\,\left(a\,x-1\right)}{a\,x+1}+4}{a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3 - ((88*(a*x - 1)^2)/(3*(a*x + 1
)^2) - (11*(a*x - 1)^3)/(a*x + 1)^3 - (25*(a*x - 1))/(a*x + 1) + 4)/(a^3*((
a*x - 1)/(a*x + 1))^(1/2) - 3*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^3*((a*x
 - 1)/(a*x + 1))^(5/2) - a^3*((a*x - 1)/(a*x + 1))^(7/2))
```

3.19 $\int e^{3\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=92

$$-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a-1/x)+3*x*(1
-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {6304, 6874, 665, 272, 44, 65, 214, 270}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x,x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a - x^(-1))) + (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a^2(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}}\right.\right.$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} - \frac{3\text{Subst}\left(\right.}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\text{Subst}\left(\right.}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 4\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2} + \frac{1}{2}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a-\frac{1}{x}\right)} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A]

time = 0.05, size = 66, normalized size = 0.72

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-14+5ax+a^2x^2)}{-1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(3*ArcCoth[a*x])*x,x]

[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 + 5*a*x + a^2*x^2))/(-1 + a*x) + 9*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(80) = 160$.
time = 0.10, size = 421, normalized size = 4.58

method	result
risch	$\frac{(ax+6)(ax-1)}{2a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{a^3\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax+1)(ax-1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-10\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+\ln\left(\frac{a^2x+\sqrt{a^2x^2-}}{\sqrt{a^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x,x,method=_RETURNVERBOSE)

[Out] -1/2/a^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-10*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-10*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+4*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+20*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x+20*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-10*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-10*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)

Maxima [A]

time = 0.26, size = 145, normalized size = 1.58

$$\frac{1}{2}\,a\left(\frac{2\,\left(\frac{15\,(ax-1)}{ax+1}-\frac{9\,(ax-1)^2}{(ax+1)^2}-4\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(15*(a*x - 1)/(a*x + 1) - 9*(a*x - 1)^2/(a*x + 1)^2 - 4)/(a^3*((a*
x - 1)/(a*x + 1))^(5/2) - 2*a^3*((a*x - 1)/(a*x + 1))^(3/2) + a^3*sqrt((a*x
 - 1)/(a*x + 1))) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^3 - 9*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Fricas [**A**]
time = 0.34, size = 103, normalized size = 1.12

$$\frac{9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^3x^3+6\,a^2x^2-9\,ax-14)\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^3x-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*(9*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x - 1)*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1) + (a^3*x^3 + 6*a^2*x^2 - 9*a*x - 14)*sqrt((a*x -
1)/(a*x + 1)))/(a^3*x - a^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.06, size = 117, normalized size = 1.27

$$\frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{\frac{9\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{15\,(a\,x-1)}{a\,x+1}+4}{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-2\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - ((9*(a*x - 1)^2)/(a*x + 1)^2 -
 (15*(a*x - 1))/(a*x + 1) + 4)/(a^2*((a*x - 1)/(a*x + 1))^(1/2) - 2*a^2*((a
*x - 1)/(a*x + 1))^(3/2) + a^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.20 $\int e^{3\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=62

$$-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*(1-1/a^2/x^2)^(1/2)/(a-1/x)+x*(1-1/a^2/x
^2)^(1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6303, 6874, 665, 270, 272, 65, 214}

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x + (3*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^2\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{4}{a(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \frac{4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\Bigg(\int$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x + (3a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.87

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5+ax)}{-1+ax} + \frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + a*x))/(-1 + a*x) + (3*Log[a*(1 + Sqrt[1 - 1/
(a^2*x^2)])*x])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(56) = 112$.
time = 0.09, size = 248, normalized size = 4.00

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{a^2\left(x-\frac{1}{a}\right)}\right)\sqrt{(ax+1)(ax-1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{-3\sqrt{(ax+1)(ax-1)}\sqrt{a^2}\,a^2x^2-3\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}+6}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(-3*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-3*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+2*((a*x+1)*(a*x-1))^(3/2)*(
a^2)^(1/2)+6*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+6*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-3*(a^2)^(1/2)*((a*x+1)*(a*x-1
))^(1/2)-3*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/(
a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.25, size = 110, normalized size = 1.77

$$-a\left(\frac{2\left(\frac{3(ax-1)}{ax+1}-2\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*(3*(a*x - 1)/(a*x + 1) - 2)/(a^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*sq
rt((a*x - 1)/(a*x + 1))) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*log
(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.37, size = 92, normalized size = 1.48

$$\frac{3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2x^2-4\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] (3*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 4*a*x - 5)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*x - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.29, size = 59, normalized size = 0.95

$$\frac{2\,a\,x+12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-10}{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*a*x + 12*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2)
- 10)/(2*a*((a*x - 1)/(a*x + 1))^(1/2))
```

3.21 $\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\csc^{-1}(ax)+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

```
[Out] arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a-1/x)
```

Rubi [A]
time = 0.56, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6304, 6874, 222, 665, 272, 65, 214}

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x,x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a - x^(-1)) + ArcCsc[a*x] + ArcTanh[Sqrt[1 - 
1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{(a-x)\sqrt{1-\frac{x^2}{a^2}}} + \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= -\left(4\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a-\frac{1}{x}} + \csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

Mathematica [**A**]
time = 0.04, size = 53, normalized size = 1.15

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}\,x}{-1+ax} + \text{ArcSin}\left(\frac{1}{ax}\right) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x,x]
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(42) = 84$.
time = 0.08, size = 372, normalized size = 8.09

method	result
default	$-\frac{-\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(-((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-ln((a^2*x+(a^2)^(1/2)*((a*x+
1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x
^2-(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^2*x^2+2*((a*x+1)*(a*x-1))^(3/2
)*(a^2)^(1/2)+2*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+2*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+2*(a^2)^(1/2)*(a^2*x^2-1)^
(1/2)*a*x+2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x-(a^2)^(1/2)*((a*x+1
)*(a*x-1))^(1/2)-a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/
2))-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2))/
(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(42) = 84$.
time = 0.46, size = 90, normalized size = 1.96

$$-a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}+\frac{4}{a\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a + 4/(a*sqrt((a*x - 1)/(a*x + 1
))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.
time = 0.35, size = 104, normalized size = 2.26

$$-\frac{2(ax-1)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -(2*(a*x - 1)*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x - 1)*log(sqrt((a*x -
 1)/(a*x + 1)) + 1) + (a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + 4*(a*x
 + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.04, size = 54, normalized size = 1.17

$$2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{4}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
- 4/((a*x - 1)/(a*x + 1))^(1/2)
```

3.22 $\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=51

$$-3a\sqrt{1-\frac{1}{a^2x^2}}-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)$$

```
[Out] 3*a*arccsc(a*x)-2*(a+1/x)^2/a/(1-1/a^2/x^2)^(1/2)-3*a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6304, 867, 683, 655, 222}

$$-\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}-3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^2,x]
```

```
[Out] -3*a*Sqrt[1 - 1/(a^2*x^2)] - (2*(a + x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
 3*a*ArcCsc[a*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{x^2}\,dx &= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\text{Subst}\left(\int \frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -3a\sqrt{1-\frac{1}{a^2x^2}} - \frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -3a\sqrt{1-\frac{1}{a^2x^2}} - \frac{2\left(a+\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}} + 3a\csc^{-1}(ax)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 0.80

$$
\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,(1-5ax)}{-1+ax} + 3a\text{ArcSin}\left(\frac{1}{ax}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^2,x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x))/(-1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(47) = 94$.
time = 0.10, size = 593, normalized size = 11.63

method	result
risch	$-\frac{ax-1}{x\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{4\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}+3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)\sqrt{(ax+1)(ax-1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3-(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)
*a^3*x^3+ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^
3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-5*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3
*x^3-3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+2*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
3/2)*a*x-2*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-2*ln((a^2*x+(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+2*(a^2)^(1/2)*(a^2*x^2-1)
^(3/2)*a*x+7*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+6*(a^2)^(1/2)*arctan(1/(
a^2*x^2-1)^(1/2))*a^2*x^2+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*a^3*x^2+((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)
-3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/
2))*a*x-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x)/x/(a^2
)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.46, size = 72, normalized size = 1.41

$$-2\,a\left(\frac{\frac{3\,(ax-1)}{ax+1}+2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}+3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*a*((3*(a*x - 1)/(a*x + 1) + 2)/(((a*x - 1)/(a*x + 1))^(3/2) + sqrt((a*x
- 1)/(a*x + 1))) + 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A]
time = 0.36, size = 74, normalized size = 1.45

$$-\frac{6\,(a^2x^2-ax)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(5\,a^2x^2+4\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
[Out] -(6*(a^2*x^2 - a*x)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (5*a^2*x^2 + 4*a*x
- 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**2,x)
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
[Out] undef
```

Mupad [B]
time = 0.05, size = 57, normalized size = 1.12

$$\frac{1}{x\sqrt{\frac{a\,x-1}{a\,x+1}}}-6\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{5\,a}{\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] 1/(x*((a*x - 1)/(a*x + 1))^(1/2)) - 6*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) -
 (5*a)/((a*x - 1)/(a*x + 1))^(1/2)
```

3.23 $\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=91

$$-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}+\frac{9}{2}a^2\csc^{-1}(ax)$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)/(a-1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a-1/x)+9/2*
a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6304, 1647, 1607, 12, 807, 679, 222}

$$-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{9}{2}a^2\csc^{-1}(ax)-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^3,x]
```

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a - x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a - x^(-1))) + (9*a^2*ArcCsc[a*x
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(-ax-x^2)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} + (3a)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{1}{2}(9a)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a-\frac{1}{x}\right)} + \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A]

time = 0.07, size = 56, normalized size = 0.62

$$\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(1+5ax-14a^2x^2)}{x(-1+ax)}+9a\text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^3,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x - 14*a^2*x^2))/(x*(-1 + a*x)) + 9*a*A
rcSin[1/(a*x)]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(79) = 158$.
time = 0.10, size = 642, normalized size = 7.05

method	result
risch	$-\frac{(ax-1)(6ax+1)}{2x^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{4a\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}+\frac{9a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)\sqrt{(ax+1)(ax-1)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+6\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^4x^4+6\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^5x^4-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^5*x^5+6*(a^2)^(1/2)*((a*x+1)*(a*x-1
))^(1/2)*a^4*x^4+6*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/
2))*a^5*x^4-6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-21*(a^2*x^2-1)^(1/2)*(a
^2)^(1/2)*a^4*x^4-9*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-6*ln((a
^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^5*x^4+4*(a^2)^(1/2)*((a*
x+1)*(a*x-1))^(3/2)*a^2*x^2-12*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-
12*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+11*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x
^3+18*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+12*ln((a^2*x+(a^2*x^2
-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+6*((a*x+1)*(a*x-1))^(1/2)*(a^2)
^(1/2)*a^2*x^2+6*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)
)*a^3*x^2-4*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-9*(a^2*x^2-1)^(1/2)*(a^2)^(1/
2)*a^2*x^2-9*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^2*x^2-6*ln((a^2*x+(a
```

```
^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2*x^2-1)^(3/2)*(a^2)^(
1/2))/x^2/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/
2)
```

Maxima [A]
time = 0.48, size = 110, normalized size = 1.21

$$-\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{15\,(ax-1)a}{ax+1}+\frac{9\,(ax-1)^2a}{(ax+1)^2}+4\,a}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) + (15*(a*x - 1)*a/(a*x + 1) + 9*(a*
x - 1)^2*a/(a*x + 1)^2 + 4*a)/(((a*x - 1)/(a*x + 1))^(5/2) + 2*((a*x - 1)/(
a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Fricas [A]
time = 0.36, size = 88, normalized size = 0.97

$$-\frac{18\,(a^3x^3-a^2x^2)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(14\,a^3x^3+9\,a^2x^2-6\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^3-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(18*(a^3*x^3 - a^2*x^2)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (14*a^3*x^
3 + 9*a^2*x^2 - 6*a*x - 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.08, size = 83, normalized size = 0.91

$$\frac{1}{2\,x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-\frac{7\,a^2}{\sqrt{\frac{a\,x-1}{a\,x+1}}}-9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{5\,a}{2\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 1/(2*x^2*((a*x - 1)/(a*x + 1))^(1/2)) - (7*a^2)/((a*x - 1)/(a*x + 1))^(1/2)
 - 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (5*a)/(2*x*((a*x - 1)/(a*x + 1
))^(1/2))
```

3.24 $\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=93

$$-\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2-\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right)+\frac{11}{2}a^3\csc^{-1}(ax)$$

```
[Out] 11/2*a^3*arccsc(a*x)-(a+1/x)^3/(1-1/a^2/x^2)^(1/2)-1/3*a*(3*a+1/x)^2*(1-1/a
^2/x^2)^(1/2)-1/6*a^2*(28*a+3/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6304, 1647, 1607, 12, 866, 1649, 1668, 794, 222}

$$\frac{11}{2}a^3\csc^{-1}(ax)-\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right)-\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2-\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/x^4,x]
```

```
[Out] -((a + x^(-1))^3/Sqrt[1 - 1/(a^2*x^2)]) - (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a + x
^(-1))^2)/3 - (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a + 3/x))/6 + (11*a^3*ArcCsc[a
*x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\,(-ax^2-x^3)}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2(3a^2+ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5-\frac{3x}{a}\right)(3a^2+ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x,\right.$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{1}{2}(11a^2)\,\text{S}$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a+\frac{1}{x}\right)^2 - \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a+\frac{3}{x}\right) + \frac{11}{2}a^3\csc^{-}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.71

$$\frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(2+7ax+19a^2x^2-52a^3x^3)}{x^2(-1+ax)}+33a^2\mathrm{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/x^4,x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 + 7*a*x + 19*a^2*x^2 - 52*a^3*x^3))/(x^2*(-1
+ a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(81) = 162.
time = 0.10, size = 666, normalized size = 7.16

method	result
risch	$-\frac{(ax-1)(28a^2x^2+9ax+2)}{6x^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{4a^2\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{x-\frac{1}{a}}+\frac{11a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)\sqrt{(ax+1)(a}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{30\sqrt{a^2}\sqrt{a^2x^2-1}\,a^6x^6+30\sqrt{(ax+1)(ax-1)}\sqrt{a^2}\,a^5x^5+30\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(30*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a^6*x^6+30*((a*x+1)*(a*x-1))^(1/2)*(
a^2)^(1/2)*a^5*x^5+30*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^
(1/2))*a^6*x^5-30*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-93*(a^2*x^2-1)^(1/2
)*(a^2)^(1/2)*a^5*x^5-33*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x^5-30
*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5+12*((a*x+1)*
(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3-60*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a
^4*x^4-60*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x
^4+51*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+96*(a^2*x^2-1)^(1/2)*(a^2)^(1/2
)*a^4*x^4+66*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+60*ln((a^2*x+(
a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^5*x^4+30*(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(1/2)*a^3*x^3+30*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a
^2)^(1/2))*a^4*x^3-14*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-33*(a^2*x^2-1)^
```

```
(1/2)*(a^2)^(1/2)*a^3*x^3-33*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2)
)-30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3-5*(a^2)^
(1/2)*(a^2*x^2-1)^(3/2)*a*x-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/x^3/(a^2)^(1/2
)/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.46, size = 154, normalized size = 1.66

$$-\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\frac{\frac{75\,(ax-1)a^2}{ax+1}+\frac{88\,(ax-1)^2a^2}{(ax+1)^2}+\frac{33\,(ax-1)^3a^2}{(ax+1)^3}+12\,a^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+\sqrt{\frac{ax-1}{ax+1}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + (75*(a*x - 1)*a^2/(a*x + 1
) + 88*(a*x - 1)^2*a^2/(a*x + 1)^2 + 33*(a*x - 1)^3*a^2/(a*x + 1)^3 + 12*a^
2)/(((a*x - 1)/(a*x + 1))^(7/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + 3*((a*x -
 1)/(a*x + 1))^(3/2) + sqrt((a*x - 1)/(a*x + 1))))*a
```

Fricas [A]
time = 0.35, size = 96, normalized size = 1.03

$$-\frac{66\,(a^4x^4-a^3x^3)\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+(52\,a^4x^4+33\,a^3x^3-26\,a^2x^2-9\,ax-2)\sqrt{\frac{ax-1}{ax+1}}}{6\,(ax^4-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(66*(a^4*x^4 - a^3*x^3)*arctan(sqrt((a*x - 1)/(a*x + 1))) + (52*a^4*x^
4 + 33*a^3*x^3 - 26*a^2*x^2 - 9*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4
- x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.24, size = 152, normalized size = 1.63

$$-\frac{4\,a^3+\frac{88\,a^3\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}+\frac{11\,a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{25\,a^3\,\left(a\,x-1\right)}{a\,x+1}}{\sqrt{\frac{a\,x-1}{a\,x+1}}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}-11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] - (4*a^3 + (88*a^3*(a*x - 1)^2)/(3*(a*x + 1)^2) + (11*a^3*(a*x - 1)^3)/(a*x
 + 1)^3 + (25*a^3*(a*x - 1))/(a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/2) + 3*((
a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x - 1)/(a*x + 1))^(5/2) + ((a*x - 1)/(a*x
 + 1))^(7/2)) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.25 $\int e^{4\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=57

$$\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}$$

```
[Out] 12*x/a^3+4*x^2/a^2+4/3*x^3/a+1/4*x^4+4/a^4/(-a*x+1)+16*ln(-a*x+1)/a^4
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}+\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}x^3\,dx = \int e^{4\tanh^{-1}(ax)}x^3\,dx$$
$$= \int \frac{x^3(1+ax)^2}{(1-ax)^2}\,dx$$
$$= \int \left(\frac{12}{a^3}+\frac{8x}{a^2}+\frac{4x^2}{a}+x^3+\frac{4}{a^3(-1+ax)^2}+\frac{16}{a^3(-1+ax)}\right)dx$$
$$= \frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 1.00

$$\frac{12x}{a^3}+\frac{4x^2}{a^2}+\frac{4x^3}{3a}+\frac{x^4}{4}+\frac{4}{a^4(1-ax)}+\frac{16\log(1-ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^3,x]
```

```
[Out] (12*x)/a^3 + (4*x^2)/a^2 + (4*x^3)/(3*a) + x^4/4 + 4/(a^4*(1 - a*x)) + (16*
Log[1 - a*x])/a^4
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.96

method	result
risch	$\frac{x^4}{4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}-\frac{4}{a^4(ax-1)}+\frac{16\ln(ax-1)}{a^4}$
norman	$\frac{\frac{13x^4}{12}+\frac{a\,x^5}{4}+\frac{8x^2}{a^2}+\frac{8x^3}{3a}-\frac{16}{a^4}}{ax-1}+\frac{16\ln(ax-1)}{a^4}$
default	$\frac{\frac{1}{4}a^3x^4+\frac{4}{3}a^2x^3+4a\,x^2+12x}{a^3}-\frac{4}{a^4(ax-1)}+\frac{16\ln(ax-1)}{a^4}$
meijerg	$\frac{\frac{xa\left(-3a^4x^4-5a^3x^3-10a^2x^2-30ax+60\right)}{-12ax+12}+5\ln(-ax+1)}{a^4}-\frac{2\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a^4}+\frac{\frac{ax\left(-2a^2x^2-6ax+1\right.}{-4ax+4}}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/4*a^3*x^4+4/3*a^2*x^3+4*a*x^2+12*x)-4/a^4/(a*x-1)+16/a^4*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 58, normalized size = 1.02

$$-\frac{4}{a^5x-a^4}+\frac{3\,a^3x^4+16\,a^2x^3+48\,ax^2+144\,x}{12\,a^3}+\frac{16\,\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="maxima")
```

```
[Out] -4/(a^5*x - a^4) + 1/12*(3*a^3*x^4 + 16*a^2*x^3 + 48*a*x^2 + 144*x)/a^3 + 1
6*log(a*x - 1)/a^4
```

Fricas [A]
time = 0.34, size = 66, normalized size = 1.16

$$\frac{3\,a^5x^5+13\,a^4x^4+32\,a^3x^3+96\,a^2x^2-144\,ax+192\,(ax-1)\log\left(ax-1\right)-48}{12\,(a^5x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^5*x^5 + 13*a^4*x^4 + 32*a^3*x^3 + 96*a^2*x^2 - 144*a*x + 192*(a*x
 - 1)*log(a*x - 1) - 48)/(a^5*x - a^4)
```

Sympy [A]
time = 0.09, size = 49, normalized size = 0.86

$$\frac{x^4}{4}-\frac{4}{a^5x-a^4}+\frac{4x^3}{3a}+\frac{4x^2}{a^2}+\frac{12x}{a^3}+\frac{16\log\left(ax-1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**3,x)
```

```
[Out] x**4/4 - 4/(a**5*x - a**4) + 4*x**3/(3*a) + 4*x**2/a**2 + 12*x/a**3 + 16*lo
g(a*x - 1)/a**4
```

Giac [A]
time = 0.40, size = 78, normalized size = 1.37

$$\frac{(ax-1)^4\left(\frac{28}{ax-1}+\frac{114}{(ax-1)^2}+\frac{300}{(ax-1)^3}+3\right)}{12\,a^4}-\frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^4}-\frac{4}{(ax-1)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(a*x - 1)^4*(28/(a*x - 1) + 114/(a*x - 1)^2 + 300/(a*x - 1)^3 + 3)/a^4
 - 16*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a^4 - 4/((a*x - 1)*a^4)
```

Mupad [B]
time = 0.04, size = 57, normalized size = 1.00

$$\frac{16\,\ln\left(a\,x-1\right)}{a^4}-\frac{4}{a\,\left(a^4\,x-a^3\right)}+\frac{12\,x}{a^3}+\frac{x^4}{4}+\frac{4\,x^3}{3\,a}+\frac{4\,x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(a*x + 1)^2)/(a*x - 1)^2,x)

[Out] (16*log(a*x - 1))/a^4 - 4/(a*(a^4*x - a^3)) + (12*x)/a^3 + x^4/4 + (4*x^3)/(3*a) + (4*x^2)/a^2

3.26 $\int e^{4\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=47

$$\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

```
[Out] 8*x/a^2+2*x^2/a+1/3*x^3+4/a^3/(-a*x+1)+12*ln(-a*x+1)/a^3
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}+\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}x^2\,dx &= \int e^{4\tanh^{-1}(ax)}x^2\,dx \\
&= \int \frac{x^2(1+ax)^2}{(1-ax)^2}\,dx \\
&= \int \left(\frac{8}{a^2}+\frac{4x}{a}+x^2+\frac{4}{a^2(-1+ax)^2}+\frac{12}{a^2(-1+ax)}\right)\,dx \\
&= \frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.00

$$\frac{8x}{a^2}+\frac{2x^2}{a}+\frac{x^3}{3}+\frac{4}{a^3(1-ax)}+\frac{12\log(1-ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^2,x]
```

```
[Out] (8*x)/a^2 + (2*x^2)/a + x^3/3 + 4/(a^3*(1 - a*x)) + (12*Log[1 - a*x])/a^3
```

Maple [A]
time = 0.13, size = 47, normalized size = 1.00

method	result	size
risch	$\frac{x^3}{3}+\frac{2x^2}{a}+\frac{8x}{a^2}-\frac{4}{a^3(ax-1)}+\frac{12\ln(ax-1)}{a^3}$	44
norman	$\frac{\frac{5x^3}{3}+\frac{a\,x^4}{3}+\frac{6x^2}{a}-\frac{12}{a^3}}{ax-1}+\frac{12\ln(ax-1)}{a^3}$	46
default	$\frac{\frac{1}{3}a^2x^3+2a\,x^2+8x}{a^2}-\frac{4}{a^3(ax-1)}+\frac{12\ln(ax-1)}{a^3}$	47
meijerg	$-\frac{-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)}{a^3}+\frac{\frac{2ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+6\ln(-ax+1)}{a^3}-\frac{-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)}{a^3}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/3*a^2*x^3+2*a*x^2+8*x)-4/a^3/(a*x-1)+12/a^3*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 49, normalized size = 1.04

$$-\frac{4}{a^4x-a^3}+\frac{a^2x^3+6\,ax^2+24\,x}{3\,a^2}+\frac{12\,\log\left(ax-1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="maxima")
[Out] -4/(a^4*x - a^3) + 1/3*(a^2*x^3 + 6*a*x^2 + 24*x)/a^2 + 12*log(a*x - 1)/a^3
```

Fricas [A]
time = 0.36, size = 57, normalized size = 1.21

$$\frac{a^4x^4 + 5\,a^3x^3 + 18\,a^2x^2 - 24\,ax + 36\,(ax-1)\log{(ax-1)} - 12}{3\,(a^4x - a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="fricas")
[Out] 1/3*(a^4*x^4 + 5*a^3*x^3 + 18*a^2*x^2 - 24*a*x + 36*(a*x - 1)*log(a*x - 1)
- 12)/(a^4*x - a^3)
```

Sympy [A]
time = 0.08, size = 39, normalized size = 0.83

$$\frac{x^3}{3} - \frac{4}{a^4x - a^3} + \frac{2x^2}{a} + \frac{8x}{a^2} + \frac{12\log{(ax-1)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**2,x)
[Out] x**3/3 - 4/(a**4*x - a**3) + 2*x**2/a + 8*x/a**2 + 12*log(a*x - 1)/a**3
```

Giac [A]
time = 0.40, size = 69, normalized size = 1.47

$$\frac{(ax-1)^3\left(\frac{9}{ax-1} + \frac{39}{(ax-1)^2} + 1\right)}{3\,a^3} - \frac{12\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a^3} - \frac{4}{(ax-1)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^2,x, algorithm="giac")
[Out] 1/3*(a*x - 1)^3*(9/(a*x - 1) + 39/(a*x - 1)^2 + 1)/a^3 - 12*log(abs(a*x - 1
)/((a*x - 1)^2*abs(a)))/a^3 - 4/((a*x - 1)*a^3)
```

Mupad [B]
time = 1.17, size = 49, normalized size = 1.04

$$\frac{12\,\ln{(a\,x-1)}}{a^3} - \frac{4}{a\,\left(a^3\,x - a^2\right)} + \frac{8\,x}{a^2} + \frac{x^3}{3} + \frac{2\,x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (12*log(a*x - 1))/a^3 - 4/(a*(a^3*x - a^2)) + (8*x)/a^2 + x^3/3 + (2*x^2)/a
```

3.27 $\int e^{4\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=39

$$\frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}$$

```
[Out] 4*x/a+1/2*x^2+4/a^2/(-a*x+1)+8*ln(-a*x+1)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6261, 78}

$$\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}+\frac{4x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x,x]
```

```
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}x\,dx &= \int e^{4\tanh^{-1}(ax)}x\,dx\\ &= \int \frac{x(1+ax)^2}{(1-ax)^2}\,dx\\ &= \int \left(\frac{4}{a}+x+\frac{4}{a(-1+ax)^2}+\frac{8}{a(-1+ax)}\right)dx\\ &= \frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{4x}{a}+\frac{x^2}{2}+\frac{4}{a^2(1-ax)}+\frac{8\log(1-ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x,x]
[Out] (4*x)/a + x^2/2 + 4/(a^2*(1 - a*x)) + (8*Log[1 - a*x])/a^2
```

Maple [A]
time = 0.12, size = 39, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{2}+\frac{4x}{a}-\frac{4}{a^2(ax-1)}+\frac{8\ln(ax-1)}{a^2}$	36
default	$\frac{\frac{1}{2}a\,x^2+4x}{a}-\frac{4}{a^2(ax-1)}+\frac{8\ln(ax-1)}{a^2}$	39
norman	$\frac{\frac{7x^2}{2}+\frac{a\,x^3}{2}-\frac{8x}{a}}{ax-1}+\frac{8\ln(ax-1)}{a^2}$	39
meijerg	$\frac{\frac{ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+3\ln(-ax+1)}{a^2}-\frac{2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a^2}+\frac{\frac{ax}{-ax+1}+\ln(-ax+1)}{a^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x,x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*a*x^2+4*x)-4/a^2/(a*x-1)+8/a^2*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 41, normalized size = 1.05

$$\frac{ax^2+8\,x}{2\,a}-\frac{4}{a^3x-a^2}+\frac{8\,\log\left(ax-1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="maxima")
[Out] 1/2*(a*x^2 + 8*x)/a - 4/(a^3*x - a^2) + 8*log(a*x - 1)/a^2
```

Fricas [A]
time = 0.35, size = 49, normalized size = 1.26

$$\frac{a^3x^3 + 7\,a^2x^2 - 8\,ax + 16\,(ax-1)\log{(ax-1)} - 8}{2\,(a^3x - a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="fricas")
[Out] 1/2*(a^3*x^3 + 7*a^2*x^2 - 8*a*x + 16*(a*x - 1)*log(a*x - 1) - 8)/(a^3*x -
a^2)
```

Sympy [A]
time = 0.06, size = 31, normalized size = 0.79

$$\frac{x^2}{2} - \frac{4}{a^3x - a^2} + \frac{4x}{a} + \frac{8\log{(ax-1)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x,x)
[Out] x**2/2 - 4/(a**3*x - a**2) + 4*x/a + 8*log(a*x - 1)/a**2
```

Giac [A]
time = 0.40, size = 64, normalized size = 1.64

$$\frac{\frac{(ax-1)^2\left(\frac{10}{ax-1}+1\right)}{a} - \frac{16\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{8}{(ax-1)a}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x,x, algorithm="giac")
[Out] 1/2*((a*x - 1)^2*(10/(a*x - 1) + 1)/a - 16*log(abs(a*x - 1)/((a*x - 1)^2*ab
s(a)))/a - 8/((a*x - 1)*a))/a
```

Mupad [B]
time = 0.04, size = 38, normalized size = 0.97

$$\frac{8\,\ln{(a\,x-1)}}{a^2} + \frac{4\,x}{a} + \frac{x^2}{2} + \frac{4}{a\,(a - a^2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (8*log(a*x - 1))/a^2 + (4*x)/a + x^2/2 + 4/(a*(a - a^2*x))
```

3.28 $\int e^{4\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$x+\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}$$

```
[Out] x+4/a/(-a*x+1)+4*ln(-a*x+1)/a
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6260, 45}

$$\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}+x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x]),x]
```

```
[Out] x + 4/(a*(1 - a*x)) + (4*Log[1 - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6260

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\,dx = \int e^{4\tanh^{-1}(ax)}\,dx$$
$$= \int \frac{(1+ax)^2}{(1-ax)^2}\,dx$$
$$= \int \left(1+\frac{4}{(-1+ax)^2}+\frac{4}{-1+ax}\right)dx$$
$$= x+\frac{4}{a(1-ax)}+\frac{4\log(1-ax)}{a}$$

Mathematica [**A**]
time = 0.02, size = 26, normalized size = 0.96

$$x-\frac{4}{a(-1+ax)}+\frac{4\log(1-ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x]),x]
```

```
[Out] x - 4/(a*(-1 + a*x)) + (4*Log[1 - a*x])/a
```

Maple [**A**]
time = 0.12, size = 26, normalized size = 0.96

method	result	size
default	$x-\frac{4}{a(ax-1)}+\frac{4\ln(ax-1)}{a}$	26
risch	$x-\frac{4}{a(ax-1)}+\frac{4\ln(ax-1)}{a}$	26
norman	$\frac{a\,x^2-5x}{ax-1}+\frac{4\ln(ax-1)}{a}$	30
meijerg	$-\frac{-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)}{a}+\frac{\frac{2ax}{-ax+1}+2\ln(-ax+1)}{a}+\frac{x}{-ax+1}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x-4/a/(a*x-1)+4/a*ln(a*x-1)
```

Maxima [**A**]
time = 0.26, size = 26, normalized size = 0.96

$$x+\frac{4\,\log\left(ax-1\right)}{a}-\frac{4}{a^2x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="maxima")
```

```
[Out] x + 4*log(a*x - 1)/a - 4/(a^2*x - a)
```

Fricas [A]
time = 0.35, size = 38, normalized size = 1.41

$$\frac{a^2x^2 - ax + 4\,(ax - 1)\log\,(ax - 1) - 4}{a^2x - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - a*x + 4*(a*x - 1)*log(a*x - 1) - 4)/(a^2*x - a)
```

Sympy [A]
time = 0.06, size = 19, normalized size = 0.70

$$x - \frac{4}{a^2x - a} + \frac{4\log\,(ax - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2,x)
```

```
[Out] x - 4/(a**2*x - a) + 4*log(a*x - 1)/a
```

Giac [A]
time = 0.41, size = 46, normalized size = 1.70

$$\frac{ax - 1}{a} - \frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{4}{(ax - 1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/a - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4/((a*x - 1)*a)
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.93

$$x - \frac{4}{a\,(a\,x - 1)} + \frac{4\,\ln\,(a\,x - 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(a*x - 1)^2,x)
```

```
[Out] x - 4/(a*(a*x - 1)) + (4*log(a*x - 1))/a
```

3.29 $\int \frac{e^{4\coth^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=13

$$\frac{4}{1-ax}+\log(x)$$

```
[Out] 4/(-a*x+1)+ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 90}

$$\frac{4}{1-ax}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{x}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{x}\,dx$$
$$= \int \frac{(1+ax)^2}{x(1-ax)^2}\,dx$$
$$= \int \left(\frac{1}{x} + \frac{4a}{(-1+ax)^2}\right)dx$$
$$= \frac{4}{1-ax} + \log(x)$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{4}{1-ax} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x,x]
```

```
[Out] 4/(1 - a*x) + Log[x]
```

Maple [A]
time = 0.13, size = 13, normalized size = 1.00

method	result	size
default	$-\frac{4}{ax-1} + \ln(x)$	13
norman	$-\frac{4ax}{ax-1} + \ln(x)$	15
risch	$-\frac{4}{ax-1} + \ln(-x)$	15
meijerg	$\frac{3ax}{-ax+1} + \frac{2ax}{-2ax+2} + 1 + \ln(x) + \ln(-a)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -4/(a*x-1)+ln(x)
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.92

$$-\frac{4}{ax-1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="maxima")
[Out] -4/(a*x - 1) + log(x)
```

Fricas [A]
time = 0.34, size = 18, normalized size = 1.38

$$\frac{(ax-1)\log(x)-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="fricas")
[Out] ((a*x - 1)*log(x) - 4)/(a*x - 1)
```

Sympy [A]
time = 0.09, size = 8, normalized size = 0.62

$$\log(x)-\frac{4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x,x)
[Out] log(x) - 4/(a*x - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.
time = 0.40, size = 57, normalized size = 4.38

$$-a\left(\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}-\frac{\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{4}{(ax-1)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x,x, algorithm="giac")
[Out] -a*(log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - log(abs(-1/(a*x - 1) - 1))/a
 + 4/((a*x - 1)*a))
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.92

$$\ln(x)-\frac{4}{a\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x*(a*x - 1)^2),x)
[Out] log(x) - 4/(a*x - 1)
```

3.30 $\int \frac{e^{4\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=32

$$-\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

```
[Out] -1/x+4*a/(-a*x+1)+4*a*ln(x)-4*a*ln(-a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 90}

$$\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^2}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{x^2}\,dx$$
$$= \int \frac{(1+ax)^2}{x^2(1-ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^2}+\frac{4a}{x}+\frac{4a^2}{(-1+ax)^2}-\frac{4a^2}{-1+ax}\right)dx$$
$$= -\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$-\frac{1}{x}+\frac{4a}{1-ax}+4a\log(x)-4a\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^2,x]
```

```
[Out] -x^(-1) + (4*a)/(1 - a*x) + 4*a*Log[x] - 4*a*Log[1 - a*x]
```

Maple [A]
time = 0.16, size = 31, normalized size = 0.97

method	result
default	$-\frac{1}{x}+4a\ln(x)-\frac{4a}{ax-1}-4a\ln(ax-1)$
risch	$\frac{-5ax+1}{x(ax-1)}+4a\ln(-x)-4a\ln(ax-1)$
norman	$\frac{-5a^2x^2+1}{x(ax-1)}+4a\ln(x)-4a\ln(ax-1)$
meijerg	$\frac{a^2x}{-ax+1}+2a\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)-a\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+4*a*ln(x)-4*a/(a*x-1)-4*a*ln(a*x-1)
```

Maxima [A]
time = 0.26, size = 34, normalized size = 1.06

$$-4\,a\log(ax-1)+4\,a\log(x)-\frac{5\,ax-1}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="maxima")
```

```
[Out] -4*a*log(a*x - 1) + 4*a*log(x) - (5*a*x - 1)/(a*x^2 - x)
```

Fricas [A]
time = 0.33, size = 55, normalized size = 1.72

$$-\frac{5\,ax + 4\,(a^2x^2 - ax)\log\left(ax - 1\right) - 4\,(a^2x^2 - ax)\log\left(x\right) - 1}{ax^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(5*a*x + 4*(a^2*x^2 - a*x)*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 1)/(a
*x^2 - x)
```

Sympy [A]
time = 0.12, size = 26, normalized size = 0.81

$$4a\left(\log\left(x\right) - \log\left(x - \frac{1}{a}\right)\right) + \frac{-5ax + 1}{ax^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**2,x)
```

```
[Out] 4*a*(log(x) - log(x - 1/a)) + (-5*a*x + 1)/(a*x**2 - x)
```

Giac [A]
time = 0.40, size = 40, normalized size = 1.25

$$4\,a\log\left(\left|-\frac{1}{ax - 1} - 1\right|\right) - \frac{4\,a}{ax - 1} + \frac{a}{\frac{1}{ax-1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^2,x, algorithm="giac")
```

```
[Out] 4*a*log(abs(-1/(a*x - 1) - 1)) - 4*a/(a*x - 1) + a/(1/(a*x - 1) + 1)
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.88

$$8\,a\,\mathrm{atanh}(2\,a\,x - 1) + \frac{5\,a\,x - 1}{x - a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x^2*(a*x - 1)^2),x)
```

```
[Out] 8*a*atanh(2*a*x - 1) + (5*a*x - 1)/(x - a*x^2)
```

3.31 $\int \frac{e^{4\coth^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=46

$$-\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)$$

```
[Out] -1/2/x^2-4*a/x+4*a^2/(-a*x+1)+8*a^2*ln(x)-8*a^2*ln(-a*x+1)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 90}

$$\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)-\frac{4a}{x}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^3}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{x^3}\,dx$$
$$= \int \frac{(1+ax)^2}{x^3(1-ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^3}+\frac{4a}{x^2}+\frac{8a^2}{x}+\frac{4a^3}{(-1+ax)^2}-\frac{8a^3}{-1+ax}\right)dx$$
$$= -\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{4a}{x}+\frac{4a^2}{1-ax}+8a^2\log(x)-8a^2\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/2*1/x^2 - (4*a)/x + (4*a^2)/(1 - a*x) + 8*a^2*Log[x] - 8*a^2*Log[1 - a*x
]
```

Maple [A]
time = 0.18, size = 43, normalized size = 0.93

method	result
default	$-\frac{1}{2x^2}-\frac{4a}{x}+8a^2\ln(x)-\frac{4a^2}{ax-1}-8a^2\ln(ax-1)$
norman	$\frac{\frac{1}{2}-8a^3x^3+\frac{7}{2}ax}{(ax-1)x^2}+8a^2\ln(x)-8a^2\ln(ax-1)$
risch	$\frac{-8a^2x^2+\frac{7}{2}ax+\frac{1}{2}}{(ax-1)x^2}-8a^2\ln(ax-1)+8a^2\ln(-x)$
meijerg	$a^2\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)-2a^2\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-4*a/x+8*a^2*ln(x)-4*a^2/(a*x-1)-8*a^2*ln(a*x-1)
```

Maxima [A]
time = 0.25, size = 48, normalized size = 1.04

$$-8\,a^2\log(ax-1)+8\,a^2\log(x)-\frac{16\,a^2x^2-7\,ax-1}{2\,(ax^3-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*log(a*x - 1) + 8*a^2*log(x) - 1/2*(16*a^2*x^2 - 7*a*x - 1)/(a*x^3 -
x^2)
```

Fricas [A]
time = 0.33, size = 73, normalized size = 1.59

$$-\frac{16\,a^2x^2-7\,ax+16\,(a^3x^3-a^2x^2)\log\left(ax-1\right)-16\,(a^3x^3-a^2x^2)\log\left(x\right)-1}{2\,(ax^3-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(16*a^2*x^2 - 7*a*x + 16*(a^3*x^3 - a^2*x^2)*log(a*x - 1) - 16*(a^3*x^
3 - a^2*x^2)*log(x) - 1)/(a*x^3 - x^2)
```

Sympy [A]
time = 0.14, size = 41, normalized size = 0.89

$$8a^2\left(\log\left(x\right)-\log\left(x-\frac{1}{a}\right)\right)+\frac{-16a^2x^2+7ax+1}{2ax^3-2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**3,x)
```

```
[Out] 8*a**2*(log(x) - log(x - 1/a)) + (-16*a**2*x**2 + 7*a*x + 1)/(2*a*x**3 - 2*
x**2)
```

Giac [A]
time = 0.40, size = 62, normalized size = 1.35

$$8\,a^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^2}{ax-1}+\frac{9\,a^2+\frac{10\,a^2}{ax-1}}{2\left(\frac{1}{ax-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^3,x, algorithm="giac")
```

```
[Out] 8*a^2*log(abs(-1/(a*x - 1) - 1)) - 4*a^2/(a*x - 1) + 1/2*(9*a^2 + 10*a^2/(a
*x - 1))/(1/(a*x - 1) + 1)^2
```

Mupad [B]
time = 1.20, size = 41, normalized size = 0.89

$$16\,a^2\,\mathrm{atanh}(2\,a\,x-1)+\frac{-8\,a^2\,x^2+\frac{7\,a\,x}{2}+\frac{1}{2}}{a\,x^3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x^3*(a*x - 1)^2),x)
[Out] 16*a^2*atanh(2*a*x - 1) + ((7*a*x)/2 - 8*a^2*x^2 + 1/2)/(a*x^3 - x^2)
```

3.32 $\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{8a^2}{x}+\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)$$

```
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+4*a^3/(-a*x+1)+12*a^3*ln(x)-12*a^3*ln(-a*x+1)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6302, 6261, 90}

$$\frac{4a^3}{1-ax}+12a^3\log(x)-12a^3\log(1-ax)-\frac{8a^2}{x}-\frac{2a}{x^2}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{x^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{x^4}\,dx$$
$$= \int \frac{(1+ax)^2}{x^4(1-ax)^2}\,dx$$
$$= \int \left(\frac{1}{x^4} + \frac{4a}{x^3} + \frac{8a^2}{x^2} + \frac{12a^3}{x} + \frac{4a^4}{(-1+ax)^2} - \frac{12a^4}{-1+ax}\right) dx$$
$$= -\frac{1}{3x^3} - \frac{2a}{x^2} - \frac{8a^2}{x} + \frac{4a^3}{1-ax} + 12a^3\log(x) - 12a^3\log(1-ax)$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 1.00

$$-\frac{1}{3x^3} - \frac{2a}{x^2} - \frac{8a^2}{x} + \frac{4a^3}{1-ax} + 12a^3\log(x) - 12a^3\log(1-ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/x^4,x]
[Out] -1/3*1/x^3 - (2*a)/x^2 - (8*a^2)/x + (4*a^3)/(1 - a*x) + 12*a^3*Log[x] - 12
*a^3*Log[1 - a*x]
```

Maple [A]
time = 0.19, size = 51, normalized size = 0.94

method	result
default	$-\frac{1}{3x^3} - \frac{2a}{x^2} - \frac{8a^2}{x} + 12a^3\ln(x) - \frac{4a^3}{ax-1} - 12a^3\ln(ax-1)$
norman	$\frac{\frac{1}{3} - 12a^4x^4 + \frac{5}{3}ax + 6a^2x^2}{(ax-1)x^3} + 12a^3\ln(x) - 12a^3\ln(ax-1)$
risch	$\frac{-12a^3x^3 + 6a^2x^2 + \frac{5}{3}ax + \frac{1}{3}}{(ax-1)x^3} - 12a^3\ln(ax-1) + 12a^3\ln(-x)$
meijerg	$-a^3\left(-\frac{3ax}{-3ax+3} + 2\ln(-ax+1) - 1 - 2\ln(x) - 2\ln(-a) + \frac{1}{ax}\right) + 2a^3\left(\frac{4ax}{-4ax+4} - 3\ln(-ax+1) +\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3/x^3-2*a/x^2-8*a^2/x+12*a^3*ln(x)-4*a^3/(a*x-1)-12*a^3*ln(a*x-1)
```

Maxima [A]
time = 0.25, size = 56, normalized size = 1.04

$$-12\,a^3\log(ax-1) + 12\,a^3\log(x) - \frac{36\,a^3x^3 - 18\,a^2x^2 - 5\,ax - 1}{3\,(ax^4 - x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="maxima")
```

```
[Out] -12*a^3*log(a*x - 1) + 12*a^3*log(x) - 1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x
 - 1)/(a*x^4 - x^3)
```

Fricas [A]
time = 0.35, size = 81, normalized size = 1.50

$$-\frac{36\,a^3x^3-18\,a^2x^2-5\,ax+36\,(a^4x^4-a^3x^3)\log{(ax-1)}-36\,(a^4x^4-a^3x^3)\log{(x)}-1}{3\,(ax^4-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(36*a^3*x^3 - 18*a^2*x^2 - 5*a*x + 36*(a^4*x^4 - a^3*x^3)*log(a*x - 1)
 - 36*(a^4*x^4 - a^3*x^3)*log(x) - 1)/(a*x^4 - x^3)
```

Sympy [A]
time = 0.17, size = 49, normalized size = 0.91

$$12a^3\left(\log{(x)}-\log{\left(x-\frac{1}{a}\right)}\right)+\frac{-36a^3x^3+18a^2x^2+5ax+1}{3ax^4-3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/x**4,x)
```

```
[Out] 12*a**3*(log(x) - log(x - 1/a)) + (-36*a**3*x**3 + 18*a**2*x**2 + 5*a*x + 1
)/(3*a*x**4 - 3*x**3)
```

Giac [A]
time = 0.40, size = 74, normalized size = 1.37

$$12\,a^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)-\frac{4\,a^3}{ax-1}+\frac{31\,a^3+\frac{69\,a^3}{ax-1}+\frac{39\,a^3}{(ax-1)^2}}{3\left(\frac{1}{ax-1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/x^4,x, algorithm="giac")
```

```
[Out] 12*a^3*log(abs(-1/(a*x - 1) - 1)) - 4*a^3/(a*x - 1) + 1/3*(31*a^3 + 69*a^3/
(a*x - 1) + 39*a^3/(a*x - 1)^2)/(1/(a*x - 1) + 1)^3
```

Mupad [B]
time = 0.06, size = 49, normalized size = 0.91

$$24\,a^3\,\mathrm{atanh}(2\,a\,x-1)+\frac{-12\,a^3\,x^3+6\,a^2\,x^2+\frac{5\,a\,x}{3}+\frac{1}{3}}{a\,x^4-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/(x^4*(a*x - 1)^2),x)
```

```
[Out] 24*a^3*atanh(2*a*x - 1) + ((5*a*x)/3 + 6*a^2*x^2 - 12*a^3*x^3 + 1/3)/(a*x^4
 - x^3)
```

3.33 $\int e^{-\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=114

$$-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] 3/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-2/3*x*(1-1/a^2/x^2)^(1/2)/a^3+3/8*x^2*
(1-1/a^2/x^2)^(1/2)/a^2-1/3*x^3*(1-1/a^2/x^2)^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)
^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6304, 849, 821, 272, 65, 214}

$$\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^3) + (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2
) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (3*
ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{4}{a}-\frac{3x}{a^2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{9}{a^2}-\frac{8x}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{24}\text{Subst}\left(\int \frac{\frac{16}{a^3}-\frac{9x}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,d\right.$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{3\text{Subst}}{}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{3\text{Subst}}{}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{3\text{Subst}}{}$$

$$= -\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^3} + \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3a} + \frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{3\tanh^{-}}{}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.60

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-16+9ax-8a^2x^2+6a^3x^3)+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{24a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^ArcCoth[a*x],x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-16 + 9*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*Log[(1
 + Sqrt[1 - 1/(a^2*x^2)])*x])/(24*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(94) = 188.
time = 0.09, size = 193, normalized size = 1.69

method	result
risch	$\frac{(6a^3x^3-8a^2x^2+9ax-16)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{24a^4}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{8a^3\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+15\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-8((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}+24a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax}{\sqrt{a}}\right.\right.}{24\sqrt{(ax+1)(ax-1)}\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+1
5*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-8*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+2
4*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))-15*ln((a^2*
x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-24*(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(1/2))/((a*x+1)*(a*x-1))^(1/2)/a^4/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(94) = 188.
time = 0.26, size = 203, normalized size = 1.78

$$-\frac{1}{24}a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-31\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*a*(2*(39*((a*x - 1)/(a*x + 1))^(7/2) - 31*((a*x - 1)/(a*x + 1))^(5/2)
 + 49*((a*x - 1)/(a*x + 1))^(3/2) - 9*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x -
1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x +
 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^5 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5)
```

Fricas [A]
time = 0.34, size = 91, normalized size = 0.80

$$\frac{(6\,a^4x^4-2\,a^3x^3+a^2x^2-7\,ax-16)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 7*a*x - 16)*sqrt((a*x - 1)/(a*x +
1)) + 9*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)
) - 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.21, size = 172, normalized size = 1.51

$$\frac{3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}-\frac{\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{49\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{31\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{13\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((3*((a*x - 1)/(a*x + 1))^
(1/2))/4 - (49*((a*x - 1)/(a*x + 1))^(3/2))/12 + (31*((a*x - 1)/(a*x + 1))^
(5/2))/12 - (13*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^2)/
(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)
^4 - (4*a^4*(a*x - 1))/(a*x + 1))
```

3.34 $\int e^{-\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] -1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+2/3*x*(1-1/a^2/x^2)^(1/2)/a^2-1/2*x^2
*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6304, 849, 821, 272, 65, 214}

$$-\frac{x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (
Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(2*a^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{1}{3}\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{2x}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{4}{a^2}-\frac{3x}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \cdots\right.}{2a^3 \cdots}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \cdots\right.}{4a^3 \cdots}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\cdots}\right.}{2a \cdots}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.67

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(4-3ax+2a^2x^2) - 3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^ArcCoth[a*x],x]
```

[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 - 3*a*x + 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(74) = 148$.
time = 0.09, size = 173, normalized size = 1.92

method	result
risch	$\frac{(2a^2x^2-3ax+4)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{6a^3}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{2a^2\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+6\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}+3\ln\left(\frac{a^2x+\sqrt{a^2}}{}\right.\right.}{6\sqrt{(ax+1)(ax-1)}\,a^3\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(2*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+6*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-6*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x+1)*(a*x-1))^(1/2)/a^3/(a^2)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(74) = 148$.
time = 0.25, size = 166, normalized size = 1.84

$$-\frac{1}{6}\,a\left(\frac{2\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")

[Out] -1/6*a*(2*(9*((a*x - 1)/(a*x + 1))^(5/2) - 4*((a*x - 1)/(a*x + 1))^(3/2) + 3*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^4 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4)

Fricas [A]
time = 0.34, size = 83, normalized size = 0.92

$$\frac{(2\,a^3x^3-a^2x^2+ax+4)\sqrt{\frac{ax-1}{ax+1}}-3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 - a^2*x^2 + a*x + 4)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt
((a*x - 1)/(a*x + 1)) + 1) + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.42, size = 86, normalized size = 0.96

$$\frac{1}{6}\sqrt{a^2x^2-1}\left(x\left(\frac{2\,x\mathrm{sgn}(ax+1)}{a}-\frac{3\,\mathrm{sgn}(ax+1)}{a^2}\right)+\frac{4\,\mathrm{sgn}(ax+1)}{a^3}\right)+\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*(x*(2*x*sgn(a*x + 1)/a - 3*sgn(a*x + 1)/a^2) + 4*sgn(
a*x + 1)/a^3) + 1/2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a
^2*abs(a))
```

Mupad [B]
time = 0.05, size = 134, normalized size = 1.49

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}-\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2) - (4*((a*x - 1)/(a*x + 1))^(3/2))/3 + 3*((a*x
- 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a*x -
 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - atanh(((a*x - 1)/(a*x +
 1))^(1/2))/a^3
```

3.35 $\int e^{-\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 1/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-x*(1-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a
^2/x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6304, 849, 821, 272, 65, 214}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/a) + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]]/(2*a^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}x\,dx &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{2}\text{Subst}\left(\int \frac{\frac{2}{a}-\frac{x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a^2}\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{2}\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.77

$$
\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+ax)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^ArcCoth[a*x],x]
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])
/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(54) = 108$.
time = 0.08, size = 152, normalized size = 2.38

method	result
risch	$\frac{(ax-2)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^2}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{2a\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+2\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-2a}{2\sqrt{(ax+1)(ax-1)}\,a^2\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+2*
(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)
)/(a^2)^(1/2))*a-2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(
1/2)))/((a*x+1)*(a*x-1))^(1/2)/a^2/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(54) = 108$.
time = 0.26, size = 130, normalized size = 2.03

$$-\frac{1}{2}\,a\left(\frac{2\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*((a*x - 1)/(a*x + 1))^(3/2) - sqrt((a*x - 1)/(a*x + 1)))/(2*(a
*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^3 + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3)
```

Fricas [A]
time = 0.34, size = 73, normalized size = 1.14

$$\frac{(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)) + log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.42, size = 71, normalized size = 1.11

$$\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{x\mathrm{sgn}(ax+1)}{a}-\frac{2\,\mathrm{sgn}(ax+1)}{a^2}\right)-\frac{\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*(x*sgn(a*x + 1)/a - 2*sgn(a*x + 1)/a^2) - 1/2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(a*abs(a))
```

Mupad [B]
time = 0.06, size = 97, normalized size = 1.52

$$\frac{\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] atanh(((a*x - 1)/(a*x + 1))^(1/2))/a^2 - (((a*x - 1)/(a*x + 1))^(1/2) - 3*(
(a*x - 1)/(a*x + 1))^(3/2))/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(
a*x - 1))/(a*x + 1))
```

3.36 $\int e^{-\coth^{-1}(ax)} dx$

Optimal. Leaf size=37

$$\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a+x*(1-1/a^2/x^2)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6303, 821, 272, 65, 214}

$$x\sqrt{1-\frac{1}{a^2x^2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^(-ArcCoth[a*x]),x]

[Out] Sqrt[1 - 1/(a^2*x^2)]*x - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/a

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x - a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)\\
&= \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.14

$$\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-ArcCoth[a*x]),x]
[Out] Sqrt[1 - 1/(a^2*x^2)]*x - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(33) = 66$.
time = 0.08, size = 98, normalized size = 2.65

method	result	size
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}\,(ax-1)}$	91
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}-a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)\right)}{\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*((a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-a*ln((
a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x+1)*(a*x-1))^
(1/2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.
time = 0.26, size = 90, normalized size = 2.43

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/a^2 - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.36, size = 64, normalized size = 1.73

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.41, size = 52, normalized size = 1.41

$$\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|} + \frac{\sqrt{a^2x^2-1}\operatorname{sgn}(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2
- 1)*sgn(a*x + 1)/a
```

Mupad [B]
time = 1.19, size = 58, normalized size = 1.57

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}} - \frac{2\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.37 $\int \frac{e^{-\coth^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=20

$$\csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right)$$

```
[Out] arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6304, 858, 222, 272, 65, 214}

$$\tanh^{-1}\left(\sqrt{1 - \frac{1}{a^2x^2}}\right) + \csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x),x]
```

```
[Out] ArcCsc[a*x] + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$

$$= \csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.70

$$\text{ArcSin}\left(\frac{1}{ax}\right) + \log\left(x\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x),x]
```

```
[Out] ArcSin[1/(a*x)] + Log[x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(18) = 36$.
time = 0.07, size = 133, normalized size = 6.65

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}-a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)-\sqrt{a^2x^2-1}\,\sqrt{a^2}\right.}{\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*((a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-a*ln(
(a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))-(a^2*x^2-1)^(1/2)*
(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1/2
)/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.
time = 0.46, size = 70, normalized size = 3.50

$$-a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a - log(sqrt((a*x - 1)/(a*x + 1)) +
 1)/a + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(18) = 36$.
time = 0.35, size = 57, normalized size = 2.85

$$-2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sqrt((a*x - 1)/(a*x + 1)) + 1) -
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(18) = 36.
time = 0.41, size = 59, normalized size = 2.95

$$-2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)-\frac{a\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) - a*log(abs(-x*abs(a)
 + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a)
```

Mupad [**B**]
time = 0.03, size = 37, normalized size = 1.85

$$2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x,x)
```

```
[Out] 2*atanh(((a*x - 1)/(a*x + 1))^(1/2)) - 2*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.38 $\int \frac{e^{-\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=25

$$-a\sqrt{1-\frac{1}{a^2x^2}}-a\csc^{-1}(ax)$$

```
[Out] -a*arccsc(a*x)-a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6304, 655, 222}

$$a\left(-\csc^{-1}(ax)\right)-a\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -(a*Sqrt[1 - 1/(a^2*x^2)]) - a*ArcCsc[a*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt{1-\frac{1}{a^2x^2}} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt{1-\frac{1}{a^2x^2}} - a\csc^{-1}(ax)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.04

$$-a\left(\sqrt{1-\frac{1}{a^2x^2}} + \text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -(a*(Sqrt[1 - 1/(a^2*x^2)] + ArcSin[1/(a*x)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(23) = 46$.
time = 0.08, size = 220, normalized size = 8.80

method	result
risch	$-\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x} - \frac{a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{ax-1}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,ax-\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-}{\sqrt{a^2}}\right.\right.}{\sqrt{(ax+1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+((a
*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1)
)^(1/2))/(a^2)^(1/2))*a^2*x+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2)^(1/2)*(a^2*
```

```
x^2-1)^(1/2)*a*x-(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x+ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x)/((a*x+1)*(a*x-1))^(1/2)/(a^2
)^(1/2)/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(23) = 46$.
time = 0.47, size = 55, normalized size = 2.20

$$-2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}-\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima")
[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*x + 1) + 1) - arctan(sqrt((a*
x - 1)/(a*x + 1))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.
time = 0.34, size = 47, normalized size = 1.88

$$\frac{2\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas")
[Out] (2*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*x + 1)*sqrt((a*x - 1)/(a*x +
1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**2,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.20, size = 55, normalized size = 2.20

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{2\,a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^2,x)
```

```
[Out] 2*a*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (2*a*((a*x - 1)/(a*x + 1))^(1/2))/(
(a*x - 1)/(a*x + 1) + 1)
```

3.39 $\int \frac{e^{-\coth^{-1}(ax)}}{x^3} dx$

Optimal. Leaf size=40

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

```
[Out] 1/2*a^2*arccsc(a*x)+1/2*a*(2*a-1/x)*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6304, 794, 222}

$$\frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right)+\frac{1}{2}a^2\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(2*a - x^(-1)))/2 + (a^2*ArcCsc[a*x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right) + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{1}{x}\right) + \frac{1}{2}a^2\csc^{-1}(ax)$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 1.02

$$\frac{a\left(\sqrt{1-\frac{1}{a^2x^2}}(-1+2ax)+ax\text{ArcSin}\left(\frac{1}{ax}\right)\right)}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^3),x]
[Out] (a*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + 2*a*x) + a*x*ArcSin[1/(a*x)]))/(2*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(34) = 68.
time = 0.09, size = 260, normalized size = 6.50

method	result
risch	$\frac{(ax+1)(2ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^2} + \frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{2ax-2}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+2\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2-2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*
x^3+2*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-2*ln((a^2*x+(a^2)^(1/2)*(
```

```
(a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2
)*a*x-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-(a^2)^(1/2)*arctan(1/(a^2*x^2-1
)^(1/2))*a^2*x^2+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^
3*x^2-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)/x^
2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.
time = 0.46, size = 93, normalized size = 2.32

$$-\left(a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{3\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -(a*arctan(sqrt((a*x - 1)/(a*x + 1))) - (3*a*((a*x - 1)/(a*x + 1))^(3/2) +
a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)
^2 + 1))*a
```

Fricas [A]
time = 0.34, size = 60, normalized size = 1.50

$$-\frac{2\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(2\,a^2x^2+ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^2*x^2 + a*x - 1)*s
qrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(34) = 68$.
time = 0.41, size = 157, normalized size = 3.92

$$-a^{2} \arctan\left(-x|a| + \sqrt{a^{2}x^{2} - 1}\right) \operatorname{sgn}(ax + 1) + \frac{\left(x|a| - \sqrt{a^{2}x^{2} - 1}\right)^{3} a^{2}\operatorname{sgn}(ax + 1) + 2\left(x|a| - \sqrt{a^{2}x^{2} - 1}\right)^{2} a|a|\operatorname{sgn}(ax + 1) - \left(x|a| - \sqrt{a^{2}x^{2} - 1}\right) a^{2}\operatorname{sgn}(ax + 1) + 2\,a|a|\operatorname{sgn}(ax + 1)}{\left(\left(x|a| - \sqrt{a^{2}x^{2} - 1}\right)^{2} + 1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -a^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + ((x*abs(a) - sqrt
(a^2*x^2 - 1))^3*a^2*sgn(a*x + 1) + 2*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*ab
s(a)*sgn(a*x + 1) - (x*abs(a) - sqrt(a^2*x^2 - 1))*a^2*sgn(a*x + 1) + 2*a*a
bs(a)*sgn(a*x + 1))/((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2
```

Mupad [B]
time = 1.20, size = 82, normalized size = 2.05

$$a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x^2}-a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+\frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^3,x)
```

```
[Out] a^2*((a*x - 1)/(a*x + 1))^(1/2) - ((a*x - 1)/(a*x + 1))^(1/2)/(2*x^2) - a^2
*atan(((a*x - 1)/(a*x + 1))^(1/2)) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(2*x)
```

3.40 $\int \frac{e^{-\coth^{-1}(ax)}}{x^4} \, dx$

Optimal. Leaf size=76

$$-a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)$$

```
[Out] 1/3*a^3*(1-1/a^2/x^2)^(3/2)-1/2*a^3*arccsc(a*x)-a^3*(1-1/a^2/x^2)^(1/2)+1/2
*a^2*(1-1/a^2/x^2)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6304, 811, 655, 201, 222}

$$-\frac{1}{2}a^3\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-a^3\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (a^3*(1 - 1/(a^2*x^2))^(3/2))/3 + (a^2*Sqrt[
1 - 1/(a^2*x^2)])/(2*x) - (a^3*ArcCsc[a*x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 811

```
Int[(x_)^2*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dis
t[1/c, Int[(f + g*x)*(a + c*x^2)^(p + 1), x], x] - Dist[a/c, Int[(f + g*x)*
(a + c*x^2)^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && EqQ[a*g^2 + f^2*c, 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{x^4}\,dx &= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\left(a^2\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\right)+a^2\text{Subst}\left(\int \left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}\,dx,x,\frac{1}{x}\right)\\
&= -a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-a^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)+a^2\text{Subst}\Bigg(\\
&= -a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-a^3\csc^{-1}(ax)+\frac{1}{2}a^2\text{Subst}\Bigg(\int\\
&= -a^3\sqrt{1-\frac{1}{a^2x^2}}+\frac{1}{3}a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{2x}-\frac{1}{2}a^3\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 0.68

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,(2-3ax+4a^2x^2)}{6x^2}-\frac{1}{2}a^3\text{ArcSin}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] -1/6*(a*Sqrt[1 - 1/(a^2*x^2)]*(2 - 3*a*x + 4*a^2*x^2))/x^2 - (a^3*ArcSin[1/
(a*x)])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(64) = 128$.
time = 0.09, size = 284, normalized size = 3.74

method	result
risch	$-\frac{(ax+1)(4a^2x^2-3ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6x^3}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{2(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3-6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a \cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x
^4+6*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-6*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x
^3+3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-6*(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2)*a^3*x^3-6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*a^4*x^3+3*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-2*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2))/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)/x^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(64) = 128$.
time = 0.46, size = 137, normalized size = 1.80

$$\frac{1}{3}\left(3\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+4\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (9*a^2*((a*x - 1)/(a*x + 1))
^(5/2) + 4*a^2*((a*x - 1)/(a*x + 1))^(3/2) + 3*a^2*sqrt((a*x - 1)/(a*x + 1)
))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x +
1)^3 + 1))*a
```

Fricas [A]
time = 0.34, size = 68, normalized size = 0.89

$$\frac{6\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(4\,a^3x^3+a^2x^2-ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (4*a^3*x^3 + a^2*x^2 - a
*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.20, size = 105, normalized size = 1.38

$$a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x^3}-\frac{2\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}-\frac{a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x}+\frac{a\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^4,x)
```

```
[Out] a^3*atan(((a*x - 1)/(a*x + 1))^(1/2)) - ((a*x - 1)/(a*x + 1))^(1/2)/(3*x^3)
 - (2*a^3*((a*x - 1)/(a*x + 1))^(1/2))/3 - (a^2*((a*x - 1)/(a*x + 1))^(1/2)
)/(6*x) + (a*((a*x - 1)/(a*x + 1))^(1/2))/(6*x^2)
```

3.41 $\int \frac{e^{-\coth^{-1}(ax)}}{x^5} dx$

Optimal. Leaf size=88

$$\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{3}{8}a^4\csc^{-1}(ax)$$

```
[Out] 3/8*a^4*arccsc(a*x)+1/24*a^3*(16*a-9/x)*(1-1/a^2/x^2)^(1/2)-1/4*a*(1-1/a^2/
x^2)^(1/2)/x^3+1/3*a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6304, 847, 794, 222}

$$\frac{3}{8}a^4\csc^{-1}(ax)+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)]*(16*a - 9/x))/24 - (a*Sqrt[1 - 1/(a^2*x^2)])/(4*
x^3) + (a^2*Sqrt[1 - 1/(a^2*x^2)])/(3*x^2) + (3*a^4*ArcCsc[a*x])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
```

```
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{x^5}\,dx &= -\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{1}{4}a^2\text{Subst}\left(\int \frac{x^2\left(\frac{3}{a}-\frac{4x}{a^2}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}-\frac{1}{12}a^4\text{Subst}\left(\int \frac{x\left(\frac{8}{a^2}-\frac{9x}{a^3}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \right.\\
&= \frac{1}{24}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(16a-\frac{9}{x}\right)-\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}+\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{3x^2}+\frac{3}{8}a^4\csc^{-1}(ax)
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 0.67

$$\frac{1}{24}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(-6+8ax-9a^2x^2+16a^3x^3\right)}{x^3}+9a^3\text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*x^5),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(-6 + 8*a*x - 9*a^2*x^2 + 16*a^3*x^3))/x^3 + 9*a
^3*ArcSin[1/(a*x)]))/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(74) = 148$.
time = 0.10, size = 308, normalized size = 3.50

method	result
risch	$\frac{(ax+1)(16a^3x^3-9a^2x^2+8ax-6)\sqrt{\frac{ax-1}{ax+1}}}{24x^4}+\frac{3a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)(ax-1)}}{8(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-24\sqrt{a^2x^2-1}\sqrt{a^2}a^5x^5+24\sqrt{a^2}\sqrt{(ax+1)(ax-1)}a^4x^4+24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^3x^3-9\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^
5*x^5+24*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+24*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*a^3*x^3-9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4-9*a^4*x^4*(a^2)^
(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-24*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1)
)^(1/2))/(a^2)^(1/2))*a^5*x^4+24*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(
a^2)^(1/2))*a^5*x^4-15*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+8*(a^2)^(1/2)*
(a^2*x^2-1)^(3/2)*a*x-6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1
/2)/x^4/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(74) = 148$.
time = 0.48, size = 173, normalized size = 1.97

$$-\frac{1}{12}\left(9\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\frac{39\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+31\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+49\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+9\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^2}{(ax+1)^2}+\frac{4\,(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (39*a^3*((a*x - 1)/(a*x +
1))^(7/2) + 31*a^3*((a*x - 1)/(a*x + 1))^(5/2) + 49*a^3*((a*x - 1)/(a*x + 1
))^(3/2) + 9*a^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x
 - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 +
 1))*a
```

Fricas [A]
time = 0.34, size = 77, normalized size = 0.88

$$-\frac{18\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(16\,a^4x^4+7\,a^3x^3-a^2x^2+2\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(18*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (16*a^4*x^4 + 7*a^3*x
^3 - a^2*x^2 + 2*a*x - 6)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(74) = 148.
time = 0.42, size = 258, normalized size = 2.93

$$-\frac{3}{4}\,a^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^7a^4\operatorname{sgn}(ax+1)+33\left(x|a|-\sqrt{a^2x^2-1}\right)^5a^4\operatorname{sgn}(ax+1)+48\left(x|a|-\sqrt{a^2x^2-1}\right)^4a^3|a|\operatorname{sgn}(ax+1)-33\left(x|a|-\sqrt{a^2x^2-1}\right)^3a^4\operatorname{sgn}(ax+1)+64\left(x|a|-\sqrt{a^2x^2-1}\right)^2a^3|a|\operatorname{sgn}(ax+1)-9\left(x|a|-\sqrt{a^2x^2-1}\right)a^4\operatorname{sgn}(ax+1)+16\,a^3|a|\operatorname{sgn}(ax+1)}{12\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -3/4*a^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1) + 1/12*(9*(x*ab
s(a) - sqrt(a^2*x^2 - 1))^7*a^4*sgn(a*x + 1) + 33*(x*abs(a) - sqrt(a^2*x^2
- 1))^5*a^4*sgn(a*x + 1) + 48*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a^3*abs(a)*s
gn(a*x + 1) - 33*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*a^4*sgn(a*x + 1) + 64*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a^3*abs(a)*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(
a^2*x^2 - 1))*a^4*sgn(a*x + 1) + 16*a^3*abs(a)*sgn(a*x + 1))/((x*abs(a) - s
qrt(a^2*x^2 - 1))^2 + 1)^4
```

Mupad [B]
time = 1.22, size = 129, normalized size = 1.47

$$\frac{2\,a^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{3}-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,x^4}-\frac{3\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}-\frac{a^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x^2}+\frac{7\,a^3\sqrt{\frac{a\,x-1}{a\,x+1}}}{24\,x}+\frac{a\sqrt{\frac{a\,x-1}{a\,x+1}}}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/x^5,x)
```

```
[Out] (2*a^4*((a*x - 1)/(a*x + 1))^(1/2))/3 - ((a*x - 1)/(a*x + 1))^(1/2)/(4*x^4)
 - (3*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - (a^2*((a*x - 1)/(a*x + 1))
^(1/2))/(24*x^2) + (7*a^3*((a*x - 1)/(a*x + 1))^(1/2))/(24*x) + (a*((a*x -
1)/(a*x + 1))^(1/2))/(12*x^3)
```

3.42 $\int e^{-2\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=42

$$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

[Out] -2*x/a^3+x^2/a^2-2/3*x^3/a+1/4*x^4+2*ln(a*x+1)/a^4

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$\frac{2\log(ax+1)}{a^4}-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[x^3/E^(2*ArcCoth[a*x]),x]

[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^3\,dx = -\int e^{-2\tanh^{-1}(ax)}x^3\,dx$$
$$= -\int \frac{x^3(1-ax)}{1+ax}\,dx$$
$$= -\int \left(\frac{2}{a^3}-\frac{2x}{a^2}+\frac{2x^2}{a}-x^3-\frac{2}{a^3(1+ax)}\right)dx$$
$$= -\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\log(1+ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(2*ArcCoth[a*x]),x]
[Out] (-2*x)/a^3 + x^2/a^2 - (2*x^3)/(3*a) + x^4/4 + (2*Log[1 + a*x])/a^4
```

Maple [A]
time = 0.10, size = 42, normalized size = 1.00

method	result	size
norman	$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\ln(ax+1)}{a^4}$	39
risch	$-\frac{2x}{a^3}+\frac{x^2}{a^2}-\frac{2x^3}{3a}+\frac{x^4}{4}+\frac{2\ln(ax+1)}{a^4}$	39
default	$\frac{\frac{1}{4}a^3x^4-\frac{2}{3}a^2x^3+a\,x^2-2x}{a^3}+\frac{2\ln(ax+1)}{a^4}$	42
meijerg	$\frac{-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)}{a^4}-\frac{\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)}{a^4}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(1/4*a^3*x^4-2/3*a^2*x^3+a*x^2-2*x)+2*ln(a*x+1)/a^4
```

Maxima [A]
time = 0.25, size = 43, normalized size = 1.02

$$\frac{3\,a^3x^4-8\,a^2x^3+12\,ax^2-24\,x}{12\,a^3}+\frac{2\,\log\left(ax+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^4 - 8*a^2*x^3 + 12*a*x^2 - 24*x)/a^3 + 2*log(a*x + 1)/a^4
```

Fricas [A]
time = 0.33, size = 42, normalized size = 1.00

$$\frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax + 24\,\log\left(ax + 1\right)}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x + 24*log(a*x + 1))/a^4
```

Sympy [A]
time = 0.04, size = 37, normalized size = 0.88

$$\frac{x^4}{4} - \frac{2x^3}{3a} + \frac{x^2}{a^2} - \frac{2x}{a^3} + \frac{2\log\left(ax + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*x-1)/(a*x+1),x)
```

```
[Out] x**4/4 - 2*x**3/(3*a) + x**2/a**2 - 2*x/a**3 + 2*log(a*x + 1)/a**4
```

Giac [A]
time = 0.41, size = 47, normalized size = 1.12

$$\frac{3\,a^4x^4 - 8\,a^3x^3 + 12\,a^2x^2 - 24\,ax}{12\,a^4} + \frac{2\,\log\left(|ax + 1|\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*x^4 - 8*a^3*x^3 + 12*a^2*x^2 - 24*a*x)/a^4 + 2*log(abs(a*x + 1)
)/a^4
```

Mupad [B]
time = 1.17, size = 38, normalized size = 0.90

$$\frac{2\,\ln\left(a\,x + 1\right)}{a^4} - \frac{2\,x}{a^3} + \frac{x^4}{4} - \frac{2\,x^3}{3\,a} + \frac{x^2}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (2*log(a*x + 1))/a^4 - (2*x)/a^3 + x^4/4 - (2*x^3)/(3*a) + x^2/a^2
```

3.43 $\int e^{-2\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=33

$$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

```
[Out] 2*x/a^2-x^2/a+1/3*x^3-2*ln(a*x+1)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6302, 6261, 78}

$$-\frac{2\log(ax+1)}{a^3}+\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}x^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^2\,dx \\ &= -\int \frac{x^2(1-ax)}{1+ax}\,dx \\ &= -\int \left(-\frac{2}{a^2}+\frac{2x}{a}-x^2+\frac{2}{a^2(1+ax)}\right)dx \\ &= \frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\log(1+ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*x)/a^2 - x^2/a + x^3/3 - (2*Log[1 + a*x])/a^3
```

Maple [A]
time = 0.08, size = 35, normalized size = 1.06

method	result	size
norman	$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\ln(ax+1)}{a^3}$	32
risch	$\frac{2x}{a^2}-\frac{x^2}{a}+\frac{x^3}{3}-\frac{2\ln(ax+1)}{a^3}$	32
default	$\frac{\frac{1}{3}a^2x^3-a\,x^2+2x}{a^2}-\frac{2\ln(ax+1)}{a^3}$	35
meijerg	$\frac{\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)}{a^3}-\frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/3*a^2*x^3-a*x^2+2*x)-2/a^3*ln(a*x+1)
```

Maxima [A]
time = 0.25, size = 34, normalized size = 1.03

$$\frac{a^2x^3-3\,ax^2+6\,x}{3\,a^2}-\frac{2\,\log\left(ax+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*x^3 - 3*a*x^2 + 6*x)/a^2 - 2*log(a*x + 1)/a^3
```

Fricas [A]
time = 0.34, size = 33, normalized size = 1.00

$$\frac{a^3x^3 - 3\,a^2x^2 + 6\,ax - 6\,\log\left(ax + 1\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x - 6*log(a*x + 1))/a^3
```

Sympy [A]
time = 0.04, size = 27, normalized size = 0.82

$$\frac{x^3}{3} - \frac{x^2}{a} + \frac{2x}{a^2} - \frac{2\log\left(ax + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x-1)/(a*x+1),x)
```

```
[Out] x**3/3 - x**2/a + 2*x/a**2 - 2*log(a*x + 1)/a**3
```

Giac [A]
time = 0.40, size = 38, normalized size = 1.15

$$\frac{a^3x^3 - 3\,a^2x^2 + 6\,ax}{3\,a^3} - \frac{2\,\log\left(|ax + 1|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/3*(a^3*x^3 - 3*a^2*x^2 + 6*a*x)/a^3 - 2*log(abs(a*x + 1))/a^3
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.94

$$\frac{2\,x}{a^2} - \frac{2\,\ln\left(a\,x + 1\right)}{a^3} + \frac{x^3}{3} - \frac{x^2}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (2*x)/a^2 - (2*log(a*x + 1))/a^3 + x^3/3 - x^2/a
```

3.44 $\int e^{-2\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=25

$$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}$$

```
[Out] -2*x/a+1/2*x^2+2*ln(a*x+1)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6261, 78}

$$\frac{2\log(ax+1)}{a^2}-\frac{2x}{a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x\,dx &= -\int e^{-2\tanh^{-1}(ax)}x\,dx \\
&= -\int \frac{x(1-ax)}{1+ax}\,dx \\
&= -\int \left(\frac{2}{a}-x-\frac{2}{a(1+ax)}\right)dx \\
&= -\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$
-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\log(1+ax)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x)/a + x^2/2 + (2*Log[1 + a*x])/a^2
```

Maple [A]
time = 0.08, size = 27, normalized size = 1.08

method	result	size
norman	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
risch	$-\frac{2x}{a}+\frac{x^2}{2}+\frac{2\ln(ax+1)}{a^2}$	24
default	$\frac{\frac{1}{2}a\,x^2-2x}{a}+\frac{2\ln(ax+1)}{a^2}$	27
meijerg	$\frac{-\frac{ax(-3ax+6)}{6}+\ln(ax+1)}{a^2}-\frac{ax-\ln(ax+1)}{a^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*a*x^2-2*x)+2*ln(a*x+1)/a^2
```

Maxima [A]
time = 0.26, size = 26, normalized size = 1.04

$$
\frac{ax^2-4\,x}{2\,a}+\frac{2\,\log\left(ax+1\right)}{a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2 - 4*x)/a + 2*log(a*x + 1)/a^2
```

Fricas [**A**]
time = 0.32, size = 25, normalized size = 1.00

$$\frac{a^2x^2 - 4\,ax + 4\,\log\left(ax + 1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*x^2 - 4*a*x + 4*log(a*x + 1))/a^2
```

Sympy [**A**]
time = 0.03, size = 20, normalized size = 0.80

$$\frac{x^2}{2} - \frac{2x}{a} + \frac{2\log\left(ax + 1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x)
```

```
[Out] x**2/2 - 2*x/a + 2*log(a*x + 1)/a**2
```

Giac [**A**]
time = 0.41, size = 30, normalized size = 1.20

$$\frac{a^2x^2 - 4\,ax}{2\,a^2} + \frac{2\,\log\left(|ax + 1|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*x^2 - 4*a*x)/a^2 + 2*log(abs(a*x + 1))/a^2
```

Mupad [**B**]
time = 0.04, size = 23, normalized size = 0.92

$$\frac{2\,\ln\left(a\,x + 1\right)}{a^2} - \frac{2\,x}{a} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (2*log(a*x + 1))/a^2 - (2*x)/a + x^2/2
```

3.45 $\int e^{-2\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=13

$$x-\frac{2\log(1+ax)}{a}$$

[Out] x-2*ln(a*x+1)/a

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6260, 45}

$$x-\frac{2\log(ax+1)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^(-2*ArcCoth[a*x]),x]

[Out] x - (2*Log[1 + a*x])/a

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6260

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_)), x_Symbol] :> Int[(1 + a*x)^(n/2)/(1 - a*x
)^(n/2), x] /; FreeQ[{a, n}, x] &&  !IntegerQ[(n - 1)/2]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\,dx\\ &= -\int \frac{1-ax}{1+ax}\,dx\\ &= -\int \left(-1+\frac{2}{1+ax}\right)dx\\ &= x-\frac{2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$x-\frac{2\log(1+ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-2*ArcCoth[a*x]),x]
[Out] x - (2*Log[1 + a*x])/a
```

Maple [A]
time = 0.08, size = 14, normalized size = 1.08

method	result	size
default	$x-\frac{2\ln(ax+1)}{a}$	14
norman	$x-\frac{2\ln(ax+1)}{a}$	14
risch	$x-\frac{2\ln(ax+1)}{a}$	14
meijerg	$\frac{ax-\ln(ax+1)}{a}-\frac{\ln(ax+1)}{a}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] x-2/a*ln(a*x+1)
```

Maxima [A]
time = 0.25, size = 13, normalized size = 1.00

$$x-\frac{2\,\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] x - 2*log(a*x + 1)/a
```

Fricas [A]
time = 0.33, size = 16, normalized size = 1.23

$$\frac{ax - 2 \log(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] (a*x - 2*log(a*x + 1))/a
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$x - \frac{2\log(ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x)
```

```
[Out] x - 2*log(a*x + 1)/a
```

Giac [A]
time = 0.41, size = 14, normalized size = 1.08

$$x - \frac{2 \log(|ax + 1|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] x - 2*log(abs(a*x + 1))/a
```

Mupad [B]
time = 0.03, size = 13, normalized size = 1.00

$$x - \frac{2 \ln(a\,x + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(a*x + 1),x)
```

```
[Out] x - (2*log(a*x + 1))/a
```

3.46 $\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=13

$$-\log(x) + 2\log(1 + ax)$$

```
[Out] -ln(x)+2*ln(a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$2\log(ax + 1) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] -Log[x] + 2*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{x}\,dx$$
$$= -\int \frac{1-ax}{x(1+ax)}\,dx$$
$$= -\int \left(\frac{1}{x} - \frac{2a}{1+ax}\right)dx$$
$$= -\log(x) + 2\log(1+ax)$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\log(x) + 2\log(1+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x),x]
[Out] -Log[x] + 2*Log[1 + a*x]
```

Maple [A]
time = 0.09, size = 14, normalized size = 1.08

method	result	size
default	$-\ln(x)+2\ln(ax+1)$	14
norman	$-\ln(x)+2\ln(ax+1)$	14
risch	$-\ln(x)+2\ln(-ax-1)$	15
meijerg	$2\ln(ax+1)-\ln(x)-\ln(a)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
[Out] -ln(x)+2*ln(a*x+1)
```

Maxima [A]
time = 0.25, size = 13, normalized size = 1.00

$$2\log(ax+1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="maxima")
[Out] 2*log(a*x + 1) - log(x)
```

Fricas [**A**]
time = 0.34, size = 13, normalized size = 1.00

$$2 \log(ax+1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="fricas")
[Out] 2*log(a*x + 1) - log(x)
```

Sympy [**A**]
time = 0.06, size = 10, normalized size = 0.77

$$-\log(x) + 2\log\left(x + \frac{1}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x)
[Out] -log(x) + 2*log(x + 1/a)
```

Giac [**A**]
time = 0.41, size = 15, normalized size = 1.15

$$2 \log(|ax+1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x,x, algorithm="giac")
[Out] 2*log(abs(a*x + 1)) - log(abs(x))
```

Mupad [**B**]
time = 0.04, size = 14, normalized size = 1.08

$$2 \ln(-3\,a\,x-3) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x*(a*x + 1)),x)
[Out] 2*log(- 3*a*x - 3) - log(x)
```

3.47 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^2} \, dx$

Optimal. Leaf size=18

$$\frac{1}{x}+2a\log(x)-2a\log(1+ax)$$

```
[Out] 1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6261, 78}

$$2a\log(x)-2a\log(ax+1)+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^2}\,dx$$
$$= -\int \frac{1-ax}{x^2(1+ax)}\,dx$$
$$= -\int \left(\frac{1}{x^2} - \frac{2a}{x} + \frac{2a^2}{1+ax}\right) dx$$
$$= \frac{1}{x} + 2a\log(x) - 2a\log(1+ax)$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{1}{x} + 2a\log(x) - 2a\log(1+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] x^(-1) + 2*a*Log[x] - 2*a*Log[1 + a*x]
```

Maple [A]
time = 0.12, size = 19, normalized size = 1.06

method	result	size
default	$\frac{1}{x}+2a\ln\left(x\right)-2a\ln\left(ax+1\right)$	19
norman	$\frac{1}{x}+2a\ln\left(x\right)-2a\ln\left(ax+1\right)$	19
risch	$\frac{1}{x}+2a\ln\left(-x\right)-2a\ln\left(ax+1\right)$	21
meijerg	$a(-\ln\left(ax+1\right)+\ln\left(x\right)+\ln\left(a\right))-a\left(\ln\left(ax+1\right)-\ln\left(x\right)-\ln\left(a\right)-\frac{1}{ax}\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/x+2*a*ln(x)-2*a*ln(a*x+1)
```

Maxima [A]
time = 0.25, size = 18, normalized size = 1.00

$$-2\,a\log\left(ax+1\right)+2\,a\log\left(x\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
[Out] -2*a*log(a*x + 1) + 2*a*log(x) + 1/x
```

Fricas [A]
time = 0.33, size = 23, normalized size = 1.28

$$-\frac{2\,ax\log\left(ax+1\right)-2\,ax\log\left(x\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
[Out] -(2*a*x*log(a*x + 1) - 2*a*x*log(x) - 1)/x
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.83

$$2a\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**2,x)
[Out] 2*a*(log(x) - log(x + 1/a)) + 1/x
```

Giac [A]
time = 0.40, size = 20, normalized size = 1.11

$$-2\,a\log\left(|ax+1|\right)+2\,a\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
[Out] -2*a*log(abs(a*x + 1)) + 2*a*log(abs(x)) + 1/x
```

Mupad [B]
time = 1.20, size = 14, normalized size = 0.78

$$\frac{1}{x}-4\,a\,\mathrm{atanh}(2\,a\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x^2*(a*x + 1)),x)
[Out] 1/x - 4*a*atanh(2*a*x + 1)
```

3.48 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=32

$$\frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)$$

```
[Out] 1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6302, 6261, 78}

$$-2a^2\log(x)+2a^2\log(ax+1)-\frac{2a}{x}+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{x^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^3}\,dx\\ &= -\int \frac{1-ax}{x^3(1+ax)}\,dx\\ &= -\int \left(\frac{1}{x^3}-\frac{2a}{x^2}+\frac{2a^2}{x}-\frac{2a^3}{1+ax}\right)dx\\ &= \frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{1}{2x^2}-\frac{2a}{x}-2a^2\log(x)+2a^2\log(1+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^3),x]
[Out] 1/(2*x^2) - (2*a)/x - 2*a^2*Log[x] + 2*a^2*Log[1 + a*x]
```

Maple [A]
time = 0.12, size = 31, normalized size = 0.97

method	result	size
norman	$\frac{\frac{1}{2}-2ax}{x^2}-2a^2\ln(x)+2a^2\ln(ax+1)$	30
default	$\frac{1}{2x^2}-\frac{2a}{x}-2a^2\ln(x)+2a^2\ln(ax+1)$	31
risch	$\frac{\frac{1}{2}-2ax}{x^2}+2a^2\ln(-ax-1)-2a^2\ln(x)$	31
meijerg	$a^2\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)-a^2\left(-\ln(ax+1)+\ln(x)+\ln(a)-\frac{1}{2a^2x^2}+\frac{1}{ax}\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2/x^2-2*a/x-2*a^2*ln(x)+2*a^2*ln(a*x+1)
```

Maxima [A]
time = 0.26, size = 30, normalized size = 0.94

$$2\,a^2\log(ax+1)-2\,a^2\log(x)-\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
[Out] 2*a^2*log(a*x + 1) - 2*a^2*log(x) - 1/2*(4*a*x - 1)/x^2
```

Fricas [A]
time = 0.35, size = 35, normalized size = 1.09

$$\frac{4\,a^2x^2\log\left(ax+1\right)-4\,a^2x^2\log\left(x\right)-4\,ax+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
[Out] 1/2*(4*a^2*x^2*log(a*x + 1) - 4*a^2*x^2*log(x) - 4*a*x + 1)/x^2
```

Sympy [A]
time = 0.08, size = 26, normalized size = 0.81

$$2a^2\left(-\log\left(x\right)+\log\left(x+\frac{1}{a}\right)\right)+\frac{-4ax+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**3,x)
[Out] 2*a**2*(-log(x) + log(x + 1/a)) + (-4*a*x + 1)/(2*x**2)
```

Giac [A]
time = 0.41, size = 32, normalized size = 1.00

$$2\,a^2\log\left(|ax+1|\right)-2\,a^2\log\left(|x|\right)-\frac{4\,ax-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
[Out] 2*a^2*log(abs(a*x + 1)) - 2*a^2*log(abs(x)) - 1/2*(4*a*x - 1)/x^2
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.75

$$4\,a^2\,\mathrm{atanh}(2\,a\,x+1)-\frac{2\,a\,x-\frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x^3*(a*x + 1)),x)
[Out] 4*a^2*atanh(2*a*x + 1) - (2*a*x - 1/2)/x^2
```

3.49 $\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=40

$$\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\log(x)-2a^3\log(1+ax)$$

```
[Out] 1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6261, 78}

$$2a^3\log(x)-2a^3\log(ax+1)+\frac{2a^2}{x}-\frac{a}{x^2}+\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{x^4}\,dx$$
$$= -\int \frac{1-ax}{x^4(1+ax)}\,dx$$
$$= -\int \left(\frac{1}{x^4} - \frac{2a}{x^3} + \frac{2a^2}{x^2} - \frac{2a^3}{x} + \frac{2a^4}{1+ax}\right)dx$$
$$= \frac{1}{3x^3} - \frac{a}{x^2} + \frac{2a^2}{x} + 2a^3\log(x) - 2a^3\log(1+ax)$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{1}{3x^3} - \frac{a}{x^2} + \frac{2a^2}{x} + 2a^3\log(x) - 2a^3\log(1+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] 1/(3*x^3) - a/x^2 + (2*a^2)/x + 2*a^3*Log[x] - 2*a^3*Log[1 + a*x]
```

Maple [A]
time = 0.12, size = 39, normalized size = 0.98

method	result
norman	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}+2a^3\ln(x)-2a^3\ln(ax+1)$
default	$\frac{1}{3x^3}-\frac{a}{x^2}+\frac{2a^2}{x}+2a^3\ln(x)-2a^3\ln(ax+1)$
risch	$\frac{\frac{1}{3}+2a^2x^2-ax}{x^3}-2a^3\ln(ax+1)+2a^3\ln(-x)$
meijerg	$a^3\left(-\ln(ax+1)+\ln(x)+\ln(a)-\frac{1}{2a^2x^2}+\frac{1}{ax}\right)-a^3\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{3x^3a^3}+\frac{1}{2a^2x\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/x^3-a/x^2+2*a^2/x+2*a^3*ln(x)-2*a^3*ln(a*x+1)
```

Maxima [A]
time = 0.25, size = 38, normalized size = 0.95

$$-2\,a^3\log(ax+1) + 2\,a^3\log(x) + \frac{6\,a^2x^2 - 3\,ax + 1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] -2*a^3*log(a*x + 1) + 2*a^3*log(x) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/x^3
```

Fricas [A]
time = 0.37, size = 43, normalized size = 1.08

$$-\frac{6\,a^3x^3\log\left(ax+1\right)-6\,a^3x^3\log\left(x\right)-6\,a^2x^2+3\,ax-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*x^3*log(a*x + 1) - 6*a^3*x^3*log(x) - 6*a^2*x^2 + 3*a*x - 1)/x^
3
```

Sympy [A]
time = 0.09, size = 34, normalized size = 0.85

$$2a^3\left(\log\left(x\right)-\log\left(x+\frac{1}{a}\right)\right)+\frac{6a^2x^2-3ax+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] 2*a**3*(log(x) - log(x + 1/a)) + (6*a**2*x**2 - 3*a*x + 1)/(3*x**3)
```

Giac [A]
time = 0.41, size = 40, normalized size = 1.00

$$-2\,a^3\log\left(|ax+1|\right)+2\,a^3\log\left(|x|\right)+\frac{6\,a^2x^2-3\,ax+1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(a*x + 1)) + 2*a^3*log(abs(x)) + 1/3*(6*a^2*x^2 - 3*a*x + 1)/
x^3
```

Mupad [B]
time = 1.21, size = 31, normalized size = 0.78

$$\frac{2\,a^2\,x^2-a\,x+\frac{1}{3}}{x^3}-4\,a^3\,\mathrm{atanh}(2\,a\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/(x^4*(a*x + 1)),x)
```

```
[Out] (2*a^2*x^2 - a*x + 1/3)/x^3 - 4*a^3*atanh(2*a*x + 1)
```

3.50 $\int e^{-3\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=136

$$-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}-\frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3}+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a}+\frac{1}{4}\sqrt{1-\frac{1}{a^2x^2}}\,x^4+\frac{51\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}$$

```
[Out] 51/8*arctanh((1-1/a^2/x^2)^(1/2))/a^4-4*(1-1/a^2/x^2)^(1/2)/a^3/(a+1/x)-6*x
*(1-1/a^2/x^2)^(1/2)/a^3+19/8*x^2*(1-1/a^2/x^2)^(1/2)/a^2-x^3*(1-1/a^2/x^2)
^(1/2)/a+1/4*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6304, 6874, 272, 44, 65, 214, 277, 270, 665}

$$\frac{19x^2\sqrt{1-\frac{1}{a^2x^2}}}{8a^2}+\frac{1}{4}x^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{x^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{51\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a^4}-\frac{6x\sqrt{1-\frac{1}{a^2x^2}}}{a^3}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a^3*(a + x^(-1))) - (6*Sqrt[1 - 1/(a^2*x^2)]*x)
/a^3 + (19*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(8*a^2) - (Sqrt[1 - 1/(a^2*x^2)]*x^3)
/a + (Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 + (51*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8
*a^4)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^5\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^4\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^3\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x^2\sqrt{1-\frac{x^2}{a^2}}} + \right.\right.$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4} + \frac{4\text{Subs}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x}{a^2}}}\,dx\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^3\left(a+\frac{1}{x}\right)} - \frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^3} + \frac{19\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{8a^2} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{a} + \frac{1}{4}\sqrt{1-}$$

Mathematica [A]

time = 0.11, size = 83, normalized size = 0.61

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-80-29ax+11a^2x^2-6a^3x^3+2a^4x^4)}{1+ax}+51\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{8a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-80 - 29*a*x + 11*a^2*x^2 - 6*a^3*x^3 + 2*a^4*
x^4))/(1 + a*x) + 51*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(8*a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(118) = 236.
time = 0.11, size = 539, normalized size = 3.96

method	result
risch	$\frac{(2a^3x^3-8a^2x^2+19ax-48)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{8a^4}+\frac{\left(\frac{51\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{8a^3\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^5\left(x+\frac{1}{a}\right)}\right)\sqrt{}}{ax-1}$
default	$-\frac{\left(-2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+8\sqrt{a^2}\,((ax+1)(ax-1))^{\frac{3}{2}}a^2x^2-4(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-21\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+16\sqrt{a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+8*(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(3/2)*a^2*x^2-4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-21*(a^2*x^2-1)^(1
/2)*(a^2)^(1/2)*a^3*x^3+16*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+72*((a*x
+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-
42*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+21*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2))/(a^2)^(1/2))*a^3*x^2-72*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2))/(a^2)^(1/2))*a^3*x^2-8*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+144*((a*x+
1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-21*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+42*l
n((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-144*ln((a^2*x+(a
^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+72*(a^2)^(1/2)*((a*x+
1)*(a*x-1))^(1/2)+21*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*
a-72*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a^4*((a
*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]
time = 0.25, size = 223, normalized size = 1.64

$$-\frac{1}{8}\,a\left(\frac{2\left(77\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-149\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-35\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^5}+\frac{51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^5}+\frac{32\sqrt{\frac{ax-1}{ax+1}}}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*a*(2*(77*((a*x - 1)/(a*x + 1))^(7/2) - 149*((a*x - 1)/(a*x + 1))^(5/2)
 + 123*((a*x - 1)/(a*x + 1))^(3/2) - 35*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x
- 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5/(a*x
 + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 51*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^5 + 51*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^5 + 32*sqrt((a*x -
1)/(a*x + 1))/a^5)
```

Fricas [A]
time = 0.35, size = 92, normalized size = 0.68

$$\frac{(2\,a^4x^4-6\,a^3x^3+11\,a^2x^2-29\,ax-80)\sqrt{\frac{ax-1}{ax+1}}+51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-51\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{8\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a^4*x^4 - 6*a^3*x^3 + 11*a^2*x^2 - 29*a*x - 80)*sqrt((a*x - 1)/(a*x
 + 1)) + 51*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 51*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.08, size = 192, normalized size = 1.41

$$\frac{51\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a^4}-\frac{\frac{35\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{123\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{149\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}-\frac{77\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}-\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (51*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a^4) - ((35*((a*x - 1)/(a*x + 1)
)^(1/2))/4 - (123*((a*x - 1)/(a*x + 1))^(3/2))/4 + (149*((a*x - 1)/(a*x + 1
))^(5/2))/4 - (77*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a^4 + (6*a^4*(a*x - 1)^2
)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x +
1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (4*((a*x - 1)/(a*x + 1))^(1/2))/a^4
```

3.51 $\int e^{-3\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=116

$$\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a}+\frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

```
[Out] -11/2*arctanh((1-1/a^2/x^2)^(1/2))/a^3+4*(1-1/a^2/x^2)^(1/2)/a^2/(a+1/x)+14
/3*x*(1-1/a^2/x^2)^(1/2)/a^2-3/2*x^2*(1-1/a^2/x^2)^(1/2)/a+1/3*x^3*(1-1/a^2
/x^2)^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6304, 6874, 277, 270, 272, 44, 65, 214, 665}

$$-\frac{3x^2\sqrt{1-\frac{1}{a^2x^2}}}{2a}+\frac{14x\sqrt{1-\frac{1}{a^2x^2}}}{3a^2}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)}+\frac{1}{3}x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{11\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a^2*(a + x^(-1))) + (14*Sqrt[1 - 1/(a^2*x^2)]*x)
/(3*a^2) - (3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(2*a) + (Sqrt[1 - 1/(a^2*x^2)]*x^3
)/3 - (11*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^4\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^3\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^3x\sqrt{1-\frac{x^2}{a^2}}} + \right.\right.$$

$$= \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3} - \frac{4\text{Subst}\left(\right.}{}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x}{a^2} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \right.}{a^3}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{3\text{Subst}\left(\right.}{}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{4\tanh^{-1}}{}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a^2\left(a+\frac{1}{x}\right)} + \frac{14\sqrt{1-\frac{1}{a^2x^2}}\,x}{3a^2} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2a} + \frac{1}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{11\tanh^{-}}{}$$

Mathematica [A]
time = 0.13, size = 75, normalized size = 0.65

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(52+19ax-7a^2x^2+2a^3x^3)}{1+ax} - 33\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{6a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(52 + 19*a*x - 7*a^2*x^2 + 2*a^3*x^3))/(1 + a*x
) - 33*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(6*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(98) = 196$.
time = 0.10, size = 471, normalized size = 4.06

method	result
risch	$\frac{(2a^2x^2-9ax+28)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{6a^3}+\frac{\left(-\frac{11\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a^2\sqrt{a^2}}+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$
default	$-\frac{\left(9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-2\sqrt{a^2}\,((ax+1)(ax-1))^{\frac{3}{2}}a^2x^2+18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-2*(a^2)^(1/2)*((a*x+1)*(a*x-1
))^(3/2)*a^2*x^2+18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-9*ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-4*(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(3/2)*a*x-42*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+42*ln((a^2*x+(
a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+9*(a^2)^(1/2)*(a^2
*x^2-1)^(1/2)*a*x-18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*
a^2*x+10*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-84*((a*x+1)*(a*x-1))^(1/2)*(a^
2)^(1/2)*a*x+84*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))
*a^2*x-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-42*(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2)+42*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2))/(a^2)^(1/2)))/a^3*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x+1)*(a*x-1
))^(1/2)/(a*x-1)
```

Maxima [A]
time = 0.26, size = 186, normalized size = 1.60

$$-\frac{1}{6}a\left(\frac{2\left(39\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-52\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^4}-\frac{33\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^4}-\frac{24\sqrt{\frac{ax-1}{ax+1}}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(39*((a*x - 1)/(a*x + 1))^(5/2) - 52*((a*x - 1)/(a*x + 1))^(3/2)
+ 21*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*
a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*log(sqrt((a*x - 1
)/(a*x + 1)) + 1)/a^4 - 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^4 - 24*sqrt
((a*x - 1)/(a*x + 1))/a^4)
```

Fricas [A]
time = 0.34, size = 84, normalized size = 0.72

$$\frac{(2\,a^3x^3 - 7\,a^2x^2 + 19\,ax + 52)\sqrt{\frac{ax-1}{ax+1}} - 33\,\log\left(\sqrt{\frac{ax-1}{ax+1}} + 1\right) + 33\,\log\left(\sqrt{\frac{ax-1}{ax+1}} - 1\right)}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*a^3*x^3 - 7*a^2*x^2 + 19*a*x + 52)*sqrt((a*x - 1)/(a*x + 1)) - 33*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 33*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.06, size = 156, normalized size = 1.34

$$\frac{7\sqrt{\frac{a\,x-1}{a\,x+1}} - \frac{52\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3} + 13\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a^3 + \frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3} - \frac{3\,a^3\,(a\,x-1)}{a\,x+1}} + \frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^3} - \frac{11\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (7*((a*x - 1)/(a*x + 1))^(1/2) - (52*((a*x - 1)/(a*x + 1))^(3/2))/3 + 13*((
a*x - 1)/(a*x + 1))^(5/2))/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^3*(a
*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (4*((a*x - 1)/(a*x
+ 1))^(1/2))/a^3 - (11*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^3
```

3.52 $\int e^{-3\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=90

$$-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a}+\frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

```
[Out] 9/2*arctanh((1-1/a^2/x^2)^(1/2))/a^2-4*(1-1/a^2/x^2)^(1/2)/a/(a+1/x)-3*x*(1
-1/a^2/x^2)^(1/2)/a+1/2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6304, 6874, 272, 44, 65, 214, 270, 665}

$$\frac{1}{2}x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)}+\frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[1 - 1/(a^2*x^2)])/(a*(a + x^(-1))) - (3*Sqrt[1 - 1/(a^2*x^2)]*x)/a
 + (Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (9*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a^
2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^3\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax^2\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a^2x\sqrt{1-\frac{x^2}{a^2}}} - \frac{4}{a^2(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right.\right.$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2} + \frac{3\text{Subst}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right) - \frac{2\text{Subst}}{}$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 4\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x,\right.$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2} +$$

$$= -\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a\left(a+\frac{1}{x}\right)} - \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{a} + \frac{1}{2}\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{9\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a^2}$$

Mathematica [A]

time = 0.04, size = 66, normalized size = 0.73

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-14-5ax+a^2x^2)}{1+ax}+9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-14 - 5*a*x + a^2*x^2))/(1 + a*x) + 9*Log[(1 +
 Sqrt[1 - 1/(a^2*x^2)])*x])/(2*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(78) = 156.
time = 0.10, size = 421, normalized size = 4.68

method	result
risch	$\frac{(ax-6)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2a^2}+\frac{\left(\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2a\sqrt{a^2}}-\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^3\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1}}{ax-1}$
default	$\frac{\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-10\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+2\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-10*((a*x+1)*(a*x-1))^(1/2)*(a^2)
^(1/2)*a^2*x^2+2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-ln((a^2*x+(a^2*x^2-1
)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2+10*ln((a^2*x+(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+4*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)
-20*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a
*x-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x+20*ln((a^2
*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-10*(a^2)^(1/2)*(
(a*x+1)*(a*x-1))^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
)*a+10*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a^2*(
(a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]

time = 0.26, size = 151, normalized size = 1.68

$$-\frac{1}{2}\,a\left(\frac{2\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^3}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^3}+\frac{8\,\sqrt{\frac{ax-1}{ax+1}}}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(7*((a*x - 1)/(a*x + 1))^(3/2) - 5*sqrt((a*x - 1)/(a*x + 1)))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 9*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^3 + 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^3 + 8
*sqrt((a*x - 1)/(a*x + 1))/a^3)
```

Fricas [A]
time = 0.33, size = 75, normalized size = 0.83

$$\frac{(a^2x^2-5\,ax-14)\sqrt{\frac{ax-1}{ax+1}}+9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2*x^2 - 5*a*x - 14)*sqrt((a*x - 1)/(a*x + 1)) + 9*log(sqrt((a*x - 1
)/(a*x + 1)) + 1) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.22, size = 120, normalized size = 1.33

$$\frac{9\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a^2}-\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a^2}-\frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}}-7\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a^2 - (4*((a*x - 1)/(a*x + 1))^(1/2)
)/a^2 - (5*((a*x - 1)/(a*x + 1))^(1/2) - 7*((a*x - 1)/(a*x + 1))^(3/2))/(a^
2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1))
```

3.53 $\int e^{-3\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=60

$$\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -3*arctanh((1-1/a^2/x^2)^(1/2))/a+4*(1-1/a^2/x^2)^(1/2)/(a+1/x)+x*(1-1/a^2/
x^2)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6303, 6874, 270, 272, 65, 214, 665}

$$x\sqrt{1-\frac{1}{a^2x^2}}+\frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) + Sqrt[1 - 1/(a^2*x^2)]*x - (3*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6303

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.)), x_Symbol] :> -Subst[Int[(1 + x/a)^((n +
1)/2)/(x^2*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x, 1/x] /; FreeQ[a
, x] && IntegerQ[(n - 1)/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}} - \frac{3}{ax\sqrt{1-\frac{x^2}{a^2}}} + \frac{4}{a(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \frac{4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int\right.$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - (3a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= \frac{4\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} + \sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.90

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(5+ax)}{1+ax} - \frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(-3*ArcCoth[a*x]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(5 + a*x))/(1 + a*x) - (3*Log[a*(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(54) = 108$.
time = 0.09, size = 248, normalized size = 4.13

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(a}}{ax-1}$
default	$-\frac{\left(-3\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-3*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+3*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+2*((a*x+1)*(a*x-1))^(3/2)*(a^2)
^(1/2)-6*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+6*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-3*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2)+3*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a
*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(54) = 108$.
time = 0.25, size = 111, normalized size = 1.85

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + 3*log(sqr
t((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2
- 4*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Fricas [A]
time = 0.33, size = 66, normalized size = 1.10

$$\frac{(ax+5)\sqrt{\frac{ax-1}{ax+1}}-3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] ((a*x + 5)*sqrt((a*x - 1)/(a*x + 1)) - 3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 3*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.04, size = 78, normalized size = 1.30

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{4\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) + (4*((a*x -
1)/(a*x + 1))^(1/2))/a - (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.54 $\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=46

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}-\csc^{-1}(ax)+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

```
[Out] -arccsc(a*x)+arctanh((1-1/a^2/x^2)^(1/2))-4*a*(1-1/a^2/x^2)^(1/2)/(a+1/x)
```

Rubi [A]
time = 0.53, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6304, 6874, 222, 272, 65, 214, 665}

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)-\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)])/(a + x^(-1)) - ArcCsc[a*x] + ArcTanh[Sqrt[1 - 
1/(a^2*x^2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}-\frac{4}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\right)dx, x, \frac{1}{x}\right)$$

$$= 4\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int\right.$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + a^2\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$= -\frac{4a\sqrt{1-\frac{1}{a^2x^2}}}{a+\frac{1}{x}} - \csc^{-1}(ax) + \tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)$$

<u>**Mathematica**</u> [**A**]
time = 0.06, size = 55, normalized size = 1.20

$$-\frac{4a\sqrt{1-\frac{1}{a^2x^2}}\,x}{1+ax} - \text{ArcSin}\left(\frac{1}{ax}\right) + \log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x),x]
[Out] (-4*a*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - ArcSin[1/(a*x)] + Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]
```

<u>**Maple**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(42) = 84.
time = 0.08, size = 369, normalized size = 8.02

method	result
default	$\frac{\left(-\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+ln((a^2*x+(a^2)^(1/2)*((a*x+1
)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^
2-(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^2*x^2+2*((a*x+1)*(a*x-1))^(3/2)
*(a^2)^(1/2)-2*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+2*ln((a^2*x+(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-2*(a^2)^(1/2)*(a^2*x^2-1)^(
1/2)*a*x-2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x-(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2)+a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2
))-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2))*(
(a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(42) = 84$.
time = 0.45, size = 89, normalized size = 1.93

$$a\left(\frac{2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a}-\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a*(2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a + log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a - 4*sqrt((a*x - 1)/(a*x + 1))/a
)
```

Fricas [**A**]
time = 0.39, size = 74, normalized size = 1.61

$$-4\sqrt{\frac{ax-1}{ax+1}}+2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -4*sqrt((a*x - 1)/(a*x + 1)) + 2*arctan(sqrt((a*x - 1)/(a*x + 1))) + log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.03, size = 54, normalized size = 1.17

$$2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)+2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-4\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/2)) + 2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
- 4*((a*x - 1)/(a*x + 1))^(1/2)
```

3.55 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=53

$$3a\sqrt{1-\frac{1}{a^2x^2}}+\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)$$

```
[Out] 3*a*arccsc(a*x)+2*(a-1/x)^2/a/(1-1/a^2/x^2)^(1/2)+3*a*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6304, 867, 683, 655, 222}

$$\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\sqrt{1-\frac{1}{a^2x^2}}+3a\csc^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] 3*a*Sqrt[1 - 1/(a^2*x^2)] + (2*(a - x^(-1))^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) +
3*a*ArcCsc[a*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 683

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((m +
p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1), x], x] /; FreeQ
[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && In
tegerQ[2*p]
```

Rule 867

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)
^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)/
(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g
, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0] && IntegerQ[n]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{x^2}\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\ &= \frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\ &= 3a\sqrt{1-\frac{1}{a^2x^2}}+\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\ &= 3a\sqrt{1-\frac{1}{a^2x^2}}+\frac{2\left(a-\frac{1}{x}\right)^2}{a\sqrt{1-\frac{1}{a^2x^2}}}+3a\csc^{-1}(ax)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 41, normalized size = 0.77

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,(1+5ax)}{1+ax}+3a\text{ArcSin}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*(1 + 5*a*x))/(1 + a*x) + 3*a*ArcSin[1/(a*x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(49) = 98$.
time = 0.10, size = 594, normalized size = 11.21

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}+\frac{\left(\frac{4\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}+3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)\sqrt{\frac{ax-1}{ax+1}}\ \sqrt{(ax+1)(ax}}{ax-1}$
default	$\frac{\left(\sqrt{a^2x^2-1}\ \sqrt{a^2}\ a^4x^4+\ln\left(\frac{a^2x+\sqrt{a^2}\ \sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3-(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\ a^2x^2+5\sqrt{a^2x^2-1}\ \surd}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(1/2))/(a^2)^(1/2))*a^4*x^3-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+5*(a^
2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-
1)^(1/2))-(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-ln((a^2*x+(a^2*x^2-1)
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+2*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(
a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+7*(
a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+6*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/
2))*a^2*x^2-2*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-2*((a*x+1)*(a*x-1))^(
1/2)*(a^2)^(1/2)*a^2*x^2-2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(
1/2))*a^3*x^2+ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a
^2*x-(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+3*(a
^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x-((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/
2)*a*x-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x)*((a*x-1
)/(a*x+1))^(3/2)/x/(a^2)^(1/2)/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]
time = 0.46, size = 72, normalized size = 1.36

$$2\,a\left(2\,\sqrt{\frac{ax-1}{ax+1}}+\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{ax-1}{ax+1}+1}-3\,\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2*a*(2*sqrt((a*x - 1)/(a*x + 1)) + sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)/(a*
x + 1) + 1) - 3*arctan(sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A]
time = 0.37, size = 49, normalized size = 0.92

$$-\frac{6\,ax\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(5\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - (5*a*x + 1)*sqrt((a*x - 1)/(a*x
 + 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.05, size = 59, normalized size = 1.11

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}+5\,a\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6\,a\,x\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^2,x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2) + 5*a*x*((a*x - 1)/(a*x + 1))^(1/2) - 6*a*x*at
an(((a*x - 1)/(a*x + 1))^(1/2)))/x
```

3.56 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^3} \, dx$

Optimal. Leaf size=87

$$-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}-\frac{9}{2}a^2\csc^{-1}(ax)$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)/(a+1/x)^3-3/2*a^3*(1-1/a^2/x^2)^(3/2)/(a+1/x)-9/2*
a^2*arccsc(a*x)-9/2*a^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6304, 1647, 1607, 12, 807, 679, 222}

$$-\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{2}a^2\csc^{-1}(ax)-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-9*a^2*Sqrt[1 - 1/(a^2*x^2)])/2 - (a^5*(1 - 1/(a^2*x^2))^(5/2))/(a + x^(-1
))^3 - (3*a^3*(1 - 1/(a^2*x^2))^(3/2))/(2*(a + x^(-1))) - (9*a^2*ArcCsc[a*x
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(ax-x^2)\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - (3a)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{1+\frac{x}{a}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{1}{2}(9a)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= -\frac{9}{2}a^2\sqrt{1-\frac{1}{a^2x^2}} - \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(a+\frac{1}{x}\right)^3} - \frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(a+\frac{1}{x}\right)} - \frac{9}{2}a^2\csc^{-1}(ax)$$

Mathematica [A]

time = 0.19, size = 56, normalized size = 0.64

$$\frac{1}{2}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(1-5ax-14a^2x^2)}{x(1+ax)}-9a\text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^3),x]

[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(1 - 5*a*x - 14*a^2*x^2))/(x*(1 + a*x)) - 9*a*ArcSin[1/(a*x)]))/2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(75) = 150.
time = 0.11, size = 641, normalized size = 7.37

method	result
risch	$-\frac{(ax+1)(6ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2x^2}+\frac{\left(-\frac{4a\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}-\frac{9a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax}}{ax-1}$
default	$-\frac{\left(6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+6\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^5x^4-6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+21\sqrt{a^2x^2-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*(6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^5*x^5+6*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4-6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+21*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+9*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-6*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^5*x^4+12*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-11*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+24*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+18*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-4*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2-12*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-12*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+6*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-4*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+9*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^2*x^2-6*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2+(a^2*x^2-1)^(3/2)*(a^2)^(

```
1/2))*((a*x-1)/(a*x+1))^(3/2)/x^2/(a^2)^(1/2)/(a*x-1)/((a*x+1)*(a*x-1))^(1/
2)
```

Maxima [A]
time = 0.46, size = 112, normalized size = 1.29

$$\left(9\,a\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-4\,a\sqrt{\frac{ax-1}{ax+1}}-\frac{7\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+5\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] (9*a*arctan(sqrt((a*x - 1)/(a*x + 1))) - 4*a*sqrt((a*x - 1)/(a*x + 1)) - (7
*a*((a*x - 1)/(a*x + 1))^(3/2) + 5*a*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1
)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Fricas [A]
time = 0.34, size = 61, normalized size = 0.70

$$\frac{18\,a^2x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(14\,a^2x^2+5\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(18*a^2*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (14*a^2*x^2 + 5*a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.06, size = 118, normalized size = 1.36

$$9\,a^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)-\frac{5\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+7\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}-4\,a^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^3,x)
```

```
[Out] 9*a^2*atan(((a*x - 1)/(a*x + 1))^(1/2)) - (5*a^2*((a*x - 1)/(a*x + 1))^(1/2
) + 7*a^2*((a*x - 1)/(a*x + 1))^(3/2))/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x -
 1))/(a*x + 1) + 1) - 4*a^2*((a*x - 1)/(a*x + 1))^(1/2)
```

3.57 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=96

$$\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{11}{2}a^3\csc^{-1}(ax)$$

```
[Out] 11/2*a^3*arccsc(a*x)+(a-1/x)^3/(1-1/a^2/x^2)^(1/2)+1/6*a^2*(28*a-3/x)*(1-1/
a^2/x^2)^(1/2)+1/3*a*(3*a-1/x)^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6304, 1647, 1607, 12, 866, 1649, 1668, 794, 222}

$$\frac{11}{2}a^3\csc^{-1}(ax)+\frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right)+\frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2+\frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (a^2*Sqrt[1 - 1/(a^2*x^2)]*(28*a - 3/x))/6 + (a - x^(-1))^3/Sqrt[1 - 1/(a^2
*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)]*(3*a - x^(-1))^2)/3 + (11*a^3*ArcCsc[a*x]
)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\,(ax^2-x^3)}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-x)x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{a^2x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2(3a^2-ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\left(-5+\frac{3x}{a}\right)(3a^2-ax)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{1}{2}(11a^2)\,\text{S}$$

$$= \frac{1}{6}a^2\sqrt{1-\frac{1}{a^2x^2}}\left(28a-\frac{3}{x}\right) + \frac{\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a\sqrt{1-\frac{1}{a^2x^2}}\left(3a-\frac{1}{x}\right)^2 + \frac{11}{2}a^3\csc^{-1}$$

Mathematica [A]
time = 0.17, size = 66, normalized size = 0.69

$$\frac{1}{6}a\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(2-7ax+19a^2x^2+52a^3x^3)}{x^2(1+ax)}+33a^2\text{ArcSin}\left(\frac{1}{ax}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (a*((Sqrt[1 - 1/(a^2*x^2)]*(2 - 7*a*x + 19*a^2*x^2 + 52*a^3*x^3))/(x^2*(1 +
 a*x)) + 33*a^2*ArcSin[1/(a*x)]))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(84) = 168.
time = 0.11, size = 666, normalized size = 6.94

method	result
risch	$\frac{(ax+1)(28a^2x^2-9ax+2)\sqrt{\frac{ax-1}{ax+1}}}{6x^3}+\frac{\left(\frac{4a^2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}+\frac{11a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$
default	$-\frac{\left(-30\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^6x^6+30\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^5x^5-30\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-30*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a^6*x^6+30*((a*x+1)*(a*x-1))^(1/2)*
(a^2)^(1/2)*a^5*x^5-30*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)
^(1/2))*a^6*x^5+30*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-93*(a^2*x^2-1)^(1/
2)*(a^2)^(1/2)*a^5*x^5-33*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x^5+3
0*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5+12*((a*x+1)
*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3+60*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*
a^4*x^4-60*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*
x^4+51*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-96*(a^2*x^2-1)^(1/2)*(a^2)^(1/
2)*a^4*x^4-66*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+60*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^5*x^4+30*(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2)*a^3*x^3-30*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(
a^2)^(1/2))*a^4*x^3+14*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-33*(a^2*x^2-1)
^(1/2)*(a^2)^(1/2)*a^3*x^3-33*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2
))+30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3-5*(a^2)
```

```
^(1/2)*(a^2*x^2-1)^(3/2)*a*x+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))*((a*x-1)/(a*x
+1))^(3/2)/x^3/(a^2)^(1/2)/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]
time = 0.47, size = 157, normalized size = 1.64

$$-\frac{1}{3}\left(33\,a^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-12\,a^2\sqrt{\frac{ax-1}{ax+1}}-\frac{39\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+52\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+21\,a^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{3\,(ax-1)}{ax+1}+\frac{3\,(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(33*a^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 12*a^2*sqrt((a*x - 1)/(a*x
 + 1)) - (39*a^2*((a*x - 1)/(a*x + 1))^(5/2) + 52*a^2*((a*x - 1)/(a*x + 1))
^(3/2) + 21*a^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x
- 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Fricas [A]
time = 0.35, size = 69, normalized size = 0.72

$$-\frac{66\,a^3x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(52\,a^3x^3+19\,a^2x^2-7\,ax+2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(66*a^3*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - (52*a^3*x^3 + 19*a^2*x
^2 - 7*a*x + 2)*sqrt((a*x - 1)/(a*x + 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.23, size = 153, normalized size = 1.59

$$\frac{7\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{52\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+13\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1}+4\,a^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-11\,a^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^4,x)
```

```
[Out] (7*a^3*((a*x - 1)/(a*x + 1))^(1/2) + (52*a^3*((a*x - 1)/(a*x + 1))^(3/2))/3
 + 13*a^3*((a*x - 1)/(a*x + 1))^(5/2))/((3*(a*x - 1)^2)/(a*x + 1)^2 + (a*x
- 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 4*a^3*((a*x - 1)/(a*x +
 1))^(1/2) - 11*a^3*atan(((a*x - 1)/(a*x + 1))^(1/2))
```

3.58 $\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=133

$$-\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{51}{8}a^4\csc^{-1}$$

```
[Out] -51/8*a^4*arccsc(a*x)-a*(a-1/x)^3/(1-1/a^2/x^2)^(1/2)-27/4*a^4*(1-1/a^2/x^2
)^(1/2)-9/8*a^3*(2*a-3/x)*(1-1/a^2/x^2)^(1/2)+1/4*a*(1-1/a^2/x^2)^(1/2)/x^3
-a^2*(1-1/a^2/x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.54, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6304, 1647, 1607, 12, 866, 1649, 1829, 27, 757, 655, 222}

$$-\frac{51}{8}a^4\csc^{-1}(ax)-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2}-\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3}-\frac{27}{4}a^4\sqrt{1-\frac{1}{a^2x^2}}-\frac{9}{8}a^3\sqrt{1-\frac{1}{a^2x^2}}\left(2a-\frac{3}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-27*a^4*Sqrt[1 - 1/(a^2*x^2)])/4 - (9*a^3*Sqrt[1 - 1/(a^2*x^2)]*(2*a - 3/x
))/8 - (a*(a - x^(-1))^3)/Sqrt[1 - 1/(a^2*x^2)] + (a*Sqrt[1 - 1/(a^2*x^2)])
/(4*x^3) - (a^2*Sqrt[1 - 1/(a^2*x^2)])/x^2 - (51*a^4*ArcCsc[a*x])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rule 6304

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 +
x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1 - x^2/a^2]), x], x
, 1/x] /; FreeQ[a, x] && IntegerQ[(n - 1)/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{x^5}\,dx = -\mathrm{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\,(ax^3-x^4)}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{(a-x)x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1+\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{a^2x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\mathrm{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1+\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\mathrm{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2(-3a^3+a^2x-ax^2)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{1}{4}a^2\mathrm{Subst}\left(\int \frac{12a-28x+\frac{27x^2}{a}-\frac{12x^3}{a^2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\mathrm{Subst}\left(\int \frac{-\frac{36}{a}+\frac{108x}{a^2}-\frac{81x}{a^3}}{\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} + \frac{1}{12}a^4\mathrm{Subst}\left(\int -\frac{9(2a-3x)^2}{a^3\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.$$

$$= -\frac{a\left(a-\frac{1}{x}\right)^3}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{4x^3} - \frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{x^2} - \frac{1}{4}(3a)\mathrm{Subst}\left(\int \frac{(2a-3x)^2}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x\right.$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 0.56

$$-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\left(-2+6ax-11a^2x^2+29a^3x^3+80a^4x^4\right)}{8x^3(1+ax)}-\frac{51}{8}a^4\text{ArcSin}\left(\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/8*(a*Sqrt[1 - 1/(a^2*x^2)]*(-2 + 6*a*x - 11*a^2*x^2 + 29*a^3*x^3 + 80*a^
4*x^4))/(x^3*(1 + a*x)) - (51*a^4*ArcSin[1/(a*x)])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(115) = 230$.
time = 0.12, size = 690, normalized size = 5.19

method	result
risch	$-\frac{(ax+1)(48a^3x^3-19a^2x^2+8ax-2)\sqrt{\frac{ax-1}{ax+1}}}{8x^4}+\frac{\left(-\frac{4a^3\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{x+\frac{1}{a}}-\frac{51a^4\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{8}\right)}{ax-1}$
default	$\frac{\left(-56\sqrt{a^2}\sqrt{a^2x^2-1}\,a^7x^7+56\sqrt{(ax+1)(ax-1)}\sqrt{a^2}\,a^6x^6-56\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-56*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a^7*x^7+56*((a*x+1)*(a*x-1))^(1/2)*(
a^2)^(1/2)*a^6*x^6-56*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^
(1/2))*a^7*x^6+56*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^5*x^5-163*(a^2)^(1/2)*(a^
2*x^2-1)^(1/2)*a^6*x^6-51*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^6*x^6+5
6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^7*x^6+16*((a*x+1)
*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4+112*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)
*a^5*x^5-112*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^
6*x^5+91*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-158*(a^2*x^2-1)^(1/2)*(a^2)^
(1/2)*a^5*x^5-102*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x^5+112*ln((a
^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5+56*(a^2)^(1/2)*((a
*x+1)*(a*x-1))^(1/2)*a^4*x^4-56*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/
2))/(a^2)^(1/2))*a^5*x^4+22*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-51*(a^2*x
^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4-51*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)
^(1/2))+56*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^5*x^4-7*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+4*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-2
```

```
*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))*((a*x-1)/(a*x+1))^(3/2)/x^4/(a^2)^(1/2)/(a*
x-1)/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]
time = 0.47, size = 193, normalized size = 1.45

$$\frac{1}{4}\left(51\,a^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-16\,a^3\sqrt{\frac{ax-1}{ax+1}}-\frac{77\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+149\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+123\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+35\,a^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4\,(ax-1)}{ax+1}+\frac{6\,(ax-1)^2}{(ax+1)^2}+\frac{4\,(ax-1)^3}{(ax+1)^3}+\frac{(ax-1)^4}{(ax+1)^4}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(51*a^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 16*a^3*sqrt((a*x - 1)/(a*x
+ 1)) - (77*a^3*((a*x - 1)/(a*x + 1))^(7/2) + 149*a^3*((a*x - 1)/(a*x + 1))
^(5/2) + 123*a^3*((a*x - 1)/(a*x + 1))^(3/2) + 35*a^3*sqrt((a*x - 1)/(a*x +
 1)))/(4*(a*x - 1)/(a*x + 1) + 6*(a*x - 1)^2/(a*x + 1)^2 + 4*(a*x - 1)^3/(a
*x + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + 1))*a
```

Fricas [A]
time = 0.33, size = 77, normalized size = 0.58

$$\frac{102\,a^4x^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-\left(80\,a^4x^4+29\,a^3x^3-11\,a^2x^2+6\,ax-2\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(102*a^4*x^4*arctan(sqrt((a*x - 1)/(a*x + 1))) - (80*a^4*x^4 + 29*a^3*x
^3 - 11*a^2*x^2 + 6*a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.24, size = 190, normalized size = 1.43

$$\frac{51\,a^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4}-4\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{\frac{35\,a^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{123\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{149\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}+\frac{77\,a^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{\frac{6\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}+\frac{4\,\left(a\,x-1\right)}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/x^5,x)
```

```
[Out] (51*a^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/4 - 4*a^4*((a*x - 1)/(a*x + 1))^
(1/2) - ((35*a^4*((a*x - 1)/(a*x + 1))^(1/2))/4 + (123*a^4*((a*x - 1)/(a*x
+ 1))^(3/2))/4 + (149*a^4*((a*x - 1)/(a*x + 1))^(5/2))/4 + (77*a^4*((a*x -
1)/(a*x + 1))^(7/2))/4)/((6*(a*x - 1)^2)/(a*x + 1)^2 + (4*(a*x - 1)^3)/(a*x
 + 1)^3 + (a*x - 1)^4/(a*x + 1)^4 + (4*(a*x - 1))/(a*x + 1) + 1)
```

3.59 $\int e^{\frac{1}{2}\coth^{-1}(ax)} x^4\,dx$

Optimal. Leaf size=253

$$\frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4}+\frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3}+\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2}+\frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+}}{40a}$$

[Out] 611/1920*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4+269/960*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^2/a^3+11/48*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2+9/40*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5+31/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+31/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5

Rubi [A]
time = 0.10, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$\frac{31\,\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{611x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{1920a^4}+\frac{269x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{960a^3}+\frac{11x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{48a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{9x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{40a}$$

Antiderivative was successfully verified.

[In] Int[E^(ArcCoth[a*x]/2)*x^4,x]

[Out] (611*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(1920*a^4) + (269*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^3)/(48*a^2) + (9*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 + (31*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (31*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^6\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{\frac{9}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{55}{4a^2} -}{x^4\sqrt[4]{1-\frac{x}{a}}\,(}\right.$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x$$

$$= \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{48a^2} + \frac{9\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{40a}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{48a^2}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{48a^2}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{48a^2}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{48a^2}$$

$$= \frac{611\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4} + \frac{269\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3} + \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{48a^2}$$

Mathematica [A]
time = 5.15, size = 173, normalized size = 0.68

$$\frac{\frac{24576e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{62976e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{64640e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{34000e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{9620e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+930\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-465\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+465\log\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{3840a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^4,x]
```

```
[Out] ((24576*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (62976*E^(ArcCoth
[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^(ArcCoth[a*x]/2))/(-1 + E^
(2*ArcCoth[a*x]))^3 + (34000*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^
2 + (9620*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 930*ArcTan[E^(Arc
Coth[a*x]/2)] - 465*Log[1 - E^(ArcCoth[a*x]/2)] + 465*Log[1 + E^(ArcCoth[a*
x]/2)])/(3840*a^5)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x)
```

Maxima [A]
time = 0.46, size = 259, normalized size = 1.02

$$-\frac{1}{3840}a\left(\frac{4\left(465\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+2405\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(465*((a*x - 1)/(a*x + 1))^(19/4) - 696*((a*x - 1)/(a*x + 1))^
(15/4) + 5090*((a*x - 1)/(a*x + 1))^(11/4) - 1120*((a*x - 1)/(a*x + 1))^(7/
4) + 2405*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/
(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 465*
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A]
time = 0.34, size = 119, normalized size = 0.47

$$\frac{2\,(384\,a^5x^5+816\,a^4x^4+872\,a^3x^3+978\,a^2x^2+1149\,ax+611)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{3840\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 + 816*a^4*x^4 + 872*a^3*x^3 + 978*a^2*x^2 + 1149*a*x
 + 611)*((a*x - 1)/(a*x + 1))^(3/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 465*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[4]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.47, size = 234, normalized size = 0.92

$$-\frac{1}{3840}\,a\left(\frac{930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{465\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{1120\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{5090\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}+\frac{696\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^3}-\frac{465\,(ax-1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^4}-2405\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^6\left(\frac{ax-1}{ax+1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 + 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(1120*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5090*(a*x -
 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 696*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(3/4)/(a*x + 1)^3 - 465*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/
(a*x + 1)^4 - 2405*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

Mupad [B]
time = 0.11, size = 229, normalized size = 0.91

$$\frac{\frac{481\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{192}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{6}+\frac{509\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{96}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{40}+\frac{31\left(\frac{a\,x-1}{a\,x+1}\right)^{19/4}}{64}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}-\frac{31\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{31\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((481*((a*x - 1)/(a*x + 1))^(3/4))/192 - (7*((a*x - 1)/(a*x + 1))^(7/4))/6
+ (509*((a*x - 1)/(a*x + 1))^(11/4))/96 - (29*((a*x - 1)/(a*x + 1))^(15/4))
/40 + (31*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*
x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)
^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (31*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(128*a^5)
```

3.60 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3}+\frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2}+\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{a}}$$

```
[Out] 83/192*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+29/96*(1-1/a/x)^(3/4)*(1+1/a/x
)^(1/4)*x^2/a^2+7/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(3
/4)*(1+1/a/x)^(1/4)*x^4+11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+1
1/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A]
time = 0.08, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$\frac{11\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{83x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{192a^3}+\frac{29x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{96a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{7x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] (83*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(192*a^3) + (29*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(96*a^2) + (7*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
 (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^5\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{29}{4a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}}\right.$$

$$= \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}$$

$$= \frac{83\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3} + \frac{29\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2} + \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a}$$

Mathematica [A]
time = 5.12, size = 149, normalized size = 0.69

$$\frac{\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{3200e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{2512e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{980e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+66\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-33\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+33\log\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{384a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^3,x]
```

```
[Out] ((1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (3200*E^(ArcCoth[a
*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^(ArcCoth[a*x]/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (980*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + 66*
ArcTan[E^(ArcCoth[a*x]/2)] - 33*Log[1 - E^(ArcCoth[a*x]/2)] + 33*Log[1 + E^
(ArcCoth[a*x]/2)])/(384*a^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x)
```

Maxima [A]
time = 0.46, size = 224, normalized size = 1.04

$$\frac{1}{384}\,a\left(\frac{4\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-279\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+107\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-245\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/384*a*(4*(33*((a*x - 1)/(a*x + 1))^(15/4) - 279*((a*x - 1)/(a*x + 1))^(11
/4) + 107*((a*x - 1)/(a*x + 1))^(7/4) - 245*((a*x - 1)/(a*x + 1))^(3/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A]
time = 0.36, size = 111, normalized size = 0.51

$$\frac{2\,(48\,a^4x^4+104\,a^3x^3+114\,a^2x^2+141\,ax+83)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 + 104*a^3*x^3 + 114*a^2*x^2 + 141*a*x + 83)*((a*x - 1)
/(a*x + 1))^(3/4) - 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x -
 1)/(a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.46, size = 203, normalized size = 0.94

$$-\frac{1}{384}\,a\left(\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{33\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{107\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{279\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}+\frac{33\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^3}-245\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^5\left(\frac{ax-1}{ax+1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a
*x + 1))^(1/4) + 1)/a^5 + 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5
+ 4*(107*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 279*(a*x - 1)^2*
((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 33*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(3/4)/(a*x + 1)^3 - 245*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a
*x + 1) - 1)^4))
```

Mupad [B]
time = 1.24, size = 192, normalized size = 0.89

$$\frac{\frac{245\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{96}-\frac{107\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{96}+\frac{93\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/4}}{32}-\frac{11\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/4}}{32}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}-\frac{11\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{64\,a^4}+\frac{11\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((245*((a*x - 1)/(a*x + 1))^(3/4))/96 - (107*((a*x - 1)/(a*x + 1))^(7/4))/9
6 + (93*((a*x - 1)/(a*x + 1))^(11/4))/32 - (11*((a*x - 1)/(a*x + 1))^(15/4)
)/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1
)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) - (11*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (11*atanh(((a*x - 1)/(a*x + 1))
^(1/4)))/(64*a^4)
```

3.61 $\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2}+\frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3+\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] 11/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2+5/12*(1-1/a/x)^(3/4)*(1+1/a/x)^
(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3+3/8*arctan((1+1/a/x)^(1
/4)/(1-1/a/x)^(1/4))/a^3+3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A]
time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{11x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{5x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) + (5*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
+ (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^4\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{11}{4a^2}-}{x^2\sqrt[4]{1-\frac{x}{a}}\,(}\right.$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 4.34, size = 399, normalized size = 2.23

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(-1070609085-946471617e^{2\coth^{-1}(ax)}+369641285e^{4\coth^{-1}(ax)}+351173641e^{6\coth^{-1}(ax)}-23818496e^{8\coth^{-1}(ax)}+1070609085\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)+732349800e^{2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)-635067810e^{4\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)-384831720e^{6\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)+60913125e^{8\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)+1280e^{8\coth^{-1}(ax)}\left(821+1346e^{2\coth^{-1}(ax)}+557e^{4\coth^{-1}(ax)}\right){}_4F_3\left(2,2,2,\frac{9}{4};1,1,\frac{21}{4};e^{2\coth^{-1}(ax)}\right)+10240e^{8\coth^{-1}(ax)}\left(23+42e^{2\coth^{-1}(ax)}+19e^{4\coth^{-1}(ax)}\right){}_5F_4\left(2,2,2,2,\frac{9}{4};1,1,1,\frac{21}{4};e^{2\coth^{-1}(ax)}\right)+20480e^{8\coth^{-1}(ax)}\,{}_6F_5\left(2,2,2,2,2,\frac{9}{4};1,1,1,1,\frac{21}{4};e^{2\coth^{-1}(ax)}\right)+40960e^{10\coth^{-1}(ax)}\,{}_6F_5\left(2,2,2,2,2,\frac{9}{4};1,1,1,1,\frac{21}{4};e^{2\coth^{-1}(ax)}\right)+20480e^{12\coth^{-1}(ax)}\,{}_6F_5\left(2,2,2,2,2,\frac{9}{4};1,1,1,1,\frac{21}{4};e^{2\coth^{-1}(ax)}\right)\right)}{1909440a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^2,x]
```

```
[Out] -1/1909440*(-1070609085 - 946471617*E^(2*ArcCoth[a*x]) + 369641285*E^(4*Arc
Coth[a*x]) + 351173641*E^(6*ArcCoth[a*x]) - 23818496*E^(8*ArcCoth[a*x]) + 1
070609085*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 732349800*E^
(2*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 63506
7810*E^(4*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])]
- 384831720*E^(6*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[
a*x])] + 60913125*E^(8*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1, 5/4, E^(2*Ar
cCoth[a*x])] + 1280*E^(8*ArcCoth[a*x])*(821 + 1346*E^(2*ArcCoth[a*x]) + 557
*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 9/4}, {1, 1, 21/4}, E^(2*A
rcCoth[a*x])] + 10240*E^(8*ArcCoth[a*x])*(23 + 42*E^(2*ArcCoth[a*x]) + 19*E
^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 21/4}, E^
(2*ArcCoth[a*x])] + 20480*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2,
 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])] + 40960*E^(10*ArcCoth[a*x
])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4}, {1, 1, 1, 1, 21/4}, E^(2*ArcCoth
[a*x])] + 20480*E^(12*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 9/4},
 {1, 1, 1, 1, 21/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((7*ArcCoth[a*x])/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x)
```

Maxima [A]
time = 0.46, size = 187, normalized size = 1.04

$$-\frac{1}{48}\,a\left(\frac{4\left(9\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+29\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(9*((a*x - 1)/(a*x + 1))^(11/4) - 6*((a*x - 1)/(a*x + 1))^(7/4)
+ 29*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^
```

```
2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x -
1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 9*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A]
time = 0.36, size = 103, normalized size = 0.58

$$\frac{2\,(8\,a^3x^3+18\,a^2x^2+21\,ax+11)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 + 18*a^2*x^2 + 21*a*x + 11)*((a*x - 1)/(a*x + 1))^(3/4)
- 18*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.45, size = 172, normalized size = 0.96

$$-\frac{1}{48}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{9\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}-29\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 9*(a*x - 1)^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 29*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B]
time = 0.08, size = 157, normalized size = 0.88

$$\frac{\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((29*((a*x - 1)/(a*x + 1))^(3/4))/12 - ((a*x - 1)/(a*x + 1))^(7/4)/2 + (3*(
(a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a
^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*x
- 1)/(a*x + 1))^(1/4)))/(8*a^3) + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8
*a^3)
```

3.62 $\int e^{\frac{1}{2}\coth^{-1}(ax)} x\,dx$

Optimal. Leaf size=142

$$\frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] 1/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)
*x^2+1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A]

time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6306, 98, 96, 95, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(3/4)*(1
 + 1/(a*x))^(5/4)*x^2)/2 + ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/
(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)} x\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^3\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3}}\right.}{8a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{2a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1-}}\right.}{4a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{\text{ta}}{}$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 0.46

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+5e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + \text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x,x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2)*(-1 + 5*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x]
))^2 + ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x,x)
```

Maxima [A]
time = 0.48, size = 149, normalized size = 1.05

$$\frac{1}{8}\,a\left(\frac{4\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="maxima")
[Out] 1/8*a*(4*(((a*x - 1)/(a*x + 1))^(7/4) - 5*((a*x - 1)/(a*x + 1))^(3/4))/(2*(
a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A]
time = 0.41, size = 93, normalized size = 0.65

$$\frac{2\,(2\,a^2x^2+5\,ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="fricas")
[Out] 1/8*(2*(2*a^2*x^2 + 5*a*x + 3)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x
 - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x
- 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.44, size = 139, normalized size = 0.98

$$-\frac{1}{8}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^3 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*((a*x
 - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 5*((a*x - 1)/(a*x + 1))^(3/4)
)/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B]
time = 0.08, size = 120, normalized size = 0.85

$$\frac{\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((5*((a*x - 1)/(a*x + 1))^(3/4))/2 - ((a*x - 1)/(a*x + 1))^(7/4)/2)/(a^2 +
(a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - atan(((a*x -
 1)/(a*x + 1))^(1/4))/(4*a^2) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2)
```

3.63 $\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=96

$$\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
+arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6305, 96, 95, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x^2\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\,(1+\frac{x}{a})^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cdots\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.53

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} + \text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2),x]
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) + ArcTan[E^(ArcCoth[a*x]/
2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A]
time = 0.46, size = 111, normalized size = 1.16

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A]
time = 0.37, size = 84, normalized size = 0.88

$$\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) - 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/4), x)
```

Giac [A]
time = 0.43, size = 108, normalized size = 1.12

$$-\frac{1}{2}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 + log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B]
time = 0.06, size = 78, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
- 1)/(a*x + 1))^(1/4))/a + atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

3.64 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$-\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\tanh$$

```
[Out] 2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/
x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/
(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2
^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$,
Rules used = {6306, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$-\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)+2\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+2\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x,x]
```

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= 4\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\text{Sub}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{1+\frac{1}{a}}}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1-}}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\,\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

$$= -\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 30, normalized size = 0.10

$$\frac{8}{5}e^{\frac{5}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{5}{8},1;\frac{13}{8};e^{4\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x,x]
```

```
[Out] (8*E^((5*ArcCoth[a*x])/2)*Hypergeometric2F1[5/8, 1, 13/8, E^(4*ArcCoth[a*x]
)])/5
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Maxima [A]
time = 0.47, size = 224, normalized size = 0.77

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

Fricas [A]
time = 0.37, size = 291, normalized size = 1.00

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\frac{1}{2}\sqrt{2}\log\left(4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)+\frac{1}{2}\sqrt{2}\log\left(-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Giac [A]
time = 0.42, size = 232, normalized size = 0.80

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}-\frac{2\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

Mupad [B]
time = 0.08, size = 101, normalized size = 0.35

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)\,2\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(1-\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(1+1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 - 1i) - 2
*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2
i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1i)
```

3.65 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2} dx$

Optimal. Leaf size=267

$$a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{a\log\left(1+\cdots\right.}{}$$

```
[Out] a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(
1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1
/4))*2^(1/2)+1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)-1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6306, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (a*ArcTan[1 - (Sqrt[2]*(1 - 1/(
a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 - 1/
(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*Log[1 + Sqrt[1 - 1/(a*x)]/
Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*
Sqrt[2]) - (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1
/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - a\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\text{Subst}\left(\int \frac{1+x}{1+x}\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}a\text{Subs}$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{a\log\left(1+\right.}{}$$

$$= a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

Mathematica [A]

time = 0.16, size = 148, normalized size = 0.55

$$a\left(\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}-\frac{\operatorname{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^2,x]
```

```
[Out] a*((2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + ArcTan[1 - Sqrt[2]*E^(
ArcCoth[a*x]/2)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)]/Sqrt[2] +
 Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]) - Log[1 +
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]/(2*Sqrt[2]))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Maxima [A]
time = 0.46, size = 186, normalized size = 0.70

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Fricas [A]
time = 0.40, size = 396, normalized size = 1.48

$$-\frac{4\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^{4}+\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{4}}a^{4}+\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{4}\right)^{\frac{1}{4}}}{a^{4}}\right)+4\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(\frac{a^{4}-\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{4}}a^{4}-\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{4}\right)^{\frac{1}{4}}}{a^{4}}\right)+\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{4}}a^{4}+\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{4}}a^{4}-\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-4\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*a^3*((a*x
- 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a
^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4))
/a^4) + 4*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(1/4)*a^3*((a*x
 - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(
a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4)
)/a^4) + sqrt(2)*(a^4)^(1/4)*x*log(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^4
)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - sqrt(2)*(a^4
)^(1/4)*x*log(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^4)*a^4 - sqrt(2)*(a^4)
^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))
^(3/4))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[4]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Giac [A]
time = 0.45, size = 186, normalized size = 0.70

$$\frac{1}{4} \left(2 \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) + 2 \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right) - \sqrt{2} \log\left(\sqrt{2} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + \sqrt{\frac{ax-1}{ax+1}} + 1\right) + \sqrt{2} \log\left(-\sqrt{2} \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + \sqrt{\frac{ax-1}{ax+1}} + 1\right) + \frac{8 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{ax-1}{ax+1} + 1}\right) a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Mupad [B]
time = 1.19, size = 87, normalized size = 0.33

$$(-1)^{1/4}\, a\, \mathrm{atan}\left((-1)^{1/4} \left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) - (-1)^{1/4}\, a\, \mathrm{atanh}\left((-1)^{1/4} \left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right) + \frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (-1)^(1/4)*a*at
anh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) + (2*a*((a*x - 1)/(a*x + 1))^(3
/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.66 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}+\frac{a^2\text{ArcTa}}{}$$

```
[Out] 1/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(
5/4)+1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}}+\frac{a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(3/4)*
(1 + 1/(a*x))^(5/4))/2 - (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{8}a\text{Subst}\left(\int \frac{}{\sqrt[4]{1-\frac{x}{a}}}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{x^2}{(2-x^4)^3}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{2}a^2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{1}{4}a^2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{1}{8}a^2\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,}\right.$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right.}{8\sqrt{2}}$$

$$= \frac{1}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{4\sqrt{2}}$$

Mathematica [A]
time = 0.15, size = 173, normalized size = 0.54

$$\frac{1}{16}a^2\left(-\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{40e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+2\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)-2\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\sqrt{2}\,\log\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^3,x]
```

```
[Out] (a^2*((-32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (40*E^(ArcCoth[
a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[
1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] - Sqrt[2]*Log[1 + Sqrt[2]*
E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Maxima [A]
time = 0.47, size = 226, normalized size = 0.71

$$\frac{1}{16}\left(\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\right)a+\frac{8\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+5a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4
))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1))*a + 8*(a*((a*x - 1)/(a*x + 1))^(7/4) + 5*a*((a*x - 1)
/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Fricas [A]
time = 0.35, size = 413, normalized size = 1.29

$$-\frac{4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right)+4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right)+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-4\left(3a^2x^2+5ax+2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(1/4)*a^6*((a
*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1)) + sq
rt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8)^(1
/4))/a^8) + 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(1/4)*a^6
*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8
)^(1/4))/a^8) + sqrt(2)*(a^8)^(1/4)*x^2*log(a^12*sqrt((a*x - 1)/(a*x + 1))
+ sqrt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - sq
rt(2)*(a^8)^(1/4)*x^2*log(a^12*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^8)*a^8 -
sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(3*a^2*x^2 + 5*a*x
 + 2)*((a*x - 1)/(a*x + 1))^(3/4))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/4)), x)
```

Giac [A]
time = 0.44, size = 223, normalized size = 0.70

$$\frac{1}{16}\left(2\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{(ax-1)a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}+5\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a
*x + 1) + 5*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B]
time = 0.07, size = 132, normalized size = 0.41

$$\frac{\frac{5\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{2}+\frac{a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}+\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] ((5*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (a^2*((a*x - 1)/(a*x + 1))^(7/4))/
2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*a^
2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - ((-1)^(1/4)*a^2*atanh((
-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

3.67 $\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$\frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x}-\frac{3a^3\text{ArcTan}\left(1-\right.}{8\sqrt{}}$$

```
[Out] 3/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/12*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^
(5/4)+1/3*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(5/4)/x+3/16*a^3*arctan(-1+(1-1/a/x
)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1/4)*
2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1
/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/32*a^3*ln(1+(1-1/a/x
)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3a^3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}+\frac{3a^3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}+\frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)/x^4,x]
```

```
[Out] (3*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 + (a^3*(1 - 1/(a*x))^(3/4
)*(1 + 1/(a*x))^(5/4))/12 + (a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(5/4))/(
3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x
))^(1/4)])/(8*Sqrt[2]) + (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (3*a^
3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{\left(-1-\frac{x}{2a}\right)\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+ \cdots}}{\sqrt[4]{1- \cdots}}\right.$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax} \cdots\right.}{3x}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax} \cdots\right.}{3x}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax} \cdots\right.}{3x}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax} \cdots\right.}{3x}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax} \cdots\right.}{3x}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax} \cdots\right.}{3x}$$

$$= \frac{3}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax} \cdots\right.}{3x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.08, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(9+6e^{2\coth^{-1}(ax)}+29e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3}+9\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/2)/x^4,x]
```

```
[Out] (a^3*((8*E^(ArcCoth[a*x]/2)*(9 + 6*E^(2*ArcCoth[a*x]) + 29*E^(4*ArcCoth[a*x
])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x] - 2*
Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ]))/96
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Maxima [A]
time = 0.47, size = 270, normalized size = 0.76

$$\frac{1}{96}\left(9\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\right)a^2+\frac{8\left(9a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+6a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+29a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3(ax-1)}{ax+1}+\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))*a^2 + 8*(9*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 6*a^2
*((a*x - 1)/(a*x + 1))^(7/4) + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x
- 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*
a
```

Fricas [A]
time = 0.42, size = 427, normalized size = 1.20

$$-\frac{36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+36\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(\frac{a^{12}-\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+9\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\log\left(729a^{18}\sqrt{\frac{ax-1}{ax+1}}+729\sqrt{a^{12}}a^{12}+729\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-9\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\log\left(729a^{18}\sqrt{\frac{ax-1}{ax+1}}+729\sqrt{a^{12}}a^{12}-729\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-4\left(11a^3x^3+21a^2x^2+18ax+8\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{96x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")

[Out] -1/96*(36*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(1/4)*a^9 *((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4))*(a^12)^(1/4))/a^12) + 36*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^ 12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4))*(a^12)^(1/4))/a^12) + 9*sqrt(2)*(a^12)^(1/4)*x^3*log(729*a^18*s qrt((a*x - 1)/(a*x + 1)) + 729*sqrt(a^12)*a^12 + 729*sqrt(2)*(a^12)^(3/4)*a ^9*((a*x - 1)/(a*x + 1))^(1/4)) - 9*sqrt(2)*(a^12)^(1/4)*x^3*log(729*a^18*s qrt((a*x - 1)/(a*x + 1)) + 729*sqrt(a^12)*a^12 - 729*sqrt(2)*(a^12)^(3/4)*a ^9*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(11*a^3*x^3 + 21*a^2*x^2 + 18*a*x + 8)* ((a*x - 1)/(a*x + 1))^(3/4))/x^3

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\sqrt[4]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)/x**4,x)

[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(1/4)), x)

Giac [A]
time = 0.45, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(18\sqrt{2}\,a^{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a^{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-9\sqrt{2}\,a^{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+9\sqrt{2}\,a^{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{6\,(ax-1)a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}+\frac{9\,(ax-1)^{2}a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^{2}}+29\,a^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^{3}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")

[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^ (1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))) - 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt ((a*x - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(6*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 9*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 29*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^ 3)*a

Mupad [B]
time = 1.21, size = 168, normalized size = 0.47

$$\frac{\frac{29\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{12}+\frac{a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}+\frac{3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/4}}{4}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1}+\frac{3\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8}-\frac{3\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(1/4)),x)
```

```
[Out] ((29*a^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (a^3*((a*x - 1)/(a*x + 1))^(7/4)
)/2 + (3*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2
+ (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + (3*(-1)^(1/4)*a^
3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (3*(-1)^(1/4)*a^3*atanh
((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

3.68 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{557\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4}+\frac{157\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3}+\frac{5\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{a}\right.}{40a}$$

[Out] 557/640*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4+157/320*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^2/a^3+5/16*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2+11/40*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5-237/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5+237/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5

Rubi [A]
time = 0.09, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$-\frac{237\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{237\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{557x\sqrt[4]{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{3/4}}{640a^4}+\frac{157x^2\sqrt[4]{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{3/4}}{320a^3}+\frac{5x^3\sqrt[4]{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{3/4}}{16a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{11x^4\sqrt[4]{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{3/4}}{40a}$$

Antiderivative was successfully verified.

[In] Int[E^((3*ArcCoth[a*x])/2)*x^4,x]

[Out] (557*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(640*a^4) + (157*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^3)/(16*a^2) + (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 - (237*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (237*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^6\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{\frac{11}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-\frac{75}{4a^2}}{x^4\left(1-\frac{x}{a}\right)^3}\right.$$

$$= \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x$$

$$= \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{16a^2} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{40a}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{16a^2}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{16a^2}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{16a^2}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{16a^2}$$

$$= \frac{557\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{640a^4} + \frac{157\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{320a^3} + \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{16a^2}$$

Mathematica [A]
time = 5.16, size = 173, normalized size = 0.68

$$\frac{\frac{8192e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{22016e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{23936e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{14032e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{5500e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-2370\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-1185\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+1185\log\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{1280a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] ((8192*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 + (22016*E^((3*A
rcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((3*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^3 + (14032*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*
ArcCoth[a*x]))^2 + (5500*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))
- 2370*ArcTan[E^(ArcCoth[a*x]/2)] - 1185*Log[1 - E^(ArcCoth[a*x]/2)] + 1185
*Log[1 + E^(ArcCoth[a*x]/2)])/(1280*a^5)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x)
```

Maxima [A]
time = 0.47, size = 259, normalized size = 1.02

$$-\frac{1}{1280}a\left(\frac{4\left(395\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+1375\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{1185\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(395*((a*x - 1)/(a*x + 1))^(17/4) - 1440*((a*x - 1)/(a*x + 1))
^(13/4) + 3710*((a*x - 1)/(a*x + 1))^(9/4) - 1992*((a*x - 1)/(a*x + 1))^(5/
4) + 1375*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
 - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^
6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 11
85*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A]
time = 0.34, size = 119, normalized size = 0.47

$$\frac{2\,(128\,a^5x^5+304\,a^4x^4+376\,a^3x^3+514\,a^2x^2+871\,ax+557)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{1280\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 + 304*a^4*x^4 + 376*a^3*x^3 + 514*a^2*x^2 + 871*a*x
+ 557)*((a*x - 1)/(a*x + 1))^(1/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4
)) + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1185*log(((a*x - 1)/(a*x +
 1))^(1/4) - 1))/a^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.53, size = 234, normalized size = 0.92

$$\frac{1}{1280}\,a\left(\frac{2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{1185\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}+\frac{4\left(\frac{1992\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{3710\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+\frac{1440\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^3}-\frac{395\,(ax-1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^4}-1375\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6\left(\frac{ax-1}{ax+1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^4,x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 - 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1992*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1440*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 395*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/
4)/(a*x + 1)^4 - 1375*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

Mupad [B]
time = 0.09, size = 229, normalized size = 0.91

$$\frac{\frac{275\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{64}-\frac{249\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{40}+\frac{371\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}{32}-\frac{9\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}{2}+\frac{79\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/4}}{64}}{a^5+\frac{10\,a^5\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a^5\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a^5\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a^5\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{5\,a^5\,\left(a\,x-1\right)}{a\,x+1}}+\frac{237\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{128\,a^5}+\frac{237\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((275*((a*x - 1)/(a*x + 1))^(1/4))/64 - (249*((a*x - 1)/(a*x + 1))^(5/4))/4
0 + (371*((a*x - 1)/(a*x + 1))^(9/4))/32 - (9*((a*x - 1)/(a*x + 1))^(13/4))
/2 + (79*((a*x - 1)/(a*x + 1))^(17/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*x
 + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)^
4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (237*ata
n(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) + (237*atanh(((a*x - 1)/(a*x + 1)
)^(1/4)))/(128*a^5)
```

3.69 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$\frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3}+\frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2}+\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3}$$

```
[Out] 63/64*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+15/32*(1-1/a/x)^(1/4)*(1+1/a/x)
^(3/4)*x^2/a^2+3/8*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(1/4
)*(1+1/a/x)^(3/4)*x^4-123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+12
3/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A]
time = 0.08, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$-\frac{123\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{63x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{64a^3}+\frac{15x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{32a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{3x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (63*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(64*a^3) + (15*(1 - 1/(a*x))
^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(32*a^2) + (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a
*x))^(3/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4 - (
123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTan
h[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^5\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\frac{9}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-\frac{45}{4a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/}}\right.$$

$$= \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}$$

$$= \frac{63\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{64a^3} + \frac{15\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{32a^2} + \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{8a}$$

Mathematica [A]
time = 5.13, size = 149, normalized size = 0.69

$$\frac{\frac{512e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{1152e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1008e^{\frac{3}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{532e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-246\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-123\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+123\log\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] ((512*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (1152*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((3*ArcCoth[a*x])/2))/
(-1 + E^(2*ArcCoth[a*x]))^2 + (532*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x])) - 246*ArcTan[E^(ArcCoth[a*x]/2)] - 123*Log[1 - E^(ArcCoth[a*x]/2)
] + 123*Log[1 + E^(ArcCoth[a*x]/2)])/(128*a^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x)
```

Maxima [A]
time = 0.46, size = 224, normalized size = 1.04

$$\frac{1}{128}a\left(\frac{4\left(41\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-183\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+147\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-133\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/128*a*(4*(41*((a*x - 1)/(a*x + 1))^(13/4) - 183*((a*x - 1)/(a*x + 1))^(9/
4) + 147*((a*x - 1)/(a*x + 1))^(5/4) - 133*((a*x - 1)/(a*x + 1))^(1/4))/(4*
(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^5
/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 123*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A]
time = 0.37, size = 111, normalized size = 0.51

$$\frac{2\left(16a^4x^4+40a^3x^3+54a^2x^2+93ax+63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{128a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 54*a^2*x^2 + 93*a*x + 63)*((a*x - 1)/(a
*x + 1))^(1/4) + 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x -
1)/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.51, size = 203, normalized size = 0.94

$$\frac{1}{128}\,a\left(\frac{246\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{123\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{123\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{4\left(\frac{147\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{183\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+\frac{41\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^3}-133\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5\left(\frac{ax-1}{ax+1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^3,x, algorithm="giac")
```

```
[Out] 1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 123*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^5 - 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
5 - 4*(147*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 183*(a*x - 1)^
2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 41*(a*x - 1)^3*((a*x - 1)/(a*x
+ 1))^(1/4)/(a*x + 1)^3 - 133*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/
(a*x + 1) - 1)^4))
```

Mupad [B]
time = 1.21, size = 192, normalized size = 0.89

$$\frac{\frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{32}-\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{32}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}+\frac{123\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{64\,a^4}+\frac{123\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((133*((a*x - 1)/(a*x + 1))^(1/4))/32 - (147*((a*x - 1)/(a*x + 1))^(5/4))/3
2 + (183*((a*x - 1)/(a*x + 1))^(9/4))/32 - (41*((a*x - 1)/(a*x + 1))^(13/4)
)/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1
)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (123*a
tan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) + (123*atanh(((a*x - 1)/(a*x + 1
))^(1/4)))/(64*a^4)
```

3.70 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2}+\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a}+\frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3-\frac{17\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

[Out] 23/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2+7/12*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3-17/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3+17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3

Rubi [A]
time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$-\frac{17\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{23x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{7x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

Antiderivative was successfully verified.

[In] Int[E^((3*ArcCoth[a*x])/2)*x^2,x]

[Out] (23*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) + (7*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) + (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^4\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{23}{4a^2} -}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\right.$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3$$

$$= \frac{23\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} + \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 7.01, size = 399, normalized size = 2.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] -1/4213440*(-1357846875 - 1400453615*E^(2*ArcCoth[a*x]) + 276606275*E^(4*Ar
cCoth[a*x]) + 438715415*E^(6*ArcCoth[a*x]) - 12962560*E^(8*ArcCoth[a*x]) +
1357846875*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 818519240*E
^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] - 9977
22110*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])]
 - 501106760*E^(6*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcCoth
[a*x])] + 137997475*E^(8*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*
ArcCoth[a*x])] + 1792*E^(8*ArcCoth[a*x])*(965 + 1618*E^(2*ArcCoth[a*x]) + 6
85*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 11/4}, {1, 1, 23/4}, E^(
2*ArcCoth[a*x])] + 14336*E^(8*ArcCoth[a*x])*(25 + 46*E^(2*ArcCoth[a*x]) + 2
1*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 11/4}, {1, 1, 1, 23/4}
, E^(2*ArcCoth[a*x])] + 28672*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2
, 2, 2, 11/4}, {1, 1, 1, 1, 23/4}, E^(2*ArcCoth[a*x])] + 57344*E^(10*ArcCot
h[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 11/4}, {1, 1, 1, 1, 23/4}, E^(2*A
rcCoth[a*x])] + 28672*E^(12*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2,
 11/4}, {1, 1, 1, 1, 23/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((5*ArcCoth[a*x])/2
))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x)
```

Maxima [A]
time = 0.47, size = 187, normalized size = 1.04

$$-\frac{1}{48}\,a\left(\frac{4\left(17\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+45\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(17*((a*x - 1)/(a*x + 1))^(9/4) - 30*((a*x - 1)/(a*x + 1))^(5/4)
 + 45*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)
^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 +
51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A]
time = 0.39, size = 103, normalized size = 0.58

$$\frac{2\left(8\,a^{3}x^{3}+22\,a^{2}x^{2}+37\,ax+23\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 37*a*x + 23)*((a*x - 1)/(a*x + 1))^(1/4)
+ 102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 51*log(((a*x - 1)/(a*x + 1))^(1
/4) + 1) - 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.48, size = 172, normalized size = 0.96

$$\frac{1}{48}\,a\left(\frac{102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{4}}+\frac{51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{4}}-\frac{51\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^{4}}+\frac{4\left(\frac{30\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{17\,(ax-1)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^{2}}-45\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{4}\left(\frac{ax-1}{ax+1}-1\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^2,x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 - 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 17*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 45*((a*x - 1)/(a*x + 1))^(1/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B]
time = 0.08, size = 157, normalized size = 0.88

$$\frac{\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,\left(a\,x-1\right)^2}{\left(a\,x+1\right)^2}-\frac{a^3\,\left(a\,x-1\right)^3}{\left(a\,x+1\right)^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}+\frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((15*((a*x - 1)/(a*x + 1))^(1/4))/4 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2 + (
17*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2
- (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*atan((
(a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) + (17*atanh(((a*x - 1)/(a*x + 1))^(1/4
)))/(8*a^3)
```

3.71 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$\frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2-\frac{9\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] 3/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)
*x^2-9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A]

time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6306, 98, 96, 95, 304, 209, 212}

$$-\frac{9\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x,x]
```

```
[Out] (3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(4*a) + ((1 - 1/(a*x))^(1/4)*
(1 + 1/(a*x))^(7/4)*x^2)/2 - (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1
/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2
)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^3\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{3\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\right.}{8a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{2a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 + \frac{9\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.}{4a^2}$$

$$= \frac{3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}x^2 - \frac{9\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{9}{}$$

Mathematica [A]
time = 0.12, size = 70, normalized size = 0.49

$$\frac{\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-3+7e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - 9\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 9\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x,x]
```

```
[Out] ((2*E^((3*ArcCoth[a*x])/2)*(-3 + 7*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[
a*x]))^2 - 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4
*a^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x,x)
```

Maxima [A]
time = 0.47, size = 152, normalized size = 1.07

$$\frac{1}{8}\, a \left(\frac{4 \left(3 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} - 7 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1} - \frac{(ax-1)^2 a^3}{(ax+1)^2} - a^3} + \frac{18 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3} + \frac{9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^3} - \frac{9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="maxima")
[Out] 1/8*a*(4*(3*((a*x - 1)/(a*x + 1))^(5/4) - 7*((a*x - 1)/(a*x + 1))^(1/4))/(2
*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan((
(a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^
3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A]
time = 0.35, size = 95, normalized size = 0.67

$$\frac{2\,(2\,a^2x^2 + 7\,ax + 5)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 18 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right) + 9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right) - 9 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="fricas")
[Out] 1/8*(2*(2*a^2*x^2 + 7*a*x + 5)*((a*x - 1)/(a*x + 1))^(1/4) + 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.48, size = 141, normalized size = 0.99

$$\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{3(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-7\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x,x, algorithm="giac")
```

```
[Out] 1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 9*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1)/a^3 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*(
3*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 7*((a*x - 1)/(a*x + 1))
^(1/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B]
time = 1.21, size = 120, normalized size = 0.85

$$\frac{\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}+\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}+\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((7*((a*x - 1)/(a*x + 1))^(1/4))/2 - (3*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^
2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) + (9*atan(
((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) + (9*atanh(((a*x - 1)/(a*x + 1))^(1/4
)))/(4*a^2)
```

3.72 $\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x-\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
/a+3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6305, 96, 95, 304, 209, 212}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a + (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{6\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 56, normalized size = 0.57

$$\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2),x]
[Out] (8*E^((3*ArcCoth[a*x])/2)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[
3/4, 2, 7/4, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A]
time = 0.46, size = 112, normalized size = 1.14

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A]
time = 0.38, size = 86, normalized size = 0.88

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) + 6*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) + 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 3*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-3/4), x)
```

Giac [A]
time = 0.45, size = 109, normalized size = 1.11

$$\frac{1}{2}\,a\left(\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 - 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B]
time = 1.19, size = 79, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/a + (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.73 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$-\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-2\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\tanh$$

```
[Out] -2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a
/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)
/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*
2^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6306, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$-\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)-2\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+2\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x,x]
```

```
[Out] -(Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]) +
Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*A
rcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1
/4)/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (S
qrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 -
 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(
1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 4\text{Sub}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \right.}{\sqrt{2}}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 30, normalized size = 0.10

$$\frac{8}{7}e^{\frac{7}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};e^{4\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x,x]
```

```
[Out] (8*E^((7*ArcCoth[a*x])/2)*Hypergeometric2F1[7/8, 1, 15/8, E^(4*ArcCoth[a*x]
)])/7
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Maxima [A]
time = 0.46, size = 224, normalized size = 0.77

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a)
```

Fricas [A]
time = 0.37, size = 291, normalized size = 1.00

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\sqrt{2}\log\left(4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-\frac{1}{2}\sqrt{2}\log\left(-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)+2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log(
((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Giac [**A**]
time = 0.45, size = 232, normalized size = 0.80

$$\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}+\frac{2\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}-\frac{2\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a)
```

Mupad [**B**]
time = 0.05, size = 101, normalized size = 0.35

$$2\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(1+1{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(1-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
```

3.74 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=268

$$a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{3a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{3a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{3a\log\left(1\right.}{}$$

[Out] a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)

Rubi [A]

time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6306, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{3a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[E^((3*ArcCoth[a*x])/2)/x^2,x]

[Out] a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4) - (3*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) + (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (6a)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (3a)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + (3a)\text{Subst}\Bigg(\int$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}(3a)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}(3a$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{3a\log\Bigg(1+}{}$$

$$= a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.05, size = 46, normalized size = 0.17

$$-8ae^{\frac{3}{2}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}}+{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] -8*a*E^((3*ArcCoth[a*x])/2)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometri
c2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

Maxima [A]
time = 0.48, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Fricas [A]
time = 0.37, size = 376, normalized size = 1.40

$$-\frac{12\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^{4}+\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}\sqrt{a^{2}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{4}}}}{a^{4}}\right)+12\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(\frac{a^{4}-\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}\sqrt{a^{2}\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{4}}}}{a^{4}}\right)-3\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(9a^{2}\sqrt{\frac{ax-1}{ax+1}}+9\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+9\sqrt{a^{4}}\right)+3\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(9a^{2}\sqrt{\frac{ax-1}{ax+1}}-9\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+9\sqrt{a^{4}}\right)-4\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1
)) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
 12*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a
*x + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - s
qrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - 3*sqr
t(2)*(a^4)^(1/4)*x*log(9*a^2*sqrt((a*x - 1)/(a*x + 1)) + 9*sqrt(2)*(a^4)^(1
/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt(a^4)) + 3*sqrt(2)*(a^4)^(1/4)*x*
log(9*a^2*sqrt((a*x - 1)/(a*x + 1)) - 9*sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a
*x + 1))^(1/4) + 9*sqrt(a^4)) - 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Giac [A]
time = 0.43, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) - 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Mupad [B]
time = 0.08, size = 88, normalized size = 0.33

$$\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1) - (-1)^(1/4)*a*
atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i - (-1)^(1/4)*a*atan((-1)^(
1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i
```

3.75 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}-\frac{9a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}+\frac{9a^2\text{ArcTan}\left(\right.}{}$$

```
[Out] 3/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(
7/4)+9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+9/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{9a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}}+\frac{9a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}}+\frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (3*a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(1/4
)*(1 + 1/(a*x))^(7/4))/2 - (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{1}{4}(3a)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{1}{8}(9a)\text{Subst}\left(\int \frac{}{\left(1-\frac{x}{a}\right)^{3}}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x}}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{1}{1+x^4}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{1}{8}\left(9a^2\right)\text{Subst}\left(\int \frac{}{1-\sqrt{2}}\right.$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right.}{8\sqrt{2}}$$

$$= \frac{3}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} - \frac{9a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{4\sqrt{2}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.06, size = 76, normalized size = 0.24

$$-\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left({}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)-3\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)+2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 1, 7/4, -E^(2*ArcCot
h[a*x])] - 3*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a*x])] + 2*Hyperg
eometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Maxima [A]
time = 0.46, size = 229, normalized size = 0.72

$$\frac{1}{16}\left(18\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(3a\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+7a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*a*((a*x - 1)/(a*x + 1))^(5/4) + 7*
a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x +
1)^2 + 1))*a
```

Fricas [A]
time = 0.36, size = 405, normalized size = 1.27

$$-\frac{36\sqrt{2}\,(a^8)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\,(a^8)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\,(a^8)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\,(a^8)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+36\sqrt{2}\,(a^8)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\,(a^8)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\,(a^8)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\,(a^8)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)-9\sqrt{2}\,(a^8)^{\frac{1}{4}}x^2\log\left(81a^4\sqrt{\frac{ax-1}{ax+1}}+81\sqrt{2}\,(a^8)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+81\sqrt{a^8}\right)+9\sqrt{2}\,(a^8)^{\frac{1}{4}}x^2\log\left(81a^4\sqrt{\frac{ax-1}{ax+1}}-81\sqrt{2}\,(a^8)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+81\sqrt{a^8}\right)-4\left(5a^2x^2+7ax+2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(36*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*((
a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*
x + 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))
/a^8) + 36*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*((
a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*
x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))
/a^8) - 9*sqrt(2)*(a^8)^(1/4)*x^2*log(81*a^4*sqrt((a*x - 1)/(a*x + 1)) + 81
*sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 81*sqrt(a^8)) + 9*sq
rt(2)*(a^8)^(1/4)*x^2*log(81*a^4*sqrt((a*x - 1)/(a*x + 1)) - 81*sqrt(2)*(a^
8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 81*sqrt(a^8)) - 4*(5*a^2*x^2 + 7
*a*x + 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Giac [**A**]
time = 0.45, size = 225, normalized size = 0.71

$$\frac{1}{16}\left(18\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{3(ax-1)a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}+7a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
- 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(3*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^
(1/4)/(a*x + 1) + 7*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1
)^2)*a
```

Mupad [**B**]
time = 0.09, size = 132, normalized size = 0.41

$$\frac{\frac{7\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{2}+\frac{3\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,9{}\mathrm{i}}{4}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,9{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] ((7*a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (3*a^2*((a*x - 1)/(a*x + 1))^(5/4)
)/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*
a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4 - ((-1)^(1/4)*a^2*at
anh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*9i)/4
```

3.76 $\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4}}{3x}-\frac{17a^3\text{ArcTan}\left(1-\frac{\sqrt{\ }}{\ }\right.}{8\sqrt{2}}$$

[Out] 17/24*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/4*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)+1/3*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(7/4)/x+17/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+17/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)

Rubi [A]
time = 0.19, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{17a^3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}+\frac{17a^3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}+\frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/4}}{3x}$$

Antiderivative was successfully verified.

[In] Int[E^((3*ArcCoth[a*x])/2)/x^4,x]

[Out] (17*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/24 + (a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/4 + (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(7/4))/(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{\left(-1-\frac{3x}{2a}\right)\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} - \frac{1}{24}\left(17a^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/}}{\left(1-\frac{x}{a}\right)^{3/}}\right.$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} -$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} -$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} -$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} -$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} -$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} -$$

$$= \frac{17}{24}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/4} + \frac{a^2\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/4}}{3x} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(17+30e^{2\coth^{-1}(ax)}+45e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3}+51\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (a^3*((8*E^((3*ArcCoth[a*x])/2)*(17 + 30*E^(2*ArcCoth[a*x]) + 45*E^(4*ArcCo
th[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*
x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1 & ]))/96
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Maxima [A]
time = 0.46, size = 277, normalized size = 0.78

$$\frac{1}{96}\left(102\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+51\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-51\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(17\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+30\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+45\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)}{ax+1}+\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(17*a^2*((a*x - 1)/(a*x +
 1))^(9/4) + 30*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 45*a^2*((a*x - 1)/(a*x +
1))^(1/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3
/(a*x + 1)^3 + 1))*a
```

Fricas [A]
time = 0.38, size = 413, normalized size = 1.16

$$-\frac{204\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\,(a^{12})^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\,(a^{12})^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+204\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\arctan\left(\frac{a^{12}-\sqrt{2}\,(a^{12})^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\,(a^{12})^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)-51\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\log\left(289\,a^6\sqrt{\frac{ax-1}{ax+1}}+289\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+289\sqrt{a^{12}}\right)+51\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\log\left(289\,a^6\sqrt{\frac{ax-1}{ax+1}}-289\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+289\sqrt{a^{12}}\right)-4\left(23\,a^3x^3+37\,a^2x^2+22\,ax+8\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^
3*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1
)/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
a^12)))/a^12) + 204*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^
(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt(
(a*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
 + sqrt(a^12)))/a^12) - 51*sqrt(2)*(a^12)^(1/4)*x^3*log(289*a^6*sqrt((a*x -
 1)/(a*x + 1)) + 289*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) +
 289*sqrt(a^12)) + 51*sqrt(2)*(a^12)^(1/4)*x^3*log(289*a^6*sqrt((a*x - 1)/(
a*x + 1)) - 289*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 289*
sqrt(a^12)) - 4*(23*a^3*x^3 + 37*a^2*x^2 + 22*a*x + 8)*((a*x - 1)/(a*x + 1)
)^(1/4))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(3/4)), x)
```

Giac [A]
time = 0.46, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(102\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+51\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-51\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{30\,(ax-1)a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}+\frac{17\,(ax-1)^2a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+45\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) + 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) - 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(1/4)/(a*x + 1) + 17*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1)^2 + 45*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

Mupad [B]
time = 1.21, size = 168, normalized size = 0.47

$$\frac{\frac{15\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{4}+\frac{5\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{2}+\frac{17\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1}-\frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,17{}\mathrm{i}}{8}-\frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,17{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(3/4)),x)
```

```
[Out] ((15*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (5*a^3*((a*x - 1)/(a*x + 1))^(5/4
))/2 + (17*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)^
2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^
3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8 - ((-1)^(1/4)*a^3*ata
nh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*17i)/8
```

3.77 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=287

$$-\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}\,x}{5\sqrt[4]{1-\frac{1}{ax}}}$$

```
[Out] -26111/1920*(1+1/a/x)^(1/4)/a^5/(1-1/a/x)^(1/4)+5533/1920*(1+1/a/x)^(1/4)*x
/a^4/(1-1/a/x)^(1/4)+1189/960*(1+1/a/x)^(1/4)*x^2/a^3/(1-1/a/x)^(1/4)+181/2
40*(1+1/a/x)^(1/4)*x^3/a^2/(1-1/a/x)^(1/4)+21/40*(1+1/a/x)^(1/4)*x^4/a/(1-1
/a/x)^(1/4)+1/5*(1+1/a/x)^(1/4)*x^5/(1-1/a/x)^(1/4)+1003/128*arctan((1+1/a/
x)^(1/4)/(1-1/a/x)^(1/4))/a^5+1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1
/4))/a^5
```

Rubi [A]
time = 0.11, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6306, 100, 156, 160, 12, 95, 218, 212, 209}

$$\frac{1003\,\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{26111\sqrt[4]{\frac{1}{ax}+1}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}}+\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{5533x\sqrt[4]{\frac{1}{ax}+1}}{1920a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{1189x^2\sqrt[4]{\frac{1}{ax}+1}}{960a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{181x^3\sqrt[4]{\frac{1}{ax}+1}}{240a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{x^5\sqrt[4]{\frac{1}{ax}+1}}{5\sqrt[4]{1-\frac{1}{ax}}}+\frac{21x^4\sqrt[4]{\frac{1}{ax}+1}}{40a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (-26111*(1 + 1/(a*x))^(1/4))/(1920*a^5*(1 - 1/(a*x))^(1/4)) + (5533*(1 + 1/
(a*x))^(1/4)*x)/(1920*a^4*(1 - 1/(a*x))^(1/4)) + (1189*(1 + 1/(a*x))^(1/4)*
x^2)/(960*a^3*(1 - 1/(a*x))^(1/4)) + (181*(1 + 1/(a*x))^(1/4)*x^3)/(240*a^2
*(1 - 1/(a*x))^(1/4)) + (21*(1 + 1/(a*x))^(1/4)*x^4)/(40*a*(1 - 1/(a*x))^(1
/4)) + ((1 + 1/(a*x))^(1/4)*x^5)/(5*(1 - 1/(a*x))^(1/4)) + (1003*ArcTan[(1
+ 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) + (1003*ArcTanh[(1 + 1/(a*
x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^6\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{20}\text{Subst}\left(\int \frac{\frac{181}{4a^2}+\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{60}\text{Subst}\left(\int \frac{-\frac{1189}{8a^3}-\frac{543}{4a}\cdots}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1-\cdots\right.}\right.$$

$$= \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^5}{5\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{120}\text{Subst}\cdots$$

$$= \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{21\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1-\frac{1}{ax}}} + \cdots$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \cdots$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \cdots$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{1920a^5\sqrt[4]{1-\frac{1}{ax}}} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1-\frac{1}{ax}}} + \cdots$$

$$= -\frac{26111\sqrt[4]{1+\frac{1}{ax}}}{\cdots} + \frac{5533\sqrt[4]{1+\frac{1}{ax}}\,x}{\cdots} + \frac{1189\sqrt[4]{1+\frac{1}{ax}}\,x^2}{\cdots} + \frac{181\sqrt[4]{1+\frac{1}{ax}}\,x^3}{\cdots} + \cdots$$

Mathematica [A]
time = 5.17, size = 198, normalized size = 0.69

$$\frac{-8e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^5}+\frac{122e^{\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{233e^{\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1661e^{\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{4117e^{\frac{1}{2}\coth^{-1}(ax)}}{192\left(-1+e^{2\coth^{-1}(ax)}\right)}+\frac{1003}{128}\mathrm{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-\frac{1003}{256}\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\frac{1003}{256}\log\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^4,x]
```

```
[Out] (-8*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]
))^5) + (122*E^(ArcCoth[a*x]/2))/(5*(-1 + E^(2*ArcCoth[a*x]))^4) + (233*E^(
ArcCoth[a*x]/2))/(6*(-1 + E^(2*ArcCoth[a*x]))^3) + (1661*E^(ArcCoth[a*x]/2)
)/(48*(-1 + E^(2*ArcCoth[a*x]))^2) + (4117*E^(ArcCoth[a*x]/2))/(192*(-1 + E
^(2*ArcCoth[a*x]))) + (1003*ArcTan[E^(ArcCoth[a*x]/2)])/128 - (1003*Log[1 -
 E^(ArcCoth[a*x]/2)])/256 + (1003*Log[1 + E^(ArcCoth[a*x]/2)])/256)/a^5
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x)
```

Maxima [A]
time = 0.47, size = 275, normalized size = 0.96

$$-\frac{1}{3840}\,a\left(\frac{4\left(\frac{58985\,(ax-1)}{ax+1}-\frac{125920\,(ax-1)^2}{(ax+1)^2}+\frac{137930\,(ax-1)^3}{(ax+1)^3}-\frac{72216\,(ax-1)^4}{(ax+1)^4}+\frac{15045\,(ax-1)^5}{(ax+1)^5}-7680\right)}{a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{21}{4}}-5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}+10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-10\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+5\,a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{30090\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}-\frac{15045\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}+\frac{15045\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(58985*(a*x - 1)/(a*x + 1) - 125920*(a*x - 1)^2/(a*x + 1)^2 +
137930*(a*x - 1)^3/(a*x + 1)^3 - 72216*(a*x - 1)^4/(a*x + 1)^4 + 15045*(a*x
 - 1)^5/(a*x + 1)^5 - 7680)/(a^6*((a*x - 1)/(a*x + 1))^(21/4) - 5*a^6*((a*x
 - 1)/(a*x + 1))^(17/4) + 10*a^6*((a*x - 1)/(a*x + 1))^(13/4) - 10*a^6*((a*
x - 1)/(a*x + 1))^(9/4) + 5*a^6*((a*x - 1)/(a*x + 1))^(5/4) - a^6*((a*x - 1
)/(a*x + 1))^(1/4)) + 30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^6)
```

Fricas [A]
time = 0.38, size = 152, normalized size = 0.53

$$-\frac{30090\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+15045\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(384\,a^6x^6+1392\,a^5x^5+2456\,a^4x^4+3826\,a^3x^3+7911\,a^2x^2-20578\,ax-26111\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{3840\left(a^6x-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="fricas")
```

```
[Out] -1/3840*(30090*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 15045*(a*x -
 1)*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*(a*x - 1)*log(((a*x - 1)/(
a*x + 1))^(1/4) - 1) - 2*(384*a^6*x^6 + 1392*a^5*x^5 + 2456*a^4*x^4 + 3826*
a^3*x^3 + 7911*a^2*x^2 - 20578*a*x - 26111)*((a*x - 1)/(a*x + 1))^(3/4))/(a
^6*x - a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x***4,x)
```

```
[Out] Integral(x**4/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A]
time = 0.48, size = 254, normalized size = 0.89

$$-\frac{1}{3840}\, a \left(\frac{30090 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6} - \frac{15045 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6} + \frac{15045 \log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6} + \frac{30720}{a^6 \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}} - \frac{4\left(\frac{49120\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} - \frac{61130\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2} + \frac{33816\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^3} - \frac{7365\,(ax-1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^4} - 20585\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^6\left(\frac{ax-1}{ax+1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^4,x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 15045*log(((a*x
- 1)/(a*x + 1))^(1/4) + 1)/a^6 + 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
- 1))/a^6 + 30720/(a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(49120*(a*x - 1)*((
a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1
))^(3/4)/(a*x + 1)^2 + 33816*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^3 - 7365*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^4 - 20585*((
a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

Mupad [B]
time = 1.26, size = 248, normalized size = 0.86

$$\frac{1003\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5} - \frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5} - \frac{\frac{787\,(a\,x-1)^2}{6\,(a\,x+1)^2} - \frac{13793\,(a\,x-1)^3}{96\,(a\,x+1)^3} + \frac{3009\,(a\,x-1)^4}{40\,(a\,x+1)^4} - \frac{1003\,(a\,x-1)^5}{64\,(a\,x+1)^5} - \frac{11797\,(a\,x-1)}{192\,(a\,x+1)} + 8}{a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - 5\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4} + 10\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4} - 10\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4} + 5\,a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4} - a^5\left(\frac{a\,x-1}{a\,x+1}\right)^{21/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (1003*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (1003*atan(((a*x - 1)
/(a*x + 1))^(1/4)))/(128*a^5) - ((787*(a*x - 1)^2)/(6*(a*x + 1)^2) - (13793
*(a*x - 1)^3)/(96*(a*x + 1)^3) + (3009*(a*x - 1)^4)/(40*(a*x + 1)^4) - (100
3*(a*x - 1)^5)/(64*(a*x + 1)^5) - (11797*(a*x - 1))/(192*(a*x + 1)) + 8)/(a
^5*((a*x - 1)/(a*x + 1))^(1/4) - 5*a^5*((a*x - 1)/(a*x + 1))^(5/4) + 10*a^5
*((a*x - 1)/(a*x + 1))^(9/4) - 10*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 5*a^5*
((a*x - 1)/(a*x + 1))^(17/4) - a^5*((a*x - 1)/(a*x + 1))^(21/4))
```

3.78 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=250

$$-\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}$$

```
[Out] -2467/192*(1+1/a/x)^(1/4)/a^4/(1-1/a/x)^(1/4)+521/192*(1+1/a/x)^(1/4)*x/a^3
/(1-1/a/x)^(1/4)+113/96*(1+1/a/x)^(1/4)*x^2/a^2/(1-1/a/x)^(1/4)+17/24*(1+1/
a/x)^(1/4)*x^3/a/(1-1/a/x)^(1/4)+1/4*(1+1/a/x)^(1/4)*x^4/(1-1/a/x)^(1/4)+47
5/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A]
time = 0.09, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6306, 100, 156, 160, 12, 95, 218, 212, 209}

$$\frac{475\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{2467\sqrt[4]{\frac{1}{ax}+1}}{192a^4\sqrt[4]{1-\frac{1}{ax}}}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{521x\sqrt[4]{\frac{1}{ax}+1}}{192a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{113x^2\sqrt[4]{\frac{1}{ax}+1}}{96a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{x^4\sqrt[4]{\frac{1}{ax}+1}}{4\sqrt[4]{1-\frac{1}{ax}}}+\frac{17x^3\sqrt[4]{\frac{1}{ax}+1}}{24a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-2467*(1 + 1/(a*x))^(1/4))/(192*a^4*(1 - 1/(a*x))^(1/4)) + (521*(1 + 1/(a*
x))^(1/4)*x)/(192*a^3*(1 - 1/(a*x))^(1/4)) + (113*(1 + 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 - 1/(a*x))^(1/4)) + (17*(1 + 1/(a*x))^(1/4)*x^3)/(24*a*(1 - 1/(a
*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^4)/(4*(1 - 1/(a*x))^(1/4)) + (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^5\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{113}{4a^2}+\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{24}\text{Subst}\left(\int \frac{-\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\cdots\right.}\right.$$

$$= \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^4}{4\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{24}\text{Subst}\left(\int \cdots\right.$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\cdots}}{4\sqrt[4]{1-\cdots}}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\cdots}}{4\sqrt[4]{1-\cdots}}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\cdots}}{4\sqrt[4]{1-\cdots}}$$

$$= -\frac{2467\sqrt[4]{1+\frac{1}{ax}}}{192a^4\sqrt[4]{1-\frac{1}{ax}}} + \frac{521\sqrt[4]{1+\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{113\sqrt[4]{1+\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{17\sqrt[4]{1+\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\cdots}}{4\sqrt[4]{1-\cdots}}$$

Mathematica [A]
time = 5.16, size = 161, normalized size = 0.64

$$\frac{-3072e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{1536e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{5248e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{7376e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{6292e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+2850\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)-1425\log\left(1-e^{\frac{1}{2}\coth^{-1}(ax)}\right)+1425\log\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{384a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^3,x]
```

```
[Out] (-3072*E^(ArcCoth[a*x]/2) + (1536*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*
x]))^4 + (5248*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (7376*E^(A
rcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x]))^2 + (6292*E^(ArcCoth[a*x]/2))/(-1
 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(ArcCoth[a*x]/2)] - 1425*Log[1 - E^(
ArcCoth[a*x]/2)] + 1425*Log[1 + E^(ArcCoth[a*x]/2)])/(384*a^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x)
```

Maxima [A]
time = 0.46, size = 238, normalized size = 0.95

$$\frac{1}{384}\,a\left(\frac{4\left(\frac{4645\,(ax-1)}{ax+1}-\frac{7483\,(ax-1)^2}{(ax+1)^2}+\frac{5415\,(ax-1)^3}{(ax+1)^3}-\frac{1425\,(ax-1)^4}{(ax+1)^4}-768\right)}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+6\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-4\,a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="maxima")
```

```
[Out] 1/384*a*(4*(4645*(a*x - 1)/(a*x + 1) - 7483*(a*x - 1)^2/(a*x + 1)^2 + 5415*
(a*x - 1)^3/(a*x + 1)^3 - 1425*(a*x - 1)^4/(a*x + 1)^4 - 768)/(a^5*((a*x -
1)/(a*x + 1))^(17/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(13/4) + 6*a^5*((a*x - 1
)/(a*x + 1))^(9/4) - 4*a^5*((a*x - 1)/(a*x + 1))^(5/4) + a^5*((a*x - 1)/(a*
x + 1))^(1/4)) - 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^5)
```

Fricas [A]
time = 0.35, size = 144, normalized size = 0.58

$$-\frac{2850\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+1425\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(48\,a^5x^5+184\,a^4x^4+362\,a^3x^3+747\,a^2x^2-1946\,ax-2467)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{384\,(a^5x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="fricas")
```

```
[Out] -1/384*(2850*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 1425*(a*x - 1)
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1425*(a*x - 1)*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1) - 2*(48*a^5*x^5 + 184*a^4*x^4 + 362*a^3*x^3 + 747*a^2*x^2
- 1946*a*x - 2467)*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*x - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A]
time = 0.46, size = 223, normalized size = 0.89

$$-\frac{1}{384}\,a\left(\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{1425\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{3072}{a^5\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{2875\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{2343\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}+\frac{657\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^3}-1573\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^5\left(\frac{ax-1}{ax+1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^3,x, algorithm="giac")
```

```
[Out] -1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^5 + 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^5 + 3072/(a^5*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(2875*(a*x - 1)*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1) - 2343*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4
)/(a*x + 1)^2 + 657*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1
573*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

Mupad [B]
time = 0.14, size = 211, normalized size = 0.84

$$\frac{475\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{7483\,(a\,x-1)^2}{96\,(a\,x+1)^2}-\frac{1805\,(a\,x-1)^3}{32\,(a\,x+1)^3}+\frac{475\,(a\,x-1)^4}{32\,(a\,x+1)^4}-\frac{4645\,(a\,x-1)}{96\,(a\,x+1)}+8}{a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}-4\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}+6\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}-4\,a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}+a^4\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (475*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (475*atan(((a*x - 1)/(a
*x + 1))^(1/4)))/(64*a^4) - ((7483*(a*x - 1)^2)/(96*(a*x + 1)^2) - (1805*(a
```

x - 1)^3)/(32(a*x + 1)^3) + (475*(a*x - 1)^4)/(32*(a*x + 1)^4) - (4645*(a*x - 1))/(96*(a*x + 1)) + 8)/(a^4*((a*x - 1)/(a*x + 1))^(1/4) - 4*a^4*((a*x - 1)/(a*x + 1))^(5/4) + 6*a^4*((a*x - 1)/(a*x + 1))^(9/4) - 4*a^4*((a*x - 1)/(a*x + 1))^(13/4) + a^4*((a*x - 1)/(a*x + 1))^(17/4))

3.79 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=213

$$-\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}}+\frac{55\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{55\tanh^{-1}\left(\right.}{8}$$

```
[Out] -287/24*(1+1/a/x)^(1/4)/a^3/(1-1/a/x)^(1/4)+61/24*(1+1/a/x)^(1/4)*x/a^2/(1-
1/a/x)^(1/4)+13/12*(1+1/a/x)^(1/4)*x^2/a/(1-1/a/x)^(1/4)+1/3*(1+1/a/x)^(1/4
)*x^3/(1-1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3+55/8
*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A]
time = 0.08, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6306, 100, 156, 160, 12, 95, 218, 212, 209}

$$\frac{55\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{287\sqrt[4]{\frac{1}{ax}+1}}{24a^3\sqrt[4]{1-\frac{1}{ax}}}+\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{61x\sqrt[4]{\frac{1}{ax}+1}}{24a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{x^3\sqrt[4]{\frac{1}{ax}+1}}{3\sqrt[4]{1-\frac{1}{ax}}}+\frac{13x^2\sqrt[4]{\frac{1}{ax}+1}}{12a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (-287*(1 + 1/(a*x))^(1/4))/(24*a^3*(1 - 1/(a*x))^(1/4)) + (61*(1 + 1/(a*x))
^(1/4)*x)/(24*a^2*(1 - 1/(a*x))^(1/4)) + (13*(1 + 1/(a*x))^(1/4)*x^2)/(12*a
*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(1/4)*x^3)/(3*(1 - 1/(a*x))^(1/4)) +
 (55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) + (55*ArcTanh
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^4\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{61}{4a^2}+\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{5/4}\left(1+\frac{x}{a}\right.}\right.$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a\text{Subst}\left(\right.$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\text{Subst}\left(\int\right.}{}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} - \frac{55\text{Subst}\left(\int\right.}{}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\text{Subst}\left(\int\right.}{}$$

$$= -\frac{287\sqrt[4]{1+\frac{1}{ax}}}{24a^3\sqrt[4]{1-\frac{1}{ax}}} + \frac{61\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{13\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\sqrt[4]{1+\frac{1}{ax}}\,x^3}{3\sqrt[4]{1-\frac{1}{ax}}} + \frac{55\tan^{-1}\left(\right.}{8}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.63, size = 441, normalized size = 2.07

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^2,x]
```

```
[Out] (-8*E^((9*ArcCoth[a*x])/2)*(-27653/195 - 899079/(512*E^(8*ArcCoth[a*x])) -
3309759/(2560*E^(6*ArcCoth[a*x])) + 8521937/(7680*E^(4*ArcCoth[a*x])) + 695
71361/(99840*E^(2*ArcCoth[a*x])) - (653*E^(2*ArcCoth[a*x]))/390 + (133407*H
ypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/512 + (899079*Hypergeome
tric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(512*E^(8*ArcCoth[a*x])) + (60267
*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(64*E^(6*ArcCoth[a*x])
) - (382227*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/(256*E^(4*A
rcCoth[a*x])) - (40827*Hypergeometric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])])/
(64*E^(2*ArcCoth[a*x])) + (E^(2*ArcCoth[a*x])*(1117 + 1906*E^(2*ArcCoth[a*x
]) + 821*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 25/4
}, E^(2*ArcCoth[a*x])])/3094 + (4*E^(2*ArcCoth[a*x])*(27 + 50*E^(2*ArcCoth[
a*x]) + 23*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{2, 2, 2, 2, 13/4}, {1, 1,
 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(2*ArcCoth[a*x])*Hypergeometric
PFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547 +
(16*E^(4*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1
, 25/4}, E^(2*ArcCoth[a*x])])/1547 + (8*E^(6*ArcCoth[a*x])*HypergeometricPF
Q[{2, 2, 2, 2, 2, 13/4}, {1, 1, 1, 1, 25/4}, E^(2*ArcCoth[a*x])])/1547))/(9
*a^3)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x)
```

Maxima [A]
time = 0.46, size = 203, normalized size = 0.95

$$-\frac{1}{48}\,a\left(\frac{4\left(\frac{425\,(ax-1)}{ax+1}-\frac{462\,(ax-1)^2}{(ax+1)^2}+\frac{165\,(ax-1)^3}{(ax+1)^3}-96\right)}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+3\,a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(425*(a*x - 1)/(a*x + 1) - 462*(a*x - 1)^2/(a*x + 1)^2 + 165*(a*
x - 1)^3/(a*x + 1)^3 - 96)/(a^4*((a*x - 1)/(a*x + 1))^(13/4) - 3*a^4*((a*x
- 1)/(a*x + 1))^(9/4) + 3*a^4*((a*x - 1)/(a*x + 1))^(5/4) - a^4*((a*x - 1)/
(a*x + 1))^(1/4)) + 330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) -
 1)/a^4)
```

Fricas [A]
time = 0.36, size = 136, normalized size = 0.64

$$-\frac{330\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\left(8\,a^4x^4+34\,a^3x^3+87\,a^2x^2-226\,ax-287\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{48\,(a^4x-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="fricas")
```

```
[Out] -1/48*(330*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*(a*x - 1)*lo
g(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 165*(a*x - 1)*log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1) - 2*(8*a^4*x^4 + 34*a^3*x^3 + 87*a^2*x^2 - 226*a*x - 287)*((a*
x - 1)/(a*x + 1))^(3/4))/(a^4*x - a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A]
time = 0.45, size = 192, normalized size = 0.90

$$-\frac{1}{48}\,a\left(\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{165\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{384}{a^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{4\left(\frac{174\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{69\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}-137\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^2,x, algorithm="giac")
```

```
[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 165*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1)/a^4 + 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^
4 + 384/(a^4*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(174*(a*x - 1)*((a*x - 1)/(a*
```

```
x + 1))^(3/4)/(a*x + 1) - 69*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x +
 1)^2 - 137*((a*x - 1)/(a*x + 1))^(3/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B]
time = 0.09, size = 176, normalized size = 0.83

$$\frac{55\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{\frac{77\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}-\frac{55\,{\left(a\,x-1\right)}^3}{4\,{\left(a\,x+1\right)}^3}-\frac{425\,\left(a\,x-1\right)}{12\,\left(a\,x+1\right)}+8}{a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}-a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (55*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (55*atan(((a*x - 1)/(a*x
+ 1))^(1/4)))/(8*a^3) - ((77*(a*x - 1)^2)/(2*(a*x + 1)^2) - (55*(a*x - 1)^3
)/(4*(a*x + 1)^3) - (425*(a*x - 1))/(12*(a*x + 1)) + 8)/(a^3*((a*x - 1)/(a*
x + 1))^(1/4) - 3*a^3*((a*x - 1)/(a*x + 1))^(5/4) + 3*a^3*((a*x - 1)/(a*x +
 1))^(9/4) - a^3*((a*x - 1)/(a*x + 1))^(13/4))
```

3.80 $\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=176

$$-\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -25/2*(1+1/a/x)^(1/4)/a^2/(1-1/a/x)^(1/4)+5/4*(1+1/a/x)^(5/4)*x/a/(1-1/a/x)
^(1/4)+1/2*(1+1/a/x)^(9/4)*x^2/(1-1/a/x)^(1/4)+25/4*arctan((1+1/a/x)^(1/4)/
(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6306, 98, 96, 95, 218, 212, 209}

$$\frac{25\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}-\frac{25\sqrt[4]{\frac{1}{ax}+1}}{2a^2\sqrt[4]{1-\frac{1}{ax}}}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(\frac{1}{ax}+1\right)^{9/4}}{2\sqrt[4]{1-\frac{1}{ax}}}+\frac{5x\left(\frac{1}{ax}+1\right)^{5/4}}{4a\sqrt[4]{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x,x]
```

```
[Out] (-25*(1 + 1/(a*x))^(1/4))/(2*a^2*(1 - 1/(a*x))^(1/4)) + (5*(1 + 1/(a*x))^(5
/4)*x)/(4*a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(9/4)*x^2)/(2*(1 - 1/(a*x
))^(1/4)) + (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^3\left(1-\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{4a}$$

$$= \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{25\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{25\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,d\ldots\right.}{8a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} - \frac{25\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+\ldots}}{\sqrt[4]{1-\ldots}}\right.}{2a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} + \frac{25\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{a\ldots}}}{\sqrt[4]{1-\frac{1}{a\ldots}}}\right.}{4a^2}$$

$$= -\frac{25\sqrt[4]{1+\frac{1}{ax}}}{2a^2\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\left(1+\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1-\frac{1}{ax}}} + \frac{25\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2} + \frac{25\,\text{ta}\ldots}{\ldots}$$

Mathematica [A]

time = 0.15, size = 80, normalized size = 0.45

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(25-45e^{2\coth^{-1}(ax)}+16e^{4\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+25\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)+25\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x,x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(25 - 45*E^(2*ArcCoth[a*x]) + 16*E^(4*ArcCoth[a*x])
))/(-1 + E^(2*ArcCoth[a*x]))^2 + 25*ArcTan[E^(ArcCoth[a*x]/2)] + 25*ArcTanh
[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x,x)
```

Maxima [A]
time = 0.46, size = 166, normalized size = 0.94

$$\frac{1}{8}\,a\left(\frac{4\left(\frac{45\,(ax-1)}{ax+1}-\frac{25\,(ax-1)^2}{(ax+1)^2}-16\right)}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-2\,a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}-\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*(45*(a*x - 1)/(a*x + 1) - 25*(a*x - 1)^2/(a*x + 1)^2 - 16)/(a^3*((
a*x - 1)/(a*x + 1))^(9/4) - 2*a^3*((a*x - 1)/(a*x + 1))^(5/4) + a^3*((a*x -
 1)/(a*x + 1))^(1/4)) - 50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4)
- 1)/a^3)
```

Fricas [A]
time = 0.37, size = 128, normalized size = 0.73

$$-\frac{50\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+25\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(2\,a^3x^3+11\,a^2x^2-34\,ax-43)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{8\,(a^3x-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="fricas")
```

```
[Out] -1/8*(50*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 25*(a*x - 1)*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) + 25*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1
/4) - 1) - 2*(2*a^3*x^3 + 11*a^2*x^2 - 34*a*x - 43)*((a*x - 1)/(a*x + 1))^(
3/4))/(a^3*x - a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A]
time = 0.47, size = 161, normalized size = 0.91

$$-\frac{1}{8}a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{64}{a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{4\left(\frac{9\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-13\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x,x, algorithm="giac")
```

```
[Out] -1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^3 + 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 +
64/(a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(9*(a*x - 1)*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) - 13*((a*x - 1)/(a*x + 1))^(3/4))/(a^3*((a*x - 1)/(a*x + 1
) - 1)^2))
```

Mupad [B]
time = 1.21, size = 139, normalized size = 0.79

$$\frac{25\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{25\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}-\frac{45\,\left(a\,x-1\right)}{2\,\left(a\,x+1\right)}+8}{a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-2\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (25*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (25*atan(((a*x - 1)/(a*x
+ 1))^(1/4)))/(4*a^2) - ((25*(a*x - 1)^2)/(2*(a*x + 1)^2) - (45*(a*x - 1))/
(2*(a*x + 1)) + 8)/(a^2*((a*x - 1)/(a*x + 1))^(1/4) - 2*a^2*((a*x - 1)/(a*x
 + 1))^(5/4) + a^2*((a*x - 1)/(a*x + 1))^(9/4))
```

3.81 $\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=130

$$-\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] -10*(1+1/a/x)^(1/4)/a/(1-1/a/x)^(1/4)+(1+1/a/x)^(5/4)*x/(1-1/a/x)^(1/4)+5*a
rctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a+5*arctanh((1+1/a/x)^(1/4)/(1-1/a/x
)^(1/4))/a
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6305, 96, 95, 218, 212, 209}

$$\frac{5\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{x\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{10\sqrt[4]{\frac{1}{ax}+1}}{a\sqrt[4]{1-\frac{1}{ax}}}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-10*(1 + 1/(a*x))^(1/4))/(a*(1 - 1/(a*x))^(1/4)) + ((1 + 1/(a*x))^(5/4)*x)
/(1 - 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/
a + (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1] || !SumSimplerQ[p, 1]) && NeQ[m, -1]

Rule 209

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 218

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 6305

Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] && !IntegerQ[n]

Rubi steps

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x^2\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{10\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{}$$

$$= -\frac{10\sqrt[4]{1+\frac{1}{ax}}}{a\sqrt[4]{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.52

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-5+4e^{2\coth^{-1}(ax)}\right)}{-1+e^{2\coth^{-1}(ax)}} + 5\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 5\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-5 + 4*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
])) + 5*ArcTan[E^(ArcCoth[a*x]/2)] + 5*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A]
time = 0.47, size = 131, normalized size = 1.01

$$-\frac{1}{2}\, a \left(\frac{4\left(\frac{5\,(ax-1)}{ax+1}-4\right)}{a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{10\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{5\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)/(a*x + 1))^(5/4) - a^
2*((a*x - 1)/(a*x + 1))^(1/4)) + 10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2
 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^2 + 5*log(((a*x - 1)/(a*x + 1))
^(1/4) - 1)/a^2)
```

Fricas [A]
time = 0.37, size = 117, normalized size = 0.90

$$-\frac{10\,(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+5\,(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\,(a^2x^2-8\,ax-9)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{2\,(a^2x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(10*(a*x - 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 5*(a*x - 1)*log(((
a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4
) - 1) - 2*(a^2*x^2 - 8*a*x - 9)*((a*x - 1)/(a*x + 1))^(3/4))/(a^2*x - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-5/4), x)
```

Giac [A]
time = 0.45, size = 141, normalized size = 1.08

$$-\frac{1}{2}\, a \left(\frac{10 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2} - \frac{5 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^2} + \frac{5 \log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right|\right)}{a^2} + \frac{4\left(\frac{5\,(ax-1)}{ax+1} - 4\right)}{a^2\left(\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1} - \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 + 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*
(5*(a*x - 1)/(a*x + 1) - 4)/(a^2*((a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*
x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))))
```

Mupad [B]
time = 0.06, size = 98, normalized size = 0.75

$$\frac{\frac{10\,(a\,x-1)}{a\,x+1} - 8}{a\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} - a\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}} - \frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a} + \frac{5\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] ((10*(a*x - 1))/(a*x + 1) - 8)/(a*((a*x - 1)/(a*x + 1))^(1/4) - a*((a*x - 1
)/(a*x + 1))^(5/4)) - (5*atan(((a*x - 1)/(a*x + 1))^(1/4)))/a + (5*atanh(((
a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.82 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=320

$$-\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}+\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\text{ArcTan}\left(\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.$$

```
[Out] -8*(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-
arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {6306, 100, 21, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)+2\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{8\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+2\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x,x]
```

```
[Out] (-8*(1 + 1/(a*x))^(1/4))/(1 - 1/(a*x))^(1/4) + Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{x\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + (4a)\text{Subst}\left(\int \frac{-\frac{1}{4a}+\frac{x}{4a^2}}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)\right.$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} - 4\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,\right.$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1-x}{1+x}\right.$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}}{-1-\sqrt{2}}\right.}{}$$

$$= -\frac{8\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}} + 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-}}{\sqrt{1+}}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 30, normalized size = 0.09

$$8e^{\frac{1}{2}\coth^{-1}(ax)}\left(-1+{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{4\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x,x]
```

```
[Out] 8*E^(ArcCoth[a*x]/2)*(-1 + Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x]
)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Maxima [A]
time = 0.47, size = 244, normalized size = 0.76

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a}+\frac{16}{a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

Fricas [A]
time = 0.36, size = 358, normalized size = 1.12

$$\frac{4\sqrt{2}(ax-1)\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}(ax-1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}(ax-1)\log\left(4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-\sqrt{2}(ax-1)\log\left(-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-4(ax-1)\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+2(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-2(ax-1)\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-16(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{2(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*(a*x - 1)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) - 1) + 4*sqrt(2)*(a*x - 1)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/
(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a
*x + 1))^(1/4) + 1) + sqrt(2)*(a*x - 1)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))
^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*(a*x - 1)*log(-4*sqrt(2
)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x + 1)) + 4) - 4*(a*x -
 1)*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 2*(a*x - 1)*log(((a*x - 1)/(a*x +
 1))^(1/4) + 1) - 2*(a*x - 1)*log(((a*x - 1)/(a*x + 1))^(1/4) - 1) - 16*(a*
x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Giac [A]
time = 0.45, size = 252, normalized size = 0.79

$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}-\frac{2\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{2\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a}+\frac{16}{a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a + 16/(a*((a*x - 1)/(a*x + 1))^(1/4)))
```

Mupad [B]
time = 1.18, size = 118, normalized size = 0.37

$$-2\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\frac{8}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}-\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1+1{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1-1{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(1
/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 -
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1
i) - 8/((a*x - 1)/(a*x + 1))^(1/4)
```

3.83 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=299

$$-5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{5a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{5a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

```
[Out] -5*a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-4*a*(1+1/a/x)^(5/4)/(1-1/a/x)^(1/4)-5/
2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arctan
(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/4*a*ln(1-(1-1/a/x)^(1
/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+5/4*a*
ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2
))*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 49, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{5a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}-\frac{5a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{4a\left(\frac{1}{ax}+1\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}}-5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] -5*a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4) - (4*a*(1 + 1/(a*x))^(5/4))/(1
 - 1/(a*x))^(1/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a
*x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1
/(a*x))^(1/4)])/Sqrt[2] - (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)]
- (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) + (5*a*Lo
g[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(
1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + 5\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x,$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - (10a)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\right.$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - (10a)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + (5a)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{2}(5a)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x,\right.$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.}{2\sqrt{2}}$$

$$= -5a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{4a\left(1+\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{5a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\,\text{t}}{}$$

Mathematica [A]
time = 0.47, size = 173, normalized size = 0.58

$$a\left(-\frac{10e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-\frac{8e^{\frac{5}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-\frac{5\text{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}+\frac{5\text{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^2,x]
```

```
[Out] a*((-10*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - (8*E^((5*ArcCoth[a*x
])/2))/(1 + E^(2*ArcCoth[a*x])) - (5*ArcTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)]
)/Sqrt[2] + (5*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (5*Log[1 -
 Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]) + (5*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Maxima [A]
time = 0.47, size = 204, normalized size = 0.68

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+5\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{5(ax-1)}{ax+1}+4\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/(((a*x - 1)/(a*x
+ 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Fricas [A]
time = 0.39, size = 451, normalized size = 1.51

$$\frac{20\sqrt{2}\,(a^4)^{\frac{1}{4}}(ax^2-x)\arctan\left(-\frac{a^4+\sqrt{2}\,(a^4)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}\,a^4+\sqrt{2}\,(a^4)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,(a^4)^{\frac{1}{4}}}{a^4}\right)+20\sqrt{2}\,(a^4)^{\frac{1}{4}}(ax^2-x)\arctan\left(\frac{a^4-\sqrt{2}\,(a^4)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^4}\,a^4-\sqrt{2}\,(a^4)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,(a^4)^{\frac{1}{4}}}{a^4}\right)+5\sqrt{2}\,(a^4)^{\frac{1}{4}}(ax^2-x)\log\left(15625\,a^6\sqrt{\frac{ax-1}{ax+1}}+15625\sqrt{a^4}\,a^4+15625\sqrt{2}\,(a^4)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5\sqrt{2}\,(a^4)^{\frac{1}{4}}(ax^2-x)\log\left(15625\,a^6\sqrt{\frac{ax-1}{ax+1}}+15625\sqrt{a^4}\,a^4-15625\sqrt{2}\,(a^4)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-4\,(9\,a^2x^2+8\,ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{4\,(ax^2-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*
a^3*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(a^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a
^4)^(1/4))/a^4) + 20*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*arctan((a^4 - sqrt(2)*
(a^4)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x -
1)/(a*x + 1)) + sqrt(a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1
))^(1/4))*(a^4)^(1/4))/a^4) + 5*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*log(15625*a
^6*sqrt((a*x - 1)/(a*x + 1)) + 15625*sqrt(a^4)*a^4 + 15625*sqrt(2)*(a^4)^(3
/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 5*sqrt(2)*(a^4)^(1/4)*(a*x^2 - x)*lo
g(15625*a^6*sqrt((a*x - 1)/(a*x + 1)) + 15625*sqrt(a^4)*a^4 - 15625*sqrt(2)
*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(9*a^2*x^2 + 8*a*x - 1)*(
(a*x - 1)/(a*x + 1))^(3/4))/(a*x^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Giac [A]
time = 0.42, size = 217, normalized size = 0.73

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+5\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{8\left(\frac{5(ax-1)}{ax+1}+4\right)}{\frac{(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) - 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) + 8*(5*(a*x - 1)/(a*x + 1) + 4)/((a*x - 1)*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Mupad [B]
time = 1.22, size = 107, normalized size = 0.36

$$5\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-5\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\frac{8\,a+\frac{10\,a\,\left(a\,x-1\right)}{a\,x+1}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] 5*(-1)^(1/4)*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 5*(-1)^(1/4)
*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (8*a + (10*a*(a*x - 1))/(
a*x + 1))/(((a*x - 1)/(a*x + 1))^(1/4) + ((a*x - 1)/(a*x + 1))^(5/4))
```

3.84 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=351

$$-\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{25a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1-}}\right)}{4\sqrt{2}}$$

```
[Out] -25/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-5/2*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)
^(5/4)-2*a^2*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-25/8*a^2*arctan(-1+(1-1/a/x)^(
1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(
1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a
/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1/a/x)
^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 79, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{25a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}}-\frac{25a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}}-\frac{2a^2\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (-25*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 - (5*a^2*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(5/4))/2 - (2*a^2*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(1
/4) + (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]
)/(4*Sqrt[2]) - (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x
)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*
a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1
/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + (5a)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \frac{1}{4}(25a)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x\right.$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \ldots$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \ldots$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \ldots$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \ldots$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \ldots$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \ldots$$

$$= -\frac{25}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^2\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \ldots$$

Mathematica [A]
time = 0.34, size = 186, normalized size = 0.53

$$\frac{1}{16}a^2\left(-128e^{\frac{1}{2}\coth^{-1}(ax)}+\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{104e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-50\sqrt{2}\,\mathrm{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)+50\sqrt{2}\,\mathrm{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)-25\sqrt{2}\,\log\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)+25\sqrt{2}\,\log\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^3,x]
```

```
[Out] (a^2*(-128*E^(ArcCoth[a*x]/2) + (32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a
*x]))^2 - (104*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 50*Sqrt[2]*Ar
cTan[1 - Sqrt[2]*E^(ArcCoth[a*x]/2)] + 50*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(Arc
Coth[a*x]/2)] - 25*Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a
*x]] + 25*Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Maxima [A]
time = 0.46, size = 244, normalized size = 0.70

$$-\frac{1}{16}\left(25\left(2\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\,\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\,\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\right)a+\frac{8\left(\frac{45(ax-1)a}{ax+1}+\frac{25(ax-1)^2a}{(ax+1)^2}+16a\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(25*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1))*a + 8*(45*(a*x - 1)*a/(a*x + 1) + 25*(a*x - 1)^2*
a/(a*x + 1)^2 + 16*a)/(((a*x - 1)/(a*x + 1))^(9/4) + 2*((a*x - 1)/(a*x + 1)
)^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Fricas [A]
time = 0.37, size = 469, normalized size = 1.34

$$\frac{100\sqrt{2}\,(a^8)^{\frac{1}{4}}(ax^3-x^2)\arctan\left(-\frac{a^8+\sqrt{2}\,(a^8)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\,\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}\,a^8+\sqrt{2}\,(a^8)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,(a^8)^{\frac{1}{4}}}{a^8}\right)+100\sqrt{2}\,(a^8)^{\frac{1}{4}}(ax^3-x^2)\arctan\left(\frac{a^8-\sqrt{2}\,(a^8)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\,\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}\,a^8-\sqrt{2}\,(a^8)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,(a^8)^{\frac{1}{4}}}{a^8}\right)+25\sqrt{2}\,(a^8)^{\frac{1}{4}}(ax^3-x^2)\log\left(244140625\,a^{12}\sqrt{\frac{ax-1}{ax+1}}+244140625\sqrt{a^8}\,a^8+244140625\sqrt{2}\,(a^8)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-25\sqrt{2}\,(a^8)^{\frac{1}{4}}(ax^3-x^2)\log\left(244140625\,a^{12}\sqrt{\frac{ax-1}{ax+1}}+244140625\sqrt{a^8}\,a^8-244140625\sqrt{2}\,(a^8)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-4\left(43a^3x^3+34a^2x^2-11ax-2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{16\,(ax^3-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(100*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)*arctan(-(a^8 + sqrt(2)*(a^8)^(1
/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x
 + 1)) + sqrt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4
))*(a^8)^(1/4))/a^8) + 100*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)*arctan((a^8 -
sqrt(2)*(a^8)^(1/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqr
t((a*x - 1)/(a*x + 1)) + sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)
/(a*x + 1))^(1/4))*(a^8)^(1/4))/a^8) + 25*sqrt(2)*(a^8)^(1/4)*(a*x^3 - x^2)
*log(244140625*a^12*sqrt((a*x - 1)/(a*x + 1)) + 244140625*sqrt(a^8)*a^8 + 2
44140625*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4)) - 25*sqrt(2)*
(a^8)^(1/4)*(a*x^3 - x^2)*log(244140625*a^12*sqrt((a*x - 1)/(a*x + 1)) + 24
4140625*sqrt(a^8)*a^8 - 244140625*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x +
 1))^(1/4)) - 4*(43*a^3*x^3 + 34*a^2*x^2 - 11*a*x - 2)*((a*x - 1)/(a*x + 1)
)^(3/4))/(a*x^3 - x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Giac [A]
time = 0.46, size = 243, normalized size = 0.69

$$-\frac{1}{16}\left(50\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-25\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+25\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{128\,a}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{8\left(\frac{9\,(ax-1)a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}+13\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) + 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 128*a/((a*x - 1)/(a*x + 1))^(1/4) +
8*(9*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) + 13*a*((a*x - 1)/(a
*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B]
time = 0.08, size = 152, normalized size = 0.43

$$\frac{25\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}-\frac{25\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}-\frac{8\,a^2+\frac{25\,a^2\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}+\frac{45\,a^2\,\left(a\,x-1\right)}{2\,\left(a\,x+1\right)}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] (25*(-1)^(1/4)*a^2*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (25*(
-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 - (8*a^2 + (2
5*a^2*(a*x - 1)^2)/(2*(a*x + 1)^2) + (45*a^2*(a*x - 1))/(2*(a*x + 1)))/(((a
*x - 1)/(a*x + 1))^(1/4) + 2*((a*x - 1)/(a*x + 1))^(5/4) + ((a*x - 1)/(a*x
+ 1))^(9/4))
```

3.85 $\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=385

$$-\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4}-\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\right.$$

```
[Out] -55/8*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-11/4*a^3*(1-1/a/x)^(3/4)*(1+1/a/x
)^(5/4)-2*a^3*(1+1/a/x)^(9/4)/(1-1/a/x)^(1/4)-1/3*a^3*(1-1/a/x)^(3/4)*(1+1/
a/x)^(9/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(
1/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-55
/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a
/x)^(1/2))*2^(1/2)+55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 91, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{55a^3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}-\frac{55a^3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{9/4}-\frac{2a^3\left(\frac{1}{ax}+1\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(\frac{1}{ax}+1\right)^{5/4}-\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{55a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{55a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] (-55*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/8 - (11*a^3*(1 - 1/(a*x))
^(3/4)*(1 + 1/(a*x))^(5/4))/4 - (2*a^3*(1 + 1/(a*x))^(9/4))/(1 - 1/(a*x))^(
1/4) - (a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(9/4))/3 + (55*a^3*ArcTan[1 -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*
ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
- (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*
x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (55*a^3*Log[1 + Sqrt[1 - 1/
(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4
)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} + \left(2a^3\right)\text{Subst}\left(\int \frac{\left(\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1-\frac{x}{a}}}\,dx,\right.$$

$$= -\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{9/4} +$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}$$

$$= -\frac{55}{8}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{3/4}\left(1+\frac{1}{ax}\right)^{5/4} - \frac{2a^3\left(1+\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1-\frac{1}{ax}}} - \frac{1}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.21, size = 104, normalized size = 0.27

$$a^3\left(-\frac{e^{\frac{1}{2}\coth^{-1}(ax)}\left(165+462e^{2\coth^{-1}(ax)}+425e^{4\coth^{-1}(ax)}+96e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)-2\log\left(e^{\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)/x^4,x]
```

```
[Out] a^3*(-1/12*(E^(ArcCoth[a*x]/2)*(165 + 462*E^(2*ArcCoth[a*x]) + 425*E^(4*Arc
Coth[a*x]) + 96*E^(6*ArcCoth[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 - (55*RootS
um[1 + #1^4 & , (ArcCoth[a*x] - 2*Log[E^(ArcCoth[a*x]/2) - #1])/#1^3 & ])/3
2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Maxima [A]
time = 0.46, size = 288, normalized size = 0.75

$$-\frac{1}{96}\left(165\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\right)a^2+\frac{8\left(\frac{425(ax-1)a^2}{ax+1}+\frac{462(ax-1)^2a^2}{(ax+1)^2}+\frac{165(ax-1)^3a^2}{(ax+1)^3}+96a^2\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(165*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1))*a^2 + 8*(425*(a*x - 1)*a^2/(a*x + 1) + 462*(a*x
- 1)^2*a^2/(a*x + 1)^2 + 165*(a*x - 1)^3*a^2/(a*x + 1)^3 + 96*a^2)/(((a*x -
 1)/(a*x + 1))^(13/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + 3*((a*x - 1)/(a*x +
 1))^(5/4) + ((a*x - 1)/(a*x + 1))^(1/4)))*a
```

Fricas [A]
time = 0.38, size = 477, normalized size = 1.24

$$\frac{660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}(ax^4-x^3)\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}(ax^4-x^3)\arctan\left(\frac{a^{12}-\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+165\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}(ax^4-x^3)\log\left(27680640625a^{18}\sqrt{\frac{ax-1}{ax+1}}+27680640625\sqrt{a^{12}}a^{12}+27680640625\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}(ax^4-x^3)\log\left(27680640625a^{18}\sqrt{\frac{ax-1}{ax+1}}+27680640625\sqrt{a^{12}}a^{12}-27680640625\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-4\left(287a^4x^4+226a^3x^3-87a^2x^2-34ax-8\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{96\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*arctan(-(a^12 + sqrt(2)*(a^12)
^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(
a*x + 1)) + sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1)
)^(1/4))*(a^12)^(1/4))/a^12) + 660*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*arcta
n((a^12 - sqrt(2)*(a^12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sq
rt(a^18*sqrt((a*x - 1)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*
a^9*((a*x - 1)/(a*x + 1))^(1/4))*(a^12)^(1/4))/a^12) + 165*sqrt(2)*(a^12)^(
1/4)*(a*x^4 - x^3)*log(27680640625*a^18*sqrt((a*x - 1)/(a*x + 1)) + 2768064
0625*sqrt(a^12)*a^12 + 27680640625*sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x
 + 1))^(1/4)) - 165*sqrt(2)*(a^12)^(1/4)*(a*x^4 - x^3)*log(27680640625*a^18
*sqrt((a*x - 1)/(a*x + 1)) + 27680640625*sqrt(a^12)*a^12 - 27680640625*sqrt
(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) - 4*(287*a^4*x^4 + 226*a^
3*x^3 - 87*a^2*x^2 - 34*a*x - 8)*((a*x - 1)/(a*x + 1))^(3/4))/(a*x^4 - x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

```
[Out] Integral(1/(x**4*((a*x - 1)/(a*x + 1))**(5/4)), x)
```

Giac [A]
time = 0.45, size = 291, normalized size = 0.76

$$-\frac{1}{96}\left(330\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-165\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+165\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\frac{768\,a^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}+\frac{8\left(\frac{174\,(ax-1)a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}+\frac{69\,(ax-1)^2a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}+137\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) - 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) + 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) + 768*a^2/((a*x - 1)/(a*x
+ 1))^(1/4) + 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) +
69*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 137*a^2*((a*x
- 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

Mupad [B]
time = 1.26, size = 188, normalized size = 0.49

$$\frac{55\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8}-\frac{55\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8}-\frac{8\,a^3+\frac{77\,a^3\,{\left(a\,x-1\right)}^2}{2\,{\left(a\,x+1\right)}^2}+\frac{55\,a^3\,{\left(a\,x-1\right)}^3}{4\,{\left(a\,x+1\right)}^3}+\frac{425\,a^3\,\left(a\,x-1\right)}{12\,\left(a\,x+1\right)}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}+{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a*x - 1)/(a*x + 1))^(5/4)),x)
```

```
[Out] (55*(-1)^(1/4)*a^3*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (55*(
-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - (8*a^3 + (7
7*a^3*(a*x - 1)^2)/(2*(a*x + 1)^2) + (55*a^3*(a*x - 1)^3)/(4*(a*x + 1)^3) +
 (425*a^3*(a*x - 1))/(12*(a*x + 1)))/(((a*x - 1)/(a*x + 1))^(1/4) + 3*((a*x
 - 1)/(a*x + 1))^(5/4) + 3*((a*x - 1)/(a*x + 1))^(9/4) + ((a*x - 1)/(a*x +
1))^(13/4))
```

3.86 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4}-\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3}+\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2}-\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.}{40a}$$

```
[Out] 611/1920*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^4-269/960*(1-1/a/x)^(1/4)*(1+1
/a/x)^(3/4)*x^2/a^3+11/48*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a^2-9/40*(1-1
/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^4/a+1/5*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^5+3
1/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-31/128*arctanh((1+1/a/x)^
(1/4)/(1-1/a/x)^(1/4))/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$\frac{31\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{31\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{611x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{1920a^4}-\frac{269x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{960a^3}+\frac{11x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{48a^2}+\frac{1}{5}x^5\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{9x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{40a}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (611*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(1920*a^4) - (269*(1 - 1/(a
*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(960*a^3) + (11*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4)*x^3)/(48*a^2) - (9*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)
*x^4)/(40*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^5)/5 + (31*ArcTan
[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (31*ArcTanh[(1 + 1/(
a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^6\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{9}{2a}+\frac{4x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-}{x^4\,(1-}\right.$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4}{40a} + \frac{1}{5}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/}$$

$$= -\frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{48a^2} - \frac{9\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right.}{40a}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)}{48a^2}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)}{48a^2}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)}{48a^2}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)}{48a^2}$$

$$= \frac{611\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{1920a^4} - \frac{269\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{960a^3} + \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)}{48a^2}$$

Mathematica [A]
time = 5.22, size = 173, normalized size = 0.68

$$\frac{\frac{24576e^{\frac{19}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}-\frac{62976e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{64640e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{34000e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{9620e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-930\text{ArcTan}\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+465\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-465\log\left(1+e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{3840a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((24576*E^((19*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (62976*E^((1
5*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (64640*E^((11*ArcCoth[a*x
])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (34000*E^((7*ArcCoth[a*x])/2))/(-1 + E
^(2*ArcCoth[a*x]))^2 + (9620*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x
])) - 930*ArcTan[E^(-1/2*ArcCoth[a*x])] + 465*Log[1 - E^(-1/2*ArcCoth[a*x])
] - 465*Log[1 + E^(-1/2*ArcCoth[a*x])])/(3840*a^5)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A]
time = 0.47, size = 259, normalized size = 1.02

$$-\frac{1}{3840}a\left(\frac{4\left(2405\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-1120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+5090\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-696\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+465\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5(ax-1)a^6}{ax+1}-\frac{10(ax-1)^2a^6}{(ax+1)^2}+\frac{10(ax-1)^3a^6}{(ax+1)^3}-\frac{5(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(2405*((a*x - 1)/(a*x + 1))^(17/4) - 1120*((a*x - 1)/(a*x + 1)
)^(13/4) + 5090*((a*x - 1)/(a*x + 1))^(9/4) - 696*((a*x - 1)/(a*x + 1))^(5/
4) + 465*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*x
- 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a^6
/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 930*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 465*l
og(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A]
time = 0.35, size = 119, normalized size = 0.47

$$\frac{2\left(384a^5x^5-48a^4x^4+8a^3x^3-98a^2x^2+73ax+611\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-930\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+465\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{3840a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 - 48*a^4*x^4 + 8*a^3*x^3 - 98*a^2*x^2 + 73*a*x + 611
)*((a*x - 1)/(a*x + 1))^(1/4) - 930*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 4
65*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 465*log(((a*x - 1)/(a*x + 1))^(1/
4) - 1))/a^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**4*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.44, size = 234, normalized size = 0.92

$$-\frac{1}{3840}\,a\left(\frac{930\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{465\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{465\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{4\left(\frac{696\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{5090\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+\frac{1120\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^3}-\frac{2405\,(ax-1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^4}-465\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6\left(\frac{ax-1}{ax+1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(930*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 465*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^6 - 465*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/
a^6 - 4*(696*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 5090*(a*x -
1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 1120*(a*x - 1)^3*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1)^3 - 2405*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1)^4 - 465*((a*x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) -
 1)^5))
```

Mupad [B]
time = 0.08, size = 229, normalized size = 0.91

$$\frac{\frac{31\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{64}-\frac{29\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{40}+\frac{509\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}{96}-\frac{7\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}{6}+\frac{481\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/4}}{192}}{a^5+\frac{10\,a^5\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a^5\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a^5\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a^5\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{5\,a^5\,\left(a\,x-1\right)}{a\,x+1}}-\frac{31\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{128\,a^5}-\frac{31\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((31*((a*x - 1)/(a*x + 1))^(1/4))/64 - (29*((a*x - 1)/(a*x + 1))^(5/4))/40
+ (509*((a*x - 1)/(a*x + 1))^(9/4))/96 - (7*((a*x - 1)/(a*x + 1))^(13/4))/6
 + (481*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*x
 + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)^
4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) - (31*atan
(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (31*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(128*a^5)
```

3.87 $\int e^{-\frac{1}{2}\coth^{-1}(ax)} x^3 \, dx$

Optimal. Leaf size=216

$$-\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3}+\frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2}-\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a}+\frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{a\ldots\right.$$

```
[Out] -83/192*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^3+29/96*(1-1/a/x)^(1/4)*(1+1/a/
x)^(3/4)*x^2/a^2-7/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3/a+1/4*(1-1/a/x)^(
1/4)*(1+1/a/x)^(3/4)*x^4-11/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+
11/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A]
time = 0.08, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$-\frac{11\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{11\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{83x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{192a^3}+\frac{29x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{96a^2}+\frac{1}{4}x^4\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{7x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] (-83*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(192*a^3) + (29*(1 - 1/(a*x
))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(96*a^2) - (7*(1 - 1/(a*x))^(1/4)*(1 + 1/
(a*x))^(3/4)*x^3)/(24*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^4)/4
- (11*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (11*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^5\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{3x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-}{x^3(1-}\right.$$

$$= \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3}{24a} + \frac{1}{4}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/}}{24a}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/}}{24a}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/}}{24a}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/}}{24a}$$

$$= -\frac{83\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{192a^3} + \frac{29\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{96a^2} - \frac{7\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/}}{24a}$$

Mathematica [A]
time = 5.16, size = 149, normalized size = 0.69

$$\frac{\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{3200e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{2512e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{980e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+66\text{ArcTan}\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-33\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+33\log\left(1+e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{384a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (3200*E^((11*
ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (2512*E^((7*ArcCoth[a*x])/2
))/(-1 + E^(2*ArcCoth[a*x]))^2 - (980*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*Ar
cCoth[a*x])) + 66*ArcTan[E^(-1/2*ArcCoth[a*x])] - 33*Log[1 - E^(-1/2*ArcCot
h[a*x])] + 33*Log[1 + E^(-1/2*ArcCoth[a*x])])/(384*a^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A]
time = 0.47, size = 224, normalized size = 1.04

$$-\frac{1}{384}a\left(\frac{4\left(245\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-107\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+279\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-33\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/384*a*(4*(245*((a*x - 1)/(a*x + 1))^(13/4) - 107*((a*x - 1)/(a*x + 1))^(
9/4) + 279*((a*x - 1)/(a*x + 1))^(5/4) - 33*((a*x - 1)/(a*x + 1))^(1/4))/(4
*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a^
5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 66*arctan(((a*x - 1)/(
a*x + 1))^(1/4))/a^5 - 33*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 33*log
(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A]
time = 0.35, size = 111, normalized size = 0.51

$$\frac{2\left(48a^4x^4-8a^3x^3+2a^2x^2-25ax-83\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+66\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-33\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{384a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 - 8*a^3*x^3 + 2*a^2*x^2 - 25*a*x - 83)*((a*x - 1)/(a*x
 + 1))^(1/4) + 66*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 33*log(((a*x - 1)/(
a*x + 1))^(1/4) + 1) - 33*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.46, size = 203, normalized size = 0.94

$$\frac{1}{384}\,a\left(\frac{66\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{33\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{33\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}+\frac{4\left(\frac{279\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{107\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+\frac{245\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^3}-33\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5\left(\frac{ax-1}{ax+1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(66*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 33*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^5 - 33*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^5 +
 4*(279*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 107*(a*x - 1)^2*(
(a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 245*(a*x - 1)^3*((a*x - 1)/(a*x +
1))^(1/4)/(a*x + 1)^3 - 33*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*
x + 1) - 1)^4))
```

Mupad [B]
time = 1.21, size = 193, normalized size = 0.89

$$\frac{11\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{64\,a^4}-\frac{\frac{11\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{32}-\frac{93\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{32}+\frac{107\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}{96}-\frac{245\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/4}}{96}}{a^4+\frac{6\,a^4\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a^4\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a^4\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a^4\,\left(a\,x-1\right)}{a\,x+1}}+\frac{11\,\mathrm{atanh}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (11*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - ((11*((a*x - 1)/(a*x + 1)
)^(1/4))/32 - (93*((a*x - 1)/(a*x + 1))^(5/4))/32 + (107*((a*x - 1)/(a*x +
```

```
1))^(9/4))/96 - (245*((a*x - 1)/(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x -
 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a
*x + 1)^4 - (4*a^4*(a*x - 1))/(a*x + 1)) + (11*atanh(((a*x - 1)/(a*x + 1))^
(1/4)))/(64*a^4)
```

3.88 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2}-\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a}+\frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3+\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

[Out] 11/24*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a^2-5/12*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^2/a+1/3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x^3+3/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3-3/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3

Rubi [A]
time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6306, 101, 156, 12, 95, 304, 209, 212}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{11x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{24a^2}+\frac{1}{3}x^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{5x^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{12a}$$

Antiderivative was successfully verified.

[In] Int[x^2/E^(ArcCoth[a*x]/2),x]

[Out] (11*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/(24*a^2) - (5*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x^3)/3 + (3*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) - (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^4\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{5}{2a}+\frac{2x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{1}{4}}{x^2\left(1-\frac{x}{a}\right.}\right.$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$= \frac{11\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{24a^2} - \frac{5\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x^2}{12a} + \frac{1}{3}\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 5.72, size = 389, normalized size = 2.17

$$-\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(-22034705-26688365e^{2\coth^{-1}(ax)}-3731255e^{4\coth^{-1}(ax)}+3122405e^{6\coth^{-1}(ax)}+22034705\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)+17244920e^{2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)-9077530e^{4\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)-7043960e^{6\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)+446985e^{8\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)+256e^{6\coth^{-1}(ax)}\left(685+1090e^{2\coth^{-1}(ax)}+437e^{4\coth^{-1}(ax)}\right){}_4F_3\left(\frac{7}{4},2,2,2;1,1,\frac{19}{4};e^{2\coth^{-1}(ax)}\right)+2048e^{6\coth^{-1}(ax)}\left(21+38e^{2\coth^{-1}(ax)}+17e^{4\coth^{-1}(ax)}\right){}_5F_4\left(\frac{7}{4},2,2,2,2;1,1,1,\frac{19}{4};e^{2\coth^{-1}(ax)}\right)+4096e^{6\coth^{-1}(ax)}\,{}_6F_5\left(\frac{7}{4},2,2,2,2,2;1,1,1,1,\frac{19}{4};e^{2\coth^{-1}(ax)}\right)+8192e^{8\coth^{-1}(ax)}\,{}_6F_5\left(\frac{7}{4},2,2,2,2,2;1,1,1,1,\frac{19}{4};e^{2\coth^{-1}(ax)}\right)+4096e^{10\coth^{-1}(ax)}\,{}_6F_5\left(\frac{7}{4},2,2,2,2,2;1,1,1,1,\frac{19}{4};e^{2\coth^{-1}(ax)}\right)\right)}{221760a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^(ArcCoth[a*x]/2),x]
```

```
[Out] -1/221760*(-22034705 - 26688365*E^(2*ArcCoth[a*x]) - 3731255*E^(4*ArcCoth[a
*x]) + 3122405*E^(6*ArcCoth[a*x]) + 22034705*Hypergeometric2F1[3/4, 1, 7/4,
 E^(2*ArcCoth[a*x])] + 17244920*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1
, 7/4, E^(2*ArcCoth[a*x])] - 9077530*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3
/4, 1, 7/4, E^(2*ArcCoth[a*x])] - 7043960*E^(6*ArcCoth[a*x])*Hypergeometric
2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 446985*E^(8*ArcCoth[a*x])*Hypergeome
tric2F1[3/4, 1, 7/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(685 + 10
90*E^(2*ArcCoth[a*x]) + 437*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2,
2, 2}, {1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])*(21 + 38
*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{7/4, 2, 2,
2, 2}, {1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[a*x])*Hyper
geometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])]
+ 8192*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1,
 1, 19/4}, E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*HypergeometricPFQ
[{7/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 19/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((5*
ArcCoth[a*x])/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A]
time = 0.46, size = 187, normalized size = 1.04

$$-\frac{1}{48}\,a\left(\frac{4\left(29\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-6\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(29*((a*x - 1)/(a*x + 1))^(9/4) - 6*((a*x - 1)/(a*x + 1))^(5/4)
+ 9*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2
```

```
*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 18*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 - 9*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A]
time = 0.35, size = 102, normalized size = 0.57

$$\frac{2\left(8\,a^3x^3-2\,a^2x^2+ax+11\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 2*a^2*x^2 + a*x + 11)*((a*x - 1)/(a*x + 1))^(1/4) - 18
*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 9*log(((a*x - 1)/(a*x + 1))^(1/4) +
1) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.44, size = 172, normalized size = 0.96

$$-\frac{1}{48}\,a\left(\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{9\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}-\frac{4\left(\frac{6\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{29\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/48*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 9*log(((a*x - 1)/(a*x
 + 1))^(1/4) + 1)/a^4 - 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 4
*(6*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 29*(a*x - 1)^2*((a*x
- 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a
*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B]
time = 1.20, size = 157, normalized size = 0.88

$$\frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}-\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] ((3*((a*x - 1)/(a*x + 1))^(1/4))/4 - ((a*x - 1)/(a*x + 1))^(5/4)/2 + (29*((
a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2 - (a^
3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (3*atan(((a*x -
 1)/(a*x + 1))^(1/4)))/(8*a^3) - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(8*
a^3)
```

3.89 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -1/4*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x/a+1/2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4
)*x^2-1/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+1/4*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A]
time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6306, 98, 96, 95, 304, 209, 212}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^(ArcCoth[a*x]/2),x]
```

```
[Out] -1/4*((1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x)/a + ((1 - 1/(a*x))^(5/4)*(
1 + 1/(a*x))^(3/4)*x^2)/2 - ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]
/(4*a^2) + ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^3\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{}\right.}{8a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x\right.}{2a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{4a^2}$$

$$= -\frac{\sqrt[4]{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/4}x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}x^2 - \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

<u>Mathematica</u> [A]
time = 0.11, size = 66, normalized size = 0.46

$$\frac{-\frac{2e^{\frac{3}{2}\coth^{-1}(ax)}\left(-5+e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} - \text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + \tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^(ArcCoth[a*x]/2),x]
```

```
[Out] ((-2*E^((3*ArcCoth[a*x])/2)*(-5 + E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a
*x]))^2 - ArcTan[E^(ArcCoth[a*x]/2)] + ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A]
time = 0.46, size = 151, normalized size = 1.06

$$-\frac{1}{8}\,a\left(\frac{4\left(5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(5*((a*x - 1)/(a*x + 1))^(5/4) - ((a*x - 1)/(a*x + 1))^(1/4))/(2*
(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 2*arctan(((a
*x - 1)/(a*x + 1))^(1/4))/a^3 - log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 +
log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A]
time = 0.33, size = 93, normalized size = 0.65

$$\frac{2\,(2\,a^2x^2-ax-3)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - a*x - 3)*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a*x -
 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt[4]{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.44, size = 140, normalized size = 0.99

$$\frac{1}{8}\,a\left(\frac{2\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}+\frac{4\left(\frac{5\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + log(((a*x - 1)/(a*x + 1)
)^(1/4) + 1)/a^3 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 + 4*(5*(a*
x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - ((a*x - 1)/(a*x + 1))^(1/4))
/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B]
time = 0.06, size = 121, normalized size = 0.85

$$\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}+\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] atan(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2) - (((a*x - 1)/(a*x + 1))^(1/4)/2
 - (5*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 -
 (2*a^2*(a*x - 1))/(a*x + 1)) + atanh(((a*x - 1)/(a*x + 1))^(1/4))/(4*a^2)
```

3.90 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=97

$$\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)*x+arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
-arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6305, 96, 95, 304, 209, 212}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-1/2*ArcCoth[a*x]),x]
```

```
[Out] (1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)*x + ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
 1/(a*x))^(1/4)]/a - ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x^2\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cdots\right)}{a}$$

$$= \sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}x + \frac{\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 33, normalized size = 0.34

$$-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-1/2*ArcCoth[a*x]),x]
[Out] (-8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, E^(2*ArcCoth[a*x]
)])/(3*a)
```

Maple [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [A]
time = 0.46, size = 111, normalized size = 1.14

$$-\frac{1}{2}a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 2*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1))^(1/4) +
1)/a^2 - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A]
time = 0.37, size = 84, normalized size = 0.87

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4) - 2*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + log(((a*x - 1)/(a*x + 1)
)^(1/4) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{\frac{ax-1}{ax+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [A]
time = 0.42, size = 108, normalized size = 1.11

$$-\frac{1}{2}a\left(\frac{2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(2*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + log(((a*x - 1)/(a*x + 1
))^(1/4) + 1)/a^2 - log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 + 4*((a*x
 - 1)/(a*x + 1))^(1/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B]
time = 1.18, size = 79, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) - atan(((a*x
- 1)/(a*x + 1))^(1/4))/a - atanh(((a*x - 1)/(a*x + 1))^(1/4))/a
```

3.91 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$\sqrt{2}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-2\mathrm{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\tanh^{-1}$$

[Out] -2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)

Rubi [A]
time = 0.16, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6306, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$\sqrt{2}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)-2\mathrm{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+2\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(E^(ArcCoth[a*x]/2)*x),x]

[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right.$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\text{Sub}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1}}{\sqrt[4]{1}}\right.}{\sqrt{2}}$$

$$= -2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

$$= \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - 2\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 30, normalized size = 0.10

$$\frac{8}{3}e^{\frac{3}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{8},1;\frac{11}{8};e^{4\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x),x]
```

```
[Out] (8*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/8, 1, 11/8, E^(4*ArcCoth[a*x]
)])/3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x,x)
```

Maxima [A]
time = 0.47, size = 224, normalized size = 0.77

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

Fricas [A]
time = 0.34, size = 291, normalized size = 1.00

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\frac{1}{2}\sqrt{2}\log\left(4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)+\frac{1}{2}\sqrt{2}\log\left(-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)+2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 2*sqrt
(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt
((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/2
*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
+ 1)) + 4) + 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) + 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x, x)
```

Giac [A]
time = 0.43, size = 232, normalized size = 0.80

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{2\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a - 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

Mupad [B]
time = 1.18, size = 101, normalized size = 0.35

$$2\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1+1{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
```

3.92 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=268

$$-a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{a\log\left(1 \right.}{}$$

```
[Out] -a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+1/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(
1/4))*2^(1/2)-1/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6306, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-a\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}+\frac{a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

```
[Out] -(a*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4)) - (a*ArcTan[1 - (Sqrt[2]*(1 -
1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (a*ArcTan[1 + (Sqrt[2]*(1 -
 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] - (a*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(2*Sqrt[2]) + (a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + (2a)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + a\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + a\text{Subst}\left(\int \frac{1+}{1+}\right.$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}a\text{Su}$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} + \frac{a\log\left(1+\right.}{}$$

$$= -a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1}}{\sqrt[4]{1+}}\right.}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 33, normalized size = 0.12

$$-\frac{8}{3}ae^{\frac{3}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^2),x]
```

```
[Out] (-8*a*E^((3*ArcCoth[a*x])/2)*Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCoth[a
*x])])/3
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^2,x)
```

Maxima [A]
time = 0.46, size = 186, normalized size = 0.69

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Fricas [A]
time = 0.35, size = 368, normalized size = 1.37

$$-\frac{4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^4+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right)+4\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\arctan\left(\frac{a^4-\sqrt{2}\left(a^4\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\left(a^4\right)^{\frac{3}{4}}\sqrt{a^2\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}}}{a^4}\right)-\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\log\left(a^2\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}\right)+\sqrt{2}\left(a^4\right)^{\frac{1}{4}}x\log\left(a^2\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^4\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^4}\right)+4\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
4*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - sqr
t(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - sqrt(2)
*(a^4)^(1/4)*x*log(a^2*sqrt((a*x - 1)/(a*x + 1)) + sqrt(2)*(a^4)^(1/4)*a*((
a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)) + sqrt(2)*(a^4)^(1/4)*x*log(a^2*sqrt
((a*x - 1)/(a*x + 1)) - sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt(a^4)) + 4*(a*x + 1)*((a*x - 1)/(a*x + 1))^(1/4))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**2, x)
```

Giac [A]
time = 0.42, size = 186, normalized size = 0.69

$$\frac{1}{4}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x +
 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x -
1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1
))*a
```

Mupad [B]
time = 1.19, size = 88, normalized size = 0.33

$$-\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,1{}\mathrm{i}-{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^2,x)
```

```
[Out] - (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (-1)^(1/4)
*a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i - (2*a*((a*x - 1)/(a*x
+ 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.93 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}-\frac{a^2\text{ArcTan}}{}$$

```
[Out] 1/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)+1/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(
3/4)-1/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-1/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+1/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)-1/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}}-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 + (a^2*(1 - 1/(a*x))^(5/4)*
(1 + 1/(a*x))^(3/4))/2 + (a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 +
 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4
))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[
1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[
2]) - (a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a
*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}a\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}a\text{Subst}\left(\int \frac{}{\left(1-\frac{x}{a}\right.}\right.$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-}}\right.$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1+x^4}\right.$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}a^2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\right.$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{8}a^2\text{Subst}\left(\int \frac{}{1-\sqrt{}}\right.$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\log\left(1+\frac{\sqrt{1-\frac{1}{a}}}{\sqrt{1+\frac{1}{a}}}\right.}{8}$$

$$= \frac{1}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1}}\right.}{4\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 56, normalized size = 0.18

$$-\frac{8}{3}a^2e^{\frac{3}{2}\coth^{-1}(ax)}\left({}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)-2\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-e^{2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^3),x]
```

```
[Out] (-8*a^2*E^((3*ArcCoth[a*x])/2)*(Hypergeometric2F1[3/4, 2, 7/4, -E^(2*ArcCot
h[a*x])] - 2*Hypergeometric2F1[3/4, 3, 7/4, -E^(2*ArcCoth[a*x])]))/3
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^3,x)
```

Maxima [A]
time = 0.46, size = 227, normalized size = 0.71

$$-\frac{1}{16}\left(2\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(5\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2\,(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*a*((a*x - 1)/(a*x + 1))^(5/4) + a*((a*x
 - 1)/(a*x + 1))^(1/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

Fricas [A]
time = 0.38, size = 396, normalized size = 1.24

$$\frac{4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+4\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}\right)+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(a^4\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}\right)+4\left(3a^2x^2+ax-2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*((a*
x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x
+ 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a
^8) + 4*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*((a*x
 - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a*x +
 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)))/a^
8) - sqrt(2)*(a^8)^(1/4)*x^2*log(a^4*sqrt((a*x - 1)/(a*x + 1)) + sqrt(2)*(a
^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)) + sqrt(2)*(a^8)^(1/4
)*x^2*log(a^4*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1
)/(a*x + 1))^(1/4) + sqrt(a^8)) + 4*(3*a^2*x^2 + a*x - 2)*((a*x - 1)/(a*x +
 1))^(1/4))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**3, x)
```

Giac [A]
time = 0.44, size = 223, normalized size = 0.70

$$-\frac{1}{16}\left(2\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{5(ax-1)a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}+a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^
(1/4))) + sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1
)/(a*x + 1)) + 1) - sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sq
rt((a*x - 1)/(a*x + 1)) + 1) - 8*(5*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)
/(a*x + 1) + a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B]
time = 0.07, size = 132, normalized size = 0.41

$$\frac{\frac{a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}+\frac{5\,a^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{\frac{(a\,x-1)^2}{(a\,x+1)^2}+\frac{2\,(a\,x-1)}{a\,x+1}+1}+\frac{(-1)^{1/4}\,a^2\,\mathrm{atan}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1{}\mathrm{i}}{4}+\frac{(-1)^{1/4}\,a^2\,\mathrm{atanh}\left((-1)^{1/4}\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^3,x)
```

```
[Out] ((a^2*((a*x - 1)/(a*x + 1))^(1/4))/2 + (5*a^2*((a*x - 1)/(a*x + 1))^(5/4))/
2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/4)*a^
2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4 + ((-1)^(1/4)*a^2*atan
h((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*1i)/4
```

3.94 $\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$-\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x}-\frac{3a^3\text{ArcTan}\left(1\right.}{}$$

```
[Out] -3/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-1/12*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)
^(3/4)+1/3*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)/x+3/16*a^3*arctan(-1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+3/16*a^3*arctan(1+(1-1/a/x)^(1/4)
*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-3/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+
1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+3/32*a^3*ln(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 92, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3a^3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}+\frac{3a^3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}-\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{3a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

```
[Out] (-3*a^3*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/8 - (a^3*(1 - 1/(a*x))^(5/
4)*(1 + 1/(a*x))^(3/4))/12 + (a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/
(3*x) - (3*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(8*Sqrt[2]) + (3*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/(8*Sqrt[2]) - (3*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)
] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) + (3*a
^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/
4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{1}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}\left(-1+\frac{x}{2a}\right)}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}}{3x} - \frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int \frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\right.}{3x}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\right.}{3x}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\right.}{3x}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\right.}{3x}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\right.}{3x}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\right.}{3x}$$

$$= -\frac{3}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{12}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\right.}{3x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(-\frac{8e^{\frac{3}{2}\coth^{-1}(ax)}\left(29+6e^{2\coth^{-1}(ax)}+9e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3}+9\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(ArcCoth[a*x]/2)*x^4),x]
```

```
[Out] (a^3*((-8*E^((3*ArcCoth[a*x])/2)*(29 + 6*E^(2*ArcCoth[a*x]) + 9*E^(4*ArcCot
h[a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 9*RootSum[1 + #1^4 & , (ArcCoth[a*x]
 + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ]))/96
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(1/4)/x^4,x)
```

Maxima [A]
time = 0.47, size = 277, normalized size = 0.78

$$\frac{1}{96}\left(18\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(29\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+6\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)}{ax+1}+\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(29*a^2*((a*x - 1)/(a*x + 1))
^(9/4) + 6*a^2*((a*x - 1)/(a*x + 1))^(5/4) + 9*a^2*((a*x - 1)/(a*x + 1))^(1
/4))/(3*(a*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x
+ 1)^3 + 1))*a
```

Fricas [A]
time = 0.38, size = 412, normalized size = 1.16

$$-\frac{36\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\,(a^{12})^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\,(a^{12})^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+36\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\arctan\left(\frac{a^{12}-\sqrt{2}\,(a^{12})^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\,(a^{12})^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)-9\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\log\left(9\,a^6\sqrt{\frac{ax-1}{ax+1}}+9\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+9\sqrt{a^{12}}\right)+9\sqrt{2}\,(a^{12})^{\frac{1}{4}}x^3\log\left(9\,a^6\sqrt{\frac{ax-1}{ax+1}}-9\sqrt{2}\,(a^{12})^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+9\sqrt{a^{12}}\right)+4\left(11\,a^3x^3+a^2x^2-2\,ax+8\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(36*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^3
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1)
/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a
^12)))/a^12) + 36*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^(3
/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a
*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt(a^12)))/a^12) - 9*sqrt(2)*(a^12)^(1/4)*x^3*log(9*a^6*sqrt((a*x - 1)/(
a*x + 1)) + 9*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt
(a^12)) + 9*sqrt(2)*(a^12)^(1/4)*x^3*log(9*a^6*sqrt((a*x - 1)/(a*x + 1)) -
9*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + 9*sqrt(a^12)) + 4*
(11*a^3*x^3 + a^2*x^2 - 2*a*x + 8)*((a*x - 1)/(a*x + 1))^(1/4))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{\frac{ax-1}{ax+1}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(1/4)/x**4, x)
```

Giac [A]
time = 0.44, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(18\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+9\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{6\,(ax-1)a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}+\frac{29\,(ax-1)^2a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+9\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(18*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/4))) + 18*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/4))) + 9*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1) - 9*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x + 1
))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(6*(a*x - 1)*a^2*((a*x - 1)/(
a*x + 1))^(1/4)/(a*x + 1) + 29*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/
(a*x + 1)^2 + 9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^
3)*a
```

Mupad [B]
time = 0.07, size = 169, normalized size = 0.47

$$-\frac{\frac{3\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{4}+\frac{a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{2}+\frac{29\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1}-\frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}}{8}-\frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,3{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/4)/x^4,x)
```

```
[Out] - ((3*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (a^3*((a*x - 1)/(a*x + 1))^(5/4)
)/2 + (29*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1)^2
 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^3
*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8 - ((-1)^(1/4)*a^3*atanh
((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4))*3i)/8
```

3.95 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=253

$$\frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4}-\frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3}+\frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2}-\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+}}{40a}$$

[Out] 557/640*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^4-157/320*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^2/a^3+5/16*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a^2-11/40*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^4/a+1/5*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^5-237/128*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5-237/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^5

Rubi [A]

time = 0.10, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$-\frac{237\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}-\frac{237\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{557x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{640a^4}-\frac{157x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{320a^3}+\frac{5x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{16a^2}+\frac{1}{5}x^5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{11x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{40a}$$

Antiderivative was successfully verified.

[In] Int[x^4/E^((3*ArcCoth[a*x])/2),x]

[Out] (557*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(640*a^4) - (157*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(320*a^3) + (5*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^3)/(16*a^2) - (11*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/(40*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^5)/5 - (237*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (237*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^6\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 - \frac{1}{5}\text{Subst}\left(\int \frac{-\frac{11}{2a}+\frac{4x}{a^2}}{x^5\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^5 + \frac{1}{20}\text{Subst}\left(\int \frac{-}{x^4\sqrt[4]{1}}\right.$$

$$= \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4}{40a} + \frac{1}{5}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$= -\frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{16a^2} - \frac{11\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+}}{40a}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{16a^2}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{16a^2}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{16a^2}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{16a^2}$$

$$= \frac{557\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{640a^4} - \frac{157\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{320a^3} + \frac{5\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{16a^2}$$

Mathematica [A]
time = 5.21, size = 173, normalized size = 0.68

$$\frac{\frac{8192e^{\frac{17}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^5}-\frac{22016e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}+\frac{23936e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{14032e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{5500e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+2370\text{ArcTan}\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+1185\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-1185\log\left(1+e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{1280a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((8192*E^((17*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^5 - (22016*E^((13
*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 + (23936*E^((9*ArcCoth[a*x])
/2))/(-1 + E^(2*ArcCoth[a*x]))^3 - (14032*E^((5*ArcCoth[a*x])/2))/(-1 + E^(
2*ArcCoth[a*x]))^2 + (5500*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) +
2370*ArcTan[E^(-1/2*ArcCoth[a*x])] + 1185*Log[1 - E^(-1/2*ArcCoth[a*x])] -
1185*Log[1 + E^(-1/2*ArcCoth[a*x])])/(1280*a^5)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A]
time = 0.47, size = 259, normalized size = 1.02

$$-\frac{1}{1280}\,a\left(\frac{4\left(1375\left(\frac{ax-1}{ax+1}\right)^{\frac{19}{4}}-1992\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}+3710\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-1440\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+395\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}-\frac{2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/1280*a*(4*(1375*((a*x - 1)/(a*x + 1))^(19/4) - 1992*((a*x - 1)/(a*x + 1)
)^(15/4) + 3710*((a*x - 1)/(a*x + 1))^(11/4) - 1440*((a*x - 1)/(a*x + 1))^(
7/4) + 395*((a*x - 1)/(a*x + 1))^(3/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10*(a*
x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)^4*a
^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) - 2370*arctan(((a*x - 1
)/(a*x + 1))^(1/4))/a^6 + 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 1
185*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6)
```

Fricas [A]
time = 0.41, size = 119, normalized size = 0.47

$$\frac{2\,(128\,a^5x^5-48\,a^4x^4+24\,a^3x^3-114\,a^2x^2+243\,ax+557)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+2370\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+1185\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{1280\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/1280*(2*(128*a^5*x^5 - 48*a^4*x^4 + 24*a^3*x^3 - 114*a^2*x^2 + 243*a*x +
557)*((a*x - 1)/(a*x + 1))^(3/4) + 2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))
 - 1185*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1185*log(((a*x - 1)/(a*x + 1
))^(1/4) - 1))/a^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x**4*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.48, size = 234, normalized size = 0.92

$$\frac{1}{1280}\, a \left(\frac{2370 \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6} - \frac{1185 \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^6} + \frac{1185 \log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right|\right)}{a^6} + \frac{4 \left(\frac{1440 (ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} - \frac{3710 (ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2} + \frac{1992 (ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^3} - \frac{1375 (ax-1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^4} - 395 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^6 \left(\frac{ax-1}{ax+1} - 1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/1280*a*(2370*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 - 1185*log(((a*x - 1
)/(a*x + 1))^(1/4) + 1)/a^6 + 1185*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^6 + 4*(1440*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3710*(a*x
 - 1)^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 1992*(a*x - 1)^3*((a*x -
1)/(a*x + 1))^(3/4)/(a*x + 1)^3 - 1375*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(3
/4)/(a*x + 1)^4 - 395*((a*x - 1)/(a*x + 1))^(3/4))/(a^6*((a*x - 1)/(a*x + 1
) - 1)^5))
```

Mupad [B]
time = 1.21, size = 229, normalized size = 0.91

$$\frac{\frac{79\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{64} - \frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2} + \frac{371\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32} - \frac{249\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{40} + \frac{275\left(\frac{a\,x-1}{a\,x+1}\right)^{19/4}}{64}}{a^5 + \frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5} - \frac{5\,a^5\,(a\,x-1)}{a\,x+1}} + \frac{237\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5} - \frac{237\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((79*((a*x - 1)/(a*x + 1))^(3/4))/64 - (9*((a*x - 1)/(a*x + 1))^(7/4))/2 +
(371*((a*x - 1)/(a*x + 1))^(11/4))/32 - (249*((a*x - 1)/(a*x + 1))^(15/4))/
40 + (275*((a*x - 1)/(a*x + 1))^(19/4))/64)/(a^5 + (10*a^5*(a*x - 1)^2)/(a*
x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*a^5*(a*x - 1)^4)/(a*x + 1)
^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x - 1))/(a*x + 1)) + (237*at
an(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5) - (237*atanh(((a*x - 1)/(a*x + 1
))^(1/4)))/(128*a^5)
```

3.96 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=216

$$-\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3}+\frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2}-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a}+\frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+$$

```
[Out] -63/64*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^3+15/32*(1-1/a/x)^(3/4)*(1+1/a/x
)^(1/4)*x^2/a^2-3/8*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3/a+1/4*(1-1/a/x)^(3/
4)*(1+1/a/x)^(1/4)*x^4+123/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+1
23/64*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A]
time = 0.08, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$\frac{123\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}+\frac{123\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{63x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{64a^3}+\frac{15x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{32a^2}+\frac{1}{4}x^4\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (-63*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(64*a^3) + (15*(1 - 1/(a*x)
)^(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(32*a^2) - (3*(1 - 1/(a*x))^(3/4)*(1 + 1/(
a*x))^(1/4)*x^3)/(8*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x^4)/4 +
(123*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (123*ArcTa
nh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^5\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{-\frac{9}{2a}+\frac{3x}{a^2}}{x^4\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^4 + \frac{1}{12}\text{Subst}\left(\int \frac{-}{x^3\sqrt[4]{1-}}\right.$$

$$= \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3}{8a} + \frac{1}{4}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{8a}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{8a}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{8a}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{8a}$$

$$= -\frac{63\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{64a^3} + \frac{15\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{32a^2} - \frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}}{8a}$$

Mathematica [A]
time = 5.17, size = 149, normalized size = 0.69

$$\frac{\frac{512e^{\frac{13}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{1152e^{\frac{9}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{1008e^{\frac{5}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{532e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}-246\text{ArcTan}\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-123\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+123\log\left(1+e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((512*E^((13*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^4 - (1152*E^((9*Ar
cCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 + (1008*E^((5*ArcCoth[a*x])/2))
/(-1 + E^(2*ArcCoth[a*x]))^2 - (532*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[
a*x])) - 246*ArcTan[E^(-1/2*ArcCoth[a*x])] - 123*Log[1 - E^(-1/2*ArcCoth[a*
x])] + 123*Log[1 + E^(-1/2*ArcCoth[a*x])])/(128*a^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A]
time = 0.48, size = 224, normalized size = 1.04

$$-\frac{1}{128}a\left(\frac{4\left(133\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{4}}-147\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+183\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-41\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{4(ax-1)a^5}{ax+1}-\frac{6(ax-1)^2a^5}{(ax+1)^2}+\frac{4(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}+\frac{246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/128*a*(4*(133*((a*x - 1)/(a*x + 1))^(15/4) - 147*((a*x - 1)/(a*x + 1))^(
11/4) + 183*((a*x - 1)/(a*x + 1))^(7/4) - 41*((a*x - 1)/(a*x + 1))^(3/4))/(
4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^3*a
^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) + 246*arctan(((a*x - 1)
/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5 + 123
*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5)
```

Fricas [A]
time = 0.41, size = 111, normalized size = 0.51

$$\frac{2\left(16a^4x^4-8a^3x^3+6a^2x^2-33ax-63\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-246\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-123\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{128a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/128*(2*(16*a^4*x^4 - 8*a^3*x^3 + 6*a^2*x^2 - 33*a*x - 63)*((a*x - 1)/(a*x
 + 1))^(3/4) - 246*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 123*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1) - 123*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x**3*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.49, size = 203, normalized size = 0.94

$$-\frac{1}{128}\, a \left(\frac{246\, \arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5} - \frac{123\, \log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} + 1\right)}{a^5} + \frac{123\, \log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}} - 1\right|\right)}{a^5} - \frac{4\left(\frac{183\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1} - \frac{147\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2} + \frac{133\,(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^3} - 41\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^5\left(\frac{ax-1}{ax+1} - 1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/128*a*(246*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 - 123*log(((a*x - 1)/
(a*x + 1))^(1/4) + 1)/a^5 + 123*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a
^5 - 4*(183*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 147*(a*x - 1)
^2*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 + 133*(a*x - 1)^3*((a*x - 1)/(a*
x + 1))^(3/4)/(a*x + 1)^3 - 41*((a*x - 1)/(a*x + 1))^(3/4))/(a^5*((a*x - 1)
/(a*x + 1) - 1)^4))
```

Mupad [B]
time = 1.18, size = 193, normalized size = 0.89

$$\frac{123\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{123\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4} - \frac{\frac{41\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{32} - \frac{183\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{32} + \frac{147\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{32} - \frac{133\left(\frac{a\,x-1}{a\,x+1}\right)^{15/4}}{32}}{a^4 + \frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4} - \frac{4\,a^4\,(a\,x-1)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (123*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (123*atan(((a*x - 1)/(a
*x + 1))^(1/4)))/(64*a^4) - ((41*((a*x - 1)/(a*x + 1))^(3/4))/32 - (183*((a
```

```
*x - 1)/(a*x + 1))^(7/4))/32 + (147*((a*x - 1)/(a*x + 1))^(11/4))/32 - (133
*((a*x - 1)/(a*x + 1))^(15/4))/32)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 -
 (4*a^4*(a*x - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(
a*x - 1))/(a*x + 1))
```

3.97 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=179

$$\frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2}-\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3-\frac{17\mathrm{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}$$

```
[Out] 23/24*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a^2-7/12*(1-1/a/x)^(3/4)*(1+1/a/x)^
(1/4)*x^2/a+1/3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x^3-17/8*arctan((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^3-17/8*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A]
time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6306, 101, 156, 12, 95, 218, 212, 209}

$$-\frac{17\mathrm{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{17\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{23x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{24a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{7x^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (23*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(24*a^2) - (7*(1 - 1/(a*x))^
(3/4)*(1 + 1/(a*x))^(1/4)*x^2)/(12*a) + ((1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^
(1/4)*x^3)/3 - (17*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
 - (17*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^4\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{2x}{a^2}}{x^3\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{2}{4}}{x^2\sqrt[4]{1-}}\right.$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

$$= \frac{23\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{24a^2} - \frac{7\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2}{12a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 6.13, size = 389, normalized size = 2.17

$$-\frac{e^{-\frac{7}{2}\coth^{-1}(ax)}\left(-15779205-17312841e^{2\coth^{-1}(ax)}-1213875e^{4\coth^{-1}(ax)}+2199249e^{6\coth^{-1}(ax)}+15779205\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)+14157000e^{2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)-2472210e^{4\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)-3598920e^{6\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)+21645e^{8\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};e^{2\coth^{-1}(ax)}\right)+256e^{6\coth^{-1}(ax)}\left(557+850e^{2\coth^{-1}(ax)}+325e^{4\coth^{-1}(ax)}\right){}_4F_3\left(\frac{5}{4},2,2,2;1,1,\frac{17}{4};e^{2\coth^{-1}(ax)}\right)+2048e^{6\coth^{-1}(ax)}\left(19+34e^{2\coth^{-1}(ax)}+15e^{4\coth^{-1}(ax)}\right){}_5F_4\left(\frac{5}{4},2,2,2,2;1,1,1,\frac{17}{4};e^{2\coth^{-1}(ax)}\right)+4096e^{6\coth^{-1}(ax)}\,{}_6F_5\left(\frac{5}{4},2,2,2,2,2;1,1,1,1,\frac{17}{4};e^{2\coth^{-1}(ax)}\right)+8192e^{8\coth^{-1}(ax)}\,{}_6F_5\left(\frac{5}{4},2,2,2,2,2;1,1,1,1,\frac{17}{4};e^{2\coth^{-1}(ax)}\right)+4096e^{10\coth^{-1}(ax)}\,{}_6F_5\left(\frac{5}{4},2,2,2,2,2;1,1,1,1,\frac{17}{4};e^{2\coth^{-1}(ax)}\right)\right)}{112320a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] -1/112320*(-15779205 - 17312841*E^(2*ArcCoth[a*x]) - 1213875*E^(4*ArcCoth[a
*x]) + 2199249*E^(6*ArcCoth[a*x]) + 15779205*Hypergeometric2F1[1/4, 1, 5/4,
 E^(2*ArcCoth[a*x])] + 14157000*E^(2*ArcCoth[a*x])*Hypergeometric2F1[1/4, 1
, 5/4, E^(2*ArcCoth[a*x])] - 2472210*E^(4*ArcCoth[a*x])*Hypergeometric2F1[1
/4, 1, 5/4, E^(2*ArcCoth[a*x])] - 3598920*E^(6*ArcCoth[a*x])*Hypergeometric
2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 21645*E^(8*ArcCoth[a*x])*Hypergeomet
ric2F1[1/4, 1, 5/4, E^(2*ArcCoth[a*x])] + 256*E^(6*ArcCoth[a*x])*(557 + 850
*E^(2*ArcCoth[a*x]) + 325*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{5/4, 2, 2,
 2}, {1, 1, 17/4}, E^(2*ArcCoth[a*x])] + 2048*E^(6*ArcCoth[a*x])*(19 + 34*E
^(2*ArcCoth[a*x]) + 15*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{5/4, 2, 2, 2,
 2}, {1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])] + 4096*E^(6*ArcCoth[a*x])*Hyperge
ometricPFQ[{5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])] +
8192*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1
, 17/4}, E^(2*ArcCoth[a*x])] + 4096*E^(10*ArcCoth[a*x])*HypergeometricPFQ[{
5/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 17/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((7*Ar
cCoth[a*x])/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A]
time = 0.46, size = 187, normalized size = 1.04

$$-\frac{1}{48}\,a\left(\frac{4\left(45\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}-30\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+17\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3\,(ax-1)a^4}{ax+1}-\frac{3\,(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}-\frac{102\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{51\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/48*a*(4*(45*((a*x - 1)/(a*x + 1))^(11/4) - 30*((a*x - 1)/(a*x + 1))^(7/4
) + 17*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1
```

```
)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) - 102*arctan(((a*x
 - 1)/(a*x + 1))^(1/4))/a^4 + 51*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 -
 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4)
```

Fricas [A]
time = 0.40, size = 103, normalized size = 0.58

$$\frac{2\,(8\,a^3x^3-6\,a^2x^2+9\,ax+23)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(8*a^3*x^3 - 6*a^2*x^2 + 9*a*x + 23)*((a*x - 1)/(a*x + 1))^(3/4) +
102*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 51*log(((a*x - 1)/(a*x + 1))^(1/4
) + 1) + 51*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.46, size = 172, normalized size = 0.96

$$\frac{1}{48}\,a\left(\frac{102\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}-\frac{51\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{51\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^4}+\frac{4\left(\frac{30\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-\frac{45\,(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}-17\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/48*a*(102*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 - 51*log(((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a^4 + 51*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 +
 4*(30*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 45*(a*x - 1)^2*((a
*x - 1)/(a*x + 1))^(3/4)/(a*x + 1)^2 - 17*((a*x - 1)/(a*x + 1))^(3/4))/(a^4
*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B]
time = 0.06, size = 157, normalized size = 0.88

$$\frac{\frac{17\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{12}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}+\frac{15\left(\frac{a\,x-1}{a\,x+1}\right)^{11/4}}{4}}{a^3+\frac{3\,a^3\,\left(a\,x-1\right)^2}{\left(a\,x+1\right)^2}-\frac{a^3\,\left(a\,x-1\right)^3}{\left(a\,x+1\right)^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}+\frac{17\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}-\frac{17\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] ((17*((a*x - 1)/(a*x + 1))^(3/4))/12 - (5*((a*x - 1)/(a*x + 1))^(7/4))/2 +
(15*((a*x - 1)/(a*x + 1))^(11/4))/4)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^2
 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (17*atan(
((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3) - (17*atanh(((a*x - 1)/(a*x + 1))^(1/
4)))/(8*a^3)
```

3.98 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=142

$$-\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2+\frac{9\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -3/4*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x/a+1/2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4
)*x^2+9/4*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2+9/4*arctanh((1+1/a/x)
^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A]
time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6306, 98, 96, 95, 218, 212, 209}

$$\frac{9\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (-3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x)/(4*a) + ((1 - 1/(a*x))^(7/4)
*(1 + 1/(a*x))^(1/4)*x^2)/2 + (9*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)])/(4*a^2) + (9*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^
2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^3\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 + \frac{3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 - \frac{9\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\,(1}\right.}{8a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 - \frac{9\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x,\right.}{2a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 + \frac{9\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x,\right.}{4a^2}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x}{4a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}\,x^2 + \frac{9\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

Mathematica [A]
time = 0.13, size = 70, normalized size = 0.49

$$\frac{-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}\left(-7+3e^{2\coth^{-1}(ax)}\right)}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2} + 9\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) + 9\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] ((-2*E^(ArcCoth[a*x]/2)*(-7 + 3*E^(2*ArcCoth[a*x])))/(-1 + E^(2*ArcCoth[a*x
]))^2 + 9*ArcTan[E^(ArcCoth[a*x]/2)] + 9*ArcTanh[E^(ArcCoth[a*x]/2)])/(4*a^
2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(3/4),x)
[Out] int(x*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A]
time = 0.46, size = 152, normalized size = 1.07

$$-\frac{1}{8}\,a\left(\frac{4\left(7\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2\,(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}+\frac{18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
[Out] -1/8*a*(4*(7*((a*x - 1)/(a*x + 1))^(7/4) - 3*((a*x - 1)/(a*x + 1))^(3/4))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) + 18*arctan(
((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^3 + 9*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3)
```

Fricas [A]
time = 0.37, size = 95, normalized size = 0.67

$$\frac{2\left(2\,a^2x^2-3\,ax-5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}-18\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-9\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
[Out] 1/8*(2*(2*a^2*x^2 - 3*a*x - 5)*((a*x - 1)/(a*x + 1))^(3/4) - 18*arctan(((a*
x - 1)/(a*x + 1))^(1/4)) + 9*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 9*log((
(a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.46, size = 141, normalized size = 0.99

$$-\frac{1}{8}a\left(\frac{18\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{9\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{9\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{4\left(\frac{7(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}-3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*a*(18*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 9*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 + 9*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 4*
(7*(a*x - 1)*((a*x - 1)/(a*x + 1))^(3/4)/(a*x + 1) - 3*((a*x - 1)/(a*x + 1)
)^(3/4))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B]
time = 1.19, size = 121, normalized size = 0.85

$$\frac{9\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{9\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{\frac{3\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{2}-\frac{7\left(\frac{a\,x-1}{a\,x+1}\right)^{7/4}}{2}}{a^2+\frac{a^2\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,a^2\,(a\,x-1)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (9*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - (9*atan(((a*x - 1)/(a*x +
1))^(1/4)))/(4*a^2) - ((3*((a*x - 1)/(a*x + 1))^(3/4))/2 - (7*((a*x - 1)/(a
*x + 1))^(7/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))
/(a*x + 1))
```

3.99 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x-\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] (1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)*x-3*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))
/a-3*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6305, 96, 95, 218, 212, 209}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+x\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-3*ArcCoth[a*x])/2),x]
```

```
[Out] (1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)*x - (3*ArcTan[(1 + 1/(a*x))^(1/4)/(
1 - 1/(a*x))^(1/4)])/a - (3*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)
])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x^2\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x + \frac{6\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x\right.}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}\,x - \frac{3\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [A]
time = 0.08, size = 55, normalized size = 0.56

$$\frac{\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}} - 3\text{ArcTan}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right) - 3\tanh^{-1}\left(e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-3*ArcCoth[a*x])/2),x]
```

```
[Out] ((2*E^(ArcCoth[a*x]/2))/(-1 + E^(2*ArcCoth[a*x])) - 3*ArcTan[E^(ArcCoth[a*x
]/2)] - 3*ArcTanh[E^(ArcCoth[a*x]/2)])/a
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [A]
time = 0.46, size = 112, normalized size = 1.14

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) - 6*a
rctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 3*log(((a*x - 1)/(a*x + 1))^(1/4)
+ 1)/a^2 - 3*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2)
```

Fricas [A]
time = 0.34, size = 86, normalized size = 0.88

$$\frac{2\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}+6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4) + 6*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - 3*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 3*log(((a*x - 1)/(a*x
+ 1))^(1/4) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [A]
time = 0.43, size = 109, normalized size = 1.11

$$\frac{1}{2}\,a\left(\frac{6\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}-\frac{3\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{3\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] 1/2*a*(6*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 - 3*log(((a*x - 1)/(a*x +
1))^(1/4) + 1)/a^2 + 3*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 4*((
a*x - 1)/(a*x + 1))^(3/4)/(a^2*((a*x - 1)/(a*x + 1) - 1)))
```

Mupad [B]
time = 1.18, size = 79, normalized size = 0.81

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{3\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}-\frac{3\,\mathrm{atanh}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(3/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (3*atan(((a
*x - 1)/(a*x + 1))^(1/4)))/a - (3*atanh(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.100 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=291

$$\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+2\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)+2\tanh^{-1}$$

[Out] 2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)

Rubi [A]

time = 0.16, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6306, 132, 65, 338, 303, 1176, 631, 210, 1179, 642, 95, 218, 212, 209}

$$\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)+2\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+2\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x),x]
```

```
[Out] Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{x\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[4]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)\right) - 4\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right.$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 4\text{Sub}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{1+\frac{1}{a}}}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1-}}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-}}{\sqrt[4]{1+\frac{}{a}}}\right.}{\sqrt{2}}$$

$$= \sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) - \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 28, normalized size = 0.10

$$8e^{\frac{1}{2}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{4\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x),x]
[Out] 8*E^(ArcCoth[a*x]/2)*Hypergeometric2F1[1/8, 1, 9/8, E^(4*ArcCoth[a*x])]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x,x)
```

Maxima [A]
time = 0.47, size = 224, normalized size = 0.77

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="maxima")
[Out] -1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a
*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2*log
(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1
)/a)
```

Fricas [A]
time = 0.42, size = 291, normalized size = 1.00

$$2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)+2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\sqrt{2}\log\left(4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-\frac{1}{2}\sqrt{2}\log\left(-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) + 2*sqrt
(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt
((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/2
*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
+ 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x, x)
```

Giac [A]
time = 0.43, size = 232, normalized size = 0.80

$$-\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-2\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}-\frac{2\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{2\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)
/(a*x + 1)) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqr
t((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a - 2
*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 2*log(abs(((a*x - 1)/(a*x + 1))^(
1/4) - 1))/a)
```

Mupad [B]
time = 1.18, size = 101, normalized size = 0.35

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1+1{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x,x)
```

```
[Out] - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*2i - 2*atan(((a*x - 1)/(a*x + 1))^(1
/4)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 -
1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 + 1
i)
```

3.101 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=269

$$-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{3a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{3a\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}+\frac{3a\log\left(\right.}{}$$

```
[Out] -a*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+3/2*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/
(1+1/a/x)^(1/4))*2^(1/2)+3/2*a*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(
1/4))*2^(1/2)+3/4*a*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-3/4*a*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)
^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6306, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3a\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}+\frac{3a\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}-a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{3a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] -(a*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4)) - (3*a*ArcTan[1 - (Sqrt[2]*(1
 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*ArcTan[1 + (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/Sqrt[2] + (3*a*Log[1 + Sqrt[1 -
1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1
/4)])/(2*Sqrt[2]) - (3*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqr
t[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + (6a)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - (3a)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + (3a)\text{Subst}$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}(3a)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right) + \frac{1}{2}$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{3a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{2\sqrt{2}} - \frac{3a\log\left(\right.}{}$$

$$= -a\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{3a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{3a\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{\sqrt{2}}$$

Mathematica [A]
time = 0.19, size = 149, normalized size = 0.55

$$a\left(-\frac{2e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}+\frac{3\text{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}-\frac{3\text{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] a*((-2*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x])) + (3*ArcTan[1 - Sqrt[2]
*E^(ArcCoth[a*x]/2)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)])/
Sqrt[2] + (3*Log[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[
2]) - (3*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]])/(2*Sqrt[2]))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^2,x)
```

Maxima [A]
time = 0.47, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Fricas [A]
time = 0.49, size = 402, normalized size = 1.49

$$-\frac{12\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^{4}+\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{4}}a^{4}+\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{4}\right)^{\frac{1}{4}}}{a^{4}}\right)+12\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(\frac{a^{4}-\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^{6}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{4}}a^{4}-\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{4}\right)^{\frac{1}{4}}}{a^{4}}\right)+3\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(729a^{6}\sqrt{\frac{ax-1}{ax+1}}+729\sqrt{a^{4}}a^{4}+729\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-3\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(729a^{6}\sqrt{\frac{ax-1}{ax+1}}+729\sqrt{a^{4}}a^{4}-729\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+4\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(12*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(1/4)*a^3*((a*x
 - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqrt(
a^4)*a^4 + sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/4)
```

```
)/a^4) + 12*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(1/4)*a^3*((a
*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^6*sqrt((a*x - 1)/(a*x + 1)) + sqr
t(a^4)*a^4 - sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4))*(a^4)^(1/
4))/a^4) + 3*sqrt(2)*(a^4)^(1/4)*x*log(729*a^6*sqrt((a*x - 1)/(a*x + 1)) +
729*sqrt(a^4)*a^4 + 729*sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
) - 3*sqrt(2)*(a^4)^(1/4)*x*log(729*a^6*sqrt((a*x - 1)/(a*x + 1)) + 729*sqr
t(a^4)*a^4 - 729*sqrt(2)*(a^4)^(3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(
a*x + 1)*((a*x - 1)/(a*x + 1))^(3/4))/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**2, x)
```

Giac [**A**]
time = 0.43, size = 187, normalized size = 0.70

$$\frac{1}{4}\left(6\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+6\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-3\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4))
) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/4))
) - 3*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x
 + 1)) + 1) + 3*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*
x - 1)/(a*x + 1)) + 1) - 8*((a*x - 1)/(a*x + 1))^(3/4)/((a*x - 1)/(a*x + 1)
 + 1))*a
```

Mupad [**B**]
time = 0.05, size = 88, normalized size = 0.33

$$3\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-3\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{\frac{a\,x-1}{a\,x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^2,x)
```

```
[Out] 3*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - 3*(-1)^(1/4)*
a*atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)) - (2*a*((a*x - 1)/(a*x + 1)
)^(3/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.102 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=319

$$\frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{9a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{4\sqrt{2}}-\frac{9a^2\text{ArcTan}}{}$$

```
[Out] 3/4*a^2*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)+1/2*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(
1/4)-9/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/8
*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-9/16*a^2*ln(
1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*
2^(1/2)+9/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/
2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{9a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}}-\frac{9a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{9a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (3*a^2*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4
)*(1 + 1/(a*x))^(1/4))/2 + (9*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/
(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x)
)^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (9*a^2*Log[1 + Sqrt[1 - 1/(a*x
)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/
(8*Sqrt[2]) + (9*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]
*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{4}(3a)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{8}(9a)\text{Subst}\left(\int \frac{}{\sqrt[4]{1-}}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x}{(2-}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{2}\left(9a^2\right)\text{Subst}\left(\int \frac{x^2}{1+x}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{4}\left(9a^2\right)\text{Subst}\left(\int \frac{1-x}{1+x}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{8}\left(9a^2\right)\text{Subst}\left(\int \frac{}{1-}\right.$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{9a^2\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}}\right.}{8\sqrt{}}$$

$$= \frac{3}{4}a^2\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{9a^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{1-}}\right.}{4\sqrt{2}}$$

Mathematica [A]

time = 0.14, size = 174, normalized size = 0.55

$$\frac{1}{16}a^2\left(\frac{32e^{\frac{1}{2}\coth^{-1}(ax)}}{\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{24e^{\frac{1}{2}\coth^{-1}(ax)}}{1+e^{2\coth^{-1}(ax)}}-18\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)+18\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}\right)-9\sqrt{2}\,\log\left(1-\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)+9\sqrt{2}\,\log\left(1+\sqrt{2}\,e^{\frac{1}{2}\coth^{-1}(ax)}+e^{\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] (a^2*((32*E^(ArcCoth[a*x]/2))/(1 + E^(2*ArcCoth[a*x]))^2 + (24*E^(ArcCoth[a
*x]/2))/(1 + E^(2*ArcCoth[a*x])) - 18*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^(ArcCoth
[a*x]/2)] + 18*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/2)] - 9*Sqrt[2]*L
og[1 - Sqrt[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]] + 9*Sqrt[2]*Log[1 + Sqr
t[2]*E^(ArcCoth[a*x]/2) + E^ArcCoth[a*x]]))/16
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^3,x)
```

Maxima [A]
time = 0.47, size = 228, normalized size = 0.71

$$-\frac{1}{16}\left(9\left(2\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\,\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\,\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\,\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\right)a-\frac{8\left(7a\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+3a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{2(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(9*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a - 8*(7*a*((a*x - 1)/(a*x + 1))^(7/4) + 3*a*((a*x
 - 1)/(a*x + 1))^(3/4))/(2*(a*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 +
1))*a
```

Fricas [A]
time = 0.37, size = 419, normalized size = 1.31

$$\frac{36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}\,a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right)+36\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^{12}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^8}\,a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^8\right)^{\frac{1}{4}}}{a^8}\right)+9\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(531441\,a^{12}\sqrt{\frac{ax-1}{ax+1}}+531441\sqrt{a^8}\,a^8+531441\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-9\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(531441\,a^{12}\sqrt{\frac{ax-1}{ax+1}}+531441\sqrt{a^8}\,a^8-531441\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+4\left(5a^2x^2+3ax-2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(36*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(1/4)*a^6*((a
*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1)) + sq
rt(a^8)*a^8 + sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^8)^(1
/4))/a^8) + 36*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(1/4)*a^
6*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^12*sqrt((a*x - 1)/(a*x + 1))
 + sqrt(a^8)*a^8 - sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1)/(a*x + 1))^(1/4))*(a^
8)^(1/4))/a^8) + 9*sqrt(2)*(a^8)^(1/4)*x^2*log(531441*a^12*sqrt((a*x - 1)/(
a*x + 1)) + 531441*sqrt(a^8)*a^8 + 531441*sqrt(2)*(a^8)^(3/4)*a^6*((a*x - 1
)/(a*x + 1))^(1/4)) - 9*sqrt(2)*(a^8)^(1/4)*x^2*log(531441*a^12*sqrt((a*x -
 1)/(a*x + 1)) + 531441*sqrt(a^8)*a^8 - 531441*sqrt(2)*(a^8)^(3/4)*a^6*((a*
x - 1)/(a*x + 1))^(1/4)) + 4*(5*a^2*x^2 + 3*a*x - 2)*((a*x - 1)/(a*x + 1))^
(3/4))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**3, x)
```

Giac [A]
time = 0.43, size = 225, normalized size = 0.71

$$-\frac{1}{16}\left(18\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+18\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-9\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+9\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{7(ax-1)a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}+3a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^3,x, algorithm="giac")
```

```
[Out] -1/16*(18*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 18*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/4))) - 9*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) + 9*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4
) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(7*(a*x - 1)*a*((a*x - 1)/(a*x + 1))
^(3/4)/(a*x + 1) + 3*a*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1) +
1)^2)*a
```

Mupad [B]
time = 1.18, size = 132, normalized size = 0.41

$$\frac{\frac{3\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{2}+\frac{7\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}-\frac{9\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}+\frac{9\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^3,x)
```

```
[Out] ((3*a^2*((a*x - 1)/(a*x + 1))^(3/4))/2 + (7*a^2*((a*x - 1)/(a*x + 1))^(7/4)
)/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) - (9*(-1)^(1/4
)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4 + (9*(-1)^(1/4)*a^2*a
tanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/4
```

3.103 $\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=356

$$-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x}-\frac{17a^3\text{ArcTan}\left(1-\right.}{8\sqrt{}}$$

[Out] -17/24*a^3*(1-1/a/x)^(3/4)*(1+1/a/x)^(1/4)-1/4*a^3*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4)+1/3*a^2*(1-1/a/x)^(7/4)*(1+1/a/x)^(1/4)/x+17/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+17/32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-17/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)

Rubi [A]

time = 0.20, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 92, 81, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{17a^3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}+\frac{17a^3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}-\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}-\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{\frac{1}{ax}+1}+\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\frac{17a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{\frac{1}{ax}+1}}{3x}$$

Antiderivative was successfully verified.

[In] Int[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]

[Out] (-17*a^3*(1 - 1/(a*x))^(3/4)*(1 + 1/(a*x))^(1/4))/24 - (a^3*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))/4 + (a^2*(1 - 1/(a*x))^(7/4)*(1 + 1/(a*x))^(1/4))/(3*x) - (17*a^3*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (17*a^3*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (17*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{3}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/4}\left(-1+\frac{3x}{2a}\right)}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}}}{3x} - \frac{1}{24}\left(17a^2\right)\text{Subst}\left(\int \frac{(1-}{(1+}\right.$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{}{a}}}{3x}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{}{a}}}{3x}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{}{a}}}{3x}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{}{a}}}{3x}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{}{a}}}{3x}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{}{a}}}{3x}$$

$$= -\frac{17}{24}a^3\left(1-\frac{1}{ax}\right)^{3/4}\sqrt[4]{1+\frac{1}{ax}} - \frac{1}{4}a^3\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{1}{ax}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/4}\sqrt[4]{1+\frac{}{a}}}{3x}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.10, size = 93, normalized size = 0.26

$$\frac{1}{96}a^3\left(-\frac{8e^{\frac{1}{2}\coth^{-1}(ax)}\left(45+30e^{2\coth^{-1}(ax)}+17e^{4\coth^{-1}(ax)}\right)}{\left(1+e^{2\coth^{-1}(ax)}\right)^3}+51\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((3*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (a^3*((-8*E^(ArcCoth[a*x]/2)*(45 + 30*E^(2*ArcCoth[a*x]) + 17*E^(4*ArcCoth[
a*x])))/(1 + E^(2*ArcCoth[a*x]))^3 + 51*RootSum[1 + #1^4 & , (ArcCoth[a*x]
+ 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1 & ]))/96
```

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(3/4)/x^4,x)
```

Maxima [A]

time = 0.47, size = 270, normalized size = 0.76

$$\frac{1}{96}\left(51\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)\right)a^2-\frac{8\left(45a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{4}}+30a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{4}}+17a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\frac{3(ax-1)}{ax+1}+\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="maxima")
```

```
[Out] 1/96*(51*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(
1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/4))) - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a
*x - 1)/(a*x + 1)) + 1))*a^2 - 8*(45*a^2*((a*x - 1)/(a*x + 1))^(11/4) + 30*
a^2*((a*x - 1)/(a*x + 1))^(7/4) + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/(3*(a
*x - 1)/(a*x + 1) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1
))*a
```

Fricas [A]

time = 0.37, size = 427, normalized size = 1.20

$$-\frac{204\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+204\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(\frac{a^{12}-\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\sqrt{a^{18}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{a^{12}}a^{12}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\left(a^{12}\right)^{\frac{1}{4}}}{a^{12}}\right)+51\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\log\left(24137569a^{18}\sqrt{\frac{ax-1}{ax+1}}+24137569\sqrt{a^{12}}a^{12}+24137569\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-51\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\log\left(24137569a^{18}\sqrt{\frac{ax-1}{ax+1}}+24137569\sqrt{a^{12}}a^{12}-24137569\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+4\left(23a^3x^3+9a^2x^2-6ax+8\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{96x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="fricas")
```

```
[Out] -1/96*(204*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(1/4)*a^
9*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*sqrt(a^18*sqrt((a*x - 1)/(a*x + 1))
 + sqrt(a^12)*a^12 + sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4))*
(a^12)^(1/4))/a^12) + 204*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(
a^12)^(1/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*sqrt(a^18*sqrt((a*x -
 1)/(a*x + 1)) + sqrt(a^12)*a^12 - sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x
 + 1))^(1/4))*(a^12)^(1/4))/a^12) + 51*sqrt(2)*(a^12)^(1/4)*x^3*log(2413756
9*a^18*sqrt((a*x - 1)/(a*x + 1)) + 24137569*sqrt(a^12)*a^12 + 24137569*sqrt
(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) - 51*sqrt(2)*(a^12)^(1/4)
*x^3*log(24137569*a^18*sqrt((a*x - 1)/(a*x + 1)) + 24137569*sqrt(a^12)*a^12
 - 24137569*sqrt(2)*(a^12)^(3/4)*a^9*((a*x - 1)/(a*x + 1))^(1/4)) + 4*(23*a
^3*x^3 + 9*a^2*x^2 - 6*a*x + 8)*((a*x - 1)/(a*x + 1))^(3/4))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/4)/x**4, x)
```

Giac [A]
time = 0.45, size = 271, normalized size = 0.76

$$\frac{1}{96}\left(102\,\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+102\,\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)-51\,\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)+51\,\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\frac{8\left(\frac{30\,(ax-1)a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{ax+1}+\frac{45\,(ax-1)^2a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}{(ax+1)^2}+17\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/4)/x^4,x, algorithm="giac")
```

```
[Out] 1/96*(102*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 102*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x
 + 1))^(1/4))) - 51*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + s
qrt((a*x - 1)/(a*x + 1)) + 1) + 51*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 8*(30*(a*x - 1)*a^2*((a*x -
 1)/(a*x + 1))^(3/4)/(a*x + 1) + 45*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(
3/4)/(a*x + 1)^2 + 17*a^2*((a*x - 1)/(a*x + 1))^(3/4))/((a*x - 1)/(a*x + 1)
 + 1)^3)*a
```

Mupad [B]
time = 0.06, size = 169, normalized size = 0.47

$$\frac{17\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8}-\frac{\frac{17\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}{12}+\frac{5\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/4}}{2}+\frac{15\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/4}}{4}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1}-\frac{17\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/4)/x^4,x)
```

```
[Out] (17*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8 - ((17*a
^3*((a*x - 1)/(a*x + 1))^(3/4))/12 + (5*a^3*((a*x - 1)/(a*x + 1))^(7/4))/2
+ (15*a^3*((a*x - 1)/(a*x + 1))^(11/4))/4)/((3*(a*x - 1)^2)/(a*x + 1)^2 + (
a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) - (17*(-1)^(1/4)*a^3*
atanh((-1)^(1/4)*((a*x - 1)/(a*x + 1))^(1/4)))/8
```

3.104 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx$

Optimal. Leaf size=287

$$\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}}+\frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}}$$

```
[Out] 26111/1920*(1-1/a/x)^(1/4)/a^5/(1+1/a/x)^(1/4)+5533/1920*(1-1/a/x)^(1/4)*x/
a^4/(1+1/a/x)^(1/4)-1189/960*(1-1/a/x)^(1/4)*x^2/a^3/(1+1/a/x)^(1/4)+181/24
0*(1-1/a/x)^(1/4)*x^3/a^2/(1+1/a/x)^(1/4)-21/40*(1-1/a/x)^(1/4)*x^4/a/(1+1/
a/x)^(1/4)+1/5*(1-1/a/x)^(1/4)*x^5/(1+1/a/x)^(1/4)+1003/128*arctan((1+1/a/x
)^(1/4)/(1-1/a/x)^(1/4))/a^5-1003/128*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/
4))/a^5
```

Rubi [A]
time = 0.11, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6306, 100, 156, 160, 12, 95, 304, 209, 212}

$$\frac{1003\,\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{\frac{1}{ax}+1}}-\frac{1003\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{128a^5}+\frac{5533x\sqrt[4]{1-\frac{1}{ax}}}{1920a^4\sqrt[4]{\frac{1}{ax}+1}}-\frac{1189x^2\sqrt[4]{1-\frac{1}{ax}}}{960a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{181x^3\sqrt[4]{1-\frac{1}{ax}}}{240a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^5\sqrt[4]{1-\frac{1}{ax}}}{5\sqrt[4]{\frac{1}{ax}+1}}-\frac{21x^4\sqrt[4]{1-\frac{1}{ax}}}{40a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (26111*(1 - 1/(a*x))^(1/4))/(1920*a^5*(1 + 1/(a*x))^(1/4)) + (5533*(1 - 1/(
a*x))^(1/4)*x)/(1920*a^4*(1 + 1/(a*x))^(1/4)) - (1189*(1 - 1/(a*x))^(1/4)*x
^2)/(960*a^3*(1 + 1/(a*x))^(1/4)) + (181*(1 - 1/(a*x))^(1/4)*x^3)/(240*a^2*
(1 + 1/(a*x))^(1/4)) - (21*(1 - 1/(a*x))^(1/4)*x^4)/(40*a*(1 + 1/(a*x))^(1/
4)) + ((1 - 1/(a*x))^(1/4)*x^5)/(5*(1 + 1/(a*x))^(1/4)) + (1003*ArcTan[(1 +
 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5) - (1003*ArcTanh[(1 + 1/(a*x
))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*a^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^4\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^6\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{5}\text{Subst}\left(\int \frac{\frac{21}{2a}-\frac{10x}{a^2}}{x^5\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{20}\text{Subst}\left(\int \frac{\frac{181}{4a^2}-\frac{42x}{a^3}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{60}\text{Subst}\left(\int \frac{\frac{1189}{8a^3} - \cdots}{x^3\left(1-\frac{x}{a}\right)^{3/4}\cdots}\right.$$

$$= -\frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^5}{5\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{120}\text{Su}\cdots$$

$$= \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{21\sqrt[4]{1-\frac{1}{ax}}\,x^4}{40a\sqrt[4]{1+\frac{1}{ax}}} + \cdots$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \cdots$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \cdots$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{1920a^5\sqrt[4]{1+\frac{1}{ax}}} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{1920a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{960a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{240a^2\sqrt[4]{1+\frac{1}{ax}}} - \cdots$$

$$= \frac{26111\sqrt[4]{1-\frac{1}{ax}}}{\cdots} + \frac{5533\sqrt[4]{1-\frac{1}{ax}}\,x}{\cdots} - \frac{1189\sqrt[4]{1-\frac{1}{ax}}\,x^2}{\cdots} + \frac{181\sqrt[4]{1-\frac{1}{ax}}\,x^3}{\cdots} - \cdots$$

Mathematica [A]
time = 5.24, size = 198, normalized size = 0.69

$$\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)}-\frac{32e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^5}-\frac{122e^{-\frac{1}{2}\coth^{-1}(ax)}}{5\left(-1+e^{-2\coth^{-1}(ax)}\right)^4}-\frac{233e^{-\frac{1}{2}\coth^{-1}(ax)}}{6\left(-1+e^{-2\coth^{-1}(ax)}\right)^3}-\frac{1661e^{-\frac{1}{2}\coth^{-1}(ax)}}{48\left(-1+e^{-2\coth^{-1}(ax)}\right)^2}-\frac{4117e^{-\frac{1}{2}\coth^{-1}(ax)}}{192\left(-1+e^{-2\coth^{-1}(ax)}\right)}-\frac{1003}{128}\mathrm{ArcTan}\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+\frac{1003}{256}\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-\frac{1003}{256}\log\left(1+e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (8/E^(ArcCoth[a*x]/2) - 32/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))
^5) - 122/(5*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^4) - 233/(6*E^(A
rcCoth[a*x]/2)*(-1 + E^(-2*ArcCoth[a*x]))^3) - 1661/(48*E^(ArcCoth[a*x]/2)*
(-1 + E^(-2*ArcCoth[a*x]))^2) - 4117/(192*E^(ArcCoth[a*x]/2)*(-1 + E^(-2*Ar
cCoth[a*x]))) - (1003*ArcTan[E^(-1/2*ArcCoth[a*x])])/128 + (1003*Log[1 - E^
(-1/2*ArcCoth[a*x])])/256 - (1003*Log[1 + E^(-1/2*ArcCoth[a*x])])/256)/a^5
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^4*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A]
time = 0.47, size = 279, normalized size = 0.97

$$-\frac{1}{3840}\,a\left(\frac{4\left(20585\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{4}}-49120\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}+61130\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-33816\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+7365\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{5\,(ax-1)a^6}{ax+1}-\frac{10\,(ax-1)^2a^6}{(ax+1)^2}+\frac{10\,(ax-1)^3a^6}{(ax+1)^3}-\frac{5\,(ax-1)^4a^6}{(ax+1)^4}+\frac{(ax-1)^5a^6}{(ax+1)^5}-a^6}+\frac{30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^6}-\frac{30720\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/3840*a*(4*(20585*((a*x - 1)/(a*x + 1))^(17/4) - 49120*((a*x - 1)/(a*x +
1))^(13/4) + 61130*((a*x - 1)/(a*x + 1))^(9/4) - 33816*((a*x - 1)/(a*x + 1)
)^(5/4) + 7365*((a*x - 1)/(a*x + 1))^(1/4))/(5*(a*x - 1)*a^6/(a*x + 1) - 10
*(a*x - 1)^2*a^6/(a*x + 1)^2 + 10*(a*x - 1)^3*a^6/(a*x + 1)^3 - 5*(a*x - 1)
^4*a^6/(a*x + 1)^4 + (a*x - 1)^5*a^6/(a*x + 1)^5 - a^6) + 30090*arctan(((a*
x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a
^6 - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^6 - 30720*((a*x - 1)/(a*x
 + 1))^(1/4)/a^6)
```

Fricas [A]
time = 0.47, size = 119, normalized size = 0.41

$$\frac{2\,(384\,a^5x^5-1008\,a^4x^4+1448\,a^3x^3-2378\,a^2x^2+5533\,ax+26111)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-30090\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+15045\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{3840\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3840*(2*(384*a^5*x^5 - 1008*a^4*x^4 + 1448*a^3*x^3 - 2378*a^2*x^2 + 5533*
a*x + 26111)*((a*x - 1)/(a*x + 1))^(1/4) - 30090*arctan(((a*x - 1)/(a*x + 1
))^(1/4)) - 15045*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 15045*log(((a*x -
1)/(a*x + 1))^(1/4) - 1))/a^5
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 254, normalized size = 0.89

$$-\frac{1}{3840}\,a\left(\frac{30090\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6}+\frac{15045\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^6}-\frac{15045\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^6}-\frac{30720\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^6}-\frac{4\left(\frac{33816(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{61130(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+\frac{49120(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^3}-\frac{20585(ax-1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^4}-7365\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^6\left(\frac{ax-1}{ax+1}-1\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/3840*a*(30090*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^6 + 15045*log(((a*x
 - 1)/(a*x + 1))^(1/4) + 1)/a^6 - 15045*log(abs(((a*x - 1)/(a*x + 1))^(1/4)
 - 1))/a^6 - 30720*((a*x - 1)/(a*x + 1))^(1/4)/a^6 - 4*(33816*(a*x - 1)*((a*
x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 61130*(a*x - 1)^2*((a*x - 1)/(a*x + 1))
^(1/4)/(a*x + 1)^2 + 49120*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1
)^3 - 20585*(a*x - 1)^4*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^4 - 7365*((a*
x - 1)/(a*x + 1))^(1/4))/(a^6*((a*x - 1)/(a*x + 1) - 1)^5))
```

Mupad [**B**]
time = 0.08, size = 253, normalized size = 0.88

$$\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^5}+\frac{\frac{491\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{64}-\frac{1409\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{40}+\frac{6113\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{307\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{6}+\frac{4117\left(\frac{a\,x-1}{a\,x+1}\right)^{17/4}}{192}}{a^5+\frac{10\,a^5\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a^5\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a^5\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a^5\,(a\,x-1)^5}{(a\,x+1)^5}-\frac{5\,a^5\,(a\,x-1)}{a\,x+1}}-\frac{1003\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{128\,a^5}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1{}\mathrm{i}\right)\,1003{}\mathrm{i}}{128\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*1003i)/(128*a^5) + (8*((a*x - 1)/(a*x
 + 1))^(1/4))/a^5 + ((491*((a*x - 1)/(a*x + 1))^(1/4))/64 - (1409*((a*x - 1
```

```
)/(a*x + 1))^(5/4))/40 + (6113*((a*x - 1)/(a*x + 1))^(9/4))/96 - (307*((a*x
 - 1)/(a*x + 1))^(13/4))/6 + (4117*((a*x - 1)/(a*x + 1))^(17/4))/192)/(a^5
+ (10*a^5*(a*x - 1)^2)/(a*x + 1)^2 - (10*a^5*(a*x - 1)^3)/(a*x + 1)^3 + (5*
a^5*(a*x - 1)^4)/(a*x + 1)^4 - (a^5*(a*x - 1)^5)/(a*x + 1)^5 - (5*a^5*(a*x
- 1))/(a*x + 1)) - (1003*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(128*a^5)
```

3.105 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx$

Optimal. Leaf size=250

$$-\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}}-\frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}}-\frac{475\text{ArcTan}\left(\frac{\sqrt[4]{1\cdots}}{\sqrt[4]{1\cdots}}\right.}{64a^4}$$

```
[Out] -2467/192*(1-1/a/x)^(1/4)/a^4/(1+1/a/x)^(1/4)-521/192*(1-1/a/x)^(1/4)*x/a^3
/(1+1/a/x)^(1/4)+113/96*(1-1/a/x)^(1/4)*x^2/a^2/(1+1/a/x)^(1/4)-17/24*(1-1/
a/x)^(1/4)*x^3/a/(1+1/a/x)^(1/4)+1/4*(1-1/a/x)^(1/4)*x^4/(1+1/a/x)^(1/4)-47
5/64*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^4+475/64*arctanh((1+1/a/x)^(
1/4)/(1-1/a/x)^(1/4))/a^4
```

Rubi [A]
time = 0.10, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6306, 100, 156, 160, 12, 95, 304, 209, 212}

$$-\frac{475\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{\frac{1}{ax}+1}}+\frac{475\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{64a^4}-\frac{521x\sqrt[4]{1-\frac{1}{ax}}}{192a^3\sqrt[4]{\frac{1}{ax}+1}}+\frac{113x^2\sqrt[4]{1-\frac{1}{ax}}}{96a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^4\sqrt[4]{1-\frac{1}{ax}}}{4\sqrt[4]{\frac{1}{ax}+1}}-\frac{17x^3\sqrt[4]{1-\frac{1}{ax}}}{24a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-2467*(1 - 1/(a*x))^(1/4))/(192*a^4*(1 + 1/(a*x))^(1/4)) - (521*(1 - 1/(a*
x))^(1/4)*x)/(192*a^3*(1 + 1/(a*x))^(1/4)) + (113*(1 - 1/(a*x))^(1/4)*x^2)/
(96*a^2*(1 + 1/(a*x))^(1/4)) - (17*(1 - 1/(a*x))^(1/4)*x^3)/(24*a*(1 + 1/(a
*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^4)/(4*(1 + 1/(a*x))^(1/4)) - (475*ArcT
an[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4) + (475*ArcTanh[(1 + 1
/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^3\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^5\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{17}{2a}-\frac{8x}{a^2}}{x^4\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{12}\text{Subst}\left(\int \frac{\frac{113}{4a^2}-\frac{51x}{2a^3}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{24}\text{Subst}\left(\int \frac{\frac{521}{8a^3}-\frac{113x}{2a^4}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1 \cdots\right.}\right.$$

$$= -\frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^4}{4\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{24}\text{Subst} \cdots$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\cdots}}{4\sqrt[4]{1\cdots}}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\cdots}}{4\sqrt[4]{1\cdots}}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\cdots}}{4\sqrt[4]{1\cdots}}$$

$$= -\frac{2467\sqrt[4]{1-\frac{1}{ax}}}{192a^4\sqrt[4]{1+\frac{1}{ax}}} - \frac{521\sqrt[4]{1-\frac{1}{ax}}\,x}{192a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{113\sqrt[4]{1-\frac{1}{ax}}\,x^2}{96a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{17\sqrt[4]{1-\frac{1}{ax}}\,x^3}{24a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\cdots}}{4\sqrt[4]{1\cdots}}$$

Mathematica [A]
time = 5.20, size = 161, normalized size = 0.64

$$\frac{-3072e^{-\frac{1}{2}\coth^{-1}(ax)}+\frac{1536e^{\frac{15}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^4}-\frac{5248e^{\frac{11}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^3}+\frac{7376e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{6292e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+2850\text{ArcTan}\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-1425\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+1425\log\left(1+e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{384a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-3072/E^(ArcCoth[a*x]/2) + (1536*E^((15*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCo
th[a*x]))^4 - (5248*E^((11*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^3 +
(7376*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x]))^2 - (6292*E^((3*Arc
Coth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) + 2850*ArcTan[E^(-1/2*ArcCoth[a*x]
)] - 1425*Log[1 - E^(-1/2*ArcCoth[a*x])] + 1425*Log[1 + E^(-1/2*ArcCoth[a*x
])])/(384*a^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^3*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A]
time = 0.46, size = 244, normalized size = 0.98

$$-\frac{1}{384}\,a\left(\frac{4\left(1573\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{4}}-2875\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+2343\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-657\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{4\,(ax-1)a^5}{ax+1}-\frac{6\,(ax-1)^2a^5}{(ax+1)^2}+\frac{4\,(ax-1)^3a^5}{(ax+1)^3}-\frac{(ax-1)^4a^5}{(ax+1)^4}-a^5}-\frac{2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}-\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}+\frac{1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^5}+\frac{3072\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/384*a*(4*(1573*((a*x - 1)/(a*x + 1))^(13/4) - 2875*((a*x - 1)/(a*x + 1))
^(9/4) + 2343*((a*x - 1)/(a*x + 1))^(5/4) - 657*((a*x - 1)/(a*x + 1))^(1/4)
)/(4*(a*x - 1)*a^5/(a*x + 1) - 6*(a*x - 1)^2*a^5/(a*x + 1)^2 + 4*(a*x - 1)^
3*a^5/(a*x + 1)^3 - (a*x - 1)^4*a^5/(a*x + 1)^4 - a^5) - 2850*arctan(((a*x
- 1)/(a*x + 1))^(1/4))/a^5 - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^5
+ 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^5 + 3072*((a*x - 1)/(a*x + 1)
)^(1/4)/a^5)
```

Fricas [A]
time = 0.39, size = 111, normalized size = 0.44

$$\frac{2\,(48\,a^4x^4-136\,a^3x^3+226\,a^2x^2-521\,ax-2467)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2850\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-1425\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{384\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/384*(2*(48*a^4*x^4 - 136*a^3*x^3 + 226*a^2*x^2 - 521*a*x - 2467)*((a*x -
1)/(a*x + 1))^(1/4) + 2850*arctan(((a*x - 1)/(a*x + 1))^(1/4)) + 1425*log((
(a*x - 1)/(a*x + 1))^(1/4) + 1) - 1425*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
)/a^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 223, normalized size = 0.89

$$\frac{1}{384}a\left(\frac{2850\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5}+\frac{1425\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^5}-\frac{1425\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^5}-\frac{3072\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^5}+\frac{4\left(\frac{2343(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{2875(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+\frac{1573(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^3}-657\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^5\left(\frac{ax-1}{ax+1}-1\right)^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/384*a*(2850*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^5 + 1425*log(((a*x - 1)
/(a*x + 1))^(1/4) + 1)/a^5 - 1425*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))
/a^5 - 3072*((a*x - 1)/(a*x + 1))^(1/4)/a^5 + 4*(2343*(a*x - 1)*((a*x - 1)/
(a*x + 1))^(1/4)/(a*x + 1) - 2875*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(
a*x + 1)^2 + 1573*(a*x - 1)^3*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^3 - 657
*((a*x - 1)/(a*x + 1))^(1/4))/(a^5*((a*x - 1)/(a*x + 1) - 1)^4))
```

Mupad [**B**]
time = 0.08, size = 217, normalized size = 0.87

$$\frac{475\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{64\,a^4}-\frac{\frac{219\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{32}-\frac{781\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{32}+\frac{2875\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{96}-\frac{1573\left(\frac{a\,x-1}{a\,x+1}\right)^{13/4}}{96}}{a^4+\frac{6\,a^4\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{4\,a^4\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{a^4\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{4\,a^4\,(a\,x-1)}{a\,x+1}}-\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^4}-\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}1\mathrm{i}\right)475\mathrm{i}}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (475*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(64*a^4) - (8*((a*x - 1)/(a*x + 1))
^(1/4))/a^4 - ((219*((a*x - 1)/(a*x + 1))^(1/4))/32 - (781*((a*x - 1)/(a*x
+ 1))^(5/4))/32 + (2875*((a*x - 1)/(a*x + 1))^(9/4))/96 - (1573*((a*x - 1)/
(a*x + 1))^(13/4))/96)/(a^4 + (6*a^4*(a*x - 1)^2)/(a*x + 1)^2 - (4*a^4*(a*x
 - 1)^3)/(a*x + 1)^3 + (a^4*(a*x - 1)^4)/(a*x + 1)^4 - (4*a^4*(a*x - 1))/(a
*x + 1)) - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*475i)/(64*a^4)
```

3.106 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=213

$$\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}}+\frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}}+\frac{55\mathrm{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}-\frac{55\tanh^{-1}\left(\right.}{8}$$

```
[Out] 287/24*(1-1/a/x)^(1/4)/a^3/(1+1/a/x)^(1/4)+61/24*(1-1/a/x)^(1/4)*x/a^2/(1+1
/a/x)^(1/4)-13/12*(1-1/a/x)^(1/4)*x^2/a/(1+1/a/x)^(1/4)+1/3*(1-1/a/x)^(1/4)
*x^3/(1+1/a/x)^(1/4)+55/8*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3-55/8*
arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^3
```

Rubi [A]
time = 0.07, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6306, 100, 156, 160, 12, 95, 304, 209, 212}

$$\frac{55\mathrm{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{\frac{1}{ax}+1}}-\frac{55\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{8a^3}+\frac{61x\sqrt[4]{1-\frac{1}{ax}}}{24a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{x^3\sqrt[4]{1-\frac{1}{ax}}}{3\sqrt[4]{\frac{1}{ax}+1}}-\frac{13x^2\sqrt[4]{1-\frac{1}{ax}}}{12a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (287*(1 - 1/(a*x))^(1/4))/(24*a^3*(1 + 1/(a*x))^(1/4)) + (61*(1 - 1/(a*x))^
(1/4)*x)/(24*a^2*(1 + 1/(a*x))^(1/4)) - (13*(1 - 1/(a*x))^(1/4)*x^2)/(12*a*
(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(1/4)*x^3)/(3*(1 + 1/(a*x))^(1/4)) +
(55*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3) - (55*ArcTanh[
(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(8*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^4\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}\text{Subst}\left(\int \frac{\frac{13}{2a}-\frac{6x}{a^2}}{x^3\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{1}{6}\text{Subst}\left(\int \frac{\frac{61}{4a^2}-\frac{13x}{a^3}}{x^2\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{6}\text{Subst}\left(\int \frac{\frac{165}{8a^3}-\frac{61x}{4a^4}}{x\left(1-\frac{x}{a}\right)^{3/4}\left(1+\frac{x}{a}\right.}\right.$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}a\text{Subst}\left(\int\right.$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\text{Subst}\left(\int \right.}{}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\text{Subst}\left(\int \right.}{}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} - \frac{55\text{Subst}\left(\int \right.}{}$$

$$= \frac{287\sqrt[4]{1-\frac{1}{ax}}}{24a^3\sqrt[4]{1+\frac{1}{ax}}} + \frac{61\sqrt[4]{1-\frac{1}{ax}}\,x}{24a^2\sqrt[4]{1+\frac{1}{ax}}} - \frac{13\sqrt[4]{1-\frac{1}{ax}}\,x^2}{12a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\sqrt[4]{1-\frac{1}{ax}}\,x^3}{3\sqrt[4]{1+\frac{1}{ax}}} + \frac{55\tan^{-1}\left(\right.}{8a^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.66, size = 389, normalized size = 1.83

$$-\frac{e^{-\frac{5}{2}\coth^{-1}(ax)}\left(-818741-1530529e^{2\coth^{-1}(ax)}-266035e^{4\coth^{-1}(ax)}+7161e^{6\coth^{-1}(ax)}+818741\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)+824824e^{2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)+248094e^{4\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)-85624e^{6\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)-2387e^{8\coth^{-1}(ax)}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};e^{2\coth^{-1}(ax)}\right)+256e^{4\coth^{-1}(ax)}\left(437+626e^{2\coth^{-1}(ax)}+221e^{4\coth^{-1}(ax)}\right)\,{}_4F_3\left(\frac{3}{4},2,2,2;1,1,\frac{15}{4};e^{2\coth^{-1}(ax)}\right)+2048e^{4\coth^{-1}(ax)}\left(17+30e^{2\coth^{-1}(ax)}+13e^{4\coth^{-1}(ax)}\right)\,{}_5F_4\left(\frac{3}{4},2,2,2,2;1,1,1,\frac{15}{4};e^{2\coth^{-1}(ax)}\right)+4096e^{4\coth^{-1}(ax)}\,{}_6F_5\left(\frac{3}{4},2,2,2,2,2;1,1,1,1,\frac{15}{4};e^{2\coth^{-1}(ax)}\right)+8192e^{6\coth^{-1}(ax)}\,{}_6F_5\left(\frac{3}{4},2,2,2,2,2;1,1,1,1,\frac{15}{4};e^{2\coth^{-1}(ax)}\right)+4096e^{8\coth^{-1}(ax)}\,{}_6F_5\left(\frac{3}{4},2,2,2,2,2;1,1,1,1,\frac{15}{4};e^{2\coth^{-1}(ax)}\right)\right)}{44352a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] -1/44352*(-818741 - 1530529*E^(2*ArcCoth[a*x]) - 266035*E^(4*ArcCoth[a*x])
+ 7161*E^(6*ArcCoth[a*x]) + 818741*Hypergeometric2F1[3/4, 1, 7/4, E^(2*ArcC
oth[a*x])] + 824824*E^(2*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E^(2*
ArcCoth[a*x])] + 248094*E^(4*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4, E
^(2*ArcCoth[a*x])] - 85624*E^(6*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7/4
, E^(2*ArcCoth[a*x])] - 2387*E^(8*ArcCoth[a*x])*Hypergeometric2F1[3/4, 1, 7
/4, E^(2*ArcCoth[a*x])] + 256*E^(4*ArcCoth[a*x])*(437 + 626*E^(2*ArcCoth[a*
x]) + 221*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{3/4, 2, 2, 2}, {1, 1, 15/4
}, E^(2*ArcCoth[a*x])] + 2048*E^(4*ArcCoth[a*x])*(17 + 30*E^(2*ArcCoth[a*x]
) + 13*E^(4*ArcCoth[a*x]))*HypergeometricPFQ[{3/4, 2, 2, 2, 2}, {1, 1, 1, 1
5/4}, E^(2*ArcCoth[a*x])] + 4096*E^(4*ArcCoth[a*x])*HypergeometricPFQ[{3/4,
 2, 2, 2, 2, 2}, {1, 1, 1, 1, 15/4}, E^(2*ArcCoth[a*x])] + 8192*E^(6*ArcCot
h[a*x])*HypergeometricPFQ[{3/4, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 15/4}, E^(2*Ar
cCoth[a*x])] + 4096*E^(8*ArcCoth[a*x])*HypergeometricPFQ[{3/4, 2, 2, 2, 2,
2}, {1, 1, 1, 1, 15/4}, E^(2*ArcCoth[a*x])])/(a^3*E^((5*ArcCoth[a*x])/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^2*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A]
time = 0.47, size = 207, normalized size = 0.97

$$-\frac{1}{48}\,a\left(\frac{4\left(137\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}-174\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+69\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{330\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^4}+\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}-\frac{165\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^4}-\frac{384\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

[Out] -1/48*a*(4*(137*((a*x - 1)/(a*x + 1))^(9/4) - 174*((a*x - 1)/(a*x + 1))^(5/4) + 69*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 - 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^4 - 384*((a*x - 1)/(a*x + 1))^(1/4)/a^4)

Fricas [A]
time = 0.36, size = 103, normalized size = 0.48

$$\frac{2\left(8\,a^{3}x^{3}-26\,a^{2}x^{2}+61\,ax+287\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{48\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")

[Out] 1/48*(2*(8*a^3*x^3 - 26*a^2*x^2 + 61*a*x + 287)*((a*x - 1)/(a*x + 1))^(1/4) - 330*arctan(((a*x - 1)/(a*x + 1))^(1/4)) - 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 165*log(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*((a*x-1)/(a*x+1))**(5/4),x)

[Out] Integral(x**2*((a*x - 1)/(a*x + 1))**(5/4), x)

Giac [A]
time = 0.45, size = 192, normalized size = 0.90

$$-\frac{1}{48}\,a\left(\frac{330\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{4}}+\frac{165\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^{4}}-\frac{165\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^{4}}-\frac{384\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^{4}}-\frac{4\left(\frac{174\,(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-\frac{137\,(ax-1)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^{2}}-69\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^{4}\left(\frac{ax-1}{ax+1}-1\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")

[Out] -1/48*a*(330*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^4 + 165*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 - 165*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^4 - 384*((a*x - 1)/(a*x + 1))^(1/4)/a^4 - 4*(174*(a*x - 1)*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) - 137*(a*x - 1)^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 - 69*((a*x - 1)/(a*x + 1))^(1/4))/(a^4*((a*x - 1)/(a*x + 1) - 1)^3))

Mupad [B]
time = 0.07, size = 181, normalized size = 0.85

$$\frac{\frac{23\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{4}-\frac{29\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}+\frac{137\left(\frac{a\,x-1}{a\,x+1}\right)^{9/4}}{12}}{a^3+\frac{3\,a^3\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a^3\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,a^3\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^3}-\frac{55\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{8\,a^3}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,55\mathrm{i}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] ((23*((a*x - 1)/(a*x + 1))^(1/4))/4 - (29*((a*x - 1)/(a*x + 1))^(5/4))/2 +
(137*((a*x - 1)/(a*x + 1))^(9/4))/12)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1)^
2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) + (atan(((
a*x - 1)/(a*x + 1))^(1/4)*1i)*55i)/(8*a^3) + (8*((a*x - 1)/(a*x + 1))^(1/4)
)/a^3 - (55*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(8*a^3)
```

3.107 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=176

$$-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}$$

```
[Out] -25/2*(1-1/a/x)^(1/4)/a^2/(1+1/a/x)^(1/4)-5/4*(1-1/a/x)^(5/4)*x/a/(1+1/a/x)
^(1/4)+1/2*(1-1/a/x)^(9/4)*x^2/(1+1/a/x)^(1/4)-25/4*arctan((1+1/a/x)^(1/4)/
(1-1/a/x)^(1/4))/a^2+25/4*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6306, 98, 96, 95, 304, 209, 212}

$$-\frac{25\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{\frac{1}{ax}+1}}+\frac{25\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{x^2\left(1-\frac{1}{ax}\right)^{9/4}}{2\sqrt[4]{\frac{1}{ax}+1}}-\frac{5x\left(1-\frac{1}{ax}\right)^{5/4}}{4a\sqrt[4]{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-25*(1 - 1/(a*x))^(1/4))/(2*a^2*(1 + 1/(a*x))^(1/4)) - (5*(1 - 1/(a*x))^(5
/4)*x)/(4*a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(9/4)*x^2)/(2*(1 + 1/(a*x
))^(1/4)) - (25*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2) +
(25*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x\,dx=-\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^3\left(1+\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}+\frac{5\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{4a}$$

$$=-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\text{Subst}\left(\int\frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$=-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\text{Subst}\left(\int\frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,d\right.}{8a^2}$$

$$=-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.}{2a^2}$$

$$=-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}+\frac{25\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\frac{\sqrt[4]{1+\frac{1}{a}}}{\sqrt[4]{1-\frac{1}{a}}}\right.}{4a^2}$$

$$=-\frac{25\sqrt[4]{1-\frac{1}{ax}}}{2a^2\sqrt[4]{1+\frac{1}{ax}}}-\frac{5\left(1-\frac{1}{ax}\right)^{5/4}x}{4a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{9/4}x^2}{2\sqrt[4]{1+\frac{1}{ax}}}-\frac{25\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{4a^2}+\frac{25\,\text{ta}}{}$$

Mathematica [A]

time = 0.19, size = 113, normalized size = 0.64

$$\frac{-64e^{-\frac{1}{2}\coth^{-1}(ax)}+\frac{16e^{\frac{7}{2}\coth^{-1}(ax)}}{\left(-1+e^{2\coth^{-1}(ax)}\right)^2}-\frac{52e^{\frac{3}{2}\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}+50\text{ArcTan}\left(e^{-\frac{1}{2}\coth^{-1}(ax)}\right)-25\log\left(1-e^{-\frac{1}{2}\coth^{-1}(ax)}\right)+25\log\left(1+e^{-\frac{1}{2}\coth^{-1}(ax)}\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (-64/E^(ArcCoth[a*x]/2) + (16*E^((7*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*
x]))^2 - (52*E^((3*ArcCoth[a*x])/2))/(-1 + E^(2*ArcCoth[a*x])) + 50*ArcTan[
E^(-1/2*ArcCoth[a*x])] - 25*Log[1 - E^(-1/2*ArcCoth[a*x])] + 25*Log[1 + E^(
-1/2*ArcCoth[a*x])])/(8*a^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A]
time = 0.46, size = 172, normalized size = 0.98

$$-\frac{1}{8}a\left(\frac{4\left(13\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}-\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^3}+\frac{64\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*(13*((a*x - 1)/(a*x + 1))^(5/4) - 9*((a*x - 1)/(a*x + 1))^(1/4))/
(2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - 50*arctan
(((a*x - 1)/(a*x + 1))^(1/4))/a^3 - 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1)
/a^3 + 25*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^3 + 64*((a*x - 1)/(a*x + 1
))^(1/4)/a^3)
```

Fricas [A]
time = 0.37, size = 95, normalized size = 0.54

$$\frac{2\left(2\,a^2x^2-9\,ax-43\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 9*a*x - 43)*((a*x - 1)/(a*x + 1))^(1/4) + 50*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + 25*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - 25*lo
g(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Integral(x*((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A]
time = 0.44, size = 161, normalized size = 0.91

$$\frac{1}{8}a\left(\frac{50\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3}+\frac{25\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{25\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^3}-\frac{64\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^3}+\frac{4\left(\frac{13(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] 1/8*a*(50*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^3 + 25*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^3 - 25*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^3 - 6
4*((a*x - 1)/(a*x + 1))^(1/4)/a^3 + 4*(13*(a*x - 1)*((a*x - 1)/(a*x + 1))^(
1/4)/(a*x + 1) - 9*((a*x - 1)/(a*x + 1))^(1/4))/(a^3*((a*x - 1)/(a*x + 1) -
 1)^2))
```

Mupad [B]
time = 1.20, size = 145, normalized size = 0.82

$$\frac{25\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{4\,a^2}-\frac{8\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a^2}-\frac{\frac{9\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{2}-\frac{13\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/4}}{2}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)\,25{}\mathrm{i}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (25*atan(((a*x - 1)/(a*x + 1))^(1/4)))/(4*a^2) - ((9*((a*x - 1)/(a*x + 1))^
(1/4))/2 - (13*((a*x - 1)/(a*x + 1))^(5/4))/2)/(a^2 + (a^2*(a*x - 1)^2)/(a*
x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (8*((a*x - 1)/(a*x + 1))^(1/4))/a
^2 - (atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*25i)/(4*a^2)
```

3.108 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=130

$$\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}}+\frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{5\text{ArcTan}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

```
[Out] 10*(1-1/a/x)^(1/4)/a/(1+1/a/x)^(1/4)+(1-1/a/x)^(5/4)*x/(1+1/a/x)^(1/4)+5*ar
ctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))/a-5*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)
^(1/4))/a
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.600$, Rules used = {6305, 96, 95, 304, 209, 212}

$$\frac{5\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}+\frac{x\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{\frac{1}{ax}+1}}-\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^((-5*ArcCoth[a*x])/2),x]
```

```
[Out] (10*(1 - 1/(a*x))^(1/4))/(a*(1 + 1/(a*x))^(1/4)) + ((1 - 1/(a*x))^(5/4)*x)/
(1 + 1/(a*x))^(1/4) + (5*ArcTan[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
 - (5*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/a
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x^2\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{10\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} + \frac{5\text{Subst}\left(\int \frac{1}{1+x^2} \ldots\right.}{}$$

$$= \frac{10\sqrt[4]{1-\frac{1}{ax}}}{a\sqrt[4]{1+\frac{1}{ax}}} + \frac{\left(1-\frac{1}{ax}\right)^{5/4}x}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{5\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a} - \frac{5\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right)}{a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 31, normalized size = 0.24

$$\frac{8e^{-\frac{1}{2}\coth^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{4}, 2; \frac{3}{4}; e^{2\coth^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((-5*ArcCoth[a*x])/2),x]
```

```
[Out] (8*Hypergeometric2F1[-1/4, 2, 3/4, E^(2*ArcCoth[a*x])])/(a*E^(ArcCoth[a*x]/
2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [A]
time = 0.46, size = 132, normalized size = 1.02

$$-\frac{1}{2}\,a\left(\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] -1/2*a*(4*((a*x - 1)/(a*x + 1))^(1/4)/((a*x - 1)*a^2/(a*x + 1) - a^2) + 10*
arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x + 1))^(1/4)
 + 1)/a^2 - 5*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)/a^2 - 16*((a*x - 1)/(a*x
 + 1))^(1/4)/a^2)
```

Fricas [A]
time = 0.35, size = 86, normalized size = 0.66

$$\frac{2\,(ax+9)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*x + 9)*((a*x - 1)/(a*x + 1))^(1/4) - 10*arctan(((a*x - 1)/(a*x +
1))^(1/4)) - 5*log(((a*x - 1)/(a*x + 1))^(1/4) + 1) + 5*log(((a*x - 1)/(a*x
 + 1))^(1/4) - 1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4), x)
```

Giac [A]
time = 0.42, size = 129, normalized size = 0.99

$$-\frac{1}{2}a\left(\frac{10\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a^2}+\frac{5\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}-\frac{5\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a^2}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2}+\frac{4\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a^2\left(\frac{ax-1}{ax+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] -1/2*a*(10*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a^2 + 5*log(((a*x - 1)/(a*x
+ 1))^(1/4) + 1)/a^2 - 5*log(abs(((a*x - 1)/(a*x + 1))^(1/4) - 1))/a^2 - 16
*((a*x - 1)/(a*x + 1))^(1/4)/a^2 + 4*((a*x - 1)/(a*x + 1))^(1/4)/(a^2*((a*x
 - 1)/(a*x + 1) - 1)))
```

Mupad [B]
time = 1.19, size = 103, normalized size = 0.79

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}+\frac{8\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}{a}-\frac{5\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\right)}{a}+\frac{\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}\,1\mathrm{i}\right)\,5\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/4))/(a - (a*(a*x - 1))/(a*x + 1)) + (atan(((a*x
 - 1)/(a*x + 1))^(1/4)*1i)*5i)/a + (8*((a*x - 1)/(a*x + 1))^(1/4))/a - (5*a
tan(((a*x - 1)/(a*x + 1))^(1/4)))/a
```

3.109 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=320

$$-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}-\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)+\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)-2\text{ArcTan}\left(\frac{\sqrt[4]{1+\cdots}}{\sqrt[4]{1-\cdots}}\right.$$

```
[Out] -8*(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-2*arctan((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)
)+2*arctanh((1+1/a/x)^(1/4)/(1-1/a/x)^(1/4))-1/2*ln(1-(1-1/a/x)^(1/4)*2^(1/
2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+1/2*ln(1+(1-1/a
/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+
arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+arctan(1+(1-1/a/
x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$,
Rules used = {6306, 100, 21, 132, 65, 246, 217, 1179, 642, 1176, 631, 210, 95, 304, 209, 212}

$$-\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)-2\text{ArcTan}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)-\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\,\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+2\tanh^{-1}\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x),x]
```

```
[Out] (-8*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4) - Sqrt[2]*ArcTan[1 - (Sqrt[2]*
(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] + Sqrt[2]*ArcTan[1 + (Sqrt[2]*(1
- 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)] - 2*ArcTan[(1 + 1/(a*x))^(1/4)/(1 -
1/(a*x))^(1/4)] + 2*ArcTanh[(1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)] - Log[
1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
+ 1/(a*x))^(1/4)]/Sqrt[2] + Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (
Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{x\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - (4a)\text{Subst}\left(\int \frac{\frac{1}{4a}+\frac{x}{4a^2}}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/4}}{x\left(1-\frac{x}{a}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\cdots}}\cdots\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 4\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\frac{1}{ax}}\right) - 4\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \cdots\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} + 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[4]{1\cdots}}{\sqrt[4]{1\cdots}}\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1-\cdots}{1+\cdots}\cdots\right)$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\cdots}{-1-\sqrt{2}\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{8\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - 2\tan^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[4]{1+\frac{1}{ax}}}{\sqrt[4]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1+\frac{\sqrt{1-\cdots}}{\sqrt{1+\cdots}}\cdots\right)}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 28, normalized size = 0.09

$$-8e^{-\frac{1}{2}\coth^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{8},1;\frac{7}{8};e^{4\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x),x]
[Out] (-8*Hypergeometric2F1[-1/8, 1, 7/8, E^(4*ArcCoth[a*x])])/E^(ArcCoth[a*x]/2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x,x)
```

Maxima [A]
time = 0.48, size = 244, normalized size = 0.76

$$\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)}{a}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="maxima")
[Out] 1/2*a*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*
x + 1)) + 1) - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1))/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*log(
((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(((a*x - 1)/(a*x + 1))^(1/4) - 1)
/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

Fricas [A]
time = 0.35, size = 308, normalized size = 0.96

$$-2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4}-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\frac{1}{2}\sqrt{2}\log\left(4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-\frac{1}{2}\sqrt{2}\log\left(-4\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+4\sqrt{\frac{ax-1}{ax+1}}+4\right)-8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+2\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((
a*x - 1)/(a*x + 1)) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) - 1) - 2*sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqr
t((a*x - 1)/(a*x + 1)) + 4) - sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/
2*sqrt(2)*log(4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sqrt((a*x - 1)/(a*x
 + 1)) + 4) - 1/2*sqrt(2)*log(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + 4*sq
rt((a*x - 1)/(a*x + 1)) + 4) - 8*((a*x - 1)/(a*x + 1))^(1/4) + 2*arctan(((a
*x - 1)/(a*x + 1))^(1/4)) + log(((a*x - 1)/(a*x + 1))^(1/4) + 1) - log(((a*
x - 1)/(a*x + 1))^(1/4) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x, x)
```

Giac [A]
time = 0.44, size = 252, normalized size = 0.79

$$\frac{1}{2}\,a\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)}{a}+\frac{\sqrt{2}\,\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}-\frac{\sqrt{2}\,\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a}+\frac{4\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{a}+\frac{2\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}-\frac{2\,\log\left(\left|\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-1\right|\right)}{a}-\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1
/4)))/a + sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/
(a*x + 1)) + 1)/a - sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt
((a*x - 1)/(a*x + 1)) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/4))/a + 2*
log(((a*x - 1)/(a*x + 1))^(1/4) + 1)/a - 2*log(abs(((a*x - 1)/(a*x + 1))^(1
/4) - 1))/a - 16*((a*x - 1)/(a*x + 1))^(1/4)/a)
```

Mupad [B]
time = 1.14, size = 118, normalized size = 0.37

$$2\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)-8\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(1+1{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(1-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x,x)
```

```
[Out] 2*atan(((a*x - 1)/(a*x + 1))^(1/4)) - atan(((a*x - 1)/(a*x + 1))^(1/4)*1i)*
2i + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 - 1i/2))*(1 + 1i
) + 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/4)*(1/2 + 1i/2))*(1 - 1i)
 - 8*((a*x - 1)/(a*x + 1))^(1/4)
```

3.110 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=299

$$\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}}+5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{5a\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}}-\frac{5a\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{a}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.}{\sqrt{2}}$$

```
[Out] 4*a*(1-1/a/x)^(5/4)/(1+1/a/x)^(1/4)+5*a*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-5/2
*a*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-5/2*a*arctan(
1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+5/4*a*ln(1-(1-1/a/x)^(1/
4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)-5/4*a*l
n(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
)*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 49, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{5a\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{\sqrt{2}}-\frac{5a\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{\sqrt{2}}+\frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{\frac{1}{ax}+1}}+5a\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}+\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}-\frac{5a\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] (4*a*(1 - 1/(a*x))^(5/4))/(1 + 1/(a*x))^(1/4) + 5*a*(1 - 1/(a*x))^(1/4)*(1
+ 1/(a*x))^(3/4) + (5*a*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*
x))^(1/4)])/Sqrt[2] - (5*a*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/
(a*x))^(1/4)])/Sqrt[2] + (5*a*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] -
 (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(2*Sqrt[2]) - (5*a*Log
[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1
 + 1/(a*x))^(1/4)])/(2*Sqrt[2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5}{2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^{3/4}\sqrt[4]{1+\frac{x}{a}}}\,dx, x,\right.$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2-x^4}}\,dx, x, \sqrt[4]{1-\right.$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (10a)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - (5a)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{2}(5a)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x,\right.$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\log\left(1+\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right.}{2\sqrt{2}}$$

$$= \frac{4a\left(1-\frac{1}{ax}\right)^{5/4}}{\sqrt[4]{1+\frac{1}{ax}}} + 5a\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{5a\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{5a\,t}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 31, normalized size = 0.10

$$8ae^{-\frac{1}{2}\coth^{-1}(ax)}\,{}_2F_1\left(-\frac{1}{4},2;\frac{3}{4};-e^{2\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^2),x]
```

```
[Out] (8*a*Hypergeometric2F1[-1/4, 2, 3/4, -E^(2*ArcCoth[a*x])])/E^(ArcCoth[a*x]/
2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^2,x)
```

Maxima [A]
time = 0.47, size = 204, normalized size = 0.68

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-32\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

Fricas [A]
time = 0.36, size = 377, normalized size = 1.26

$$\frac{20\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(-\frac{a^{4}+\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}\sqrt{a^{2}\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{4}}}}{a^{4}}\right)+20\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\arctan\left(\frac{a^{4}-\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\left(a^{4}\right)^{\frac{3}{4}}\sqrt{a^{2}\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{4}}}}{a^{4}}\right)-5\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(25a^{2}\sqrt{\frac{ax-1}{ax+1}}+25\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+25\sqrt{a^{4}}\right)+5\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}x\log\left(25a^{2}\sqrt{\frac{ax-1}{ax+1}}-25\sqrt{2}\left(a^{4}\right)^{\frac{1}{4}}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+25\sqrt{a^{4}}\right)+4\left(9ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(20*sqrt(2)*(a^4)^(1/4)*x*arctan(-(a^4 + sqrt(2)*(a^4)^(3/4)*a*((a*x -
1)/(a*x + 1))^(1/4) - sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)
) + sqrt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) +
20*sqrt(2)*(a^4)^(1/4)*x*arctan((a^4 - sqrt(2)*(a^4)^(3/4)*a*((a*x - 1)/(a*
x + 1))^(1/4) + sqrt(2)*(a^4)^(3/4)*sqrt(a^2*sqrt((a*x - 1)/(a*x + 1)) - sq
rt(2)*(a^4)^(1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^4)))/a^4) - 5*sqrt
(2)*(a^4)^(1/4)*x*log(25*a^2*sqrt((a*x - 1)/(a*x + 1)) + 25*sqrt(2)*(a^4)^(
1/4)*a*((a*x - 1)/(a*x + 1))^(1/4) + 25*sqrt(a^4)) + 5*sqrt(2)*(a^4)^(1/4)*
x*log(25*a^2*sqrt((a*x - 1)/(a*x + 1)) - 25*sqrt(2)*(a^4)^(1/4)*a*((a*x - 1
)/(a*x + 1))^(1/4) + 25*sqrt(a^4)) + 4*(9*a*x + 1)*((a*x - 1)/(a*x + 1))^(1
/4))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**2,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**2, x)
```

Giac [A]
time = 0.41, size = 204, normalized size = 0.68

$$-\frac{1}{4}\left(10\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+10\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+5\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-32\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\frac{8\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/4
))) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/
4))) + 5*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x - 1)/(
a*x + 1)) + 1) - 5*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(
(a*x - 1)/(a*x + 1)) + 1) - 32*((a*x - 1)/(a*x + 1))^(1/4) - 8*((a*x - 1)/(
a*x + 1))^(1/4)/((a*x - 1)/(a*x + 1) + 1))*a
```

Mupad [B]
time = 1.18, size = 106, normalized size = 0.35

$$8\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+5\,{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)+\frac{2\,a\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{\frac{a\,x-1}{a\,x+1}+1}+{\left(-1\right)}^{1/4}\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,5{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^2,x)
```

```
[Out] 8*a*((a*x - 1)/(a*x + 1))^(1/4) + (-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(
a*x + 1))^(1/4))*5i + 5*(-1)^(1/4)*a*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))^
(1/4)*1i) + (2*a*((a*x - 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)
```

3.111 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=351

$$-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}}-\frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}-\frac{25a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{1\cdots}}\right.}{4\sqrt{2}}$$

[Out] -2*a^2*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)-25/4*a^2*(1-1/a/x)^(1/4)*(1+1/a/x)^(3/4)-5/2*a^2*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+25/8*a^2*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+25/8*a^2*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)-25/16*a^2*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)+25/16*a^2*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 79, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{25a^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{4\sqrt{2}}+\frac{25a^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{4\sqrt{2}}-\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{5}{2}a^2\left(\frac{1}{ax}+1\right)^{3/4}\left(1-\frac{1}{ax}\right)^{5/4}-\frac{25}{4}a^2\left(\frac{1}{ax}+1\right)^{3/4}\sqrt[4]{1-\frac{1}{ax}}-\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{25a^2\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]

[Out] (-2*a^2*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) - (25*a^2*(1 - 1/(a*x))^(1/4)*(1 + 1/(a*x))^(3/4))/4 - (5*a^2*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4))/2 - (25*a^2*ArcTan[1 - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) + (25*a^2*ArcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(4*Sqrt[2]) - (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) + (25*a^2*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - (5a)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{4}(25a)\text{Subst}\left(\int \frac{\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x\right.$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{1}{8}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{4}$$

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{8}$$

2

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - -$$

2

$$= -\frac{2a^2\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{25}{4}a^2\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} - \frac{5}{2}a^2\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} - -$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 94, normalized size = 0.27

$$a^2\left(-\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(16+45e^{2\coth^{-1}(ax)}+25e^{4\coth^{-1}(ax)}\right)}{2\left(1+e^{2\coth^{-1}(ax)}\right)^2}+\frac{25}{16}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^3),x]
```

```
[Out] a^2*(-1/2*(16 + 45*E^(2*ArcCoth[a*x]) + 25*E^(4*ArcCoth[a*x]))/(E^(ArcCoth[
a*x]/2)*(1 + E^(2*ArcCoth[a*x]))^2) + (25*RootSum[1 + #1^4 & , (ArcCoth[a*x
] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ])/16)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^3,x)
```

Maxima [A]
time = 0.47, size = 247, normalized size = 0.70

$$\frac{1}{16}\left(50\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+25\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-25\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-128\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\frac{8\left(13\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+9\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{2(ax-1)}{ax+1}+\frac{(ax-1)^2}{(ax+1)^2}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*a*((a*x - 1)/(a*x + 1))^(5/4) + 9*a*((a*x - 1)/(a*x + 1))^(1/4))/(2*(a
*x - 1)/(a*x + 1) + (a*x - 1)^2/(a*x + 1)^2 + 1))*a
```

Fricas [A]
time = 0.37, size = 405, normalized size = 1.15

$$-\frac{100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(-\frac{a^8+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)+100\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\arctan\left(\frac{a^8-\sqrt{2}\left(a^8\right)^{\frac{3}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\left(a^8\right)^{\frac{3}{4}}\sqrt{a^4\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^8}}}{a^8}\right)-25\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(625\,a^4\sqrt{\frac{ax-1}{ax+1}}+625\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+625\sqrt{a^8}\right)+25\sqrt{2}\left(a^8\right)^{\frac{1}{4}}x^2\log\left(625\,a^4\sqrt{\frac{ax-1}{ax+1}}-625\sqrt{2}\left(a^8\right)^{\frac{1}{4}}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+625\sqrt{a^8}\right)+4\left(43\,a^2x^2+9\,ax-2\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(100*sqrt(2)*(a^8)^(1/4)*x^2*arctan(-(a^8 + sqrt(2)*(a^8)^(3/4)*a^2*(
(a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(a
*x + 1)) + sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8))
)/a^8) + 100*sqrt(2)*(a^8)^(1/4)*x^2*arctan((a^8 - sqrt(2)*(a^8)^(3/4)*a^2*
((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^8)^(3/4)*sqrt(a^4*sqrt((a*x - 1)/(
a*x + 1)) - sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a^8)
))/a^8) - 25*sqrt(2)*(a^8)^(1/4)*x^2*log(625*a^4*sqrt((a*x - 1)/(a*x + 1))
+ 625*sqrt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 625*sqrt(a^8))
+ 25*sqrt(2)*(a^8)^(1/4)*x^2*log(625*a^4*sqrt((a*x - 1)/(a*x + 1)) - 625*sq
rt(2)*(a^8)^(1/4)*a^2*((a*x - 1)/(a*x + 1))^(1/4) + 625*sqrt(a^8)) + 4*(43*
a^2*x^2 + 9*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/4))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**3,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**3, x)
```

Giac [A]
time = 0.41, size = 243, normalized size = 0.69

$$\frac{1}{16}\left(50\sqrt{2}\,a\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+50\sqrt{2}\,a\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+25\sqrt{2}\,a\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-25\sqrt{2}\,a\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-128\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\frac{8\left(\frac{13\,(ax-1)a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}+9\,a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(50*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/4))) + 50*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))
^(1/4))) + 25*sqrt(2)*a*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) + sqrt((a*x
 - 1)/(a*x + 1)) + 1) - 25*sqrt(2)*a*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/
4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 128*a*((a*x - 1)/(a*x + 1))^(1/4) - 8
*(13*(a*x - 1)*a*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) + 9*a*((a*x - 1)/(a*
x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B]
time = 0.07, size = 153, normalized size = 0.44

$$-8\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}-\frac{\frac{9\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{2}+\frac{13\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{2}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}-\frac{{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,25{}\mathrm{i}}{4}-\frac{25\,{\left(-1\right)}^{1/4}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^3,x)
```

```
[Out] - 8*a^2*((a*x - 1)/(a*x + 1))^(1/4) - ((9*a^2*((a*x - 1)/(a*x + 1))^(1/4))/
2 + (13*a^2*((a*x - 1)/(a*x + 1))^(5/4))/2)/((a*x - 1)^2/(a*x + 1)^2 + (2*(
a*x - 1))/(a*x + 1) + 1) - ((-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x
+ 1))^(1/4))*25i)/4 - (25*(-1)^(1/4)*a^2*atan((-1)^(1/4)*((a*x - 1)/(a*x +
1))^(1/4)*1i))/4
```

3.112 $\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=385

$$\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}}+\frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right.$$

```
[Out] 2*a^3*(1-1/a/x)^(9/4)/(1+1/a/x)^(1/4)+55/8*a^3*(1-1/a/x)^(1/4)*(1+1/a/x)^(3
/4)+11/4*a^3*(1-1/a/x)^(5/4)*(1+1/a/x)^(3/4)+1/3*a^3*(1-1/a/x)^(9/4)*(1+1/a
/x)^(3/4)-55/16*a^3*arctan(-1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1
/2)-55/16*a^3*arctan(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4))*2^(1/2)+55/
32*a^3*ln(1-(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/2)/(1+1/a/
x)^(1/2))*2^(1/2)-55/32*a^3*ln(1+(1-1/a/x)^(1/4)*2^(1/2)/(1+1/a/x)^(1/4)+(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 91, 81, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{55a^3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}\right)}{8\sqrt{2}}-\frac{55a^3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{8\sqrt{2}}+\frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/4}+\frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{55a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}-\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}-\frac{55a^3\log\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}}+\frac{\sqrt{2}\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+1\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] (2*a^3*(1 - 1/(a*x))^(9/4))/(1 + 1/(a*x))^(1/4) + (55*a^3*(1 - 1/(a*x))^(1/
4)*(1 + 1/(a*x))^(3/4))/8 + (11*a^3*(1 - 1/(a*x))^(5/4)*(1 + 1/(a*x))^(3/4)
)/4 + (a^3*(1 - 1/(a*x))^(9/4)*(1 + 1/(a*x))^(3/4))/3 + (55*a^3*ArcTan[1 -
(Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) - (55*a^3*A
rcTan[1 + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)])/(8*Sqrt[2]) +
 (55*a^3*Log[1 + Sqrt[1 - 1/(a*x)]/Sqrt[1 + 1/(a*x)] - (Sqrt[2]*(1 - 1/(a*x
))^(1/4))/(1 + 1/(a*x))^(1/4)])/(16*Sqrt[2]) - (55*a^3*Log[1 + Sqrt[1 - 1/(
a*x)]/Sqrt[1 + 1/(a*x)] + (Sqrt[2]*(1 - 1/(a*x))^(1/4))/(1 + 1/(a*x))^(1/4)
])/(16*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\frac{5}{2}\coth^{-1}(ax)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} - \left(2a^3\right)\text{Subst}\left(\int \frac{\left(-\frac{5}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{2}\left(11a^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/4}}{\sqrt[4]{1+\frac{x}{a}}}\,dx\right.$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{1}{3}a^3\left(1-\frac{1}{ax}\right)^{9/4}\left(1+\frac{1}{ax}\right)^{3/4}$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} +$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} +$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} +$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} +$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} +$$

$$= \frac{2a^3\left(1-\frac{1}{ax}\right)^{9/4}}{\sqrt[4]{1+\frac{1}{ax}}} + \frac{55}{8}a^3\sqrt[4]{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/4} + \frac{11}{4}a^3\left(1-\frac{1}{ax}\right)^{5/4}\left(1+\frac{1}{ax}\right)^{3/4} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 104, normalized size = 0.27

$$a^3\left(\frac{e^{-\frac{1}{2}\coth^{-1}(ax)}\left(96+425e^{2\coth^{-1}(ax)}+462e^{4\coth^{-1}(ax)}+165e^{6\coth^{-1}(ax)}\right)}{12\left(1+e^{2\coth^{-1}(ax)}\right)^3}-\frac{55}{32}\text{RootSum}\left[1+\#1^4\&,\frac{\coth^{-1}(ax)+2\log\left(e^{-\frac{1}{2}\coth^{-1}(ax)}-\#1\right)}{\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^((5*ArcCoth[a*x])/2)*x^4),x]
```

```
[Out] a^3*((96 + 425*E^(2*ArcCoth[a*x]) + 462*E^(4*ArcCoth[a*x]) + 165*E^(6*ArcCo
th[a*x]))/(12*E^(ArcCoth[a*x]/2)*(1 + E^(2*ArcCoth[a*x]))^3) - (55*RootSum[
1 + #1^4 & , (ArcCoth[a*x] + 2*Log[E^(-1/2*ArcCoth[a*x]) - #1])/#1^3 & ])/3
2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

```
[Out] int(((a*x-1)/(a*x+1))^(5/4)/x^4,x)
```

Maxima [A]
time = 0.47, size = 297, normalized size = 0.77

$$-\frac{1}{96}\left(330\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+165\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-165\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-768\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\frac{8\left(137\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{4}}+174\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}+69\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\frac{3(ax-1)}{ax+1}+\frac{3(ax-1)^2}{(ax+1)^2}+\frac{(ax-1)^3}{(ax+1)^3}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(137*a^2*((a*x - 1)/(a*x + 1))^(9/4) + 174*a^2*((a*x - 1)/(
a*x + 1))^(5/4) + 69*a^2*((a*x - 1)/(a*x + 1))^(1/4))/(3*(a*x - 1)/(a*x + 1
) + 3*(a*x - 1)^2/(a*x + 1)^2 + (a*x - 1)^3/(a*x + 1)^3 + 1))*a
```

Fricas [A]
time = 0.36, size = 413, normalized size = 1.07

$$\frac{660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{a^{12}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}+\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)+660\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\arctan\left(\frac{a^{12}-\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{2}\left(a^{12}\right)^{\frac{3}{4}}\sqrt{a^6\sqrt{\frac{ax-1}{ax+1}}-\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{a^{12}}}}{a^{12}}\right)-165\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\log\left(3025\,a^6\sqrt{\frac{ax-1}{ax+1}}+3025\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3025\sqrt{a^{12}}\right)+165\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}x^3\log\left(3025\,a^6\sqrt{\frac{ax-1}{ax+1}}-3025\sqrt{2}\left(a^{12}\right)^{\frac{1}{4}}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+3025\sqrt{a^{12}}\right)+4\left(287\,a^3x^3+61\,a^2x^2-26\,ax+8\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="fricas")
```

```
[Out] 1/96*(660*sqrt(2)*(a^12)^(1/4)*x^3*arctan(-(a^12 + sqrt(2)*(a^12)^(3/4)*a^3
*((a*x - 1)/(a*x + 1))^(1/4) - sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((a*x - 1)
/(a*x + 1)) + sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(a
^12)))/a^12) + 660*sqrt(2)*(a^12)^(1/4)*x^3*arctan((a^12 - sqrt(2)*(a^12)^(
3/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4) + sqrt(2)*(a^12)^(3/4)*sqrt(a^6*sqrt((
a*x - 1)/(a*x + 1)) - sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
+ sqrt(a^12)))/a^12) - 165*sqrt(2)*(a^12)^(1/4)*x^3*log(3025*a^6*sqrt((a*x
- 1)/(a*x + 1)) + 3025*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
 + 3025*sqrt(a^12)) + 165*sqrt(2)*(a^12)^(1/4)*x^3*log(3025*a^6*sqrt((a*x -
 1)/(a*x + 1)) - 3025*sqrt(2)*(a^12)^(1/4)*a^3*((a*x - 1)/(a*x + 1))^(1/4)
+ 3025*sqrt(a^12)) + 4*(287*a^3*x^3 + 61*a^2*x^2 - 26*a*x + 8)*((a*x - 1)/(
a*x + 1))^(1/4))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(5/4)/x**4,x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(5/4)/x**4, x)
```

Giac [A]
time = 0.44, size = 291, normalized size = 0.76

$$-\frac{1}{96}\left(330\sqrt{2}\,a^2\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+330\sqrt{2}\,a^2\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)\right)+165\sqrt{2}\,a^2\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-165\sqrt{2}\,a^2\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+\sqrt{\frac{ax-1}{ax+1}}+1\right)-768\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}-\frac{8\left(\frac{174\,(ax-1)a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{ax+1}+\frac{137\,(ax-1)^2a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}{(ax+1)^2}+69\,a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(5/4)/x^4,x, algorithm="giac")
```

```
[Out] -1/96*(330*sqrt(2)*a^2*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1)
)^(1/4))) + 330*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*
x + 1))^(1/4))) + 165*sqrt(2)*a^2*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/4) +
 sqrt((a*x - 1)/(a*x + 1)) + 1) - 165*sqrt(2)*a^2*log(-sqrt(2)*((a*x - 1)/(
a*x + 1))^(1/4) + sqrt((a*x - 1)/(a*x + 1)) + 1) - 768*a^2*((a*x - 1)/(a*x
+ 1))^(1/4) - 8*(174*(a*x - 1)*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1) +
137*(a*x - 1)^2*a^2*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1)^2 + 69*a^2*((a*x
- 1)/(a*x + 1))^(1/4))/((a*x - 1)/(a*x + 1) + 1)^3)*a
```

Mupad [B]
time = 0.07, size = 188, normalized size = 0.49

$$\frac{\frac{23\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}}{4}+\frac{29\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}}{2}+\frac{137\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/4}}{12}}{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{3\,\left(a\,x-1\right)}{a\,x+1}+1}+8\,a^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}+\frac{{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\right)\,55{}\mathrm{i}}{8}+\frac{55\,{\left(-1\right)}^{1/4}\,a^3\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/4}\,1{}\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(5/4)/x^4,x)
```

```
[Out] ((23*a^3*((a*x - 1)/(a*x + 1))^(1/4))/4 + (29*a^3*((a*x - 1)/(a*x + 1))^(5/
4))/2 + (137*a^3*((a*x - 1)/(a*x + 1))^(9/4))/12)/((3*(a*x - 1)^2)/(a*x + 1
)^2 + (a*x - 1)^3/(a*x + 1)^3 + (3*(a*x - 1))/(a*x + 1) + 1) + 8*a^3*((a*x
- 1)/(a*x + 1))^(1/4) + ((-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1
))^(1/4))*55i)/8 + (55*(-1)^(1/4)*a^3*atan((-1)^(1/4)*((a*x - 1)/(a*x + 1))
^(1/4)*1i))/8
```

3.113 $\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx$

Optimal. Leaf size=285

$$\frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x+\frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^2+\frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3-\frac{19\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}}{\cdots}\right)}{54\sqrt{3}}$$

```
[Out] 11/27*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+7/18*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*
x^2+1/3*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x^3+19/81*arctanh((1+1/x)^(1/6)/((-1
+x)/x)^(1/6))-19/324*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1/x)^(1/6)/((-1
+x)/x)^(1/6))+19/324*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)^(1/6)/((-1
+x)/x)^(1/6))-19/162*arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2
))*3^(1/2)+19/162*arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*
3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6306, 101, 156, 12, 95, 216, 648, 632, 210, 642, 212}

$$-\frac{19\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{19\text{ArcTan}\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right)}{54\sqrt{3}}+\frac{1}{3}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^3+\frac{7}{18}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{11}{27}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{19}{324}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{19}{81}\tanh^{-1}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] (11*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/27 + (7*(1 + x^(-1))^(1/6)*((-
1 + x)/x)^(5/6)*x^2)/18 + ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x^3)/3 - (
19*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])/(54*Sqr
t[3]) + (19*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]]
)/(54*Sqrt[3]) + (19*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/81 - (
19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/324 + (19*Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (
1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)])/324
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{7}{3}+2x}{\sqrt[6]{1-x}\,x^3(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{22}{9} - }{\sqrt[6]{1-x}\,x^2(1}\right.$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

$$= \frac{11}{27}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{7}{18}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x^2 + \frac{1}{3}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 5.44, size = 340, normalized size = 1.19

$$-\frac{e^{-\frac{11}{3}\coth^{-1}(x)}\left(-296480275-249661425e^{2\coth^{-1}(x)}+115464425e^{4\coth^{-1}(x)}+98482225e^{6\coth^{-1}(x)}-7754850e^{8\coth^{-1}(x)}+296480275\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{2\coth^{-1}(x)}\right)+207307100e^{2\coth^{-1}(x)}\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{2\coth^{-1}(x)}\right)-167885900e^{4\coth^{-1}(x)}\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{2\coth^{-1}(x)}\right)-105382550e^{6\coth^{-1}(x)}\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{2\coth^{-1}(x)}\right)+14826175e^{8\coth^{-1}(x)}\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};e^{2\coth^{-1}(x)}\right)+126e^{8\coth^{-1}(x)}\left(1795+2930e^{2\coth^{-1}(x)}+1207e^{4\coth^{-1}(x)}\right){}_4F_3\left(2,2,2,\frac{13}{6};1,1,\frac{31}{6};e^{2\coth^{-1}(x)}\right)+3024e^{8\coth^{-1}(x)}\left(17+31e^{2\coth^{-1}(x)}+14e^{4\coth^{-1}(x)}\right){}_5F_4\left(2,2,2,2,\frac{13}{6};1,1,1,\frac{31}{6};e^{2\coth^{-1}(x)}\right)+4536e^{8\coth^{-1}(x)}\,{}_6F_5\left(2,2,2,2,2,\frac{13}{6};1,1,1,1,\frac{31}{6};e^{2\coth^{-1}(x)}\right)+9072e^{10\coth^{-1}(x)}\,{}_6F_5\left(2,2,2,2,2,\frac{13}{6};1,1,1,1,\frac{31}{6};e^{2\coth^{-1}(x)}\right)+4536e^{12\coth^{-1}(x)}\,{}_6F_5\left(2,2,2,2,2,\frac{13}{6};1,1,1,1,\frac{31}{6};e^{2\coth^{-1}(x)}\right)\right)}{389025}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[x]/3)*x^2,x]
```

```
[Out] -1/389025*(-296480275 - 249661425*E^(2*ArcCoth[x]) + 115464425*E^(4*ArcCoth
[x]) + 98482225*E^(6*ArcCoth[x]) - 7754850*E^(8*ArcCoth[x]) + 296480275*Hyp
ergeometric2F1[1/6, 1, 7/6, E^(2*ArcCoth[x])] + 207307100*E^(2*ArcCoth[x])*
Hypergeometric2F1[1/6, 1, 7/6, E^(2*ArcCoth[x])] - 167885900*E^(4*ArcCoth[x
])*Hypergeometric2F1[1/6, 1, 7/6, E^(2*ArcCoth[x])] - 105382550*E^(6*ArcCot
h[x])*Hypergeometric2F1[1/6, 1, 7/6, E^(2*ArcCoth[x])] + 14826175*E^(8*ArcC
oth[x])*Hypergeometric2F1[1/6, 1, 7/6, E^(2*ArcCoth[x])] + 126*E^(8*ArcCoth
[x])*(1795 + 2930*E^(2*ArcCoth[x]) + 1207*E^(4*ArcCoth[x]))*HypergeometricP
FQ[{2, 2, 2, 13/6}, {1, 1, 31/6}, E^(2*ArcCoth[x])] + 3024*E^(8*ArcCoth[x])
*(17 + 31*E^(2*ArcCoth[x]) + 14*E^(4*ArcCoth[x]))*HypergeometricPFQ[{2, 2,
2, 2, 13/6}, {1, 1, 1, 31/6}, E^(2*ArcCoth[x])] + 4536*E^(8*ArcCoth[x])*Hyp
ergeometricPFQ[{2, 2, 2, 2, 2, 13/6}, {1, 1, 1, 1, 31/6}, E^(2*ArcCoth[x])]
 + 9072*E^(10*ArcCoth[x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 13/6}, {1, 1, 1
, 1, 31/6}, E^(2*ArcCoth[x])] + 4536*E^(12*ArcCoth[x])*HypergeometricPFQ[{2
, 2, 2, 2, 2, 13/6}, {1, 1, 1, 1, 31/6}, E^(2*ArcCoth[x])])/E^((11*ArcCoth[
x])/3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.75, size = 705, normalized size = 2.47

method	result
trager	$\frac{(1+x)(18x^2+21x+22)\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}}{54}+\frac{19\ln\left(3\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}x-9\,\mathrm{RootOf}\left(9\_Z^2+3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+3\left(-\frac{1-x}{1+x}\right)^{\frac{5}{6}}-9\,\mathrm{RootOf}\left(9\_\right.\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(1+x)*(18*x^2+21*x+22)*(-(1-x)/(1+x))^(5/6)+19/162*ln(3*(-(1-x)/(1+x))
^(5/6)*x-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5
/6)-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x-1
8*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)-18*Ro
otOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/2)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(
1+x))^(1/3)*x-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x)
)^(1/3)*x-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(
```

```
1/3)-9*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-
3*(-(1-x)/(1+x))^(1/6)-3*RootOf(9*_Z^2+3*_Z+1)-2)+19/54*RootOf(9*_Z^2+3*_Z+
1)*ln(3*(-(1-x)/(1+x))^(5/6)*x-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3
)*x+3*(-(1-x)/(1+x))^(5/6)-18*RootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*
(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(2/3)-6*(-(1-x)/(1+x))^(1/2)*x+18*R
ootOf(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-6*(-(1-x)/(1+x))^(1/2)+18*RootO
f(9*_Z^2+3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/(1
+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^(1/6)-6*RootOf(9*_Z^2+
3*_Z+1)-1)
```

Maxima [A]
time = 0.47, size = 220, normalized size = 0.77

$$-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}-8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="maxima")
```

```
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/27*(19*((
x - 1)/(x + 1))^(17/6) - 8*((x - 1)/(x + 1))^(11/6) + 61*((x - 1)/(x + 1))^
(5/6))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1
) + 19/324*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/
324*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log
(((x - 1)/(x + 1))^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Fricas [A]
time = 0.34, size = 173, normalized size = 0.61

$$\frac{1}{54}\left(18x^3+39x^2+43x+22\right)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{19}{162}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)-\frac{19}{162}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="fricas")
```

```
[Out] 1/54*(18*x^3 + 39*x^2 + 43*x + 22)*((x - 1)/(x + 1))^(5/6) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 19/162*sqrt(3)
*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 19/324*log(((x
 - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x - 1)/(
x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x + 1))
^(1/6) + 1) - 19/162*log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**2,x)
```

```
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/6), x)
```

Giac [A]
time = 0.44, size = 215, normalized size = 0.75

$$-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{19}{162}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)+\frac{\frac{8(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-\frac{19(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2}-61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}-1\right)^3}+\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{324}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{19}{162}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{19}{162}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^2,x, algorithm="giac")
```

```
[Out] -19/162*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 19/16
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/27*(8*(x
- 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) - 19*(x - 1)^2*((x - 1)/(x + 1))^(5/6)
/(x + 1)^2 - 61*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1) - 1)^3 + 19/324*l
og(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 19/324*log(((x
- 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 19/162*log(((x - 1)/(x
 + 1))^(1/6) + 1) - 19/162*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

Mupad [B]
time = 0.13, size = 168, normalized size = 0.59

$$-\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)19\mathrm{i}}{81}-\frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27}-\frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27}+\frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}4952198\mathrm{i}}{14348907\left(-\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19\sqrt{3}}{162}-\frac{19}{162}\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}4952198\mathrm{i}}{14348907\left(\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19\sqrt{3}}{162}+\frac{19}{162}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] - (atan(((x - 1)/(x + 1))^(1/6)*1i)*19i)/81 - ((61*((x - 1)/(x + 1))^(5/6))
/27 - (8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/(
(3*(x - 1))/(x + 1) - (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) -
atan((((x - 1)/(x + 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348
907 - 2476099/14348907)))*((19*3^(1/2))/162 - 19i/162) - atan((((x - 1)/(x
+ 1))^(1/6)*4952198i)/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/1434
8907)))*((19*3^(1/2))/162 + 19i/162)
```

3.114 $\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx$

Optimal. Leaf size=258

$$\frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

```
[Out] 1/6*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+1/2*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)*x^2
+1/9*arctanh((1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/36*ln(1+(1+1/x)^(1/3)/((-1+x
)/x)^(1/3)-(1+1/x)^(1/6)/((-1+x)/x)^(1/6))+1/36*ln(1+(1+1/x)^(1/3)/((-1+x)/
x)^(1/3)+(1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/18*arctan(1/3*(1-2*(1+1/x)^(1/6)
/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/18*arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1
+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6306, 98, 96, 95, 216, 648, 632, 210, 642, 212}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right)}{6\sqrt{3}}+\frac{1}{2}\left(\frac{1}{x}+1\right)^{7/6}\left(\frac{x-1}{x}\right)^{5/6}x^2+\frac{1}{6}\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{36}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x)/6 + ((1 + x^(-1))^(7/6)*((-1 + x)
/x)^(5/6)*x^2)/2 - ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/S
qrt[3]]/(6*Sqrt[3]) + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6)
)/Sqrt[3]]/(6*Sqrt[3]) + ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/9 -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/36 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-
1))^(1/6)/((-1 + x)/x)^(1/6)]/36
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,x\ldots}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx\ldots\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,\ldots\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\ldots\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 + \frac{1}{9}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\ldots\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}x^2 - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right)\ldots}{6\sqrt{3}}$$

Mathematica [A]

time = 0.23, size = 167, normalized size = 0.65

$$\frac{1}{36}\left(\frac{72e^{\frac{1}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{84e^{\frac{1}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\,\text{ArcTan}\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)+2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)+2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)-\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[x]/3)*x,x]
```

```
[Out] ((72*E^(ArcCoth[x]/3))/(-1 + E^(2*ArcCoth[x]))^2 + (84*E^(ArcCoth[x]/3))/(-
1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]]
 + 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E^(ArcCot
h[x]/3)] + 2*Log[1 + E^(ArcCoth[x]/3)] - Log[1 - E^(ArcCoth[x]/3) + E^((2*A
rcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/36
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.45, size = 1158, normalized size = 4.49

method	result	size
trager	Expression too large to display	1158
risch	Expression too large to display	2770

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)*(4+3*x)*(-(1-x)/(1+x))^(5/6)+1/18*ln(3*(-(1-x)/(1+x))^(5/6)*x+9*R
ootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*RootOf
(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_
Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*
x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+9*
RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)+9*RootO
f(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1
+x))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)-1/6*ln(3*(-(1-x)/(1+x))^(5/6)*x+9*Roo
tOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^
2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+3*(-(1-x)/(1+x))^(2/3)+18*RootOf(9*_Z^2-3*
_Z+1)*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+
18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+9*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x))^(1/3)+9*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/(1+x))^(1/6)*x-3*(-(1-x)/(1+x
))^(1/6)+3*RootOf(9*_Z^2-3*_Z+1)-2)*RootOf(9*_Z^2-3*_Z+1)+1/6*RootOf(9*_Z^2
-3*_Z+1)*ln(3*(-(1-x)/(1+x))^(5/6)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))
^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3
)+6*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)*x+
6*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/2)+6*(-(1
-x)/(1+x))^(1/2)*x-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x+6*(-(1-x
)/(1+x))^(1/2)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x
))^(1/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+x))^
(1/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)-3*RootOf(9*_Z^2-3*_Z+1)-
1)
```

Maxima [A]
time = 0.47, size = 194, normalized size = 0.75

$$-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2\,(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) + 1/3*(((x - 1)/(
x + 1))^(11/6) - 7*((x - 1)/(x + 1))^(5/6))/(2*(x - 1)/(x + 1) - (x - 1)^2/
(x + 1)^2 - 1) + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6)
 + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1
/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(((x - 1)/(x + 1))^(1/6) - 1
)
```

Fricas [A]
time = 0.34, size = 168, normalized size = 0.65

$$\frac{1}{6}\,(3\,x^2+7\,x+4)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\,\sqrt{3}\right)-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\,\sqrt{3}\right)+\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 7*x + 4)*((x - 1)/(x + 1))^(5/6) - 1/18*sqrt(3)*arctan(2/3*sqr
t(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - 1/18*sqrt(3)*arctan(2/3*sqrt(
3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) + 1/36*log(((x - 1)/(x + 1))^(1/3
) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x -
 1)/(x + 1))^(1/6) + 1) + 1/18*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(
((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x,x)
```

```
[Out] Integral(x/((x - 1)/(x + 1))**(1/6), x)
```

Giac [A]
time = 0.43, size = 191, normalized size = 0.74

$$-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{18}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}-7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}-1\right)^2}+\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{36}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{1}{18}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{18}\,\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x,x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/18*sq
rt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/3*((x - 1)*((
x - 1)/(x + 1))^(5/6)/(x + 1) - 7*((x - 1)/(x + 1))^(5/6))/((x - 1)/(x + 1)
 - 1)^2 + 1/36*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) -
 1/36*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + 1/18*log
(((x - 1)/(x + 1))^(1/6) + 1) - 1/18*log(abs(((x - 1)/(x + 1))^(1/6) - 1))
```

Mupad [B]
time = 1.22, size = 142, normalized size = 0.55

$$\frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3}-\frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2\,(x-1)}{x+1}+1}-\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)1\mathrm{i}}{9}-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(-\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{\sqrt{3}}{18}-\frac{1}{18}\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}2\mathrm{i}}{243\left(\frac{1}{243}+\frac{\sqrt{3}\,1\mathrm{i}}{243}\right)}\right)\left(\frac{\sqrt{3}}{18}+\frac{1}{18}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] ((7*((x - 1)/(x + 1))^(5/6))/3 - ((x - 1)/(x + 1))^(11/6)/3)/((x - 1)^2/(x
+ 1)^2 - (2*(x - 1))/(x + 1) + 1) - (atan(((x - 1)/(x + 1))^(1/6)*1i)*1i)/9
 - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243 - 1/243)))*(3^(
1/2)/18 - 1i/18) - atan((((x - 1)/(x + 1))^(1/6)*2i)/(243*((3^(1/2)*1i)/243
 + 1/243)))*(3^(1/2)/18 + 1i/18)
```

3.115 $\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx$

Optimal. Leaf size=223

$$\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}x-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

```
[Out] (1+1/x)^(1/6)*((-1+x)/x)^(5/6)*x+2/3*arctanh((1+1/x)^(1/6)/((-1+x)/x)^(1/6)
)-1/6*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1/x)^(1/6)/((-1+x)/x)^(1/6))+1
/6*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/3*
arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)+1/3*arctan
(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {6305, 96, 95, 216, 648, 632, 210, 642, 212}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}x-\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{6}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3),x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6)*x - ArcTan[(1 - (2*(1 + x^(-1))^(1/6)
)/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + (2*(1 + x^(-1))^(1/6))
/((-1 + x)/x)^(1/6))/Sqrt[3]]/Sqrt[3] + (2*ArcTanh[(1 + x^(-1))^(1/6)/((-1
+ x)/x)^(1/6)])/3 - Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^
(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)
^(1/3) + (1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)]/6
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{1}{3}\coth^{-1}(x)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x - 2\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1-}{1-x-}\right.$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \right.$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{2}{3}\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{6}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \right.$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} x + \frac{\tan^{-1}\left(\frac{-1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{2}{3}\tan$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 35, normalized size = 0.16

$$2e^{\frac{1}{3}\coth^{-1}(x)}\left(\frac{1}{-1+e^{2\coth^{-1}(x)}} + {}_2F_1\left(\frac{1}{6}, 1; \frac{7}{6}; e^{2\coth^{-1}(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[x]/3),x]
```

```
[Out] 2*E^(ArcCoth[x]/3)*((-1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6, 1
, 7/6, E^(2*ArcCoth[x])])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.77, size = 987, normalized size = 4.43

method	result	size
trager	Expression too large to display	987
risch	Expression too large to display	2764

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)*(-(1-x)/(1+x))^(5/6)-1/3*ln(3*(-(1-x)/(1+x))^(5/6)*x+9*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+9*RootOf(9*_Z^2-3*_Z+
1)*(-(1-x)/(1+x))^(2/3)-6*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(9*_Z^2-3*_Z+1)*(
-(1-x)/(1+x))^(1/2)*x-6*(-(1-x)/(1+x))^(2/3)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-
x)/(1+x))^(1/2)+6*(-(1-x)/(1+x))^(1/2)*x+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(
1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)+18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x)
)^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/
6)*x-3*(-(1-x)/(1+x))^(1/3)-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/6)+3*
RootOf(9*_Z^2-3*_Z+1)+1)+1/3*ln(3*(-(1-x)/(1+x))^(5/6)*x+18*RootOf(9*_Z^2-3
*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+18*RootOf(9*_Z^2-3*_Z+
1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(2/3)-6*(
-(1-x)/(1+x))^(1/2)*x-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-6*(-(
1-x)/(1+x))^(1/2)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(
1+x))^(1/3)*x+3*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x)/(1+
x))^(1/6)+6*RootOf(9*_Z^2-3*_Z+1)-1)-ln(3*(-(1-x)/(1+x))^(5/6)*x+18*RootOf(
9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1-x)/(1+x))^(5/6)+18*RootOf(9*_Z
^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(
2/3)-6*(-(1-x)/(1+x))^(1/2)*x-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)
*x-6*(-(1-x)/(1+x))^(1/2)-18*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)+3*(
-(1-x)/(1+x))^(1/3)*x+3*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x+3*(-(
1-x)/(1+x))^(1/6)+6*RootOf(9*_Z^2-3*_Z+1)-1)*RootOf(9*_Z^2-3*_Z+1)
```

Maxima [A]
time = 0.46, size = 167, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}-1}+\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1
))^(1/6) - 1)
```

Fricas [A]
time = 0.34, size = 160, normalized size = 0.72

$$(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}} - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + \frac{1}{3}\sqrt{3}\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right) + \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="fricas")
```

```
[Out] (x + 1)*((x - 1)/(x + 1))^(5/6) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/6) + 1/3*sqrt(3)) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x +
 1))^(1/6) - 1/3*sqrt(3)) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x +
 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6)
 + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(((x - 1)/(x + 1))^(1
/6) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6),x)
```

```
[Out] Integral(((x - 1)/(x + 1))**(-1/6), x)
```

Giac [A]
time = 0.42, size = 168, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right)\right) - \frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1} - 1} + \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{6}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) + \frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} + 1\right) - \frac{1}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - 1/3*sqrt
(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 2*((x - 1)/(x + 1
))^(5/6)/((x - 1)/(x + 1) - 1) + 1/6*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)
/(x + 1))^(1/6) + 1) - 1/6*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^
```

```
(1/6) + 1) + 1/3*log(((x - 1)/(x + 1))^(1/6) + 1) - 1/3*log(abs(((x - 1)/(x
 + 1))^(1/6) - 1))
```

Mupad [B]
time = 0.10, size = 115, normalized size = 0.52

$$-\frac{\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\,1\mathrm{i}\right)\,2\mathrm{i}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}-1}-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}64\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)/(x + 1))^(1/6),x)
```

```
[Out] - (atan(((x - 1)/(x + 1))^(1/6)*1i)*2i)/3 - (2*((x - 1)/(x + 1))^(5/6))/((x
 - 1)/(x + 1) - 1) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i - 32))
*(3^(1/2)/3 - 1i/3) - atan((((x - 1)/(x + 1))^(1/6)*64i)/(3^(1/2)*32i + 32)
)*(3^(1/2)/3 + 1i/3)
```

3.116 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=402

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\,\text{ArcTan}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}}{\sqrt{3}}\right)-\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\text{ArcT}$$

```
[Out] 2*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^
(1/6)-3^(1/2))+arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+2*arctanh((
1+1/x)^(1/6)/((-1+x)/x)^(1/6))-1/2*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)-(1+1
/x)^(1/6)/((-1+x)/x)^(1/6))+1/2*ln(1+(1+1/x)^(1/3)/((-1+x)/x)^(1/3)+(1+1/x)
^(1/6)/((-1+x)/x)^(1/6))-arctan(1/3*(1-2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^
(1/2))*3^(1/2)+arctan(1/3*(1+2*(1+1/x)^(1/6)/((-1+x)/x)^(1/6))*3^(1/2))*3^(
1/2)+1/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/
x)^(1/6))*3^(1/2)-1/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*
3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {6306, 132, 65, 338, 301, 648, 632, 210, 642, 209, 95, 216, 212}

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}}{\sqrt{3}}\right)+\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1}{\sqrt{3}}\right)-\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\text{ArcTan}\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right)+2\text{ArcTan}\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)-\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}-\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt[3]{\frac{1}{x}+1}}{\sqrt[3]{\frac{x-1}{x}}}+\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)+2\tanh^{-1}\left(\frac{\sqrt[6]{\frac{1}{x}+1}}{\sqrt[6]{\frac{x-1}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]])
+ Sqrt[3]*ArcTan[(1 + (2*(1 + x^(-1))^(1/6))/((-1 + x)/x)^(1/6))/Sqrt[3]] -
 ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + ArcTan[Sqrt[
3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)] + 2*ArcTan[((-1 + x)/x)^(1/
6)/(1 + x^(-1))^(1/6)] + 2*ArcTanh[(1 + x^(-1))^(1/6)/((-1 + x)/x)^(1/6)] -
 Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) - (1 + x^(-1))^(1/6)/((-1 +
x)/x)^(1/6)]/2 + Log[1 + (1 + x^(-1))^(1/3)/((-1 + x)/x)^(1/3) + (1 + x^(-1
))^(1/6)/((-1 + x)/x)^(1/6)]/2 + (Sqrt[3]*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1/
6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - (Sqrt[
```

```
3]*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(
1/3)/(1 + x^(-1))^(1/3)])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}\,x}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,x(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= 6\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right) - 6\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right)$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + 2\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + 2$$

$$= 2\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{\frac{-1+x}{x}}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{}{1+}\right.$$

$$= 2\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}\right) - \frac{1}{2}\log\left(1+\frac{\sqrt[3]{1+\frac{1}{x}}}{\sqrt[3]{-\frac{1-x}{x}}} - \frac{\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-}{x}}}\right.$$

$$= -\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + 2\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right.$$

$$= -\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) + \sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{1+\frac{1}{x}}}{\sqrt[6]{-\frac{1-x}{x}}}}{\sqrt{3}}\right) - \tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{}}{\sqrt[6]{}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.03, size = 26, normalized size = 0.06

$$\frac{12}{7}e^{\frac{7}{3}\coth^{-1}(x)}\,{}_2F_1\left(\frac{7}{12},1;\frac{19}{12};e^{4\coth^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[x]/3)/x,x]
```

```
[Out] (12*E^((7*ArcCoth[x])/3)*Hypergeometric2F1[7/12, 1, 19/12, E^(4*ArcCoth[x])
])/7
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.82, size = 2714, normalized size = 6.75

method	result	size
trager	Expression too large to display	2714

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 27*RootOf(81*_Z^4-9*_Z^2+1)^3*ln(-(27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1
+x))^(2/3)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+
x))^(5/6)*x+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(81*
_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-
x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(
1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+27*RootOf(81
*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-x)/(1+x))^(1/2)*x-6*RootOf(81
*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)
/(1+x))^(1/6)*x-18*RootOf(81*_Z^4-9*_Z^2+1)^3*x-2*(-(1-x)/(1+x))^(1/2)-6*Ro
otOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-
(1-x)/(1+x))^(1/6)+RootOf(81*_Z^4-9*_Z^2+1)*x)/x)-9*ln(-27*RootOf(81*_Z^4-9
*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(5/6)*x-27*RootOf(81*_Z^
4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)-54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1
+x))^(1/2)*x-3*(-(1-x)/(1+x))^(5/6)-3*(-(1-x)/(1+x))^(2/3)*x-54*RootOf(81*_
Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/
(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/3)-54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)
/(1+x))^(1/3)-27*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x+3*(-(1-x
)/(1+x))^(1/3)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)+3*(-(1-
x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2+3*(-(
1-x)/(1+x))^(1/6)+2)*RootOf(81*_Z^4-9*_Z^2+1)^2+9*RootOf(81*_Z^4-9*_Z^2+1)^
2*ln(27*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^
(5/6)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)+54*RootOf(81*_Z^
4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x-3*(-(1-x)/(1+x))^(5/6)-6*(-(1-x)/(1+x)
)^(2/3)*x+54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+54*RootOf(81*_
```

```
Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/3)*x-6*(-(1-x)/(1+x))^(2/3)-6*(-(1-x)/(1+
x))^(1/2)*x+54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/3)+27*RootOf(81
*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-6*(-(1-x)/(1+x))^(1/2)-3*(-(1-x)/(
1+x))^(1/3)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-3*(-(1-x)/
(1+x))^(1/3)+9*RootOf(81*_Z^4-9*_Z^2+1)^2+1)-3*RootOf(81*_Z^4-9*_Z^2+1)*ln(
-(27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+27*RootOf(81*_Z^4-9*
_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x+3*RootOf(81*_Z^4-9*_Z
^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(
1/2)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^
(5/6)+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_
Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^
(1/3)-2*(-(1-x)/(1+x))^(1/2)*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1
/3)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-18*RootOf(81*_Z^4
-9*_Z^2+1)^3*x-2*(-(1-x)/(1+x))^(1/2)-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1
+x))^(1/3)-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)+RootOf(81*_Z^4
-9*_Z^2+1)*x)/x)-3*RootOf(81*_Z^4-9*_Z^2+1)*ln(-(-54*RootOf(81*_Z^4-9*_Z^2+
1)^3*(-(1-x)/(1+x))^(2/3)*x-54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2
/3)+(-(1-x)/(1+x))^(5/6)*x+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*
x-18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x+27*RootOf(81*_Z^4-9*
_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)+3*RootOf(81*_Z^4-9*_Z
^2+1)*(-(1-x)/(1+x))^(2/3)-18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/
2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-6*RootOf(81*_Z^4-9*_Z
^2+1)*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1
/6)*x+9*RootOf(81*_Z^4-9*_Z^2+1)^3*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+
x))^(1/3)+9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-(-(1-x)/(1+x))^
(1/6)*x+RootOf(81*_Z^4-9*_Z^2+1)*x-(-(1-x)/(1+x))^(1/6))/x)+ln(-27*RootOf(8
1*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(5/6)*x-27*RootO
f(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(2/3)-54*RootOf(81*_Z^4-9*_Z^2+1)^2*(-
(1-x)/(1+x))^(1/2)*x-3*(-(1-x)/(1+x))^(5/6)-3*(-(1-x)/(1+x))^(2/3)*x-54*Roo
tOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-54*RootOf(81*_Z^4-9*_Z^2+1)^2*
(-(1-x)/(1+x))^(1/3)*x-3*(-(1-x)/(1+x))^(2/3)-54*RootOf(81*_Z^4-9*_Z^2+1)^2
*(-(1-x)/(1+x))^(1/3)-27*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x+
3*(-(1-x)/(1+x))^(1/3)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)
+3*(-(1-x)/(1+x))^(1/3)+3*(-(1-x)/(1+x))^(1/6)*x-9*RootOf(81*_Z^4-9*_Z^2+1)
^2+3*(-(1-x)/(1+x))^(1/6)+2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="maxima")
[Out] integrate(1/(x*((x - 1)/(x + 1))^(1/6)), x)
```

Fricas [A]
time = 0.37, size = 340, normalized size = 0.85

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\frac{1}{3}\sqrt{3}\right)-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)+\frac{1}{2}\sqrt{3}\log\left(-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)-2\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-2\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1/3*sqrt(3)) - sqrt(3
)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/6) - 1/3*sqrt(3)) - 1/2*sqrt(3)*l
og(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) +
1/2*sqrt(3)*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^
(1/3) + 16) - 2*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3)*((x - 1)/(x + 1))^(1/
6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/(x + 1))^(1/6)) - 2*arct
an(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1
/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) +
 1/2*log(((x - 1)/(x + 1))^(1/3) + ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log((
(x - 1)/(x + 1))^(1/3) - ((x - 1)/(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1
))^(1/6) + 1) - log(((x - 1)/(x + 1))^(1/6) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x,x)
```

```
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A]
time = 0.41, size = 261, normalized size = 0.65

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)\right)-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right)\right)-\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)+\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+1\right)-\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) + 1)) - sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/6) - 1)) - 1/2*sqrt(3)*log(sqrt(3)*(
(x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*sqrt(3)*log(-sq
rt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) + arctan(sqrt(
3) + 2*((x - 1)/(x + 1))^(1/6)) + arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/
```

```
6)) + 2*arctan(((x - 1)/(x + 1))^(1/6)) + 1/2*log(((x - 1)/(x + 1))^(1/3) +
 ((x - 1)/(x + 1))^(1/6) + 1) - 1/2*log(((x - 1)/(x + 1))^(1/3) - ((x - 1)/
(x + 1))^(1/6) + 1) + log(((x - 1)/(x + 1))^(1/6) + 1) - log(abs(((x - 1)/(
x + 1))^(1/6) - 1))
```

Mupad [B]
time = 1.27, size = 167, normalized size = 0.42

$$2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)-\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}1\mathrm{i}\right)2\mathrm{i}-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}1486016741376\mathrm{i}}{-743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(\sqrt{3}-\mathrm{i}\right)-\mathrm{atan}\left(\frac{\left(\frac{x-1}{x+1}\right)^{1/6}1486016741376\mathrm{i}}{743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(\sqrt{3}+1\mathrm{i}\right)-\mathrm{atan}\left(\frac{1486016741376\left(\frac{x-1}{x+1}\right)^{1/6}}{-743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)-\mathrm{atan}\left(\frac{1486016741376\left(\frac{x-1}{x+1}\right)^{1/6}}{743008370688+\sqrt{3}\,743008370688\mathrm{i}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] 2*atan(((x - 1)/(x + 1))^(1/6)) - atan(((x - 1)/(x + 1))^(1/6)*1i)*2i - ata
n((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(1/2)*743008370688i - 7430083
70688))*(3^(1/2) - 1i) - atan((((x - 1)/(x + 1))^(1/6)*1486016741376i)/(3^(
1/2)*743008370688i + 743008370688))*(3^(1/2) + 1i) - atan((1486016741376*((
x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i - 743008370688))*(3^(1/2)*1i
+ 1) - atan((1486016741376*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*743008370688i
+ 743008370688))*(3^(1/2)*1i - 1)
```

3.117 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx$

Optimal. Leaf size=233

$$\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}-\frac{1}{3}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{3}\text{ArcTan}\left(\sqrt{3}+\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{2}{3}\text{ArcTan}\left(\right.$$

```
[Out] (1+1/x)^(1/6)*((-1+x)/x)^(5/6)+2/3*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+1
/3*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)-3^(1/2))+1/3*arctan(2*((-1+x)/x)
^(1/6)/(1+1/x)^(1/6)+3^(1/2))+1/6*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+
x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)-1/6*ln(1+((-1+x)/x)^(1/3)/(1+1/x
)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6306, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$-\frac{1}{3}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{3}\text{ArcTan}\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right)+\frac{2}{3}\text{ArcTan}\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\sqrt[6]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{5/6}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] (1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6) - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6)]/3 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x
^(-1))^(1/6)]/3 + (2*ArcTan[((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/3 + Log
[1 - (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(
1 + x^(-1))^(1/3)]/(2*Sqrt[3]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 +
x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(2*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,(1+x)^{5/6}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x, \sqrt[6]{\frac{-1+x}{x}}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + 2\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{2}{3}\text{Subst}\left(\int \frac{-\frac{1}{2}+ \cdots}{1-\sqrt{3} \cdots}\right.$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx, x, \cdots\right.$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{2}{3}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{\log\left(1-\frac{\sqrt{3}\,\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}+\frac{\sqrt[3]{-\frac{1-\cdots}{x}}}{\sqrt[3]{1+\cdots}}\right.}{2\sqrt{3}\cdots}$$

$$= \sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{3}\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{3}\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{-\frac{1-\cdots}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 39, normalized size = 0.17

$$-2e^{\frac{1}{3}\coth^{-1}(x)}\left(-\frac{1}{1+e^{2\coth^{-1}(x)}}+{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-e^{2\coth^{-1}(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^2,x]
```

```
[Out] -2*E^(ArcCoth[x]/3)*(-(1 + E^(2*ArcCoth[x]))^(-1) + Hypergeometric2F1[1/6,
1, 7/6, -E^(2*ArcCoth[x])])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.95, size = 1487, normalized size = 6.38

method	result	size
trager	Expression too large to display	1487
risch	Expression too large to display	3472

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)*(-(1-x)/(1+x))^(5/6)/x-9*RootOf(81*_Z^4-9*_Z^2+1)^3*ln((-27*RootOf(81
*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1
-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/
(1+x))^(2/3)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x-27*Root
Of(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)-3*RootOf
(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-
x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-x
)/(1+x))^(1/2)*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf
(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^3*x
-2*(-(1-x)/(1+x))^(1/2)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*R
ootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-RootOf(81*_Z^4-9*_Z^2+1)*x)/
x)+RootOf(81*_Z^4-9*_Z^2+1)*ln((54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x)
)^(2/3)*x+54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))
^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*RootOf(81*_Z^
4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/
(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x
))^(2/3)-18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z
^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x
))^(1/3)*x+9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-9*RootOf(81*
_Z^4-9*_Z^2+1)^3*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)+9*RootOf
(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-(-(1-x)/(1+x))^(1/6)*x-RootOf(81*
_Z^4-9*_Z^2+1)*x-(-(1-x)/(1+x))^(1/6))/x)+RootOf(81*_Z^4-9*_Z^2+1)*ln((-27*
RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x-27*RootOf(81*_Z^4-9*_Z^2+
1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)
```

```
*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*
x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)
-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1
)^2*(-(1-x)/(1+x))^(1/2)-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)
-2*(-(1-x)/(1+x))^(1/2)*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x
-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x+18*RootOf(81*_Z^4-9*_Z
^2+1)^3*x-2*(-(1-x)/(1+x))^(1/2)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^
(1/3)-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-RootOf(81*_Z^4-9*_Z
^2+1)*x)/x)
```

Maxima [A]
time = 0.49, size = 152, normalized size = 0.65

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{3}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
+ 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arctan(
sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)/(x
+ 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

Fricas [A]
time = 0.36, size = 223, normalized size = 0.96

$$-\frac{\sqrt{3}\,x\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)-\sqrt{3}\,x\log\left(-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)+4\,x\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+4\,x\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-4\,x\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-6\,(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x*log(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1
))^(1/3) + 16) - sqrt(3)*x*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x
 - 1)/(x + 1))^(1/3) + 16) + 4*x*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(3)*((x
- 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/(x + 1)
)^(1/6)) + 4*x*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^(1/6) + (
(x - 1)/(x + 1))^(1/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) - 4*x*arctan(((x -
 1)/(x + 1))^(1/6)) - 6*(x + 1)*((x - 1)/(x + 1))^(5/6))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{2}\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**2,x)
```

```
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A]
time = 0.41, size = 152, normalized size = 0.65

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{3}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{2}{3}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
+ 1) + 1/6*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))
^(1/3) + 1) + 2*((x - 1)/(x + 1))^(5/6)/((x - 1)/(x + 1) + 1) + 1/3*arctan(
sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/3*arctan(-sqrt(3) + 2*((x - 1)/(x
+ 1))^(1/6)) + 2/3*arctan(((x - 1)/(x + 1))^(1/6))
```

Mupad [B]
time = 1.23, size = 109, normalized size = 0.47

$$\frac{2\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/6}}{\frac{x-1}{x+1}+1}-\mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\mathrm{atan}\left(\frac{64\left(\frac{x-1}{x+1}\right)^{1/6}}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] (2*atan(((x - 1)/(x + 1))^(1/6)))/3 + (2*((x - 1)/(x + 1))^(5/6))/((x - 1)/
(x + 1) + 1) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i - 32))*((3^(1
/2)*1i)/3 + 1/3) - atan((64*((x - 1)/(x + 1))^(1/6))/(3^(1/2)*32i + 32))*((
3^(1/2)*1i)/3 - 1/3)
```

3.118 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=260

$$\frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}-\frac{1}{18}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{-1+x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right)+\frac{1}{18}\text{ArcTan}\left(\sqrt{3}\right.$$

[Out] 1/6*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)+1/2*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)+1/9*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1/6))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)-3^(1/2))+1/18*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+1/36*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)-1/36*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)

Rubi [A]
time = 0.28, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6306, 81, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$-\frac{1}{18}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{18}\text{ArcTan}\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right)+\frac{1}{9}\text{ArcTan}\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{2}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{1}{6}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}-\frac{\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{12\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[E^(ArcCoth[x]/3)/x^3,x]

[Out] ((1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/6 + ((1 + x^(-1))^(7/6)*((-1 + x)/x)^(5/6))/2 - ArcTan[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18 + ArcTan[Sqrt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)]/18 + ArcTan[((-1 + x)/x)^(1/6)/(1 + x^(-1))^(1/6)]/9 + Log[1 - (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3]) - Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/(12*Sqrt[3])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{6}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x}\,(1+x)\cdots}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(2-x^6)^{5/6}}\,dx, x\cdots\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}\right.$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + \frac{1}{36\cdots}$$

lo

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} + \frac{1}{9}\tan^{-1}\left(\frac{\sqrt[6]{-\frac{1-x}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\right) + -\cdots$$

$$= \frac{1}{6}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6} - \frac{1}{18}\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[6]{-\frac{1-\cdots}{x}}}{\sqrt[6]{1+\frac{1}{x}}}\cdots\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.20, size = 124, normalized size = 0.48

$$\frac{1}{54}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(1+7e^{2\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^2}-6\text{ArcTan}\left(e^{\frac{1}{3}\coth^{-1}(x)}\right)+\text{RootSum}\left[1-\#1^2+\#1^4\&,\frac{2\coth^{-1}(x)-6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)-\coth^{-1}(x)\#1^2+3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-\#1+2\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^3,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(1 + 7*E^(2*ArcCoth[x])))/(1 + E^(2*ArcCoth[x]))^2 -
6*ArcTan[E^(ArcCoth[x]/3)] + RootSum[1 - #1^2 + #1^4 & , (2*ArcCoth[x] - 6*
Log[E^(ArcCoth[x]/3) - #1] - ArcCoth[x]*#1^2 + 3*Log[E^(ArcCoth[x]/3) - #1]
*#1^2)/(-#1 + 2*#1^3) & ])/54
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 13.77, size = 896, normalized size = 3.45

method	result	size
trager	Expression too large to display	896
risch	Expression too large to display	3478

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)*(4*x+3)/x^2*(-(1-x)/(1+x))^(5/6)+1/18*RootOf(_Z^2+1)*ln((3*(-(1-x
)/(1+x))^(5/6)*x+9*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/
3)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(-3*_Z*RootOf(_Z^2+1)
+9*_Z^2-1)*RootOf(_Z^2+1)*x*(-(1-x)/(1+x))^(1/2)+3*(-(1-x)/(1+x))^(5/6)+9*R
ootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2/3)+3*RootOf(_Z^2+1)*
(-(1-x)/(1+x))^(2/3)+18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1
)*(-(1-x)/(1+x))^(1/2)-18*(-(1-x)/(1+x))^(1/3)*RootOf(-3*_Z*RootOf(_Z^2+1)+
9*_Z^2-1)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootOf(-3*_Z*RootOf(_
Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/6)*x-18*(-(1-x)/(1+x))^(1
/3)*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(
1/3)-9*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*RootOf(_Z^2+1)*(-(1-x)/(1+x))^
(1/6)-3*(-(1-x)/(1+x))^(1/6)*x+3*x*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)-2*
x*RootOf(_Z^2+1)-3*(-(1-x)/(1+x))^(1/6))/x)+1/6*RootOf(-3*_Z*RootOf(_Z^2+1)
+9*_Z^2-1)*ln((3*(-(1-x)/(1+x))^(5/6)*x-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z
^2-1)*(-(1-x)/(1+x))^(2/3)*x+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+3*(-(1
-x)/(1+x))^(5/6)-18*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*(-(1-x)/(1+x))^(2
/3)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(2/3)+6*(-(1-x)/(1+x))^(1/2)*x-18*(-(1-
x)/(1+x))^(1/3)*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)*x+3*RootOf(_Z^2+1)*(-
(1-x)/(1+x))^(1/3)*x+6*(-(1-x)/(1+x))^(1/2)-18*(-(1-x)/(1+x))^(1/3)*RootOf(
-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)+3*RootOf(_Z^2+1)*(-(1-x)/(1+x))^(1/3)+3*(-(1
-x)/(1+x))^(1/6)*x-6*x*RootOf(-3*_Z*RootOf(_Z^2+1)+9*_Z^2-1)+x*RootOf(_Z^2+
1)+3*(-(1-x)/(1+x))^(1/6))/x)
```

Maxima [A]
time = 0.47, size = 178, normalized size = 0.68

$$-\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)}+\frac{1}{18}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{18}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="maxima")
```

```
[Out] -1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 1/3*(((x - 1)/(x + 1))^(11/6) + 7*((x - 1)/(x + 1))^(5/6))/
(2*(x - 1)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) + 1/18*arctan(sqrt(3) + 2*((x
 - 1)/(x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) +
 1/9*arctan(((x - 1)/(x + 1))^(1/6))
```

Fricas [A]
time = 0.37, size = 240, normalized size = 0.92

$$-\frac{\sqrt{3}x^2\log\left(16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)-\sqrt{3}x^2\log\left(-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16\right)+4x^2\arctan\left(\sqrt{3}+\frac{1}{2}\sqrt{-16\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+16\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+16}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+4x^2\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-4x^2\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-6(4x^2+7x+3)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{36x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(sqrt(3)*x^2*log(16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x
+ 1))^(1/3) + 16) - sqrt(3)*x^2*log(-16*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 1
6*((x - 1)/(x + 1))^(1/3) + 16) + 4*x^2*arctan(sqrt(3) + 1/2*sqrt(-16*sqrt(
3)*((x - 1)/(x + 1))^(1/6) + 16*((x - 1)/(x + 1))^(1/3) + 16) - 2*((x - 1)/
(x + 1))^(1/6)) + 4*x^2*arctan(-sqrt(3) + 2*sqrt(sqrt(3)*((x - 1)/(x + 1))^
(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) - 2*((x - 1)/(x + 1))^(1/6)) - 4*x^2*a
rctan(((x - 1)/(x + 1))^(1/6)) - 6*(4*x^2 + 7*x + 3)*((x - 1)/(x + 1))^(5/6
))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**3,x)
```

```
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A]
time = 0.43, size = 175, normalized size = 0.67

$$-\frac{1}{36}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{1}{36}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}+7\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{3\left(\frac{x-1}{x+1}+1\right)^2}+\frac{1}{18}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{18}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{1}{9}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^3,x, algorithm="giac")
```

```
[Out] -1/36*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3)
 + 1) + 1/36*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1
))^(1/3) + 1) + 1/3*((x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 7*((x - 1)/(
x + 1))^(5/6))/((x - 1)/(x + 1) + 1)^2 + 1/18*arctan(sqrt(3) + 2*((x - 1)/(
x + 1))^(1/6)) + 1/18*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 1/9*ar
ctan(((x - 1)/(x + 1))^(1/6))
```

Mupad [B]
time = 0.11, size = 136, normalized size = 0.52

$$\frac{\operatorname{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{9}+\frac{\frac{7\left(\frac{x-1}{x+1}\right)^{5/6}}{3}+\frac{\left(\frac{x-1}{x+1}\right)^{11/6}}{3}}{\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1}-\operatorname{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(-\frac{1}{243}+\frac{\sqrt{3}\,1i}{243}\right)}\right)\left(\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)-\operatorname{atan}\left(\frac{2\left(\frac{x-1}{x+1}\right)^{1/6}}{243\left(\frac{1}{243}+\frac{\sqrt{3}\,1i}{243}\right)}\right)\left(-\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] atan(((x - 1)/(x + 1))^(1/6))/9 + ((7*((x - 1)/(x + 1))^(5/6))/3 + ((x - 1)
/(x + 1))^(11/6)/3)/((2*(x - 1))/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - atan(
(2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 - 1/243)))*((3^(1/2)*1i)
/18 + 1/18) - atan((2*((x - 1)/(x + 1))^(1/6))/(243*((3^(1/2)*1i)/243 + 1/2
43)))*((3^(1/2)*1i)/18 - 1/18)
```

3.119 $\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx$

Optimal. Leaf size=287

$$\frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}+\frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x}-\frac{19}{162}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}\right.$$

```
[Out] 19/54*(1+1/x)^(1/6)*((-1+x)/x)^(5/6)+1/18*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)+1/
3*(1+1/x)^(7/6)*((-1+x)/x)^(5/6)/x+19/81*arctan(((-1+x)/x)^(1/6)/(1+1/x)^(1
/6))+19/162*arctan(2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)-3^(1/2))+19/162*arctan(
2*((-1+x)/x)^(1/6)/(1+1/x)^(1/6)+3^(1/2))+19/324*ln(1+((-1+x)/x)^(1/3)/(1+1
/x)^(1/3)-((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)-19/324*ln(1+((-1+
x)/x)^(1/3)/(1+1/x)^(1/3)+((-1+x)/x)^(1/6)*3^(1/2)/(1+1/x)^(1/6))*3^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6306, 92, 81, 52, 65, 338, 301, 648, 632, 210, 642, 209}

$$-\frac{19}{162}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{19}{162}\text{ArcTan}\left(\frac{2\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+\sqrt{3}\right)+\frac{19}{81}\text{ArcTan}\left(\frac{\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}\right)+\frac{1}{18}\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}+\frac{\left(\frac{x-1}{x}\right)^{5/6}\left(\frac{1}{x}+1\right)^{7/6}}{3x}+\frac{19}{54}\left(\frac{x-1}{x}\right)^{5/6}\sqrt[6]{\frac{1}{x}+1}+\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}-\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}-\frac{19\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+\frac{\sqrt{3}\sqrt[6]{\frac{x-1}{x}}}{\sqrt[6]{\frac{1}{x}+1}}+1\right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)/x^4,x]
```

```
[Out] (19*(1 + x^(-1))^(1/6)*((-1 + x)/x)^(5/6))/54 + ((1 + x^(-1))^(7/6)*((-1 +
x)/x)^(5/6))/18 + ((1 + x^(-1))^(7/6)*((-1 + x)/x)^(5/6))/(3*x) - (19*ArcTa
n[Sqrt[3] - (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[Sq
rt[3] + (2*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6)])/162 + (19*ArcTan[((-1 +
 x)/x)^(1/6)/(1 + x^(-1))^(1/6)])/81 + (19*Log[1 - (Sqrt[3]*((-1 + x)/x)^(1
/6))/(1 + x^(-1))^(1/6) + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt
[3]) - (19*Log[1 + (Sqrt[3]*((-1 + x)/x)^(1/6))/(1 + x^(-1))^(1/6) + ((-1 +
 x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/(108*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
```

```
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{3}\coth^{-1}(x)}}{x^4}\,dx = -\text{Subst}\left(\int \frac{x^2\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{1}{3}\text{Subst}\left(\int \frac{\left(-1-\frac{x}{3}\right)\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{54}\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt[6]{1-x}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{162}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{S}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{27}\text{S}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\text{S}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\text{t}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} + \frac{19}{81}\text{t}$$

$$= \frac{19}{54}\sqrt[6]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{1}{18}\left(1+\frac{1}{x}\right)^{7/6}\left(-\frac{1-x}{x}\right)^{5/6} + \frac{\left(1+\frac{1}{x}\right)^{7/6}\left(\frac{-1+x}{x}\right)^{5/6}}{3x} - \frac{19}{162}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.14, size = 133, normalized size = 0.46

$$\frac{1}{486}\left(\frac{18e^{\frac{1}{3}\coth^{-1}(x)}\left(19+8e^{2\coth^{-1}(x)}+61e^{4\coth^{-1}(x)}\right)}{\left(1+e^{2\coth^{-1}(x)}\right)^3}-114\text{ArcTan}\left(e^{\frac{1}{3}\coth^{-1}(x)}\right)-19\text{RootSum}\left[1-\#1^2+\#1^4\&,\frac{-2\coth^{-1}(x)+6\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)+\coth^{-1}(x)\#1^2-3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-\#1+2\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[x]/3)/x^4,x]
```

```
[Out] ((18*E^(ArcCoth[x]/3)*(19 + 8*E^(2*ArcCoth[x]) + 61*E^(4*ArcCoth[x])))/(1 +
 E^(2*ArcCoth[x]))^3 - 114*ArcTan[E^(ArcCoth[x]/3)] - 19*RootSum[1 - #1^2 +
 #1^4 & , (-2*ArcCoth[x] + 6*Log[E^(ArcCoth[x]/3) - #1] + ArcCoth[x]*#1^2 -
 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-#1 + 2*#1^3) & ])/486
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 15.27, size = 1498, normalized size = 5.22

method	result	size
trager	Expression too large to display	1498
risch	Expression too large to display	2166

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(1+x)*(22*x^2+21*x+18)/x^3*(-(1-x)/(1+x))^(5/6)+19/6*RootOf(81*_Z^4-9*
_Z^2+1)^3*ln((27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+27*RootO
f(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x+3*RootOf(
81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1
-x)/(1+x))^(1/2)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(
1-x)/(1+x))^(5/6)+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf
(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(
1-x)/(1+x))^(1/3)-2*(-(1-x)/(1+x))^(1/2)*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-
x)/(1+x))^(1/3)*x-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-18*Ro
otOf(81*_Z^4-9*_Z^2+1)^3*x-2*(-(1-x)/(1+x))^(1/2)-6*RootOf(81*_Z^4-9*_Z^2+1
)*(-(1-x)/(1+x))^(1/3)-9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)+Ro
otOf(81*_Z^4-9*_Z^2+1)*x)/x)-19/54*RootOf(81*_Z^4-9*_Z^2+1)*ln((27*RootOf(8
1*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)*x+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(
1-x)/(1+x))^(2/3)+(-(1-x)/(1+x))^(5/6)*x+3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)
/(1+x))^(2/3)*x+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x+27*Roo
tOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)+3*RootO
f(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)+18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1
-x)/(1+x))^(1/2)+27*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)-2*(-(1-
x)/(1+x))^(1/2)*x-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)*x-9*RootO
f(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-18*RootOf(81*_Z^4-9*_Z^2+1)^3*
```

```
x-2*(-(1-x)/(1+x))^(1/2)-6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)-9*
RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)+RootOf(81*_Z^4-9*_Z^2+1)*x)
/x)+19/54*RootOf(81*_Z^4-9*_Z^2+1)*ln((54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x
)/(1+x))^(2/3)*x+54*RootOf(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(2/3)+(-(1-x)
/(1+x))^(5/6)*x-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(2/3)*x-18*RootOf
(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)*x-27*RootOf(81*_Z^4-9*_Z^2+1)^3*(
-(1-x)/(1+x))^(1/3)*x+(-(1-x)/(1+x))^(5/6)-3*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-
x)/(1+x))^(2/3)-18*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/2)-27*RootO
f(81*_Z^4-9*_Z^2+1)^3*(-(1-x)/(1+x))^(1/3)+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-
x)/(1+x))^(1/3)*x+9*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)*x-9*Roo
tOf(81*_Z^4-9*_Z^2+1)^3*x+6*RootOf(81*_Z^4-9*_Z^2+1)*(-(1-x)/(1+x))^(1/3)+9
*RootOf(81*_Z^4-9*_Z^2+1)^2*(-(1-x)/(1+x))^(1/6)-(-(1-x)/(1+x))^(1/6)*x-Roo
tOf(81*_Z^4-9*_Z^2+1)*x-(-(1-x)/(1+x))^(1/6))/x)
```

Maxima [A]
time = 0.47, size = 205, normalized size = 0.71

$$-\frac{19}{324}\,\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19}{324}\,\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19\left(\frac{x-1}{x+1}\right)^{\frac{17}{6}}+8\left(\frac{x-1}{x+1}\right)^{\frac{11}{6}}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{3(x-1)}{x+1}+\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}+1\right)}+\frac{19}{162}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{19}{162}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="maxima")
```

```
[Out] -19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/
3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x
 + 1))^(1/3) + 1) + 1/27*(19*((x - 1)/(x + 1))^(17/6) + 8*((x - 1)/(x + 1))
^(11/6) + 61*((x - 1)/(x + 1))^(5/6))/(3*(x - 1)/(x + 1) + 3*(x - 1)^2/(x +
 1)^2 + (x - 1)^3/(x + 1)^3 + 1) + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x +
1))^(1/6)) + 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*ar
ctan(((x - 1)/(x + 1))^(1/6))
```

Fricas [A]
time = 0.37, size = 246, normalized size = 0.86

$$-\frac{19\,\sqrt{3}\,x^3\log\left(5776\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+5776\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+5776\right)-19\,\sqrt{3}\,x^3\log\left(-5776\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+5776\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+5776\right)+76\,x^3\arctan\left(\sqrt{3}+\frac{1}{38}\sqrt{-5776\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+5776\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+5776}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+76\,x^3\arctan\left(-\sqrt{3}+2\sqrt{\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1}-2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-76\,x^3\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)-6\,(22\,x^3+43\,x^2+39\,x+18)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{324\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="fricas")
```

```
[Out] -1/324*(19*sqrt(3)*x^3*log(5776*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 5776*((x
- 1)/(x + 1))^(1/3) + 5776) - 19*sqrt(3)*x^3*log(-5776*sqrt(3)*((x - 1)/(x
+ 1))^(1/6) + 5776*((x - 1)/(x + 1))^(1/3) + 5776) + 76*x^3*arctan(sqrt(3)
+ 1/38*sqrt(-5776*sqrt(3)*((x - 1)/(x + 1))^(1/6) + 5776*((x - 1)/(x + 1))^
(1/3) + 5776) - 2*((x - 1)/(x + 1))^(1/6)) + 76*x^3*arctan(-sqrt(3) + 2*sqr
t(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/3) + 1) - 2*((x -
1)/(x + 1))^(1/6)) - 76*x^3*arctan(((x - 1)/(x + 1))^(1/6)) - 6*(22*x^3 + 4
3*x^2 + 39*x + 18)*((x - 1)/(x + 1))^(5/6))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[6]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)/x**4,x)
```

```
[Out] Integral(1/(x**4*((x - 1)/(x + 1))**(1/6)), x)
```

Giac [A]
time = 0.44, size = 199, normalized size = 0.69

$$-\frac{19}{324}\sqrt{3}\log\left(\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{19}{324}\sqrt{3}\log\left(-\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+\frac{\frac{8(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{x+1}+\frac{19(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{(x+1)^2}+61\left(\frac{x-1}{x+1}\right)^{\frac{5}{6}}}{27\left(\frac{x-1}{x+1}+1\right)^3}+\frac{19}{162}\arctan\left(\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{19}{162}\arctan\left(-\sqrt{3}+2\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)+\frac{19}{81}\arctan\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)/x^4,x, algorithm="giac")
```

```
[Out] -19/324*sqrt(3)*log(sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x + 1))^(1/
3) + 1) + 19/324*sqrt(3)*log(-sqrt(3)*((x - 1)/(x + 1))^(1/6) + ((x - 1)/(x
 + 1))^(1/3) + 1) + 1/27*(8*(x - 1)*((x - 1)/(x + 1))^(5/6)/(x + 1) + 19*(x
 - 1)^2*((x - 1)/(x + 1))^(5/6)/(x + 1)^2 + 61*((x - 1)/(x + 1))^(5/6))/((x
 - 1)/(x + 1) + 1)^3 + 19/162*arctan(sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) +
 19/162*arctan(-sqrt(3) + 2*((x - 1)/(x + 1))^(1/6)) + 19/81*arctan(((x - 1
)/(x + 1))^(1/6))
```

Mupad [B]
time = 1.26, size = 161, normalized size = 0.56

$$\frac{19\,\mathrm{atan}\left(\left(\frac{x-1}{x+1}\right)^{1/6}\right)}{81}+\frac{\frac{61\left(\frac{x-1}{x+1}\right)^{5/6}}{27}+\frac{8\left(\frac{x-1}{x+1}\right)^{11/6}}{27}+\frac{19\left(\frac{x-1}{x+1}\right)^{17/6}}{27}}{\frac{3(x-1)}{x+1}+\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}+1}-\mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(-\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(\frac{19}{162}+\frac{\sqrt{3}\,19\mathrm{i}}{162}\right)-\mathrm{atan}\left(\frac{4952198\left(\frac{x-1}{x+1}\right)^{1/6}}{14348907\left(\frac{2476099}{14348907}+\frac{\sqrt{3}\,2476099\mathrm{i}}{14348907}\right)}\right)\left(-\frac{19}{162}+\frac{\sqrt{3}\,19\mathrm{i}}{162}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((x - 1)/(x + 1))^(1/6)),x)
```

```
[Out] (19*atan(((x - 1)/(x + 1))^(1/6)))/81 + ((61*((x - 1)/(x + 1))^(5/6))/27 +
(8*((x - 1)/(x + 1))^(11/6))/27 + (19*((x - 1)/(x + 1))^(17/6))/27)/((3*(x
- 1))/(x + 1) + (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 + 1) - atan((
4952198*((x - 1)/(x + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 - 2
476099/14348907)))*((3^(1/2)*19i)/162 + 19/162) - atan((4952198*((x - 1)/(x
 + 1))^(1/6))/(14348907*((3^(1/2)*2476099i)/14348907 + 2476099/14348907)))*
((3^(1/2)*19i)/162 - 19/162)
```

3.120 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx$

Optimal. Leaf size=157

$$\frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x+\frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3-\frac{22\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\cdots\right)}{27\sqrt{3}\cdots}$$

```
[Out] 14/27*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x+4/9*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x
^2+1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x^3-11/27*ln((1+1/x)^(1/3)-((-1+x)/x)
^(1/3))-11/81*ln(x)-22/81*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(
1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6306, 101, 156, 12, 93}

$$-\frac{22\text{ArcTan}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{27\sqrt{3}}+\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^3+\frac{4}{9}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{14}{27}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{11}{27}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{11\log(x)}{81}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x^2,x]
```

```
[Out] (14*(1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/27 + (4*(1 + x^(-1))^(1/3)*((-
1 + x)/x)^(2/3)*x^2)/9 + ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x^3)/3 - (2
2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/
(27*Sqrt[3]) - (11*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/27 - (11*L
og[x])/81
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
```

```
a, b, c, d, e, f}, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{8}{3}+2x}{\sqrt[3]{1-x}\,x^3(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{28}{9}-\frac{8}{3}\cdots}{\sqrt[3]{1-x}\,x^2(1-\cdots}\right.$$

$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x\cdots$$

$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x\cdots$$

$$= \frac{14}{27}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{4}{9}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x^2 + \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x\cdots$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 4.90, size = 340, normalized size = 2.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^2,x]
```

```
[Out] -1/49140*(-22750000 - 20915440*E^(2*ArcCoth[x]) + 7026175*E^(4*ArcCoth[x])
+ 7394140*E^(6*ArcCoth[x]) - 433485*E^(8*ArcCoth[x]) + 22750000*Hypergeomet
ric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 15227940*E^(2*ArcCoth[x])*Hypergeom
etric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] - 14083160*E^(4*ArcCoth[x])*Hyperge
ometric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] - 8250060*E^(6*ArcCoth[x])*Hyperg
eometric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 1456000*E^(8*ArcCoth[x])*Hyper
geometric2F1[1/3, 1, 4/3, E^(2*ArcCoth[x])] + 54*E^(8*ArcCoth[x])*(475 + 78
2*E^(2*ArcCoth[x]) + 325*E^(4*ArcCoth[x]))*HypergeometricPFQ[{2, 2, 2, 7/3}
, {1, 1, 16/3}, E^(2*ArcCoth[x])] + 162*E^(8*ArcCoth[x])*(35 + 64*E^(2*ArcC
oth[x]) + 29*E^(4*ArcCoth[x]))*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1,
1, 16/3}, E^(2*ArcCoth[x])] + 486*E^(8*ArcCoth[x])*HypergeometricPFQ[{2, 2,
 2, 2, 2, 7/3}, {1, 1, 1, 1, 16/3}, E^(2*ArcCoth[x])] + 972*E^(10*ArcCoth[x
])*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/3}, {1, 1, 1, 1, 16/3}, E^(2*ArcCoth
```

```
[x])] + 486*E^(12*ArcCoth[x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/3}, {1, 1
, 1, 1, 16/3}, E^(2*ArcCoth[x])])/E^((10*ArcCoth[x])/3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.95, size = 613, normalized size = 3.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*(9*x^2+12*x+14)*(-1+x)/((-1+x)/(1+x))^(1/3)+(22/81*RootOf(_Z^2-_Z+1)*l
n((2*RootOf(_Z^2-_Z+1)^2*x^2+2*RootOf(_Z^2-_Z+1)^2*x-3*RootOf(_Z^2-_Z+1)*(x
^3+x^2-x-1)^(2/3)-3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x-5*RootOf(_Z^2-_
Z+1)*x^2-3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)-4*RootOf(_Z^2-_Z+1)*x+2*x^
2+RootOf(_Z^2-_Z+1)-2)/(1+x))-22/81*ln((2*RootOf(_Z^2-_Z+1)^2*x^2+2*RootOf(
_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+3*RootOf(_Z^2-_Z+1)*
(x^3+x^2-x-1)^(1/3)*x+RootOf(_Z^2-_Z+1)*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-
1)^(1/3)-3*(x^3+x^2-x-1)^(2/3)-3*(x^3+x^2-x-1)^(1/3)*x-x^2-RootOf(_Z^2-_Z+1
)-3*(x^3+x^2-x-1)^(1/3)-2*x-1)/(1+x))*RootOf(_Z^2-_Z+1)+22/81*ln((2*RootOf(
_Z^2-_Z+1)^2*x^2+2*RootOf(_Z^2-_Z+1)^2*x+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^
(2/3)+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x+RootOf(_Z^2-_Z+1)*x^2+3*Roo
tOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)-3*(x^3+x^2-x-1)^(2/3)-3*(x^3+x^2-x-1)^(1
/3)*x-x^2-RootOf(_Z^2-_Z+1)-3*(x^3+x^2-x-1)^(1/3)-2*x-1)/(1+x)))/((-1+x)/(1
+x))^(1/3)*((1+x)^2*(-1+x))^(1/3)/(1+x)
```

Maxima [A]
time = 0.49, size = 149, normalized size = 0.95

$$-\frac{22}{81}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(11\left(\frac{x-1}{x+1}\right)^{\frac{8}{3}}-10\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{22}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="maxima")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/27*(
11*((x - 1)/(x + 1))^(8/3) - 10*((x - 1)/(x + 1))^(5/3) + 35*((x - 1)/(x +
1))^(2/3))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^3
 - 1) + 11/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) -
22/81*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Fricas [A]
time = 0.34, size = 100, normalized size = 0.64

$$\frac{1}{27}\left(9x^3+21x^2+26x+14\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{22}{81}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{22}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^3 + 21*x^2 + 26*x + 14)*((x - 1)/(x + 1))^(2/3) - 22/81*sqrt(3)*a
rctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 11/81*log(((x -
1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(((x - 1)/(x +
1))^(1/3) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**2,x)
```

```
[Out] Integral(x**2/((x - 1)/(x + 1))**(1/3), x)
```

Giac [A]
time = 0.42, size = 144, normalized size = 0.92

$$-\frac{22}{81}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\frac{10\,(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-\frac{11\,(x-1)^2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{(x+1)^2}-35\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{27\left(\frac{x-1}{x+1}-1\right)^3}+\frac{11}{81}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{22}{81}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^2,x, algorithm="giac")
```

```
[Out] -22/81*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/27*(
10*(x - 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 11*(x - 1)^2*((x - 1)/(x + 1))
^(2/3)/(x + 1)^2 - 35*((x - 1)/(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)^3 + 11
/81*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 22/81*log(
abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B]
time = 1.19, size = 171, normalized size = 1.09

$$-\frac{22\,\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-\frac{484}{729}\right)}{81}-\frac{\frac{70\left(\frac{x-1}{x+1}\right)^{2/3}}{27}-\frac{20\left(\frac{x-1}{x+1}\right)^{5/3}}{27}+\frac{22\left(\frac{x-1}{x+1}\right)^{8/3}}{27}}{\frac{3\,(x-1)}{x+1}-\frac{3\,(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}-\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(-\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)^2\right)\left(-\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)+\ln\left(\frac{484\left(\frac{x-1}{x+1}\right)^{1/3}}{729}-9\left(\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)^2\right)\left(\frac{11}{81}+\frac{\sqrt{3}\,11\mathrm{i}}{81}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] log((484*((x - 1)/(x + 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 + 11/81)^2)*((3
^(1/2)*11i)/81 + 11/81) - ((70*((x - 1)/(x + 1))^(2/3))/27 - (20*((x - 1)/(
x + 1))^(5/3))/27 + (22*((x - 1)/(x + 1))^(8/3))/27)/((3*(x - 1))/(x + 1) -
 (3*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1) - log((484*((x - 1)/(x
+ 1))^(1/3))/729 - 9*((3^(1/2)*11i)/81 - 11/81)^2)*((3^(1/2)*11i)/81 - 11/8
1) - (22*log((484*((x - 1)/(x + 1))^(1/3))/729 - 484/729))/81
```

3.121 $\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2-\frac{2\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(\sqrt[3]{\right.$$

```
[Out] 1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x+1/2*(1+1/x)^(4/3)*((-1+x)/x)^(2/3)*x^2
-1/3*ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-1/9*ln(x)-2/9*arctan(1/3*3^(1/2)+2/
3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6306, 98, 96, 93}

$$-\frac{2\text{ArcTan}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\left(\frac{1}{x}+1\right)^{4/3}\left(\frac{x-1}{x}\right)^{2/3}x^2+\frac{1}{3}\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\frac{1}{3}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)*x,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x)/3 + ((1 + x^(-1))^(4/3)*((-1 + x)
/x)^(2/3)*x^2)/2 - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1
 + x^(-1))^(1/3))])/(3*Sqrt[3]) - Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/
3)]/3 - Log[x]/9
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1 \ldots$$

$$= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3}x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}x^2 - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{-\ldots}}{\sqrt{3}\sqrt[3]{1\ldots}}\right)}{3\sqrt{3}} \ldots$$

Mathematica [A]
time = 0.23, size = 165, normalized size = 1.27

$$\frac{1}{9}\left(\frac{18e^{\frac{2}{3}\coth^{-1}(x)}}{\left(-1+e^{2\coth^{-1}(x)}\right)^2}+\frac{24e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\,\text{ArcTan}\left(\frac{-1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2e^{\frac{1}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}\right)-2\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}\right)+\log\left(1-e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{1}{3}\coth^{-1}(x)}+e^{\frac{2}{3}\coth^{-1}(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x,x]
```

```
[Out] ((18*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x]))^2 + (24*E^((2*ArcCoth[x]
)/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(-1 + 2*E^(ArcCoth[x]/3))/
Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*E^(ArcCoth[x]/3))/Sqrt[3]] - 2*Log[1 - E
^(ArcCoth[x]/3)] - 2*Log[1 + E^(ArcCoth[x]/3)] + Log[1 - E^(ArcCoth[x]/3) +
 E^((2*ArcCoth[x])/3)] + Log[1 + E^(ArcCoth[x]/3) + E^((2*ArcCoth[x])/3)])/
9
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.95, size = 403, normalized size = 3.10

method	result
trager	$\frac{(1+x)(5+3x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6} - \frac{2\ln\left(-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x-9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}(\right.}{}$
risch	$\frac{(5+3x)(-1+x)}{6\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}} + \frac{\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+2\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x-3\,\mathrm{RootOf}\left(\_Z^2-\_Z\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)*(5+3*x)*(-(1-x)/(1+x))^(2/3)-2/9*ln(-9*RootOf(9*_Z^2-3*_Z+1)*(-(1
-x)/(1+x))^(2/3)*x-9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+9*RootOf(9*
_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)*x-36*RootOf(9*_Z^2-3*_Z+1)^2*x+9*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(-(1-x)/(1+x))^(1/3)*x+12*RootOf(9*_Z^
2-3*_Z+1)*x-3*(-(1-x)/(1+x))^(1/3)-6*RootOf(9*_Z^2-3*_Z+1)-x+1)+2/3*RootOf(
9*_Z^2-3*_Z+1)*ln(9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9
*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*x+18*RootOf(9*_Z^
2-3*_Z+1)^2*x-3*(-(1-x)/(1+x))^(2/3)+3*(-(1-x)/(1+x))^(1/3)*x-15*RootOf(9*_
Z^2-3*_Z+1)*x+3*(-(1-x)/(1+x))^(1/3)-3*RootOf(9*_Z^2-3*_Z+1)+2*x+2)
```

Maxima [A]
time = 0.47, size = 123, normalized size = 0.95

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + 2/3*(((x
 - 1)/(x + 1))^(5/3) - 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^
(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Fricas [A]
time = 0.34, size = 95, normalized size = 0.73

$$\frac{1}{6}\left(3 x^{2}+8 x+5\right)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{9} \sqrt{3} \arctan\left(\frac{2}{3} \sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3} \sqrt{3}\right)+\frac{1}{9} \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9} \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 + 8*x + 5)*((x - 1)/(x + 1))^(2/3) - 2/9*sqrt(3)*arctan(2/3*sqrt
(3)*((x - 1)/(x + 1))^(1/3) + 1/3*sqrt(3)) + 1/9*log(((x - 1)/(x + 1))^(2/3
) + ((x - 1)/(x + 1))^(1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x,x)
```

```
[Out] Integral(x/((x - 1)/(x + 1))**(1/3), x)
```

Giac [A]
time = 0.42, size = 120, normalized size = 0.92

$$-\frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}-4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}-1\right)^{2}}+\frac{1}{9} \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9} \log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2/3*((x
- 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) - 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/
(x + 1) - 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3)
+ 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B]
time = 0.05, size = 145, normalized size = 1.12

$$\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}-\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}-\frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-\frac{4}{9}\right)}{9}-\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}-9\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] ((8*((x - 1)/(x + 1))^(2/3))/3 - (2*((x - 1)/(x + 1))^(5/3))/3)/((x - 1)^2/
(x + 1)^2 - (2*(x - 1))/(x + 1) + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))/9
 - 4/9))/9 - log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*1i)/9 - 1/9)^2
)*((3^(1/2)*1i)/9 - 1/9) + log((4*((x - 1)/(x + 1))^(1/3))/9 - 9*((3^(1/2)*
1i)/9 + 1/9)^2)*((3^(1/2)*1i)/9 + 1/9)
```

3.122 $\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx$

Optimal. Leaf size=96

$$\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x-\frac{2\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+x}{x}}\right)-\frac{\log(x)}{3}$$

```
[Out] (1+1/x)^(1/3)*((-1+x)/x)^(2/3)*x-ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-1/3*ln(
x)-2/3*arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6305, 96, 93}

$$-\frac{2\text{ArcTan}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}x-\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3),x]
```

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3)*x - (2*ArcTan[1/Sqrt[3] + (2*((-1 + x
)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[(1 + x^(-1))^(1/3)
 - ((-1 + x)/x)^(1/3)] - Log[x]/3
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{2}{3}\coth^{-1}(x)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x^2}\,dx,x,\frac{1}{x}\right)$$

$$= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x-\frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx,x,\frac{1}{x}\right)$$

$$= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}x-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{-\frac{1}{}}\right.$$

Mathematica [A]
time = 0.11, size = 85, normalized size = 0.89

$$\frac{1}{3}\left(\frac{6e^{\frac{2}{3}\coth^{-1}(x)}}{-1+e^{2\coth^{-1}(x)}}+2\sqrt{3}\,\text{ArcTan}\left(\frac{1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)-2\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}\right)+\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*ArcCoth[x])/3),x]
```

```
[Out] ((6*E^((2*ArcCoth[x])/3))/(-1 + E^(2*ArcCoth[x])) + 2*Sqrt[3]*ArcTan[(1 + 2
*E^((2*ArcCoth[x])/3))/Sqrt[3]] - 2*Log[1 - E^((2*ArcCoth[x])/3)] + Log[1 +
 E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)])/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.97, size = 397, normalized size = 4.14

method	result

risch	$\frac{-1+x}{\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2x^2+3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-3\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3+x^2-x\right.}{}\right.}{}\right.}{}$
trager	$(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}+\frac{2\ln\left(9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}-3\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-1+x)/(1+x))^(1/3),x,method=_RETURNVERBOSE)

[Out] (-1+x)/((-1+x)/(1+x))^(1/3)+(-2/3*ln(-(4*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)-3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)*x+4*RootOf(_Z^2-_Z+1)^2*x-4*RootOf(_Z^2-_Z+1)*x^2-3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(1/3)+3*(x^3+x^2-x-1)^(1/3)*x-2*RootOf(_Z^2-_Z+1)*x+x^2+3*(x^3+x^2-x-1)^(1/3)+2*RootOf(_Z^2-_Z+1)-1)/(1+x))+2/3*RootOf(_Z^2-_Z+1)*ln((2*RootOf(_Z^2-_Z+1)^2*x^2+3*RootOf(_Z^2-_Z+1)*(x^3+x^2-x-1)^(2/3)+2*RootOf(_Z^2-_Z+1)^2*x-5*RootOf(_Z^2-_Z+1)*x^2-3*(x^3+x^2-x-1)^(2/3)+3*(x^3+x^2-x-1)^(1/3)*x-6*RootOf(_Z^2-_Z+1)*x+2*x^2+3*(x^3+x^2-x-1)^(1/3)-RootOf(_Z^2-_Z+1)+4*x+2)/(1+x)))/((-1+x)/(1+x))^(1/3)*((1+x)^2*(-1+x))^(1/3)/(1+x)

Maxima [A]
time = 0.47, size = 96, normalized size = 1.00

$$-\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1}+\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="maxima")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x - 1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)

Fricas [A]
time = 0.35, size = 87, normalized size = 0.91

$$(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{2}{3}\,\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+\frac{1}{3}\,\sqrt{3}\right)+\frac{1}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\,\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="fricas")

```
[Out] (x + 1)*((x - 1)/(x + 1))^(2/3) - 2/3*sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(
x + 1))^(1/3) + 1/3*sqrt(3)) + 1/3*log(((x - 1)/(x + 1))^(2/3) + ((x - 1)/(
x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\frac{x-1}{x+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3),x)
```

```
[Out] Integral(((x - 1)/(x + 1))**(-1/3), x)
```

Giac [A]
time = 0.42, size = 97, normalized size = 1.01

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}-1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) - 2*((x -
1)/(x + 1))^(2/3)/((x - 1)/(x + 1) - 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) +
 ((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) - 1))
```

Mupad [B]
time = 0.05, size = 118, normalized size = 1.23

$$-\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-4\right)}{3}-\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)+\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}-9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 + 1/3)^2)*((3^(1/2)*1i)/3
 + 1/3) - log(4*((x - 1)/(x + 1))^(1/3) - 9*((3^(1/2)*1i)/3 - 1/3)^2)*((3^(
1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) - 4))/3 - (2*((x - 1)/
(x + 1))^(2/3))/((x - 1)/(x + 1) - 1)
```

3.123 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx$

Optimal. Leaf size=155

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)-\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)-\frac{3}{2}\log\left(\sqrt[3]{1+\frac{1}{x}}-\sqrt[3]{\frac{-1+}{x}}\right.$$

```
[Out] -3/2*ln((1+1/x)^(1/3)-((-1+x)/x)^(1/3))-3/2*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(
1/3))-1/2*ln(1+1/x)-1/2*ln(x)+arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1
/x)^(1/3)*3^(1/2))*3^(1/2)-arctan(1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^
(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6306, 132, 62, 93}

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)-\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}+\frac{1}{\sqrt{3}}\right)-\frac{3}{2}\log\left(\sqrt[3]{\frac{1}{x}+1}-\sqrt[3]{\frac{x-1}{x}}\right)-\frac{3}{2}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{2}\log\left(\frac{1}{x}+1\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x,x]
```

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(
1/3))]) - Sqrt[3]*ArcTan[1/Sqrt[3] + (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x
^(-1))^(1/3))] - (3*Log[(1 + x^(-1))^(1/3) - ((-1 + x)/x)^(1/3)])/2 - (3*Lo
g[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)])/2 - Log[1 + x^(-1)]/2 - Log[x
]/2
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 93

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, Simp[(-Sqr
t[3])*q*(ArcTan[1/Sqrt[3] + 2*q*((a + b*x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))
]/(d*e - c*f)), x] + (Simp[q*(Log[e + f*x]/(2*(d*e - c*f))), x] - Simp[3*q*
```

```
(Log[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)]/(2*(d*e - c*f))), x])] /; FreeQ[{
a, b, c, d, e, f}, x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x}\,dx &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}\,x}\,dx, x, \frac{1}{x}\right)\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right) - \text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\ &= -\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right) - \frac{3}{2}\log\left(\right.\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 26, normalized size = 0.17

$$\frac{3}{2}e^{\frac{8}{3}\coth^{-1}(x)}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};e^{4\coth^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x,x]
[Out] (3*E^((8*ArcCoth[x])/3)*Hypergeometric2F1[2/3, 1, 5/3, E^(4*ArcCoth[x])])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.88, size = 1038, normalized size = 6.70

method	result	size
trager	Expression too large to display	1038

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*RootOf(9*_Z^2-3*_Z+1)*ln((945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*
x^2+1890*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(2
/3)*x^2+945*(-(1-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2-1224*RootOf(9*_Z
^2-3*_Z+1)^2*x^2+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-336*(-(1-x)
/(1+x))^(2/3)*x-168*(-(1-x)/(1+x))^(1/3)*x^2+3060*RootOf(9*_Z^2-3*_Z+1)^2*x
+1353*RootOf(9*_Z^2-3*_Z+1)*x^2-168*(-(1-x)/(1+x))^(2/3)-945*RootOf(9*_Z^2-
3*_Z+1)*(-(1-x)/(1+x))^(1/3)-1224*RootOf(9*_Z^2-3*_Z+1)^2-1062*RootOf(9*_Z^
2-3*_Z+1)*x-304*x^2+168*(-(1-x)/(1+x))^(1/3)+1353*RootOf(9*_Z^2-3*_Z+1)+32*
x-304)/x)-3*ln(-(945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1890*Ro
otOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-147*(-(1-x)/(1+x))^(2/3)*x^2+945
*(-(1-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1224*RootOf(9*_Z^2-3*_Z+1)^
2*x^2+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-294*(-(1-x)/(1+x))^(2/
3)*x-147*(-(1-x)/(1+x))^(1/3)*x^2-3060*RootOf(9*_Z^2-3*_Z+1)^2*x+537*RootOf
(9*_Z^2-3*_Z+1)*x^2-147*(-(1-x)/(1+x))^(2/3)-945*RootOf(9*_Z^2-3*_Z+1)*(-(1
-x)/(1+x))^(1/3)+1224*RootOf(9*_Z^2-3*_Z+1)^2+978*RootOf(9*_Z^2-3*_Z+1)*x-1
1*x^2+147*(-(1-x)/(1+x))^(1/3)+537*RootOf(9*_Z^2-3*_Z+1)-18*x-11)/x)*RootOf
(9*_Z^2-3*_Z+1)+ln(-(945*(-(1-x)/(1+x))^(2/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+189
0*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x-147*(-(1-x)/(1+x))^(2/3)*x^2
+945*(-(1-x)/(1+x))^(1/3)*RootOf(9*_Z^2-3*_Z+1)*x^2+1224*RootOf(9*_Z^2-3*_Z
+1)^2*x^2+945*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)-294*(-(1-x)/(1+x))
^(2/3)*x-147*(-(1-x)/(1+x))^(1/3)*x^2-3060*RootOf(9*_Z^2-3*_Z+1)^2*x+537*Ro
otOf(9*_Z^2-3*_Z+1)*x^2-147*(-(1-x)/(1+x))^(2/3)-945*RootOf(9*_Z^2-3*_Z+1)*
(-(1-x)/(1+x))^(1/3)+1224*RootOf(9*_Z^2-3*_Z+1)^2+978*RootOf(9*_Z^2-3*_Z+1)
*x-11*x^2+147*(-(1-x)/(1+x))^(1/3)+537*RootOf(9*_Z^2-3*_Z+1)-18*x-11)/x)
```

Maxima [A]
time = 0.48, size = 140, normalized size = 0.90

$$-\sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right)\right) + \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)\right) + \frac{1}{2} \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} + \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) + \frac{1}{2} \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}} - \left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} + 1\right) - \log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) + 1)) + sqrt(3)*arct
an(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 1/2*log(((x - 1)/(x + 1))
^(2/3) + ((x - 1)/(x + 1))^(1/3) + 1) + 1/2*log(((x - 1)/(x + 1))^(2/3) - (
```

```
(x - 1)/(x + 1))^(1/3) + 1) - log(((x - 1)/(x + 1))^(1/3) + 1) - log(((x -
1)/(x + 1))^(1/3) - 1)
```

Fricas [A]
time = 0.38, size = 86, normalized size = 0.55

$$\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-1\right)+\frac{1}{2}\log\left(\frac{(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+x+1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="fricas")
[Out] sqrt(3)*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(2/3) + 1/3*sqrt(3)) - log(((x
 - 1)/(x + 1))^(2/3) - 1) + 1/2*log(((x + 1)*((x - 1)/(x + 1))^(2/3) + (x -
 1)*((x - 1)/(x + 1))^(1/3) + x + 1)/(x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x,x)
[Out] Integral(1/(x*((x - 1)/(x + 1))**(1/3)), x)
```

Giac [A]
time = 0.42, size = 79, normalized size = 0.51

$$\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+1\right)\right)+\frac{1}{2}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}+\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}}{x+1}+1\right)-\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x,x, algorithm="giac")
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(2/3) + 1)) + 1/2*log(((x -
 1)/(x + 1))^(2/3) + (x - 1)*((x - 1)/(x + 1))^(1/3)/(x + 1) + 1) - log(abs
(((x - 1)/(x + 1))^(2/3) - 1))
```

Mupad [B]
time = 1.42, size = 82, normalized size = 0.53

$$-\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}-1296\right)-\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648-\sqrt{3}\,648\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(1296\left(\frac{x-1}{x+1}\right)^{2/3}+648+\sqrt{3}\,648\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] log(3^(1/2)*648i + 1296*((x - 1)/(x + 1))^(2/3) + 648)*((3^(1/2)*1i)/2 + 1/
2) - log(1296*((x - 1)/(x + 1))^(2/3) - 3^(1/2)*648i + 648)*((3^(1/2)*1i)/2
 - 1/2) - log(1296*((x - 1)/(x + 1))^(2/3) - 1296)
```

3.124 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2} dx$

Optimal. Leaf size=99

$$\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}-\frac{2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}}-\log\left(1+\frac{\sqrt[3]{\frac{-1+x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right)-\frac{1}{3}\log\left(1+\frac{1}{x}\right)$$

```
[Out] (1+1/x)^(1/3)*((-1+x)/x)^(2/3)-ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3))-1/3*ln(
1+1/x)+2/3*arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*
3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6306, 52, 62}

$$-\frac{2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{\sqrt{3}}+\sqrt[3]{\frac{1}{x}+1}\left(\frac{x-1}{x}\right)^{2/3}-\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{3}\log\left(\frac{1}{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x^2,x]
```

```
[Out] (1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3) - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/
x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/Sqrt[3] - Log[1 + ((-1 + x)/x)^(1/
3)/(1 + x^(-1))^(1/3)] - Log[1 + x^(-1)]/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^2}\,dx &= -\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{\sqrt{3}} - \log\left(1+\frac{\sqrt[3]{-\frac{1-x}{x}}}{\sqrt[3]{1+\frac{1}{x}}}\right) -\end{aligned}$$

Mathematica [A]
time = 0.10, size = 87, normalized size = 0.88

$$\frac{2e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - \frac{2\text{ArcTan}\left(\frac{-1+2e^{\frac{2}{3}\coth^{-1}(x)}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) + \frac{1}{3}\log\left(1-e^{\frac{2}{3}\coth^{-1}(x)}+e^{\frac{4}{3}\coth^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^2,x]
```

```
[Out] (2*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x])) - (2*ArcTan[(-1 + 2*E^((2*A
rcCoth[x])/3))/Sqrt[3]])/Sqrt[3] - (2*Log[1 + E^((2*ArcCoth[x])/3)])/3 + Lo
g[1 - E^((2*ArcCoth[x])/3) + E^((4*ArcCoth[x])/3)]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.15, size = 407, normalized size = 4.11

method	result
trager	$\frac{(1+x)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{x}-\frac{2\ln\left(-\frac{9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)/x*(-(1-x)/(1+x))^(2/3)-2/3*ln(-(9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x)
)^(2/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)+9*RootOf(9*_Z^2-3*_Z
+1)*(-(1-x)/(1+x))^(1/3)*x+9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(1/3)-3*(
-(1-x)/(1+x))^(1/3)*x+36*RootOf(9*_Z^2-3*_Z+1)^2+6*RootOf(9*_Z^2-3*_Z+1)*x-
3*(-(1-x)/(1+x))^(1/3)-12*RootOf(9*_Z^2-3*_Z+1)-x+1)/x)+2*RootOf(9*_Z^2-3*_
Z+1)*ln((9*RootOf(9*_Z^2-3*_Z+1)*(-(1-x)/(1+x))^(2/3)*x+9*RootOf(9*_Z^2-3*_
Z+1)*(-(1-x)/(1+x))^(2/3)-3*(-(1-x)/(1+x))^(2/3)*x-3*(-(1-x)/(1+x))^(2/3)-3
*(-(1-x)/(1+x))^(1/3)*x+18*RootOf(9*_Z^2-3*_Z+1)^2-3*RootOf(9*_Z^2-3*_Z+1)*
x-3*(-(1-x)/(1+x))^(1/3)-15*RootOf(9*_Z^2-3*_Z+1)+2*x+2)/x)
```

Maxima [A]
time = 0.47, size = 98, normalized size = 0.99

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Fricas [A]
time = 0.33, size = 97, normalized size = 0.98

$$\frac{2\sqrt{3}\,x\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-2\,x\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\,(x+1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*x*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3))
+ x*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 2*x*log(((
x - 1)/(x + 1))^(1/3) + 1) + 3*(x + 1)*((x - 1)/(x + 1))^(2/3))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**2,x)
[Out] Integral(1/(x**2*((x - 1)/(x + 1))**(1/3)), x)
```

Giac [A]
time = 0.40, size = 99, normalized size = 1.00

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{\frac{x-1}{x+1}+1}+\frac{1}{3}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{3}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^2,x, algorithm="giac")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2*((x - 1
)/(x + 1))^(2/3)/((x - 1)/(x + 1) + 1) + 1/3*log(((x - 1)/(x + 1))^(2/3) -
((x - 1)/(x + 1))^(1/3) + 1) - 2/3*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

Mupad [B]
time = 0.02, size = 118, normalized size = 1.19

$$\frac{2\left(\frac{x-1}{x+1}\right)^{2/3}}{\frac{x-1}{x+1}+1}-\ln\left(9\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)+\ln\left(9\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)^2+4\left(\frac{x-1}{x+1}\right)^{1/3}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)-\frac{2\ln\left(4\left(\frac{x-1}{x+1}\right)^{1/3}+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((x - 1)/(x + 1))^(1/3)),x)
[Out] log(9*((3^(1/2)*1i)/3 + 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(1/2)*1i)/3
 + 1/3) - log(9*((3^(1/2)*1i)/3 - 1/3)^2 + 4*((x - 1)/(x + 1))^(1/3))*((3^(
1/2)*1i)/3 - 1/3) - (2*log(4*((x - 1)/(x + 1))^(1/3) + 4))/3 + (2*((x - 1)/
(x + 1))^(2/3))/((x - 1)/(x + 1) + 1)
```

3.125 $\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(\frac{-1+x}{x}\right)^{2/3}+\frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3}-\frac{2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{-1+x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right)}{3\sqrt{3}}-\frac{1}{3}\log\left(1+\ldots\right.$$

[Out] 1/3*(1+1/x)^(1/3)*((-1+x)/x)^(2/3)+1/2*(1+1/x)^(4/3)*((-1+x)/x)^(2/3)-1/3*ln(1+((-1+x)/x)^(1/3)/(1+1/x)^(1/3))-1/9*ln(1+1/x)+2/9*arctan(-1/3*3^(1/2)+2/3*((-1+x)/x)^(1/3)/(1+1/x)^(1/3)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6306, 81, 52, 62}

$$-\frac{2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{\frac{x-1}{x}}}{\sqrt{3}\sqrt[3]{\frac{1}{x}+1}}\right)}{3\sqrt{3}}+\frac{1}{2}\left(\frac{x-1}{x}\right)^{2/3}\left(\frac{1}{x}+1\right)^{4/3}+\frac{1}{3}\left(\frac{x-1}{x}\right)^{2/3}\sqrt[3]{\frac{1}{x}+1}-\frac{1}{3}\log\left(\frac{\sqrt[3]{\frac{x-1}{x}}}{\sqrt[3]{\frac{1}{x}+1}}+1\right)-\frac{1}{9}\log\left(\frac{1}{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] ((1 + x^(-1))^(1/3)*((-1 + x)/x)^(2/3))/3 + ((1 + x^(-1))^(4/3)*((-1 + x)/x)^(2/3))/2 - (2*ArcTan[1/Sqrt[3] - (2*((-1 + x)/x)^(1/3))/(Sqrt[3]*(1 + x^(-1))^(1/3))])/(3*Sqrt[3]) - Log[1 + ((-1 + x)/x)^(1/3)/(1 + x^(-1))^(1/3)]/3 - Log[1 + x^(-1)]/9
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
```

```
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\frac{2}{3}\coth^{-1}(x)}}{x^3}\,dx &= -\text{Subst}\left(\int \frac{x\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)^{2/3}}\right.\\
&= \frac{1}{3}\sqrt[3]{1+\frac{1}{x}}\left(-\frac{1-x}{x}\right)^{2/3} + \frac{1}{2}\left(1+\frac{1}{x}\right)^{4/3}\left(\frac{-1+x}{x}\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{-\frac{1-x}{x}}}{\sqrt{3}\sqrt[3]{1+\frac{1}{x}}}\right.}{3\sqrt{3}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.17, size = 134, normalized size = 1.03

$$-\frac{2}{27}\left(\frac{27e^{\frac{2}{3}\coth^{-1}(x)}}{\left(1+e^{2\coth^{-1}(x)}\right)^2} - \frac{36e^{\frac{2}{3}\coth^{-1}(x)}}{1+e^{2\coth^{-1}(x)}} - 2\coth^{-1}(x) + 3\log\left(1+e^{\frac{2}{3}\coth^{-1}(x)}\right) - \text{RootSum}\left[1-\#1^2+\#1^4\&, \frac{\coth^{-1}(x) - 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right) + \coth^{-1}(x)\#1^2 - 3\log\left(e^{\frac{1}{3}\coth^{-1}(x)}-\#1\right)\#1^2}{-2+\#1^2}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((2*ArcCoth[x])/3)/x^3,x]
```

```
[Out] (-2*((27*E^((2*ArcCoth[x])/3))/(1 + E^(2*ArcCoth[x]))^2 - (36*E^((2*ArcCoth
[x])/3))/(1 + E^(2*ArcCoth[x])) - 2*ArcCoth[x] + 3*Log[1 + E^((2*ArcCoth[x]
)/3)] - RootSum[1 - #1^2 + #1^4 & , (ArcCoth[x] - 3*Log[E^(ArcCoth[x]/3) -
#1] + ArcCoth[x]*#1^2 - 3*Log[E^(ArcCoth[x]/3) - #1]*#1^2)/(-2 + #1^2) & ])
)/27
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.16, size = 507, normalized size = 3.90

method	result
risch	$\frac{(-1+x)(5x+3)}{6x^2\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}+\frac{\left(-\frac{2\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)^2x^2+27\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\left(x^3+x^2-x-1\right)^{\frac{2}{3}}-45\,\mathrm{RootOf}\left(\_Z^2-3\_Z+9\right)\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$
trager	$\frac{(1+x)(5x+3)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}}{6x^2}-\frac{2\ln\left(\frac{9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{RootOf}\left(9\_Z^2-3\_Z+1\right)\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}-3\left(-\frac{1-x}{1+x}\right)^{\frac{2}{3}}x+9\,\mathrm{Ro}\cdots}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-1+x)*(5*x+3)/x^2/((-1+x)/(1+x))^(1/3)+(-2/9*ln((8*RootOf(_Z^2-3*_Z+9)
^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(2/3)-45*RootOf(_Z^2-3*_Z+9)*(x
^3+x^2-x-1)^(1/3)*x-8*RootOf(_Z^2-3*_Z+9)^2*x-30*RootOf(_Z^2-3*_Z+9)*x^2-21
6*(x^3+x^2-x-1)^(2/3)-45*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)-81*(x^3+x^
2-x-1)^(1/3)*x-16*RootOf(_Z^2-3*_Z+9)^2-54*RootOf(_Z^2-3*_Z+9)*x-27*x^2-81*
(x^3+x^2-x-1)^(1/3)-24*RootOf(_Z^2-3*_Z+9)-36*x-9)/x/(1+x))+2/27*RootOf(_Z^
2-3*_Z+9)*ln(-(-2*RootOf(_Z^2-3*_Z+9)^2*x^2+27*RootOf(_Z^2-3*_Z+9)*(x^3+x^2
-x-1)^(2/3)+72*RootOf(_Z^2-3*_Z+9)*(x^3+x^2-x-1)^(1/3)*x+2*RootOf(_Z^2-3*_Z
+9)^2*x-27*RootOf(_Z^2-3*_Z+9)*x^2+135*(x^3+x^2-x-1)^(2/3)+72*RootOf(_Z^2-3
*_Z+9)*(x^3+x^2-x-1)^(1/3)-81*(x^3+x^2-x-1)^(1/3)*x+4*RootOf(_Z^2-3*_Z+9)^2
+6*RootOf(_Z^2-3*_Z+9)*x-36*x^2-81*(x^3+x^2-x-1)^(1/3)+33*RootOf(_Z^2-3*_Z+
9)-216*x-180)/x/(1+x)))/((-1+x)/(1+x))^(1/3)*((1+x)^2*(-1+x))^(1/3)/(1+x)
```

Maxima [A]
time = 0.48, size = 124, normalized size = 0.95

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\left(\frac{x-1}{x+1}\right)^{\frac{5}{3}}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{2(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1\right)}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*(((x
- 1)/(x + 1))^(5/3) + 4*((x - 1)/(x + 1))^(2/3))/(2*(x - 1)/(x + 1) + (x -
1)^2/(x + 1)^2 + 1) + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(
1/3) + 1) - 2/9*log(((x - 1)/(x + 1))^(1/3) + 1)
```

Fricas [A]
time = 0.33, size = 111, normalized size = 0.85

$$\frac{4\sqrt{3}\,x^2\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-4\,x^2\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)+3\,(5\,x^2+8\,x+3)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{18\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*x^2*arctan(2/3*sqrt(3)*((x - 1)/(x + 1))^(1/3) - 1/3*sqrt(3
)) + 2*x^2*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) + 1) - 4*x
^2*log(((x - 1)/(x + 1))^(1/3) + 1) + 3*(5*x^2 + 8*x + 3)*((x - 1)/(x + 1))
^(2/3))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)/x**3,x)
```

```
[Out] Integral(1/(x**3*((x - 1)/(x + 1))**(1/3)), x)
```

Giac [A]
time = 0.42, size = 122, normalized size = 0.94

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}-1\right)\right)+\frac{2\left(\frac{(x-1)\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}}{x+1}+4\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}\right)}{3\left(\frac{x-1}{x+1}+1\right)^2}+\frac{1}{9}\log\left(\left(\frac{x-1}{x+1}\right)^{\frac{2}{3}}-\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right)-\frac{2}{9}\log\left(\left|\left(\frac{x-1}{x+1}\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*((x - 1)/(x + 1))^(1/3) - 1)) + 2/3*((x -
 1)*((x - 1)/(x + 1))^(2/3)/(x + 1) + 4*((x - 1)/(x + 1))^(2/3))/((x - 1)/(
x + 1) + 1)^2 + 1/9*log(((x - 1)/(x + 1))^(2/3) - ((x - 1)/(x + 1))^(1/3) +
 1) - 2/9*log(abs(((x - 1)/(x + 1))^(1/3) + 1))
```

Mupad [B]
time = 0.03, size = 145, normalized size = 1.12

$$\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{2/3}}{3}+\frac{2\left(\frac{x-1}{x+1}\right)^{5/3}}{3}}{\frac{2\,(x-1)}{x+1}+\frac{(x-1)^2}{(x+1)^2}+1}-\frac{2\ln\left(\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}+\frac{4}{9}\right)}{9}-\ln\left(9\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(9\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2+\frac{4\left(\frac{x-1}{x+1}\right)^{1/3}}{9}\right)\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((x - 1)/(x + 1))^(1/3)),x)
```

```
[Out] ((8*((x - 1)/(x + 1))^(2/3))/3 + (2*((x - 1)/(x + 1))^(5/3))/3)/((2*(x - 1)
)/(x + 1) + (x - 1)^2/(x + 1)^2 + 1) - (2*log((4*((x - 1)/(x + 1))^(1/3))/9
 + 4/9))/9 - log(9*((3^(1/2)*1i)/9 - 1/9)^2 + (4*((x - 1)/(x + 1))^(1/3))/9
)*((3^(1/2)*1i)/9 - 1/9) + log(9*((3^(1/2)*1i)/9 + 1/9)^2 + (4*((x - 1)/(x
+ 1))^(1/3))/9)*((3^(1/2)*1i)/9 + 1/9)
```

3.126 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=429

$$\frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2}+\frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3-\frac{11\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[8]{\cdots}}{\sqrt[8]{1\cdots}}\right)}{64\sqrt{2}\,a^3}$$

```
[Out] 37/96*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a^2+3/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(
1/8)*x^2/a+1/3*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x^3+11/64*arctan((1+1/a/x)^(
1/8)/(1-1/a/x)^(1/8))/a^3+11/64*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^
3-11/128*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)+11/1
28*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)-11/256*ln(
1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/
a^3*2^(1/2)+11/256*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a/x)^(1/8)*2^(
1/2)/(1-1/a/x)^(1/8))/a^3*2^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {6306, 101, 156, 12, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$-\frac{11\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64\sqrt{2}\,a^3}+\frac{11\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{64\sqrt{2}\,a^3}+\frac{11\text{ArcTan}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}-\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}\,a^3}+\frac{11\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{128\sqrt{2}\,a^3}+\frac{11\tanh^{-1}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{64a^3}+\frac{37x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{96a^2}+\frac{1}{3}x^3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{3x^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x^2,x]
```

```
[Out] (37*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(96*a^2) + (3*(1 - 1/(a*x))^
(7/8)*(1 + 1/(a*x))^(1/8)*x^2)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(
1/8)*x^3)/3 - (11*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1
/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 -
1/(a*x))^(1/8)])/(64*Sqrt[2]*a^3) + (11*ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(
a*x))^(1/8)])/(64*a^3) + (11*ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8
)])/(64*a^3) - (11*Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8
) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3) + (11*Log[1
 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/
(1 - 1/(a*x))^(1/4)])/(128*Sqrt[2]*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^2\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^4\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{9}{4a}+\frac{2x}{a^2}}{x^3\sqrt[8]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3 + \frac{1}{6}\text{Subst}\left(\int \frac{-\frac{37}{16a^2}-}{x^2\sqrt[8]{1-\frac{x}{a}}\,(}\right.$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3$$

$$= \frac{37\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{96a^2} + \frac{3\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^2}{8a} + \frac{1}{3}\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x^3$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.11, size = 399, normalized size = 0.93

$$-\frac{e^{\frac{9}{4}\coth^{-1}(ax)}\left(20905836325-62455078125e^{-6\coth^{-1}(ax)}-51095314325e^{-4\coth^{-1}(ax)}+25918688125e^{-2\coth^{-1}(ax)}-1776332800e^{2\coth^{-1}(ax)}-22053358800\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{2\coth^{-1}(ax)}\right)+62455078125e^{-6\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{2\coth^{-1}(ax)}\right)+44155861200e^{-4\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{2\coth^{-1}(ax)}\right)-34498723050e^{-2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{2\coth^{-1}(ax)}\right)+2913904125e^{2\coth^{-1}(ax)}\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{2\coth^{-1}(ax)}\right)+12288e^{2\coth^{-1}(ax)}\left(3145+5122e^{2\coth^{-1}(ax)}+2105e^{4\coth^{-1}(ax)}\right){}_4F_3\left(2,2,2,\frac{17}{8};1,1,\frac{41}{8};e^{2\coth^{-1}(ax)}\right)+196608e^{2\coth^{-1}(ax)}\left(45+82e^{2\coth^{-1}(ax)}+37e^{4\coth^{-1}(ax)}\right){}_5F_4\left(2,2,2,2,\frac{17}{8};1,1,1,\frac{41}{8};e^{2\coth^{-1}(ax)}\right)+786432e^{2\coth^{-1}(ax)}\,{}_6F_5\left(2,2,2,2,2,\frac{17}{8};1,1,1,1,\frac{41}{8};e^{2\coth^{-1}(ax)}\right)+1572864e^{4\coth^{-1}(ax)}\,{}_6F_5\left(2,2,2,2,2,\frac{17}{8};1,1,1,1,\frac{41}{8};e^{2\coth^{-1}(ax)}\right)+786432e^{6\coth^{-1}(ax)}\,{}_6F_5\left(2,2,2,2,2,\frac{17}{8};1,1,1,1,\frac{41}{8};e^{2\coth^{-1}(ax)}\right)\right)}{64627200a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^2,x]
```

```
[Out] -1/64627200*(E^((9*ArcCoth[a*x])/4)*(20905836325 - 62455078125/E^(6*ArcCoth
[a*x]) - 51095314325/E^(4*ArcCoth[a*x]) + 25918688125/E^(2*ArcCoth[a*x]) -
1776332800*E^(2*ArcCoth[a*x]) - 22053358800*Hypergeometric2F1[1/8, 1, 9/8,
E^(2*ArcCoth[a*x])] + (62455078125*Hypergeometric2F1[1/8, 1, 9/8, E^(2*ArcC
oth[a*x])])/E^(6*ArcCoth[a*x]) + (44155861200*Hypergeometric2F1[1/8, 1, 9/8
, E^(2*ArcCoth[a*x])])/E^(4*ArcCoth[a*x]) - (34498723050*Hypergeometric2F1[
1/8, 1, 9/8, E^(2*ArcCoth[a*x])])/E^(2*ArcCoth[a*x]) + 2913904125*E^(2*ArcC
oth[a*x])*Hypergeometric2F1[1/8, 1, 9/8, E^(2*ArcCoth[a*x])] + 12288*E^(2*A
rcCoth[a*x])*(3145 + 5122*E^(2*ArcCoth[a*x]) + 2105*E^(4*ArcCoth[a*x]))*Hyp
ergeometricPFQ[{2, 2, 2, 17/8}, {1, 1, 41/8}, E^(2*ArcCoth[a*x])] + 196608*
E^(2*ArcCoth[a*x])*(45 + 82*E^(2*ArcCoth[a*x]) + 37*E^(4*ArcCoth[a*x]))*Hyp
ergeometricPFQ[{2, 2, 2, 2, 17/8}, {1, 1, 1, 41/8}, E^(2*ArcCoth[a*x])] + 7
86432*E^(2*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 17/8}, {1, 1, 1,
 1, 41/8}, E^(2*ArcCoth[a*x])] + 1572864*E^(4*ArcCoth[a*x])*HypergeometricP
FQ[{2, 2, 2, 2, 2, 17/8}, {1, 1, 1, 1, 41/8}, E^(2*ArcCoth[a*x])] + 786432*
E^(6*ArcCoth[a*x])*HypergeometricPFQ[{2, 2, 2, 2, 2, 17/8}, {1, 1, 1, 1, 41
/8}, E^(2*ArcCoth[a*x])]))/a^3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x)
```

Maxima [A]
time = 0.47, size = 341, normalized size = 0.79

$$-\frac{1}{768}\,a\left(\frac{16\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{23}{8}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+105\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{3(ax-1)a^4}{ax+1}-\frac{3(ax-1)^2a^4}{(ax+1)^2}+\frac{(ax-1)^3a^4}{(ax+1)^3}-a^4}+\frac{33\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)\right)}{a^4}+\frac{132\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)}{a^4}-\frac{66\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a^4}+\frac{66\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)}{a^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="maxima")
```

[Out] -1/768*a*(16*(33*((a*x - 1)/(a*x + 1))^(23/8) - 10*((a*x - 1)/(a*x + 1))^(1
5/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(3*(a*x - 1)*a^4/(a*x + 1) - 3*(a*x
 - 1)^2*a^4/(a*x + 1)^2 + (a*x - 1)^3*a^4/(a*x + 1)^3 - a^4) + 33*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*l
og(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
 sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(
1/4) + 1))/a^4 + 132*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x
 - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a
^4)

Fricas [A]
time = 0.37, size = 457, normalized size = 1.07

$$\frac{132\sqrt{2}\,a^{3}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\,a^{9}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+a^{6}\sqrt{\frac{1}{a^{12}}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,a^{3}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\sqrt{2}\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-1\right)+132\sqrt{2}\,a^{3}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{-\sqrt{2}\,a^{9}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+a^{6}\sqrt{\frac{1}{a^{12}}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,a^{3}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}-\sqrt{2}\,a^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}+1\right)+33\sqrt{2}\,a^{3}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,a^{9}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+a^{6}\sqrt{\frac{1}{a^{12}}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-33\sqrt{2}\,a^{3}\left(\frac{1}{a^{12}}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,a^{9}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^{12}}\right)^{\frac{3}{4}}+a^{6}\sqrt{\frac{1}{a^{12}}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+8\left(32a^{3}x^{3}+68a^{2}x^{2}+73ax+37\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}-132\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+66\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)-66\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)}{768\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="fricas")

[Out] 1/768*(132*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^9*((a*
x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-12)) + ((a*x - 1)/(a
*x + 1))^(1/4))*a^3*(a^(-12))^(1/4) - sqrt(2)*a^3*((a*x - 1)/(a*x + 1))^(1/
8)*(a^(-12))^(1/4) - 1) + 132*sqrt(2)*a^3*(a^(-12))^(1/4)*arctan(sqrt(2)*sq
rt(-sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-
12)) + ((a*x - 1)/(a*x + 1))^(1/4))*a^3*(a^(-12))^(1/4) - sqrt(2)*a^3*((a*x
 - 1)/(a*x + 1))^(1/8)*(a^(-12))^(1/4) + 1) + 33*sqrt(2)*a^3*(a^(-12))^(1/4
)*log(sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^
(-12)) + ((a*x - 1)/(a*x + 1))^(1/4)) - 33*sqrt(2)*a^3*(a^(-12))^(1/4)*log(
-sqrt(2)*a^9*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-12))^(3/4) + a^6*sqrt(a^(-12)
) + ((a*x - 1)/(a*x + 1))^(1/4)) + 8*(32*a^3*x^3 + 68*a^2*x^2 + 73*a*x + 37
)*((a*x - 1)/(a*x + 1))^(7/8) - 132*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + 6
6*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 66*log(((a*x - 1)/(a*x + 1))^(1/8)
 - 1))/a^3

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt[8]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**2,x)

[Out] Integral(x**2/((a*x - 1)/(a*x + 1))**(1/8), x)

Giac [A]
time = 0.45, size = 308, normalized size = 0.72

$$-\frac{1}{768}\,a\left(\frac{66\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}+\frac{66\,\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^4}-\frac{33\,\sqrt{2}\,\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{33\,\sqrt{2}\,\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^4}+\frac{132\,\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)}{a^4}-\frac{66\,\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a^4}+\frac{66\,\log\left(-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a^4}+\frac{16\left(33\left(\frac{ax-1}{ax+1}\right)^{\frac{23}{8}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+105\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{a^4\left(\frac{ax-1}{ax+1}-1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^2,x, algorithm="giac")
```

```
[Out] -1/768*a*(66*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^
(1/8)))/a^4 + 66*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x +
 1))^(1/8)))/a^4 - 33*sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a
*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 33*sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x
 + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^4 + 132*arctan(((a*x - 1)
/(a*x + 1))^(1/8))/a^4 - 66*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 66*l
og(-((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^4 + 16*(33*((a*x - 1)/(a*x + 1))^(23
/8) - 10*((a*x - 1)/(a*x + 1))^(15/8) + 105*((a*x - 1)/(a*x + 1))^(7/8))/(a
^4*((a*x - 1)/(a*x + 1) - 1)^3))
```

Mupad [B]
time = 1.31, size = 227, normalized size = 0.53

$$\frac{\frac{35\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{16}-\frac{5\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{24}+\frac{11\left(\frac{a\,x-1}{a\,x+1}\right)^{23/8}}{16}}{a^3+\frac{3\,a^3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a^3\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{3\,a^3\,\left(a\,x-1\right)}{a\,x+1}}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}\right)\,11{}\mathrm{i}}{64\,a^3}-\frac{11\,\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\right)}{64\,a^3}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{11}{128}+\frac{11}{128}{}\mathrm{i}\right)}{a^3}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{11}{128}-\frac{11}{128}{}\mathrm{i}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] ((35*((a*x - 1)/(a*x + 1))^(7/8))/16 - (5*((a*x - 1)/(a*x + 1))^(15/8))/24
+ (11*((a*x - 1)/(a*x + 1))^(23/8))/16)/(a^3 + (3*a^3*(a*x - 1)^2)/(a*x + 1
)^2 - (a^3*(a*x - 1)^3)/(a*x + 1)^3 - (3*a^3*(a*x - 1))/(a*x + 1)) - (atan(
((a*x - 1)/(a*x + 1))^(1/8)*1i)*11i)/(64*a^3) - (11*atan(((a*x - 1)/(a*x +
1))^(1/8)))/(64*a^3) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1
/2 - 1i/2))*(11/128 - 11i/128))/a^3 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x
 + 1))^(1/8)*(1/2 + 1i/2))*(11/128 + 11i/128))/a^3
```

3.127 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx$

Optimal. Leaf size=392

$$\frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a}+\frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}\,a^2}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{}}{}\right.}{16\sqrt{}}$$

```
[Out] 1/8*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x/a+1/2*(1-1/a/x)^(7/8)*(1+1/a/x)^(9/8)
*x^2+1/16*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a^2+1/16*arctanh((1+1/a/x
)^(1/8)/(1-1/a/x)^(1/8))/a^2-1/32*arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x
)^(1/8))/a^2*2^(1/2)+1/32*arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))
/a^2*2^(1/2)-1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1
/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)+1/64*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+
(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a^2*2^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {6306, 98, 96, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16\sqrt{2}\,a^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{16\sqrt{2}\,a^2}+\frac{\text{ArcTan}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}\,a^2}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{32\sqrt{2}\,a^2}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2}+\frac{1}{2}x^2\left(1-\frac{1}{ax}\right)^{7/8}\left(\frac{1}{ax}+1\right)^{9/8}+\frac{x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] ((1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x)/(8*a) + ((1 - 1/(a*x))^(7/8)*(1
 + 1/(a*x))^(9/8)*x^2)/2 - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/
(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(
1 - 1/(a*x))^(1/8)]/(16*Sqrt[2]*a^2) + ArcTan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a
*x))^(1/8)]/(16*a^2) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)]/(16
*a^2) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/
(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(32*Sqrt[2]*a^2) + Log[1 + (Sqrt[2]*(1 +
1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/
4)]/(32*Sqrt[2]*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^3\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8a}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{7}\cdots}\right.}{32a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1\cdots}}{\sqrt[8]{1\cdots}}\right.}{4a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1-\cdots}}{\sqrt[8]{1-\cdots}}\right.}{8a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1-\cdots}}{\sqrt[8]{1-\cdots}}\right.}{16a^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\text{ta}\cdots}{\cdots}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{16a^2} + \frac{\text{ta}\cdots}{\cdots}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x}{8a} + \frac{1}{2}\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}x^2 - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\cdots\right.}{16\sqrt{2}\,a^2}$$

Mathematica [A]
time = 0.55, size = 319, normalized size = 0.81

$$\frac{\frac{2}{\left(-1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{-1+e^{\frac{1}{4}\coth^{-1}(ax)}}-\frac{2}{\left(1+e^{\frac{1}{4}\coth^{-1}(ax)}\right)^2}+\frac{6}{1+e^{\frac{1}{4}\coth^{-1}(ax)}}+\frac{8e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(1+e^{\frac{1}{2}\coth^{-1}(ax)}\right)^2}-\frac{12e^{\frac{1}{4}\coth^{-1}(ax)}}{1+e^{\frac{1}{2}\coth^{-1}(ax)}}+\frac{32e^{\frac{1}{4}\coth^{-1}(ax)}}{\left(1+e^{\coth^{-1}(ax)}\right)^2}-\frac{40e^{\frac{1}{4}\coth^{-1}(ax)}}{1+e^{\coth^{-1}(ax)}}+4\text{ArcTan}\left(e^{\frac{1}{4}\coth^{-1}(ax)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}e^{\frac{1}{4}\coth^{-1}(ax)}\right)+2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}e^{\frac{1}{4}\coth^{-1}(ax)}\right)+4\tanh^{-1}\left(e^{\frac{1}{4}\coth^{-1}(ax)}\right)-\sqrt{2}\log\left(1-\sqrt{2}e^{\frac{1}{4}\coth^{-1}(ax)}+e^{\frac{1}{2}\coth^{-1}(ax)}\right)+\sqrt{2}\log\left(1+\sqrt{2}e^{\frac{1}{4}\coth^{-1}(ax)}+e^{\frac{1}{2}\coth^{-1}(ax)}\right)}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x,x]
```

```
[Out] (2/(-1 + E^(ArcCoth[a*x]/4))^2 + 6/(-1 + E^(ArcCoth[a*x]/4)) - 2/(1 + E^(Ar
cCoth[a*x]/4))^2 + 6/(1 + E^(ArcCoth[a*x]/4)) + (8*E^(ArcCoth[a*x]/4))/(1 +
 E^(ArcCoth[a*x]/2))^2 - (12*E^(ArcCoth[a*x]/4))/(1 + E^(ArcCoth[a*x]/2)) +
 (32*E^(ArcCoth[a*x]/4))/(1 + E^ArcCoth[a*x])^2 - (40*E^(ArcCoth[a*x]/4))/(
1 + E^ArcCoth[a*x]) + 4*ArcTan[E^(ArcCoth[a*x]/4)] - 2*Sqrt[2]*ArcTan[1 - S
qrt[2]*E^(ArcCoth[a*x]/4)] + 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*E^(ArcCoth[a*x]/4
)] + 4*ArcTanh[E^(ArcCoth[a*x]/4)] - Sqrt[2]*Log[1 - Sqrt[2]*E^(ArcCoth[a*x
]/4) + E^(ArcCoth[a*x]/2)] + Sqrt[2]*Log[1 + Sqrt[2]*E^(ArcCoth[a*x]/4) + E
^(ArcCoth[a*x]/2)])/(64*a^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x,x)
```

Maxima [A]
time = 0.48, size = 304, normalized size = 0.78

$$\frac{1}{64}a\left(\frac{16\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\frac{2(ax-1)a^3}{ax+1}-\frac{(ax-1)^2a^3}{(ax+1)^2}-a^3}-\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}-\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)}{a^3}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a^3}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="maxima")
```

```
[Out] 1/64*a*(16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x - 1)/(a*x + 1))^(7/8))/(
2*(a*x - 1)*a^3/(a*x + 1) - (a*x - 1)^2*a^3/(a*x + 1)^2 - a^3) - (2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sqrt(2)*lo
g(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) +
sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1))/a^3 - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^3 + 2*log(((a*x - 1
)/(a*x + 1))^(1/8) + 1)/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1)/a^3)
```

Fricas [A]
time = 0.36, size = 448, normalized size = 1.14

$$\frac{4\sqrt{2}a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}+a^4\sqrt{\frac{1}{a^8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}-\sqrt{2}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{-\sqrt{2}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}+a^4\sqrt{\frac{1}{a^8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}-\sqrt{2}a^2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^8}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\log\left(\sqrt{2}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}+a^4\sqrt{\frac{1}{a^8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-\sqrt{2}a^2\left(\frac{1}{a^8}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}a^6\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^8}\right)^{\frac{3}{4}}+a^4\sqrt{\frac{1}{a^8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+8\left(4a^2x^2+9ax+5\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}-4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)-2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)}{64a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="fricas")
```

```
[Out] 1/64*(4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^6*((a*x -
1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(a*x + 1
))^(1/4))*a^2*(a^(-8))^(1/4) - sqrt(2)*a^2*((a*x - 1)/(a*x + 1))^(1/8)*(a^(
-8))^(1/4) - 1) + 4*sqrt(2)*a^2*(a^(-8))^(1/4)*arctan(sqrt(2)*sqrt(-sqrt(2)
*a^6*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x
- 1)/(a*x + 1))^(1/4))*a^2*(a^(-8))^(1/4) - sqrt(2)*a^2*((a*x - 1)/(a*x + 1
))^(1/8)*(a^(-8))^(1/4) + 1) + sqrt(2)*a^2*(a^(-8))^(1/4)*log(sqrt(2)*a^6*(
(a*x - 1)/(a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(
a*x + 1))^(1/4)) - sqrt(2)*a^2*(a^(-8))^(1/4)*log(-sqrt(2)*a^6*((a*x - 1)/(
a*x + 1))^(1/8)*(a^(-8))^(3/4) + a^4*sqrt(a^(-8)) + ((a*x - 1)/(a*x + 1))^(
1/4)) + 8*(4*a^2*x^2 + 9*a*x + 5)*((a*x - 1)/(a*x + 1))^(7/8) - 4*arctan(((
a*x - 1)/(a*x + 1))^(1/8)) + 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log
(((a*x - 1)/(a*x + 1))^(1/8) - 1))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[8]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x,x)
```

```
[Out] Integral(x/((a*x - 1)/(a*x + 1))**(1/8), x)
```

Giac [A]
time = 0.45, size = 288, normalized size = 0.73

$$-\frac{1}{64}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a^3}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^3}+\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)}{a^3}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a^3}+\frac{2\log\left(-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a^3}+\frac{16\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}-9\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{a^3\left(\frac{ax-1}{ax+1}-1\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x,x, algorithm="giac")
```

```
[Out] -1/64*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1
/8)))/a^3 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1)
)^(1/8)))/a^3 - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1
```

```
)/(a*x + 1))^(1/4) + 1)/a^3 + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1
/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/a^3 + 4*arctan(((a*x - 1)/(a*x + 1))
^(1/8))/a^3 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a^3 + 2*log(-((a*x - 1
)/(a*x + 1))^(1/8) + 1)/a^3 + 16*(((a*x - 1)/(a*x + 1))^(15/8) - 9*((a*x -
1)/(a*x + 1))^(7/8))/(a^3*((a*x - 1)/(a*x + 1) - 1)^2))
```

Mupad [B]
time = 1.28, size = 190, normalized size = 0.48

$$\frac{\frac{9\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{4}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{15/8}}{4}}{a^2+\frac{a^2\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,a^2\,\left(a\,x-1\right)}{a\,x+1}}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{16\,a^2}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\right)}{16\,a^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}+\frac{1}{32}{}\mathrm{i}\right)}{a^2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{32}-\frac{1}{32}{}\mathrm{i}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] ((9*((a*x - 1)/(a*x + 1))^(7/8))/4 - ((a*x - 1)/(a*x + 1))^(15/8)/4)/(a^2 +
 (a^2*(a*x - 1)^2)/(a*x + 1)^2 - (2*a^2*(a*x - 1))/(a*x + 1)) - (atan(((a*x
 - 1)/(a*x + 1))^(1/8)*1i)*1i)/(16*a^2) - atan(((a*x - 1)/(a*x + 1))^(1/8))
/(16*a^2) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))
*(1/32 - 1i/32))/a^2 - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1
/2 + 1i/2))*(1/32 + 1i/32))/a^2
```

3.128 $\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=352

$$\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a}+\frac{\text{ArcTan}\left(\frac{\sqrt[8]{1\cdots}}{\sqrt[8]{1\cdots}}\right)}{2a}$$

```
[Out] (1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)*x+1/2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8
))/a+1/2*arctanh((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))/a-1/4*arctan(1-(1+1/a/x)^
(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/4*arctan(1+(1+1/a/x)^(1/8)*2^(1/
2)/(1-1/a/x)^(1/8))/a*2^(1/2)-1/8*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1
/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)+1/8*ln(1+(1+1/a/x)^(1/4)/(1-
1/a/x)^(1/4)+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))/a*2^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {6305, 96, 95, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{2\sqrt{2}\,a}+\frac{\text{ArcTan}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}+x\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}-\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}-\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt[4]{\frac{1}{ax}+1}}{\sqrt[4]{1-\frac{1}{ax}}}+\frac{\sqrt{2}\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}+1\right)}{4\sqrt{2}\,a}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{\frac{1}{ax}+1}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4),x]
```

```
[Out] (1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8)*x - ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x
))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*(1 + 1/(
a*x))^(1/8))/(1 - 1/(a*x))^(1/8)]/(2*Sqrt[2]*a) + ArcTan[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)]/(2*a) + ArcTanh[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1
/8)]/(2*a) - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1
 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/(4*Sqrt[2]*a) + Log[1 + (Sqrt[2]*(1
+ 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(
1/4)]/(4*Sqrt[2]*a)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x^2\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \cdots\right.}{a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cdots\right.}{2a}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\text{Subst}\left(\cdots\right.}{}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x + \frac{\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2a} - \frac{\log\left(1-\cdots\right.}{}$$

$$= \left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}\,x - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)}{2\sqrt{2}\,a} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right.}{2\sqrt{2}\,a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 56, normalized size = 0.16

$$\frac{2e^{\frac{1}{4}\coth^{-1}(ax)}\left(1+\left(-1+e^{2\coth^{-1}(ax)}\right)\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};e^{2\coth^{-1}(ax)}\right)\right)}{a\left(-1+e^{2\coth^{-1}(ax)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/4),x]
```

```
[Out] (2*E^(ArcCoth[a*x]/4)*(1 + (-1 + E^(2*ArcCoth[a*x]))*Hypergeometric2F1[1/8,
 1, 9/8, E^(2*ArcCoth[a*x])]))/(a*(-1 + E^(2*ArcCoth[a*x])))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8),x)
```

Maxima [A]
time = 0.47, size = 265, normalized size = 0.75

$$-\frac{1}{8}a\left(\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a^2}+\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)}{a^2}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a^2}+\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="maxima")
```

```
[Out] -1/8*a*(16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)*a^2/(a*x + 1) - a^2) + (2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/8))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(1/8))) - sq
rt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4)
 + 1) + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x
+ 1))^(1/4) + 1))/a^2 + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a^2 - 2*log((
(a*x - 1)/(a*x + 1))^(1/8) + 1)/a^2 + 2*log(((a*x - 1)/(a*x + 1))^(1/8) - 1
)/a^2)
```

Fricas [A]
time = 0.41, size = 423, normalized size = 1.20

$$\frac{4\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+a^2\sqrt{\frac{1}{a^4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-1\right)+4\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{2}\sqrt{-\sqrt{2}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+a^2\sqrt{\frac{1}{a^4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\sqrt{2}a\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+1\right)+\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(\sqrt{2}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+a^2\sqrt{\frac{1}{a^4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)-\sqrt{2}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}a^3\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+a^2\sqrt{\frac{1}{a^4}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\right)+8(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}-4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)+2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)-2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}-1\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*sqrt(sqrt(2)*a^3*((a*x - 1)/
(a*x + 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^
(1/4))*a*(a^(-4))^(1/4) - sqrt(2)*a*((a*x - 1)/(a*x + 1))^(1/8)*(a^(-4))^(1
/4) - 1) + 4*sqrt(2)*a*(a^(-4))^(1/4)*arctan(sqrt(2)*sqrt(-sqrt(2)*a^3*((a*
x - 1)/(a*x + 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x
 + 1))^(1/4))*a*(a^(-4))^(1/4) - sqrt(2)*a*((a*x - 1)/(a*x + 1))^(1/8)*(a^(
-4))^(1/4) + 1) + sqrt(2)*a*(a^(-4))^(1/4)*log(sqrt(2)*a^3*((a*x - 1)/(a*x
+ 1))^(1/8)*(a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^(1/4)
) - sqrt(2)*a*(a^(-4))^(1/4)*log(-sqrt(2)*a^3*((a*x - 1)/(a*x + 1))^(1/8)*(
a^(-4))^(3/4) + a^2*sqrt(a^(-4)) + ((a*x - 1)/(a*x + 1))^(1/4)) + 8*(a*x +
1)*((a*x - 1)/(a*x + 1))^(7/8) - 4*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + 2*
log(((a*x - 1)/(a*x + 1))^(1/8) + 1) - 2*log(((a*x - 1)/(a*x + 1))^(1/8) -
1))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(-1/8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(-1/8), x)
```

Mupad [B]
time = 1.26, size = 149, normalized size = 0.42

$$\frac{2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,a}-\frac{\mathrm{atan}\left({\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\right)}{2\,a}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}+\frac{1}{4}{}\mathrm{i}\right)}{a}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}-\frac{1}{4}{}\mathrm{i}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(7/8))/(a - (a*(a*x - 1))/(a*x + 1)) - (atan(((a*x
 - 1)/(a*x + 1))^(1/8)*1i)*1i)/(2*a) - atan(((a*x - 1)/(a*x + 1))^(1/8))/(2
*a) - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4
- 1i/4))/a - (2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 + 1i/2)
)*(1/4 + 1i/4))/a
```

3.129 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx$

Optimal. Leaf size=919

$$-\sqrt{2+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\sqrt{2-\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2-\sqrt{2}}}\right)+\cdots$$

```
[Out] 2*arctan((1+1/a/x)^(1/8)/(1-1/a/x)^(1/8))+2*arctanh((1+1/a/x)^(1/8)/(1-1/a/
x)^(1/8))-1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)-(1+1/a/x)^(1/8)*2^(1/2)/
(1-1/a/x)^(1/8))*2^(1/2)+1/2*ln(1+(1+1/a/x)^(1/4)/(1-1/a/x)^(1/4)+(1+1/a/x)
^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))*2^(1/2)-arctan(1-(1+1/a/x)^(1/8)*2^(1/2)/(1
-1/a/x)^(1/8))*2^(1/2)+arctan(1+(1+1/a/x)^(1/8)*2^(1/2)/(1-1/a/x)^(1/8))*2^
(1/2)-arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1
/2))^(1/2))*(2-2^(1/2))^(1/2)+arctan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+
2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/2*ln(1+(1-1/a/x)^(1/
4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^
(1/2))^(1/2)-1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^
(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-arctan((-2*(1-1/a/x)^(1/8)/
(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+arc
tan((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
)*(2+2^(1/2))^(1/2)+1/2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8
)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/2*ln(1+(1-1/a/x)^(
1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+
2^(1/2))^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 919, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {6306, 132, 65, 338, 305, 1136, 1183, 648, 632, 210, 642, 95, 220, 218, 212, 209, 217, 1179, 1176, 631}

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] -(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]]) - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 +
```

```
Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]]
+ Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1
+ 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + S
qrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]] -
 Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + Sq
rt[2]*ArcTan[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8)] + 2*Arc
Tan[(1 + 1/(a*x))^(1/8)/(1 - 1/(a*x))^(1/8)] + 2*ArcTanh[(1 + 1/(a*x))^(1/8
)/(1 - 1/(a*x))^(1/8)] + (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1
+ 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1
/8)])/2 - (Sqrt[2 - Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4
) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 + (Sqrt
[2 + Sqrt[2]]*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + S
qrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - (Sqrt[2 + Sqrt[2]]*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/2 - Log[1 - (Sqrt[2]*(1 + 1/(a*x))^(1/8
))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2] +
 Log[1 + (Sqrt[2]*(1 + 1/(a*x))^(1/8))/(1 - 1/(a*x))^(1/8) + (1 + 1/(a*x))^
(1/4)/(1 - 1/(a*x))^(1/4)]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
```

```
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x}\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{x\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{1}{x\sqrt[8]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= 8\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right) - 8\text{Subst}\left(\int \frac{1}{-1+x^8}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right)$$

$$= 4\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 4\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 8\text{Subs}$$

$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\text{Subs}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \frac{\sqrt[8]{1+}}{\sqrt[8]{1-}}\right.}{\sqrt{2}}$$

$$= 2\tan^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tanh^{-1}\left(\frac{\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) - \frac{\log\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}+\frac{\sqrt[4]{1+}}{\sqrt[4]{1-}}\right.}{\sqrt{2}}$$

$$= -\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + \sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{\sqrt[8]{1-\frac{1}{ax}}}\right) + 2\tan^{-1}\left(\frac{\sqrt[8]{}}{\sqrt[8]{}}\right.$$

$$\left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{}\right) \quad \left(\frac{\sqrt{2}\sqrt[8]{1+\frac{1}{ax}}}{}\right) \quad \left(\sqrt[8]{}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 30, normalized size = 0.03

$$\frac{16}{9}e^{\frac{9}{4}\coth^{-1}(ax)}\,{}_2F_1\left(\frac{9}{16},1;\frac{25}{16};e^{4\coth^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x,x]
```

```
[Out] (16*E^((9*ArcCoth[a*x])/4)*Hypergeometric2F1[9/16, 1, 25/16, E^(4*ArcCoth[a
*x])])/9
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2289 vs. 2(736) = 1472.
time = 0.42, size = 2289, normalized size = 2.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-((sqr
t(2) + 2)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(2*(sqrt(2
)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2
```

) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)
/(a*x + 1))^(1/4) + 4) + 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 3*sqrt(sqr
t(2) + 2))/((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt
(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) +
2))*arctan(((sqrt(2) + 2)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + sqrt(2
)*sqrt(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*
sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/
8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 2*sqrt(2)*((a*x - 1)/(a*x + 1))^(
1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt
(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)
*sqrt(-sqrt(2) + 2))*arctan(((sqrt(2) + 2)^(3/2) + (sqrt(2) + 1)*sqrt(-sqrt
(2) + 2) - sqrt(2)*sqrt(2*(sqrt(2)*(sqrt(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2)
+ 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1
)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) + 2*sqrt(2)*((a*x -
1)/(a*x + 1))^(1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2)^(3/2) - (sqrt(2)
+ 1)*sqrt(-sqrt(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2
) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-((sqrt(2) + 2)^(3/2) + (sqrt(2
) + 1)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) +
(sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(
2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) -
2*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) - 3*sqrt(sqrt(2) + 2))/((sqrt(2) + 2
)^(3/2) - (sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 3*sqrt(sqrt(2) + 2))) - 1/8*(s
qrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*(sqrt
(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*s
qrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x +
1))^(1/4) + 4) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) +
2))*log(-2*(sqrt(2)*(sqrt(2) + 2)^(3/2) + (sqrt(2)*(sqrt(2) + 2) - sqrt(2))
*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1
/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) + 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) -
sqrt(2)*sqrt(-sqrt(2) + 2))*log(2*(sqrt(2)*(sqrt(2) + 2)^(3/2) - (sqrt(2)*
(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sqrt(2)*sqrt(sqrt(2) + 2))*
((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 1/8*(sqr
t(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(-2*(sqrt(2)*(sqrt(
2) + 2)^(3/2) - (sqrt(2)*(sqrt(2) + 2) - sqrt(2))*sqrt(-sqrt(2) + 2) - 3*sq
rt(2)*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x +
1))^(1/4) + 4) + 2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*((a*x - 1)/(a*x + 1)
)^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - sqrt(2)*((a*x - 1)/(a*x + 1))^
(1/8) - 1) + 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*((a*x - 1)/(a*x +
1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - sqrt(2)*((a*x - 1)/(a*x +
1))^(1/8) + 1) - sqrt(-sqrt(2) + 2)*arctan(-((sqrt(2) + 1)*sqrt(-sqrt(2) +
2) - 2*sqrt((sqrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) +
((a*x - 1)/(a*x + 1))^(1/4) + 1) + 2*((a*x - 1)/(a*x + 1))^(1/8))/((sqrt(2
) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))) - sqrt(-sqrt(2) + 2)*arctan(((sqrt(2)
+ 1)*sqrt(-sqrt(2) + 2) + 2*sqrt(-(sqrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x -
1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 2*((a*x - 1)/(a*x

```
+ 1))^(1/8))/((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))) - sqrt(sqrt(2) +
2)*arctan(-((sqrt(2) + 2)^(3/2) - 2*sqrt(((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt
(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) -
3*sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/((sqrt(2) + 1)*sqrt(-s
qrt(2) + 2))) - sqrt(sqrt(2) + 2)*arctan(((sqrt(2) + 2)^(3/2) + 2*sqrt(-((s
qrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8) + ((a*
x - 1)/(a*x + 1))^(1/4) + 1) - 3*sqrt(sqrt(2) + 2) - 2*((a*x - 1)/(a*x + 1)
)^(1/8))/((sqrt(2) + 1)*sqrt(-sqrt(2) + 2))) - 1/4*sqrt(sqrt(2) + 2)*log((s
qrt(2) + 1)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*
x + 1))^(1/4) + 1) + 1/4*sqrt(sqrt(2) + 2)*log(-(sqrt(2) + 1)*sqrt(-sqrt(2)
 + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) - 1/4*
sqrt(-sqrt(2) + 2)*log(((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x -
1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/4*sqrt(-sqrt(2)
+ 2)*log(-((sqrt(2) + 2)^(3/2) - 3*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))
^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + 1/2*sqrt(2)*log(4*sqrt(2)*((a*x
 - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) + 4) - 1/2*sqrt(2)*l
og(-4*sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + 4*((a*x - 1)/(a*x + 1))^(1/4) +
 4) - 2*arctan(((a*x - 1)/(a*x + 1))^(1/8)) + log(((a*x - 1)/(a*x + 1))^(1/
8) + 1) - log(((a*x - 1)/(a*x + 1))^(1/8) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[8]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x,x)
```

```
[Out] Integral(1/(x*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Giac [A]
time = 1.10, size = 661, normalized size = 0.72

$$-\frac{1}{2}a\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}+\frac{2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)\right)}{a}-\frac{\sqrt{2}\log\left(\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{\sqrt{2}\log\left(-\sqrt{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a}+\frac{4\arctan\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}\right)}{a}-\frac{2\log\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a}+\frac{2\log\left(-\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+1\right)}{a}-\frac{4\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)}{a\sqrt{2\sqrt{2}+4}}-\frac{4\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)}{a\sqrt{2\sqrt{2}+4}}-\frac{4\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}}\right)}{a\sqrt{-2\sqrt{2}+4}}-\frac{4\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}}\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\log\left(\sqrt{\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a\sqrt{-2\sqrt{2}+4}}-\frac{2\log\left(-\sqrt{\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a\sqrt{-2\sqrt{2}+4}}+\frac{2\log\left(\sqrt{-\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a\sqrt{2\sqrt{2}+4}}-\frac{2\log\left(-\sqrt{-\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)}{a\sqrt{2\sqrt{2}+4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*((a*x - 1)/(a*x + 1))^(1/
8)))/a + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*((a*x - 1)/(a*x + 1))^(
1/8)))/a - sqrt(2)*log(sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*
x + 1))^(1/4) + 1)/a + sqrt(2)*log(-sqrt(2)*((a*x - 1)/(a*x + 1))^(1/8) + (
(a*x - 1)/(a*x + 1))^(1/4) + 1)/a + 4*arctan(((a*x - 1)/(a*x + 1))^(1/8))/a
 - 2*log(((a*x - 1)/(a*x + 1))^(1/8) + 1)/a + 2*log(-((a*x - 1)/(a*x + 1))^
```

```
(1/8) + 1)/a - 4*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))
/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2) + 4)) - 4*arctan(-(sqrt(sqrt(2) + 2)
 - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2))/(a*sqrt(2*sqrt(2) + 4
)) - 4*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqr
t(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) - 4*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a
*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2))/(a*sqrt(-2*sqrt(2) + 4)) + 2*l
og(sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1
/4) + 1)/(a*sqrt(-2*sqrt(2) + 4)) - 2*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*
x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(-2*sqrt(2) + 4)) +
 2*log(sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1
))^(1/4) + 1)/(a*sqrt(2*sqrt(2) + 4)) - 2*log(-sqrt(-sqrt(2) + 2)*((a*x - 1
)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1)/(a*sqrt(2*sqrt(2) + 4
)))
```

Mupad [B]
time = 1.40, size = 648, normalized size = 0.71

$$-\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}-2\,\mathrm{atan}\left(\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1+1{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)+\mathrm{atan}\left(-\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{\sqrt{2}-2}}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{\sqrt{2}+2}}+\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{\sqrt{2}+2}}\right)\,\left(\sqrt{\sqrt{2}-2}\,1{}\mathrm{i}+\sqrt{\sqrt{2}+2}\,1{}\mathrm{i}\right)-\mathrm{atan}\left(\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{\sqrt{2}-2}}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{\sqrt{2}+2}}-\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{\sqrt{2}-2}}+\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{\sqrt{2}+2}}\right)\,\left(\sqrt{\sqrt{2}-2}\,1{}\mathrm{i}-\sqrt{\sqrt{2}+2}\,1{}\mathrm{i}\right)-\mathrm{atan}\left(\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{-\sqrt{2}-2}}-\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{-\sqrt{2}-2}}+\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{2-\sqrt{2}}}\right)\,\left(\sqrt{-\sqrt{2}-2}\,1{}\mathrm{i}+\sqrt{2-\sqrt{2}}\,1{}\mathrm{i}\right)-\mathrm{atan}\left(\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{-\sqrt{2}-2}}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{-\sqrt{2}-2}}-\frac{\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}}{2\,\sqrt{2-\sqrt{2}}}\right)\,\left(\sqrt{-\sqrt{2}-2}\,1{}\mathrm{i}-\sqrt{2-\sqrt{2}}\,1{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (((a*x - 1)/(a*
x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8
)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2
*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i + (2^(1/2) + 2)^(1/2)*1i) -
2*atan(((a*x - 1)/(a*x + 1))^(1/8)) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x
+ 1))^(1/8)*(1/2 - 1i/2))*(1 - 1i) - 2^(1/2)*atan(2^(1/2)*((a*x - 1)/(a*x +
 1))^(1/8)*(1/2 + 1i/2))*(1 + 1i) - atan(((a*x - 1)/(a*x + 1))^(1/8)*1i)*2i
 - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(2^(1/2) - 2)^(1/2) + (((a*x - 1)/
(a*x + 1))^(1/8)*1i)/(2^(1/2) + 2)^(1/2) - (2^(1/2)*((a*x - 1)/(a*x + 1))^(
1/8)*1i)/(2*(2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*1i)
/(2*(2^(1/2) + 2)^(1/2)))*((2^(1/2) - 2)^(1/2)*1i - (2^(1/2) + 2)^(1/2)*1i)
 - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2) - (((a*x - 1
)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) + (2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)
*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i + (2 - 2^(1/2))^(1/
2)*1i) - atan((((a*x - 1)/(a*x + 1))^(1/8)*1i)/(- 2^(1/2) - 2)^(1/2) + (((a
*x - 1)/(a*x + 1))^(1/8)*1i)/(2 - 2^(1/2))^(1/2) + (2^(1/2)*((a*x - 1)/(a*x
 + 1))^(1/8)*1i)/(2*(- 2^(1/2) - 2)^(1/2)) - (2^(1/2)*((a*x - 1)/(a*x + 1))
^(1/8)*1i)/(2*(2 - 2^(1/2))^(1/2)))*((- 2^(1/2) - 2)^(1/2)*1i - (2 - 2^(1/2
))^(1/2)*1i)
```

3.130 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=676

$$a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}-\frac{1}{4}\sqrt{2+\sqrt{2}}\,a\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2-\sqrt{2}}\,a\text{ArcTan}$$

```
[Out] a*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x
)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*a*arcta
n((2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*
(2-2^(1/2))^(1/2)+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8
)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/8*a*ln(1+(1-1/a/x)
^(1/4)/(1+1/a/x)^(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(
2-2^(1/2))^(1/2)-1/4*a*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2
))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/4*a*arctan((2*(1-1/a/x)^(1
/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
+1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/
2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/8*a*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6306, 52, 65, 338, 305, 1136, 1183, 648, 632, 210, 642}

$$-\frac{1}{4}\sqrt{2+\sqrt{2}}\,a\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{4}\sqrt{2-\sqrt{2}}\,a\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,a\text{ArcTan}\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,a\text{ArcTan}\left(\frac{\frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)+a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{\frac{1}{ax}+1}+\frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{\sqrt{2-\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)+\frac{1}{8}\sqrt{2+\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}-\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)-\frac{1}{8}\sqrt{2+\sqrt{2}}\,a\log\left(\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{\frac{1}{ax}+1}}+\frac{\sqrt{2+\sqrt{2}}\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{\frac{1}{ax}+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x^2,x]
```

```
[Out] a*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8) - (Sqrt[2 + Sqrt[2]]*a*ArcTan[(Sq
rt[2 - Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/4 - (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x
))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqrt[2]]*a
*ArcTan[(Sqrt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/S
qrt[2 + Sqrt[2]]])/4 + (Sqrt[2 - Sqrt[2]]*a*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*
(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 -
```

```
 Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqr
t[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 - Sqrt[2]]*a*L
og[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/
(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/8 + (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1
/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))
/(1 + 1/(a*x))^(1/8)])/8 - (Sqrt[2 + Sqrt[2]]*a*Log[1 + (1 - 1/(a*x))^(1/4)
/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x)
)^(1/8)])/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
```

```
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt[8]{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{7/8}}\,dx, x, \frac{1}{x}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^6}{(2-x^8)^{7/8}}\,dx, x, \sqrt[8]{1-\frac{1}{ax}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + (2a)\text{Subst}\left(\int \frac{x^6}{1+x^8}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{a\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}\,x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} - \frac{a\text{Subst}\left(\int \frac{\ldots}{1+\ldots}\right.}{\ldots}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{a\text{Subst}\left(\int \frac{1-\sqrt{2}\,x^2}{1-\sqrt{2}\,x^2+x^4}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{\sqrt{2}} + \frac{a\text{Subst}\left(\int \frac{1\ldots}{1+\ldots}\right.}{\ldots}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{a\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}-\left(1-\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx, x, \frac{\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{a\text{S}\ldots}{\ldots}$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{4}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\,a\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,d\ldots\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{8}\sqrt{2-\sqrt{2}}\,a\log\left(1+\frac{\sqrt[4]{1-\frac{1}{ax}}}{\sqrt[4]{1+\frac{1}{ax}}} - \frac{\sqrt{2-\sqrt{2}}\,\sqrt[8]{1-\frac{1}{a\ldots}}}{\sqrt[8]{1+\frac{1}{ax}}}\right.$$

$$= a\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} - \frac{1}{4}\sqrt{2+\sqrt{2}}\,a\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}} - \frac{2\sqrt[8]{1-\frac{1}{ax}}}{\sqrt[8]{1+\frac{1}{ax}}}}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{4}\sqrt{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 46, normalized size = 0.07

$$-2ae^{\frac{1}{4}\coth^{-1}(ax)}\left(-\frac{1}{1+e^{2\coth^{-1}(ax)}}+{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^2,x]
```

```
[Out] -2*a*E^(ArcCoth[a*x]/4)*(-(1 + E^(2*ArcCoth[a*x]))^(-1) + Hypergeometric2F1
[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2874 vs. 2(528) = 1056.
time = 0.43, size = 2874, normalized size = 4.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^8)^(1/8)*a^7*((a*x
- 1)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - 2*sq
rt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 + (a^8)^(7/8)*(a^7*(s
```

qrt(2) + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1
/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2))) + 8*(a^8)^(1/8)*x
*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x + 1))^(1/8)
- (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - 2*sqrt(a^14*((a*x - 1)/(a*
x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqr
t(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2
)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2))) + 8*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*arc
tan(-(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*(a^8)^(1/8)*a^7
*((a*x - 1)/(a*x + 1))^(1/8) - 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a
^8)^(3/4)*a^8 + (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(
7/8)*((a*x - 1)/(a*x + 1))^(1/8))*(a^8)^(1/8))/((a^8*(sqrt(2) + 2) - a^8)*s
qrt(-sqrt(2) + 2))) + 8*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*arctan((a^8*(sqrt(2
) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*(a^8)^(1/8)*a^7*((a*x - 1)/(a*x
+ 1))^(1/8) + 2*sqrt(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (
a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(
a*x + 1))^(1/8))*(a^8)^(1/8))/((a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2)
)) + 2*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4)
 + (a^8)^(3/4)*a^8 + (a^8)^(7/8)*(a^7*(sqrt(2) + 2) - a^7)*sqrt(-sqrt(2) +
2)*((a*x - 1)/(a*x + 1))^(1/8)) - 2*(a^8)^(1/8)*x*sqrt(sqrt(2) + 2)*log(a^1
4*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^8)^(7/8)*(a^7*(sqrt(2)
 + 2) - a^7)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)) + 2*(a^8)^(1/8
)*x*sqrt(-sqrt(2) + 2)*log(a^14*((a*x - 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a
^8 + (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x
- 1)/(a*x + 1))^(1/8)) - 2*(a^8)^(1/8)*x*sqrt(-sqrt(2) + 2)*log(a^14*((a*x
- 1)/(a*x + 1))^(1/4) + (a^8)^(3/4)*a^8 - (a^7*(sqrt(2) + 2)^(3/2) - 3*a^7*
sqrt(sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) + 4*(a^8)^(1/8)
*(sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(a^8*
(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^8)^(1/8)*a^7*(
(a*x - 1)/(a*x + 1))^(1/8) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) -
 sqrt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(s
qrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)
a^7(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1
)/(a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt
(2) + 2) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(
sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((a^8*(sq
rt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*
x - 1)/(a*x + 1))^(1/8) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) + sq
rt(2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt
(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) - (sqrt(2)*a^
7*(sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(
a*x + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2)
 + 2) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(sqr
t(2)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan((a^8*(sqrt(
2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x -
 1)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) - sqrt(

```
2)*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)
*a^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(
sqrt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x
 + 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) +
2) - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + 4*(a^8)^(1/8)*(sqrt(2
)*x*sqrt(sqrt(2) + 2) - sqrt(2)*x*sqrt(-sqrt(2) + 2))*arctan(-(a^8*(sqrt(2)
 + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^8)^(1/8)*a^7*((a*x - 1
)/(a*x + 1))^(1/8) + (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2) + sqrt(2)
*sqrt(4*a^14*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2*(sqrt(2)*a
^7*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sq
rt(2) + 2) - sqrt(2)*a^7)*sqrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x +
 1))^(1/8))*(a^8)^(1/8))/(a^8*(sqrt(2) + 2)^(3/2) - 3*a^8*sqrt(sqrt(2) + 2)
 - (a^8*(sqrt(2) + 2) - a^8)*sqrt(-sqrt(2) + 2))) + (a^8)^(1/8)*(sqrt(2)*x*
sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4*a^14*((a*x - 1)/(a*
x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 + 2*(sqrt(2)*a^7*(sqrt(2) + 2)^(3/2) - 3*
sqrt(2)*a^7*sqrt(sqrt(2) + 2) + (sqrt(2)*a^7*(sqrt(2) + 2) - sqrt(2)*a^7)*s
qrt(-sqrt(2) + 2))*(a^8)^(7/8)*((a*x - 1)/(a*x + 1))^(1/8)) - (a^8)^(1/8)*(
sqrt(2)*x*sqrt(sqrt(2) + 2) + sqrt(2)*x*sqrt(-sqrt(2) + 2))*log(4*a^14*((a*
x - 1)/(a*x + 1))^(1/4) + 4*(a^8)^(3/4)*a^8 - 2...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[8]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**2,x)
```

```
[Out] Integral(1/(x**2*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Giac [**A**]
time = 0.53, size = 432, normalized size = 0.64

$$\frac{1}{8}\left(2\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}}\right)+2\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}}\right)-\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\frac{16\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}}{\frac{ax-1}{ax+1}+1}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1
))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2)
+ 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*sqrt(sqrt(2)
+ 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(
2) + 2)) + 2*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a*x - 1)/(
a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) +
```

```
2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(sq
rt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/
(a*x + 1))^(1/4) + 1) - sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*((a*x - 1
)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + sqrt(-sqrt(2) + 2)*
log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))
^(1/4) + 1) + 16*((a*x - 1)/(a*x + 1))^(7/8)/((a*x - 1)/(a*x + 1) + 1))*a
```

Mupad [B]
time = 1.25, size = 162, normalized size = 0.24

$$\frac{(-1)^{1/8}\,a\,\mathrm{atan}\left((-1)^{1/8}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\right)}{2}+\frac{(-1)^{1/8}\,a\,\mathrm{atan}\left((-1)^{1/8}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{2\,a\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/8}}{\frac{a\,x-1}{a\,x+1}+1}+(-1)^{1/8}\,\sqrt{2}\,a\,\mathrm{atan}\left((-1)^{1/8}\,\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+(-1)^{1/8}\,\sqrt{2}\,a\,\mathrm{atan}\left((-1)^{1/8}\,\sqrt{2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] ((-1)^(1/8)*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/2 + ((-1)^(1/8)
*a*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/2 + (2*a*((a*x - 1)/
(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1) + (-1)^(1/8)*2^(1/2)*a*atan((-1
)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/4 - 1i/4) + (-
1)^(1/8)*2^(1/2)*a*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2
 + 1i/2))*(1/4 + 1i/4)
```

3.131 $\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=731

$$\frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}}+\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8}-\frac{1}{32}\sqrt{2+\sqrt{2}}\,a^2\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-\frac{2\sqrt[8]{\cdots}}{\sqrt[8]{\cdots}}}{\sqrt{2+\sqrt{2}}}\cdots\right.$$

```
[Out] 1/8*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(1/8)+1/2*a^2*(1-1/a/x)^(7/8)*(1+1/a/x)^(
9/8)-1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))
/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8)/(1
+1/a/x)^(1/8)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/64*
a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/
(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^
(1/4)+(1-1/a/x)^(1/8)*(2-2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2-2^(1/2))^(1/2)-
1/32*a^2*arctan((-2*(1-1/a/x)^(1/8)/(1+1/a/x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/32*a^2*arctan((2*(1-1/a/x)^(1/8)/(1+1/a/
x)^(1/8)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/64*a^2*l
n(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)-(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/
a/x)^(1/8))*(2+2^(1/2))^(1/2)-1/64*a^2*ln(1+(1-1/a/x)^(1/4)/(1+1/a/x)^(1/4)
+(1-1/a/x)^(1/8)*(2+2^(1/2))^(1/2)/(1+1/a/x)^(1/8))*(2+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6306, 81, 52, 65, 338, 305, 1136, 1183, 648, 632, 210, 642}

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(7/8)*(1 + 1/(a*x))^(1/8))/8 + (a^2*(1 - 1/(a*x))^(7/8)*
(1 + 1/(a*x))^(9/8))/2 - (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sqrt[2 - Sqrt[2]] -
 (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqrt[2]]])/32 - (Sqr
t[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] - (2*(1 - 1/(a*x))^(1/8))/(1 +
 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 + Sqrt[2]]*a^2*ArcTan[(Sq
rt[2 - Sqrt[2]] + (2*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 + Sqr
t[2]]])/32 + (Sqrt[2 - Sqrt[2]]*a^2*ArcTan[(Sqrt[2 + Sqrt[2]] + (2*(1 - 1/(
```

```
a*x))^(1/8))/(1 + 1/(a*x))^(1/8))/Sqrt[2 - Sqrt[2]]])/32 + (Sqrt[2 - Sqrt[2
]]*a^2*Log[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 - Sqrt[2]]
*(1 - 1/(a*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 - Sqrt[2]]*a^2*Log
[1 + (1 - 1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 - Sqrt[2]]*(1 - 1/(a
*x))^(1/8))/(1 + 1/(a*x))^(1/8)])/64 + (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 -
1/(a*x))^(1/4)/(1 + 1/(a*x))^(1/4) - (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8)
)/(1 + 1/(a*x))^(1/8)])/64 - (Sqrt[2 + Sqrt[2]]*a^2*Log[1 + (1 - 1/(a*x))^(
1/4)/(1 + 1/(a*x))^(1/4) + (Sqrt[2 + Sqrt[2]]*(1 - 1/(a*x))^(1/8))/(1 + 1/(
a*x))^(1/8)])/64
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 305

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Dist[s^3/(2*Sqrt[2]*b*r), Int[x^(
m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Dist[s^3/(2*Sq
rt[2]*b*r), Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x],
```

```
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && G
tQ[a/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\frac{1}{4}\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int \frac{x\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{8}a\text{Subst}\left(\int \frac{\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{1}{32}a\text{Subst}\left(\int \frac{}{\sqrt[8]{1-}}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\text{Subst}\left(\int \frac{x^6}{(2-x^8}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{4}a^2\text{Subst}\left(\int \frac{x^6}{1+x^8}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{a^2\text{Subst}\left(\int \frac{x^4}{1-\sqrt{2}\,x^2+}\right.}{8\sqrt{}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} - \frac{a^2\text{Subst}\left(\int \frac{1-\sqrt{2}\,x^2}{1-\sqrt{2}\,x^2+}\right.}{8\sqrt{}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}}{1-\sqrt{2-}}\right.}{16\sqrt{}}$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{32}\left(\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\right.$$

$$= \frac{1}{8}a^2\left(1-\frac{1}{ax}\right)^{7/8}\sqrt[8]{1+\frac{1}{ax}} + \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{7/8}\left(1+\frac{1}{ax}\right)^{9/8} + \frac{1}{64}\sqrt{2-\sqrt{2}}\,a^2\log\left(\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 72, normalized size = 0.10

$$-\frac{a^2e^{\frac{1}{4}\coth^{-1}(ax)}\left(-1-9e^{2\coth^{-1}(ax)}+\left(1+e^{2\coth^{-1}(ax)}\right)^2 {}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{2\coth^{-1}(ax)}\right)\right)}{4\left(1+e^{2\coth^{-1}(ax)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(ArcCoth[a*x]/4)/x^3,x]
```

```
[Out] -1/4*(a^2*E^(ArcCoth[a*x]/4)*(-1 - 9*E^(2*ArcCoth[a*x]) + (1 + E^(2*ArcCoth
[a*x]))^2*Hypergeometric2F1[1/8, 1, 9/8, -E^(2*ArcCoth[a*x])]))/(1 + E^(2*A
rcCoth[a*x]))^2
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2931 vs. 2(575) = 1150.
time = 0.43, size = 2931, normalized size = 4.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="fricas")
```

```
[Out] -1/256*(8*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^16)^(1/8)*a^14*
((a*x - 1)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2
) - 2*sqrt(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 + (a^16)^(7
/8)*(a^14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1
/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2))) +
 8*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*arctan(-(2*(a^16)^(1/8)*a^14*((a*x -
 1)/(a*x + 1))^(1/8) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) - 2*s
qrt(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^
14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8))*(a
^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2))) + 8*(a^1
6)^(1/8)*x^2*sqrt(sqrt(2) + 2)*arctan(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*s
qrt(sqrt(2) + 2) + 2*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) - 2*sqrt
(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 + (a^16)^(7/8)*(a^14*
(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8)
)*(a^16)^(1/8))/((a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 8*(a^16
)^(1/8)*x^2*sqrt(sqrt(2) + 2)*arctan((a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqr
t(sqrt(2) + 2) - 2*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) + 2*sqrt(a
^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14*(s
qrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*
(a^16)^(1/8))/((a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 2*(a^16)^
(1/8)*x^2*sqrt(sqrt(2) + 2)*log(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(
3/4)*a^16 + (a^16)^(7/8)*(a^14*(sqrt(2) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a
*x - 1)/(a*x + 1))^(1/8)) - 2*(a^16)^(1/8)*x^2*sqrt(sqrt(2) + 2)*log(a^28*(
(a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14*(sqrt(2
) + 2) - a^14)*sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)) + 2*(a^16)^(
1/8)*x^2*sqrt(-sqrt(2) + 2)*log(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(
3/4)*a^16 + (a^16)^(7/8)*(a^14*(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) +
2))*((a*x - 1)/(a*x + 1))^(1/8)) - 2*(a^16)^(1/8)*x^2*sqrt(-sqrt(2) + 2)*lo
g(a^28*((a*x - 1)/(a*x + 1))^(1/4) + (a^16)^(3/4)*a^16 - (a^16)^(7/8)*(a^14
*(sqrt(2) + 2)^(3/2) - 3*a^14*sqrt(sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8
)) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + sqrt(2)*x^2*sqrt(-sqrt
(2) + 2))*arctan(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + 2*
sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) - (a^16*(sqrt(2) + 2)
 - a^16)*sqrt(-sqrt(2) + 2) - sqrt(2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/
4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2)
 - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)
*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16
*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + (a^16*(sqrt(2) + 2) - a^1
6)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + s
qrt(2)*x^2*sqrt(-sqrt(2) + 2))*arctan((a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sq
rt(sqrt(2) + 2) - 2*sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) -
 (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^28*((a*x
 - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14
*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2) - (sqrt(2)*a^14*(sq
rt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))
```

```
*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + (a^16
*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^(1/8)*(sqrt(2)*x^2*s
qrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*arctan((a^16*(sqrt(2) +
2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) + 2*sqrt(2)*(a^16)^(1/8)*a^14*((a*x - 1
)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2) - sqrt(
2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16 + 2*(a^16)
^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(sqrt(2) + 2)
 + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-sqrt(2) + 2))*((a*x -
1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(
sqrt(2) + 2) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(2) + 2))) + 4*(a^16)^
(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) - sqrt(2)*x^2*sqrt(-sqrt(2) + 2))*arct
an(-(a^16*(sqrt(2) + 2)^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) - 2*sqrt(2)*(a^16)
^(1/8)*a^14*((a*x - 1)/(a*x + 1))^(1/8) + (a^16*(sqrt(2) + 2) - a^16)*sqrt(
-sqrt(2) + 2) + sqrt(2)*sqrt(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^
(3/4)*a^16 - 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a
^14*sqrt(sqrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqrt(2)*a^14)*sqrt(-s
qrt(2) + 2))*((a*x - 1)/(a*x + 1))^(1/8))*(a^16)^(1/8))/(a^16*(sqrt(2) + 2)
^(3/2) - 3*a^16*sqrt(sqrt(2) + 2) - (a^16*(sqrt(2) + 2) - a^16)*sqrt(-sqrt(
2) + 2))) + (a^16)^(1/8)*(sqrt(2)*x^2*sqrt(sqrt(2) + 2) + sqrt(2)*x^2*sqrt(
-sqrt(2) + 2))*log(4*a^28*((a*x - 1)/(a*x + 1))^(1/4) + 4*(a^16)^(3/4)*a^16
 + 2*(a^16)^(7/8)*(sqrt(2)*a^14*(sqrt(2) + 2)^(3/2) - 3*sqrt(2)*a^14*sqrt(s
qrt(2) + 2) + (sqrt(2)*a^14*(sqrt(2) + 2) - sqr...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[8]{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)/x**3,x)
```

```
[Out] Integral(1/(x**3*((a*x - 1)/(a*x + 1))**(1/8)), x)
```

Giac [**A**]
time = 0.55, size = 461, normalized size = 0.63

$$\frac{1}{64}\left(2a\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2a\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}}\right)+2a\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}}\right)+2a\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}}\right)-a\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+a\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)-a\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+a\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}+\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}+1\right)+\frac{16\left(a\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{8}}+9a\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{8}}\right)}{\left(\frac{ax-1}{ax+1}+1\right)^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*(2*a*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*((a*x - 1)/(a*x
+ 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqr
t(2) + 2) - 2*((a*x - 1)/(a*x + 1))^(1/8))/sqrt(-sqrt(2) + 2)) + 2*a*sqrt(s
```

```
qrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*((a*x - 1)/(a*x + 1))^(1/8))/sqr
t(sqrt(2) + 2)) + 2*a*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*((a
*x - 1)/(a*x + 1))^(1/8))/sqrt(sqrt(2) + 2)) - a*sqrt(sqrt(2) + 2)*log(sqrt
(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1
) + a*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8)
+ ((a*x - 1)/(a*x + 1))^(1/4) + 1) - a*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2)
 + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*x - 1)/(a*x + 1))^(1/4) + 1) + a*sq
rt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*((a*x - 1)/(a*x + 1))^(1/8) + ((a*
x - 1)/(a*x + 1))^(1/4) + 1) + 16*(a*((a*x - 1)/(a*x + 1))^(15/8) + 9*a*((a
*x - 1)/(a*x + 1))^(7/8))/((a*x - 1)/(a*x + 1) + 1)^2)*a
```

Mupad [B]
time = 1.26, size = 210, normalized size = 0.29

$$\frac{\frac{9\,a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/8}}{4}+\frac{a^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/8}}{4}}{\frac{{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{2\,\left(a\,x-1\right)}{a\,x+1}+1}+\frac{{\left(-1\right)}^{1/8}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/8}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\right)}{16}+\frac{{\left(-1\right)}^{1/8}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/8}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{16}+{\left(-1\right)}^{1/8}\,\sqrt{2}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/8}\,\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(\frac{1}{32}-\frac{1}{32}{}\mathrm{i}\right)+{\left(-1\right)}^{1/8}\,\sqrt{2}\,a^2\,\mathrm{atan}\left({\left(-1\right)}^{1/8}\,\sqrt{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{1/8}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(\frac{1}{32}+\frac{1}{32}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a*x - 1)/(a*x + 1))^(1/8)),x)
```

```
[Out] ((9*a^2*((a*x - 1)/(a*x + 1))^(7/8))/4 + (a^2*((a*x - 1)/(a*x + 1))^(15/8))
/4)/((a*x - 1)^2/(a*x + 1)^2 + (2*(a*x - 1))/(a*x + 1) + 1) + ((-1)^(1/8)*a
^2*atan((-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)))/16 + ((-1)^(1/8)*a^2*atan(
(-1)^(1/8)*((a*x - 1)/(a*x + 1))^(1/8)*1i)*1i)/16 + (-1)^(1/8)*2^(1/2)*a^2*
atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1))^(1/8)*(1/2 - 1i/2))*(1/32 - 1
i/32) + (-1)^(1/8)*2^(1/2)*a^2*atan((-1)^(1/8)*2^(1/2)*((a*x - 1)/(a*x + 1)
)^(1/8)*(1/2 + 1i/2))*(1/32 + 1i/32)
```

3.132 $\int e^{4\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=45

$$\frac{x^{1+m}}{1+m}+\frac{4x^{1+m}}{1-ax}-4x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)$$

```
[Out] x^(1+m)/(1+m)+4*x^(1+m)/(-a*x+1)-4*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6302, 6261, 91, 81, 66}

$$-4x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)+\frac{4x^{m+1}}{1-ax}+\frac{x^{m+1}}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*x^m,x]
```

```
[Out] x^(1 + m)/(1 + m) + (4*x^(1 + m))/(1 - a*x) - 4*x^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, a*x]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1]))
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}x^m\,dx &= \int e^{4\tanh^{-1}(ax)}x^m\,dx \\
&= \int \frac{x^m(1+ax)^2}{(1-ax)^2}\,dx \\
&= \frac{4x^{1+m}}{1-ax} - \frac{\int \frac{x^m(a^2(3+4m)+a^3x)}{1-ax}\,dx}{a^2} \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - (4(1+m))\int \frac{x^m}{1-ax}\,dx \\
&= \frac{x^{1+m}}{1+m} + \frac{4x^{1+m}}{1-ax} - 4x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.04

$$\frac{x^{1+m}(-5-4m+ax-4(1+m)(-1+ax)\,{}_2F_1(1,1+m;2+m;ax))}{(1+m)(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(-5 - 4*m + a*x - 4*(1 + m)*(-1 + a*x)*Hypergeometric2F1[1, 1 +
m, 2 + m, a*x]))/((1 + m)*(-1 + a*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.22, size = 201, normalized size = 4.47

method	result
meijerg	$-\frac{(-a)^{-m}\left(\frac{x^{m}(-a)^{m}\left(a^{2}m\,x^{2}+amx+2ax-m^{2}-3m-2\right)}{(1+m)m(-ax+1)}+x^{m}(-a)^{m}(2+m)\Phi(ax,1,m)\right)}{a}+\frac{2(-a)^{-m}\left(-\frac{x^{m}(-a)^{m}(ax-m-1)}{m(-ax+1)}-x^{m}(-a)^{m}\cdots\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*x^m,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(-m)/a*(x^m*(-a)^m*(a^2*m*x^2+a*m*x+2*a*x-m^2-3*m-2)/(1+m)/m/(-a*x+1)
+x^m*(-a)^m*(2+m)*LerchPhi(a*x,1,m))+2*(-a)^(-m)/a*(-x^m*(-a)^m*(a*x-m-1)/m
/(-a*x+1)-x^m*(-a)^m*(1+m)*LerchPhi(a*x,1,m))-(-a)^(-m)/a*(1/(1+m)*x^m*(-a)
^m*(-1-m)/(-a*x+1)+x^m*(-a)^m*m*LerchPhi(a*x,1,m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}(ax+1)^{2}}{(ax-1)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*x**m,x)
```

```
[Out] Integral(x**m*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*x^m,x, algorithm="giac")
[Out] integrate((a*x + 1)^2*x^m/(a*x - 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,{\left(a\,x+1\right)}^2}{{\left(a\,x-1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] int((x^m*(a*x + 1)^2)/(a*x - 1)^2, x)
```

3.133 $\int e^{3\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=151

$$-\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}+$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-x^m*hyp
ergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2, -1/
2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+4*x^m*hypergeom([3/2, -1/2*m],[1-1/2*
m],1/a^2/x^2)/a/m
```

Rubi [A]
time = 0.93, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6307, 6874, 371, 864, 822}

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{4x^m\,{}_2F_1\left(\frac{3}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) - (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m) +
 (4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) + (4*x^m*Hypergeometric2F1[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
```

```
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}} - \frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}} + \frac{4x^{-2-m}}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right) d$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-}}{\left(1-\frac{x}{a}\right)}\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^{m}\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x^{m}\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,{}_2F_1\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 0.22, size = 228, normalized size = 1.51

$$\frac{x^{1+m}\left(3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{1-ax}\,\sqrt{\frac{1+ax}{a^2}}\,F_1\left(m;-\frac{1}{2},\frac{1}{2};1+m;-ax,ax\right)-2(1+m)\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{1-ax}\,\sqrt{\frac{1+ax}{a^2}}\,F_1\left(m;-\frac{1}{2},\frac{3}{2};1+m;-ax,ax\right)+m\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{-\frac{1}{a^2}+x^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2};\frac{1}{2}-\frac{m}{2};\frac{1}{a^2x^2}\right)\right)}{m(1+m)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{-\frac{1}{a^2}+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^
2]*AppellF1[m, -1/2, 1/2, 1 + m, -(a*x), a*x] - 2*(1 + m)*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[1 - a*x]*Sqrt[(1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, -(a*x)
, a*x] + m*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[-a^(-2) + x^2]*Hypergeometric2
F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]))/(m*(1 + m)*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*Sqrt[-a^(-2) + x^2])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a^2*x^2 - 2*a
*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m,x)
```

```
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.134 $\int e^{2\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=35

$$\frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)}{1+m}$$

[Out] x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],a*x)/(1+m)

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6302, 6261, 81, 66}

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;ax)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*x^m,x]

[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(1 + m)

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] && !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^m\,dx &= -\int e^{2\tanh^{-1}(ax)}x^m\,dx\\ &= -\int \frac{x^m(1+ax)}{1-ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-2\int \frac{x^m}{1-ax}\,dx\\ &= \frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;ax)}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.74

$$\frac{x^{1+m}(1-2\,{}_2F_1(1,1+m;2+m;ax))}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, a*x]))/(1 + m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.14, size = 106, normalized size = 3.03

method	result	size
meijerg	$-\frac{(-a)^{-m}\left(-\frac{x^m(-a)^m(amx+m+1)}{(1+m)m}+x^m(-a)^m\Phi(ax,1,m)\right)}{a}+\frac{(-a)^{-m}\left(-\frac{x^m(-a)^m(-1-m)}{(1+m)m}-x^m(-a)^m\Phi(ax,1,m)\right)}{a}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^m,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a)^(-m)/a*(-x^m*(-a)^m*(a*m*x+m+1)/(1+m)/m+x^m*(-a)^m*LerchPhi(a*x,1,m))
+(-a)^(-m)/a*(-1/(1+m)*x^m*(-a)^m*(-1-m)/m-x^m*(-a)^m*LerchPhi(a*x,1,m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(26) = 52$.
time = 2.20, size = 100, normalized size = 2.86

$$-\frac{amx^2x^m\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}-\frac{2ax^2x^m\Phi(ax,1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}-\frac{mxx^m\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}-\frac{xx^m\Phi(ax,1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**m,x)
```

```
[Out] -a*m*x**2*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - 2*a*x**2
*x**m*lerchphi(a*x, 1, m + 2)*gamma(m + 2)/gamma(m + 3) - m*x*x**m*lerchphi
(a*x, 1, m + 1)*gamma(m + 1)/gamma(m + 2) - x*x**m*lerchphi(a*x, 1, m + 1)*
gamma(m + 1)/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*x^m/(a*x - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^m*(a*x + 1))/(a*x - 1), x)
```

3.135 $\int e^{\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=74

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}+\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+x^m*hyperg
eom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6307, 822, 371}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) + (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\left(1+\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\right) - \frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}}\right.}{a}$$

$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.28, size = 128, normalized size = 1.73

$$x^{1+m}\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{-\frac{1}{a^2}+x^2}\,F_1\left(m;-\frac{1}{2},\frac{1}{2};1+m;-ax,ax\right)}{m\sqrt{-1+ax}\sqrt{\frac{1+ax}{a^2}}\sqrt{1-a^2x^2}} + \frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2};\frac{1}{2}-\frac{m}{2};\frac{1}{a^2x^2}\right)}{1+m}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*x^m,x]
```

```
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[-a^(-2) + x^2]*AppellF1[m, -1/2, 1
/2, 1 + m, -(a*x), a*x])/(m*Sqrt[-1 + a*x]*Sqrt[(1 + a*x)/a^2]*Sqrt[1 - a^2
*x^2])) + Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 +
m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="fricas")
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m,x)
[Out] Integral(x**m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m,x, algorithm="giac")
[Out] integrate(x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.136 $\int e^{-\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=75

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

```
[Out] x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-x^m*hyperg
eom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6307, 822, 371}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}-\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^ArcCoth[a*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(1 +
 m) - (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right) + \frac{\left(\left(\frac{1}{x}\right)^m x^m\right)\operatorname{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{a^2}}} \ldots\right)}{a}$$

$$= \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} - \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.17, size = 115, normalized size = 1.53

$$x^{1+m}\left(-\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}\,F_1\left(m;-\frac{1}{2},\frac{1}{2};1+m;ax,-ax\right)}{m\sqrt{1-ax}\sqrt{-\frac{1}{a^2}+x^2}} + \frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2};\frac{1}{2}-\frac{m}{2};\frac{1}{a^2x^2}\right)}{1+m}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^ArcCoth[a*x],x]
```

```
[Out] x^(1 + m)*(-((Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[m, -1/2,
1/2, 1 + m, a*x, -(a*x)])/(m*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])) + Hypergeo
metric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x^2)]/(1 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(x**m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.137 $\int e^{-2\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=36

$$\frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;-ax)}{1+m}$$

```
[Out] x^(1+m)/(1+m)-2*x^(1+m)*hypergeom([1, 1+m],[2+m],-a*x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6302, 6261, 81, 66}

$$\frac{x^{m+1}}{m+1}-\frac{2x^{m+1}\,{}_2F_1(1,m+1;m+2;-ax)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(2*ArcCoth[a*x]),x]
```

```
[Out] x^(1 + m)/(1 + m) - (2*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]
)/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6261

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> Int[x^m*((1 + a*x
)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, m, n}, x] &&  !IntegerQ[(n - 1)/2
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x^m\,dx &= -\int e^{-2\tanh^{-1}(ax)}x^m\,dx\\
&= -\int \frac{x^m(1-ax)}{1+ax}\,dx\\
&= \frac{x^{1+m}}{1+m}-2\int \frac{x^m}{1+ax}\,dx\\
&= \frac{x^{1+m}}{1+m}-\frac{2x^{1+m}\,{}_2F_1(1,1+m;2+m;-ax)}{1+m}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.75

$$\frac{x^{1+m}(1-2\,{}_2F_1(1,1+m;2+m;-ax))}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*(1 - 2*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(1 + m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.09, size = 93, normalized size = 2.58

method	result	size
meijerg	$a^{-1-m}\left(\frac{x^m a^m(amx-m-1)}{(1+m)m}+x^m a^m\Phi(-ax,1,m)\right)-a^{-1-m}\left(\frac{x^m a^m}{m}+\frac{x^m a^m(-1-m)\Phi(-ax,1,m)}{1+m}\right)$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] a^(-1-m)*(x^m*a^m*(a*m*x-m-1)/(1+m)/m+x^m*a^m*LerchPhi(-a*x,1,m))-a^(-1-m)*
(x^m*a^m/m+1/(1+m)*x^m*a^m*(-1-m)*LerchPhi(-a*x,1,m))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.10, size = 119, normalized size = 3.31

$$\frac{amx^2x^m\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}+\frac{2ax^2x^m\Phi(axe^{i\pi},1,m+2)\,\Gamma(m+2)}{\Gamma(m+3)}-\frac{mxx^m\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}-\frac{xx^m\Phi(axe^{i\pi},1,m+1)\,\Gamma(m+1)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a*x-1)/(a*x+1),x)
```

```
[Out] a*m*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/gamma(m
+ 3) + 2*a*x**2*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 2)*gamma(m + 2)/g
amma(m + 3) - m*x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1)
/gamma(m + 2) - x*x**m*lerchphi(a*x*exp_polar(I*pi), 1, m + 1)*gamma(m + 1)
/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)*x^m/(a*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^m*(a*x - 1))/(a*x + 1), x)
```

3.138 $\int e^{-3\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=150

$$-\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}+\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}+\frac{4x^{1+m}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m}-$$

```
[Out] -3*x^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)+x^m*hyp
ergeom([1/2, -1/2*m],[1-1/2*m],1/a^2/x^2)/a/m+4*x^(1+m)*hypergeom([3/2, -1/
2-1/2*m],[1/2-1/2*m],1/a^2/x^2)/(1+m)-4*x^m*hypergeom([3/2, -1/2*m],[1-1/2*
m],1/a^2/x^2)/a/m
```

Rubi [A]
time = 0.80, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6307, 6874, 371, 864, 822}

$$-\frac{3x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{4x^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-m-1);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{m+1}+\frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}-\frac{4x^m\,{}_2F_1\left(\frac{3}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) + (x^m*Hypergeometric2F1[1/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m) +
 (4*x^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, 1/(a^2*x^2)])/(
1 + m) - (4*x^m*Hypergeometric2F1[3/2, -1/2*m, 1 - m/2, 1/(a^2*x^2)])/(a*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
```

```
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6307

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)
^m, Subst[Int[(1 + x/a)^((n + 1)/2)/(x^(m + 2)*(1 - x/a)^((n - 1)/2)*Sqrt[1
 - x^2/a^2]), x], x, 1/x], x] /; FreeQ[{a, m}, x] && IntegerQ[(n - 1)/2] &&
  !IntegerQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \left(-\frac{3x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}+\frac{x^{-1-m}}{a\sqrt{1-\frac{x^2}{a^2}}}+\frac{4x^{-2-m}}{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\right)\right.\right.$$

$$= \left(3\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right) - \left(4\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{x}{a}\right.}\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} - \left(4\left(\frac{1}{x}\right)^m x\right.$$

$$= -\frac{3x^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-m);\frac{1-m}{2};\frac{1}{a^2x^2}\right)}{1+m} + \frac{x^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{a^2x^2}\right)}{am} + \frac{4x^{1+m}\,{}_2F_1(}{}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

time = 0.17, size = 192, normalized size = 1.28

$$\frac{x^{1+m}\left(-3(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}F_1\left(m;-\frac{1}{2},\frac{1}{2};1+m;ax,-ax\right)+2(1+m)\sqrt{1-\frac{1}{a^2x^2}}\sqrt{\frac{-1+ax}{a^2}}F_1\left(m;-\frac{1}{2},\frac{3}{2};1+m;ax,-ax\right)+m\sqrt{1-ax}\sqrt{-\frac{1}{a^2}+x^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-\frac{m}{2};\frac{1}{2}-\frac{m}{2};\frac{1}{a^2x^2}\right)\right)}{m(1+m)\sqrt{1-ax}\sqrt{-\frac{1}{a^2}+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*(-3*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-1 + a*x)/a^2]*AppellF1[
m, -1/2, 1/2, 1 + m, a*x, -(a*x)] + 2*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[(-
1 + a*x)/a^2]*AppellF1[m, -1/2, 3/2, 1 + m, a*x, -(a*x)] + m*Sqrt[1 - a*x]*
Sqrt[-a^(-2) + x^2]*Hypergeometric2F1[-1/2, -1/2 - m/2, 1/2 - m/2, 1/(a^2*x
^2)]))/(m*(1 + m)*Sqrt[1 - a*x]*Sqrt[-a^(-2) + x^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.139 $\int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=41

$$\frac{x^{1+m} F_1\left(-1-m;\frac{5}{4},-\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,5/4,-5/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6308, 138}

$$\frac{x^{m+1} F_1\left(-m-1;\frac{5}{4},-\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 5/4, -5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{5}{2}\coth^{-1}(ax)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{5/4}}{\left(1-\frac{x}{a}\right)^{5/4}} \, dx, x, \frac{1}{x}\right)\right)\\ &= \frac{x^{1+m} F_1\left(-1-m;\frac{5}{4},-\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int e^{\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^((5*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((5*ArcCoth[a*x])/2)*x^m, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a^2*x^2 - 2
*a*x + 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(5/4)*x**m,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(5/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.140 $\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{1+m}F_1\left(-1-m;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,3/4,-3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 3/4, -3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1+\frac{x}{a}\right)^{3/4}}{\left(1-\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;\frac{3}{4},-\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int e^{\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^((3*ArcCoth[a*x])/2)*x^m,x]
```

```
[Out] Integrate[E^((3*ArcCoth[a*x])/2)*x^m, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/4)*x**m,x)
```

```
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(3/4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.141 $\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=41

$$\frac{x^{1+m} F_1\left(-1-m;\frac{1}{4},-\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/4,-1/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6308, 138}

$$\frac{x^{m+1} F_1\left(-m-1;\frac{1}{4},-\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/2)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/4, -1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{1}{2}\coth^{-1}(ax)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[4]{1+\frac{x}{a}}}{\sqrt[4]{1-\frac{x}{a}}} \, dx, x, \frac{1}{x}\right)\right)\\
&= \frac{x^{1+m} F_1\left(-1-m;\frac{1}{4},-\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}
\end{aligned}$$

<u>**Mathematica**</u> [**F**]
time = 0.35, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(ArcCoth[a*x]/2)*x^m,x]
```

```
[Out] Integrate[E^(ArcCoth[a*x]/2)*x^m, x]
```

<u>**Maple**</u> [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x)
```

<u>**Maxima**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

<u>**Fricas**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(3/4)/(a*x - 1), x)
```

<u>**Sympy**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt[4]{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/4)*x**m,x)
```

```
[Out] Integral(x**m/((a*x - 1)/(a*x + 1))**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/4)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.142 $\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{1+m}F_1\left(-1-m;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-1/4,1/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^(ArcCoth[a*x]/2),x]
[Out] (x^(1 + m)*AppellF1[-1 - m, -1/4, 1/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\sqrt[4]{1-\frac{x}{a}}}{\sqrt[4]{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;-\frac{1}{4},\frac{1}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [**F**]
time = 0.39, size = 0, normalized size = 0.00

$$\int e^{-\frac{1}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/E^(ArcCoth[a*x]/2),x]
```

```
[Out] Integrate[x^m/E^(ArcCoth[a*x]/2), x]
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(1/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(1/4),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(1/4),x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(\frac{a\,x-1}{a\,x+1}\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(1/4),x)
```

```
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(1/4), x)
```

3.143 $\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{1+m}F_1\left(-1-m;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-3/4,3/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6308, 138}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((3*ArcCoth[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -3/4, 3/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{3/4}}{\left(1+\frac{x}{a}\right)^{3/4}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;-\frac{3}{4},\frac{3}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int e^{-\frac{3}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/E^((3*ArcCoth[a*x])/2),x]
[Out] Integrate[x^m/E^((3*ArcCoth[a*x])/2), x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
[Out] int(x^m*((a*x-1)/(a*x+1))^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="maxima")
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="fricas")
[Out] integral(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(3/4),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \left(\frac{a\,x-1}{a\,x+1}\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(3/4),x)
```

```
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(3/4), x)
```

3.144 $\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{1+m}F_1\left(-1-m;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,-5/4,5/4,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6308, 138}

$$\frac{x^{m+1}F_1\left(-m-1;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, -5/4, 5/4, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\left(1-\frac{x}{a}\right)^{5/4}}{\left(1+\frac{x}{a}\right)^{5/4}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;-\frac{5}{4},\frac{5}{4};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int e^{-\frac{5}{2}\coth^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^m/E^((5*ArcCoth[a*x])/2),x]
```

```
[Out] Integrate[x^m/E^((5*ArcCoth[a*x])/2), x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^m\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

```
[Out] int(x^m*((a*x-1)/(a*x+1))^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*x^m*((a*x - 1)/(a*x + 1))^(1/4)/(a*x + 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*((a*x-1)/(a*x+1))**(5/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*((a*x-1)/(a*x+1))^(5/4),x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*((a*x - 1)/(a*x + 1))^(5/4),x)
```

```
[Out] int(x^m*((a*x - 1)/(a*x + 1))^(5/4), x)
```

3.145 $\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx$

Optimal. Leaf size=34

$$\frac{x^{1+m}F_1\left(-1-m;\frac{1}{3},-\frac{1}{3};-m;\frac{1}{x},-\frac{1}{x}\right)}{1+m}$$

[Out] x^(1+m)*AppellF1(-1-m,1/3,-1/3,-m,1/x,-1/x)/(1+m)

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6308, 138}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{3},-\frac{1}{3};-m;\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Int[E^((2*ArcCoth[x])/3)*x^m,x]

[Out] (x^(1 + m)*AppellF1[-1 - m, 1/3, -1/3, -m, x^(-1), -x^(-1)])/(1 + m)

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\sqrt[3]{1+x}}{\sqrt[3]{1-x}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;\frac{1}{3},-\frac{1}{3};-m;\frac{1}{x},-\frac{1}{x}\right)}{1+m}\end{aligned}$$

Mathematica [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int e^{\frac{2}{3}\coth^{-1}(x)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^((2*ArcCoth[x])/3)*x^m,x]
[Out] Integrate[E^((2*ArcCoth[x])/3)*x^m, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{-1+x}{1+x}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/3)*x^m,x)
[Out] int(1/((-1+x)/(1+x))^(1/3)*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="fricas")
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(2/3)/(x - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt[3]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/3)*x**m,x)
```

```
[Out] Integral(x**m/((x - 1)/(x + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/3)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(\frac{x-1}{x+1}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((x - 1)/(x + 1))^(1/3),x)
```

```
[Out] int(x^m/((x - 1)/(x + 1))^(1/3), x)
```

3.146 $\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx$

Optimal. Leaf size=34

$$\frac{x^{1+m} F_1\left(-1-m;\frac{1}{6},-\frac{1}{6};-m;\frac{1}{x},-\frac{1}{x}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/6,-1/6,-m,1/x,-1/x)/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6308, 138}

$$\frac{x^{m+1} F_1\left(-m-1;\frac{1}{6},-\frac{1}{6};-m;\frac{1}{x},-\frac{1}{x}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[x]/3)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/6, -1/6, -m, x^(-1), -x^(-1)])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[n] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\frac{1}{3}\coth^{-1}(x)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int \frac{x^{-2-m}\sqrt[6]{1+x}}{\sqrt[6]{1-x}} \, dx, x, \frac{1}{x}\right)\right)\\
&= \frac{x^{1+m} F_1\left(-1-m;\frac{1}{6},-\frac{1}{6};-m;\frac{1}{x},-\frac{1}{x}\right)}{1+m}
\end{aligned}$$

Mathematica [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{3}\coth^{-1}(x)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(ArcCoth[x]/3)*x^m,x]
[Out] Integrate[E^(ArcCoth[x]/3)*x^m, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{-1+x}{1+x}\right)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/6)*x^m,x)
[Out] int(1/((-1+x)/(1+x))^(1/6)*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="maxima")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="fricas")
[Out] integral((x + 1)*x^m*((x - 1)/(x + 1))^(5/6)/(x - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt[6]{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/6)*x**m,x)
[Out] Integral(x**m/((x - 1)/(x + 1))**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/6)*x^m,x, algorithm="giac")
[Out] integrate(x^m/((x - 1)/(x + 1))^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(\frac{x-1}{x+1}\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((x - 1)/(x + 1))^(1/6),x)
[Out] int(x^m/((x - 1)/(x + 1))^(1/6), x)
```

3.147 $\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$

Optimal. Leaf size=41

$$\frac{x^{1+m}F_1\left(-1-m;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/8,-1/8,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6308, 138}

$$\frac{x^{m+1}F_1\left(-m-1;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(ArcCoth[a*x]/4)*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, 1/8, -1/8, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int\frac{x^{-2-m}\sqrt[8]{1+\frac{x}{a}}}{\sqrt[8]{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\ &= \frac{x^{1+m}F_1\left(-1-m;\frac{1}{8},-\frac{1}{8};-m;\frac{1}{ax},-\frac{1}{ax}\right)}{1+m}\end{aligned}$$

Mathematica [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int e^{\frac{1}{4}\coth^{-1}(ax)}x^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(ArcCoth[a*x]/4)*x^m,x]
```

```
[Out] Integrate[E^(ArcCoth[a*x]/4)*x^m, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(\frac{ax-1}{ax+1}\right)^{\frac{1}{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*x^m*((a*x - 1)/(a*x + 1))^(7/8)/(a*x - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/8)*x**m,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/8)*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\left(\frac{a\,x-1}{a\,x+1}\right)^{1/8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a*x - 1)/(a*x + 1))^(1/8),x)
```

```
[Out] int(x^m/((a*x - 1)/(a*x + 1))^(1/8), x)
```

3.148 $\int e^{n \coth^{-1}(ax)} x^m \, dx$

Optimal. Leaf size=45

$$\frac{x^{1+m} F_1\left(-1-m; \frac{n}{2}, -\frac{n}{2}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}$$

```
[Out] x^(1+m)*AppellF1(-1-m,1/2*n,-1/2*n,-m,1/a/x,-1/a/x)/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6308, 138}

$$\frac{x^{m+1} F_1\left(-m-1; \frac{n}{2}, -\frac{n}{2}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x^m,x]
```

```
[Out] (x^(1 + m)*AppellF1[-1 - m, n/2, -1/2*n, -m, 1/(a*x), -(1/(a*x))])/(1 + m)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 6308

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_), x_Symbol] :> Dist[(-x^m)*(1/x)^
m, Subst[Int[(1 + x/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x], x] /
; FreeQ[{a, m, n}, x] &&  !IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{n \coth^{-1}(ax)} x^m \, dx &= -\left(\left(\left(\frac{1}{x}\right)^m x^m\right) \text{Subst}\left(\int x^{-2-m} \left(1-\frac{x}{a}\right)^{-n/2} \left(1+\frac{x}{a}\right)^{n/2} \, dx, x, \frac{1}{x}\right)\right) \\
&= \frac{x^{1+m} F_1\left(-1-m; \frac{n}{2}, -\frac{n}{2}; -m; \frac{1}{ax}, -\frac{1}{ax}\right)}{1+m}
\end{aligned}$$

Mathematica [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int e^{n \coth^{-1}(ax)} x^m \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^m,x]
```

```
[Out] Integrate[E^(n*ArcCoth[a*x])*x^m, x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)} x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^m,x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="maxima")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="fricas")
```

```
[Out] integral(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m e^{n\,\mathrm{acoth}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**m,x)
```

```
[Out] Integral(x**m*exp(n*acoth(a*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^m,x, algorithm="giac")
```

```
[Out] integrate(x^m*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*exp(n*acoth(a*x)),x)
```

```
[Out] int(x^m*exp(n*acoth(a*x)), x)
```

3.149 $\int e^{n\coth^{-1}(ax)}x^2\,dx$

Optimal. Leaf size=174

$$\frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3+\frac{2(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\,{}_2F_1}{3a^3(2-n)}$$

```
[Out] 1/6*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/a+1/3*(1-1/a/x)^(1-1/2*n)
*(1+1/a/x)^(1+1/2*n)*x^3+2/3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*
n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^3/(2-n)
```

Rubi [A]
time = 0.06, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6306, 105, 156, 12, 133}

$$\frac{2(n^2+2)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{3a^3(2-n)}+\frac{1}{3}x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{nx^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/(6*a) + ((1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3)/3 + (2*(2 + n^2)*(1 - 1/(a*x
))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n
/2, (a - x^(-1))/(a + x^(-1))])/(3*a^3*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}x^2\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx,x,\frac{1}{x}\right)\\
&= \frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3+\frac{1}{3}\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx\right.\\
&= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3-\frac{1}{6}\text{Subst}\left(\int \frac{(2+\right.\right.\\
&= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3-\frac{(2+n^2)\,\text{Subst}\left(\right.}{}\\
&= \frac{n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{6a}+\frac{1}{3}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3+\frac{2(2+n^2)\left(1-\frac{1}{a}\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.56, size = 118, normalized size = 0.68

$$\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(2+n^2)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(an^2x+2a^3x^3+n(-1+a^2x^2)+(2+n^2)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{6a^3(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometric2F1[1, 1
+ n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n^2*x + 2*a^3*x^3 + n*(-1
+ a^2*x^2) + (2 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x]
)])))/(6*a^3*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2,x)
```

```
[Out] Integral(x**2*exp(n*acoth(a*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(n*acoth(a*x)),x)
```

```
[Out] int(x^2*exp(n*acoth(a*x)), x)
```

3.150 $\int e^{n \coth^{-1}(ax)} x \, dx$

Optimal. Leaf size=122

$$\frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x^2+\frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)} {}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}$$

```
[Out] 1/2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2+2*n*(1-1/a/x)^(1-1/2*n)*(1+
1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n],(a-1/x)/(a+1/x))/a^2/(2-
n)
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6306, 98, 133}

$$\frac{2n\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} {}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(2-n)}+\frac{1}{2}x^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*x,x]
```

```
[Out] ((1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/2 + (2*n*(1 - 1/(a*
x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, (a - x^(-1))/(a + x^(-1))])/(a^2*(2 - n))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}x\,dx = -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 - \frac{n\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{1}{2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2 + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}{}_2F_1\left(2,1-\frac{n}{2};2-\right.}{a^2(2-n)}$$

Mathematica [A]
time = 0.21, size = 98, normalized size = 0.80

$$\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n^2\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+a^2x^2+n\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{2a^2(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*x,x]
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + a^2*x^2 + n*Hypergeomet
ric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(2*a^2*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{n\,\text{arccoth}(ax)}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x,x)
[Out] int(exp(n*arccoth(a*x))*x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="maxima")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="fricas")
[Out] integral(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{n\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x,x)
[Out] Integral(x*exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x,x, algorithm="giac")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(n*acoth(a*x)),x)
[Out] int(x*exp(n*acoth(a*x)), x)
```

3.151 $\int e^{n\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=78

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] 4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n]
,(a-1/x)/(a+1/x))/a/(2-n)
```

Rubi [A]

time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6305, 133}

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 82, normalized size = 1.05

$$\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(ax+{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{a(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x)),x)
[Out] int(exp(n*arccoth(a*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x)),x)
[Out] Integral(exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x)),x)
[Out] int(exp(n*acoth(a*x)), x)
```

3.152 $\int \frac{e^{n \coth^{-1}(ax)}}{x} \, dx$

Optimal. Leaf size=127

$$-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}+\frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{n}$$

```
[Out] -2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/n/((1
-1/a/x)^(1/2*n))+2^(1+1/2*n)*hypergeom([-1/2*n, -1/2*n],[1-1/2*n],1/2*(a-1/
x)/a)/n/((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6306, 132, 71, 133}

$$\frac{2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(-\frac{n}{2},-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{n}-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (-2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/
(a + x^(-1))])/(n*(1 - 1/(a*x))^(n/2)) + (2^(1 + n/2)*Hypergeometric2F1[-1/
2*n, -1/2*n, 1 - n/2, (a - x^(-1))/(2*a)])/(n*(1 - 1/(a*x))^(n/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] || !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x}\,dx &= -\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)\\ &= \frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a} - \text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x,\right.\\ &= -\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{n} + \frac{2^{1+\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\,{}_2F_1\left(-\frac{n}{2},-\frac{n}{2}\right.}{n}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 142, normalized size = 1.12

$$\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)-(2+n)\left({}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)-{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{n(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, -E^(2*ArcCoth[a*x])] + E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] - (2 + n)*(Hypergeometric2F1[1, n/2, 1 +
n/2, -E^(2*ArcCoth[a*x])] - Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth
[a*x])])))/(n*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x,x)
[Out] int(exp(n*acoth(a*x))/x, x)
```

3.153 $\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx$

Optimal. Leaf size=70

$$\frac{2^{1+\frac{n}{2}}a\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

```
[Out] 2^(1+1/2*n)*a*(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2
*(a-1/x)/a)/(2-n)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6306, 71}

$$\frac{a2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^2,x]
```

```
[Out] (2^(1 + n/2)*a*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2
 - n/2, (a - x^(-1))/(2*a)])/(2 - n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x^2}\,dx &= -\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{2^{1+\frac{n}{2}}a\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.63

$$-\frac{4ae^{(2+n)\coth^{-1}(ax)}\,{}_2F_1\left(2,1+\frac{n}{2};2+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)}{2+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^2,x]
```

```
[Out] (-4*a*E^((2 + n)*ArcCoth[a*x])*Hypergeometric2F1[2, 1 + n/2, 2 + n/2, -E^(2
*ArcCoth[a*x])])/(2 + n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**2,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^2,x)
```

```
[Out] int(exp(n*acoth(a*x))/x^2, x)
```

3.154 $\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx$

Optimal. Leaf size=114

$$\frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

```
[Out] 1/2*a^2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+2^(1/2*n)*a^2*n*(1-1/a/x)^(
1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

Rubi [A]
time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6306, 81, 71}

$$\frac{a^2 2^{n/2}n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}+\frac{1}{2}a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^3,x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/2 + (2^(n/2)*a^2*n*
(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a - x^
(-1))/(2*a)])/(2 - n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
```

```
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x^3}\,dx = -\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} - \frac{1}{2}(an)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} + \frac{2^{n/2}a^2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{2-n}$$

Mathematica [A]
time = 0.46, size = 107, normalized size = 0.94

$$-\frac{a^2e^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n^2\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+\frac{1}{a^2x^2}+\frac{n}{ax}+n\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)\right)\right)}{2(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^3,x]
```

```
[Out] -1/2*(a^2*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1,
 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-1 + 1/(a^2*x^2) + n/(a
*x) + n*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}{(ax)}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**3,x)
[Out] Integral(exp(n*acoth(a*x))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^3,x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}\left(a\,x\right)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^3,x)
[Out] int(exp(n*acoth(a*x))/x^3, x)
```

3.155 $\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx$

Optimal. Leaf size=167

$$\frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}+\frac{2^{n/2}a^3(2+n^2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2}\right.}{3(2-n)}$$

```
[Out] 1/6*a^3*n*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)+1/3*a^2*(1-1/a/x)^(1-1/2*
n)*(1+1/a/x)^(1+1/2*n)/x+1/3*2^(1/2*n)*a^3*(n^2+2)*(1-1/a/x)^(1-1/2*n)*hype
rgeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2-n)
```

Rubi [A]

time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6306, 92, 81, 71}

$$\frac{a^3 2^{n/2}(n^2+2)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{6}a^3n\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^4,x]
```

```
[Out] (a^3*n*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/6 + (a^2*(1 - 1/(
a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(3*x) + (2^(n/2)*a^3*(2 + n^2)*(
1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a - x^(
-1))/(2*a)])/(3*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{x^4}\,dx &= -\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}+\frac{1}{3}a^2\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{nx}{a}\right)\,dx,x\right.\\
&= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}-\frac{1}{6}\left(a^2\left(2+n^2\right)\right)\text{Subs}\\
&= \frac{1}{6}a^3n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{3x}+\frac{2^{n/2}a^3\left(2+n^2\right)\left(1-\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.43, size = 132, normalized size = 0.79

$$-\frac{a^3e^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n\left(2+n^2\right)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-\left(\left(1-\frac{1}{a^2x^2}\right)\left(n+\frac{2}{ax}\right)\right)+\frac{2+n^2}{ax}+\left(2+n^2\right)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)\right)\right)}{6(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^4,x]
```

```
[Out] -1/6*(a^3*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*(2 + n^2)*Hypergeometr
ic2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]) + (2 + n)*(-((1 - 1/(a^2*x
^2))*(n + 2/(a*x))) + (2 + n^2)/(a*x) + (2 + n^2)*Hypergeometric2F1[1, n/2,
 1 + n/2, -E^(2*ArcCoth[a*x])])))/(2 + n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^4,x)
```

```
[Out] int(exp(n*arccoth(a*x))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**4,x)
```

```
[Out] Integral(exp(n*acoth(a*x))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{acoth}(a x)}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^4,x)
```

```
[Out] int(exp(n*acoth(a*x))/x^4, x)
```

3.156 $\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx$

Optimal. Leaf size=183

$$\frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a(6+n^2)+\frac{2n}{x}\right)+\frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2}+\frac{2^{-2+\frac{n}{2}}a^4n(8+n^2)\left(1-\right.}{}$$

```
[Out] 1/24*a^3*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(a*(n^2+6)+2*n/x)+1/4*a^2*
(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)/x^2+1/3*2^(-2+1/2*n)*a^4*n*(n^2+8)*
(1-1/a/x)^(1-1/2*n)*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/(2
-n)
```

Rubi [A]
time = 0.09, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6306, 102, 152, 71}

$$\frac{a^4 2^{\frac{n}{2}-2}n(n^2+8)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{3(2-n)}+\frac{1}{24}a^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(a(n^2+6)+\frac{2n}{x}\right)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}+\frac{a^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/x^5,x]
```

```
[Out] (a^3*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(a*(6 + n^2) + (2*n)
/x))/24 + (a^2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(4*x^2) +
 (2^(-2 + n/2)*a^4*n*(8 + n^2)*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1
- n/2, -1/2*n, 2 - n/2, (a - x^(-1))/(2*a)])/(3*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 6306

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_)^(m_.), x_Symbol] :> -Subst[Int[(1 + x
/a)^(n/2)/(x^(m + 2)*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&
!IntegerQ[n] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{n\coth^{-1}(ax)}}{x^5}\,dx &= -\text{Subst}\left(\int x^3\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)\\ &= \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} + \frac{1}{4}a^2\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}\left(-2-\frac{nx}{a}\right)\,dx\right.\\ &= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right) + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} -\\ &= \frac{1}{24}a^3\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(a\left(6+n^2\right)+\frac{2n}{x}\right) + \frac{a^2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{4x^2} +\end{aligned}$$

Mathematica [A]
time = 0.35, size = 148, normalized size = 0.81

$$-\frac{1}{24}a^4e^{n\coth^{-1}(ax)}\left(-6-n^2+\frac{6}{a^4x^4}+\frac{2n}{a^3x^3}+\frac{n^2}{a^2x^2}+\frac{6n}{ax}+\frac{n^3}{ax}-\frac{e^{2\coth^{-1}(ax)}n^2(8+n^2)\;{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)}{2+n}+n\left(8+n^2\right)\;{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/x^5,x]
```

```
[Out] -1/24*(a^4*E^(n*ArcCoth[a*x])*(-6 - n^2 + 6/(a^4*x^4) + (2*n)/(a^3*x^3) + n
^2/(a^2*x^2) + (6*n)/(a*x) + n^3/(a*x) - (E^(2*ArcCoth[a*x])*n^2*(8 + n^2)*
```

```
Hypergeometric2F1[1, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])])/(2 + n) + n*(8
 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])]))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x^5,x)
[Out] int(exp(n*arccoth(a*x))/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}{\left(ax \right)}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x**5,x)
[Out] Integral(exp(n*acoth(a*x))/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/x^5,x)
```

```
[Out] int(exp(n*acoth(a*x))/x^5, x)
```

3.157 $\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p}{1+p}+\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\sqrt{1+\frac{1}{ax}}\,(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2-p)*(-a*c*x+c)^p*hypergeom([-p, 1/2-p],[1-p],2/(a+1/x
)/x)*(1+1/a/x)^(1/2)/a/p/(1+p)/(1-1/a/x)^(1/2)+x*(-a*c*x+c)^p*(1-1/a/x)^(1/
2)*(1+1/a/x)^(1/2)/(1+p)
```

Rubi [A]
time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 134}

$$\frac{\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{ap(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{x\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}\,(c-acx)^p}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^p)/(1 + p) + (((a - x^(-
1))/(a + x^(-1)))^(1/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric2
F1[1/2 - p, -p, 1 - p, 2/((a + x^(-1))*x)])/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}\right.\right.\right.$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p}{1+p} - \frac{\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int \frac{x^{-}}{}\right.}{a(1+p)}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p}{1+p} + \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\sqrt{1+\frac{1}{ax}}\,(c-acx)^p\,{}_2F_1\left(\frac{1}{2}-p\right.}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.06, size = 131, normalized size = 0.92

$$\frac{\sqrt{1+\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left(p(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^p+\sqrt{\frac{-1+ax}{1+ax}}\,{}_2F_1\left(\frac{1}{2}-p,-p;1-p;\frac{2}{1+ax}\right)\right)}{ap(1+p)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*(p*(-1 + a*x)*((-1 + a*x)/(1 + a*x))^p + S
qrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1/2 - p, -p, 1 - p, 2/(1 + a*x)
]))/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^p)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^{p}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{p}}{\sqrt{\frac{ax-1}{ax+1}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c-a\,c\,x)^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.158 $\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=132

$$-\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5+\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] 17/15*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3-3/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1
/5*a^4*c^4*(1-1/a^2/x^2)^(3/2)*x^5+7/8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a-7
/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6310, 6313, 1821, 821, 272, 43, 65, 214}

$$-\frac{7}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{17}{15}a^2c^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (-7*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (17*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2
)*x^3)/15 - (3*a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (a^4*c^4*(1 - 1/(a^
2*x^2))^(3/2)*x^5)/5 + (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^4\,dx = \left(a^4c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4 x^4\,dx$$

$$= -\left(\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5 + \frac{1}{5}\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(\frac{15}{a}-\frac{17x}{a^2}+\frac{5x^2}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx, x,$$

$$= -\frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5 - \frac{1}{20}\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(\frac{68}{a^2}\right.}{}\right.$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5 +$$

$$= \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5 +$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 +$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 +$$

$$= -\frac{7}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{17}{15}a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{3}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 +$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 0.61

$$\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-136-15ax+112a^2x^2-90a^3x^3+24a^4x^4\right)+105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A]
time = 0.09, size = 183, normalized size = 1.39

method	result
risch	$\frac{(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136)(ax-1)c^4}{120a\sqrt{\frac{ax-1}{ax+1}}}+\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax+1)(ax-1)}}{8\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^4\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-90\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+120((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-105\sqrt{a^2}\right.}{120a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(a*x-1)*c^4/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-90*(a^2)^(1/2
)*(a^2*x^2-1)^(3/2)*a*x+120*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+16*(a^2*x^2
-1)^(3/2)*(a^2)^(1/2)-105*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+105*ln((a^2*x+(
a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x
+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(112) = 224.
time = 0.26, size = 259, normalized size = 1.96

$$\frac{1}{120}\left(\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+490\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^4\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{5(ax-1)a^2}{ax+1}-\frac{10(ax-1)^2a^2}{(ax+1)^2}+\frac{10(ax-1)^3a^2}{(ax+1)^3}-\frac{5(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5a^2}{(ax+1)^5}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^4*((a*x - 1)/(a*x + 1))^(9/2) + 790*
c^4*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 490
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a
*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2
/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5
- a^2))*a
```

Fricas [A]
time = 0.39, size = 125, normalized size = 0.95

$$\frac{105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^4x^5-66\,a^4c^4x^4+22\,a^3c^4x^3+97\,a^2c^4x^2-151\,ac^4x-136\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^4*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^4*x^5 - 66*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 9
7*a^2*c^4*x^2 - 151*a*c^4*x - 136*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(6*a*
*2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**3*x**3/sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**4*x**4/sqrt(a*x/(a*x + 1) -
1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.43, size = 138, normalized size = 1.05

$$-\frac{7\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)}-\frac{1}{120}\sqrt{a^2x^2-1}\left(\left(\left(\frac{15\,c^4}{\mathrm{sgn}\,(ax+1)}-2\left(\frac{56\,ac^4}{\mathrm{sgn}\,(ax+1)}+3\left(\frac{4\,a^3c^4x}{\mathrm{sgn}\,(ax+1)}-\frac{15\,a^2c^4}{\mathrm{sgn}\,(ax+1)}\right)x\right)x\right)x+\frac{136\,c^4}{a\mathrm{sgn}\,(ax+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -7/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1/
120*sqrt(a^2*x^2 - 1)*((15*c^4/sgn(a*x + 1) - 2*(56*a*c^4/sgn(a*x + 1) + 3*
(4*a^3*c^4*x/sgn(a*x + 1) - 15*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 136*c^4/(a*s
gn(a*x + 1)))
```

Mupad [B]
time = 0.13, size = 214, normalized size = 1.62

$$\frac{\frac{49\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}-\frac{7\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{15}+\frac{79\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{6}+\frac{7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}+\frac{7\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((49*c^4*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^4*((a*x - 1)/(a*x + 1))^(1/2
))/4 - (224*c^4*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^4*((a*x - 1)/(a*x +
 1))^(7/2))/6 + (7*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*c^4*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.159 $\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=105

$$-\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] 2/3*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3-1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4+5/8
*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-5/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6310, 6313, 1821, 821, 272, 43, 65, 214}

$$-\frac{5}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{2}{3}a^2c^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (-5*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + (2*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)
*x^3)/3 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^4)/4 + (5*c^3*ArcTanh[Sqrt[1 -
 1/(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(\frac{8}{a}-\frac{5x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{4}\left(5ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-}}{x^3}\right.$$

$$= \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}\left(5ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-}}{x^2}\right.$$

$$(5$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - -$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{8}$$

$$5a$$

$$= -\frac{5}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{2}{3}a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + -$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.70

$$\frac{c^3\left(-a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(16+9ax-16a^2x^2+6a^3x^3\right)+15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(-(a*Sqrt[1 - 1/(a^2*x^2)]*x*(16 + 9*a*x - 16*a^2*x^2 + 6*a^3*x^3)) +
15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Maple [A]
time = 0.08, size = 141, normalized size = 1.34

method	result
risch	$-\frac{(6a^3x^3-16a^2x^2+9ax+16)(ax-1)c^3}{24a\sqrt{\frac{ax-1}{ax+1}}}+\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax+1)(ax-1)}}{8\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$-\frac{(ax-1)c^3\left(6\sqrt{a^2}(a^2x^2-1)^{\frac{3}{2}}ax+15\sqrt{a^2}\sqrt{a^2x^2-1}\,ax-16((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a}}{\sqrt{a^2}}\right.\right.}{24a\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)(ax-1)}\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(a*x-1)*c^3/a*(6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+15*(a^2)^(1/2)*(a^
2*x^2-1)^(1/2)*a*x-16*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-15*ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)
*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(89) = 178.
time = 0.27, size = 221, normalized size = 2.10

$$\frac{1}{24}\left(\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(15\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+73\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^2}{ax+1}-\frac{6(ax-1)^2a^2}{(ax+1)^2}+\frac{4(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 + 2*(15*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 73*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - 55*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^3*sq
rt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*
x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^
2))*a
```

Fricas [A]
time = 0.43, size = 115, normalized size = 1.10

$$\frac{15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^3x^4-10\,a^3c^3x^3-7\,a^2c^3x^2+25\,ac^3x+16\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(15*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^3*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (6*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 7*a^2*c^3*x^2 + 25*a*c
^3*x + 16*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{3ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^3x^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a
**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3/sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(a*x +
1)), x))
```

Giac [A]
time = 0.41, size = 118, normalized size = 1.12

$$-\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\left(ax+1\right)}-\frac{1}{24}\sqrt{a^2x^2-1}\left(\left(2\left(\frac{3\,a^2c^3x}{\mathrm{sgn}\left(ax+1\right)}-\frac{8\,ac^3}{\mathrm{sgn}\left(ax+1\right)}\right)x+\frac{9\,c^3}{\mathrm{sgn}\left(ax+1\right)}\right)x+\frac{16\,c^3}{a\mathrm{sgn}\left(ax+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -5/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1/
24*sqrt(a^2*x^2 - 1)*((2*(3*a^2*c^3*x/sgn(a*x + 1) - 8*a*c^3/sgn(a*x + 1))*
x + 9*c^3/sgn(a*x + 1))*x + 16*c^3/(a*sgn(a*x + 1)))
```

Mupad [B]
time = 0.08, size = 177, normalized size = 1.69

$$\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{55\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{73\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}+\frac{5\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a) - ((5*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/4 - (55*c^3*((a*x - 1)/(a*x + 1))^(3/2))/12 + (73*c^3*((a*x -
1)/(a*x + 1))^(5/2))/12 + (5*c^3*((a*x - 1)/(a*x + 1))^(7/2))/4)/(a - (4*a*
(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2 - (4*a*(a*x - 1)^3)/(a
*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4)
```

3.160 $\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=78

$$-\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] 1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3+1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a-
1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6310, 6313, 821, 272, 43, 65, 214}

$$-\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

```
[Out] -1/2*(a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x
^3)/3 + (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^2\,dx = \left(a^2c^2\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x\right.}{4a}$$

$$= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\right.$$

$$= -\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [**A**]
time = 0.11, size = 64, normalized size = 0.82

$$\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-2-3ax+2a^2x^2)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - 3*a*x + 2*a^2*x^2) + 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [**A**]
time = 0.09, size = 121, normalized size = 1.55

method	result	size
risch	$\frac{(2a^2x^2-3ax-2)(ax-1)c^2}{6a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$	112
default	$-\frac{(ax-1)c^2\left(3\sqrt{a^2}\sqrt{a^2x^2-1}\,ax-2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)*c^2*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-2*((a*x+1)*(a*x-1))^(
3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)
/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(66) = 132$.
time = 0.27, size = 181, normalized size = 2.32

$$\frac{1}{6}a\left(\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 8*c^2*((a*
x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2
/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
a^2))
```

Fricas [A]
time = 0.34, size = 103, normalized size = 1.32

$$\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2a^3c^2x^3-a^2c^2x^2-5ac^2x-2c^2)\sqrt{\frac{ax-1}{ax+1}}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) + (2*a^3*c^2*x^3 - a^2*c^2*x^2 - 5*a*c^2*x - 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.43, size = 98, normalized size = 1.26

$$\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,ac^2x}{\mathrm{sgn}\,(ax+1)}-\frac{3\,c^2}{\mathrm{sgn}\,(ax+1)}\right)x-\frac{2\,c^2}{a\mathrm{sgn}\,(ax+1)}\right)-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) - 3*c^2/sgn(a*x + 1))*x - 2*
c^2/(a*sgn(a*x + 1))) - 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(ab
s(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.21, size = 138, normalized size = 1.77

$$\frac{\frac{8\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - c^2*((a*x - 1)/(a*x + 1))^(1/2) +
c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (c^2*atanh(((a*x - 1)
/(a*x + 1))^(1/2)))/a
```

3.161 $\int e^{\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=47

$$-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

[Out] 1/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6310, 6313, 272, 43, 65, 214}

$$\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]*(c - a*c*x),x]

[Out] -1/2*(a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)\,dx = -\left((ac)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$= (ac)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{2}(ac)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$
$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{2}(ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)$$
$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [**A**]
time = 0.06, size = 51, normalized size = 1.09

$$\frac{c\left(-a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x),x]
```

```
[Out] (c*(-(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]
))/(2*a)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(39) = 78$.
time = 0.04, size = 93, normalized size = 1.98

method	result	size

risch	$-\frac{x(ax-1)c}{2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	92
default	$-\frac{(ax-1)c\left(x\sqrt{a^2x^2-1}\,\sqrt{a^2}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x-1)*c*(x*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)
*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)
/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(39) = 78$.
time = 0.27, size = 132, normalized size = 2.81

$$\frac{1}{2}\,a\left(\frac{2\left(c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(c*((a*x - 1)/(a*x + 1))^(3/2) + c*sqrt((a*x - 1)/(a*x + 1)))/(2*(
a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + c*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.38, size = 77, normalized size = 1.64

$$\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2+acx)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - c*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (a^2*c*x^2 + a*c*x)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{ax}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a
*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.41, size = 58, normalized size = 1.23

$$-\frac{\sqrt{a^2x^2-1}\,cx}{2\,\mathrm{sgn}\left(ax+1\right)}-\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2*x^2 - 1)*c*x/sgn(a*x + 1) - 1/2*c*log(abs(-x*abs(a) + sqrt(a^
2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.20, size = 94, normalized size = 2.00

$$\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2) +
 c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2*a*(a*x - 1))/(a*x + 1) + (a*(a*x -
1)^2)/(a*x + 1)^2)
```

3.162 $\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=51

$$\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6310, 6313, 866, 1819, 272, 65, 214}

$$\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x),x]
```

```
[Out] (2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$

$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 1.18

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\,x+(1-ax)\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*x + (1 - a*x)*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x])/(a*c*(-1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(47) = 94$.
time = 0.13, size = 249, normalized size = 4.88

method	result
default	$\frac{-\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2-\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}+2\sqrt{(}}{a\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-ln((a^2*x+(a^2)^(1/2)*((a
*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1
/2)+2*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+2*ln((a^2*x+(a^2)^(1/2)*((a*x
+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-
a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/(a^2)^(1/2)/
(a*x-1)/c/((a*x+1)*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.27, size = 78, normalized size = 1.53

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2}{a^2c\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2/(a^2*c*sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A]
time = 0.39, size = 87, normalized size = 1.71

$$-\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - 2*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x - a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.07, size = 48, normalized size = 0.94

$$\frac{2}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 2/(a*c*((a*x - 1)/(a*x + 1))^(1/2)) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2))
)/(a*c)
```

3.163 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

```
[Out] -1/3*a^2*(1-1/a^2/x^2)^(3/2)/c^2/(a-1/x)^3
```

Rubi [A]
time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {6310, 6313, 665}

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
```

```
[Out] -1/3*(a^2*(1 - 1/(a^2*x^2))^(3/2))/(c^2*(a - x^(-1))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3c^2\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A]
time = 0.07, size = 34, normalized size = 1.03

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)}{3c^2(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^2,x]
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x))/(c^2*(-1 + a*x)^2)
```

Maple [A]
time = 0.12, size = 36, normalized size = 1.09

method	result	size
gosper	$-\frac{ax+1}{3(ax-1)c^2\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
default	$-\frac{ax+1}{3(ax-1)c^2\sqrt{\frac{ax-1}{ax+1}}\,a}$	36
trager	$-\frac{(ax+1)^2\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^2(ax-1)^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(1/2)/a
```

Maxima [A]
time = 0.26, size = 23, normalized size = 0.70

$$-\frac{1}{3\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

Fricas [A]
time = 0.36, size = 57, normalized size = 1.73

$$-\frac{(a^2x^2+2\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*c
^2*x + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

Giac [A]
time = 0.42, size = 49, normalized size = 1.48

$$-\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)
^3*a*c^2)
```

Mupad [B]
time = 1.18, size = 23, normalized size = 0.70

$$-\frac{1}{3\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -1/(3*a*c^2*((a*x - 1)/(a*x + 1))^(3/2))
```

3.164 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=67

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

```
[Out] 1/5*a^3*(1-1/a^2/x^2)^(3/2)/c^3/(a-1/x)^4-4/15*a^2*(1-1/a^2/x^2)^(3/2)/c^3/
(a-1/x)^3
```

Rubi [A]
time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6310, 6313, 807, 665}

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2))/(5*c^3*(a - x^(-1))^4) - (4*a^2*(1 - 1/(a^2*x
^2))^(3/2))/(15*c^3*(a - x^(-1))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3x^3}\,dx}{a^3c^3}\\ &= \frac{\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx,x,\frac{1}{x}\right)}{a^3c^3}\\ &= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{5a^2c^3}\\ &= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5c^3\left(a-\frac{1}{x}\right)^4}-\frac{4a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15c^3\left(a-\frac{1}{x}\right)^3}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.63

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-4-3ax+a^2x^2)}{15c^3(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^3,x]
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(-4 - 3*a*x + a^2*x^2))/(c^3*(-1 + a*x)^3)
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.61

method	result	size
gosper	$-\frac{(ax-4)(ax+1)}{15(ax-1)^{2}c^{3}\sqrt{\frac{ax-1}{ax+1}}\,a}$	41
default	$-\frac{(ax-4)(ax+1)}{15(ax-1)^{2}c^{3}\sqrt{\frac{ax-1}{ax+1}}\,a}$	41
trager	$-\frac{(ax+1)\left(a^{2}x^{2}-3ax-4\right)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^{3}(ax-1)^{3}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/15*(a*x-4)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(1/2)/a
```

Maxima [A]
time = 0.26, size = 39, normalized size = 0.58

$$-\frac{\frac{5\,(ax-1)}{ax+1}-3}{30\,ac^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/30*(5*(a*x - 1)/(a*x + 1) - 3)/(a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

Fricas [A]
time = 0.37, size = 77, normalized size = 1.15

$$-\frac{\left(a^{3}x^{3}-2\,a^{2}x^{2}-7\,ax-4\right)\sqrt{\frac{ax-1}{ax+1}}}{15\,\left(a^{4}c^{3}x^{3}-3\,a^{3}c^{3}x^{2}+3\,a^{2}c^{3}x-ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/15*(a^3*x^3 - 2*a^2*x^2 - 7*a*x - 4)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^3*
x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x**2*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Giac [A]
time = 0.42, size = 85, normalized size = 1.27

$$\frac{2\left(15\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 5*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
5*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^3)
```

Mupad [B]
time = 1.17, size = 39, normalized size = 0.58

$$-\frac{\frac{a\,x-1}{3\,\left(a\,x+1\right)}-\frac{1}{5}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -((a*x - 1)/(3*(a*x + 1)) - 1/5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

3.165 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=100

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}$$

```
[Out] -1/7*a^4*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^5+12/35*a^3*(1-1/a^2/x^2)^(3/2)/c^
4/(a-1/x)^4-23/105*a^2*(1-1/a^2/x^2)^(3/2)/c^4/(a-1/x)^3
```

Rubi [A]
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$-\frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5}+\frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -1/7*(a^4*(1 - 1/(a^2*x^2))^(3/2))/(c^4*(a - x^(-1))^5) + (12*a^3*(1 - 1/(a
^2*x^2))^(3/2))/(35*c^4*(a - x^(-1))^4) - (23*a^2*(1 - 1/(a^2*x^2))^(3/2))/
(105*c^4*(a - x^(-1))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
```

```
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4} \\
&= -\frac{\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^4c^4} \\
&= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{\text{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{3x}{a^3}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{c^4} \\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{7a^2c^4} \\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{35a^2c^4} \\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7c^4\left(a-\frac{1}{x}\right)^5} + \frac{12a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35c^4\left(a-\frac{1}{x}\right)^4} - \frac{23a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^4\left(a-\frac{1}{x}\right)^3}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.51

$$
-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(23+13ax-8a^2x^2+2a^3x^3)}{105c^4(-1+ax)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^4,x]
```

```
[Out] -1/105*(Sqrt[1 - 1/(a^2*x^2)]*x*(23 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^4
*(-1 + a*x)^4)
```

Maple [A]
time = 0.13, size = 50, normalized size = 0.50

method	result	size
gosper	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
default	$-\frac{(2a^2x^2-10ax+23)(ax+1)}{105(ax-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	50
trager	$-\frac{(ax+1)(2a^3x^3-8a^2x^2+13ax+23)\sqrt{-\frac{-ax+1}{ax+1}}}{105a\,c^4(ax-1)^4}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(2*a^2*x^2-10*a*x+23)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(1/2)/
a
```

Maxima [A]
time = 0.27, size = 55, normalized size = 0.55

$$\frac{\frac{42\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}-15}{420\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/420*(42*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 - 15)/(a*c^4*((a
*x - 1)/(a*x + 1))^(7/2))
```

Fricas [A]
time = 0.34, size = 96, normalized size = 0.96

$$-\frac{(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+36\,ax+23)\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + 36*a*x + 23)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx \Big/ c^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**3*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))), x)/c**4
```

Giac [A]

time = 0.45, size = 105, normalized size = 1.05

$$-\frac{4\left(70\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{4}x^{4}+35\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{3}x^{3}+21\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{2}x^{2}-7\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)x+1\right)}{105\left(\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)x-1\right)^{7}ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 35*(a + sqrt(a^2 - 1/x^2))^3*x^3
 + 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 7*(a + sqrt(a^2 - 1/x^2))*x + 1)/(((a
 + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^4)
```

Mupad [B]

time = 0.04, size = 56, normalized size = 0.56

$$-\frac{\frac{(a\,x-1)^2}{3\,(a\,x+1)^2}-\frac{2\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -((a*x - 1)^2/(3*(a*x + 1)^2) - (2*(a*x - 1))/(5*(a*x + 1)) + 1/7)/(4*a*c^4
*((a*x - 1)/(a*x + 1))^(7/2))
```

3.166 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=133

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6}-\frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5}+\frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}-\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

```
[Out] 1/9*a^5*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^6-8/21*a^4*(1-1/a^2/x^2)^(3/2)/c^5/
(a-1/x)^5+47/105*a^3*(1-1/a^2/x^2)^(3/2)/c^5/(a-1/x)^4-58/315*a^2*(1-1/a^2/
x^2)^(3/2)/c^5/(a-1/x)^3
```

Rubi [A]
time = 0.23, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$-\frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}+\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6}-\frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5}+\frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(3/2))/(9*c^5*(a - x^(-1))^6) - (8*a^4*(1 - 1/(a^2*x
^2))^(3/2))/(21*c^5*(a - x^(-1))^5) + (47*a^3*(1 - 1/(a^2*x^2))^(3/2))/(105
*c^5*(a - x^(-1))^4) - (58*a^2*(1 - 1/(a^2*x^2))^(3/2))/(315*c^5*(a - x^(-1
))^3)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{\operatorname{Subst}\left(\int \frac{\left(\frac{4}{a^2}-\frac{7x}{a^3}+\frac{2x^2}{a^4}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{a^4\operatorname{Subst}\left(\int \frac{\left(\frac{18}{a^6}-\frac{20x}{a^7}\right)\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^6}\,dx, x, \frac{1}{x}\right)}{2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} + \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{29\operatorname{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{3a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\operatorname{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right.}{21a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58\operatorname{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x,\right.}{105a^2c^5}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9c^5\left(a-\frac{1}{x}\right)^6} - \frac{8a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21c^5\left(a-\frac{1}{x}\right)^5} + \frac{47a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105c^5\left(a-\frac{1}{x}\right)^4} - \frac{58a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315c^5\left(a-\frac{1}{x}\right)^3}$$

Mathematica [A]

time = 0.05, size = 59, normalized size = 0.44

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-58-25ax+21a^2x^2-10a^3x^3+2a^4x^4)}{315c^5(-1+ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^5,x]
```

```
[Out] -1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(-58 - 25*a*x + 21*a^2*x^2 - 10*a^3*x^3 + 2
*a^4*x^4))/(c^5*(-1 + a*x)^5)
```

Maple [A]
time = 0.13, size = 58, normalized size = 0.44

method	result	size
gosper	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
default	$-\frac{(2a^3x^3-12a^2x^2+33ax-58)(ax+1)}{315(ax-1)^4c^5\sqrt{\frac{ax-1}{ax+1}}\,a}$	58
trager	$-\frac{(ax+1)(2a^4x^4-10a^3x^3+21a^2x^2-25ax-58)\sqrt{-\frac{-ax+1}{ax+1}}}{315a\,c^5(ax-1)^5}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(2*a^3*x^3-12*a^2*x^2+33*a*x-58)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a*x
+1))^(1/2)/a
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.53

$$-\frac{\frac{135\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{105\,(ax-1)^3}{(ax+1)^3}-35}{2520\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/2520*(135*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 105*(a*x -
 1)^3/(a*x + 1)^3 - 35)/(a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

Fricas [A]
time = 0.33, size = 116, normalized size = 0.87

$$-\frac{\left(2\,a^{5}x^{5}-8\,a^{4}x^{4}+11\,a^{3}x^{3}-4\,a^{2}x^{2}-83\,ax-58\right)\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^{6}c^{5}x^{5}-5\,a^{5}c^{5}x^{4}+10\,a^{4}c^{5}x^{3}-10\,a^{3}c^{5}x^{2}+5\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 - 4*a^2*x^2 - 83*a*x - 58)*sqrt(
(a*x - 1)/(a*x + 1))/(a^6*c^5*x^5 - 5*a^5*c^5*x^4 + 10*a^4*c^5*x^3 - 10*a^3
*c^5*x^2 + 5*a^2*c^5*x - a*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{5}x^{5}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-5a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+10a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-10a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 5*a**4*x**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 10*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 5*a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**5
```

Giac [A]
time = 0.47, size = 125, normalized size = 0.94

$$\frac{4\left(315\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{5}x^{5}+189\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{4}x^{4}+84\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{3}x^{3}-36\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)^{2}x^{2}+9\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)x-1\right)}{315\left(\left(a+\sqrt{a^{2}-\frac{1}{x^{2}}}\right)x-1\right)^{9}ac^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 4/315*(315*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 189*(a + sqrt(a^2 - 1/x^2))^4*x^
4 + 84*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 9
*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^5)
```

Mupad [B]
time = 1.17, size = 72, normalized size = 0.54

$$\frac{\frac{3\,{\left(a\,x-1\right)}^2}{5\,{\left(a\,x+1\right)}^2}-\frac{{\left(a\,x-1\right)}^3}{3\,{\left(a\,x+1\right)}^3}-\frac{3\,\left(a\,x-1\right)}{7\,\left(a\,x+1\right)}+\frac{1}{9}}{8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((3*(a*x - 1)^2)/(5*(a*x + 1)^2) - (a*x - 1)^3/(3*(a*x + 1)^3) - (3*(a*x -
1))/(7*(a*x + 1)) + 1/9)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(9/2))
```

3.167 $\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=42

$$\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

[Out] 2*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(1+p)/a/c/(1+p)

Rubi [A]

time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {6302, 6265, 21, 45}

$$\frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]

[Out] (2*(c - a*c*x)^p)/(a*p) - (c - a*c*x)^(1 + p)/(a*c*(1 + p))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^p\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^p\,dx \\
&= -\int \frac{(1+ax)(c-acx)^p}{1-ax}\,dx \\
&= -\left(c\int (1+ax)(c-acx)^{-1+p}\,dx\right) \\
&= -\left(c\int \left(2(c-acx)^{-1+p}-\frac{(c-acx)^p}{c}\right)dx\right) \\
&= \frac{2(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.67

$$\frac{(c-acx)^p(2+p+apx)}{ap(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^p,x]
[Out] ((c - a*c*x)^p*(2 + p + a*p*x))/(a*p*(1 + p))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.69

method	result	size
gosper	$\frac{(apx+p+2)(-acx+c)^p}{ap(1+p)}$	29
risch	$\frac{(apx+p+2)(-acx+c)^p}{ap(1+p)}$	29
norman	$\frac{x\,\mathrm{e}^{p\ln(-acx+c)}}{1+p}+\frac{(2+p)\mathrm{e}^{p\ln(-acx+c)}}{ap(1+p)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)
[Out] (a*p*x+p+2)*(-a*c*x+c)^p/a/p/(1+p)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 1.17

$$\frac{(ac^p px+c^p)(-ax+1)^p}{(p^2+p)a}+\frac{(-ax+1)^p c^p}{ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] (a*c^p*p*x + c^p)*(-a*x + 1)^p/((p^2 + p)*a) + (-a*x + 1)^p*c^p/(a*p)
```

Fricas [A]
time = 0.34, size = 28, normalized size = 0.67

$$\frac{(apx + p + 2)(-acx + c)^p}{ap^2 + ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] (a*p*x + p + 2)*(-a*c*x + c)^p/(a*p^2 + a*p)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(29) = 58$.
time = 0.59, size = 124, normalized size = 2.95

$$\begin{cases} -c^p x & \text{for}\: a = 0 \\ -\frac{ax \log\left(x - \frac{1}{a}\right)}{a^2cx - ac} + \frac{\log\left(x - \frac{1}{a}\right)}{a^2cx - ac} + \frac{2}{a^2cx - ac} & \text{for}\: p = -1 \\ x + \frac{2\log\left(x - \frac{1}{a}\right)}{a} & \text{for}\: p = 0 \\ \frac{apx(-acx + c)^p}{ap^2 + ap} + \frac{p(-acx + c)^p}{ap^2 + ap} + \frac{2(-acx + c)^p}{ap^2 + ap} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((-c**p*x, Eq(a, 0)), (-a*x*log(x - 1/a)/(a**2*c*x - a*c) + log(x
- 1/a)/(a**2*c*x - a*c) + 2/(a**2*c*x - a*c), Eq(p, -1)), (x + 2*log(x - 1/
a)/a, Eq(p, 0)), (a*p*x*(-a*c*x + c)**p/(a*p**2 + a*p) + p*(-a*c*x + c)**p/
(a*p**2 + a*p) + 2*(-a*c*x + c)**p/(a*p**2 + a*p), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)*(-a*c*x + c)^p/(a*x - 1), x)
```

Mupad [B]
time = 1.21, size = 28, normalized size = 0.67

$$\frac{(c - a\,c\,x)^p\,(p + a\,p\,x + 2)}{a\,p\,(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x + 1))/(a*x - 1),x)
[Out] ((c - a*c*x)^p*(p + a*p*x + 2))/(a*p*(p + 1))
```

3.168 $\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=37

$$\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

```
[Out] 2/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 45}

$$\frac{2c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] (2*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}(c-acx)^5\,dx = -\int e^{2\tanh^{-1}(ax)}(c-acx)^5\,dx$$
$$= -\left(c^5\int (1-ax)^4(1+ax)\,dx\right)$$
$$= -\left(c^5\int \left(2(1-ax)^4-(1-ax)^5\right)\,dx\right)$$
$$= \frac{2c^5(1-ax)^5}{5a} - \frac{c^5(1-ax)^6}{6a}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.62

$$-\frac{c^5(-1+ax)^5(7+5ax)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -1/30*(c^5*(-1 + a*x)^5*(7 + 5*a*x))/a
```

Maple [A]
time = 0.15, size = 47, normalized size = 1.27

method	result
gosper	$-\frac{x\left(5a^5x^5-18a^4x^4+15a^3x^3+20a^2x^2-45ax+30\right)c^5}{30}$
default	$c^5\left(-\frac{1}{6}a^5x^6+\frac{3}{5}a^4x^5-\frac{1}{2}a^3x^4-\frac{2}{3}a^2x^3+\frac{3}{2}a\,x^2-x\right)$
norman	$-c^5x-\frac{2}{3}a^2c^5x^3-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6+\frac{3}{2}c^5a\,x^2$
risch	$-c^5x-\frac{2}{3}a^2c^5x^3-\frac{1}{2}a^3c^5x^4+\frac{3}{5}a^4c^5x^5-\frac{1}{6}a^5c^5x^6+\frac{3}{2}c^5a\,x^2$
meijerg	$-\frac{c^5\left(\frac{ax\left(70a^5x^5+84a^4x^4+105a^3x^3+140a^2x^2+210ax+420\right)}{420}+\ln(-ax+1)\right)}{a}-\frac{4c^5\left(-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\ln(-ax+\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(-1/6*a^5*x^6+3/5*a^4*x^5-1/2*a^3*x^4-2/3*a^2*x^3+3/2*a*x^2-x)
```

Maxima [A]
time = 0.27, size = 60, normalized size = 1.62

$$-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Fricas [A]
time = 0.32, size = 60, normalized size = 1.62

$$-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.
time = 0.03, size = 66, normalized size = 1.78

$$-\frac{a^5c^5x^6}{6}+\frac{3a^4c^5x^5}{5}-\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}+\frac{3ac^5x^2}{2}-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + 3*a**4*c**5*x**5/5 - a**3*c**5*x**4/2 - 2*a**2*c**5*x**
3/3 + 3*a*c**5*x**2/2 - c**5*x
```

Giac [A]
time = 0.40, size = 60, normalized size = 1.62

$$-\frac{1}{6}\,a^5c^5x^6+\frac{3}{5}\,a^4c^5x^5-\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3+\frac{3}{2}\,ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/6*a^5*c^5*x^6 + 3/5*a^4*c^5*x^5 - 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 + 3/
2*a*c^5*x^2 - c^5*x
```

Mupad [B]
time = 0.03, size = 60, normalized size = 1.62

$$-\frac{a^5\,c^5\,x^6}{6}+\frac{3\,a^4\,c^5\,x^5}{5}-\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}+\frac{3\,a\,c^5\,x^2}{2}-c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (3*a*c^5*x^2)/2 - c^5*x - (2*a^2*c^5*x^3)/3 - (a^3*c^5*x^4)/2 + (3*a^4*c^5*
x^5)/5 - (a^5*c^5*x^6)/6
```

3.169 $\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=37

$$\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

```
[Out] 1/2*c^4*(-a*x+1)^4/a-1/5*c^4*(-a*x+1)^5/a
```

Rubi [A]

time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6302, 6264, 45}

$$\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(1 - a*x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^4\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^4\,dx \\
&= -\left(c^4\int(1-ax)^3(1+ax)\,dx\right) \\
&= -\left(c^4\int\left(2(1-ax)^3-(1-ax)^4\right)\,dx\right) \\
&= \frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.81

$$\frac{1}{10}c^4x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.81

method	result
gosper	$\frac{x\left(2a^4x^4-5a^3x^3+10ax-10\right)c^4}{10}$
default	$c^4\left(\frac{1}{5}a^4x^5-\frac{1}{2}a^3x^4+a\,x^2-x\right)$
norman	$c^4a\,x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
risch	$c^4a\,x^2-c^4x-\frac{1}{2}a^3c^4x^4+\frac{1}{5}a^4c^4x^5$
meijerg	$-\frac{c^4\left(-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\ln(-ax+1)\right)}{a}-\frac{3c^4\left(\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)\right)}{a}-\frac{2c^4\left(-\frac{ax(4\ldots}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(1/5*a^4*x^5-1/2*a^3*x^4+a*x^2-x)
```

Maxima [A]
time = 0.26, size = 37, normalized size = 1.00

$$\frac{1}{5}\,a^4c^4x^5-\frac{1}{2}\,a^3c^4x^4+ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Fricas [A]
time = 0.33, size = 37, normalized size = 1.00

$$\frac{1}{5}a^4c^4x^5 - \frac{1}{2}a^3c^4x^4 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Sympy [A]
time = 0.02, size = 36, normalized size = 0.97

$$\frac{a^4c^4x^5}{5} - \frac{a^3c^4x^4}{2} + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**4,x)
```

```
[Out] a**4*c**4*x**5/5 - a**3*c**4*x**4/2 + a*c**4*x**2 - c**4*x
```

Giac [A]
time = 0.40, size = 37, normalized size = 1.00

$$\frac{1}{5}a^4c^4x^5 - \frac{1}{2}a^3c^4x^4 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/5*a^4*c^4*x^5 - 1/2*a^3*c^4*x^4 + a*c^4*x^2 - c^4*x
```

Mupad [B]
time = 0.05, size = 37, normalized size = 1.00

$$\frac{a^4\,c^4\,x^5}{5} - \frac{a^3\,c^4\,x^4}{2} + a\,c^4\,x^2 - c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] a*c^4*x^2 - c^4*x - (a^3*c^4*x^4)/2 + (a^4*c^4*x^5)/5
```

3.170 $\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=37

$$\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

[Out] 2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a

Rubi [A]

time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]

[Out] (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x)^4)/(4*a)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^3\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^3\,dx \\
&= -\left(c^3\int (1-ax)^2(1+ax)\,dx\right) \\
&= -\left(c^3\int \left(2(1-ax)^2-(1-ax)^3\right)\,dx\right) \\
&= \frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.81

$$-\frac{1}{12}c^3x\left(12-6ax-4a^2x^2+3a^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -1/12*(c^3*x*(12 - 6*a*x - 4*a^2*x^2 + 3*a^3*x^3))
```

Maple [A]
time = 0.13, size = 31, normalized size = 0.84

method	result
gosper	$-\frac{x\left(3a^3x^3-4a^2x^2-6ax+12\right)c^3}{12}$
default	$c^3\left(-\frac{1}{4}a^3x^4+\frac{1}{3}a^2x^3+\frac{1}{2}a\,x^2-x\right)$
norman	$-c^3x+\frac{1}{3}a^2c^3x^3-\frac{1}{4}a^3c^3x^4+\frac{1}{2}c^3a\,x^2$
risch	$-c^3x+\frac{1}{3}a^2c^3x^3-\frac{1}{4}a^3c^3x^4+\frac{1}{2}c^3a\,x^2$
meijerg	$-\frac{c^3\left(\frac{ax\left(15a^3x^3+20a^2x^2+30ax+60\right)}{60}+\ln(-ax+1)\right)}{a}-\frac{2c^3\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}+\frac{2c^3(-ax-\ln(-ax+1))}{a}+\frac{c^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/4*a^3*x^4+1/3*a^2*x^3+1/2*a*x^2-x)
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.03

$$-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Fricas [A]
time = 0.33, size = 38, normalized size = 1.03

$$-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Sympy [A]
time = 0.02, size = 37, normalized size = 1.00

$$-\frac{a^3c^3x^4}{4}+\frac{a^2c^3x^3}{3}+\frac{ac^3x^2}{2}-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**3,x)
```

```
[Out] -a**3*c**3*x**4/4 + a**2*c**3*x**3/3 + a*c**3*x**2/2 - c**3*x
```

Giac [A]
time = 0.40, size = 38, normalized size = 1.03

$$-\frac{1}{4}\,a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*a^3*c^3*x^4 + 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 - c^3*x
```

Mupad [B]
time = 0.05, size = 38, normalized size = 1.03

$$-\frac{a^3\,c^3\,x^4}{4}+\frac{a^2\,c^3\,x^3}{3}+\frac{a\,c^3\,x^2}{2}-c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (a*c^3*x^2)/2 - c^3*x + (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

3.171 $\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=20

$$-c^2x+\frac{1}{3}a^2c^2x^3$$

```
[Out] -c^2*x+1/3*a^2*c^2*x^3
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 41}

$$\frac{1}{3}a^2c^2x^3-c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*x) + (a^2*c^2*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^2\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^2\,dx\\ &= -\left(c^2\int(1-ax)(1+ax)\,dx\right)\\ &= -\left(c^2\int\left(1-a^2x^2\right)\,dx\right)\\ &= -c^2x+\frac{1}{3}a^2c^2x^3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.85

$$-c^2\left(x-\frac{a^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] -(c^2*(x - (a^2*x^3)/3))
```

Maple [A]
time = 0.13, size = 17, normalized size = 0.85

method	result	size
gosper	$\frac{x(a^2x^2-3)c^2}{3}$	16
default	$c^2\left(\frac{1}{3}a^2x^3-x\right)$	17
norman	$-c^2x+\frac{1}{3}a^2c^2x^3$	19
risch	$-c^2x+\frac{1}{3}a^2c^2x^3$	19
meijerg	$-\frac{c^2\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}-\frac{c^2\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c^2(-ax-\ln(-ax+1))}{a}+\frac{c^2\ln(-ax+1)}{a}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/3*a^2*x^3-x)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.90

$$\frac{1}{3}\,a^2c^2x^3-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Fricas [A]
time = 0.32, size = 18, normalized size = 0.90

$$\frac{1}{3}\,a^2c^2x^3-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Sympy [A]
time = 0.02, size = 15, normalized size = 0.75

$$\frac{a^2c^2x^3}{3}-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 - c**2*x
```

Giac [A]
time = 0.41, size = 18, normalized size = 0.90

$$\frac{1}{3}\,a^2c^2x^3-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*c^2*x^3 - c^2*x
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.75

$$\frac{c^2\,x\,\left(a^2\,x^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^2*x*(a^2*x^2 - 3))/3
```

3.172 $\int e^{2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=14

$$-cx-\frac{1}{2}acx^2$$

[Out] -c*x-1/2*a*c*x^2

Rubi [C] Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.86, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2326}

$$\frac{c(1-a^2x^2)\,e^{2\coth^{-1}(ax)}}{2a}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x),x]

[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}(c-acx)\,dx=\frac{ce^{2\coth^{-1}(ax)}(1-a^2x^2)}{2a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.
time = 0.01, size = 26, normalized size = 1.86

$$\frac{ce^{2\coth^{-1}(ax)}(1-a^2x^2)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x),x]

[Out] (c*E^(2*ArcCoth[a*x])*(1 - a^2*x^2))/(2*a)

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{cx(ax+2)}{2}$	10
default	$c\left(-\frac{1}{2}a\,x^2-x\right)$	13
norman	$-cx-\frac{1}{2}ac\,x^2$	13
risch	$-cx-\frac{1}{2}ac\,x^2$	13
meijerg	$-\frac{c\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a}+\frac{c\ln(-ax+1)}{a}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*a*x^2-x)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$-\frac{1}{2}\,acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

Fricas [A]
time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{2}\,acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.86

$$-\frac{acx^2}{2}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x)
```

```
[Out] -a*c*x**2/2 - c*x
```

Giac [A]
time = 0.40, size = 12, normalized size = 0.86

$$-\frac{1}{2}\,acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*a*c*x^2 - c*x
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.64

$$-\frac{c\,x\,(a\,x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] -(c*x*(a*x + 2))/2
```

3.173 $\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=32

$$-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

```
[Out] -2/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6302, 6264, 45}

$$-\frac{2}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -2/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{c-acx}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{c-acx}\,dx$$
$$= -\frac{\int \frac{1+ax}{(1-ax)^2}\,dx}{c}$$
$$= -\frac{\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{c}$$
$$= -\frac{2}{ac(1-ax)} - \frac{\log(1-ax)}{ac}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.94

$$-\frac{\frac{2}{a(1-ax)}+\frac{\log(1-ax)}{a}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -((2/(a*(1 - a*x)) + Log[1 - a*x]/a)/c)
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\frac{2}{a(ax-1)}-\frac{\ln(ax-1)}{a}}{c}$	29
norman	$\frac{2x}{c(ax-1)}-\frac{\ln(ax-1)}{ac}$	29
risch	$\frac{2}{ac(ax-1)}-\frac{\ln(ax-1)}{ac}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(2/a/(a*x-1)-1/a*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 30, normalized size = 0.94

$$\frac{2}{a^2cx-ac} - \frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 2/(a^2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Fricas [A]
time = 0.33, size = 29, normalized size = 0.91

$$-\frac{(ax-1)\log(ax-1)-2}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -((a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.62

$$\frac{2}{a^2cx-ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x)
```

```
[Out] 2/(a**2*c*x - a*c) - log(a*x - 1)/(a*c)
```

Giac [A]
time = 0.41, size = 31, normalized size = 0.97

$$-\frac{\log(|ax-1|)}{ac}+\frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -log(abs(a*x - 1))/(a*c) + 2/((a*x - 1)*a*c)
```

Mupad [B]
time = 1.20, size = 29, normalized size = 0.91

$$-\frac{2}{a\,\left(c-a\,c\,x\right)}-\frac{\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)*(a*x - 1)),x)
```

```
[Out] - 2/(a*(c - a*c*x)) - log(a*x - 1)/(a*c)
```

3.174 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{x}{c^2(1-ax)^2}$$

[Out] -x/c^2/(-a*x+1)^2

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 34}

$$-\frac{x}{c^2(1-ax)^2}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]

[Out] -(x/(c^2*(1 - a*x)^2))

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$$
$$= -\frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c^2}$$
$$= -\frac{x}{c^2(1-ax)^2}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.79

$$-\frac{(1+ax)^2}{4ac^2(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -1/4*(1 + a*x)^2/(a*c^2*(1 - a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.
time = 0.12, size = 30, normalized size = 2.14

method	result	size
gosper	$-\frac{x}{c^2(ax-1)^2}$	14
norman	$-\frac{x}{c^2(ax-1)^2}$	14
risch	$-\frac{x}{c^2(ax-1)^2}$	14
default	$\frac{-\frac{1}{a(ax-1)}-\frac{1}{a(ax-1)^2}}{c^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/a/(a*x-1)-1/a/(a*x-1)^2)
```

Maxima [A]
time = 0.26, size = 26, normalized size = 1.86

$$-\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Fricas [A]
time = 0.34, size = 26, normalized size = 1.86

$$-\frac{x}{a^2c^2x^2 - 2\,ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -x/(a^2*c^2*x^2 - 2*a*c^2*x + c^2)
```

Sympy [A]
time = 0.09, size = 24, normalized size = 1.71

$$-\frac{x}{a^2c^2x^2 - 2ac^2x + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**2,x)
```

```
[Out] -x/(a**2*c**2*x**2 - 2*a*c**2*x + c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.
time = 0.41, size = 34, normalized size = 2.43

$$-\frac{1}{(acx-c)^2a} - \frac{1}{(acx-c)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c)
```

Mupad [B]
time = 1.19, size = 13, normalized size = 0.93

$$-\frac{x}{c^2\,(a\,x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^2*(a*x - 1)),x)
```

```
[Out] -x/(c^2*(a*x - 1)^2)
```

3.175 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=37

$$-\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

```
[Out] -2/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 45}

$$\frac{1}{2ac^3(1-ax)^2}-\frac{2}{3ac^3(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -2/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$$
$$= -\frac{\int \frac{1+ax}{(1-ax)^4}\,dx}{c^3}$$
$$= -\frac{\int \left(\frac{2}{(-1+ax)^4}+\frac{1}{(-1+ax)^3}\right)dx}{c^3}$$
$$= -\frac{2}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.62

$$\frac{1+3ax}{6ac^3(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (1 + 3*a*x)/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.81

method	result	size
risch	$\frac{\frac{x}{2}+\frac{1}{6a}}{(ax-1)^3c^3}$	21
gosper	$\frac{3ax+1}{6a\,c^3(ax-1)^3}$	22
default	$\frac{\frac{2}{3a(ax-1)^3}+\frac{1}{2a(ax-1)^2}}{c^3}$	30
norman	$\frac{\frac{x}{c}-\frac{a\,x^2}{2c}+\frac{a^2x^3}{6c}}{(ax-1)^3c^2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(2/3/a/(a*x-1)^3+1/2/a/(a*x-1)^2)
```

Maxima [A]
time = 0.25, size = 47, normalized size = 1.27

$$\frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.27

$$\frac{3\,ax+1}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] 1/6*(3*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [A]
time = 0.12, size = 49, normalized size = 1.32

$$-\frac{-3ax-1}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**3,x)
[Out] -(-3*a*x - 1)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a**2*c**3*x - 6*a*
c**3)
```

Giac [A]
time = 0.39, size = 21, normalized size = 0.57

$$\frac{3\,ax+1}{6\,(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
[Out] 1/6*(3*a*x + 1)/((a*x - 1)^3*a*c^3)
```

Mupad [B]
time = 1.20, size = 46, normalized size = 1.24

$$-\frac{\frac{x}{2}+\frac{1}{6\,a}}{-a^3\,c^3\,x^3+3\,a^2\,c^3\,x^2-3\,a\,c^3\,x+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^3*(a*x - 1)),x)
[Out] -(x/2 + 1/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^3*x)
```

3.176 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

```
[Out] -1/2/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 45}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/2*1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx \\
&= -\frac{\int \frac{1+ax}{(1-ax)^5}\,dx}{c^4} \\
&= -\frac{\int \left(-\frac{2}{(-1+ax)^5}-\frac{1}{(-1+ax)^4}\right)dx}{c^4} \\
&= -\frac{1}{2ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.62

$$
-\frac{1+2ax}{6ac^4(-1+ax)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/6*(1 + 2*a*x)/(a*c^4*(-1 + a*x)^4)
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.81

method	result	size
risch	$\frac{-\frac{x}{3}-\frac{1}{6a}}{(ax-1)^4c^4}$	21
gosper	$-\frac{2ax+1}{6a\,c^4(ax-1)^4}$	22
default	$\frac{-\frac{1}{2a(ax-1)^4}-\frac{1}{3a(ax-1)^3}}{c^4}$	30
norman	$\frac{\frac{a\,x^2}{c}-\frac{x}{c}-\frac{2a^2x^3}{3c}+\frac{a^3x^4}{6c}}{(ax-1)^4c^3}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2/a/(a*x-1)^4-1/3/a/(a*x-1)^3)
```

Maxima [A]
time = 0.25, size = 57, normalized size = 1.54

$$
-\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

Fricas [A]
time = 0.34, size = 57, normalized size = 1.54

$$-\frac{2\,ax+1}{6\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*x + 1)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x
 + a*c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.
time = 0.18, size = 60, normalized size = 1.62

$$\frac{-2ax-1}{6a^5c^4x^4-24a^4c^4x^3+36a^3c^4x^2-24a^2c^4x+6ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] (-2*a*x - 1)/(6*a**5*c**4*x**4 - 24*a**4*c**4*x**3 + 36*a**3*c**4*x**2 - 24
*a**2*c**4*x + 6*a*c**4)
```

Giac [A]
time = 0.41, size = 21, normalized size = 0.57

$$-\frac{2\,ax+1}{6\,(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*a*x + 1)/((a*x - 1)^4*a*c^4)
```

Mupad [B]
time = 0.09, size = 56, normalized size = 1.51

$$-\frac{\frac{x}{3}+\frac{1}{6\,a}}{a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+6\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^4*(a*x - 1)),x)
```

```
[Out] -(x/3 + 1/(6*a))/(c^4 + 6*a^2*c^4*x^2 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c
^4*x)
```

3.177 $\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=202

$$\frac{3\sqrt{1+\frac{1}{ax}}\,(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}}-\frac{3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\sqrt{1+\frac{1}{ax}}\,(c-acx)^p\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;\right.}{a^2p\left(1-p^2\right)\left(1-\frac{1}{ax}\right)^{3/2}x}$$

```
[Out] (1+1/a/x)^(3/2)*x*(-a*c*x+c)^p/(1+p)/(1-1/a/x)^(1/2)-3*((a-1/x)/(a+1/x))^(3
/2-p)*(-a*c*x+c)^p*hypergeom([1-p, 3/2-p],[2-p],2/(a+1/x)/x)*(1+1/a/x)^(1/2
)/a^2/p/(-p^2+1)/(1-1/a/x)^(3/2)/x+3*(-a*c*x+c)^p*(1+1/a/x)^(1/2)/a/p/(1+p)
/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6311, 6316, 96, 134}

$$-\frac{3\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}(c-acx)^p\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{a^2p\left(1-p^2\right)x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^p}{(p+1)\sqrt{1-\frac{1}{ax}}}+\frac{3\sqrt{\frac{1}{ax}+1}\,(c-acx)^p}{ap(p+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p)/(a*p*(1 + p)*Sqrt[1 - 1/(a*x)]) + ((1 +
 1/(a*x))^(3/2)*x*(c - a*c*x)^p)/((1 + p)*Sqrt[1 - 1/(a*x)]) - (3*((a - x^(
-1))/(a + x^(-1)))^(3/2 - p)*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*Hypergeometric
2F1[1 - p, 3/2 - p, 2 - p, 2/((a + x^(-1))*x)])/(a^2*p*(1 - p^2)*(1 - 1/(a*
x))^(3/2)*x)
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
```

```
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(1+\frac{x}{a}\right)^{3/}\right.\right.$$

$$= \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-1-p}(1-\right.}{a(1+p)}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\,(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-ac\right.}{}$$

$$= \frac{3\sqrt{1+\frac{1}{ax}}\,(c-acx)^p}{ap(1+p)\sqrt{1-\frac{1}{ax}}} + \frac{\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^p}{(1+p)\sqrt{1-\frac{1}{ax}}} - \frac{3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\sqrt{1+\frac{1}{ax}}\,(c-}{a^2p\,(1}$$

Mathematica [A]
time = 0.09, size = 155, normalized size = 0.77

$$\frac{\sqrt{1+\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-p}(c-acx)^p\left((-1+p)\left(\frac{-1+ax}{1+ax}\right)^p(1+ax)(3+p+apx)+3\sqrt{\frac{-1+ax}{1+ax}}\,{}_2F_1\left(1-p,\frac{3}{2}-p;2-p;\frac{2}{1+ax}\right)\right)}{a(-1+p)p(1+p)\sqrt{1-\frac{1}{ax}}(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - a*c*x)^p*((-1 + p)*((-1 + a*x)/(1 + a*x))^p*(1 + a*
x)*(3 + p + a*p*x) + 3*Sqrt[(-1 + a*x)/(1 + a*x)]*Hypergeometric2F1[1 - p,
3/2 - p, 2 - p, 2/(1 + a*x)]))/(a*(-1 + p)*p*(1 + p)*Sqrt[1 - 1/(a*x)]*((-1
 + a*x)/(1 + a*x))^p*(1 + a*x))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-acx+c)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x^2 - 2*a*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c-a\,c\,x)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a*c*x)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.178 $\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=105

$$\frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4+\frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] -1/4*a^3*c^4*(1-1/a^2/x^2)^(3/2)*x^4+1/5*a^4*c^4*(1-1/a^2/x^2)^(5/2)*x^5-3/
8*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+3/8*a*c^4*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.389$, Rules used = {6310, 6313, 821, 272, 43, 65, 214}

$$\frac{3}{8}ac^4x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{1}{5}a^4c^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}-\frac{1}{4}a^3c^4x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^4*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 + (a^4*c^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/5 - (3*c^4*ArcTanh[Sqrt[1 - 1/
(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^4\,dx = \left(a^4c^4\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^4 x^4\,dx$$

$$= -\left(\left(a^4c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 + \left(a^3c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 + \frac{1}{2}\left(a^3c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 - \frac{1}{8}\left(3ac^4\right)\text{Subst}\left(\int \sqrt{\right.$$

(

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 + -$$

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 - \frac{1}{8}$$

3

$$= \frac{3}{8}ac^4\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{1}{5}a^4c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5 - -$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 0.76

$$\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(8+25ax-16a^2x^2-10a^3x^3+8a^4x^4)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a
^4*x^4) - 15*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(89) = 178$.
time = 0.08, size = 192, normalized size = 1.83

method	result
risch	$\frac{(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8)(ax-1)c^4}{40a\sqrt{\frac{ax-1}{ax+1}}} - \frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^4\sqrt{(ax+1)(ax-1)}}{8\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)^2c^4\left(-24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+30\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+40((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-45\sqrt{a^2}\right.}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(a*x-1)^2*c^4/a*(-24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+30*(a^2)^
(1/2)*(a^2*x^2-1)^(3/2)*a*x+40*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-16*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)-45*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+45*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(3/2)/(a*x
+1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(89) = 178$.
time = 0.26, size = 259, normalized size = 2.47

$$-\frac{1}{40}\left(\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-15\,c^4\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5a^2}{(ax+1)^5}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^4*
((a*x - 1)/(a*x + 1))^(7/2) - 128*c^4*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^4*
((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x - 1
)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
+ 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2)
)*a
```

Fricas [A]
time = 0.34, size = 126, normalized size = 1.20

$$-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(8\,a^5c^4x^5-2\,a^4c^4x^4-26\,a^3c^4x^3+9\,a^2c^4x^2+33\,ac^4x+8\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/40*(15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^4*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (8*a^5*c^4*x^5 - 2*a^4*c^4*x^4 - 26*a^3*c^4*x^3 + 9*a^2
*c^4*x^2 + 33*a*c^4*x + 8*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2*x**2/(a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x) + Integral(-4*a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Inte
gral(a**4*x**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x))
```

Giac [A]
time = 0.44, size = 138, normalized size = 1.31

$$\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\left(ax+1\right)}+\frac{1}{40}\sqrt{a^2x^2-1}\left(\left(\frac{25\,c^4}{\mathrm{sgn}\left(ax+1\right)}-2\left(\frac{8\,ac^4}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{4\,a^3c^4x}{\mathrm{sgn}\left(ax+1\right)}-\frac{5\,a^2c^4}{\mathrm{sgn}\left(ax+1\right)}\right)x\right)x\right)x+\frac{8\,c^4}{a\mathrm{sgn}\left(ax+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] 3/8*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + 1/4
0*sqrt(a^2*x^2 - 1)*((25*c^4/sgn(a*x + 1) - 2*(8*a*c^4/sgn(a*x + 1) - (4*a^
3*c^4*x/sgn(a*x + 1) - 5*a^2*c^4/sgn(a*x + 1))*x)*x)*x + 8*c^4/(a*sgn(a*x +
 1)))
```

Mupad [B]
time = 0.09, size = 214, normalized size = 2.04

$$\frac{\frac{3\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}+\frac{32\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{5}+\frac{7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{2}-\frac{3\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}-\frac{3\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((3*c^4*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^4*((a*x - 1)/(a*x + 1))^(3/2)
)/2 + (32*c^4*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^4*((a*x - 1)/(a*x + 1))
^(7/2))/2 - (3*c^4*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^4*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.179 $\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=78

$$\frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

```
[Out] -1/4*a^3*c^3*(1-1/a^2/x^2)^(3/2)*x^4-3/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a
+3/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6310, 6313, 272, 43, 65, 214}

$$\frac{3}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}-\frac{1}{4}a^3c^3x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (3*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 - (a^3*c^3*(1 - 1/(a^2*x^2))^(3/2)*x^
4)/4 - (3*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a^2}\right)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{8}\left(3ac^3\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 + \frac{\left(3c^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,d\right.}{16a}$$

$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{1}{8}\left(3ac^3\right)\text{Subst}\left(\int \frac{1}{a^2-a^2x}\right.$$

$$= \frac{3}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{4}a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4 - \frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}$$

Mathematica [A]
time = 0.12, size = 64, normalized size = 0.82

$$\frac{c^3\left(a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2(5-2a^2x^2)-3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{8a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (c^3*(a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*(5 - 2*a^2*x^2) - 3*Log[a*(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(8*a)
```

Maple [A]
time = 0.09, size = 124, normalized size = 1.59

method	result	size
risch	$-\frac{x(2a^2x^2-5)(ax-1)c^3}{8\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax+1)(ax-1)}}{8\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	106
default	$-\frac{(ax-1)^2c^3\left(2x(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3x\sqrt{a^2x^2-1}\,\sqrt{a^2}+3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)\right)}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(a*x-1)^2*c^3*(2*x*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*x*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)+3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)))/((a*x
-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(66) = 132$.
time = 0.27, size = 221, normalized size = 2.83

$$-\frac{1}{8}\left(\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1)/a^2 + 2*(3*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 11*c^3*((a*
x - 1)/(a*x + 1))^(5/2) - 11*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((
a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x +
1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*
a
```

Fricas [A]
time = 0.35, size = 109, normalized size = 1.40

$$-\frac{3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2\,a^4c^3x^4+2\,a^3c^3x^3-5\,a^2c^3x^2-5\,ac^3x)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) + (2*a^4*c^3*x^4 + 2*a^3*c^3*x^3 - 5*a^2*c^3*x^2 - 5*a*c^3*x
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int \frac{3ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int \frac{a^3x^3}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2*x**2/(a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x) + Integral(a**3*x**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [A]
time = 0.43, size = 84, normalized size = 1.08

$$-\frac{1}{8}\left(\frac{2\,a^2c^3x^2}{\operatorname{sgn}\left(ax+1\right)}-\frac{5\,c^3}{\operatorname{sgn}\left(ax+1\right)}\right)\sqrt{a^2x^2-1}\,x+\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\operatorname{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*c^3*x^2/sgn(a*x + 1) - 5*c^3/sgn(a*x + 1))*sqrt(a^2*x^2 - 1)*x
+ 3/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.20, size = 176, normalized size = 2.26

$$\frac{\frac{3\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}-\frac{11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}+\frac{3\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((3*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(3/2
))/4 - (11*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 + (3*c^3*((a*x - 1)/(a*x + 1)
)^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x + 1)^2
- (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (3*c^3*ata
nh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.180 $\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=78

$$\frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] 1/3*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3-1/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+
1/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6310, 6313, 864, 821, 272, 43, 65, 214}

$$\frac{1}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)
/3 - (c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^2\,dx = (a^2c^2)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$
$$= -\left((a^2c^2)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left((a^2c^2)\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - (ac^2)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{2}(ac^2)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a^2}}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$
$$= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \ldots\right.}{4a}$$
$$= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{1}{2}(ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,d\ldots\right.$$
$$= \frac{1}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3 - \frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

<u>Mathematica</u> [**A**]
time = 0.11, size = 64, normalized size = 0.82

$$\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+3ax+2a^2x^2) - 3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*a*x + 2*a^2*x^2) - 3*Log[a*(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A]
time = 0.08, size = 130, normalized size = 1.67

method	result	size
risch	$\frac{(2a^2x^2+3ax-2)(ax-1)c^2}{6a\sqrt{\frac{ax-1}{ax+1}}}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	112
default	$\frac{(ax-1)^2c^2\left(3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x-1)^2*c^2*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+2*((a*x+1)*(a*x-1))^
(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a
)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(66) = 132.
time = 0.26, size = 181, normalized size = 2.32

$$-\frac{1}{6}a\left(\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1)/a^2 - 2*(3*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 8*c^2*((a
*x - 1)/(a*x + 1))^(3/2) - 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^
2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 -
 a^2))
```

Fricas [A]
time = 0.36, size = 103, normalized size = 1.32

$$-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2a^3c^2x^3+5a^2c^2x^2+ac^2x-2c^2)\sqrt{\frac{ax-1}{ax+1}}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c^2*log(sqrt((a*x - 1)/(
a*x + 1)) - 1) - (2*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + a*c^2*x - 2*c^2)*sqrt((a*
x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\left(-\frac{2ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^{2}x^{2}}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2*x**2/(a*x*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [A]
time = 0.41, size = 98, normalized size = 1.26

$$\frac{1}{6}\sqrt{a^{2}x^{2}-1}\left(\left(\frac{2\,ac^{2}x}{\mathrm{sgn}\left(ax+1\right)}+\frac{3\,c^{2}}{\mathrm{sgn}\left(ax+1\right)}\right)x-\frac{2\,c^{2}}{a\mathrm{sgn}\left(ax+1\right)}\right)+\frac{c^{2}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)}{2\,|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a^2*x^2 - 1)*((2*a*c^2*x/sgn(a*x + 1) + 3*c^2/sgn(a*x + 1))*x - 2*
c^2/(a*sgn(a*x + 1))) + 1/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(ab
s(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.20, size = 139, normalized size = 1.78

$$\frac{c^{2}\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c^{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c^{2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^{2}}{{\left(a\,x+1\right)}^{2}}-\frac{a\,{\left(a\,x-1\right)}^{3}}{{\left(a\,x+1\right)}^{3}}}-\frac{c^{2}\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (c^2*((a*x - 1)/(a*x + 1))^(1/2) + (8*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 -
c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x
 - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^2*atanh(((a*x - 1)
/(a*x + 1))^(1/2)))/a
```

3.181 $\int e^{3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=65

$$-2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a-2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2
*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6310, 6313, 866, 1821, 821, 272, 65, 214}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}-2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)\,dx = -\left((ac)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$= (ac)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)$$
$$= (ac)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{2}(ac)\text{Subst}\left(\int \frac{-\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{4a}$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{2}(3ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1$$
$$= -2c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 0.82

$$-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^2
*x^2)])*x]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(55) = 110$.

time = 0.08, size = 162, normalized size = 2.49

method	result
risch	$-\frac{(ax+4)(ax-1)c}{2a\sqrt{\frac{ax-1}{ax+1}}}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c\left(\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+4\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}-\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+4a\ln\left(\frac{a^2x+}{}\right.\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x-1)^2*c*((a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+4*(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(1/2)-ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+4*a*
ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+
1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.

time = 0.26, size = 135, normalized size = 2.08

$$-\frac{1}{2}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(3/2) - 5*c*sqrt((a*x - 1)/(a*x + 1)))
/(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 3*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
/a^2)
```

Fricas [A]

time = 0.34, size = 81, normalized size = 1.25

$$-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2+5\,acx+4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 + 5*a*c*x + 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\frac{ax}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c),x)
```

```
[Out] -c*(Integral(a*x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x))
```

Giac [A]
time = 0.42, size = 74, normalized size = 1.14

$$-\frac{1}{2}\sqrt{a^2x^2-1}\left(\frac{cx}{\mathrm{sgn}\,(ax+1)}+\frac{4\,c}{a\mathrm{sgn}\,(ax+1)}\right)+\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2*x^2 - 1)*(c*x/sgn(a*x + 1) + 4*c/(a*sgn(a*x + 1))) + 3/2*c*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.20, size = 97, normalized size = 1.49

$$-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-3\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (5*c*((a*x - 1)/(a*x + 1))^(1/2) - 3*c*((a*x - 1)/(a*x + 1))^(3/2))/(a -
(2*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x
 - 1)/(a*x + 1))^(1/2)))/a
```

3.182 $\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=80

$$\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] 8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)-arctanh((1-1/a^2/x^2)^(1/2))/a/c+4/3/
a^2/c/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6310, 6313, 866, 1819, 12, 272, 65, 214}

$$\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{4}{3a^2cx\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) + 4/(3*a^2*c*Sqrt[1 - 1/
(a^2*x^2)]*x) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^4}{x\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-3-\frac{4x}{a}+\frac{3x^2}{a^2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{3}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{4}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.79

$$\frac{\frac{4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+2ax)}{(-1+ax)^2}-\frac{3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] ((4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x))/(-1 + a*x)^2 - (3*Log[a*(1 + Sqrt
[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(70) = 140.
time = 0.16, size = 345, normalized size = 4.31

method	result
default	$-\frac{3\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3+3\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+}}{\sqrt{a^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a*(3*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x
^3+3*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-9*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-3*(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(3/2)*a*x-9*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+9*ln((a^2*x+(a^2)^
(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+((a*x+1)*(a*x-1))^(3/2)*(
a^2)^(1/2)+9*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-3*a*ln((a^2*x+(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))-3*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2))/(a^2)^(1/2)/(a*x-1)/c/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1
))^(3/2)
```

Maxima [A]
time = 0.26, size = 95, normalized size = 1.19

$$-\frac{1}{3}\,a\left(\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{3\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\left(\frac{3\,(ax-1)}{ax+1}+1\right)}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 3*log(sqrt((a*x - 1)
/(a*x + 1)) - 1)/(a^2*c) - 2*(3*(a*x - 1)/(a*x + 1) + 1)/(a^2*c*((a*x - 1)/
(a*x + 1))^(3/2)))
```

Fricas [A]
time = 0.36, size = 120, normalized size = 1.50

$$-\frac{3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\left(a^{2}x^{2}-2\,ax+1\right)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-4\left(2\,a^{2}x^{2}+ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x
^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - 4*(2*a^2*x^2 + a*x - 1
)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{1}{\frac{a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x +
 1))/(a*x + 1)), x)/c
```

Giac [A]
time = 0.41, size = 35, normalized size = 0.44

$$\frac{\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)}{c|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 0.07, size = 63, normalized size = 0.79

$$\frac{\frac{2\,(a\,x-1)}{a\,x+1}+\frac{2}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((2*(a*x - 1))/(a*x + 1) + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2)) - (2*atan
h(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.183 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

[Out] -1/5*a^4*(1-1/a^2/x^2)^(5/2)/c^2/(a-1/x)^5

Rubi [A]
time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6313, 665}

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

Antiderivative was successfully verified.

[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]

[Out] -1/5*(a^4*(1 - 1/(a^2*x^2))^(5/2))/(c^2*(a - x^(-1))^5)

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5c^2\left(a-\frac{1}{x}\right)^5}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 1.09

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2}{5c^2(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -1/5*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2)/(c^2*(-1 + a*x)^3)
```

Maple [A]
time = 0.14, size = 36, normalized size = 1.09

method	result	size
gosper	$-\frac{ax+1}{5(ax-1)c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
default	$-\frac{ax+1}{5(ax-1)c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	36
trager	$-\frac{(ax+1)\left(a^2x^2+2ax+1\right)\sqrt{-\frac{-ax+1}{ax+1}}}{5a\,c^2(ax-1)^3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/5*(a*x+1)/(a*x-1)/c^2/((a*x-1)/(a*x+1))^(3/2)/a
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.70

$$-\frac{1}{5\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(29) = 58$.
time = 0.32, size = 77, normalized size = 2.33

$$-\frac{(a^3x^3 + 3\,a^2x^2 + 3\,ax + 1)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^2x^3 - 3\,a^3c^2x^2 + 3\,a^2c^2x - ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*x
^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1} - \frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1} + \frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1} - \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**2*
x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x
)/c**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.
time = 0.44, size = 69, normalized size = 2.09

$$-\frac{2\left(5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{5\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -2/5*(5*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 10*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^2)
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.70

$$-\frac{1}{5\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -1/(5*a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.184 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=67

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

```
[Out] 1/7*a^5*(1-1/a^2/x^2)^(5/2)/c^3/(a-1/x)^6-6/35*a^4*(1-1/a^2/x^2)^(5/2)/c^3/
(a-1/x)^5
```

Rubi [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6310, 6313, 807, 665}

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2))/(7*c^3*(a - x^(-1))^6) - (6*a^4*(1 - 1/(a^2*x
^2))^(5/2))/(35*c^3*(a - x^(-1))^5)
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3x^3}\,dx}{a^3c^3}\\ &= \frac{\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^6}\,dx,x,\frac{1}{x}\right)}{a^3c^3}\\ &= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^5}\,dx,x,\frac{1}{x}\right)}{7a^2c^3}\\ &= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7c^3\left(a-\frac{1}{x}\right)^6}-\frac{6a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{35c^3\left(a-\frac{1}{x}\right)^5}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 0.61

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-6+ax)(1+ax)^2}{35c^3(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-6 + a*x)*(1 + a*x)^2)/(c^3*(-1 + a*x)^4)
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.61

method	result	size
gosper	$-\frac{(ax-6)(ax+1)}{35(ax-1)^{2} c^{3} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a}$	41
default	$-\frac{(ax-6)(ax+1)}{35(ax-1)^{2} c^{3} \left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} a}$	41
trager	$-\frac{(ax+1)\left(a^{3} x^{3}-4 a^{2} x^{2}-11 a x-6\right) \sqrt{-\frac{-ax+1}{ax+1}}}{35 a\, c^{3} (ax-1)^{4}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(a*x-6)*(a*x+1)/(a*x-1)^2/c^3/((a*x-1)/(a*x+1))^(3/2)/a
```

Maxima [A]
time = 0.25, size = 39, normalized size = 0.58

$$-\frac{\frac{7\,(ax-1)}{ax+1}-5}{70\,ac^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/70*(7*(a*x - 1)/(a*x + 1) - 5)/(a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

Fricas [A]
time = 0.35, size = 95, normalized size = 1.42

$$-\frac{\left(a^{4}x^{4}-3\,a^{3}x^{3}-15\,a^{2}x^{2}-17\,ax-6\right)\sqrt{\frac{ax-1}{ax+1}}}{35\,\left(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(a^4*x^4 - 3*a^3*x^3 - 15*a^2*x^2 - 17*a*x - 6)*sqrt((a*x - 1)/(a*x +
 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**3
*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(59) = 118$.
time = 0.47, size = 125, normalized size = 1.87

$$\frac{2\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+70\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+14\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/35*(35*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 35*(a + sqrt(a^2 - 1/x^2))^4*x^4 +
 70*(a + sqrt(a^2 - 1/x^2))^3*x^3 + 14*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 7*(a
 + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^3)
```

Mupad [B]
time = 1.18, size = 39, normalized size = 0.58

$$-\frac{\frac{a\,x-1}{5\,\left(a\,x+1\right)}-\frac{1}{7}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -((a*x - 1)/(5*(a*x + 1)) - 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2))
```

3.185 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=94

$$-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

```
[Out] -47/315*(a+1/x)^5/a^6/c^4/(1-1/a^2/x^2)^(5/2)+16/63*(a+1/x)^6/a^7/c^4/(1-1/
a^2/x^2)^(7/2)-1/9*(a+1/x)^7/a^8/c^4/(1-1/a^2/x^2)^(9/2)
```

Rubi [A]
time = 0.18, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6310, 6313, 866, 1649, 803, 665}

$$-\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] (-47*(a + x^(-1))^5)/(315*a^6*c^4*(1 - 1/(a^2*x^2))^(5/2)) + (16*(a + x^(-1
))^6)/(63*a^7*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (a + x^(-1))^7/(9*a^8*c^4*(1 -
 1/(a^2*x^2))^(9/2))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
```

```
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^7}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^2\left(1+\frac{x}{a}\right)^7}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$
$$= -\frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6(7a^2+9ax)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{9a^4c^4}$$
$$= \frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{47\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^5}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{63a^2c^4}$$
$$= -\frac{47\left(a+\frac{1}{x}\right)^5}{315a^6c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{16\left(a+\frac{1}{x}\right)^6}{63a^7c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\left(a+\frac{1}{x}\right)^7}{9a^8c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.53

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2\left(47-14ax+2a^2x^2\right)}{315c^4(-1+ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/315*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(47 - 14*a*x + 2*a^2*x^2))/(c^4
*(-1 + a*x)^5)
```

Maple [A]
time = 0.14, size = 50, normalized size = 0.53

method	result	size
gosper	$-\frac{(2a^2x^2-14ax+47)(ax+1)}{315(ax-1)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50

default	$-\frac{(2a^2x^2-14ax+47)(ax+1)}{315(ax-1)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
trager	$-\frac{(ax+1)(2a^4x^4-10a^3x^3+21a^2x^2+80ax+47)\sqrt{-\frac{-ax+1}{ax+1}}}{315a\,c^4(ax-1)^5}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(2*a^2*x^2-14*a*x+47)*(a*x+1)/(a*x-1)^3/c^4/((a*x-1)/(a*x+1))^(3/2)/
a
```

Maxima [A]
time = 0.26, size = 55, normalized size = 0.59

$$\frac{\frac{90\,(ax-1)}{ax+1}-\frac{63\,(ax-1)^2}{(ax+1)^2}-35}{1260\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*(90*(a*x - 1)/(a*x + 1) - 63*(a*x - 1)^2/(a*x + 1)^2 - 35)/(a*c^4*((
a*x - 1)/(a*x + 1))^(9/2))
```

Fricas [A]
time = 0.35, size = 116, normalized size = 1.23

$$-\frac{(2\,a^5x^5-8\,a^4x^4+11\,a^3x^3+101\,a^2x^2+127\,ax+47)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/315*(2*a^5*x^5 - 8*a^4*x^4 + 11*a^3*x^3 + 101*a^2*x^2 + 127*a*x + 47)*sq
rt((a*x - 1)/(a*x + 1))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*
a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 5*a**4*
x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Giac [A]
time = 0.48, size = 145, normalized size = 1.54

$$-\frac{4\left(210\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6x^6+315\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+441\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+126\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3+36\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-9\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{315\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^9ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/315*(210*(a + sqrt(a^2 - 1/x^2))^6*x^6 + 315*(a + sqrt(a^2 - 1/x^2))^5*x
^5 + 441*(a + sqrt(a^2 - 1/x^2))^4*x^4 + 126*(a + sqrt(a^2 - 1/x^2))^3*x^3
+ 36*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 9*(a + sqrt(a^2 - 1/x^2))*x + 1)/(((a
+ sqrt(a^2 - 1/x^2))*x - 1)^9*a*c^4)
```

Mupad [B]
time = 1.19, size = 56, normalized size = 0.60

$$-\frac{\frac{{\left(a\,x-1\right)}^2}{5\,{\left(a\,x+1\right)}^2}-\frac{2\,\left(a\,x-1\right)}{7\,\left(a\,x+1\right)}+\frac{1}{9}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -((a*x - 1)^2/(5*(a*x + 1)^2) - (2*(a*x - 1))/(7*(a*x + 1)) + 1/9)/(4*a*c^4
*((a*x - 1)/(a*x + 1))^(9/2))
```

3.186 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=125

$$-\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}$$

```
[Out] -152/1155*(a+1/x)^5/a^6/c^5/(1-1/a^2/x^2)^(5/2)+79/231*(a+1/x)^6/a^7/c^5/(1
-1/a^2/x^2)^(7/2)-10/33*(a+1/x)^7/a^8/c^5/(1-1/a^2/x^2)^(9/2)+1/11*(a+1/x)^
8/a^9/c^5/(1-1/a^2/x^2)^(11/2)
```

Rubi [A]
time = 0.25, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6310, 6313, 866, 1649, 803, 665}

$$\frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}}-\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

```
[Out] (-152*(a + x^(-1))^5)/(1155*a^6*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (79*(a + x^(
-1))^6)/(231*a^7*c^5*(1 - 1/(a^2*x^2))^(7/2)) - (10*(a + x^(-1))^7)/(33*a^8
*c^5*(1 - 1/(a^2*x^2))^(9/2)) + (a + x^(-1))^8/(11*a^9*c^5*(1 - 1/(a^2*x^2)
)^(11/2))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
```

```
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^8}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^8}{\left(1-\frac{x^2}{a^2}\right)^{13/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^7(8a^3+11a^2x+11ax^2)}{\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{11a^5c^5}$$

$$= -\frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^6(138a^3+99a^2x)}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{99a^5c^5}$$

$$= \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right)^8}{11a^9c^5\left(1-\frac{1}{a^2x^2}\right)^{11/2}} - \frac{152\text{Subst}\left(\int \frac{(1}{(1-}\right.}{231a}$$

$$= -\frac{152\left(a+\frac{1}{x}\right)^5}{1155a^6c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{79\left(a+\frac{1}{x}\right)^6}{231a^7c^5\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{10\left(a+\frac{1}{x}\right)^7}{33a^8c^5\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\left(a+\frac{1}{x}\right.}{11a^9c^5\left(1-\right.}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.46

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(1+ax)^2\left(-152+61ax-16a^2x^2+2a^3x^3\right)}{1155c^5(-1+ax)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^5,x]
```

```
[Out] -1/1155*(Sqrt[1 - 1/(a^2*x^2)]*x*(1 + a*x)^2*(-152 + 61*a*x - 16*a^2*x^2 +
2*a^3*x^3))/(c^5*(-1 + a*x)^6)
```

Maple [A]
time = 0.13, size = 58, normalized size = 0.46

method	result	size
gosper	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
default	$-\frac{(2a^3x^3-16a^2x^2+61ax-152)(ax+1)}{1155(ax-1)^4c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
trager	$-\frac{(ax+1)(2a^5x^5-12a^4x^4+31a^3x^3-46a^2x^2-243ax-152)\sqrt{-\frac{-ax+1}{ax+1}}}{1155a\,c^5(ax-1)^6}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1155*(2*a^3*x^3-16*a^2*x^2+61*a*x-152)*(a*x+1)/(a*x-1)^4/c^5/((a*x-1)/(a
*x+1))^(3/2)/a
```

Maxima [A]
time = 0.26, size = 71, normalized size = 0.57

$$-\frac{\frac{385\,(ax-1)}{ax+1}-\frac{495\,(ax-1)^2}{(ax+1)^2}+\frac{231\,(ax-1)^3}{(ax+1)^3}-105}{9240\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/9240*(385*(a*x - 1)/(a*x + 1) - 495*(a*x - 1)^2/(a*x + 1)^2 + 231*(a*x -
 1)^3/(a*x + 1)^3 - 105)/(a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

Fricas [A]
time = 0.36, size = 134, normalized size = 1.07

$$-\frac{(2\,a^6x^6-10\,a^5x^5+19\,a^4x^4-15\,a^3x^3-289\,a^2x^2-395\,ax-152)\sqrt{\frac{ax-1}{ax+1}}}{1155\,(a^7c^5x^6-6\,a^6c^5x^5+15\,a^5c^5x^4-20\,a^4c^5x^3+15\,a^3c^5x^2-6\,a^2c^5x+ac^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/1155*(2*a^6*x^6 - 10*a^5*x^5 + 19*a^4*x^4 - 15*a^3*x^3 - 289*a^2*x^2 - 3
95*a*x - 152)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 15*a
^5*c^5*x^4 - 20*a^4*c^5*x^3 + 15*a^3*c^5*x^2 - 6*a^2*c^5*x + a*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{20a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{15a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 6*a**5
*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 15*a**4*x**4*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 20*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))/(a*x + 1) + 15*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - 6*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1)), x)/c**5
```

Giac [A]
time = 0.53, size = 165, normalized size = 1.32

$$\frac{4\left(1155\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^7x^7+2079\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^6x^6+2541\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^5x^5+825\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^4x^4+165\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3-55\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+11\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{1155\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^{11}ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 4/1155*(1155*(a + sqrt(a^2 - 1/x^2))^7*x^7 + 2079*(a + sqrt(a^2 - 1/x^2))^6
*x^6 + 2541*(a + sqrt(a^2 - 1/x^2))^5*x^5 + 825*(a + sqrt(a^2 - 1/x^2))^4*x
^4 + 165*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 55*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 11*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^11*a*c^
5)
```

Mupad [B]
time = 0.04, size = 72, normalized size = 0.58

$$\frac{\frac{3\,{\left(a\,x-1\right)}^2}{7\,{\left(a\,x+1\right)}^2}-\frac{{\left(a\,x-1\right)}^3}{5\,{\left(a\,x+1\right)}^3}-\frac{a\,x-1}{3\,\left(a\,x+1\right)}+\frac{1}{11}}{8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((3*(a*x - 1)^2)/(7*(a*x + 1)^2) - (a*x - 1)^3/(5*(a*x + 1)^3) - (a*x - 1)/
(3*(a*x + 1)) + 1/11)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(11/2))
```

3.187 $\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=66

$$\frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}$$

```
[Out] 4*c*(-a*c*x+c)^(-1+p)/a/(1-p)+4*(-a*c*x+c)^p/a/p-(-a*c*x+c)^(1+p)/a/c/(1+p)
```

Rubi [A]

time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6265, 21, 45}

$$\frac{4c(c-acx)^{p-1}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{p+1}}{ac(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] (4*c*(c - a*c*x)^(-1 + p))/(a*(1 - p)) + (4*(c - a*c*x)^p)/(a*p) - (c - a*c
*x)^(1 + p)/(a*c*(1 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^p\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^p\,dx\\
&= \int \frac{(1+ax)^2(c-acx)^p}{(1-ax)^2}\,dx\\
&= c^2\int (1+ax)^2(c-acx)^{-2+p}\,dx\\
&= c^2\int \left(4(c-acx)^{-2+p}-\frac{4(c-acx)^{-1+p}}{c}+\frac{(c-acx)^p}{c^2}\right)dx\\
&= \frac{4c(c-acx)^{-1+p}}{a(1-p)}+\frac{4(c-acx)^p}{ap}-\frac{(c-acx)^{1+p}}{ac(1+p)}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.76

$$\frac{(c-acx)^p\left(\frac{4+3p}{p(1+p)}+\frac{ax}{1+p}+\frac{4}{(-1+p)(-1+ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^p,x]
```

```
[Out] ((c - a*c*x)^p*((4 + 3*p)/(p*(1 + p)) + (a*x)/(1 + p) + 4/((-1 + p)*(-1 + a
*x))))/a
```

Maple [A]
time = 0.16, size = 74, normalized size = 1.12

method	result	size
gosper	$\frac{(-acx+c)^{p}\left(a^{2}p^{2}x^{2}-a^{2}x^{2}p+2a\,p^{2}x+2apx-4ax+p^{2}+3p+4\right)}{(p^{2}-1)ap(ax-1)}$	74
risch	$\frac{\left(a^{2}p^{2}x^{2}-a^{2}x^{2}p+2a\,p^{2}x+2apx-4ax+p^{2}+3p+4\right)(-acx+c)^{p}}{ap(1+p)(-1+p)(ax-1)}$	77
norman	$\frac{\frac{a\,x^{2}e^{p\ln(-acx+c)}}{1+p}+\frac{\left(p^{2}+3p+4\right)e^{p\ln(-acx+c)}}{ap\left(p^{2}-1\right)}+\frac{2(2+p)x\,e^{p\ln(-acx+c)}}{p(1+p)}}{ax-1}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x,method=_RETURNVERBOSE)
```

```
[Out] (-a*c*x+c)^p*(a^2*p^2*x^2-a^2*p*x^2+2*a*p^2*x+2*a*p*x-4*a*x+p^2+3*p+4)/(p^2
-1)/a/p/(a*x-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

time = 0.28, size = 153, normalized size = 2.32

$$\frac{((p^2-p)a^2c^px^2+2\,ac^p(p-1)x+2\,c^p)(-ax+1)^pa^2}{(p^3-p)a^4x-(p^3-p)a^3}+\frac{2\,(ac^p(p-1)x+c^p)(-ax+1)^pa}{(p^2-p)a^3x-(p^2-p)a^2}+\frac{(-ax+1)^pc^p}{a^2(p-1)x-a(p-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 - p)*a^2*c^p*x^2 + 2*a*c^p*(p - 1)*x + 2*c^p)*(-a*x + 1)^p*a^2/((p^3
- p)*a^4*x - (p^3 - p)*a^3) + 2*(a*c^p*(p - 1)*x + c^p)*(-a*x + 1)^p*a/((p^
2 - p)*a^3*x - (p^2 - p)*a^2) + (-a*x + 1)^p*c^p/(a^2*(p - 1)*x - a*(p - 1)
)
```

Fricas [A]

time = 0.36, size = 81, normalized size = 1.23

$$-\frac{((a^2p^2-a^2p)x^2+p^2+2\,(ap^2+ap-2\,a)x+3\,p+4)(-acx+c)^p}{ap^3-ap-(a^2p^3-a^2p)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="fricas")
```

```
[Out] -((a^2*p^2 - a^2*p)*x^2 + p^2 + 2*(a*p^2 + a*p - 2*a)*x + 3*p + 4)*(-a*c*x
+ c)^p/(a*p^3 - a*p - (a^2*p^3 - a^2*p)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(48) = 96$.

time = 0.56, size = 530, normalized size = 8.03

$$\begin{cases} c^px & \text{for}\: a=0 \\ -\frac{a^2x^2\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{2ax\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}+\frac{4ax}{a^3cx^2-2a^2cx+ac}-\frac{\log\left(x-\frac{1}{a}\right)}{a^3cx^2-2a^2cx+ac}-\frac{2}{a^3cx^2-2a^2cx+ac} & \text{for}\: p=-1 \\ \frac{a^2x^2}{a^2x-a}+\frac{4ax\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{4\log\left(x-\frac{1}{a}\right)}{a^2x-a}-\frac{5}{a^2x-a} & \text{for}\: p=0 \\ -\frac{acx^2}{2}-3cx-\frac{4c\log\left(x-\frac{1}{a}\right)}{a} & \text{for}\: p=1 \\ \frac{a^2p^2x^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{a^2px^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2ap^2x(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{2apx(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}-\frac{4ax(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{p^2(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{3p(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap}+\frac{4(-acx+c)^p}{a^2p^3x-a^2px-ap^3+ap} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**p,x)
```

```
[Out] Piecewise((c**p*x, Eq(a, 0)), (-a**2*x**2*log(x - 1/a)/(a**3*c*x**2 - 2*a**
2*c*x + a*c) + 2*a*x*log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + 4*a*x/
(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(x - 1/a)/(a**3*c*x**2 - 2*a**2*c*x +
 a*c) - 2/(a**3*c*x**2 - 2*a**2*c*x + a*c), Eq(p, -1)), (a**2*x**2/(a**2*x
- a) + 4*a*x*log(x - 1/a)/(a**2*x - a) - 4*log(x - 1/a)/(a**2*x - a) - 5/(a
**2*x - a), Eq(p, 0)), (-a*c*x**2/2 - 3*c*x - 4*c*log(x - 1/a)/a, Eq(p, 1))
, (a**2*p**2*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) -
```

```
 a**2*p*x**2*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*
p**2*x*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + 2*a*p*x*(-
a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) - 4*a*x*(-a*c*x + c)*
*p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*p) + p**2*(-a*c*x + c)**p/(a**2*p**
3*x - a**2*p*x - a*p**3 + a*p) + 3*p*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*
x - a*p**3 + a*p) + 4*(-a*c*x + c)**p/(a**2*p**3*x - a**2*p*x - a*p**3 + a*
p), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)^2*(-a*c*x + c)^p/(a*x - 1)^2, x)
```

Mupad [B]
time = 1.35, size = 57, normalized size = 0.86

$$\frac{4\,{\left(c-a\,c\,x\right)}^p}{a\,\left(a\,x-1\right)\,\left(p-1\right)}+\frac{{\left(c-a\,c\,x\right)}^p\,\left(3\,p+a\,p\,x+4\right)}{a\,p\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (4*(c - a*c*x)^p)/(a*(a*x - 1)*(p - 1)) + ((c - a*c*x)^p*(3*p + a*p*x + 4))
/(a*p*(p + 1))
```

3.188 $\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx$

Optimal. Leaf size=53

$$-\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

```
[Out] -c^5*(-a*x+1)^4/a+4/5*c^5*(-a*x+1)^5/a-1/6*c^5*(-a*x+1)^6/a
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$-\frac{c^5(1-ax)^6}{6a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^4}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
```

```
[Out] -((c^5*(1 - a*x)^4)/a) + (4*c^5*(1 - a*x)^5)/(5*a) - (c^5*(1 - a*x)^6)/(6*a
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}(c-acx)^5\,dx = \int e^{4\tanh^{-1}(ax)}(c-acx)^5\,dx$$
$$= c^5\int (1-ax)^3(1+ax)^2\,dx$$
$$= c^5\int \left(4(1-ax)^3-4(1-ax)^4+(1-ax)^5\right)\,dx$$
$$= -\frac{c^5(1-ax)^4}{a}+\frac{4c^5(1-ax)^5}{5a}-\frac{c^5(1-ax)^6}{6a}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.58

$$-\frac{c^5(-1+ax)^4\left(11+14ax+5a^2x^2\right)}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^5,x]
[Out] -1/30*(c^5*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2))/a
```

Maple [A]
time = 0.18, size = 45, normalized size = 0.85

method	result
gosper	$-\frac{x\left(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30\right)c^5}{30}$
default	$c^5\left(-\frac{1}{6}a^5x^6+\frac{1}{5}a^4x^5+\frac{1}{2}a^3x^4-\frac{2}{3}a^2x^3-\frac{1}{2}a\,x^2+x\right)$
risch	$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3-\frac{1}{2}c^5a\,x^2+c^5x$
norman	$\frac{-c^5x+\frac{1}{6}a^2c^5x^3-\frac{7}{6}a^3c^5x^4+\frac{3}{10}a^4c^5x^5+\frac{11}{30}a^5c^5x^6-\frac{1}{6}a^6c^5x^7+\frac{3}{2}c^5a\,x^2}{ax-1}$
meijerg	$-\frac{c^5\left(\frac{ax\left(-20a^6x^6-28a^5x^5-42a^4x^4-70a^3x^3-140a^2x^2-420ax+840\right)}{-120ax+120}+7\ln(-ax+1)\right)}{a}-\frac{3c^5\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-70a^2x^2-21\right.}{70(-ax+1)}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
[Out] c^5*(-1/6*a^5*x^6+1/5*a^4*x^5+1/2*a^3*x^4-2/3*a^2*x^3-1/2*a*x^2+x)
```

Maxima [A]
time = 0.25, size = 59, normalized size = 1.11

$$-\frac{1}{6}\,a^5c^5x^6+\frac{1}{5}\,a^4c^5x^5+\frac{1}{2}\,a^3c^5x^4-\frac{2}{3}\,a^2c^5x^3-\frac{1}{2}\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Fricas [A]
time = 0.35, size = 59, normalized size = 1.11

$$-\frac{1}{6}a^5c^5x^6+\frac{1}{5}a^4c^5x^5+\frac{1}{2}a^3c^5x^4-\frac{2}{3}a^2c^5x^3-\frac{1}{2}ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/6*a^5*c^5*x^6 + 1/5*a^4*c^5*x^5 + 1/2*a^3*c^5*x^4 - 2/3*a^2*c^5*x^3 - 1/
2*a*c^5*x^2 + c^5*x
```

Sympy [A]
time = 0.03, size = 63, normalized size = 1.19

$$-\frac{a^5c^5x^6}{6}+\frac{a^4c^5x^5}{5}+\frac{a^3c^5x^4}{2}-\frac{2a^2c^5x^3}{3}-\frac{ac^5x^2}{2}+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**5,x)
```

```
[Out] -a**5*c**5*x**6/6 + a**4*c**5*x**5/5 + a**3*c**5*x**4/2 - 2*a**2*c**5*x**3/
3 - a*c**5*x**2/2 + c**5*x
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.79

$$-\frac{\left(5\,c^5+\frac{24\,c^5}{ax-1}+\frac{30\,c^5}{(ax-1)^2}\right)(ax-1)^6}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/30*(5*c^5 + 24*c^5/(a*x - 1) + 30*c^5/(a*x - 1)^2)*(a*x - 1)^6/a
```

Mupad [B]
time = 1.19, size = 59, normalized size = 1.11

$$-\frac{a^5\,c^5\,x^6}{6}+\frac{a^4\,c^5\,x^5}{5}+\frac{a^3\,c^5\,x^4}{2}-\frac{2\,a^2\,c^5\,x^3}{3}-\frac{a\,c^5\,x^2}{2}+c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^5*x - (a*c^5*x^2)/2 - (2*a^2*c^5*x^3)/3 + (a^3*c^5*x^4)/2 + (a^4*c^5*x^5)
/5 - (a^5*c^5*x^6)/6
```

3.189 $\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=32

$$c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$$

[Out] c^4*x-2/3*a^2*c^4*x^3+1/5*a^4*c^4*x^5

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {6302, 6264, 41, 200}

$$\frac{1}{5}a^4c^4x^5-\frac{2}{3}a^2c^4x^3+c^4x$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]

[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}(c-acx)^4\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^4\,dx\\ &= c^4\int (1-ax)^2(1+ax)^2\,dx\\ &= c^4\int \left(1-a^2x^2\right)^2\,dx\\ &= c^4\int \left(1-2a^2x^2+a^4x^4\right)\,dx\\ &= c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^4,x]
[Out] c^4*x - (2*a^2*c^4*x^3)/3 + (a^4*c^4*x^5)/5
```

Maple [A]
time = 0.18, size = 23, normalized size = 0.72

method	result
default	$c^4\left(\frac{1}{5}a^4x^5-\frac{2}{3}a^2x^3+x\right)$
gosper	$\frac{x\left(3a^4x^4-10a^2x^2+15\right)c^4}{15}$
risch	$c^4x-\frac{2}{3}a^2c^4x^3+\frac{1}{5}a^4c^4x^5$
norman	$\frac{-c^4x+c^4a\,x^2+\frac{2}{3}a^2c^4x^3-\frac{2}{3}a^3c^4x^4-\frac{1}{5}a^4c^4x^5+\frac{1}{5}a^5c^4x^6}{ax-1}$
meijerg	$-\frac{c^4\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-70a^2x^2-210ax+420\right)}{70(-ax+1)}-6\ln(-ax+1)\right)}{a}-\frac{2c^4\left(\frac{ax\left(-3a^4x^4-5a^3x^3-10a^2x^2-30ax+60\right)}{-12ax+12}+5\ln(-a\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
[Out] c^4*(1/5*a^4*x^5-2/3*a^2*x^3+x)
```

Maxima [A]
time = 0.25, size = 28, normalized size = 0.88

$$\frac{1}{5}\,a^4c^4x^5-\frac{2}{3}\,a^2c^4x^3+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="maxima")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Fricas [A]
time = 0.32, size = 28, normalized size = 0.88

$$\frac{1}{5}\,a^4c^4x^5 - \frac{2}{3}\,a^2c^4x^3 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="fricas")
[Out] 1/5*a^4*c^4*x^5 - 2/3*a^2*c^4*x^3 + c^4*x
```

Sympy [A]
time = 0.02, size = 29, normalized size = 0.91

$$\frac{a^4c^4x^5}{5} - \frac{2a^2c^4x^3}{3} + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**4,x)
[Out] a**4*c**4*x**5/5 - 2*a**2*c**4*x**3/3 + c**4*x
```

Giac [A]
time = 0.39, size = 42, normalized size = 1.31

$$\frac{\left(3\,c^4 + \frac{15\,c^4}{ax-1} + \frac{20\,c^4}{(ax-1)^2}\right)(ax-1)^5}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^4,x, algorithm="giac")
[Out] 1/15*(3*c^4 + 15*c^4/(a*x - 1) + 20*c^4/(a*x - 1)^2)*(a*x - 1)^5/a
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.75

$$\frac{c^4\,x\,(3\,a^4\,x^4 - 10\,a^2\,x^2 + 15)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] (c^4*x*(3*a^4*x^4 - 10*a^2*x^2 + 15))/15
```

3.190 $\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=35

$$\frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}$$

```
[Out] 2/3*c^3*(a*x+1)^3/a-1/4*c^3*(a*x+1)^4/a
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6302, 6264, 45}

$$\frac{2c^3(ax+1)^3}{3a}-\frac{c^3(ax+1)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] (2*c^3*(1 + a*x)^3)/(3*a) - (c^3*(1 + a*x)^4)/(4*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^3\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^3\,dx \\
&= c^3\int (1-ax)(1+ax)^2\,dx \\
&= c^3\int \left(2(1+ax)^2-(1+ax)^3\right)\,dx \\
&= \frac{2c^3(1+ax)^3}{3a}-\frac{c^3(1+ax)^4}{4a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.86

$$-\frac{1}{12}c^3x\left(-12-6ax+4a^2x^2+3a^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -1/12*(c^3*x*(-12 - 6*a*x + 4*a^2*x^2 + 3*a^3*x^3))
```

Maple [A]
time = 0.17, size = 29, normalized size = 0.83

method	result
gosper	$-\frac{x\left(3a^3x^3+4a^2x^2-6ax-12\right)c^3}{12}$
default	$c^3\left(-\frac{1}{4}a^3x^4-\frac{1}{3}a^2x^3+\frac{1}{2}a\,x^2+x\right)$
risch	$-\frac{1}{4}a^3c^3x^4-\frac{1}{3}a^2c^3x^3+\frac{1}{2}c^3a\,x^2+c^3x$
norman	$\frac{-c^3x+\frac{5}{6}a^2c^3x^3-\frac{1}{12}a^3c^3x^4-\frac{1}{4}a^4c^3x^5+\frac{1}{2}c^3a\,x^2}{ax-1}$
meijerg	$-\frac{c^3\left(\frac{ax\left(-3a^4x^4-5a^3x^3-10a^2x^2-30ax+60\right)}{-12ax+12}+5\ln(-ax+1)\right)}{a}-\frac{c^3\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}+\frac{2c^3\left(\frac{ax\left(-2\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/4*a^3*x^4-1/3*a^2*x^3+1/2*a*x^2+x)
```

Maxima [A]
time = 0.26, size = 37, normalized size = 1.06

$$-\frac{1}{4}\,a^3c^3x^4-\frac{1}{3}\,a^2c^3x^3+\frac{1}{2}\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Fricas [A]
time = 0.34, size = 37, normalized size = 1.06

$$-\frac{1}{4}a^3c^3x^4 - \frac{1}{3}a^2c^3x^3 + \frac{1}{2}ac^3x^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/4*a^3*c^3*x^4 - 1/3*a^2*c^3*x^3 + 1/2*a*c^3*x^2 + c^3*x
```

Sympy [A]
time = 0.03, size = 37, normalized size = 1.06

$$-\frac{a^3c^3x^4}{4} - \frac{a^2c^3x^3}{3} + \frac{ac^3x^2}{2} + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**3,x)
[Out] -a**3*c**3*x**4/4 - a**2*c**3*x**3/3 + a*c**3*x**2/2 + c**3*x
```

Giac [A]
time = 0.40, size = 42, normalized size = 1.20

$$-\frac{\left(3\,c^3 + \frac{16\,c^3}{ax-1} + \frac{24\,c^3}{(ax-1)^2}\right)(ax-1)^4}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^3,x, algorithm="giac")
[Out] -1/12*(3*c^3 + 16*c^3/(a*x - 1) + 24*c^3/(a*x - 1)^2)*(a*x - 1)^4/a
```

Mupad [B]
time = 0.05, size = 37, normalized size = 1.06

$$-\frac{a^3\,c^3\,x^4}{4} - \frac{a^2\,c^3\,x^3}{3} + \frac{a\,c^3\,x^2}{2} + c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c^3*x + (a*c^3*x^2)/2 - (a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4
```

3.191 $\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(1+ax)^3}{3a}$$

[Out] 1/3*c^2*(a*x+1)^3/a

Rubi [A]

time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 32}

$$\frac{c^2(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]

[Out] (c^2*(1 + a*x)^3)/(3*a)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}(c-acx)^2\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)^2\,dx\\
&= c^2\int (1+ax)^2\,dx\\
&= \frac{c^2(1+ax)^3}{3a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 1.59

$$c^2x + ac^2x^2 + \frac{1}{3}a^2c^2x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x)^2,x]
[Out] c^2*x + a*c^2*x^2 + (a^2*c^2*x^3)/3
```

Maple [A]
time = 0.16, size = 16, normalized size = 0.94

method	result	size
default	$\frac{c^2(ax+1)^3}{3a}$	16
gosper	$\frac{x(a^2x^2+3ax+3)c^2}{3}$	20
risch	$\frac{a^2c^2x^3}{3} + a\,c^2x^2 + c^2x + \frac{c^2}{3a}$	34
norman	$\frac{-\frac{c^2}{a}+\frac{2a^2c^2x^3}{3}+\frac{a^3c^2x^4}{3}}{ax-1}$	40
meijerg	$-\frac{c^2\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}+\frac{2c^2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c^2x}{-ax+1}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*c^2*(a*x+1)^3/a
```

Maxima [A]
time = 0.25, size = 25, normalized size = 1.47

$$\frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="maxima")
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Fricas [A]
time = 0.36, size = 25, normalized size = 1.47

$$\frac{1}{3}\,a^2c^2x^3 + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*a^2*c^2*x^3 + a*c^2*x^2 + c^2*x
```

Sympy [A]
time = 0.02, size = 24, normalized size = 1.41

$$\frac{a^2c^2x^3}{3} + ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + a*c**2*x**2 + c**2*x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(15) = 30.
time = 0.40, size = 40, normalized size = 2.35

$$\frac{\left(c^2 + \frac{6\,c^2}{ax-1} + \frac{12\,c^2}{(ax-1)^2}\right)(ax-1)^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(c^2 + 6*c^2/(a*x - 1) + 12*c^2/(a*x - 1)^2)*(a*x - 1)^3/a
```

Mupad [B]
time = 0.03, size = 19, normalized size = 1.12

$$\frac{c^2\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^2*x*(3*a*x + a^2*x^2 + 3))/3
```

3.192 $\int e^{4\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=27

$$-3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}$$

```
[Out] -3*c*x-1/2*a*c*x^2-4*c*ln(-a*x+1)/a
```

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6302, 6264, 45}

$$-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}-3cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -3*c*x - (a*c*x^2)/2 - (4*c*Log[1 - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}(c-acx)\,dx &= \int e^{4\tanh^{-1}(ax)}(c-acx)\,dx\\ &= c\int \frac{(1+ax)^2}{1-ax}\,dx\\ &= c\int \left(-3-ax+\frac{4}{1-ax}\right)dx\\ &= -3cx-\frac{1}{2}acx^2-\frac{4c\log(1-ax)}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.96

$$c\left(-3x-\frac{ax^2}{2}-\frac{4\log(1-ax)}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a*c*x),x]
[Out] c*(-3*x - (a*x^2)/2 - (4*Log[1 - a*x])/a)
```

Maple [A]
time = 0.22, size = 24, normalized size = 0.89

method	result	size
default	$c\left(-\frac{a\,x^2}{2}-3x-\frac{4\ln(ax-1)}{a}\right)$	24
risch	$-\frac{ac\,x^2}{2}-3cx-\frac{4c\ln(ax-1)}{a}$	25
norman	$\frac{3cx-\frac{5}{2}ac\,x^2-\frac{1}{2}a^2c\,x^3}{ax-1}-\frac{4c\ln(ax-1)}{a}$	43
meijerg	$-\frac{c\left(\frac{ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+3\ln(-ax+1)\right)}{a}+\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}+\frac{cx}{-ax+1}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] c*(-1/2*a*x^2-3*x-4/a*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.89

$$-\frac{1}{2}\,acx^2-3\,cx-\frac{4\,c\log\left(ax-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Fricas [A]
time = 0.35, size = 28, normalized size = 1.04

$$-\frac{a^2cx^2 + 6\,acx + 8\,c\log\left(ax - 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2 + 6*a*c*x + 8*c*log(a*x - 1))/a
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.96

$$-\frac{acx^2}{2} - 3cx - \frac{4c\log\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a*c*x+c),x)
```

```
[Out] -a*c*x**2/2 - 3*c*x - 4*c*log(a*x - 1)/a
```

Giac [A]
time = 0.41, size = 50, normalized size = 1.85

$$-\frac{\left(ax - 1\right)^2\left(c + \frac{8\,c}{ax - 1}\right)}{2\,a} + \frac{4\,c\log\left(\frac{|ax - 1|}{\left(ax - 1\right)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a*c*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*x - 1)^2*(c + 8*c/(a*x - 1))/a + 4*c*log(abs(a*x - 1)/((a*x - 1)^2*
abs(a)))/a
```

Mupad [B]
time = 1.18, size = 26, normalized size = 0.96

$$-\frac{c\left(8\,\ln\left(a\,x - 1\right) + 6\,a\,x + a^2\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] -(c*(8*log(a*x - 1) + 6*a*x + a^2*x^2))/(2*a)
```

3.193 $\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=48

$$\frac{2}{ac(1-ax)^2}-\frac{4}{ac(1-ax)}-\frac{\log(1-ax)}{ac}$$

```
[Out] 2/a/c/(-a*x+1)^2-4/a/c/(-a*x+1)-ln(-a*x+1)/a/c
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 45}

$$-\frac{4}{ac(1-ax)}+\frac{2}{ac(1-ax)^2}-\frac{\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] 2/(a*c*(1 - a*x)^2) - 4/(a*c*(1 - a*x)) - Log[1 - a*x]/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{c-acx}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{c-acx}\,dx$$
$$= \frac{\int \frac{(1+ax)^2}{(1-ax)^3}\,dx}{c}$$
$$= \frac{\int \left(\frac{1}{1-ax} - \frac{4}{(-1+ax)^3} - \frac{4}{(-1+ax)^2}\right)dx}{c}$$
$$= \frac{2}{ac(1-ax)^2} - \frac{4}{ac(1-ax)} - \frac{\log(1-ax)}{ac}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.75

$$\frac{-2+4ax-(-1+ax)^2\log(1-ax)}{ac(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x),x]
[Out] (-2 + 4*a*x - (-1 + a*x)^2*Log[1 - a*x])/(a*c*(-1 + a*x)^2)
```

Maple [A]
time = 0.17, size = 41, normalized size = 0.85

method	result	size
norman	$\frac{2a\,x^2}{c(ax-1)^2} - \frac{\ln(ax-1)}{ac}$	32
risch	$\frac{4x-\frac{2}{a}}{(ax-1)^2 c} - \frac{\ln(ax-1)}{ac}$	36
default	$\frac{\frac{4}{a(ax-1)}+\frac{2}{a(ax-1)^2}-\frac{\ln(ax-1)}{a}}{c}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] 1/c*(4/a/(a*x-1)+2/a/(a*x-1)^2-1/a*ln(a*x-1))
```

Maxima [A]
time = 0.25, size = 44, normalized size = 0.92

$$\frac{2\,(2\,ax-1)}{a^3cx^2-2\,a^2cx+ac} - \frac{\log\,(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] 2*(2*a*x - 1)/(a^3*c*x^2 - 2*a^2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Fricas [A]
time = 0.35, size = 49, normalized size = 1.02

$$\frac{4\,ax-(a^2x^2-2\,ax+1)\log(ax-1)-2}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (4*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x - 1) - 2)/(a^3*c*x^2 - 2*a^2*c*x + a
*c)
```

Sympy [A]
time = 0.12, size = 37, normalized size = 0.77

$$-\frac{-4ax+2}{a^3cx^2-2a^2cx+ac}-\frac{\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c),x)
```

```
[Out] -(-4*a*x + 2)/(a**3*c*x**2 - 2*a**2*c*x + a*c) - log(a*x - 1)/(a*c)
```

Giac [A]
time = 0.39, size = 57, normalized size = 1.19

$$\frac{\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}+\frac{2\left(\frac{2\,ac}{ax-1}+\frac{ac}{(ax-1)^2}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) + 2*(2*a*c/(a*x - 1) + a*c/(a*
x - 1)^2)/(a^2*c^2)
```

Mupad [B]
time = 0.06, size = 42, normalized size = 0.88

$$\frac{4\,x-\frac{2}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}-\frac{\ln(a\,x-1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)*(a*x - 1)^2),x)
```

```
[Out] (4*x - 2/a)/(c + a^2*c*x^2 - 2*a*c*x) - log(a*x - 1)/(a*c)
```

3.194 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=25

$$\frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

```
[Out] 1/6*(a*x+1)^3/a/c^2/(-a*x+1)^3
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 37}

$$\frac{(ax+1)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$$
$$= \frac{\int \frac{(1+ax)^2}{(1-ax)^4}\,dx}{c^2}$$
$$= \frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{(1+ax)^3}{6ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] (1 + a*x)^3/(6*a*c^2*(1 - a*x)^3)
```

Maple [A]
time = 0.15, size = 42, normalized size = 1.68

method	result	size
risch	$\frac{-a\,x^2-\frac{1}{3a}}{(ax-1)^3c^2}$	24
gosper	$-\frac{3a^2x^2+1}{3(ax-1)^3a\,c^2}$	26
norman	$\frac{-\frac{x}{c}-\frac{a^2x^3}{3c}}{(ax-1)^3c}$	30
default	$\frac{-\frac{1}{a(ax-1)}-\frac{4}{3a(ax-1)^3}-\frac{2}{a(ax-1)^2}}{c^2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/a/(a*x-1)-4/3/a/(a*x-1)^3-2/a/(a*x-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.
time = 0.26, size = 51, normalized size = 2.04

$$-\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.
time = 0.37, size = 51, normalized size = 2.04

$$-\frac{3\,a^2x^2+1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="fricas")
[Out] -1/3*(3*a^2*x^2 + 1)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.
time = 0.14, size = 51, normalized size = 2.04

$$\frac{-3a^2x^2-1}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**2,x)
[Out] (-3*a**2*x**2 - 1)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3
*a*c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(22) = 44$.
time = 0.41, size = 50, normalized size = 2.00

$$-\frac{2}{(acx-c)^2a}-\frac{1}{(acx-c)ac}-\frac{4\,c}{3\,(acx-c)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^2,x, algorithm="giac")
[Out] -2/((a*c*x - c)^2*a) - 1/((a*c*x - c)*a*c) - 4/3*c/((a*c*x - c)^3*a)
```

Mupad [B]
time = 1.19, size = 25, normalized size = 1.00

$$-\frac{3\,a^2\,x^2+1}{3\,a\,c^2\,(a\,x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)^2*(a*x - 1)^2),x)
[Out] -(3*a^2*x^2 + 1)/(3*a*c^2*(a*x - 1)^3)
```

3.195 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=52

$$\frac{1}{ac^3(1-ax)^4}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{2ac^3(1-ax)^2}$$

```
[Out] 1/a/c^3/(-a*x+1)^4-4/3/a/c^3/(-a*x+1)^3+1/2/a/c^3/(-a*x+1)^2
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 45}

$$\frac{1}{2ac^3(1-ax)^2}-\frac{4}{3ac^3(1-ax)^3}+\frac{1}{ac^3(1-ax)^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] 1/(a*c^3*(1 - a*x)^4) - 4/(3*a*c^3*(1 - a*x)^3) + 1/(2*a*c^3*(1 - a*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^3}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^3}\,dx$$
$$= \frac{\int \frac{(1+ax)^2}{(1-ax)^5}\,dx}{c^3}$$
$$= \frac{\int \left(-\frac{4}{(-1+ax)^5} - \frac{4}{(-1+ax)^4} - \frac{1}{(-1+ax)^3}\right)dx}{c^3}$$
$$= \frac{1}{ac^3(1-ax)^4} - \frac{4}{3ac^3(1-ax)^3} + \frac{1}{2ac^3(1-ax)^2}$$

Mathematica [**A**]
time = 0.02, size = 31, normalized size = 0.60

$$\frac{1+2ax+3a^2x^2}{6ac^3(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^3,x]
[Out] (1 + 2*a*x + 3*a^2*x^2)/(6*a*c^3*(-1 + a*x)^4)
```

Maple [**A**]
time = 0.15, size = 41, normalized size = 0.79

method	result	size
risch	$\frac{\frac{a\,x^2}{2}+\frac{x}{3}+\frac{1}{6a}}{(ax-1)^4c^3}$	27
gosper	$\frac{3a^2x^2+2ax+1}{6(ax-1)^4a\,c^3}$	30
default	$\frac{\frac{1}{a(ax-1)^4}+\frac{4}{3a(ax-1)^3}+\frac{1}{2a(ax-1)^2}}{c^3}$	41
norman	$\frac{\frac{x}{c}-\frac{a\,x^2}{2c}+\frac{2a^2x^3}{3c}-\frac{a^3x^4}{6c}}{(ax-1)^4c^2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^3*(1/a/(a*x-1)^4+4/3/a/(a*x-1)^3+1/2/a/(a*x-1)^2)
```

Maxima [**A**]
time = 0.26, size = 65, normalized size = 1.25

$$\frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

Fricas [A]
time = 0.34, size = 65, normalized size = 1.25

$$\frac{3\,a^2x^2+2\,ax+1}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*x^2 + 2*a*x + 1)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 -
4*a^2*c^3*x + a*c^3)
```

Sympy [A]
time = 0.19, size = 70, normalized size = 1.35

$$-\frac{-3a^2x^2-2ax-1}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**3,x)
```

```
[Out] -(-3*a**2*x**2 - 2*a*x - 1)/(6*a**5*c**3*x**4 - 24*a**4*c**3*x**3 + 36*a**3
*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3)
```

Giac [A]
time = 0.40, size = 42, normalized size = 0.81

$$\frac{\frac{3}{(ax-1)^2a}+\frac{8}{(ax-1)^3a}+\frac{6}{(ax-1)^4a}}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*(3/((a*x - 1)^2*a) + 8/((a*x - 1)^3*a) + 6/((a*x - 1)^4*a))/c^3
```

Mupad [B]
time = 1.23, size = 29, normalized size = 0.56

$$\frac{3\,a^2\,x^2+2\,a\,x+1}{6\,a\,c^3\,{\left(a\,x-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)^3*(a*x - 1)^2),x)
```

```
[Out] (2*a*x + 3*a^2*x^2 + 1)/(6*a*c^3*(a*x - 1)^4)
```

3.196 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=53

$$\frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

```
[Out] 4/5/a/c^4/(-a*x+1)^5-1/a/c^4/(-a*x+1)^4+1/3/a/c^4/(-a*x+1)^3
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6302, 6264, 45}

$$\frac{1}{3ac^4(1-ax)^3}-\frac{1}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] 4/(5*a*c^4*(1 - a*x)^5) - 1/(a*c^4*(1 - a*x)^4) + 1/(3*a*c^4*(1 - a*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{(c-acx)^4}\,dx$$
$$= \frac{\int \frac{(1+ax)^2}{(1-ax)^6}\,dx}{c^4}$$
$$= \frac{\int \left(\frac{4}{(-1+ax)^6}+\frac{4}{(-1+ax)^5}+\frac{1}{(-1+ax)^4}\right)dx}{c^4}$$
$$= \frac{4}{5ac^4(1-ax)^5}-\frac{1}{ac^4(1-ax)^4}+\frac{1}{3ac^4(1-ax)^3}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.58

$$-\frac{2+5ax+5a^2x^2}{15ac^4(-1+ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/15*(2 + 5*a*x + 5*a^2*x^2)/(a*c^4*(-1 + a*x)^5)
```

Maple [A]
time = 0.16, size = 42, normalized size = 0.79

method	result	size
risch	$\frac{-\frac{a\,x^2}{3}-\frac{x}{3}-\frac{2}{15a}}{(ax-1)^5 c^4}$	27
gosper	$-\frac{5a^2x^2+5ax+2}{15(ax-1)^5 a\,c^4}$	30
default	$\frac{-\frac{4}{5a(ax-1)^5}-\frac{1}{a(ax-1)^4}-\frac{1}{3a(ax-1)^3}}{c^4}$	42
norman	$\frac{\frac{a\,x^2}{c}-\frac{x}{c}-\frac{4a^2x^3}{3c}+\frac{2a^3x^4}{3c}-\frac{2a^4x^5}{15c}}{(ax-1)^5 c^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-4/5/a/(a*x-1)^5-1/a/(a*x-1)^4-1/3/a/(a*x-1)^3)
```

Maxima [A]
time = 0.26, size = 77, normalized size = 1.45

$$-\frac{5\,a^2x^2+5\,ax+2}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Fricas [A]
time = 0.33, size = 77, normalized size = 1.45

$$-\frac{5\,a^{2}x^{2}+5\,ax+2}{15\,(a^{6}c^{4}x^{5}-5\,a^{5}c^{4}x^{4}+10\,a^{4}c^{4}x^{3}-10\,a^{3}c^{4}x^{2}+5\,a^{2}c^{4}x-ac^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(5*a^2*x^2 + 5*a*x + 2)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3
 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A]
time = 0.21, size = 80, normalized size = 1.51

$$\frac{-5a^{2}x^{2}-5ax-2}{15a^{6}c^{4}x^{5}-75a^{5}c^{4}x^{4}+150a^{4}c^{4}x^{3}-150a^{3}c^{4}x^{2}+75a^{2}c^{4}x-15ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a*c*x+c)**4,x)
```

```
[Out] (-5*a**2*x**2 - 5*a*x - 2)/(15*a**6*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**
4*c**4*x**3 - 150*a**3*c**4*x**2 + 75*a**2*c**4*x - 15*a*c**4)
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.79

$$-\frac{\frac{5}{(ax-1)^{3}a}+\frac{15}{(ax-1)^{4}a}+\frac{12}{(ax-1)^{5}a}}{15\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/15*(5/((a*x - 1)^3*a) + 15/((a*x - 1)^4*a) + 12/((a*x - 1)^5*a))/c^4
```

Mupad [B]
time = 1.24, size = 29, normalized size = 0.55

$$-\frac{5\,a^2\,x^2+5\,a\,x+2}{15\,a\,c^4\,{\left(a\,x-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a*c*x)^4*(a*x - 1)^2),x)
```

```
[Out] -(5*a*x + 5*a^2*x^2 + 2)/(15*a*c^4*(a*x - 1)^5)
```

3.197 $\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=94

$$\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p\,{}_2F_1\left(-1-p,-\frac{1}{2}-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

```
[Out] ((a-1/x)/(a+1/x))^(-1/2-p)*x*(-a*c*x+c)^p*hypergeom([-1-p, -1/2-p],[-p],2/(
a+1/x)/x)*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(1+p)
```

Rubi [A]
time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6311, 6316, 134}

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}(c-acx)^p\,{}_2F_1\left(-p-1,-p-\frac{1}{2};-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^p*Hypergeometric2F1[-1 - p, -1/2 - p, -p, 2/((a + x^(-1))*x)
])/(1 + p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$
$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x(c-acx)^p\,{}_2F_1\left(-1-p,-\frac{1}{2}-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.81

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(c-acx)^p\,{}_2F_1\left(-1-p,-\frac{1}{2}-p;-p;\frac{2}{1+ax}\right)}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^ArcCoth[a*x],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(c - a*c*x)^p*Hy
pergeometric2F1[-1 - p, -1/2 - p, -p, 2/(1 + a*x)])/(1 + p)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int(-acx+c)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,(-c(ax-1))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c-a\,c\,x)^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.198 $\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=127

$$\frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4-\frac{35c^3\tanh^{-1}\left(\sqrt{1-\right.}{8a}$$

```
[Out] -35/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+20/3*c^3*x*(1-1/a^2/x^2)^(1/2)-27/
8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+4/3*a^2*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a^3
*c^3*x^4*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6310, 6313, 1821, 821, 272, 65, 214}

$$-\frac{27}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{20}{3}c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{35c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+\frac{4}{3}a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

```
[Out] (20*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (27*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8
 + (4*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]
*x^4)/4 - (35*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{16}{a}-\frac{27x}{a^2}+\frac{16x^2}{a^3}-\frac{4x^3}{a^4}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{12}\left(a^3c^3\right)\text{Subst}\left(\int \frac{\frac{81}{a^2}-\frac{80x}{a^3}\ldots}{x^3\sqrt{1-\ldots}}\right.$$

$$= -\frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{24}(\ldots$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{\ldots}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{\ldots}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{\ldots}$$

$$= \frac{20}{3}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{27}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{4}{3}a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{\ldots}$$

<u>**Mathematica**</u> [A]

time = 0.09, size = 72, normalized size = 0.57

$$\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(160-81ax+32a^2x^2-6a^3x^3)-105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(160 - 81*a*x + 32*a^2*x^2 - 6*a^3*x^3) - 1
05*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(24*a)
```

Maple [A]
time = 0.13, size = 196, normalized size = 1.54

method	result
risch	$-\frac{(6a^3x^3-32a^2x^2+81ax-160)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{24a}-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{8\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(6\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+87\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-32((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-87\ln\left(\frac{a^2x+\sqrt{a^2x^2-}}{\sqrt{a^2}}\right.\right.}{24a\sqrt{(ax+1)(ax-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3/a*(6*(a^2)^(1/2)*(a^2*x^2-1)^(3/2
)*a*x+87*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-32*((a*x+1)*(a*x-1))^(3/2)*(a^2)
^(1/2)-87*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-192*(a^2)
^(1/2)*((a*x+1)*(a*x-1))^(1/2)+192*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1)
)^(1/2))/(a^2)^(1/2)))/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(107) = 214.
time = 0.26, size = 221, normalized size = 1.74

$$-\frac{1}{24}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(279\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-511\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+385\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4(ax-1)a^2}{ax+1}-\frac{6(ax-1)^2a^2}{(ax+1)^2}+\frac{4(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(279*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 511*
```

```
c^3*((a*x - 1)/(a*x + 1))^(5/2) + 385*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105
*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*
a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)
^4 - a^2))*a
```

Fricas [A]
time = 0.40, size = 114, normalized size = 0.90

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^3x^4-26\,a^3c^3x^3+49\,a^2c^3x^2-79\,ac^3x-160\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (6*a^4*c^3*x^4 - 26*a^3*c^3*x^3 + 49*a^2*c^3*x^2 - 79
*a*c^3*x - 160*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-3*a
**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**3*x**3*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
), x))
```

Giac [A]
time = 0.40, size = 109, normalized size = 0.86

$$\frac{35\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{8\,|a|}+\frac{1}{24}\sqrt{a^2x^2-1}\left(\frac{160\,c^3\mathrm{sgn}(ax+1)}{a}-\left(81\,c^3\mathrm{sgn}(ax+1)+2\left(3\,a^2c^3x\mathrm{sgn}(ax+1)-16\,ac^3\mathrm{sgn}(ax+1)\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 35/8*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/24
*sqrt(a^2*x^2 - 1)*(160*c^3*sgn(a*x + 1)/a - (81*c^3*sgn(a*x + 1) + 2*(3*a^
2*c^3*x*sgn(a*x + 1) - 16*a*c^3*sgn(a*x + 1))*x)*x)
```

Mupad [B]
time = 1.23, size = 176, normalized size = 1.39

$$\frac{\frac{35\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{385\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{511\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{12}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{35\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((35*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (385*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/12 + (511*c^3*((a*x - 1)/(a*x + 1))^(5/2))/12 - (93*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x +
 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) - (35*
c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.199 $\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=100

$$\frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -5/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+11/3*c^2*x*(1-1/a^2/x^2)^(1/2)-3/2*
a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3*a^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.389$, Rules used = {6310, 6313, 1821, 821, 272, 65, 214}

$$-\frac{3}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{11}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

```
[Out] (11*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)/3 - (3*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2
+ (a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^3)/3 - (5*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^2\,dx = (a^2c^2)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left((a^2c^2)\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{1}{3}(a^2c^2)\,\text{Subst}\left(\int \frac{\frac{9}{a}-\frac{11x}{a^2}+\frac{3x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{6}(a^2c^2)\,\text{Subst}\left(\int \frac{\frac{22}{a^2}-\frac{15}{a^3}\cdots}{x^2\sqrt{1-\cdots}}\right.$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{(5c^2)\,\text{Sub}\cdots}{\cdots}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{(5c^2)\,\text{Sub}\cdots}{\cdots}$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{2}(5ac^2)\,\text{S}\cdots$$

$$= \frac{11}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{5c^2\tanh^{\cdots}}{\cdots}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 0.64

$$\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(22-9ax+2a^2x^2)-15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 - 9*a*x + 2*a^2*x^2) - 15*Log[a*(1 + Sq
rt[1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(84) = 168$.
time = 0.12, size = 176, normalized size = 1.76

method	result
risch	$\frac{(2a^2x^2-9ax+22)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a} - \frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(9\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{a^2}\ \cdots\right.}{6\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(9*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a
*x-2*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*a-24*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+24*a*ln((a^2
*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x+1)*(a*x-1))^(1/
2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(84) = 168$.
time = 0.26, size = 181, normalized size = 1.81

$$-\frac{1}{6}\,a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{2\left(33\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 + 2*(33*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 40*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x -
1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1
)^3 - a^2))
```

Fricas [A]
time = 0.38, size = 104, normalized size = 1.04

$$-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-7\,a^2c^2x^2+13\,ac^2x+22\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 7*a^2*c^2*x^2 + 13*a*c^2*x + 22*c^2)*sq
rt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**2
*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) -
 1/(a*x + 1)), x))
```

Giac [A]
time = 0.42, size = 90, normalized size = 0.90

$$\frac{5\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{2\,|a|}+\frac{1}{6}\,\sqrt{a^2x^2-1}\left(\left(2\,ac^2x\mathrm{sgn}(ax+1)-9\,c^2\mathrm{sgn}(ax+1)\right)x+\frac{22\,c^2\mathrm{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/2*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/6*s
qrt(a^2*x^2 - 1)*((2*a*c^2*x*sgn(a*x + 1) - 9*c^2*sgn(a*x + 1))*x + 22*c^2*
sgn(a*x + 1)/a)
```

Mupad [B]
time = 1.21, size = 140, normalized size = 1.40

$$\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+11\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{5\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (40*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + 11*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*
a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c^2*atanh(((
a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.200 $\int e^{-\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=65

$$2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -3/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+2*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2
*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6310, 6313, 1821, 821, 272, 65, 214}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+2cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^ArcCoth[a*x],x]
```

```
[Out] 2*c*Sqrt[1 - 1/(a^2*x^2)]*x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (3*c*ArcT
anh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)\,dx = -\left((ac)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$
$$=(ac)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$
$$=-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}(ac)\text{Subst}\left(\int\frac{\frac{4}{a}-\frac{3x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$
$$=2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(3c)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$
$$=2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{(3c)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{4a}$$
$$=2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{1}{2}(3ac)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1\right.$$
$$=2c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.82

$$-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-4+ax)+3\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-4 + a*x) + 3*Log[a*(1 + Sqrt[1 - 1/(a^
2*x^2)])*x]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(55) = 110$.
time = 0.10, size = 153, normalized size = 2.35

method	result
risch	$-\frac{(ax-4)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+4\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a-4a\ln\right.}{2\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+4
*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*a-4*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^
(1/2)))/((a*x+1)*(a*x-1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(55) = 110$.
time = 0.27, size = 135, normalized size = 2.08

$$\frac{1}{2}\,a\left(\frac{2\left(5\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(5*c*((a*x - 1)/(a*x + 1))^(3/2) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/
(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 3*c*log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
a^2)
```

Fricas [A]
time = 0.36, size = 81, normalized size = 1.25

$$-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(a^2cx^2-3\,acx-4\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (a^2*c*x^2 - 3*a*c*x - 4*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.42, size = 68, normalized size = 1.05

$$\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}-\frac{1}{2}\sqrt{a^2x^2-1}\left(cx\operatorname{sgn}(ax+1)-\frac{4\,c\operatorname{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/2*sqr
t(a^2*x^2 - 1)*(c*x*sgn(a*x + 1) - 4*c*sgn(a*x + 1)/a)
```

Mupad [B]
time = 0.06, size = 96, normalized size = 1.48

$$\frac{3\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{3\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (3*c*((a*x - 1)/(a*x + 1))^(1/2) - 5*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2
*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (3*c*atanh(((a*x -
 1)/(a*x + 1))^(1/2)))/a
```

3.201 $\int \frac{e^{-\coth^{-1}(ax)}}{c-acx} dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c

Rubi [A]

time = 0.07, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6310, 6313, 272, 65, 214}

$$-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]

[Out] -(ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{c-acx}\,dx &= -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}\\
&= -\frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 1.48

$$
-\frac{\log\left(ax\left(1+\sqrt{\frac{-1+a^2x^2}{a^2x^2}}\right)\right)}{ac}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)),x]
[Out] -(Log[a*x*(1 + Sqrt[(-1 + a^2*x^2)/(a^2*x^2)])]/(a*c))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(21) = 42$.
time = 0.15, size = 76, normalized size = 3.30

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^{2}}}\right)}{\sqrt{(ax+1)(ax-1)}\,c\sqrt{a^{2}}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1
/2))/(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1/2)/c/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.
time = 0.26, size = 55, normalized size = 2.39

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="maxima")
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.
time = 0.33, size = 47, normalized size = 2.04

$$-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] -(log(sqrt((a*x - 1)/(a*x + 1)) + 1) - log(sqrt((a*x - 1)/(a*x + 1)) - 1))/
(a*c)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{ax-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c),x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Giac [**A**]
time = 0.41, size = 33, normalized size = 1.43

$$\frac{\log\left(\left|-x|a| + \sqrt{a^2x^2 - 1}\right|\right)\operatorname{sgn}(ax+1)}{c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a))
```

Mupad [**B**]
time = 0.06, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x),x)
```

```
[Out] -(2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.202 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2} \, dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

```
[Out] -(1-1/a^2/x^2)^(1/2)/c^2/(a-1/x)
```

Rubi [A]
time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6313, 665}

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

```
[Out] -(Sqrt[1 - 1/(a^2*x^2)]/(c^2*(a - x^(-1))))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}$$

Mathematica [A]
time = 0.07, size = 27, normalized size = 0.96

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^2),x]
```

```
[Out] -((Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(-1 + a*x)))
```

Maple [A]
time = 0.16, size = 36, normalized size = 1.29

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)c^2a}$	36
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{(ax-1)c^2a}$	36
trager	$-\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^2(ax-1)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/(a*x-1)/c^2/a

Maxima [A]
time = 0.26, size = 23, normalized size = 0.82

$$-\frac{1}{ac^2\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/(a*c^2*sqrt((a*x - 1)/(a*x + 1)))
```

Fricas [A]
time = 0.35, size = 39, normalized size = 1.39

$$-\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.82

$$-\frac{1}{a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^2,x)
[Out] -1/(a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

3.203 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=62

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

```
[Out] 1/3*a*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)^2-2/3*(1-1/a^2/x^2)^(1/2)/c^3/(a-1/x)
```

Rubi [A]
time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6310, 6313, 807, 665}

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)])/(3*c^3*(a - x^(-1))^2) - (2*Sqrt[1 - 1/(a^2*x^2)]
)/(3*c^3*(a - x^(-1)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3x^3}\,dx}{a^3c^3}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3a^2c^3}$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)^2} - \frac{2\sqrt{1-\frac{1}{a^2x^2}}}{3c^3\left(a-\frac{1}{x}\right)}$$

Mathematica [A]
time = 0.08, size = 34, normalized size = 0.55

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+ax)}{3c^3(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^3),x]
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(c^3*(-1 + a*x)^2)
```

Maple [A]
time = 0.16, size = 41, normalized size = 0.66

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax-2)(ax+1)}{3(ax-1)^{2}c^{3}a}$	41
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax-2)(ax+1)}{3(ax-1)^{2}c^{3}a}$	41
trager	$-\frac{(ax-2)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{3}(ax-1)^{2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(a*x-2)*(a*x+1)/(a*x-1)^2/c^3/a
```

Maxima [A]
time = 0.25, size = 39, normalized size = 0.63

$$-\frac{\frac{3\,(ax-1)}{ax+1}-1}{6\,ac^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="maxima")
[Out] -1/6*(3*(a*x - 1)/(a*x + 1) - 1)/(a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

Fricas [A]
time = 0.40, size = 57, normalized size = 0.92

$$-\frac{(a^{2}x^{2}-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^{3}c^{3}x^{2}-2\,a^{2}c^{3}x+ac^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="fricas")
[Out] -1/3*(a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^3*x^2 - 2*a^2*c^3
*x + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{3}x^{3}-3a^{2}x^{2}+3ax-1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**3,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x**2 + 3*a*
x - 1), x)/c**3
```

Giac [A]
time = 0.43, size = 45, normalized size = 0.73

$$\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^3*a*
c^3)
```

Mupad [B]
time = 0.03, size = 38, normalized size = 0.61

$$-\frac{\frac{a\,x-1}{a\,x+1}-\frac{1}{3}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^3,x)
```

```
[Out] -((a*x - 1)/(a*x + 1) - 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

3.204 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=95

$$-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

```
[Out] -1/5*a^2*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)^3+8/15*a*(1-1/a^2/x^2)^(1/2)/c^4/(
a-1/x)^2-7/15*(1-1/a^2/x^2)^(1/2)/c^4/(a-1/x)
```

Rubi [A]

time = 0.16, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$-\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3}+\frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
```

```
[Out] -1/5*(a^2*Sqrt[1 - 1/(a^2*x^2)])/(c^4*(a - x^(-1))^3) + (8*a*Sqrt[1 - 1/(a^
2*x^2)])/(15*c^4*(a - x^(-1))^2) - (7*Sqrt[1 - 1/(a^2*x^2)])/(15*c^4*(a - x
^(-1)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
```

```
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4x^4}\,dx}{a^4c^4}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{x}{a^3}}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5a^2c^4}$$

$$= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{15a^2c^4}$$

$$= -\frac{a^2\sqrt{1-\frac{1}{a^2x^2}}}{5c^4\left(a-\frac{1}{x}\right)^3} + \frac{8a\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)^2} - \frac{7\sqrt{1-\frac{1}{a^2x^2}}}{15c^4\left(a-\frac{1}{x}\right)}$$

Mathematica [A]
time = 0.08, size = 43, normalized size = 0.45

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(7-6ax+2a^2x^2)}{15c^4(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^4),x]
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^4*(-1 + a*x)^3)
```

Maple [A]
time = 0.17, size = 50, normalized size = 0.53

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3c^4a}$	50
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2-6ax+7)(ax+1)}{15(ax-1)^3c^4a}$	50
trager	$-\frac{(2a^2x^2-6ax+7)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^4(ax-1)^3}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2-6*a*x+7)*(a*x+1)/(a*x-1)^3/c^4/a
```

Maxima [A]
time = 0.25, size = 55, normalized size = 0.58

$$\frac{\frac{10\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-3}{60\,ac^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^4*((a*x
 - 1)/(a*x + 1))^(5/2))
```

Fricas [A]
time = 0.43, size = 77, normalized size = 0.81

$$-\frac{(2\,a^3x^3-4\,a^2x^2+ax+7)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^4*
x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x**3 + 6*a**
2*x**2 - 4*a*x + 1), x)/c**4
```

Giac [A]
time = 0.43, size = 65, normalized size = 0.68

$$-\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^4)
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.58

$$-\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^4,x)
```

```
[Out] -((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^4*((a
*x - 1)/(a*x + 1))^(5/2))
```

3.205 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=128

$$\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}-\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3}+\frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2}-\frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

```
[Out] 1/7*a^3*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^4-18/35*a^2*(1-1/a^2/x^2)^(1/2)/c^5
/(a-1/x)^3+23/35*a*(1-1/a^2/x^2)^(1/2)/c^5/(a-1/x)^2-12/35*(1-1/a^2/x^2)^(1
/2)/c^5/(a-1/x)
```

Rubi [A]

time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6310, 6313, 1653, 807, 673, 665}

$$-\frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3}+\frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2}-\frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}+\frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

```
[Out] (a^3*Sqrt[1 - 1/(a^2*x^2)])/(7*c^5*(a - x^(-1))^4) - (18*a^2*Sqrt[1 - 1/(a^
2*x^2)])/(35*c^5*(a - x^(-1))^3) + (23*a*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a
- x^(-1))^2) - (12*Sqrt[1 - 1/(a^2*x^2)])/(35*c^5*(a - x^(-1)))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 673

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-e)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m + p + 1))), x] + Dist[Simpl
ify[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x]
, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ
[p] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 807

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(m
 + p + 1))), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2}{a^2}-\frac{3x}{a^3}}{\left(1-\frac{x}{a}\right)^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{18\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{7a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{a\sqrt{1-\frac{1}{a^2x^2}}}{c^5\left(a-\frac{1}{x}\right)^2} - \frac{36\text{Subst}\Bigg(\int \frac{1}{\left(1-\frac{x}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,d}{35a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\text{Subst}\Bigg(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}}{35a^2c^5}$$

$$= \frac{a^3\sqrt{1-\frac{1}{a^2x^2}}}{7c^5\left(a-\frac{1}{x}\right)^4} - \frac{18a^2\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^3} + \frac{23a\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)^2} - \frac{12\sqrt{1-\frac{1}{a^2x^2}}}{35c^5\left(a-\frac{1}{x}\right)}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 0.40

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-12+13ax-8a^2x^2+2a^3x^3)}{35c^5(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^5),x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-12 + 13*a*x - 8*a^2*x^2 + 2*a^3*x^3))/(c^5
*(-1 + a*x)^4)
```

Maple [A]
time = 0.17, size = 58, normalized size = 0.45

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^3x^3-8a^2x^2+13ax-12)(ax+1)}{35(ax-1)^4c^5a}$	58
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^3x^3-8a^2x^2+13ax-12)(ax+1)}{35(ax-1)^4c^5a}$	58
trager	$-\frac{(2a^3x^3-8a^2x^2+13ax-12)(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^5(ax-1)^4}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(2*a^3*x^3-8*a^2*x^2+13*a*x-12)*(a*x+1)/(a*x-
1)^4/c^5/a
```

Maxima [A]
time = 0.25, size = 71, normalized size = 0.55

$$-\frac{\frac{21\,(ax-1)}{ax+1}-\frac{35\,(ax-1)^2}{(ax+1)^2}+\frac{35\,(ax-1)^3}{(ax+1)^3}-5}{280\,ac^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/280*(21*(a*x - 1)/(a*x + 1) - 35*(a*x - 1)^2/(a*x + 1)^2 + 35*(a*x - 1)^
3/(a*x + 1)^3 - 5)/(a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

Fricas [A]
time = 0.37, size = 95, normalized size = 0.74

$$-\frac{(2\,a^4x^4-6\,a^3x^3+5\,a^2x^2+ax-12)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^5x^4-4\,a^4c^5x^3+6\,a^3c^5x^2-4\,a^2c^5x+ac^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/35*(2*a^4*x^4 - 6*a^3*x^3 + 5*a^2*x^2 + a*x - 12)*sqrt((a*x - 1)/(a*x +
1))/(a^5*c^5*x^4 - 4*a^4*c^5*x^3 + 6*a^3*c^5*x^2 - 4*a^2*c^5*x + a*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**5,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 5*a**4*x**4 + 10*a
**3*x**3 - 10*a**2*x**2 + 5*a*x - 1), x)/c**5
```

Giac [A]
time = 0.47, size = 85, normalized size = 0.66

$$\frac{4\left(35\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^3x^3-21\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+7\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)}{35\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^7ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] 4/35*(35*(a + sqrt(a^2 - 1/x^2))^3*x^3 - 21*(a + sqrt(a^2 - 1/x^2))^2*x^2 +
 7*(a + sqrt(a^2 - 1/x^2))*x - 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^7*a*c^5)
```

Mupad [B]
time = 1.19, size = 71, normalized size = 0.55

$$\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{(a\,x-1)^3}{(a\,x+1)^3}-\frac{3\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{7}}{8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^5,x)
```

```
[Out] ((a*x - 1)^2/(a*x + 1)^2 - (a*x - 1)^3/(a*x + 1)^3 - (3*(a*x - 1))/(5*(a*x
+ 1)) + 1/7)/(8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2))
```

3.206 $\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=44

$$\frac{(c-acx)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}$$

```
[Out] 1/2*(-a*c*x+c)^(2+p)*hypergeom([1, 2+p],[3+p],-1/2*a*x+1/2)/a/c^2/(2+p)
```

Rubi [A]

time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6265, 21, 70}

$$\frac{(c-acx)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{1}{2}(1-ax)\right)}{2ac^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - a*c*x)^(2 + p)*Hypergeometric2F1[1, 2 + p, 3 + p, (1 - a*x)/2])/(2*a*
c^2*(2 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^p\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^p\,dx \\
&= -\int \frac{(1-ax)(c-acx)^p}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{1+p}}{1+ax}\,dx}{c} \\
&= \frac{(c-acx)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{1}{2}(1-ax)\right)}{2ac^2(2+p)}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.00

$$
-\frac{(-1+ax)(c-acx)^p\left(-1+{}_2F_1\left(1,1+p;2+p;\frac{1}{2}(1-ax)\right)\right)}{a(1+p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -(((-1 + a*x)*(c - a*c*x)^p*(-1 + Hypergeometric2F1[1, 1 + p, 2 + p, (1 - a
*x)/2]))/(a*(1 + p)))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$
\int \frac{(-acx+c)^p\,(ax-1)}{ax+1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

```
[Out] int((-a*c*x+c)^p*(a*x-1)/(a*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c(ax-1)\right)^{p}\left(ax-1\right)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**p*(a*x-1)/(a*x+1),x)
[Out] Integral((-c*(a*x - 1))**p*(a*x - 1)/(a*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(-a*c*x + c)^p/(a*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c-a\,c\,x\right)}^p\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a*c*x)^p*(a*x - 1))/(a*x + 1), x)
```

3.207 $\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx$

Optimal. Leaf size=91

$$16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log(1+ax)}{a}$$

```
[Out] 16*c^4*x-4*c^4*(-a*x+1)^2/a-4/3*c^4*(-a*x+1)^3/a-1/2*c^4*(-a*x+1)^4/a-1/5*c
^4*(-a*x+1)^5/a-32*c^4*ln(a*x+1)/a
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6302, 6264, 45}

$$-\frac{c^4(1-ax)^5}{5a}-\frac{c^4(1-ax)^4}{2a}-\frac{4c^4(1-ax)^3}{3a}-\frac{4c^4(1-ax)^2}{a}-\frac{32c^4\log(ax+1)}{a}+16c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 16*c^4*x - (4*c^4*(1 - a*x)^2)/a - (4*c^4*(1 - a*x)^3)/(3*a) - (c^4*(1 - a*
x)^4)/(2*a) - (c^4*(1 - a*x)^5)/(5*a) - (32*c^4*Log[1 + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^4\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)^4\,dx$$
$$= -\left(c^4\int\frac{(1-ax)^5}{1+ax}\,dx\right)$$
$$= -\left(c^4\int\left(-16-8(1-ax)-4(1-ax)^2-2(1-ax)^3-(1-ax)^4+\frac{32}{1+ax}\right.\right.$$
$$= 16c^4x-\frac{4c^4(1-ax)^2}{a}-\frac{4c^4(1-ax)^3}{3a}-\frac{c^4(1-ax)^4}{2a}-\frac{c^4(1-ax)^5}{5a}-\frac{32c^4\log}{}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.62

$$\frac{c^4(-181+930ax-390a^2x^2+160a^3x^3-45a^4x^4+6a^5x^5-960\log(1+ax))}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^4/E^(2*ArcCoth[a*x]),x]
[Out] (c^4*(-181 + 930*a*x - 390*a^2*x^2 + 160*a^3*x^3 - 45*a^4*x^4 + 6*a^5*x^5 -
 960*Log[1 + a*x]))/(30*a)
```

Maple [A]
time = 0.14, size = 50, normalized size = 0.55

method	result
default	$c^4\left(\frac{a^4x^5}{5}-\frac{3a^3x^4}{2}+\frac{16a^2x^3}{3}-13a\,x^2+31x-\frac{32\ln(ax+1)}{a}\right)$
norman	$31c^4x+\frac{16a^2c^4x^3}{3}-\frac{3a^3c^4x^4}{2}+\frac{a^4c^4x^5}{5}-13c^4a\,x^2-\frac{32c^4\ln(ax+1)}{a}$
risch	$31c^4x+\frac{16a^2c^4x^3}{3}-\frac{3a^3c^4x^4}{2}+\frac{a^4c^4x^5}{5}-13c^4a\,x^2-\frac{32c^4\ln(ax+1)}{a}$
meijerg	$\frac{c^4\left(\frac{ax\left(12a^4x^4-15a^3x^3+20a^2x^2-30ax+60\right)}{60}-\ln(ax+1)\right)}{a}-\frac{5c^4\left(-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)\right)}{a}+\frac{10c^4\left(\frac{ax\left(4a^2x^2-6\right.}{12}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^4*(1/5*a^4*x^5-3/2*a^3*x^4+16/3*a^2*x^3-13*a*x^2+31*x-32/a*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 63, normalized size = 0.69

$$\frac{1}{5}\,a^4c^4x^5-\frac{3}{2}\,a^3c^4x^4+\frac{16}{3}\,a^2c^4x^3-13\,ac^4x^2+31\,c^4x-\frac{32\,c^4\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/5*a^4*c^4*x^5 - 3/2*a^3*c^4*x^4 + 16/3*a^2*c^4*x^3 - 13*a*c^4*x^2 + 31*c^
4*x - 32*c^4*log(a*x + 1)/a
```

Fricas [A]
time = 0.35, size = 68, normalized size = 0.75

$$\frac{6\,a^5c^4x^5 - 45\,a^4c^4x^4 + 160\,a^3c^4x^3 - 390\,a^2c^4x^2 + 930\,ac^4x - 960\,c^4\log\left(ax+1\right)}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(6*a^5*c^4*x^5 - 45*a^4*c^4*x^4 + 160*a^3*c^4*x^3 - 390*a^2*c^4*x^2 +
930*a*c^4*x - 960*c^4*log(a*x + 1))/a
```

Sympy [A]
time = 0.07, size = 68, normalized size = 0.75

$$\frac{a^4c^4x^5}{5} - \frac{3a^3c^4x^4}{2} + \frac{16a^2c^4x^3}{3} - 13ac^4x^2 + 31c^4x - \frac{32c^4\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] a**4*c**4*x**5/5 - 3*a**3*c**4*x**4/2 + 16*a**2*c**4*x**3/3 - 13*a*c**4*x**
2 + 31*c**4*x - 32*c**4*log(a*x + 1)/a
```

Giac [A]
time = 0.40, size = 75, normalized size = 0.82

$$-\frac{32\,c^4\log\left(|ax+1|\right)}{a} + \frac{6\,a^9c^4x^5 - 45\,a^8c^4x^4 + 160\,a^7c^4x^3 - 390\,a^6c^4x^2 + 930\,a^5c^4x}{30\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*c^4*log(abs(a*x + 1))/a + 1/30*(6*a^9*c^4*x^5 - 45*a^8*c^4*x^4 + 160*a^
7*c^4*x^3 - 390*a^6*c^4*x^2 + 930*a^5*c^4*x)/a^5
```

Mupad [B]
time = 1.17, size = 63, normalized size = 0.69

$$31\,c^4\,x - 13\,a\,c^4\,x^2 + \frac{16\,a^2\,c^4\,x^3}{3} - \frac{3\,a^3\,c^4\,x^4}{2} + \frac{a^4\,c^4\,x^5}{5} - \frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] 31*c^4*x - 13*a*c^4*x^2 + (16*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + (a^4*c^4
*x^5)/5 - (32*c^4*log(a*x + 1))/a
```

3.208 $\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=73

$$8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}$$

```
[Out] 8*c^3*x-2*c^3*(-a*x+1)^2/a-2/3*c^3*(-a*x+1)^3/a-1/4*c^3*(-a*x+1)^4/a-16*c^3
*ln(a*x+1)/a
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6302, 6264, 45}

$$-\frac{c^3(1-ax)^4}{4a}-\frac{2c^3(1-ax)^3}{3a}-\frac{2c^3(1-ax)^2}{a}-\frac{16c^3\log(ax+1)}{a}+8c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 8*c^3*x - (2*c^3*(1 - a*x)^2)/a - (2*c^3*(1 - a*x)^3)/(3*a) - (c^3*(1 - a*x
)^4)/(4*a) - (16*c^3*Log[1 + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^3\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)^3\,dx$$
$$= -\left(c^3\int \frac{(1-ax)^4}{1+ax}\,dx\right)$$
$$= -\left(c^3\int \left(-8-4(1-ax)-2(1-ax)^2-(1-ax)^3+\frac{16}{1+ax}\right)dx\right)$$
$$= 8c^3x-\frac{2c^3(1-ax)^2}{a}-\frac{2c^3(1-ax)^3}{3a}-\frac{c^3(1-ax)^4}{4a}-\frac{16c^3\log(1+ax)}{a}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.66

$$-\frac{c^3(35-180ax+66a^2x^2-20a^3x^3+3a^4x^4+192\log(1+ax))}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^3/E^(2*ArcCoth[a*x]),x]
[Out] -1/12*(c^3*(35 - 180*a*x + 66*a^2*x^2 - 20*a^3*x^3 + 3*a^4*x^4 + 192*Log[1
+ a*x]))/a
```

Maple [A]
time = 0.16, size = 42, normalized size = 0.58

method	result
default	$c^3\left(-\frac{a^3x^4}{4}+\frac{5a^2x^3}{3}-\frac{11a\,x^2}{2}+15x-\frac{16\ln(ax+1)}{a}\right)$
norman	$15c^3x+\frac{5a^2c^3x^3}{3}-\frac{a^3c^3x^4}{4}-\frac{11c^3a\,x^2}{2}-\frac{16c^3\ln(ax+1)}{a}$
risch	$15c^3x+\frac{5a^2c^3x^3}{3}-\frac{a^3c^3x^4}{4}-\frac{11c^3a\,x^2}{2}-\frac{16c^3\ln(ax+1)}{a}$
meijerg	$-\frac{c^3\left(-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)\right)}{a}+\frac{4c^3\left(\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)\right)}{a}-\frac{6c^3\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^3*(-1/4*a^3*x^4+5/3*a^2*x^3-11/2*a*x^2+15*x-16/a*ln(a*x+1))
```

Maxima [A]
time = 0.25, size = 52, normalized size = 0.71

$$-\frac{1}{4}\,a^3c^3x^4+\frac{5}{3}\,a^2c^3x^3-\frac{11}{2}\,ac^3x^2+15\,c^3x-\frac{16\,c^3\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*c^3*x^4 + 5/3*a^2*c^3*x^3 - 11/2*a*c^3*x^2 + 15*c^3*x - 16*c^3*log
(a*x + 1)/a
```

Fricas [A]
time = 0.58, size = 57, normalized size = 0.78

$$-\frac{3\,a^4c^3x^4-20\,a^3c^3x^3+66\,a^2c^3x^2-180\,ac^3x+192\,c^3\log\left(ax+1\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*c^3*x^4 - 20*a^3*c^3*x^3 + 66*a^2*c^3*x^2 - 180*a*c^3*x + 192*
c^3*log(a*x + 1))/a
```

Sympy [A]
time = 0.06, size = 56, normalized size = 0.77

$$-\frac{a^3c^3x^4}{4}+\frac{5a^2c^3x^3}{3}-\frac{11ac^3x^2}{2}+15c^3x-\frac{16c^3\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**3*c**3*x**4/4 + 5*a**2*c**3*x**3/3 - 11*a*c**3*x**2/2 + 15*c**3*x - 16*
c**3*log(a*x + 1)/a
```

Giac [A]
time = 0.40, size = 64, normalized size = 0.88

$$-\frac{16\,c^3\log\left(|ax+1|\right)}{a}-\frac{3\,a^7c^3x^4-20\,a^6c^3x^3+66\,a^5c^3x^2-180\,a^4c^3x}{12\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*c^3*log(abs(a*x + 1))/a - 1/12*(3*a^7*c^3*x^4 - 20*a^6*c^3*x^3 + 66*a^5
*c^3*x^2 - 180*a^4*c^3*x)/a^4
```

Mupad [B]
time = 0.04, size = 52, normalized size = 0.71

$$15\,c^3\,x-\frac{11\,a\,c^3\,x^2}{2}+\frac{5\,a^2\,c^3\,x^3}{3}-\frac{a^3\,c^3\,x^4}{4}-\frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] 15*c^3*x - (11*a*c^3*x^2)/2 + (5*a^2*c^3*x^3)/3 - (a^3*c^3*x^4)/4 - (16*c^3
*log(a*x + 1))/a
```

3.209 $\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=55

$$4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}$$

```
[Out] 4*c^2*x-c^2*(-a*x+1)^2/a-1/3*c^2*(-a*x+1)^3/a-8*c^2*ln(a*x+1)/a
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6302, 6264, 45}

$$-\frac{c^2(1-ax)^3}{3a}-\frac{c^2(1-ax)^2}{a}-\frac{8c^2\log(ax+1)}{a}+4c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 4*c^2*x - (c^2*(1 - a*x)^2)/a - (c^2*(1 - a*x)^3)/(3*a) - (8*c^2*Log[1 + a*
x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^2\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^2\,dx \\
&= -\left(c^2\int \frac{(1-ax)^3}{1+ax}\,dx\right) \\
&= -\left(c^2\int \left(-4-2(1-ax)-(1-ax)^2+\frac{8}{1+ax}\right)dx\right) \\
&= 4c^2x-\frac{c^2(1-ax)^2}{a}-\frac{c^2(1-ax)^3}{3a}-\frac{8c^2\log(1+ax)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.71

$$\frac{c^2(-4+21ax-6a^2x^2+a^3x^3-24\log(1+ax))}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^2/E^(2*ArcCoth[a*x]),x]
[Out] (c^2*(-4 + 21*a*x - 6*a^2*x^2 + a^3*x^3 - 24*Log[1 + a*x]))/(3*a)
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.62

method	result	size
default	$c^2\left(\frac{a^2x^3}{3}-2a\,x^2+7x-\frac{8\ln(ax+1)}{a}\right)$	34
norman	$7c^2x-2a\,c^2x^2+\frac{a^2c^2x^3}{3}-\frac{8c^2\ln(ax+1)}{a}$	42
risch	$7c^2x-2a\,c^2x^2+\frac{a^2c^2x^3}{3}-\frac{8c^2\ln(ax+1)}{a}$	42
meijerg	$\frac{c^2\left(\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)\right)}{a}-\frac{3c^2\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}+\frac{3c^2(ax-\ln(ax+1))}{a}-\frac{c^2\ln(ax+1)}{a}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^2*(1/3*a^2*x^3-2*a*x^2+7*x-8/a*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.75

$$\frac{1}{3}\,a^2c^2x^3-2\,ac^2x^2+7\,c^2x-\frac{8\,c^2\log(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/3*a^2*c^2*x^3 - 2*a*c^2*x^2 + 7*c^2*x - 8*c^2*log(a*x + 1)/a
```

Fricas [A]
time = 0.38, size = 45, normalized size = 0.82

$$\frac{a^3c^2x^3 - 6\,a^2c^2x^2 + 21\,ac^2x - 24\,c^2\log\left(ax+1\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(a^3*c^2*x^3 - 6*a^2*c^2*x^2 + 21*a*c^2*x - 24*c^2*log(a*x + 1))/a
```

Sympy [A]
time = 0.06, size = 41, normalized size = 0.75

$$\frac{a^2c^2x^3}{3} - 2ac^2x^2 + 7c^2x - \frac{8c^2\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*(a*x-1)/(a*x+1),x)
[Out] a**2*c**2*x**3/3 - 2*a*c**2*x**2 + 7*c**2*x - 8*c**2*log(a*x + 1)/a
```

Giac [A]
time = 0.40, size = 52, normalized size = 0.95

$$-\frac{8\,c^2\log\left(|ax+1|\right)}{a} + \frac{a^5c^2x^3 - 6\,a^4c^2x^2 + 21\,a^3c^2x}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] -8*c^2*log(abs(a*x + 1))/a + 1/3*(a^5*c^2*x^3 - 6*a^4*c^2*x^2 + 21*a^3*c^2*
x)/a^3
```

Mupad [B]
time = 0.04, size = 41, normalized size = 0.75

$$7\,c^2\,x - 2\,a\,c^2\,x^2 + \frac{a^2\,c^2\,x^3}{3} - \frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^2*(a*x - 1))/(a*x + 1),x)
[Out] 7*c^2*x - 2*a*c^2*x^2 + (a^2*c^2*x^3)/3 - (8*c^2*log(a*x + 1))/a
```

3.210 $\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=26

$$3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

```
[Out] 3*c*x-1/2*a*c*x^2-4*c*ln(a*x+1)/a
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6302, 6264, 45}

$$-\frac{1}{2}acx^2-\frac{4c\log(ax+1)}{a}+3cx$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{1+ax}\,dx\right)$$
$$= -\left(c\int \left(-3+ax+\frac{4}{1+ax}\right)\,dx\right)$$
$$= 3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$3cx-\frac{1}{2}acx^2-\frac{4c\log(1+ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] 3*c*x - (a*c*x^2)/2 - (4*c*Log[1 + a*x])/a
```

Maple [A]
time = 0.09, size = 24, normalized size = 0.92

method	result	size
default	$c\left(-\frac{a\,x^2}{2}+3x-\frac{4\ln(ax+1)}{a}\right)$	24
norman	$3cx-\frac{ac\,x^2}{2}-\frac{4c\ln(ax+1)}{a}$	25
risch	$3cx-\frac{ac\,x^2}{2}-\frac{4c\ln(ax+1)}{a}$	25
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a}+\frac{2c(ax-\ln(ax+1))}{a}-\frac{c\ln(ax+1)}{a}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*a*x^2+3*x-4/a*ln(a*x+1))
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.92

$$-\frac{1}{2}\,acx^2+3\,cx-\frac{4\,c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*a*c*x^2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Fricas [A]
time = 0.53, size = 28, normalized size = 1.08

$$-\frac{a^2cx^2 - 6\,acx + 8\,c\log\left(ax+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*x^2 - 6*a*c*x + 8*c*log(a*x + 1))/a
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.92

$$-\frac{acx^2}{2} + 3cx - \frac{4c\log\left(ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a*c*x**2/2 + 3*c*x - 4*c*log(a*x + 1)/a
```

Giac [A]
time = 0.39, size = 35, normalized size = 1.35

$$-\frac{4\,c\log\left(|ax+1|\right)}{a} - \frac{a^3cx^2 - 6\,a^2cx}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*c*log(abs(a*x + 1))/a - 1/2*(a^3*c*x^2 - 6*a^2*c*x)/a^2
```

Mupad [B]
time = 0.04, size = 26, normalized size = 1.00

$$-\frac{c\left(8\,\ln\left(a\,x+1\right)-6\,a\,x+a^2\,x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] -(c*(8*log(a*x + 1) - 6*a*x + a^2*x^2))/(2*a)
```

3.211 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=14

$$-\frac{\log(1+ax)}{ac}$$

[Out] -ln(a*x+1)/a/c

Rubi [A]

time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6264, 31}

$$-\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]

[Out] -(Log[1 + a*x]/(a*c))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{c-acx}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-acx}\,dx\\ &= -\frac{\int \frac{1}{1+ax}\,dx}{c}\\ &= -\frac{\log(1+ax)}{ac}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{\log(1+ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)),x]
[Out] -(Log[1 + a*x]/(a*c))
```

Maple [A]
time = 0.13, size = 15, normalized size = 1.07

method	result	size
default	$-\frac{\ln(ax+1)}{ac}$	15
norman	$-\frac{\ln(ax+1)}{ac}$	15
risch	$-\frac{\ln(ax+1)}{ac}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c),x,method=_RETURNVERBOSE)
[Out] -ln(a*x+1)/a/c
```

Maxima [A]
time = 0.25, size = 14, normalized size = 1.00

$$-\frac{\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="maxima")
[Out] -log(a*x + 1)/(a*c)
```

Fricas [A]
time = 0.40, size = 14, normalized size = 1.00

$$-\frac{\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="fricas")
[Out] -log(a*x + 1)/(a*c)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{\log(acx+c)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x)
[Out] -log(a*c*x + c)/(a*c)
```

Giac [A]
time = 0.39, size = 15, normalized size = 1.07

$$-\frac{\log(|ax+1|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c),x, algorithm="giac")
[Out] -log(abs(a*x + 1))/(a*c)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 1.00

$$-\frac{\ln(a\,x+1)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)*(a*x + 1)),x)
[Out] -log(a*x + 1)/(a*c)
```

3.212 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

[Out] -arctanh(a*x)/a/c^2

Rubi [A]
time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6264, 35, 212}

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]

[Out] -(ArcTanh[a*x]/(a*c^2))

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^2}\,dx$$
$$= -\frac{\int \frac{1}{(1-ax)(1+ax)}\,dx}{c^2}$$
$$= -\frac{\int \frac{1}{1-a^2x^2}\,dx}{c^2}$$
$$= -\frac{\tanh^{-1}(ax)}{ac^2}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] -(ArcTanh[a*x]/(a*c^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.
time = 0.13, size = 28, normalized size = 2.33

method	result	size
default	$\frac{-\frac{\ln(ax+1)}{2a}+\frac{\ln(ax-1)}{2a}}{c^2}$	28
norman	$\frac{\ln(ax-1)}{2a\,c^2}-\frac{\ln(ax+1)}{2a\,c^2}$	30
risch	$-\frac{\ln(ax+1)}{2a\,c^2}+\frac{\ln(-ax+1)}{2a\,c^2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2/a*ln(a*x+1)+1/2/a*ln(a*x-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.
time = 0.26, size = 29, normalized size = 2.42

$$-\frac{\log\left(ax+1\right)}{2\,ac^2}+\frac{\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Fricas [**A**]
time = 0.41, size = 23, normalized size = 1.92

$$-\frac{\log\left(ax+1\right)-\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(log(a*x + 1) - log(a*x - 1))/(a*c^2)
```

Sympy [**A**]
time = 0.07, size = 20, normalized size = 1.67

$$\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**2,x)
```

```
[Out] (log(x - 1/a)/2 - log(x + 1/a)/2)/(a*c**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.
time = 0.39, size = 25, normalized size = 2.08

$$-\frac{\log\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^2)
```

Mupad [**B**]
time = 0.06, size = 12, normalized size = 1.00

$$-\frac{\mathrm{atanh}(a\,x)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^2*(a*x + 1)),x)
```

```
[Out] -atanh(a*x)/(a*c^2)
```

3.213 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}$$

```
[Out] -1/2/a/c^3/(-a*x+1)-1/2*arctanh(a*x)/a/c^3
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6264, 46, 213}

$$-\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] -1/2*1/(a*c^3*(1 - a*x)) - ArcTanh[a*x]/(2*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^3}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^2(1+ax)}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2(-1+a^2x^2)}\right)dx}{c^3}\\
&= -\frac{1}{2ac^3(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{2c^3}\\
&= -\frac{1}{2ac^3(1-ax)}-\frac{\tanh^{-1}(ax)}{2ac^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.97

$$-\frac{\frac{1}{2a(1-ax)}+\frac{\tanh^{-1}(ax)}{2a}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^3),x]
[Out] -((1/(2*a*(1 - a*x)) + ArcTanh[a*x]/(2*a))/c^3)
```

Maple [A]
time = 0.13, size = 40, normalized size = 1.21

method	result	size
default	$\frac{-\frac{\ln(ax+1)}{4a}+\frac{1}{2a(ax-1)}+\frac{\ln(ax-1)}{4a}}{c^3}$	40
risch	$\frac{1}{2a(ax-1)c^3}-\frac{\ln(ax+1)}{4a\,c^3}+\frac{\ln(-ax+1)}{4a\,c^3}$	46
norman	$\frac{-\frac{x}{2c}+\frac{a\,x^2}{2c}}{c^2(ax-1)^2}+\frac{\ln(ax-1)}{4c^3a}-\frac{\ln(ax+1)}{4a\,c^3}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/c^3*(-1/4/a*ln(a*x+1)+1/2/a/(a*x-1)+1/4/a*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 48, normalized size = 1.45

$$\frac{1}{2\,(a^2c^3x - ac^3)} - \frac{\log\left(ax+1\right)}{4\,ac^3} + \frac{\log\left(ax-1\right)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2/(a^2*c^3*x - a*c^3) - 1/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)/(a*c^
3)
```

Fricas [A]
time = 0.33, size = 46, normalized size = 1.39

$$-\frac{(ax-1)\log\left(ax+1\right) - (ax-1)\log\left(ax-1\right) - 2}{4\,(a^2c^3x - ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a*x - 1)*log(a*x + 1) - (a*x - 1)*log(a*x - 1) - 2)/(a^2*c^3*x - a*c
^3)
```

Sympy [A]
time = 0.12, size = 39, normalized size = 1.18

$$\frac{1}{2a^2c^3x - 2ac^3} - \frac{-\frac{\log\left(x-\frac{1}{a}\right)}{4} + \frac{\log\left(x+\frac{1}{a}\right)}{4}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**3,x)
```

```
[Out] 1/(2*a**2*c**3*x - 2*a*c**3) - (-log(x - 1/a)/4 + log(x + 1/a)/4)/(a*c**3)
```

Giac [A]
time = 0.39, size = 46, normalized size = 1.39

$$-\frac{\log\left(|ax+1|\right)}{4\,ac^3} + \frac{\log\left(|ax-1|\right)}{4\,ac^3} + \frac{1}{2\,(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) + 1/2/((a*x
- 1)*a*c^3)
```

Mupad [B]
time = 0.07, size = 31, normalized size = 0.94

$$-\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}-\frac{\mathrm{atanh}(a\,x)}{2\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^3*(a*x + 1)),x)
[Out] - 1/(2*a*(c^3 - a*c^3*x)) - atanh(a*x)/(2*a*c^3)
```

3.214 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4} dx$

Optimal. Leaf size=51

$$-\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^4}$$

```
[Out] -1/4/a/c^4/(-a*x+1)^2-1/4/a/c^4/(-a*x+1)-1/4*arctanh(a*x)/a/c^4
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6264, 46, 213}

$$-\frac{1}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] -1/4*1/(a*c^4*(1 - a*x)^2) - 1/(4*a*c^4*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^4}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^4} \\
&= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^4} \\
&= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^4} \\
&= -\frac{1}{4ac^4(1-ax)^2}-\frac{1}{4ac^4(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^4}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.69

$$\frac{-2+ax-(-1+ax)^2\tanh^{-1}(ax)}{4ac^4(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^4),x]
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^4*(-1 + a*x)^2)
```

Maple [A]
time = 0.13, size = 52, normalized size = 1.02

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{(ax-1)^2c^4}+\frac{\ln(-ax+1)}{8a\,c^4}-\frac{\ln(ax+1)}{8a\,c^4}$	51
default	$\frac{-\frac{\ln(ax+1)}{8a}-\frac{1}{4a(ax-1)^2}+\frac{1}{4a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^4}$	52
norman	$\frac{\frac{3x}{4c}-\frac{5a\,x^2}{4c}+\frac{a^2x^3}{2c}}{c^3(ax-1)^3}+\frac{\ln(ax-1)}{8c^4a}-\frac{\ln(ax+1)}{8a\,c^4}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
[Out] 1/c^4*(-1/8/a*ln(a*x+1)-1/4/a/(a*x-1)^2+1/4/a/(a*x-1)+1/8/a*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 63, normalized size = 1.24

$$\frac{ax-2}{4\left(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4}\right)}-\frac{\log\left(ax+1\right)}{8\,ac^{4}}+\frac{\log\left(ax-1\right)}{8\,ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x - 2)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) - 1/8*log(a*x + 1)/(a*c^4
) + 1/8*log(a*x - 1)/(a*c^4)
```

Fricas [A]
time = 0.33, size = 76, normalized size = 1.49

$$\frac{2\,ax-\left(a^{2}x^{2}-2\,ax+1\right)\log\left(ax+1\right)+\left(a^{2}x^{2}-2\,ax+1\right)\log\left(ax-1\right)-4}{8\left(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A]
time = 0.15, size = 54, normalized size = 1.06

$$\frac{ax-2}{4a^{3}c^{4}x^{2}-8a^{2}c^{4}x+4ac^{4}}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**4,x)
```

```
[Out] (a*x - 2)/(4*a**3*c**4*x**2 - 8*a**2*c**4*x + 4*a*c**4) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**4)
```

Giac [A]
time = 0.41, size = 51, normalized size = 1.00

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^{4}}+\frac{\log\left(|ax-1|\right)}{8\,ac^{4}}+\frac{ax-2}{4\left(ax-1\right)^{2}ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^4) + 1/8*log(abs(a*x - 1))/(a*c^4) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^4)
```

Mupad [B]
time = 0.07, size = 46, normalized size = 0.90

$$\frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}-\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^4*(a*x + 1)),x)
[Out] (x/4 - 1/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) - atanh(a*x)/(4*a*c^4)
```

3.215 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{6ac^5(1-ax)^3}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{8ac^5(1-ax)}-\frac{\tanh^{-1}(ax)}{8ac^5}$$

```
[Out] -1/6/a/c^5/(-a*x+1)^3-1/8/a/c^5/(-a*x+1)^2-1/8/a/c^5/(-a*x+1)-1/8*arctanh(a
*x)/a/c^5
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6264, 46, 213}

$$-\frac{1}{8ac^5(1-ax)}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{6ac^5(1-ax)^3}-\frac{\tanh^{-1}(ax)}{8ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] -1/6*1/(a*c^5*(1 - a*x)^3) - 1/(8*a*c^5*(1 - a*x)^2) - 1/(8*a*c^5*(1 - a*x)
) - ArcTanh[a*x]/(8*a*c^5)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^5}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^5}\,dx\\ &= -\frac{\int \frac{1}{(1-ax)^4(1+ax)}\,dx}{c^5}\\ &= -\frac{\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8(-1+a^2x^2)}\right)dx}{c^5}\\ &= -\frac{1}{6ac^5(1-ax)^3}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{8ac^5(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{8c^5}\\ &= -\frac{1}{6ac^5(1-ax)^3}-\frac{1}{8ac^5(1-ax)^2}-\frac{1}{8ac^5(1-ax)}-\frac{\tanh^{-1}(ax)}{8ac^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.64

$$\frac{10-9ax+3a^2x^2-3(-1+ax)^3\tanh^{-1}(ax)}{24ac^5(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^5),x]
[Out] (10 - 9*a*x + 3*a^2*x^2 - 3*(-1 + a*x)^3*ArcTanh[a*x])/(24*a*c^5*(-1 + a*x)
^3)
```

Maple [A]
time = 0.13, size = 64, normalized size = 0.93

method	result	size
risch	$\frac{\frac{a\,x^2}{8}-\frac{3x}{8}+\frac{5}{12a}}{(ax-1)^3 c^5}+\frac{\ln(-ax+1)}{16c^5a}-\frac{\ln(ax+1)}{16c^5a}$	57
default	$\frac{-\frac{\ln(ax+1)}{16a}+\frac{1}{6a(ax-1)^3}-\frac{1}{8a(ax-1)^2}+\frac{1}{8a(ax-1)}+\frac{\ln(ax-1)}{16a}}{c^5}$	64
norman	$\frac{-\frac{7x}{8c}+\frac{2a\,x^2}{c}-\frac{37a^2x^3}{24c}+\frac{5a^3x^4}{12c}}{c^4(ax-1)^4}+\frac{\ln(ax-1)}{16c^5a}-\frac{\ln(ax+1)}{16c^5a}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-1/16/a*ln(a*x+1)+1/6/a/(a*x-1)^3-1/8/a/(a*x-1)^2+1/8/a/(a*x-1)+1/16
/a*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 84, normalized size = 1.22

$$\frac{3\,a^2x^2-9\,ax+10}{24\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}-\frac{\log{(ax+1)}}{16\,ac^5}+\frac{\log{(ax-1)}}{16\,ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*x^2 - 9*a*x + 10)/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x -
a*c^5) - 1/16*log(a*x + 1)/(a*c^5) + 1/16*log(a*x - 1)/(a*c^5)
```

Fricas [A]
time = 0.34, size = 113, normalized size = 1.64

$$\frac{6\,a^2x^2-18\,ax-3\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log{(ax+1)}+3\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log{(ax-1)}+20}{48\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(6*a^2*x^2 - 18*a*x - 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x + 1)
 + 3*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a*x - 1) + 20)/(a^4*c^5*x^3 - 3*
a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [A]
time = 0.21, size = 78, normalized size = 1.13

$$-\frac{-3a^2x^2+9ax-10}{24a^4c^5x^3-72a^3c^5x^2+72a^2c^5x-24ac^5}-\frac{-\frac{\log{\left(x-\frac{1}{a}\right)}}{16}+\frac{\log{\left(x+\frac{1}{a}\right)}}{16}}{ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**5,x)
```

```
[Out] -(-3*a**2*x**2 + 9*a*x - 10)/(24*a**4*c**5*x**3 - 72*a**3*c**5*x**2 + 72*a*
*2*c**5*x - 24*a*c**5) - (-log(x - 1/a)/16 + log(x + 1/a)/16)/(a*c**5)
```

Giac [A]
time = 0.41, size = 89, normalized size = 1.29

$$-\frac{\log{\left(\left|-\frac{2\,c}{acx-c}-1\right|\right)}}{16\,ac^5}+\frac{\frac{3\,a^2c^2}{acx-c}-\frac{3\,a^2c^3}{(acx-c)^2}+\frac{4\,a^2c^4}{(acx-c)^3}}{24\,a^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/16*log(abs(-2*c/(a*c*x - c) - 1))/(a*c^5) + 1/24*(3*a^2*c^2/(a*c*x - c)
- 3*a^2*c^3/(a*c*x - c)^2 + 4*a^2*c^4/(a*c*x - c)^3)/(a^3*c^6)
```

Mupad [B]
time = 1.20, size = 65, normalized size = 0.94

$$-\frac{\frac{a\,x^2}{8}-\frac{3\,x}{8}+\frac{5}{12\,a}}{-a^3\,c^5\,x^3+3\,a^2\,c^5\,x^2-3\,a\,c^5\,x+c^5}-\frac{\mathrm{atanh}(a\,x)}{8\,a\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^5*(a*x + 1)),x)
```

```
[Out] - ((a*x^2)/8 - (3*x)/8 + 5/(12*a))/(c^5 + 3*a^2*c^5*x^2 - a^3*c^5*x^3 - 3*a
*c^5*x) - atanh(a*x)/(8*a*c^5)
```

3.216 $\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=94

$$\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2}x(c-acx)^p\,{}_2F_1\left(-\frac{3}{2}-p,-1-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}$$

```
[Out] ((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2)*x*(-a*c*x+c)^p*hypergeom([-1-p,
-3/2-p],[-p],2/(a+1/x)/x)/(1+p)/(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6311, 6316, 134}

$$\frac{x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}(c-acx)^p\,{}_2F_1\left(-p-\frac{3}{2},-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(p+1)\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2)*x*(c - a*c*x)^p
*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*Sqrt
[1 + 1/(a*x)])
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int\frac{x^{-2-p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{3/2}x(c-acx)^p\,{}_2F_1\left(-\frac{3}{2}-p,-1-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(1+p)\sqrt{1+\frac{1}{ax}}}$$

Mathematica [A]
time = 0.08, size = 96, normalized size = 1.02

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{-1+ax}{1+ax}\right)^{-\frac{1}{2}-p}(1+ax)(c-acx)^p\,{}_2F_1\left(-\frac{3}{2}-p,-1-p;-p;\frac{2}{1+ax}\right)}{a(1+p)\sqrt{1+\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((-1 + a*x)/(1 + a*x))^(-1/2 - p)*(1 + a*x)*(c - a*c*x)^
p*Hypergeometric2F1[-3/2 - p, -1 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*Sqrt[1 +
 1/(a*x)])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int(-acx+c)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

[Out] int((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")

[Out] integrate((-a*c*x + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] integral((a*x - 1)*(-a*c*x + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)**p*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Timed out

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sa

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c - a\,c\,x)^p\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.217 $\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=152

$$\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2+2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4-\frac{315}{}$$

[Out] -315/8*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+30*c^3*x*(1-1/a^2/x^2)^(1/2)-67/8*a*c^3*x^2*(1-1/a^2/x^2)^(1/2)+2*a^2*c^3*x^3*(1-1/a^2/x^2)^(1/2)-1/4*a^3*c^3*x^4*(1-1/a^2/x^2)^(1/2)

Rubi [A]
time = 0.29, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6310, 6313, 1819, 1821, 821, 272, 65, 214}

$$-\frac{67}{8}ac^3x^2\sqrt{1-\frac{1}{a^2x^2}}+30c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{315c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{8a}+2a^2c^3x^3\sqrt{1-\frac{1}{a^2x^2}}-\frac{1}{4}a^3c^3x^4\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]

[Out] (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 30*c^3*Sqrt[1 - 1/(a^2*x^2)]*x - (67*a*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)/8 + 2*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^3 - (a^3*c^3*Sqrt[1 - 1/(a^2*x^2)]*x^4)/4 - (315*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(8*a)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^3\,dx = -\left(\left(a^3c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3 x^3\,dx\right)$$

$$= \left(a^3c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^6}{x^5\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \left(a^3c^3\right)\text{Subst}\left(\int \frac{-1+\frac{6x}{a}-\frac{16x^2}{a^2}+\frac{26x^3}{a^3}-\frac{31x^4}{a^4}}{x^5\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 + \frac{1}{4}\left(a^3c^3\right)\text{Subst}\left(\int \frac{-\frac{24}{a}+\frac{67x}{a^2}-\frac{104x}{a^3}}{x^4\sqrt{1-\frac{x}{a}}}\right.$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^4 - \frac{1}{12}\left(a^3c^3\right)\text{Sub}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{4}a^3c^3\sqrt{1-}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 30c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{67}{8}ac^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + 2a^2c^3\sqrt{1-\frac{1}{a^2}}$$

Mathematica [A]
time = 0.11, size = 86, normalized size = 0.57

$$\frac{1}{8}c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(496+173ax-51a^2x^2+14a^3x^3-2a^4x^4)}{1+ax}-\frac{315\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*x*(496 + 173*a*x - 51*a^2*x^2 + 14*a^3*x^3 - 2
*a^4*x^4))/(1 + a*x) - (315*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(134) = 268.
time = 0.14, size = 542, normalized size = 3.57

method	result
risch	$-\frac{(2a^3x^3-16a^2x^2+67ax-240)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{8a}-\frac{\left(\frac{315\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{8\sqrt{a^2}}-\frac{32\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\right.}}{a^2\left(x+\frac{1}{a}\right)}\right.}{ax-1}$
default	$\frac{\left(-2(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+16\sqrt{a^2}\,((ax+1)(ax-1))^{\frac{3}{2}}a^2x^2-4(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-69\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+32\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+16*(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(3/2)*a^2*x^2-4*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-69*(a^2*x^2-1)^(1
/2)*(a^2)^(1/2)*a^3*x^3+32*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+384*((a*
x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-2*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x
-138*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+69*ln((a^2*x+(a^2*x^2-1)^(1/2)*(
a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-384*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1)
)^(1/2))/(a^2)^(1/2))*a^3*x^2-112*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+768*(
(a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-69*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x
+138*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-768*ln((a^
2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+384*(a^2)^(1/2)
*((a*x+1)*(a*x-1))^(1/2)+69*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^
(1/2))*a-384*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))
/a*c^3*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]

time = 0.26, size = 244, normalized size = 1.61

$$-\frac{1}{8}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{256\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}-\frac{2\left(325\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-765\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+643\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-187\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{4\,(ax-1)a^2}{ax+1}-\frac{6\,(ax-1)^2a^2}{(ax+1)^2}+\frac{4\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 256*c^3*sqrt((a*x - 1)/(a*x + 1))/a^2 - 2*(325
*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 765*c^3*((a*x - 1)/(a*x + 1))^(5/2) + 64
3*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 187*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(
a*x - 1)*a^2/(a*x + 1) - 6*(a*x - 1)^2*a^2/(a*x + 1)^2 + 4*(a*x - 1)^3*a^2/
(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 - a^2))*a
```

Fricas [A]

time = 0.35, size = 114, normalized size = 0.75

$$-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^4c^3x^4-14\,a^3c^3x^3+51\,a^2c^3x^2-173\,ac^3x-496\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
1)/(a*x + 1)) - 1) + (2*a^4*c^3*x^4 - 14*a^3*c^3*x^3 + 51*a^2*c^3*x^2 - 173
*a*c^3*x - 496*c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
-4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(6*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**3*x**3*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x**4*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.09, size = 199, normalized size = 1.31

$$\frac{\frac{187\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{643\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{765\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{4}-\frac{325\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{4}}{a-\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{6\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{4\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{315\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((187*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 - (643*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/4 + (765*c^3*((a*x - 1)/(a*x + 1))^(5/2))/4 - (325*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/4)/(a - (4*a*(a*x - 1))/(a*x + 1) + (6*a*(a*x - 1)^2)/(a*x +
 1)^2 - (4*a*(a*x - 1)^3)/(a*x + 1)^3 + (a*(a*x - 1)^4)/(a*x + 1)^4) + (32*
c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (315*c^3*atanh(((a*x - 1)/(a*x + 1))^(
1/2)))/(4*a)
```

3.218 $\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=129

$$\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2+\frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3-\frac{35c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -35/2*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^2/(1-1/a^2/x^2)^(
1/2)+35/3*c^2*x*(1-1/a^2/x^2)^(1/2)-5/2*a*c^2*x^2*(1-1/a^2/x^2)^(1/2)+1/3*a
^2*c^2*x^3*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6310, 6313, 1819, 1821, 821, 272, 65, 214}

$$-\frac{5}{2}ac^2x^2\sqrt{1-\frac{1}{a^2x^2}}+\frac{35}{3}c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{35c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}+\frac{1}{3}a^2c^2x^3\sqrt{1-\frac{1}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (35*c^2*Sqrt[1 - 1/(a^2
*x^2)]*x)/3 - (5*a*c^2*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 + (a^2*c^2*Sqrt[1 - 1/(
a^2*x^2)]*x^3)/3 - (35*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^2\,dx = \left(a^2c^2\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2 x^2\,dx$$

$$= -\left(\left(a^2c^2\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^5}{x^4\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \left(a^2c^2\right)\text{Subst}\left(\int \frac{-1+\frac{5x}{a}-\frac{11x^2}{a^2}+\frac{15x^3}{a^3}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 - \frac{1}{3}\left(a^2c^2\right)\text{Subst}\left(\int \frac{-\frac{15}{a}+\frac{35x}{a^2}-\frac{45x^2}{a^3}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x^3 + \frac{1}{6}\left(a^2c^2\right)\text{Subs}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2}}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{35}{3}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{5}{2}ac^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{3}a^2c^2\sqrt{1-\frac{1}{a^2}}$$

Mathematica [A]

time = 0.09, size = 78, normalized size = 0.60

$$\frac{1}{6}c^2\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(166+55ax-13a^2x^2+2a^3x^3)}{1+ax}-\frac{105\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*((Sqrt[1 - 1/(a^2*x^2)]*x*(166 + 55*a*x - 13*a^2*x^2 + 2*a^3*x^3))/(1
+ a*x) - (105*Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(111) = 222$.
time = 0.14, size = 474, normalized size = 3.67

method	result
risch	$\frac{(2a^2x^2-15ax+70)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{6a}+\frac{\left(-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c^2\sqrt{}}{ax-1}$
default	$-\frac{\left(15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3-2\sqrt{a^2}\,((ax+1)(ax-1))^{\frac{3}{2}}a^2x^2+30\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2-15\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(15*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-2*(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(3/2)*a^2*x^2+30*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-15*ln((a^2*x+(a^
2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-4*(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(3/2)*a*x-120*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+120*ln((a^2
*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+15*(a^2)^(1/2)
*(a^2*x^2-1)^(1/2)*a*x-30*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*a^2*x+46*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-240*((a*x+1)*(a*x-1))^(1/
2)*(a^2)^(1/2)*a*x+240*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)
^(1/2))*a^2*x-15*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a-12
0*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+120*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(1/2))/(a^2)^(1/2)))/a*c^2*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a
*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]
time = 0.26, size = 204, normalized size = 1.58

$$-\frac{1}{6}\,a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{96\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{2\left(87\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-136\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+57\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 96*c^2*sqrt((a*x - 1)/(a*x + 1))/a^2 + 2*(87
*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 136*c^2*((a*x - 1)/(a*x + 1))^(3/2) + 57
*c^2*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^2*
a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2))
```

Fricas [A]
time = 0.36, size = 104, normalized size = 0.81

$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^2x^3-13\,a^2c^2x^2+55\,ac^2x+166\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
1)/(a*x + 1)) - 1) - (2*a^3*c^2*x^3 - 13*a^2*c^2*x^2 + 55*a*c^2*x + 166*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.19, size = 163, normalized size = 1.26

$$\frac{19\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{136\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{35\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (19*c^2*((a*x - 1)/(a*x + 1))^(1/2) - (136*c^2*((a*x - 1)/(a*x + 1))^(3/2))
/3 + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (
3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (16*c^2*((a*x
 - 1)/(a*x + 1))^(1/2))/a - (35*c^2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.219 $\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=92

$$\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+4c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

```
[Out] -15/2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^2/x^2)^(1/2)+
4*c*x*(1-1/a^2/x^2)^(1/2)-1/2*a*c*x^2*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6310, 6313, 1819, 1821, 821, 272, 65, 214}

$$-\frac{1}{2}acx^2\sqrt{1-\frac{1}{a^2x^2}}+4cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{15c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + 4*c*Sqrt[1 - 1/(a^2*x^2)]*
x - (a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2)/2 - (15*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]
])/(2*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)\,dx = -\left((ac)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)$$

$$= (ac)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - (ac)\text{Subst}\left(\int \frac{-1+\frac{4x}{a}-\frac{7x^2}{a^2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{1}{2}(ac)\text{Subst}\left(\int \frac{-\frac{8}{a}+\frac{15x}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 4c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{(15c)\text{Subst}\left(\int \frac{}{x\sqrt{1}}\right.}{2a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 4c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 + \frac{(15c)\text{Subst}\left(\int \frac{}{x\sqrt{1}}\right.}{4a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 4c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{1}{2}(15ac)\text{Subst}\left(\int \frac{}{a}\right.$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + 4c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{1}{2}ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2 - \frac{15c\tanh^{-1}\left(\sqrt{1-}\right.}{2a}$$

Mathematica [A]

time = 0.09, size = 68, normalized size = 0.74

$$\frac{1}{2}c\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(24+7ax-a^2x^2)}{1+ax}-\frac{15\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a*c*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*((Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 7*a*x - a^2*x^2))/(1 + a*x) - (15*Log[a*
(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(80) = 160$.
time = 0.10, size = 422, normalized size = 4.59

method	result
risch	$-\frac{(ax-8)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{2a}-\frac{\left(\frac{15\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{2\sqrt{a^2}}-\frac{8\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^2\left(x+\frac{1}{a}\right)}\right)c\sqrt{\frac{ax-1}{ax+1}}\ \sqrt{(a}}{ax-1}$
default	$\frac{\left(-\sqrt{a^2x^2-1}\ \sqrt{a^2}\ a^3x^3+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\ \sqrt{a^2}}{\sqrt{a^2}}\right)a^3x^2-2\sqrt{a^2x^2-1}\ \sqrt{a^2}\ a^2x^2+16\sqrt{(ax+1)(ax}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2))/(a^2)^(1/2))*a^3*x^2-2*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+16*(
(a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-16*ln((a^2*x+(a^2)^(1/2)*((a*x+1
)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^
(1/2))/(a^2)^(1/2))*a^2*x-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-8*((a*x+1)*(a*x
-1))^(3/2)*(a^2)^(1/2)+32*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-32*ln((a^
2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+16*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2)-16*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*c*
((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]

time = 0.25, size = 156, normalized size = 1.70

$$\frac{1}{2}\,a\left(\frac{2\left(9\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-7\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{16\,c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*(9*c*((a*x - 1)/(a*x + 1))^(3/2) - 7*c*sqrt((a*x - 1)/(a*x + 1)))/
(2*(a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 15*c*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/a^2 + 16*c*sqrt((a*x - 1)/(a*x + 1))/a^2)
```

Fricas [A]
time = 0.33, size = 81, normalized size = 0.88

$$-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2-7\,acx-24\,c)\sqrt{\frac{ax-1}{ax+1}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (a^2*c*x^2 - 7*a*c*x - 24*c)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2*
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.07, size = 117, normalized size = 1.27

$$\frac{7\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-9\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{15\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (7*c*((a*x - 1)/(a*x + 1))^(1/2) - 9*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (2
*a*(a*x - 1))/(a*x + 1) + (a*(a*x - 1)^2)/(a*x + 1)^2) - (15*c*atanh(((a*x
- 1)/(a*x + 1))^(1/2)))/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

3.220 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=53

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6310, 6313, 1819, 272, 65, 214}

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^
2)]]/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 1.02

$$\frac{\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{1+ax} - \frac{\log\left(a\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)),x]
[Out] ((2*Sqrt[1 - 1/(a^2*x^2)]*x)/(1 + a*x) - Log[a*(1 + Sqrt[1 - 1/(a^2*x^2)])*
x]/a)/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(49) = 98$.

time = 0.12, size = 248, normalized size = 4.68

method	result
default	$-\frac{\left(-\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(-((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+ln((a^2*x+(a^2)^(1/2)*((a*x+
1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)
-2*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+2*ln((a^2*x+(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+a*l
n((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x
+1))^(3/2)/(a^2)^(1/2)/c/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]

time = 0.26, size = 78, normalized size = 1.47

$$-a\left(\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{2\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -a*(log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x +
1)) - 1)/(a^2*c) - 2*sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Fricas [A]

time = 0.41, size = 63, normalized size = 1.19

$$\frac{2\sqrt{\frac{ax-1}{ax+1}}-\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] (2*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) + log(sqr
t((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Integra
l(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.17, size = 48, normalized size = 0.91

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2
)))/(a*c)
```

3.221 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=28

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (a-1/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6313, 651}

$$\frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] (a - x^(-1))/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= \frac{a-\frac{1}{x}}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 0.93

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(1 + a*x))
```

Maple [A]
time = 0.16, size = 35, normalized size = 1.25

method	result	size
trager	$\frac{\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^2}$	25
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^2}$	35
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}{(ax-1)a\,c^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)/(a*x-1)/a/c^2
```

Maxima [A]
time = 0.26, size = 22, normalized size = 0.79

$$\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="maxima")

[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)

Fricas [A]
time = 0.47, size = 22, normalized size = 0.79

$$\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="fricas")

[Out] sqrt((a*x - 1)/(a*x + 1))/(a*c^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**2,x)

[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x**2 - a*x + 1), x))/c**2

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^2,x, algorithm="giac")

[Out] undef

Mupad [B]
time = 0.03, size = 22, normalized size = 0.79

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^2,x)
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(a*c^2)
```

3.222 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=21

$$\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/a/c^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6313, 267}

$$\frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] 1/(a*c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^3}\,dx = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3 x^3}\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^3c^3}$$
$$= \frac{1}{ac^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 1.57

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{c^3\left(-1+a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^3),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x^2)/(c^3*(-1 + a^2*x^2))
```

Maple [A]
time = 0.16, size = 33, normalized size = 1.57

method	result	size
trager	$\frac{x\sqrt{-\frac{-ax+1}{ax+1}}}{c^3(ax-1)}$	30
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} x(ax+1)}{(ax-1)^2 c^3}$	33
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} x(ax+1)}{(ax-1)^2 c^3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*x*(a*x+1)/(a*x-1)^2/c^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

time = 0.27, size = 48, normalized size = 2.29

$$\frac{1}{2} a \left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3} + \frac{1}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*(sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + 1/(a^2*c^3*sqrt((a*x - 1)/(a*x
 + 1))))
```

Fricas [A]
time = 0.39, size = 31, normalized size = 1.48

$$\frac{x\sqrt{\frac{ax-1}{ax+1}}}{ac^3x - c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] x*sqrt((a*x - 1)/(a*x + 1))/(a*c^3*x - c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{a^4x^4 - 2a^3x^3 + 2ax - 1} \right) dx + \int \frac{ax\sqrt{\frac{ax}{ax+1} - \frac{1}{ax+1}}}{a^4x^4 - 2a^3x^3 + 2ax - 1}\, dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**3,x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*x**3 + 2*
a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 -
2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

Giac [A]
time = 0.41, size = 22, normalized size = 1.05

$$\frac{x\,\mathrm{sgn}(ax+1)}{\sqrt{a^2x^2-1}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] x*sgn(a*x + 1)/(sqrt(a^2*x^2 - 1)*c^3)
```

Mupad [B]
time = 1.17, size = 38, normalized size = 1.81

$$\frac{\frac{a\,x-1}{a\,x+1}+1}{2\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^3,x)
```

```
[Out] ((a*x - 1)/(a*x + 1) + 1)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(1/2))
```

3.223 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=61

$$\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,\left(a-\frac{1}{x}\right)x^2}$$

```
[Out] 2/3/a/c^4/(1-1/a^2/x^2)^(1/2)-1/3/a^2/c^4/(a-1/x)/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6310, 6313, 869, 12, 267}

$$\frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4x^2\sqrt{1-\frac{1}{a^2x^2}}\,\left(a-\frac{1}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] 2/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) - 1/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a - 
x^(-1))*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 869

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^n*((a + c*x^2)^(p + 1)/(2*a*e*p*(d + e*x
))), x] - Dist[1/(2*d*e*p), Int[(f + g*x)^(n - 1)*(a + c*x^2)^p*Simp[d*g*n
- e*f*(2*p + 1) - e*g*(n + 2*p + 1)*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && IG
tQ[n, 0] && ILtQ[n + 2*p, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^4}\,dx &= \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4x^4}\,dx}{a^4c^4}\\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{\left(1-\frac{x}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^4c^4}\\
&= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}+\frac{\text{Subst}\left(\int \frac{2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}\\
&= -\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}+\frac{2\text{Subst}\left(\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a^3c^4}\\
&= \frac{2}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{1}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x^2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.82

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1-2ax+2a^2x^2)}{3c^4(-1+ax)^2(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^4),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^4*(-1 + a*x)^2*(1 +
 a*x))
```

Maple [A]
time = 0.17, size = 45, normalized size = 0.74

method	result	size
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2a^{3}x^{3}-3ax-1\right)}{3(ax-1)^{3}c^{4}a}$	45
trager	$\frac{\left(2a^{2}x^{2}-2ax-1\right)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^{4}\left(ax-1\right)^{2}}$	47
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\left(2a^{2}x^{2}-2ax-1\right)\left(ax+1\right)}{3(ax-1)^{3}c^{4}a}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a*x-1)/(a*x+1))^(3/2)*(2*a^3*x^3-3*a*x-1)/(a*x-1)^3/c^4/a
```

Maxima [A]
time = 0.26, size = 65, normalized size = 1.07

$$\frac{1}{12}\,a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{4}}+\frac{\frac{6\,(ax-1)}{ax+1}-1}{a^{2}c^{4}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A]
time = 0.37, size = 58, normalized size = 0.95

$$\frac{\left(2\,a^{2}x^{2}-2\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}}{3\,\left(a^{3}c^{4}x^{2}-2\,a^{2}c^{4}x+ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^4*x^2 - 2*a^2*
c^4*x + a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**4,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a
**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x
+ 1), x))/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^4, x)
```

Mupad [B]
time = 1.23, size = 50, normalized size = 0.82

$$\frac{-2\,a^2\,x^2+2\,a\,x+1}{\left(3\,a\,c^4-3\,a^3\,c^4\,x^2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^4,x)
```

```
[Out] (2*a*x - 2*a^2*x^2 + 1)/((3*a*c^4 - 3*a^3*c^4*x^2)*((a*x - 1)/(a*x + 1))^(1
/2))
```

3.224 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5} dx$

Optimal. Leaf size=94

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -4/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(3/2)+1/5*(a+1/x)^2/a^3/c^5/(1-1/a^2/x^2
)^(5/2)+1/5*(5*a+2/x)/a^2/c^5/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6310, 6313, 866, 1649, 651}

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
```

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) + (a + x^(-1))^2/(5*a
^3*c^5*(1 - 1/(a^2*x^2))^(5/2)) + (5*a + 2/x)/(5*a^2*c^5*Sqrt[1 - 1/(a^2*x^
2)])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
```

```
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^5}\,dx = -\frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^5 x^5}\,dx}{a^5c^5}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^3}{\left(1-\frac{x}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^2}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a^5c^5}$$

$$= \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)(2a^3+5a^2x+5ax^2)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5a^5c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\operatorname{Subst}\left(\int \frac{6a^3+15a^2x}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15a^5c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\left(a+\frac{1}{x}\right)^2}{5a^3c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{5a+\frac{2}{x}}{5a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.05, size = 57, normalized size = 0.61

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax-4a^2x^2+2a^3x^3)}{5c^5(-1+ax)^3(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^5),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3))/(5*c^5*(-1 + a*
x)^3*(1 + a*x))
```

Maple [A]
time = 0.17, size = 61, normalized size = 0.65

method	result	size

trager	$\frac{(2a^3x^3-4a^2x^2+ax+2)\sqrt{-\frac{-ax+1}{ax+1}}}{5a\,c^5(ax-1)^3}$	54
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^3x^3-4a^2x^2+ax+2)(ax+1)}{5(ax-1)^4c^5a}$	57
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^4x^4-2a^3x^3-3a^2x^2+3ax+2)}{5(ax-1)^4c^5a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*((a*x-1)/(a*x+1))^(3/2)*(2*a^4*x^4-2*a^3*x^3-3*a^2*x^2+3*a*x+2)/(a*x-1)
^4/c^5/a
```

Maxima [A]
time = 0.25, size = 82, normalized size = 0.87

$$\frac{1}{40}\,a\left(\frac{5\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5}-\frac{\frac{5\,(ax-1)}{ax+1}-\frac{15\,(ax-1)^2}{(ax+1)^2}-1}{a^2c^5\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/40*a*(5*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^5) - (5*(a*x - 1)/(a*x + 1) - 15
*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(5/2)))
```

Fricas [A]
time = 0.36, size = 77, normalized size = 0.82

$$\frac{(2\,a^3x^3-4\,a^2x^2+ax+2)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^5*x^
3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx+\int \frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**5,x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*
a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*a*x
 - 1), x))/c**5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^5, x)
```

Mupad [B]
time = 1.21, size = 51, normalized size = 0.54

$$\frac{2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2}{5\,a\,c^5\,{\left(a\,x+1\right)}^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^5,x)
```

```
[Out] (a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2)/(5*a*c^5*(a*x + 1)^3*((a*x - 1)/(a*x + 1)
)^(5/2))
```

3.225 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx$

Optimal. Leaf size=125

$$-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -46/35*(a+1/x)/a^2/c^6/(1-1/a^2/x^2)^(3/2)+24/35*(a+1/x)^2/a^3/c^6/(1-1/a^2
/x^2)^(5/2)-1/7*(a+1/x)^3/a^4/c^6/(1-1/a^2/x^2)^(7/2)+1/35*(35*a+13/x)/a^2/
c^6/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6310, 6313, 866, 1649, 651}

$$-\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (-46*(a + x^(-1)))/(35*a^2*c^6*(1 - 1/(a^2*x^2))^(3/2)) + (24*(a + x^(-1))^
2)/(35*a^3*c^6*(1 - 1/(a^2*x^2))^(5/2)) - (a + x^(-1))^3/(7*a^4*c^6*(1 - 1/
(a^2*x^2))^(7/2)) + (35*a + 13/x)/(35*a^2*c^6*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 && !(IGtQ[n, 0] && ILtQ[m + n, 0] && !GtQ[p, 1])
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
```

```
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^6}\,dx = \frac{\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^6 x^6}\,dx}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(1-\frac{x}{a}\right)^3\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4\left(1+\frac{x}{a}\right)^3}{\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a^6c^6}$$

$$= -\frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^2(3a^4+7a^3x+7a^2x^2+7ax^3)}{\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7a^6c^6}$$

$$= \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)(33a^4+70a^3x+35a^2x^2)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35a^6c^6}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{39a^4+10\ldots}{\left(1-\frac{x^2}{a^2}\right)\ldots}\right)}{105a^6\ldots}$$

$$= -\frac{46\left(a+\frac{1}{x}\right)}{35a^2c^6\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{24\left(a+\frac{1}{x}\right)^2}{35a^3c^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\left(a+\frac{1}{x}\right)^3}{7a^4c^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{35a+\frac{13}{x}}{35a^2c^6\sqrt{1-\frac{1}{a^2x\ldots}}}$$

Mathematica [A]
time = 0.06, size = 66, normalized size = 0.53

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-13+4ax+20a^2x^2-24a^3x^3+8a^4x^4)}{35c^6(-1+ax)^4(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^6),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^6*(-1 + a*x)^4*(1 + a*x))
```

Maple [A]
time = 0.18, size = 69, normalized size = 0.55

method	result	size
trager	$\frac{\left(8 a^{4} x^{4}-24 a^{3} x^{3}+20 a^{2} x^{2}+4 a x -13\right) \sqrt{-\frac{-a x +1}{a x +1}}}{35 a \,c^{6} \left(a x -1\right)^{4}}$	63
gosper	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(8 a^{4} x^{4}-24 a^{3} x^{3}+20 a^{2} x^{2}+4 a x -13\right) \left(a x +1\right)}{35 \left(a x -1\right)^{5} c^{6} a}$	66
default	$\frac{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(8 a^{5} x^{5}-16 a^{4} x^{4}-4 a^{3} x^{3}+24 a^{2} x^{2}-9 a x -13\right)}{35 \left(a x -1\right)^{5} c^{6} a}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^5*x^5-16*a^4*x^4-4*a^3*x^3+24*a^2*x^2-9*a
*x-13)/(a*x-1)^5/c^6/a
```

Maxima [A]
time = 0.27, size = 97, normalized size = 0.78

$$\frac{1}{560} \, a {\left(\frac{35 \, \sqrt{\frac{a x - 1}{a x + 1}}}{a^{2} c^{6}} + \frac{\frac{28 \, {\left(a x - 1\right)}}{a x + 1} - \frac{70 \, {\left(a x - 1\right)}^{2}}{{\left(a x + 1\right)}^{2}} + \frac{140 \, {\left(a x - 1\right)}^{3}}{{\left(a x + 1\right)}^{3}} - 5}{a^{2} c^{6} \left(\frac{a x - 1}{a x + 1}\right)^{\frac{7}{2}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^6) + (28*(a*x - 1)/(a*x + 1) -
 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^6*((a
*x - 1)/(a*x + 1))^(7/2)))
```

Fricas [A]
time = 0.35, size = 96, normalized size = 0.77

$$\frac{{\left(8 \, a^{4} x^{4} - 24 \, a^{3} x^{3} + 20 \, a^{2} x^{2} + 4 \, a x - 13\right)} \sqrt{\frac{a x - 1}{a x + 1}}}{35 \, {\left(a^{5} c^{6} x^{4} - 4 \, a^{4} c^{6} x^{3} + 6 \, a^{3} c^{6} x^{2} - 4 \, a^{2} c^{6} x + a c^{6}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^6*x^4 - 4*a^4*c^6*x^3 + 6*a^3*c^6*x^2 - 4*a^2*c^6*x + a*c^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7-5a^6x^6+9a^5x^5-5a^4x^4-5a^3x^3+9a^2x^2-5ax+1}\,dx}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**6,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a
**5*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x) + Integ
ral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 - 5*a**6*x**6 + 9*a**5
*x**5 - 5*a**4*x**4 - 5*a**3*x**3 + 9*a**2*x**2 - 5*a*x + 1), x))/c**6
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a*c*x - c)^6, x)
```

Mupad [B]
time = 0.07, size = 60, normalized size = 0.48

$$\frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^6\,{\left(a\,x+1\right)}^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^6,x)
```

```
[Out] (4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^6*(a*x + 1)^4*((
a*x - 1)/(a*x + 1))^(7/2))
```

3.226 $\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=254

$$-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}-\frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{128(1-}{231}$$

```
[Out] -32/99*(a-1/x)^3*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)+9088/
3465*(1+1/a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)/x^3-768/385*(1+1/
a/x)^(3/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+128/231*(1+1/a/x)^(3/2)
*(-a*c*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^4*(1+1/a/x)^(3/2)*x*(-
a*c*x+c)^(9/2)/a^4/(1-1/a/x)^(9/2)
```

Rubi [A]
time = 0.15, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\frac{9088\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{3465a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{32\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^4\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{768\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{385a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{128\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{9/2}}{231a^2x\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (-32*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1/(
a*x))^(9/2)) + (9088*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(3465*a^4*(1 -
1/(a*x))^(9/2)*x^3) - (768*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(385*a^3*
(1 - 1/(a*x))^(9/2)*x^2) + (128*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(9/2))/(231
*a^2*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^4*(1 + 1/(a*x))^(3/2)*x*(c -
a*c*x)^(9/2))/(11*a^4*(1 - 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n])
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{(c-acx)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^{13/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(16\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{(1-}{}\right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2\left(a-\frac{1}{x}\right)^4\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{9/2}}{11a^4\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{128\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{231a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2(a-}{}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{768\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{385a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{128(1}{2}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9088\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{9/2}}{3465a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3}-\frac{768(}{3}$$

<u>**Mathematica**</u> [**A**]
time = 0.04, size = 83, normalized size = 0.33

$$\frac{2c^4\sqrt{1+\frac{1}{ax}}(1+ax)\sqrt{c-acx}\left(5419-6396ax+4530a^2x^2-1820a^3x^3+315a^4x^4\right)}{3465a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(5419 - 6396*a*x + 4530*
a^2*x^2 - 1820*a^3*x^3 + 315*a^4*x^4))/(3465*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.13, size = 69, normalized size = 0.27

method	result	size
default	$\frac{2\sqrt{-c\left(ax-1\right)}\, c^{4}(ax+1)(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419)}{3465\sqrt{\frac{ax-1}{ax+1}}\, a}$	69
gosper	$\frac{2(ax+1)(315a^{4}x^{4}-1820a^{3}x^{3}+4530a^{2}x^{2}-6396ax+5419)(-acx+c)^{\frac{9}{2}}}{3465a(ax-1)^{4}\sqrt{\frac{ax-1}{ax+1}}}$	72
risch	$-\frac{2c^{5}(ax-1)(315a^{5}x^{5}-1505a^{4}x^{4}+2710a^{3}x^{3}-1866a^{2}x^{2}-977ax+5419)}{3465\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(ax-1\right)}\, a}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3465/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^4*(a*x+1)*(315*a^4*x^4-
1820*a^3*x^3+4530*a^2*x^2-6396*a*x+5419)/a
```

Maxima [A]
time = 0.26, size = 99, normalized size = 0.39

$$\frac{2\left(315\,a^{5}\sqrt{-c}\,c^{4}x^{5}-1505\,a^{4}\sqrt{-c}\,c^{4}x^{4}+2710\,a^{3}\sqrt{-c}\,c^{4}x^{3}-1866\,a^{2}\sqrt{-c}\,c^{4}x^{2}-977\,a\sqrt{-c}\,c^{4}x+5419\,\sqrt{-c}\,c^{4}\right)\sqrt{ax+1}}{3465\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*a^5*sqrt(-c)*c^4*x^5 - 1505*a^4*sqrt(-c)*c^4*x^4 + 2710*a^3*sqr
t(-c)*c^4*x^3 - 1866*a^2*sqrt(-c)*c^4*x^2 - 977*a*sqrt(-c)*c^4*x + 5419*sqr
t(-c)*c^4)*sqrt(a*x + 1)/a
```

Fricas [A]
time = 0.34, size = 105, normalized size = 0.41

$$\frac{2\left(315\,a^{6}c^{4}x^{6}-1190\,a^{5}c^{4}x^{5}+1205\,a^{4}c^{4}x^{4}+844\,a^{3}c^{4}x^{3}-2843\,a^{2}c^{4}x^{2}+4442\,ac^{4}x+5419\,c^{4}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3465\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*a^6*c^4*x^6 - 1190*a^5*c^4*x^5 + 1205*a^4*c^4*x^4 + 844*a^3*c^4
*x^3 - 2843*a^2*c^4*x^2 + 4442*a*c^4*x + 5419*c^4)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 147, normalized size = 0.58

$$\frac{2\left(4096\sqrt{2}\sqrt{-c}\,c^3-\frac{315\,(acx+c)^5\sqrt{-acx-c}-3080\,(acx+c)^4\sqrt{-acx-c}\,c+11880\,(acx+c)^3\sqrt{-acx-c}\,c^2-22176\,(acx+c)^2\sqrt{-acx-c}\,c^3-18480\,(-acx-c)^{\frac{3}{2}}c^4}{c^2}\right)c^2}{3465\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] 2/3465*(4096*sqrt(2)*sqrt(-c)*c^3 - (315*(a*c*x + c)^5*sqrt(-a*c*x - c) - 3
080*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 11880*(a*c*x + c)^3*sqrt(-a*c*x - c)
*c^2 - 22176*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3 - 18480*(-a*c*x - c)^(3/2)*
c^4)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [**B**]
time = 1.46, size = 76, normalized size = 0.30

$$\frac{2\,c^4\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(315\,a^4\,x^4-1820\,a^3\,x^3+4530\,a^2\,x^2-6396\,a\,x+5419\right)}{3465\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(4530*a^2*
x^2 - 6396*a*x - 1820*a^3*x^3 + 315*a^4*x^4 + 5419))/(3465*a*(a*x - 1))
```

3.227 $\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=197

$$-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2}+\frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}}{9a^3\left(1-\frac{1}{ax}\right.}$$

```
[Out] -8/21*(1+1/a/x)^(3/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)-568/315*(1+1/a/x)^
(3/2)*(-a*c*x+c)^(7/2)/a^3/(1-1/a/x)^(7/2)/x^2+48/35*(1+1/a/x)^(3/2)*(-a*c*
x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(a-1/x)^3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)
^(7/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A]
time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$-\frac{568\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{315a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{48\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
```

```
[Out] (-8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(21*a*(1 - 1/(a*x))^(7/2)) - (56
8*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(315*a^3*(1 - 1/(a*x))^(7/2)*x^2)
+ (48*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(7/2))/(35*a^2*(1 - 1/(a*x))^(7/2)*x)
 + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(7/2))/(9*a^3*(1 - 1
/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{(c-acx)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{}\right.}{3a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{7/2}}{9a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(8\left(\frac{1}{x}\right)^{7/2}\right.}{}$$

$$=-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{35a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right.}{9a^3\left(1-\right.}$$

$$=-\frac{8\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{21a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{568\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{7/2}}{315a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2}+\frac{48\left(1+\frac{1}{ax}\right)^{3/2}(c-}{35a^2\left(1-\frac{1}{ax}\right.}$$

Mathematica [A]
time = 0.03, size = 75, normalized size = 0.38

$$-\frac{2c^3\sqrt{1+\frac{1}{ax}}\,(1+ax)\sqrt{c-acx}\,(-319+321ax-165a^2x^2+35a^3x^3)}{315a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(7/2),x]
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(-319 + 321*a*x - 165*a
^2*x^2 + 35*a^3*x^3))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.13, size = 61, normalized size = 0.31

method	result	size
default	$-\frac{2\sqrt{-c\left(ax-1\right)}\, c^{3}(ax+1)\left(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319\right)}{315\sqrt{\frac{ax-1}{ax+1}}\, a}$	61
gosper	$\frac{2(ax+1)\left(35a^{3}x^{3}-165a^{2}x^{2}+321ax-319\right)\left(-acx+c\right)^{\frac{7}{2}}}{315a\left(ax-1\right)^{3}\sqrt{\frac{ax-1}{ax+1}}}$	64
risch	$\frac{2c^{4}(ax-1)\left(35a^{4}x^{4}-130a^{3}x^{3}+156a^{2}x^{2}+2ax-319\right)}{315\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(ax-1\right)}\, a}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^3*(a*x+1)*(35*a^3*x^3-1
65*a^2*x^2+321*a*x-319)/a
```

Maxima [A]
time = 0.27, size = 83, normalized size = 0.42

$$-\frac{2\left(35\,a^{4}\sqrt{-c}\,c^{3}x^{4}-130\,a^{3}\sqrt{-c}\,c^{3}x^{3}+156\,a^{2}\sqrt{-c}\,c^{3}x^{2}+2\,a\sqrt{-c}\,c^{3}x-319\,\sqrt{-c}\,c^{3}\right)\sqrt{ax+1}}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 130*a^3*sqrt(-c)*c^3*x^3 + 156*a^2*sqrt(-
c)*c^3*x^2 + 2*a*sqrt(-c)*c^3*x - 319*sqrt(-c)*c^3)*sqrt(a*x + 1)/a
```

Fricas [A]
time = 0.33, size = 94, normalized size = 0.48

$$-\frac{2\left(35\,a^{5}c^{3}x^{5}-95\,a^{4}c^{3}x^{4}+26\,a^{3}c^{3}x^{3}+158\,a^{2}c^{3}x^{2}-317\,ac^{3}x-319\,c^{3}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{315\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 95*a^4*c^3*x^4 + 26*a^3*c^3*x^3 + 158*a^2*c^3*x^2
- 317*a*c^3*x - 319*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
- a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.43, size = 102, normalized size = 0.52

$$\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-35\,a^4\,x^4+60\,a^3\,x^3+34\,a^2\,x^2-124\,a\,x+193\right)}{315\,a}+\frac{1024\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(34*a^2*x^2 - 124*a*x
+ 60*a^3*x^3 - 35*a^4*x^4 + 193))/(315*a) + (1024*c^3*(c - a*c*x)^(1/2)*((a
*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.228 $\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=115

$$\frac{64a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{105(c-acx)^{3/2}}+\frac{16a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{35\sqrt{c-acx}}+\frac{2}{7}a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\sqrt{c-acx}$$

```
[Out] 64/105*a^2*c^4*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+16/35*a^2*c^3*(1-1/
a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)+2/7*a^2*c^2*(1-1/a^2/x^2)^(3/2)*x^3*(-a
*c*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6311, 6316, 91, 79, 37}

$$\frac{142\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{105a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{36\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (-36*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (1
42*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(5/2))/(105*a^2*(1 - 1/(a*x))^(5/2)*x) +
 (2*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{9}{a}+\frac{7x}{2a^2}\right)\sqrt{1+\cdots}}{x^{7/2}}\cdots\right.}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{\left(71\left(\frac{1}{x}\right)^{5/2}(c-\cdots\right.}{\cdots}$$

$$= -\frac{36\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{142\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{5/2}}{105a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\cdots}{7\left(1-\cdots\right.}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.58

$$\frac{2c^2\sqrt{1+\frac{1}{ax}}\,(1+ax)\sqrt{c-acx}\,(71-54ax+15a^2x^2)}{105a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(71 - 54*a*x + 15*a^2*x^
2))/(105*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.13, size = 53, normalized size = 0.46

method	result	size

default	$\frac{2\sqrt{-c\,(ax-1)}\;c^{2}(ax+1)(15a^{2}x^{2}-54ax+71)}{105\sqrt{\frac{ax-1}{ax+1}}\;a}$	53
gosper	$\frac{2(ax+1)(15a^{2}x^{2}-54ax+71)(-acx+c)^{\frac{5}{2}}}{105a(ax-1)^{2}\sqrt{\frac{ax-1}{ax+1}}}$	56
risch	$-\frac{2c^{3}(ax-1)(15a^{3}x^{3}-39a^{2}x^{2}+17ax+71)}{105\sqrt{\frac{ax-1}{ax+1}}\;\sqrt{-c\,(ax-1)}\;a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c^2*(a*x+1)*(15*a^2*x^2-54
*a*x+71)/a
```

Maxima [A]
time = 0.26, size = 67, normalized size = 0.58

$$\frac{2\left(15\,a^{3}\sqrt{-c}\,c^{2}x^{3}-39\,a^{2}\sqrt{-c}\,c^{2}x^{2}+17\,a\sqrt{-c}\,c^{2}x+71\,\sqrt{-c}\,c^{2}\right)\sqrt{ax+1}}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*a^3*sqrt(-c)*c^2*x^3 - 39*a^2*sqrt(-c)*c^2*x^2 + 17*a*sqrt(-c)*c^
2*x + 71*sqrt(-c)*c^2)*sqrt(a*x + 1)/a
```

Fricas [A]
time = 0.36, size = 83, normalized size = 0.72

$$\frac{2\,(15\,a^{4}c^{2}x^{4}-24\,a^{3}c^{2}x^{3}-22\,a^{2}c^{2}x^{2}+88\,ac^{2}x+71\,c^{2})\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^4*c^2*x^4 - 24*a^3*c^2*x^3 - 22*a^2*c^2*x^2 + 88*a*c^2*x + 71*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 97, normalized size = 0.84

$$\frac{2\left(64\sqrt{2}\sqrt{-c}\,c-\frac{15\,(acx+c)^3\sqrt{-acx-c}-84\,(acx+c)^2\sqrt{-acx-c}\,c-140\,(-acx-c)^{\frac{3}{2}}c^2}{c^2}\right)c^2}{105\,a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/105*(64*sqrt(2)*sqrt(-c)*c - (15*(a*c*x + c)^3*sqrt(-a*c*x - c) - 84*(a*c
*x + c)^2*sqrt(-a*c*x - c)*c - 140*(-a*c*x - c)^(3/2)*c^2)/c^2)*c^2/(a*abs(
c)*sgn(a*x + 1))
```

Mupad [B]
time = 1.41, size = 60, normalized size = 0.52

$$\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-54\,a\,x+71\right)}{105\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^
2 - 54*a*x + 71))/(105*a*(a*x - 1))
```

3.229 $\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=77

$$\frac{8a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{15(c-acx)^{3/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{5\sqrt{c-acx}}$$

```
[Out] 8/15*a^2*c^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*c*x+c)^(3/2)+2/5*a^2*c^2*(1-1/a^2/
x^2)^(3/2)*x^3/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6311, 6316, 79, 37}

$$\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\left(\frac{1}{ax}+1\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-14*(1 + 1/(a*x))^(3/2)*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (2
*(1 + 1/(a*x))^(3/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= \frac{(c-acx)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}} \\
&= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}} \\
&= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\left(7\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,dx,\right.}{5a\left(1-\frac{1}{ax}\right)^{3/2}} \\
&= -\frac{14\left(1+\frac{1}{ax}\right)^{3/2}(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.74

$$-\frac{2c\sqrt{1+\frac{1}{ax}}\,(1+ax)(-7+3ax)\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a*Sqrt[
1 - 1/(a*x)])
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.56

method	result	size
default	$-\frac{2\sqrt{-c\left(ax-1\right)}\, c(ax+1)(3ax-7)}{15\sqrt{\frac{ax-1}{ax+1}}\, a}$	43
gosper	$\frac{2(ax+1)(3ax-7)(-acx+c)^{\frac{3}{2}}}{15a(ax-1)\sqrt{\frac{ax-1}{ax+1}}}$	48
risch	$\frac{2c^{2}(ax-1)\left(3a^{2}x^{2}-4ax-7\right)}{15\sqrt{\frac{ax-1}{ax+1}}\, \sqrt{-c\left(ax-1\right)}\, a}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*c*(a*x+1)*(3*a*x-7)/a
```

Maxima [A]
time = 0.26, size = 45, normalized size = 0.58

$$-\frac{2\left(3\,a^{2}\sqrt{-c}\,cx^{2}-4\,a\sqrt{-c}\,cx-7\,\sqrt{-c}\,c\right)\sqrt{ax+1}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^2*sqrt(-c)*c*x^2 - 4*a*sqrt(-c)*c*x - 7*sqrt(-c)*c)*sqrt(a*x + 1
)/a
```

Fricas [A]
time = 0.38, size = 64, normalized size = 0.83

$$-\frac{2\left(3\,a^{3}cx^{3}-a^{2}cx^{2}-11\,acx-7\,c\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - a^2*c*x^2 - 11*a*c*x - 7*c)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1))^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.37, size = 50, normalized size = 0.65

$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\left(3\,a\,x-7\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] -(2*c*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 7)*((a*x - 1)/(a*x + 1))^(1/2)
)/(15*a*(a*x - 1))
```

3.230 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=29

$$\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6309}

$$\frac{2(ax+1)\sqrt{c-acx}\,e^{\coth^{-1}(ax)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.48

$$\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.08, size = 36, normalized size = 1.24

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$	35
default	$\frac{2\sqrt{-c\,(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\, a}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(a*x+1)/a
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.90

$$\frac{2\left(a\sqrt{-c}\,x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

Fricas [A]
time = 0.38, size = 50, normalized size = 1.72

$$\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.42, size = 49, normalized size = 1.69

$$\frac{2\,c^2\left(\frac{2\,\sqrt{2}\,\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x +
 1))
```

Mupad [B]
time = 1.31, size = 43, normalized size = 1.48

$$\frac{2\,\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1
))
```

3.231 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}}-\frac{2\sqrt{2}\,\sqrt{1-\frac{1}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}$$

```
[Out] 2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)-2*arctanh(2^(1/2)*(1/x
)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1/2
)/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{2x\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}{\sqrt{c-acx}}-\frac{2\sqrt{2}\,\sqrt{1-\frac{1}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - a*c*x] - (2*Sqrt[2]*Sqrt
[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{x}\right)\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\,\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}} - \frac{\left(4\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{a\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-acx}} - \frac{2\sqrt{2}\,\sqrt{1-\frac{1}{ax}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 0.84

$$\frac{2\sqrt{1-\frac{1}{ax}}\,x\left(\sqrt{a}\,\sqrt{1+\frac{1}{ax}} - \sqrt{2}\,\sqrt{\frac{1}{x}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\,\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*x*(Sqrt[a]*Sqrt[1 + 1/(a*x)] - Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[c
 - a*c*x])
```

Maple [A]
time = 0.18, size = 83, normalized size = 0.70

method	result	size
default	$\frac{2\sqrt{-c\left(ax-1\right)}\,\left(\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\left(ax+1\right)}\,\sqrt{2}}{2\sqrt{c}}\right)-\sqrt{-c\left(ax+1\right)}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(ax+1\right)}\,ca}$	83
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(ax-1\right)}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c\left(ax+1\right)}\,(ax-1)}{a\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\left(ax-1\right)}}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*2^(1/2)*arctan(1/2*(-
c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/c/
a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.36, size = 239, normalized size = 2.03

$$\left[\frac{\sqrt{2}\,(acx-c)\sqrt{-\frac{1}{c}}\,\log\left(-\frac{a^2x^2-2\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)-2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac},\ -\frac{2\left(\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}-\frac{\sqrt{2}\,(acx-c)\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{\sqrt{c}}\right)}{a^2cx-ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a
*x + 1)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x
 - a*c), -2*(sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)) - sqrt(2)
*(a*c*x - c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*
x - 1)*sqrt(c)))/sqrt(c))/(a^2*c*x - a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))), x)
```

Giac [A]
time = 0.42, size = 94, normalized size = 0.80

$$\frac{2\,c\left(\frac{\sqrt{2}\left(\sqrt{c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{-c}\right)}{c}-\frac{\sqrt{2}\,\sqrt{c}\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-\sqrt{-acx-c}}{c}\right)}{a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*(sqrt(2)*(sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - sqrt(-c))/c - (sqrt(2)*sqr
t(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - sqrt(-a*c*x - c))/c)/(a
*abs(c)*sgn(a*x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.232 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -1/2*(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a
^(1/2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-a*(1-1/a/x)^(3/2)*x*(1+1/a/x)^(
1/2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - a*c*x)^(3/
2))) - (Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]
*Sqrt[1 + 1/(a*x)])])/(Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Mathematica [A]
time = 0.07, size = 116, normalized size = 0.91

$$\frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2\sqrt{a}\,c(-1+ax)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(-
1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*S
qrt[a]*c*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.16, size = 118, normalized size = 0.92

method	result
default	$-\frac{\sqrt{-c\left(ax-1\right)}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\left(ax+1\right)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\left(ax+1\right)}\,\sqrt{2}}{2\sqrt{c}}\right)c+2\sqrt{-c}\right.}{2\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)\sqrt{-c\left(ax+1\right)}\,c^{\frac{5}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a*x-1))^(1/2)*(2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2^(1/2)*arctan(1/2*(-c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/c
^(5/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.39, size = 281, normalized size = 2.20

$$\left[-\frac{\sqrt{2}\,\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{-c}\,\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,\left(ax+1\right)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+4\,\sqrt{-acx+c}\,\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{4\,\left(a^{3}c^{2}x^{2}-2\,a^{2}c^{2}x+ac^{2}\right)},\ -\frac{\sqrt{2}\,\left(a^{2}x^{2}-2\,ax+1\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+2\,\sqrt{-acx+c}\,\left(ax+1\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,\left(a^{3}c^{2}x^{2}-2\,a^{2}c^{2}x+ac^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/2*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
 + 1))/(a*c*x - c)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1))**(3/2)), x)
```

Giac [A]
time = 0.44, size = 60, normalized size = 0.47

$$\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{-acx-c}}{acx-c}}{2\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + 2*sqrt(
-a*c*x - c)/(a*c*x - c))/(a*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a\,c\,x\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.233 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}-\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] -1/4*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)+1/1
6*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/
2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/8*a^2*(1-1/a/x)^(5/2)*x^2*(1+1/a
/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A]
time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(8*(a - x^(-1))*(c - a*c*x)
^(5/2)) - (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(4*(a - x^(-1))
^2*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[
x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(8*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x
)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\Bigg(}{16\left(\frac{1}{x}\right)^{5/}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\Bigg(}{8\left(\frac{1}{x}\right)^{5/2}(}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{8\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\Bigg(}{8\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c}$$

Mathematica [A]
time = 0.11, size = 123, normalized size = 0.64

$$\frac{\sqrt{1-\frac{1}{ax}}\,x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}(3+ax)+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{16\sqrt{a}\,c^2(-1+ax)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(5/2),x]

[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(3 + a*x) + Sqrt[2]*Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(16*Sqrt[a]*c^2*(-1 + a*x)^2*Sqrt[c - a*c*x])

Maple [A]
time = 0.16, size = 167, normalized size = 0.87

method	result
default	$\frac{\sqrt{-c\left(ax-1\right)}\,\left(-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\left(ax+1\right)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+2\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\left(ax+1\right)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+2ax\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,\left(ax-1\right)^{2}c^{\frac{7}{2}}\,\sqrt{-c\left(ax\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/16*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2+2*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+2*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)-2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+6*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2/c^(7/2)/(-c*(a*x+1))^(1/2)/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((-a*c*x + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)

Fricas [A]
time = 0.44, size = 337, normalized size = 1.75

$$\left[-\frac{\sqrt{2}\,(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,(a^2x^2+4\,ax+3)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{32\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)},\;-\frac{\sqrt{2}\,(a^3x^3-3\,a^2x^2+3\,ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(a^2x^2+4\,ax+3)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")

[Out] [-1/32*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*

```
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3), -1/16*(sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*arctan
(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) -
2*(a^2*x^2 + 4*a*x + 3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^
3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 78, normalized size = 0.40

$$\frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left((-acx-c)^{\frac{3}{2}}-2\sqrt{-acx-c}\,c\right)}{(acx-c)^2c}}{16\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) + 2*((-a
*c*x - c)^(3/2) - 2*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c))/(a*abs(c))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c-a\,c\,x)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.234 $\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}+\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\mathrm{ta}}{32\sqrt{2}\,(}$$

```
[Out] -1/6*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(7/2)+1/1
6*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)-1/64*a
^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))
/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/32*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/a/x
)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A]

time = 0.15, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{32\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}+\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] -1/6*(a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^2)/((a - x^(-1))^3*(c -
 a*c*x)^(7/2)) - (a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(32*(a - x
^(-1))*(c - a*c*x)^(7/2)) + (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^
3)/(16*(a - x^(-1))^2*(c - a*c*x)^(7/2)) - (a^(5/2)*(1 - 1/(a*x))^(7/2)*Arc
Tanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(32*Sqrt[2]*(x^(-
1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}\sqrt{1+\frac{x}{a}}}{\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{16\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}}{32\left(\frac{1}{x}\right)^{7/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3}}{16\left(a-\frac{1}{x}\right)^2(c-acx)}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3}}{16\left(a-\frac{1}{x}\right)^2(c-acx)}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{32\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} + \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3}}{16\left(a-\frac{1}{x}\right)^2(c-acx)}$$

Mathematica [A]

time = 0.11, size = 139, normalized size = 0.56

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}\,(25+10ax-3a^2x^2)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}\,(-1+ax)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{192\sqrt{a}\,c^3\sqrt{\frac{1}{x}}\,(-1+ax)^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(25 + 10*a*x - 3*a^2*x^2))
/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])]))/(192*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 + a*x)^3*Sqrt[c
 - a*c*x])
```

Maple [A]
time = 0.16, size = 219, normalized size = 0.88

method	result
default	$\frac{\sqrt{-c\,(ax-1)}\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3+9\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*a^3*c*x^3+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))
*a^2*c*x^2+6*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)-9*2^(1/2)*arctan(1/2*(-c*(a
*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-20*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)+3*2^(
1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-50*(-c*(a*x+1))^(1/2)
*c^(1/2))/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3/c^(9/2)/(-c*(a*x+1))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.39, size = 393, normalized size = 1.57

$$\left[-\frac{3\,\sqrt{2}\,{\left(a^{4} x^{4} - 4\,a^{3} x^{3} + 6\,a^{2} x^{2} - 4\,a x + 1\right)}\sqrt{-c}\,\log\left(-\frac{a^{2} c x^{2} + 2\,a c x - 2\,\sqrt{2}\,\sqrt{-a c x + c}\,{\left(a x + 1\right)}\sqrt{-c}\,\sqrt{\frac{a x - 1}{a x + 1}} - 3\,c}{a^{2} x^{2} - 2\,a x + 1}\right) - 4\,{\left(3\,a^{3} x^{3} - 7\,a^{2} x^{2} - 35\,a x - 25\right)}\sqrt{-a c x + c}\,\sqrt{\frac{a x - 1}{a x + 1}}}{384\,{\left(a^{5} c^{4} x^{4} - 4\,a^{4} c^{4} x^{3} + 6\,a^{3} c^{4} x^{2} - 4\,a^{2} c^{4} x + a c^{4}\right)}},\ -\frac{3\,\sqrt{2}\,{\left(a^{4} x^{4} - 4\,a^{3} x^{3} + 6\,a^{2} x^{2} - 4\,a x + 1\right)}\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a c x + c}\,\sqrt{c}\,\sqrt{\frac{a x - 1}{a x + 1}}}{a c x - c}\right) - 2\,{\left(3\,a^{3} x^{3} - 7\,a^{2} x^{2} - 35\,a x - 25\right)}\sqrt{-a c x + c}\,\sqrt{\frac{a x - 1}{a x + 1}}}{192\,{\left(a^{5} c^{4} x^{4} - 4\,a^{4} c^{4} x^{3} + 6\,a^{3} c^{4} x^{2} - 4\,a^{2} c^{4} x + a c^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 - 7*a^
2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x
^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/192*(3*sqrt(2
)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt
(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3
- 7*a^2*x^2 - 35*a*x - 25)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5
*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [A]
time = 0.46, size = 105, normalized size = 0.42

$$\frac{\frac{3\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx - c}}{2\,\sqrt{c}}\right)}{c^{\frac{5}{2}}} - \frac{2\,\left(3\,(acx+c)^{2}\sqrt{-acx - c} + 16\,(-acx-c)^{\frac{3}{2}}c - 12\,\sqrt{-acx - c}\,c^{2}\right)}{(acx-c)^{3}c^{2}}}{192\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/192*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*(
3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*x
 - c)*c^2)/((a*c*x - c)^3*c^2))/(a*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c - a\,c\,x)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.235 $\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

```
[Out] 4/7*(-a*c*x+c)^(7/2)/a-2/9*(-a*c*x+c)^(9/2)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6302, 6265, 21, 45}

$$\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx\\ &= -\int \frac{(1+ax)(c-acx)^{7/2}}{1-ax}\,dx\\ &= -\left(c\int (1+ax)(c-acx)^{5/2}\,dx\right)\\ &= -\left(c\int \left(2(c-acx)^{5/2}-\frac{(c-acx)^{7/2}}{c}\right)\,dx\right)\\ &= \frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.85

$$-\frac{2c^3(-1+ax)^3(11+7ax)\sqrt{c-acx}}{63a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*(-1 + a*x)^3*(11 + 7*a*x)*Sqrt[c - a*c*x])/(63*a)
```

Maple [A]
time = 0.16, size = 33, normalized size = 0.82

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{7}{2}}(7ax+11)}{63a}$	21
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}\right)}{ca}$	33
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}\right)}{ca}$	33
trager	$-\frac{2c^3(7a^4x^4-10a^3x^3-12a^2x^2+26ax-11)\sqrt{-acx+c}}{63a}$	48
risch	$\frac{2c^4(7a^4x^4-10a^3x^3-12a^2x^2+26ax-11)(ax-1)}{63a\sqrt{-c(ax-1)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/9*(-a*c*x+c)^(9/2)-2/7*c*(-a*c*x+c)^(7/2))
```

Maxima [A]
time = 0.26, size = 32, normalized size = 0.80

$$-\frac{2\left(7\left(-acx+c\right)^{\frac{9}{2}}-18\left(-acx+c\right)^{\frac{7}{2}}c\right)}{63\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
[Out] -2/63*(7*(-a*c*x + c)^(9/2) - 18*(-a*c*x + c)^(7/2)*c)/(a*c)
```

Fricas [A]
time = 0.35, size = 60, normalized size = 1.50

$$-\frac{2\left(7\,a^{4}c^{3}x^{4}-10\,a^{3}c^{3}x^{3}-12\,a^{2}c^{3}x^{2}+26\,ac^{3}x-11\,c^{3}\right)\sqrt{-acx+c}}{63\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
[Out] -2/63*(7*a^4*c^3*x^4 - 10*a^3*c^3*x^3 - 12*a^2*c^3*x^2 + 26*a*c^3*x - 11*c^
3)*sqrt(-a*c*x + c)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(31)=62$.
time = 7.03, size = 170, normalized size = 4.25

$$\begin{cases}\frac{-c^{3}\left(\begin{cases}0 & \text{for}\: c=0\\-\frac{2\left(-acx+c\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)-\frac{2c^{2}\left(-acx+c\right)^{\frac{3}{2}}}{3}+2c\left(-\frac{c\left(-acx+c\right)^{\frac{3}{2}}}{3}+\frac{\left(-acx+c\right)^{\frac{5}{2}}}{5}\right)+\frac{4c\left(-acx+c\right)^{\frac{5}{2}}}{5}-\frac{2\left(-acx+c\right)^{\frac{7}{2}}}{7}-\frac{2\left(-\frac{c^{3}\left(-acx+c\right)^{\frac{3}{2}}}{3}+\frac{3c^{2}\left(-acx+c\right)^{\frac{5}{2}}}{5}-\frac{3c\left(-acx+c\right)^{\frac{7}{2}}}{7}+\frac{\left(-acx+c\right)^{\frac{9}{2}}}{9}\right)}{c}}{a} & \text{for}\: a\neq 0\\-c^{\frac{7}{2}}x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(7/2),x)
[Out] Piecewise(((-c**3*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), T
rue)) - 2*c**2*(-a*c*x + c)**(3/2)/3 + 2*c*(-c*(-a*c*x + c)**(3/2)/3 + (-a*
c*x + c)**(5/2)/5) + 4*c*(-a*c*x + c)**(5/2)/5 - 2*(-a*c*x + c)**(7/2)/7 -
2*(-c**3*(-a*c*x + c)**(3/2)/3 + 3*c**2*(-a*c*x + c)**(5/2)/5 - 3*c*(-a*c*x
 + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/c)/a, Ne(a, 0)), (-c**(7/2)*x, True
))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(32)=64$.
time = 0.41, size = 205, normalized size = 5.12

$$\frac{2\left(90\left(acx-c\right)^{3}\sqrt{-acx+c}+378\left(acx-c\right)^{2}\sqrt{-acx+c}\,c-630\left(-acx+c\right)^{\frac{3}{2}}c^{2}+945\sqrt{-acx+c}\,c^{3}+210\left(\left(-acx+c\right)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c\right)c^{2}-\frac{35\left(acx-c\right)^{4}\sqrt{-acx+c}+180\left(acx-c\right)^{3}\sqrt{-acx+c}\,c+378\left(acx-c\right)^{2}\sqrt{-acx+c}\,c^{2}-420\left(-acx+c\right)^{\frac{3}{2}}c^{3}+315\sqrt{-acx+c}\,c^{4}}{c}\right)}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/315*(90*(a*c*x - c)^3*sqrt(-a*c*x + c) + 378*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c - 630*(-a*c*x + c)^(3/2)*c^2 + 945*sqrt(-a*c*x + c)*c^3 + 210*((-a*c*x
 + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c^2 - (35*(a*c*x - c)^4*sqrt(-a*c*x + c
) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 378*(a*c*x - c)^2*sqrt(-a*c*x +
c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sqrt(-a*c*x + c)*c^4)/c)/a
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.80

$$\frac{4\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(9*a*c)
```

3.236 $\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

```
[Out] 4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6302, 6265, 21, 45}

$$\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx \\
&= -\int \frac{(1+ax)(c-acx)^{5/2}}{1-ax}\,dx \\
&= -\left(c\int (1+ax)(c-acx)^{3/2}\,dx\right) \\
&= -\left(c\int \left(2(c-acx)^{3/2}-\frac{(c-acx)^{5/2}}{c}\right)\,dx\right) \\
&= \frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.85

$$\frac{2c^2(-1+ax)^2(9+5ax)\sqrt{c-acx}}{35a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*c^2*(-1 + a*x)^2*(9 + 5*a*x)*Sqrt[c - a*c*x])/(35*a)
```

Maple [A]
time = 0.16, size = 33, normalized size = 0.82

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{5}{2}}(5ax+9)}{35a}$	21
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}\right)}{ca}$	33
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}\right)}{ca}$	33
trager	$\frac{2c^2(5a^3x^3-a^2x^2-13ax+9)\sqrt{-acx+c}}{35a}$	40
risch	$-\frac{2c^3(5a^3x^3-a^2x^2-13ax+9)(ax-1)}{35a\sqrt{-c(ax-1)}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/7*(-a*c*x+c)^(7/2)-2/5*c*(-a*c*x+c)^(5/2))
```

Maxima [A]
time = 0.25, size = 32, normalized size = 0.80

$$-\frac{2\left(5\,(-acx+c)^{\frac{7}{2}}-14\,(-acx+c)^{\frac{5}{2}}c\right)}{35\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] -2/35*(5*(-a*c*x + c)^(7/2) - 14*(-a*c*x + c)^(5/2)*c)/(a*c)
```

Fricas [A]
time = 0.32, size = 49, normalized size = 1.22

$$\frac{2\,(5\,a^3c^2x^3-a^2c^2x^2-13\,ac^2x+9\,c^2)\sqrt{-acx+c}}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/35*(5*a^3*c^2*x^3 - a^2*c^2*x^2 - 13*a*c^2*x + 9*c^2)*sqrt(-a*c*x + c)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(31) = 62.
time = 4.69, size = 80, normalized size = 2.00

$$\begin{cases} \frac{-c^2\left(\begin{cases} 0 & \text{for}\; c=0 \\ -\frac{2(-acx+c)^{\frac{3}{2}}}{3c} & \text{otherwise} \end{cases}\right)-\frac{2\left(\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{c}}{a} & \text{for}\; a\neq 0 \\ -c^{\frac{5}{2}}x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(5/2),x)
[Out] Piecewise(((-c**2*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), T
rue)) - 2*(c**2*(-a*c*x + c)**(3/2)/3 - 2*c*(-a*c*x + c)**(5/2)/5 + (-a*c*x
 + c)**(7/2)/7)/c)/a, Ne(a, 0)), (-c**(5/2)*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(32) = 64.
time = 0.40, size = 141, normalized size = 3.52

$$-\frac{2\left(21\,(acx-c)^2\sqrt{-acx+c}-70\,(-acx+c)^{\frac{3}{2}}c-35\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c\right)c-\frac{3\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}\,c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\sqrt{-acx+c}\,c^3\right)}{c}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(5/2),x, algorithm="giac")

[Out] -2/105*(21*(a*c*x - c)^2*sqrt(-a*c*x + c) - 70*(-a*c*x + c)^(3/2)*c - 35*((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)*c - 3*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*x + c)*c^3)/c)/a

Mupad [B]
time = 0.03, size = 32, normalized size = 0.80

$$\frac{4\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a*c*x)^(5/2)*(a*x + 1))/(a*x - 1),x)

[Out] (4*(c - a*c*x)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c)

3.237 $\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=40

$$\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

```
[Out] 4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6302, 6265, 21, 45}

$$\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx \\
&= -\int \frac{(1+ax)(c-acx)^{3/2}}{1-ax}\,dx \\
&= -\left(c\int (1+ax)\sqrt{c-acx}\,dx\right) \\
&= -\left(c\int \left(2\sqrt{c-acx}-\frac{(c-acx)^{3/2}}{c}\right)dx\right) \\
&= \frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.75

$$
-\frac{2c(-1+ax)(7+3ax)\sqrt{c-acx}}{15a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*c*(-1 + a*x)*(7 + 3*a*x)*Sqrt[c - a*c*x])/(15*a)
```

Maple [A]
time = 0.16, size = 33, normalized size = 0.82

method	result	size
gosper	$\frac{2(-acx+c)^{\frac{3}{2}}(3ax+7)}{15a}$	21
trager	$-\frac{2c(3a^2x^2+4ax-7)\sqrt{-acx+c}}{15a}$	30
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}-\frac{2c(-acx+c)^{\frac{3}{2}}}{3}\right)}{ca}$	33
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}-\frac{2c(-acx+c)^{\frac{3}{2}}}{3}\right)}{ca}$	33
risch	$\frac{2c^2(3a^2x^2+4ax-7)(ax-1)}{15a\sqrt{-c(ax-1)}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] -2/c/a*(1/5*(-a*c*x+c)^(5/2)-2/3*c*(-a*c*x+c)^(3/2))

Maxima [A]
time = 0.26, size = 32, normalized size = 0.80

$$-\frac{2\left(3\left(-acx+c\right)^{\frac{5}{2}}-10\left(-acx+c\right)^{\frac{3}{2}}c\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*(-a*c*x + c)^(5/2) - 10*(-a*c*x + c)^(3/2)*c)/(a*c)
```

Fricas [A]
time = 0.33, size = 32, normalized size = 0.80

$$-\frac{2\left(3\,a^2cx^2+4\,acx-7\,c\right)\sqrt{-acx+c}}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^2*c*x^2 + 4*a*c*x - 7*c)*sqrt(-a*c*x + c)/a
```

Sympy [A]
time = 3.85, size = 61, normalized size = 1.52

$$\begin{cases}\frac{-c\left(\begin{cases}0 & \text{for}\: c=0\\-\frac{2(-acx+c)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)-\frac{2\left(-\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{c}}{a} & \text{for}\: a\neq 0\\-c^{\frac{3}{2}}x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(3/2),x)
```

```
[Out] Piecewise(((-c*Piecewise((0, Eq(c, 0)), (-2*(-a*c*x + c)**(3/2)/(3*c), True
)) - 2*(-c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/c)/a, Ne(a, 0)),
(-c**(3/2)*x, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(32) = 64.
time = 0.40, size = 71, normalized size = 1.78

$$\frac{2\left(15\sqrt{-acx+c}\,c-\frac{3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\sqrt{-acx+c}\,c^2}{c}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(-a*c*x + c)*c - (3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*
x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/c)/a
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.80

$$\frac{4\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/2))/(5*a*c)
```

3.238 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a

Rubi [A]

time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]

[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$= -\int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx$$
$$= -\left(c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\right)$$
$$= -\left(c\int \left(\frac{2}{\sqrt{c-acx}} - \frac{\sqrt{c-acx}}{c}\right)dx\right)$$
$$= \frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.61

$$\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.87

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)-2*c*(-a*c*x+c)^(1/2))
```

Maxima [A]
time = 0.25, size = 30, normalized size = 0.79

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.50

$$\frac{2\sqrt{-acx+c}\,(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a
```

Sympy [A]
time = 1.57, size = 31, normalized size = 0.82

$$-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)
[Out] -2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A]
time = 0.41, size = 44, normalized size = 1.16

$$\frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c}{c}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.84

$$\frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] (4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

3.239 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=36

$$-\frac{4}{a\sqrt{c-acx}}-\frac{2\sqrt{c-acx}}{ac}$$

```
[Out] -4/a/(-a*c*x+c)^(1/2)-2*(-a*c*x+c)^(1/2)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6302, 6265, 21, 45}

$$-\frac{2\sqrt{c-acx}}{ac}-\frac{4}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] -4/(a*Sqrt[c - a*c*x]) - (2*Sqrt[c - a*c*x])/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx\\
&= -\int \frac{1+ax}{(1-ax)\sqrt{c-acx}}\,dx\\
&= -\left(c\int \frac{1+ax}{(c-acx)^{3/2}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{(c-acx)^{3/2}}-\frac{1}{c\sqrt{c-acx}}\right)dx\right)\\
&= -\frac{4}{a\sqrt{c-acx}}-\frac{2\sqrt{c-acx}}{ac}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 0.58

$$\frac{-6+2ax}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
[Out] (-6 + 2*a*x)/(a*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.14, size = 31, normalized size = 0.86

method	result	size
gosper	$\frac{2ax-6}{a\sqrt{-acx+c}}$	20
trager	$-\frac{2(ax-3)\sqrt{-acx+c}}{ca(ax-1)}$	30
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ca}$	31
default	$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ca}$	31
risch	$\frac{2ax-2}{a\sqrt{-c\,(ax-1)}}-\frac{4}{a\sqrt{-c\,(ax-1)}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/c/a*((-a*c*x+c)^(1/2)+2*c/(-a*c*x+c)^(1/2))
```

Maxima [A]
time = 0.25, size = 30, normalized size = 0.83

$$-\frac{2\left(\sqrt{-acx+c}+\frac{2c}{\sqrt{-acx+c}}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2*(sqrt(-a*c*x + c) + 2*c/sqrt(-a*c*x + c))/(a*c)
```

Fricas [A]
time = 0.34, size = 29, normalized size = 0.81

$$-\frac{2\sqrt{-acx+c}\,(ax-3)}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x - 3)/(a^2*c*x - a*c)
```

Sympy [A]
time = 7.73, size = 49, normalized size = 1.36

$$\begin{cases} \frac{-\frac{2}{\sqrt{-acx+c}}+\frac{2\left(-\frac{c}{\sqrt{-acx+c}}-\sqrt{-acx+c}\right)}{c}}{a} & \text{for } a \neq 0 \\ -\frac{x}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(1/2),x)
[Out] Piecewise(((-2/sqrt(-a*c*x + c) + 2*(-c/sqrt(-a*c*x + c) - sqrt(-a*c*x + c)
)/c)/a, Ne(a, 0)), (-x/sqrt(c), True))
```

Giac [A]
time = 0.41, size = 32, normalized size = 0.89

$$-\frac{4}{\sqrt{-acx+c}\,a}-\frac{2\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -4/(sqrt(-a*c*x + c)*a) - 2*sqrt(-a*c*x + c)/(a*c)
```

Mupad [B]
time = 1.21, size = 19, normalized size = 0.53

$$\frac{2\,a\,x-6}{a\,\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(1/2)*(a*x - 1)),x)
```

```
[Out] (2*a*x - 6)/(a*(c - a*c*x)^(1/2))
```

3.240 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{4}{3a(c-acx)^{3/2}}+\frac{2}{ac\sqrt{c-acx}}$$

[Out] -4/3/a/(-a*c*x+c)^(3/2)+2/a/c/(-a*c*x+c)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\frac{2}{ac\sqrt{c-acx}}-\frac{4}{3a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]

[Out] -4/(3*a*(c - a*c*x)^(3/2)) + 2/(a*c*Sqrt[c - a*c*x])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx \\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{3/2}}\,dx \\
&= -\left(c\int \frac{1+ax}{(c-acx)^{5/2}}\,dx\right) \\
&= -\left(c\int \left(\frac{2}{(c-acx)^{5/2}}-\frac{1}{c(c-acx)^{3/2}}\right)dx\right) \\
&= -\frac{4}{3a(c-acx)^{3/2}}+\frac{2}{ac\sqrt{c-acx}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 0.89

$$-\frac{2(-1+3ax)\sqrt{c-acx}}{3ac^2(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
[Out] (-2*(-1 + 3*a*x)*Sqrt[c - a*c*x])/(3*a*c^2*(-1 + a*x)^2)
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.87

method	result	size
gosper	$-\frac{2(3ax-1)}{3a(-acx+c)^{\frac{3}{2}}}$	21
trager	$-\frac{2(3ax-1)\sqrt{-acx+c}}{3c^2(ax-1)^2a}$	31
derivativedivides	$-\frac{2\left(\frac{2c}{3(-acx+c)^{\frac{3}{2}}}-\frac{1}{\sqrt{-acx+c}}\right)}{ca}$	33
default	$-\frac{2\left(\frac{2c}{3(-acx+c)^{\frac{3}{2}}}-\frac{1}{\sqrt{-acx+c}}\right)}{ca}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(2/3*c/(-a*c*x+c)^(3/2)-1/(-a*c*x+c)^(1/2))
```

Maxima [A]
time = 0.25, size = 26, normalized size = 0.68

$$-\frac{2\,(3\,acx-c)}{3\,(-acx+c)^{\frac{3}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2/3*(3*a*c*x - c)/((-a*c*x + c)^(3/2)*a*c)
```

Fricas [A]
time = 0.41, size = 44, normalized size = 1.16

$$-\frac{2\,\sqrt{-acx+c}\,(3\,ax-1)}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] -2/3*sqrt(-a*c*x + c)*(3*a*x - 1)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A]
time = 16.77, size = 29, normalized size = 0.76

$$-\frac{4}{3a\,(-acx+c)^{\frac{3}{2}}}+\frac{2}{ac\sqrt{-acx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(3/2),x)
[Out] -4/(3*a*(-a*c*x + c)**(3/2)) + 2/(a*c*sqrt(-a*c*x + c))
```

Giac [A]
time = 0.39, size = 36, normalized size = 0.95

$$\frac{2\,(3\,acx-c)}{3\,(acx-c)\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] 2/3*(3*a*c*x - c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c)
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.53

$$-\frac{6\,a\,x-2}{3\,a\,{\left(c-a\,c\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(3/2)*(a*x - 1)),x)
[Out] -(6*a*x - 2)/(3*a*(c - a*c*x)^(3/2))
```

3.241 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{4}{5a(c-acx)^{5/2}}+\frac{2}{3ac(c-acx)^{3/2}}$$

```
[Out] -4/5/a/(-a*c*x+c)^(5/2)+2/3/a/c/(-a*c*x+c)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\frac{2}{3ac(c-acx)^{3/2}}-\frac{4}{5a(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -4/(5*a*(c - a*c*x)^(5/2)) + 2/(3*a*c*(c - a*c*x)^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx\\ &= -\int \frac{1+ax}{(1-ax)(c-acx)^{5/2}}\,dx\\ &= -\left(c\int \frac{1+ax}{(c-acx)^{7/2}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{(c-acx)^{7/2}}-\frac{1}{c(c-acx)^{5/2}}\right)\,dx\right)\\ &= -\frac{4}{5a(c-acx)^{5/2}}+\frac{2}{3ac(c-acx)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.85

$$\frac{2(1+5ax)\sqrt{c-acx}}{15ac^3(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
[Out] (2*(1 + 5*a*x)*Sqrt[c - a*c*x])/(15*a*c^3*(-1 + a*x)^3)
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.82

method	result	size
gosper	$-\frac{2(5ax+1)}{15a(-acx+c)^{\frac{5}{2}}}$	21
trager	$\frac{2(5ax+1)\sqrt{-acx+c}}{15c^3(ax-1)^3a}$	31
derivativedivides	$-\frac{2\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	33
default	$-\frac{2\left(\frac{2c}{5(-acx+c)^{\frac{5}{2}}}-\frac{1}{3(-acx+c)^{\frac{3}{2}}}\right)}{ca}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/c/a*(2/5*c/(-a*c*x+c)^(5/2)-1/3/(-a*c*x+c)^(3/2))
```

Maxima [A]
time = 0.25, size = 24, normalized size = 0.60

$$-\frac{2\,(5\,acx+c)}{15\,(-acx+c)^{\frac{5}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*a*c*x + c)/((-a*c*x + c)^(5/2)*a*c)
```

Fricas [A]
time = 0.34, size = 56, normalized size = 1.40

$$\frac{2\,\sqrt{-acx+c}\,(5\,ax+1)}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(-a*c*x + c)*(5*a*x + 1)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*
x - a*c^3)
```

Sympy [A]
time = 12.62, size = 31, normalized size = 0.78

$$-\frac{4}{5a\,(-acx+c)^{\frac{5}{2}}}+\frac{2}{3ac\,(-acx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] -4/(5*a*(-a*c*x + c)**(5/2)) + 2/(3*a*c*(-a*c*x + c)**(3/2))
```

Giac [A]
time = 0.42, size = 34, normalized size = 0.85

$$-\frac{2\,(5\,acx+c)}{15\,(acx-c)^2\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a*c*x + c)/((a*c*x - c)^2*sqrt(-a*c*x + c)*a*c)
```

Mupad [B]
time = 1.21, size = 20, normalized size = 0.50

$$-\frac{10\,a\,x+2}{15\,a\,{\left(c-a\,c\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(5/2)*(a*x - 1)),x)
[Out] -(10*a*x + 2)/(15*a*(c - a*c*x)^(5/2))
```

3.242 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{4}{7a(c-acx)^{7/2}}+\frac{2}{5ac(c-acx)^{5/2}}$$

```
[Out] -4/7/a/(-a*c*x+c)^(7/2)+2/5/a/c/(-a*c*x+c)^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6302, 6265, 21, 45}

$$\frac{2}{5ac(c-acx)^{5/2}}-\frac{4}{7a(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -4/(7*a*(c - a*c*x)^(7/2)) + 2/(5*a*c*(c - a*c*x)^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx\\
&= -\int \frac{1+ax}{(1-ax)(c-acx)^{7/2}}\,dx\\
&= -\left(c\int \frac{1+ax}{(c-acx)^{9/2}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{(c-acx)^{9/2}}-\frac{1}{c(c-acx)^{7/2}}\right)dx\right)\\
&= -\frac{4}{7a(c-acx)^{7/2}}+\frac{2}{5ac(c-acx)^{5/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 34, normalized size = 0.85

$$-\frac{2(3+7ax)\sqrt{c-acx}}{35ac^4(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
[Out] (-2*(3 + 7*a*x)*Sqrt[c - a*c*x])/(35*a*c^4*(-1 + a*x)^4)
```

Maple [**A**]
time = 0.14, size = 33, normalized size = 0.82

method	result	size
gosper	$-\frac{2(7ax+3)}{35a(-acx+c)^{\frac{7}{2}}}$	21
trager	$-\frac{2(7ax+3)\sqrt{-acx+c}}{35c^4(ax-1)^4 a}$	31
derivativedivides	$-\frac{2\left(-\frac{1}{5(-acx+c)^{\frac{5}{2}}}+\frac{2c}{7(-acx+c)^{\frac{7}{2}}}\right)}{ca}$	33
default	$-\frac{2\left(-\frac{1}{5(-acx+c)^{\frac{5}{2}}}+\frac{2c}{7(-acx+c)^{\frac{7}{2}}}\right)}{ca}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/c/a*(-1/5/(-a*c*x+c)^(5/2)+2/7*c/(-a*c*x+c)^(7/2))
```

Maxima [A]
time = 0.25, size = 26, normalized size = 0.65

$$-\frac{2\,(7\,acx+3\,c)}{35\,(-acx+c)^{\frac{7}{2}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/35*(7*a*c*x + 3*c)/((-a*c*x + c)^(7/2)*a*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.
time = 0.35, size = 66, normalized size = 1.65

$$-\frac{2\,\sqrt{-acx+c}\,(7\,ax+3)}{35\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/35*sqrt(-a*c*x + c)*(7*a*x + 3)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4
*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A]
time = 23.29, size = 31, normalized size = 0.78

$$-\frac{4}{7a\,(-acx+c)^{\frac{7}{2}}}+\frac{2}{5ac\,(-acx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] -4/(7*a*(-a*c*x + c)**(7/2)) + 2/(5*a*c*(-a*c*x + c)**(5/2))
```

Giac [A]
time = 0.41, size = 36, normalized size = 0.90

$$\frac{2\,(7\,acx+3\,c)}{35\,(acx-c)^3\sqrt{-acx+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/35*(7*a*c*x + 3*c)/((a*c*x - c)^3*sqrt(-a*c*x + c)*a*c)
```

Mupad [B]
time = 1.19, size = 20, normalized size = 0.50

$$-\frac{14\,a\,x+6}{35\,a\,{\left(c-a\,c\,x\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a*c*x)^(7/2)*(a*x - 1)),x)
[Out] -(14*a*x + 6)/(35*a*(c - a*c*x)^(7/2))
```

3.243 $\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=197

$$-\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2}+\frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x}+\frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}}{11a^3\left(1-\frac{1}{ax}\right.}$$

```
[Out] -8/33*(1+1/a/x)^(5/2)*(-a*c*x+c)^(9/2)/a/(1-1/a/x)^(9/2)-856/1155*(1+1/a/x)
^(5/2)*(-a*c*x+c)^(9/2)/a^3/(1-1/a/x)^(9/2)/x^2+16/21*(1+1/a/x)^(5/2)*(-a*c
*x+c)^(9/2)/a^2/(1-1/a/x)^(9/2)/x+2/11*(a-1/x)^3*(1+1/a/x)^(5/2)*x*(-a*c*x+
c)^(9/2)/a^3/(1-1/a/x)^(9/2)
```

Rubi [A]
time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$-\frac{856\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{1155a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{16\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{21a^2x\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{8\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (-8*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(33*a*(1 - 1/(a*x))^(9/2)) - (85
6*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(1155*a^3*(1 - 1/(a*x))^(9/2)*x^2)
 + (16*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(9/2))/(21*a^2*(1 - 1/(a*x))^(9/2)*x
) + (2*(a - x^(-1))^3*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(9/2))/(11*a^3*(1 -
 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{(c-acx)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\left(1+\frac{x}{a}\right)^{3/2}}{x^{13/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(12\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{(1-\frac{x}{a}}{}\right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{9/2}}{11a^3\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(8\left(\frac{1}{x}\right)^{9/}\right.}{}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{21a^2\left(1-\frac{1}{ax}\right)^{9/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\left(1+\frac{1}{a}\right.}{11a^3\left(1\right.}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{33a\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{856\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{9/2}}{1155a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{16\left(1+\frac{1}{ax}\right)^{5/2}(c\right.}{21a^2\left(1-\frac{1}{a}\right.}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.39

$$\frac{2c^4\sqrt{1+\frac{1}{ax}}\,(1+ax)^2\sqrt{c-acx}\,(-533+755ax-455a^2x^2+105a^3x^3)}{1155a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(9/2),x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(-533 + 755*a*x - 455*
a^2*x^2 + 105*a^3*x^3))/(1155*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.14, size = 66, normalized size = 0.34

method	result	size
gosper	$\frac{2(ax+1)(105a^3x^3-455a^2x^2+755ax-533)(-acx+c)^{\frac{9}{2}}}{1155a(ax-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64

default	$\frac{2(ax-1)(ax+1)\sqrt{-c\,(ax-1)}\,c^{4}(105a^{3}x^{3}-455a^{2}x^{2}+755ax-533)}{1155\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	66
risch	$-\frac{2c^{5}(ax-1)(105a^{5}x^{5}-245a^{4}x^{4}-50a^{3}x^{3}+522a^{2}x^{2}-311ax-533)}{1155\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\,a}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-c*(a*x-1))^(1/2)*c^4*(105*
a^3*x^3-455*a^2*x^2+755*a*x-533)/a
```

Maxima [A]
time = 0.27, size = 106, normalized size = 0.54

$$\frac{2\,\left(105\,a^{5}\sqrt{-c}\,c^{4}x^{5}-455\,a^{4}\sqrt{-c}\,c^{4}x^{4}+650\,a^{3}\sqrt{-c}\,c^{4}x^{3}-78\,a^{2}\sqrt{-c}\,c^{4}x^{2}-755\,a\sqrt{-c}\,c^{4}x+533\,\sqrt{-c}\,c^{4}\right)(ax+1)^{\frac{3}{2}}}{1155\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*a^5*sqrt(-c)*c^4*x^5 - 455*a^4*sqrt(-c)*c^4*x^4 + 650*a^3*sqrt(
-c)*c^4*x^3 - 78*a^2*sqrt(-c)*c^4*x^2 - 755*a*sqrt(-c)*c^4*x + 533*sqrt(-c)
*c^4)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Fricas [A]
time = 0.36, size = 105, normalized size = 0.53

$$\frac{2\,\left(105\,a^{6}c^{4}x^{6}-140\,a^{5}c^{4}x^{5}-295\,a^{4}c^{4}x^{4}+472\,a^{3}c^{4}x^{3}+211\,a^{2}c^{4}x^{2}-844\,ac^{4}x-533\,c^{4}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{1155\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*a^6*c^4*x^6 - 140*a^5*c^4*x^5 - 295*a^4*c^4*x^4 + 472*a^3*c^4*x
^3 + 211*a^2*c^4*x^2 - 844*a*c^4*x - 533*c^4)*sqrt(-a*c*x + c)*sqrt((a*x -
1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(9/2),x)
```

[Out] Timed out

Giac [A]
time = 0.44, size = 130, normalized size = 0.66

$$-\frac{2\left(512\sqrt{2}\sqrt{-c}\,c^3+\frac{105\,(acx+c)^5\sqrt{-acx-c}-770\,(acx+c)^4\sqrt{-acx-c}\,c+1980\,(acx+c)^3\sqrt{-acx-c}\,c^2-1848\,(acx+c)^2\sqrt{-acx-c}\,c^3}{c^2}\right)c^2}{1155\,a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(9/2),x, algorithm="giac")

[Out] -2/1155*(512*sqrt(2)*sqrt(-c)*c^3 + (105*(a*c*x + c)^5*sqrt(-a*c*x - c) - 770*(a*c*x + c)^4*sqrt(-a*c*x - c)*c + 1980*(a*c*x + c)^3*sqrt(-a*c*x - c)*c^2 - 1848*(a*c*x + c)^2*sqrt(-a*c*x - c)*c^3)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))

Mupad [B]
time = 1.45, size = 110, normalized size = 0.56

$$\frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(105\,a^5\,x^5-35\,a^4\,x^4-330\,a^3\,x^3+142\,a^2\,x^2+353\,a\,x-491\right)}{1155\,a}-\frac{2048\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{1155\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - a*c*x)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)

[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(353*a*x + 142*a^2*x^2 - 330*a^3*x^3 - 35*a^4*x^4 + 105*a^5*x^5 - 491))/(1155*a) - (2048*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(1155*a*(a*x - 1))

3.244 $\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$-\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

```
[Out] -44/63*(1+1/a/x)^(5/2)*(-a*c*x+c)^(7/2)/a/(1-1/a/x)^(7/2)+214/315*(1+1/a/x)
^(5/2)*(-a*c*x+c)^(7/2)/a^2/(1-1/a/x)^(7/2)/x+2/9*(1+1/a/x)^(5/2)*x*(-a*c*x
+c)^(7/2)/(1-1/a/x)^(7/2)
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 91, 79, 37}

$$\frac{214\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{315a^2x\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{44\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-44*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(63*a*(1 - 1/(a*x))^(7/2)) + (2
14*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(7/2))/(315*a^2*(1 - 1/(a*x))^(7/2)*x) +
 (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(7/2))/(9*(1 - 1/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(-\frac{11}{a}+\frac{9x}{2a^2}\right)\left(1+\frac{x}{a}\right)}{x^{9/2}}\right.}{9\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{7/2}}{9\left(1-\frac{1}{ax}\right)^{7/2}} - \frac{\left(107\left(\frac{1}{x}\right)^{7/2}(c-\right.}{}$$

$$= -\frac{44\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{63a\left(1-\frac{1}{ax}\right)^{7/2}} + \frac{214\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{7/2}}{315a^2\left(1-\frac{1}{ax}\right)^{7/2}x} + \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(}{9\left(1-\frac{1}{a}\right.}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.50

$$-\frac{2c^3\sqrt{1+\frac{1}{ax}}\,(1+ax)^2\sqrt{c-acx}\,(107-110ax+35a^2x^2)}{315a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2),x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*(1 + a*x)^2*Sqrt[c - a*c*x]*(107 - 110*a*x + 35*a
^2*x^2))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.12, size = 58, normalized size = 0.42

method	result	size
gosper	$\frac{2(ax+1)(35a^2x^2-110ax+107)(-acx+c)^{\frac{7}{2}}}{315a(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	56
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c\,(ax-1)}\,c^3(35a^2x^2-110ax+107)}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	58
risch	$\frac{2c^4(ax-1)(35a^4x^4-40a^3x^3-78a^2x^2+104ax+107)}{315\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\,a}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-c*(a*x-1))^(1/2)*c^3*(35*a
^2*x^2-110*a*x+107)/a
```

Maxima [A]
time = 0.27, size = 90, normalized size = 0.66

$$-\frac{2\left(35\,a^4\sqrt{-c}\,c^3x^4-110\,a^3\sqrt{-c}\,c^3x^3+72\,a^2\sqrt{-c}\,c^3x^2+110\,a\sqrt{-c}\,c^3x-107\,\sqrt{-c}\,c^3\right)(ax+1)^{\frac{3}{2}}}{315\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^4*sqrt(-c)*c^3*x^4 - 110*a^3*sqrt(-c)*c^3*x^3 + 72*a^2*sqrt(-c
)*c^3*x^2 + 110*a*sqrt(-c)*c^3*x - 107*sqrt(-c)*c^3)*(a*x + 1)^(3/2)/((a*x
- 1)*a)
```

Fricas [A]
time = 0.35, size = 94, normalized size = 0.69

$$-\frac{2\left(35 a^{5} c^{3} x^{5}-5 a^{4} c^{3} x^{4}-118 a^{3} c^{3} x^{3}+26 a^{2} c^{3} x^{2}+211 a c^{3} x+107 c^{3}\right) \sqrt{-a c x+c} \sqrt{\frac{a x-1}{a x+1}}}{315\left(a^{2} x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 5*a^4*c^3*x^4 - 118*a^3*c^3*x^3 + 26*a^2*c^3*x^2 +
 211*a*c^3*x + 107*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x -
 a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.44, size = 102, normalized size = 0.74

$$-\frac{2 c^{3} \sqrt{c-a c x} \sqrt{\frac{a x-1}{a x+1}}\left(35 a^{4} x^{4}+30 a^{3} x^{3}-88 a^{2} x^{2}-62 a x+149\right)}{315 a}-\frac{512 c^{3} \sqrt{c-a c x} \sqrt{\frac{a x-1}{a x+1}}}{315 a\left(a x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(30*a^3*x^3 - 88*a^2
*x^2 - 62*a*x + 35*a^4*x^4 + 149))/(315*a) - (512*c^3*(c - a*c*x)^(1/2)*((a
*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.245 $\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -18/35*(1+1/a/x)^(5/2)*(-a*c*x+c)^(5/2)/a/(1-1/a/x)^(5/2)+2/7*(1+1/a/x)^(5/
2)*x*(-a*c*x+c)^(5/2)/(1-1/a/x)^(5/2)
```

Rubi [A]
time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6311, 6316, 79, 37}

$$\frac{2x\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{18\left(\frac{1}{ax}+1\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (-18*(1 + 1/(a*x))^(5/2)*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) + (2
*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(5/2))/(7*(1 - 1/(a*x))^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx &= \frac{(c-acx)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\\
&= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= \frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(9\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\mathrm{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}}\,dx,x,\right.}{7a\left(1-\frac{1}{ax}\right)^{5/2}}\\
&= -\frac{18\left(1+\frac{1}{ax}\right)^{5/2}(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{5/2}}{7\left(1-\frac{1}{ax}\right)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.66

$$\frac{2\sqrt{1+\frac{1}{ax}}\,(-9+5ax)\sqrt{c-acx}\,(c+acx)^2}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(-9 + 5*a*x)*Sqrt[c - a*c*x]*(c + a*c*x)^2)/(35*a*Sqrt
[1 - 1/(a*x)])
```

Maple [A]
time = 0.12, size = 50, normalized size = 0.56

method	result	size
gosper	$\frac{2(ax+1)(5ax-9)(-acx+c)^{\frac{5}{2}}}{35a(ax-1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	48
default	$\frac{2(ax-1)(ax+1)\sqrt{-c\,(ax-1)}\,c^{2}(5ax-9)}{35\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	50
risch	$-\frac{2c^{3}(ax-1)(5a^{3}x^{3}+a^{2}x^{2}-13ax-9)}{35\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\,a}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-c*(a*x-1))^(1/2)*c^2*(5*a*x-
9)/a
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.83

$$\frac{2\left(5\,a^{3}\sqrt{-c}\,c^{2}x^{3}-9\,a^{2}\sqrt{-c}\,c^{2}x^{2}-5\,a\sqrt{-c}\,c^{2}x+9\,\sqrt{-c}\,c^{2}\right)(ax+1)^{\frac{3}{2}}}{35\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^3*sqrt(-c)*c^2*x^3 - 9*a^2*sqrt(-c)*c^2*x^2 - 5*a*sqrt(-c)*c^2*x
+ 9*sqrt(-c)*c^2)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Fricas [A]
time = 0.33, size = 83, normalized size = 0.93

$$\frac{2\,(5\,a^{4}c^{2}x^{4}+6\,a^{3}c^{2}x^{3}-12\,a^{2}c^{2}x^{2}-22\,ac^{2}x-9\,c^{2})\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 + 6*a^3*c^2*x^3 - 12*a^2*c^2*x^2 - 22*a*c^2*x - 9*c^2)*
sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 80, normalized size = 0.90

$$-\frac{2\left(16\sqrt{2}\sqrt{-c}\,c+\frac{5\,(acx+c)^3\sqrt{-acx-c}-14\,(acx+c)^2\sqrt{-acx-c}\,c}{c^2}\right)c^2}{35\,a|c|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/35*(16*sqrt(2)*sqrt(-c)*c + (5*(a*c*x + c)^3*sqrt(-a*c*x - c) - 14*(a*c*
x + c)^2*sqrt(-a*c*x - c)*c)/c^2)*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [B]
time = 1.39, size = 93, normalized size = 1.04

$$-\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(-5\,a^3\,x^3-11\,a^2\,x^2+a\,x+23\right)}{35\,a}-\frac{64\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a*x - 11*a^2*x^2 -
5*a^3*x^3 + 23))/(35*a) - (64*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(
1/2))/(35*a*(a*x - 1))
```

3.246 $\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

```
[Out] 2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-a*c*x+c)^(3/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6309}

$$\frac{2(ax+1)(c-acx)^{3/2}e^{3\coth^{-1}(ax)}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*E^(3*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(3/2))/(5*a)
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{2e^{3\coth^{-1}(ax)}(1+ax)(c-acx)^{3/2}}{5a}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.39

$$\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(5/2)*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Maple [A]
time = 0.08, size = 42, normalized size = 1.35

method	result	size
gosper	$\frac{2(ax+1)(-acx+c)^{\frac{3}{2}}}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	35
default	$-\frac{2(ax-1)(ax+1)\sqrt{-c\left(ax-1\right)}\,c}{5\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	42
risch	$\frac{2c^{2}(ax-1)\left(a^{2}x^{2}+2ax+1\right)}{5\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(ax-1\right)}\,a}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-c*(a*x-1))^(1/2)*c/a
```

Maxima [A]
time = 0.27, size = 41, normalized size = 1.32

$$-\frac{2\left(a^{2}\sqrt{-c}\,cx^{2}-\sqrt{-c}\,c\right)(ax+1)^{\frac{3}{2}}}{5\,(ax-1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^2*sqrt(-c)*c*x^2 - sqrt(-c)*c)*(a*x + 1)^(3/2)/((a*x - 1)*a)
```

Fricas [A]
time = 0.34, size = 61, normalized size = 1.97

$$-\frac{2\left(a^{3}cx^{3}+3\,a^{2}cx^{2}+3\,acx+c\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^3*c*x^3 + 3*a^2*c*x^2 + 3*a*c*x + c)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^2*x - a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(32) = 64$.

time = 149.58, size = 129, normalized size = 4.16

$$-\frac{8c\sqrt{-acx+c}}{5a\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}+\frac{8(-acx+c)^{\frac{3}{2}}}{5a\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}-\frac{2(-acx+c)^{\frac{5}{2}}}{5ac\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(3/2),x)
```

```
[Out] -8*c*sqrt(-a*c*x + c)/(5*a*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))) + 8*
(-a*c*x + c)**(3/2)/(5*a*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))) - 2*(-
a*c*x + c)**(5/2)/(5*a*c*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c)))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.38, size = 81, normalized size = 2.61

$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2+4\,a\,x+7\right)}{5\,a}-\frac{16\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + a^2*x^2 + 7))
/(5*a) - (16*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x - 1
))
```

3.247 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=163

$$\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1
/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Sub}}{a}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Sub}}{a^2}$$

$$= \frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tan}{a^{3/2}\sqrt{1}}$$

Mathematica [A]

time = 0.04, size = 105, normalized size = 0.64

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\,\sqrt{1+\frac{1}{ax}}\,(7+ax)-6\sqrt{2}\,\sqrt{\frac{1}{x}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.16, size = 107, normalized size = 0.66

method	result
default	$-\frac{2(ax-1)\sqrt{-c\,(ax-1)}\left(6\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)-ax\sqrt{-c\,(ax+1)}-7\sqrt{-c\,(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax+1)}\,a}$
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}-\frac{4\sqrt{2}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c\,(ax+1)}\,(ax-1)}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(6*c^(1/2)*
2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))-a*x*(-c*(a*x+1))^(1/
2)-7*(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.41, size = 250, normalized size = 1.53

$$\left[\frac{2\left(3\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+(a^2x^2+8ax+7)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)},\ -\frac{2\left(6\sqrt{2}\,(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(a^2x^2+8ax+7)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a
*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.248 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=177

$$-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}+\frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

```
[Out] 2*a*(1+1/a/x)^(3/2)*x*(1-1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-6*(1-1/a/x)^
(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(1/2)-3*arctanh(2^(1/2)*(1/x)^(1/2
)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a^(1/2)/(1/x)^(1/2)/(-a*
c*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{2ax\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}}-\frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (-6*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - a*c*x]) + (
2*a*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x)/((a - x^(-1))*Sqrt[c - a*c*x])
 - (3*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqr
t[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{}{\sqrt{x}\,(}\right.}{a\sqrt{\frac{1}{x}}\sqrt{c-}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{\left(6\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\right.}{a\sqrt{\frac{1}{x}}\sqrt{c-ac}}$$

$$= -\frac{6\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} + \frac{2a\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x}{\left(a-\frac{1}{x}\right)\sqrt{c-acx}} - \frac{3\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{a}\,\sqrt{}}\right.}{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Mathematica [A]

time = 0.10, size = 116, normalized size = 0.66

$$\frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\,\sqrt{1+\frac{1}{ax}}\,(-2+ax)-3\sqrt{2}\,\sqrt{\frac{1}{x}}\,(-1+ax)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\,(-1+ax)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 3*Sqrt[2]*Sq
rt[x^(-1)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])]))/(Sqrt[a]*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.16, size = 135, normalized size = 0.76

method	result
default	$\frac{\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx-2ax\sqrt{-c\,(ax+1)}\,\sqrt{c}-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(\cdots}{\cdots}\right.\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)c^{\frac{3}{2}}\sqrt{-c\,(ax+1)}\,a}$
risch	$\frac{2ax-2}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}+\frac{\left(-\frac{2\sqrt{-acx-c}}{a(-acx+c)}+\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{a\sqrt{c}}\right)\sqrt{-c\,(ax+1)}\,(ax-\cdots}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(-c*(a*x-1))^(1/2)*(3*2^(1/2)*arctan(1/2*
(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-2*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)-
3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c+4*(-c*(a*x+1))^(
1/2)*c^(1/2))/c^(3/2)/(-c*(a*x+1))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

[Out] integrate(1/(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Fricas [A]
time = 0.36, size = 288, normalized size = 1.63

$$\left[\frac{3\sqrt{2}\left(a^{2}cx^{2}-2\,acx+c\right)\sqrt{-\frac{1}{c}}\log\left(-\frac{a^{2}x^{2}-2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}+2\,ax-3}{a^{2}x^{2}-2\,ax+1}\right)-4\left(a^{2}x^{2}-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{2\left(a^{3}cx^{2}-2\,a^{2}cx+ac\right)},-\frac{2\left(a^{2}x^{2}-ax-2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}-\frac{3\sqrt{2}\left(a^{2}cx^{2}-2\,acx+c\right)\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{\sqrt{c}}}{a^{3}cx^{2}-2\,a^{2}cx+ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/2*(3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*log(-(a^2*x^2 - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c), -(2*(a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)) - 3*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*sqrt(c)))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\,(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)

[Out] Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1))), x)

Giac [A]
time = 0.43, size = 75, normalized size = 0.42

$$-\frac{3\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)-2\sqrt{-acx-c}+\frac{2\sqrt{-acx-c}\,c}{acx-c}}{a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")

[Out] -(3*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) - 2*sqrt(-a*c*x - c) + 2*sqrt(-a*c*x - c)*c/(a*c*x - c))/(a*abs(c))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.249 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x/(a-1/x)^2/(-a*c*x+c)^(3/2)-3/8*(
1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1/2)
/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)-3/4*a*(1-1/a/x)^(3/2)*x*(1+1/a/x)^(1/
2)/(a-1/x)/(-a*c*x+c)^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}}-\frac{3ax\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}}-\frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-3*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(4*(a - x^(-1))*(c - a*c*x)^
(3/2)) - (a^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x)/(2*(a - x^(-1))^2*
(c - a*c*x)^(3/2)) - (3*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(3/2)*(c - a*c*x)
^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2} x^{3/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2} x^{3/2}}\,dx}{(c-acx)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \right.}{8\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \right.}{4\left(\frac{1}{x}\right)^{3/2}(c}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{4\left(a-\frac{1}{x}\right)(c-acx)^{3/2}} - \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x}{2\left(a-\frac{1}{x}\right)^2(c-acx)^{3/2}} - \frac{3\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}}{4\sqrt{2}\left(\frac{1}{x}\right)^{3/2}(c-}$$

Mathematica [A]
time = 0.10, size = 125, normalized size = 0.67

$$\frac{\sqrt{1-\frac{1}{ax}}\,x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(-1+5ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{8\sqrt{a}\,c(-1+ax)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(-1 + 5*a*x) + 3*Sqrt[2]*Sqrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(8*Sqrt[a]*c*(-1 + a*x)^2*Sqrt[c - a*c*x])

Maple [A]
time = 0.16, size = 174, normalized size = 0.93

method	result
default	$-\frac{\sqrt{-c\,(ax-1)}\,\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}-6\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+1\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)c^{\frac{5}{2}}\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/8/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(5/2)*(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-6*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+10*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)+3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-2*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((-a*c*x + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Fricas [A]
time = 0.36, size = 341, normalized size = 1.82

$$\left[-\frac{3\sqrt{2}\,(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{-c}\,\log\left(-\frac{a^{2}cx^{2}+2\,acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+4\,(5\,a^{2}x^{2}+4\,ax-1)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2})},\ -\frac{3\sqrt{2}\,(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+2\,(5\,a^{2}x^{2}+4\,ax-1)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{8\,(a^{4}c^{2}x^{3}-3\,a^{3}c^{2}x^{2}+3\,a^{2}c^{2}x-ac^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")

[Out] [-1/16*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*

```
c^2*x - a*c^2), -1/8*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*a
rctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) + 2*(5*a^2*x^2 + 4*a*x - 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
[Out] Timed out
```

Giac [A]
time = 0.44, size = 78, normalized size = 0.42

$$\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5(-acx-c)^{\frac{3}{2}}+6\sqrt{-acx-c}\,c\right)}{(acx-c)^2}}{8a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] 1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - 2*(5*
(-a*c*x - c)^(3/2) + 6*sqrt(-a*c*x - c)*c)/(a*c*x - c)^2)/(a*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c-a\,c\,x)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.250 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}-\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\text{ta}}{16\sqrt{2}\,\left(\frac{1}{x}\right.}$$

```
[Out] 1/24*a^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/2)*x^2/(a-1/x)^2/(-a*c*x+c)^(5/2)-1/6
*a^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^3/(-a*c*x+c)^(5/2)+1/32*a^
(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))/
(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+1/16*a^2*(1-1/a/x)^(5/2)*x^2*(1+1/a/x)
^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A]
time = 0.16, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^4x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}+\frac{a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}}+\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/(16*(a - x^(-1))*(c - a*c*x
)^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^2)/(24*(a - x^(-1
))^2*(c - a*c*x)^(5/2)) - (a^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^2)
/(6*(a - x^(-1))^3*(c - a*c*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))
^(5/2)*(c - a*c*x)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^{4}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)^{3}}\,dx, x, \frac{1}{x}\right)}{12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\right.}{16\left(\frac{1}{x}\right)^{5/2}(\ldots}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/\ldots}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/\ldots}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/\ldots}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/\ldots}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{16\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^2}{24\left(a-\frac{1}{x}\right)^2(c-acx)^{5/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/\ldots}}{6\left(a-\frac{1}{x}\right)^3(c-acx)^{5/\ldots}}$$

Mathematica [A]

time = 0.10, size = 142, normalized size = 0.57

$$-\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{a}\,\sqrt{1+\frac{1}{ax}}\,\sqrt{\frac{1}{x}}\,(7+22ax+3a^2x^2)-\frac{3\sqrt{2}\,(-1+ax)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)}{x}\right)}{96\sqrt{a}\,c^2\left(\frac{1}{x}\right)^{3/2}(-1+ax)^3\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -1/96*(Sqrt[1 - 1/(a*x)]*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(7 + 22*
a*x + 3*a^2*x^2) - (3*Sqrt[2]*(-1 + a*x)^3*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])])/x))/(Sqrt[a]*c^2*(x^(-1))^(3/2)*(-1 + a*x)^3*S
qrt[c - a*c*x])
```

Maple [A]
time = 0.15, size = 226, normalized size = 0.90

method	result
default	$-\frac{\sqrt{-c\,(ax-1)}\left(3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3-9\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)*(-c*(a*x-1))^(1/2)/c^(7/2)*
(3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^3*c*x^3-9*2^(1/
2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-6*a^2*x^2*(-c*(
a*x+1))^(1/2)*c^(1/2)+9*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/
2))*a*c*x-44*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1
))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/
2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 393, normalized size = 1.57

$$\left[-\frac{3\sqrt{2}\left(a^{4}x^{4}-4a^{3}x^{3}+6a^{2}x^{2}-4ax+1\right)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2acx-2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^{2}x^{2}-2ax+1}\right)-4\left(3a^{3}x^{3}+25a^{2}x^{2}+29ax+7\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{192\left(a^{5}c^{3}x^{4}-4a^{4}c^{3}x^{3}+6a^{3}c^{3}x^{2}-4a^{2}c^{3}x+ac^{3}\right)},\ -\frac{3\sqrt{2}\left(a^{4}x^{4}-4a^{3}x^{3}+6a^{2}x^{2}-4ax+1\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\left(3a^{3}x^{3}+25a^{2}x^{2}+29ax+7\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{96\left(a^{5}c^{3}x^{4}-4a^{4}c^{3}x^{3}+6a^{3}c^{3}x^{2}-4a^{2}c^{3}x+ac^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*l
og(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^3*x^3 + 25*a
^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x
^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), -1/96*(3*sqrt(2)
*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^3*x^3 +
 25*a^2*x^2 + 29*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*
c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 105, normalized size = 0.42

$$\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{2\left(3(acx+c)^{2}\sqrt{-acx-c}-16(-acx-c)^{\frac{3}{2}}c-12\sqrt{-acx-c}\,c^{2}\right)}{(acx-c)^{3}c}}{96\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - 2*(3
*(a*c*x + c)^2*sqrt(-a*c*x - c) - 16*(-a*c*x - c)^(3/2)*c - 12*sqrt(-a*c*x
- c)*c^2)/((a*c*x - c)^3*c))/(a*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c-a\,c\,x)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.251 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}-\frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}+\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\right.}{32\left(a-\frac{1}{x}\right)^3\left(\right.}$$

```
[Out] -1/8*a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^2/(a-1/x)^4/(-a*c*x+c)^(7/2)-1/1
28*a^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^3/(a-1/x)^2/(-a*c*x+c)^(7/2)+1/32*
a^5*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^3/(a-1/x)^3/(-a*c*x+c)^(7/2)-3/512*a^
(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))/
(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-3/256*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/a/x
)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A]

time = 0.16, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{256\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}+\frac{a^5x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}-\frac{a^5x^2\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}-\frac{a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}-\frac{3a^3x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -1/8*(a^5*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^2)/((a - x^(-1))^4*(c -
 a*c*x)^(7/2)) - (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(256*(a
- x^(-1))*(c - a*c*x)^(7/2)) - (a^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)
*x^3)/(128*(a - x^(-1))^2*(c - a*c*x)^(7/2)) + (a^5*(1 - 1/(a*x))^(7/2)*(1
+ 1/(a*x))^(5/2)*x^3)/(32*(a - x^(-1))^3*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1
 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(256*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^{5}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{\left(1-\frac{x}{a}\right)^{4}}\,dx, x, \frac{1}{x}\right)}{16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^3}{32\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}} - \frac{\left(a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subs}\cdots}{64\left(\frac{1}{x}\right)^{7/2}\cdots}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3}{128\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}} + \frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\cdots\right.}{32\left(a-\frac{1}{x}\right)^3(c-ac\cdots}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\cdots\right.}{128\left(a-\frac{1}{x}\right)^2(c-a\cdots}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\cdots\right.}{128\left(a-\frac{1}{x}\right)^2(c-a\cdots}$$

$$= -\frac{a^5\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^2}{8\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}} - \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{256\left(a-\frac{1}{x}\right)(c-acx)^{7/2}} - \frac{a^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\cdots\right.}{128\left(a-\frac{1}{x}\right)^2(c-a\cdots}$$

Mathematica [A]

time = 0.13, size = 147, normalized size = 0.48

$$\frac{\sqrt{1-\frac{1}{ax}}\left(\frac{2\sqrt{a}\sqrt{1+\frac{1}{ax}}\,(39+79ax+13a^2x^2-3a^3x^3)}{\sqrt{\frac{1}{x}}}+3\sqrt{2}\,(-1+ax)^4\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{512\sqrt{a}\,c^3\sqrt{\frac{1}{x}}\,(-1+ax)^4\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*((2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(39 + 79*a*x + 13*a^2*x^2
- 3*a^3*x^3))/Sqrt[x^(-1)] + 3*Sqrt[2]*(-1 + a*x)^4*ArcTanh[(Sqrt[2]*Sqrt[x
^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(512*Sqrt[a]*c^3*Sqrt[x^(-1)]*(-1 +
a*x)^4*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.15, size = 278, normalized size = 0.91

method	result
default	$\frac{\sqrt{-c\,(ax-1)}\,\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^4cx^4+12\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/
c^(1/2))*a^4*c*x^4+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2)
)*a^3*c*x^3+6*a^3*x^3*(-c*(a*x+1))^(1/2)*c^(1/2)-18*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2-26*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(
1/2)+12*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x-158*a*
x*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2
)/c^(1/2))*c-78*(-c*(a*x+1))^(1/2)*c^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)
^3/(a*x+1)/c^(9/2)/(-c*(a*x+1))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*c*x + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.40, size = 449, normalized size = 1.46

$$\left[-\frac{3\sqrt{2}\left(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\left(3a^4x^4-10a^3x^3-92a^2x^2-118ax-39\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{1024\left(a^6c^4x^5-5a^5c^4x^4+10a^4c^4x^3-10a^3c^4x^2+5a^2c^4x-ac^4\right)},\ -\frac{3\sqrt{2}\left(a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\left(3a^4x^4-10a^3x^3-92a^2x^2-118ax-39\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{512\left(a^6c^4x^5-5a^5c^4x^4+10a^4c^4x^3-10a^3c^4x^2+5a^2c^4x-ac^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/1024*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x
- 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x +
 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3
*a^4*x^4 - 10*a^3*x^3 - 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a
*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4), -1/512*(3*sqrt(2)*(a^5*x^5 - 5*a^4*x^4 + 10
*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*(3*a^4*x^4 - 10*a^3*x^3
- 92*a^2*x^2 - 118*a*x - 39)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a
^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x
- a*c^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [A]
time = 0.46, size = 129, normalized size = 0.42

$$\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}-\frac{2\left(3(acx+c)^3\sqrt{-acx-c}-22(acx+c)^2\sqrt{-acx-c}\,c+44(-acx-c)^{\frac{3}{2}}c^2+24\sqrt{-acx-c}\,c^3\right)}{(acx-c)^4c^2}}{512\,a|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/512*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) - 2*(
3*(a*c*x + c)^3*sqrt(-a*c*x - c) - 22*(a*c*x + c)^2*sqrt(-a*c*x - c)*c + 44
```

```
*(-a*c*x - c)^(3/2)*c^2 + 24*sqrt(-a*c*x - c)*c^3)/((a*c*x - c)^4*c^2))/(a*
abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c-a\,c\,x)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int(1/((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.252 $\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=194

$$\frac{16384c^5\sqrt{1-\frac{1}{a^2x^2}}\,x}{693\sqrt{c-acx}}+\frac{4096}{693}c^4\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{512}{231}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}+\frac{640}{693}c^2\sqrt{1-\frac{1}{a^2x^2}}$$

```
[Out] 512/231*c^3*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+640/693*c^2*x*(-a*c*x+c)
^(5/2)*(1-1/a^2/x^2)^(1/2)+40/99*c*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)+2
/11*x*(-a*c*x+c)^(9/2)*(1-1/a^2/x^2)^(1/2)+16384/693*c^5*x*(1-1/a^2/x^2)^(1
/2)/(-a*c*x+c)^(1/2)+4096/693*c^4*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 311, normalized size of antiderivative = 1.60, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$-\frac{22016\sqrt{\frac{1}{ax}+1}\,(c-acx)^{9/2}}{693a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\,\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{40\sqrt{\frac{1}{ax}+1}\,\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{640\sqrt{\frac{1}{ax}+1}\,\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{693a^5x\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{1024\sqrt{\frac{1}{ax}+1}\,(c-acx)^{9/2}}{99a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{512\sqrt{\frac{1}{ax}+1}\,(c-acx)^{9/2}}{231a^3x^2\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-40*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^5*(1 - 1/(a*
x))^(9/2)) - (22016*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(693*a^5*(1 - 1/(a
*x))^(9/2)*x^4) + (1024*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(99*a^4*(1 - 1
/(a*x))^(9/2)*x^3) - (512*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(9/2))/(231*a^3*(1
- 1/(a*x))^(9/2)*x^2) + (640*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(
9/2))/(693*a^5*(1 - 1/(a*x))^(9/2)*x) + (2*(a - x^(-1))^5*Sqrt[1 + 1/(a*x)]
*x*(c - a*c*x)^(9/2))/(11*a^5*(1 - 1/(a*x))^(9/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{9/2}\,dx = \frac{(c-acx)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^5}{x^{13/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{\left(20\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int \frac{}{x^{11/}} \right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{2\left(a-\frac{1}{x}\right)^5\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{9/2}}{11a^5\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{640\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{512\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{231a^3\left(1-\frac{1}{ax}\right)^{9/2}x^2} + \frac{640(a}{}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} + \frac{1024\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{99a^4\left(1-\frac{1}{ax}\right)^{9/2}x^3} - \frac{512}{23}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{99a^5\left(1-\frac{1}{ax}\right)^{9/2}} - \frac{22016\sqrt{1+\frac{1}{ax}}\,(c-acx)^{9/2}}{693a^5\left(1-\frac{1}{ax}\right)^{9/2}x^4} + \frac{102}{}$$

Mathematica [A]

time = 0.03, size = 86, normalized size = 0.44

$$\frac{2c^4\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}\,(-11531+5419ax-3198a^2x^2+1510a^3x^3-455a^4x^4+63a^5x^5)}{693a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (2*c^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-11531 + 5419*a*x - 3198*a^2*x^2
+ 1510*a^3*x^3 - 455*a^4*x^4 + 63*a^5*x^5))/(693*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.08, size = 84, normalized size = 0.43

method	result	size
risch	$-\frac{2c^5\sqrt{\frac{ax-1}{ax+1}}\,(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531)(ax+1)}{693\sqrt{-c(ax-1)}\,a}$	77
gosper	$\frac{2(ax+1)(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531)(-acx+c)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}}{693a(ax-1)^5}$	80
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}\,c^4(63a^5x^5-455a^4x^4+1510a^3x^3-3198a^2x^2+5419ax-11531)}{693(ax-1)a}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/693*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*c^4*(63*a^5*x^5-45
5*a^4*x^4+1510*a^3*x^3-3198*a^2*x^2+5419*a*x-11531)/(a*x-1)/a
```

Maxima [A]
time = 0.28, size = 128, normalized size = 0.66

$$\frac{2\,(63\,a^6\sqrt{-c}\,c^4x^6-392\,a^5\sqrt{-c}\,c^4x^5+1055\,a^4\sqrt{-c}\,c^4x^4-1688\,a^3\sqrt{-c}\,c^4x^3+2221\,a^2\sqrt{-c}\,c^4x^2-6112\,a\sqrt{-c}\,c^4x-11531\,\sqrt{-c}\,c^4)(ax-1)}{693\,(a^2x-a)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*a^6*sqrt(-c)*c^4*x^6 - 392*a^5*sqrt(-c)*c^4*x^5 + 1055*a^4*sqrt(-
c)*c^4*x^4 - 1688*a^3*sqrt(-c)*c^4*x^3 + 2221*a^2*sqrt(-c)*c^4*x^2 - 6112*a
*sqrt(-c)*c^4*x - 11531*sqrt(-c)*c^4)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Fricas [A]
time = 0.36, size = 105, normalized size = 0.54

$$\frac{2\,(63\,a^{6}c^{4}x^{6}-392\,a^{5}c^{4}x^{5}+1055\,a^{4}c^{4}x^{4}-1688\,a^{3}c^{4}x^{3}+2221\,a^{2}c^{4}x^{2}-6112\,ac^{4}x-11531\,c^{4})\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{693\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^6*c^4*x^6 - 392*a^5*c^4*x^5 + 1055*a^4*c^4*x^4 - 1688*a^3*c^4*x
^3 + 2221*a^2*c^4*x^2 - 6112*a*c^4*x - 11531*c^4)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.41, size = 110, normalized size = 0.57

$$\frac{2\,c^{4}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(63\,a^{5}\,x^{5}-329\,a^{4}\,x^{4}+726\,a^{3}\,x^{3}-962\,a^{2}\,x^{2}+1259\,a\,x-4853\right)}{693\,a}-\frac{32768\,c^{4}\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{693\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(1259*a*x - 962*a^2*x^
2 + 726*a^3*x^3 - 329*a^4*x^4 + 63*a^5*x^5 - 4853))/(693*a) - (32768*c^4*(c
 - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(693*a*(a*x - 1))
```

3.253 $\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=161

$$\frac{4096c^4\sqrt{1-\frac{1}{a^2x^2}}\,x}{315\sqrt{c-acx}}+\frac{1024}{315}c^3\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{128}{105}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}+\frac{32}{63}c\sqrt{1-\frac{1}{a^2x^2}}\,x(c$$

```
[Out] 128/105*c^2*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+32/63*c*x*(-a*c*x+c)^(5/
2)*(1-1/a^2/x^2)^(1/2)+2/9*x*(-a*c*x+c)^(7/2)*(1-1/a^2/x^2)^(1/2)+4096/315*
c^4*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+1024/315*c^3*x*(1-1/a^2/x^2)^(1/
2)*(-a*c*x+c)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 254, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\frac{5504\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}{315a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\,\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{32\sqrt{\frac{1}{ax}+1}\,\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{256\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}{45a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{128\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}{105a^2x\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-32*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*
x))^(7/2)) + (5504*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(315*a^4*(1 - 1/(a*
x))^(7/2)*x^3) - (256*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(45*a^3*(1 - 1/(
a*x))^(7/2)*x^2) + (128*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(7/2))/(105*a^2*(1 -
1/(a*x))^(7/2)*x) + (2*(a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(7/2)
)/(9*a^4*(1 - 1/(a*x))^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{7/2}\,dx=\frac{(c-acx)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{11/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(16\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int\frac{(}{x^{9/2}}\right.}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{2\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{7/2}}{9a^4\left(1-\frac{1}{ax}\right)^{7/2}}-$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{128\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}{105a^2\left(1-\frac{1}{ax}\right)^{7/2}x}+\frac{2\left(a-\frac{1}{x}\right.}{}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{256\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}{45a^3\left(1-\frac{1}{ax}\right)^{7/2}x^2}+\frac{128\sqrt{1}}{105a}$$

$$=-\frac{32\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{5504\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}{315a^4\left(1-\frac{1}{ax}\right)^{7/2}x^3}-\frac{256\sqrt{}}{45a}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.48

$$-\frac{2c^3\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}\,(2867-1276ax+642a^2x^2-220a^3x^3+35a^4x^4)}{315a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(2867 - 1276*a*x + 642*a^2*x^2 -
220*a^3*x^3 + 35*a^4*x^4))/(315*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.08, size = 76, normalized size = 0.47

method	result	size
risch	$\frac{2c^{4}\sqrt{\frac{ax-1}{ax+1}}\,(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)(ax+1)}{315\sqrt{-c\left(ax-1\right)}\,a}$	69
gosper	$\frac{2(ax+1)(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)(-acx+c)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}{315a(ax-1)^{4}}$	72
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\left(ax-1\right)}\,c^{3}(35a^{4}x^{4}-220a^{3}x^{3}+642a^{2}x^{2}-1276ax+2867)}{315(ax-1)a}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*c^3*(35*a^4*x^4-2
20*a^3*x^3+642*a^2*x^2-1276*a*x+2867)/(a*x-1)/a
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.70

$$-\frac{2\left(35\,a^{5}\sqrt{-c}\,c^{3}x^{5}-185\,a^{4}\sqrt{-c}\,c^{3}x^{4}+422\,a^{3}\sqrt{-c}\,c^{3}x^{3}-634\,a^{2}\sqrt{-c}\,c^{3}x^{2}+1591\,a\sqrt{-c}\,c^{3}x+2867\,\sqrt{-c}\,c^{3}\right)(ax-1)}{315\,(a^{2}x-a)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/315*(35*a^5*sqrt(-c)*c^3*x^5 - 185*a^4*sqrt(-c)*c^3*x^4 + 422*a^3*sqrt(-
c)*c^3*x^3 - 634*a^2*sqrt(-c)*c^3*x^2 + 1591*a*sqrt(-c)*c^3*x + 2867*sqrt(-
c)*c^3)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Fricas [A]
time = 0.34, size = 94, normalized size = 0.58

$$-\frac{2\left(35\,a^{5}c^{3}x^{5}-185\,a^{4}c^{3}x^{4}+422\,a^{3}c^{3}x^{3}-634\,a^{2}c^{3}x^{2}+1591\,ac^{3}x+2867\,c^{3}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*a^5*c^3*x^5 - 185*a^4*c^3*x^4 + 422*a^3*c^3*x^3 - 634*a^2*c^3*x^
2 + 1591*a*c^3*x + 2867*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^
2*x - a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.36, size = 102, normalized size = 0.63

$$-\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(35\,a^4\,x^4-150\,a^3\,x^3+272\,a^2\,x^2-362\,a\,x+1229\right)}{315\,a}-\frac{8192\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{315\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(272*a^2*x^2 - 362*a
*x - 150*a^3*x^3 + 35*a^4*x^4 + 1229))/(315*a) - (8192*c^3*(c - a*c*x)^(1/2
)*((a*x - 1)/(a*x + 1))^(1/2))/(315*a*(a*x - 1))
```

3.254 $\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=128

$$\frac{256c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}{35\sqrt{c-acx}}+\frac{64}{35}c^2\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{24}{35}c\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}+\frac{2}{7}\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{5/2}$$

```
[Out] 24/35*c*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+2/7*x*(-a*c*x+c)^(5/2)*(1-1/
a^2/x^2)^(1/2)+256/35*c^3*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+64/35*c^2*
x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.54, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$-\frac{344\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}{35a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\,\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{16\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}{5a^2x\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-24*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a*(1 - 1/(a*x))^(5/2)) - (344
*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(35*a^3*(1 - 1/(a*x))^(5/2)*x^2) + (1
6*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2))/(5*a^2*(1 - 1/(a*x))^(5/2)*x) + (2*(
a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*x*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(
5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^3}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(12\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int \frac{\cdots}{x^{7/\cdots}}\cdots\right.}{7a\left(1-\frac{1}{ax}\right)^{5/2}\cdots}$$

$$= -\frac{24\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\left(24\left(\frac{1}{x}\right)\cdots\right.}{\cdots}$$

$$= -\frac{24\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{16\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{5a^2\left(1-\frac{1}{ax}\right)^{5/2}x} + \frac{2\left(a-\frac{1}{x}\right)^3\sqrt{1+\cdots}}{7a^3\left(1\cdots\right.}$$

$$= -\frac{24\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{344\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}{35a^3\left(1-\frac{1}{ax}\right)^{5/2}x^2} + \frac{16\sqrt{1+\frac{1}{ax}}\,(c\cdots}{5a^2\left(1-\frac{1}{ax}\cdots\right.}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.55

$$\frac{2c^2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}\,(-177+71ax-27a^2x^2+5a^3x^3)}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^ArcCoth[a*x],x]
```

[Out] (2*c^2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-177 + 71*a*x - 27*a^2*x^2 + 5*a^3*x^3))/(35*a*Sqrt[1 - 1/(a*x)])

Maple [A]
time = 0.08, size = 68, normalized size = 0.53

method	result	size
risch	$-\frac{2c^{3}\sqrt{\frac{ax-1}{ax+1}}\,\left(5a^{3}x^{3}-27a^{2}x^{2}+71ax-177\right)\left(ax+1\right)}{35\sqrt{-c\left(ax-1\right)}\,a}$	61
gosper	$\frac{2\left(ax+1\right)\left(5a^{3}x^{3}-27a^{2}x^{2}+71ax-177\right)\left(-acx+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{35\left(ax-1\right)^{3}a}$	64
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)\sqrt{-c\left(ax-1\right)}\,c^{2}\left(5a^{3}x^{3}-27a^{2}x^{2}+71ax-177\right)}{35\left(ax-1\right)a}$	68

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/35*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*c^2*(5*a^3*x^3-27*a^2*x^2+71*a*x-177)/(a*x-1)/a

Maxima [A]
time = 0.28, size = 96, normalized size = 0.75

$$\frac{2\left(5\,a^{4}\sqrt{-c}\,c^{2}x^{4}-22\,a^{3}\sqrt{-c}\,c^{2}x^{3}+44\,a^{2}\sqrt{-c}\,c^{2}x^{2}-106\,a\sqrt{-c}\,c^{2}x-177\,\sqrt{-c}\,c^{2}\right)\left(ax-1\right)}{35\left(a^{2}x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")

[Out] 2/35*(5*a^4*sqrt(-c)*c^2*x^4 - 22*a^3*sqrt(-c)*c^2*x^3 + 44*a^2*sqrt(-c)*c^2*x^2 - 106*a*sqrt(-c)*c^2*x - 177*sqrt(-c)*c^2)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))

Fricas [A]
time = 0.32, size = 83, normalized size = 0.65

$$\frac{2\left(5\,a^{4}c^{2}x^{4}-22\,a^{3}c^{2}x^{3}+44\,a^{2}c^{2}x^{2}-106\,ac^{2}x-177\,c^{2}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

```
[Out] 2/35*(5*a^4*c^2*x^4 - 22*a^3*c^2*x^3 + 44*a^2*c^2*x^2 - 106*a*c^2*x - 177*c
^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.37, size = 94, normalized size = 0.73

$$\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-17\,a^2\,x^2+27\,a\,x-79\right)}{35\,a}-\frac{512\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(27*a*x - 17*a^2*x^2 +
 5*a^3*x^3 - 79))/(35*a) - (512*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))
^(1/2))/(35*a*(a*x - 1))
```

3.255 $\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{64c^2\sqrt{1-\frac{1}{a^2x^2}}\,x}{15\sqrt{c-acx}}+\frac{16}{15}c\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}+\frac{2}{5}\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}$$

```
[Out] 2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)+64/15*c^2*x*(1-1/a^2/x^2)^(1/2)/
(-a*c*x+c)^(1/2)+16/15*c*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 91, 79, 37}

$$\frac{86\sqrt{\frac{1}{ax}+1}\,(c-acx)^{3/2}}{15a^2x\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{2x\sqrt{\frac{1}{ax}+1}\,(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{28\sqrt{\frac{1}{ax}+1}\,(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-28*Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a*(1 - 1/(a*x))^(3/2)) + (86*
Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))/(15*a^2*(1 - 1/(a*x))^(3/2)*x) + (2*Sq
rt[1 + 1/(a*x)]*x*(c - a*c*x)^(3/2))/(5*(1 - 1/(a*x))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{(c-acx)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int \frac{-\frac{7}{a}+\frac{5x}{2a^2}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,d\right.}{5\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}\,(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x(c-acx)^{3/2}}{5\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(c-ac\right.}{}$$

$$= -\frac{28\sqrt{1+\frac{1}{ax}}\,(c-acx)^{3/2}}{15a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{86\sqrt{1+\frac{1}{ax}}\,(c-acx)^{3/2}}{15a^2\left(1-\frac{1}{ax}\right)^{3/2}x} + \frac{2\sqrt{1+\frac{1}{ax}}\,x(c-}{5\left(1-\frac{1}{ax}\right)^{3/}}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.63

$$-\frac{2c\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}\,(43-14ax+3a^2x^2)}{15a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^ArcCoth[a*x],x]
[Out] (-2*c*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(43 - 14*a*x + 3*a^2*x^2))/(15*a*Sq
rt[1 - 1/(a*x)])
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.61

method	result	size
risch	$\frac{2c^{2}\sqrt{\frac{ax-1}{ax+1}}\,(3a^{2}x^{2}-14ax+43)(ax+1)}{15\sqrt{-c\,(ax-1)}\,a}$	53
gosper	$\frac{2(ax+1)(3a^{2}x^{2}-14ax+43)(-acx+c)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{15a(ax-1)^{2}}$	56
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,c(3a^{2}x^{2}-14ax+43)}{15(ax-1)a}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*c*(3*a^2*x^2-14*a*
x+43)/(a*x-1)/a
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.76

$$-\frac{2\left(3\,a^{3}\sqrt{-c}\,cx^{3}-11\,a^{2}\sqrt{-c}\,cx^{2}+29\,a\sqrt{-c}\,cx+43\,\sqrt{-c}\,c\right)(ax-1)}{15\,(a^{2}x-a)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(3*a^3*sqrt(-c)*c*x^3 - 11*a^2*sqrt(-c)*c*x^2 + 29*a*sqrt(-c)*c*x + 4
3*sqrt(-c)*c)*(a*x - 1)/((a^2*x - a)*sqrt(a*x + 1))
```

Fricas [A]
time = 0.34, size = 64, normalized size = 0.67

$$-\frac{2\left(3\,a^{3}cx^{3}-11\,a^{2}cx^{2}+29\,acx+43\,c\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*a^3*c*x^3 - 11*a^2*c*x^2 + 29*a*c*x + 43*c)*sqrt(-a*c*x + c)*sqrt(
(a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.34, size = 82, normalized size = 0.86

$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^2\,x^2-8\,a\,x+21\right)}{15\,a}-\frac{128\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 8*a*x + 2
1))/(15*a) - (128*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(15*a*(a
*x - 1))
```

3.256 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=62

$$\frac{8c\sqrt{1-\frac{1}{a^2x^2}}\,x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}$$

```
[Out] 8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a*
c*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 79, 37}

$$\frac{2x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \right.}{3a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [**A**]

time = 0.02, size = 50, normalized size = 0.81

$$\frac{2\sqrt{1+\frac{1}{ax}}\,(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.08, size = 48, normalized size = 0.77

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c\left(ax-1\right)}\,a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\left(ax-1\right)}\,(ax-5)}{3(ax-1)a}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(a*x-5)/(a*x-1)/a
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.87

$$\frac{2\left(a^2\sqrt{-c}\,x^2-4\,a\sqrt{-c}\,x-5\,\sqrt{-c}\,\right)(ax-1)}{3\left(a^2x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

Fricas [A]
time = 0.34, size = 50, normalized size = 0.81

$$\frac{2\left(a^2x^2-4\,ax-5\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Giac [A]
time = 0.41, size = 43, normalized size = 0.69

$$\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2}+\frac{4\,\sqrt{-acx-c}\,|c|}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

Mupad [B]
time = 1.28, size = 71, normalized size = 1.15

$$\frac{2\,\sqrt{c-a\,c\,x}\,\left(a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.257 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=29

$$\frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

```
[Out] 2*(a*x+1)/a*((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6309}

$$\frac{2(ax+1)e^{-\coth^{-1}(ax)}}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*(1 + a*x))/(a*E^ArcCoth[a*x]*Sqrt[c - a*c*x])
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{2e^{-\coth^{-1}(ax)}(1+ax)}{a\sqrt{c-acx}}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.97

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-acx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - a*c*x]
```

Maple [A]
time = 0.08, size = 46, normalized size = 1.59

method	result	size
gosper	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-acx+c}}$	35
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c\,(ax-1)}\,a}$	36
default	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}}{(ax-1)ca}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)/(a*x-1)/c/a
```

Maxima [A]
time = 0.26, size = 29, normalized size = 1.00

$$-\frac{2\left(a\sqrt{-c}\,x+\sqrt{-c}\right)}{\sqrt{ax+1}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))/(sqrt(a*x + 1)*a*c)
```

Fricas [A]
time = 0.34, size = 44, normalized size = 1.52

$$-\frac{2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\,(ax-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.25, size = 34, normalized size = 1.17

$$\frac{\left(2\,x+\frac{2}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(1/2),x)
```

```
[Out] ((2*x + 2/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

3.258 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{2}\,\sqrt{a}\,\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

```
[Out] -(1-1/a/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*a^(1/
2)/(1/x)^(3/2)/(-a*c*x+c)^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6311, 6316, 95, 212}

$$-\frac{\sqrt{2}\,\sqrt{a}\,\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]
```

```
[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt
[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}\\
&= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}\\
&= -\frac{\left(2\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}\\
&= -\frac{\sqrt{2}\sqrt{a}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 76, normalized size = 1.00

$$-\frac{\sqrt{2}\,\sqrt{a}\,\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(3/2)),x]

[Out] -((Sqrt[2]*Sqrt[a]*(1 - 1/(a*x))^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/((x^(-1))^(3/2)*(c - a*c*x)^(3/2)))

Maple [A]
time = 0.15, size = 78, normalized size = 1.03

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)}{(ax-1)\sqrt{-c\,(ax+1)}\,c^{\frac{3}{2}}\,a}$	78

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)/c^(3/2)/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(3/2), x)

Fricas [A]
time = 0.35, size = 141, normalized size = 1.86

$$\left[\frac{\sqrt{2}\,\sqrt{-\frac{1}{c}}\,\log\left(-\frac{a^2x^2+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-\frac{1}{c}}+2\,ax-3}{a^2x^2-2\,ax+1}\right)}{2\,ac},\ -\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)\sqrt{c}}\right)}{ac^{\frac{3}{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/2*sqrt(2)*sqrt(-1/c)*log(-(a^2*x^2 + 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c) + 2*a*x - 3)/(a^2*x^2 - 2*a*x + 1))/(a*c), -sqrt(2)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*sqrt(c)))/(a*c^(3/2))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-c\left(ax-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(3/2),x)

[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1))**(3/2), x)

Giac [A]
time = 0.43, size = 64, normalized size = 0.84

$$\frac{\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{a\sqrt{c}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)|c|\mathrm{sgn}(ax+1)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")

[Out] (sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*sqrt(c)) - sqrt(2)*arctan(sqrt(-c)/sqrt(c))/(a*sqrt(c)))*abs(c)*sgn(a*x + 1)/c^2

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2),x)

[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(3/2), x)

3.259 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}+\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] 1/4*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(
1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(5/2)*x^2*(1+1
/a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^2)/((a - x^(-1))*(c - a*c
*x)^(5/2)) + (a^(3/2)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(S
qrt[a]*Sqrt[1 + 1/(a*x)])])/(2*Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{4\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}{2\left(a-\frac{1}{x}\right)(c-acx)^{5/2}} + \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{2\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A]
time = 0.08, size = 116, normalized size = 0.85

$$\frac{\sqrt{1-\frac{1}{ax}}\,x\left(-2\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{4\sqrt{a}\,c^2(-1+ax)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(-2*Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[2]*Sqrt[x^(-1)]*(
-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(4*
Sqrt[a]*c^2*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.15, size = 123, normalized size = 0.90

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\left(-\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{}}{2\sqrt{c}}\right.\right.}{4c^{\frac{7}{2}}\,(ax-1)^2\sqrt{-c\,(ax+1)}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-2^(1/2)*arctan(1/2
*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+2^(1/2)*arctan(1/2*(-c*(a*x+1))^
(1/2)*2^(1/2)/c^(1/2))*c+2*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(7/2)/(a*x-1)^2/(-
c*(a*x+1))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(5/2), x)
```

Fricas [A]
time = 0.35, size = 281, normalized size = 2.07

$$\left[-\frac{\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)-4\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{8\,(a^3c^3x^2-2\,a^2c^3x+ac^3)},\ -\frac{\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{4\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) - 4*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*
x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), -1/4*(sqrt(2)*(a^2*x^2 - 2*a*
x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))
)/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(5/2), x)
```

3.260 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}+\frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] -3/32*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)
^(1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/4*a^3*(1-1/a/x)^(7/2)*x^2*(1
+1/a/x)^(1/2)/(a-1/x)^2/(-a*c*x+c)^(7/2)+3/16*a^3*(1-1/a/x)^(7/2)*x^3*(1+1/
a/x)^(1/2)/(a-1/x)/(-a*c*x+c)^(7/2)
```

Rubi [A]
time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}+\frac{3a^3x^3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)(c-acx)^{7/2}}-\frac{a^3x^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/4*(a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^2)/((a - x^(-1))^2*(c - a
*c*x)^(7/2)) + (3*a^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x^3)/(16*(a - x
^(-1))*(c - a*c*x)^(7/2)) - (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]
*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(16*Sqrt[2]*(x^(-1))^(7/2)*(c
- a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2} x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2} \text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^3 \sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2} \sqrt{1+\frac{1}{ax}}\, x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right) \text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)^2 \sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{8\left(\frac{1}{x}\right)^{7/2} (c-acx)^{7/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2} \sqrt{1+\frac{1}{ax}}\, x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2} \sqrt{1+\frac{1}{ax}}\, x^3}{16\left(a-\frac{1}{x}\right) (c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right) \text{Subs}}{32\,(\frac{1}{x}}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2} \sqrt{1+\frac{1}{ax}}\, x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2} \sqrt{1+\frac{1}{ax}}\, x^3}{16\left(a-\frac{1}{x}\right) (c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right) \text{Subs}}{16\left(\frac{1}{x}\right)}$$

$$= -\frac{a^3\left(1-\frac{1}{ax}\right)^{7/2} \sqrt{1+\frac{1}{ax}}\, x^2}{4\left(a-\frac{1}{x}\right)^2 (c-acx)^{7/2}} + \frac{3a^3\left(1-\frac{1}{ax}\right)^{7/2} \sqrt{1+\frac{1}{ax}}\, x^3}{16\left(a-\frac{1}{x}\right) (c-acx)^{7/2}} - \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2} \tanh^{-}}{16\sqrt{2}\left(\frac{1}{x}\right)^{7/2}}$$

Mathematica [A]
time = 0.11, size = 125, normalized size = 0.65

$$\frac{\sqrt{1-\frac{1}{ax}}\, x\left(2\sqrt{a}\sqrt{1+\frac{1}{ax}}(7-3ax)+3\sqrt{2}\sqrt{\frac{1}{x}}(-1+ax)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{32\sqrt{a}\,c^3(-1+ax)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*x*(2*Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 - 3*a*x) + 3*Sqrt[2]*S
qrt[x^(-1)]*(-1 + a*x)^2*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1
/(a*x)])]))/(32*Sqrt[a]*c^3*(-1 + a*x)^2*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.15, size = 172, normalized size = 0.89

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\left(-3\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^{2}c\,x^{2}+6\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{2\sqrt{c}}\right.\right.}{32c^{\frac{9}{2}}\,(ax-1)^{3}\sqrt{-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*2^(1/2)*arctan(
1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2+6*2^(1/2)*arctan(1/2*(-c*
(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+6*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)-3*2^
(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*c-14*(-c*(a*x+1))^(1/2
)*c^(1/2))/c^(9/2)/(a*x-1)^3/(-c*(a*x+1))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a*c*x + c)^(7/2), x)
```

Fricas [A]
time = 0.41, size = 341, normalized size = 1.77

$$\left[-\frac{3\sqrt{2}\,(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2acx-2\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)-4\,(3a^2x^2-4ax-7)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{64\,(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4)},\ -\frac{3\sqrt{2}\,(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,(3a^2x^2-4ax-7)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{32\,(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*log(-(a^2*c*x^
2 + 2*a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/
(a*x + 1)) - 3*c)/(a^2*x^2 - 2*a*x + 1)) - 4*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-
```

```
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*
c^4*x - a*c^4), -1/32*(3*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*
arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x -
c)) - 2*(3*a^2*x^2 - 4*a*x - 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))
/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a*c*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**A**]
time = 0.43, size = 88, normalized size = 0.46

$$\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(-acx-c\right)^{\frac{3}{2}}+10\sqrt{-acx-c}\,c\right)}{\left(acx-c\right)^{2}c^{2}}\right)|c|\mathrm{sgn}(ax+1)}{32\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/32*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/c^(5/2) + 2*(3
*(-a*c*x - c)^(3/2) + 10*sqrt(-a*c*x - c)*c)/((a*c*x - c)^2*c^2))*abs(c)*sg
n(a*x + 1)/(a*c^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a\,c\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a*c*x)^(7/2), x)
```

3.261 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=137

$$-\frac{32c^3\sqrt{c-acx}}{a}-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{8c(c-acx)^{5/2}}{5a}-\frac{4(c-acx)^{7/2}}{7a}-\frac{2(c-acx)^{9/2}}{9ac}+\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.}{a}$$

```
[Out] -16/3*c^2*(-a*c*x+c)^(3/2)/a-8/5*c*(-a*c*x+c)^(5/2)/a-4/7*(-a*c*x+c)^(7/2)/
a-2/9*(-a*c*x+c)^(9/2)/a/c+32*c^(7/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/
c^(1/2))*2^(1/2)/a-32*c^3*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{32c^3\sqrt{c-acx}}{a}-\frac{16c^2(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{9/2}}{9ac}-\frac{4(c-acx)^{7/2}}{7a}-\frac{8c(c-acx)^{5/2}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-32*c^3*Sqrt[c - a*c*x])/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c -
a*c*x)^(5/2))/(5*a) - (4*(c - a*c*x)^(7/2))/(7*a) - (2*(c - a*c*x)^(9/2))/(
9*a*c) + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{7/2}\,dx$$
$$= -\int \frac{(1-ax)(c-acx)^{7/2}}{1+ax}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{9/2}}{1+ax}\,dx}{c}$$
$$= -\frac{2(c-acx)^{9/2}}{9ac} - 2\int \frac{(c-acx)^{7/2}}{1+ax}\,dx$$
$$= -\frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (4c)\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= -\frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (8c^2)\int \frac{(c-acx)^{3/2}}{1+ax}\,d$$
$$= -\frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - \frac{2(c-acx)^{9/2}}{9ac} - (16$$
$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - 2$$
$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - 2$$
$$= -\frac{32c^3\sqrt{c-acx}}{a} - \frac{16c^2(c-acx)^{3/2}}{3a} - \frac{8c(c-acx)^{5/2}}{5a} - \frac{4(c-acx)^{7/2}}{7a} - 2$$

Mathematica [A]
time = 0.05, size = 88, normalized size = 0.64

$$\frac{2c^3\left(\sqrt{c-acx}\,(-6257+1754ax-732a^2x^2+230a^3x^3-35a^4x^4)+5040\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)\right)}{315a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*c^3*(Sqrt[c - a*c*x]*(-6257 + 1754*a*x - 732*a^2*x^2 + 230*a^3*x^3 - 35*
a^4*x^4) + 5040*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])
)/(315*a)
```

Maple [A]
time = 0.20, size = 101, normalized size = 0.74

method	result
risch	$\frac{2(35a^4x^4-230a^3x^3+732a^2x^2-1754ax+6257)(ax-1)c^4}{315a\sqrt{-c(ax-1)}}+\frac{32c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{8c^3(-acx+c)^{\frac{3}{2}}}{3}+16c^4\sqrt{-acx+c}-16c^{\frac{9}{2}}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-a}\right.\right.}{ca}$
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{9}{2}}}{9}+\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{4c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{8c^3(-acx+c)^{\frac{3}{2}}}{3}+16c^4\sqrt{-acx+c}-16c^{\frac{9}{2}}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-a}\right.\right.}{ca}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/9*(-a*c*x+c)^(9/2)+2/7*c*(-a*c*x+c)^(7/2)+4/5*c^2*(-a*c*x+c)^(5/2
)+8/3*c^3*(-a*c*x+c)^(3/2)+16*c^4*(-a*c*x+c)^(1/2)-16*c^(9/2)*2^(1/2)*arcta
nh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]

time = 0.49, size = 123, normalized size = 0.90

$$-\frac{2\left(2520\,\sqrt{2}\,c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+35\,(-acx+c)^{\frac{9}{2}}+90\,(-acx+c)^{\frac{7}{2}}c+252\,(-acx+c)^{\frac{5}{2}}c^2+840\,(-acx+c)^{\frac{3}{2}}c^3+5040\,\sqrt{-acx+c}\,c^4\right)}{315\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/315*(2520*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqr
t(2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) + 90*(-a*c*x + c)
^(7/2)*c + 252*(-a*c*x + c)^(5/2)*c^2 + 840*(-a*c*x + c)^(3/2)*c^3 + 5040*s
qrt(-a*c*x + c)*c^4)/(a*c)
```

Fricas [A]

time = 0.34, size = 204, normalized size = 1.49

$$\left[\frac{2\left(2520\,\sqrt{2}\,c^{\frac{7}{2}}\log\left(\frac{acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{ax+1}\right)-(35\,a^4c^3x^4-230\,a^3c^3x^3+732\,a^2c^3x^2-1754\,ac^3x+6257\,c^3)\sqrt{-acx+c}\right)}{315\,a},\,-\frac{2\left(5040\,\sqrt{2}\,\sqrt{-c}\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2\,c}\right)+(35\,a^4c^3x^4-230\,a^3c^3x^3+732\,a^2c^3x^2-1754\,ac^3x+6257\,c^3)\sqrt{-acx+c}\right)}{315\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(2520*sqrt(2)*c^(7/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c
) - 3*c)/(a*x + 1)) - (35*a^4*c^3*x^4 - 230*a^3*c^3*x^3 + 732*a^2*c^3*x^2 -
 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c))/a, -2/315*(5040*sqrt(2)*sqrt(-c
)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*c^3*x^4 - 2
```

```
30*a^3*c^3*x^3 + 732*a^2*c^3*x^2 - 1754*a*c^3*x + 6257*c^3)*sqrt(-a*c*x + c
))/a]
```

Sympy [A]
time = 48.22, size = 129, normalized size = 0.94

$$-\frac{32\sqrt{2}\,c^4\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{32c^3\sqrt{-acx+c}}{a}-\frac{16c^2(-acx+c)^{\frac{3}{2}}}{3a}-\frac{8c(-acx+c)^{\frac{5}{2}}}{5a}-\frac{4(-acx+c)^{\frac{7}{2}}}{7a}-\frac{2(-acx+c)^{\frac{9}{2}}}{9ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -32*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
 32*c**3*sqrt(-a*c*x + c)/a - 16*c**2*(-a*c*x + c)**(3/2)/(3*a) - 8*c*(-a*c
*x + c)**(5/2)/(5*a) - 4*(-a*c*x + c)**(7/2)/(7*a) - 2*(-a*c*x + c)**(9/2)/
(9*a*c)
```

Giac [A]
time = 0.40, size = 161, normalized size = 1.18

$$-\frac{32\,\sqrt{2}\,c^4\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\,\left(35\,(acx-c)^4\sqrt{-acx+c}\,a^8c^8-90\,(acx-c)^3\sqrt{-acx+c}\,a^8c^9+252\,(acx-c)^2\sqrt{-acx+c}\,a^8c^{10}+840\,(-acx+c)^{\frac{3}{2}}a^8c^{11}+5040\,\sqrt{-acx+c}\,a^8c^{12}\right)}{315\,a^9c^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -32*sqrt(2)*c^4*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
- 2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^8*c^8 - 90*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^8*c^9 + 252*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^10 + 840*(-a
*c*x + c)^(3/2)*a^8*c^11 + 5040*sqrt(-a*c*x + c)*a^8*c^12)/(a^9*c^9)
```

Mupad [B]
time = 0.08, size = 112, normalized size = 0.82

$$-\frac{4\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a}-\frac{8\,c\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}-\frac{32\,c^3\,\sqrt{c-a\,c\,x}}{a}-\frac{16\,c^2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}-\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a\,c}-\frac{\sqrt{2}\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,32{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(7/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(7/2))/(7*a) - (8*c*(c - a*c*x)^(5/2))/(5*a) - (32*c^3*(c
- a*c*x)^(1/2))/a - (16*c^2*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(9/2)
)/(9*a*c) - (2^(1/2)*c^(7/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)
))*32i)/a
```

3.262 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=116

$$-\frac{16c^2\sqrt{c-acx}}{a}-\frac{8c(c-acx)^{3/2}}{3a}-\frac{4(c-acx)^{5/2}}{5a}-\frac{2(c-acx)^{7/2}}{7ac}+\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

```
[Out] -8/3*c*(-a*c*x+c)^(3/2)/a-4/5*(-a*c*x+c)^(5/2)/a-2/7*(-a*c*x+c)^(7/2)/a/c+1
6*c^(5/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-16*c^2*(-
a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{16c^2\sqrt{c-acx}}{a}-\frac{2(c-acx)^{7/2}}{7ac}-\frac{4(c-acx)^{5/2}}{5a}-\frac{8c(c-acx)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-16*c^2*Sqrt[c - a*c*x])/a - (8*c*(c - a*c*x)^(3/2))/(3*a) - (4*(c - a*c*x
)^(5/2))/(5*a) - (2*(c - a*c*x)^(7/2))/(7*a*c) + (16*Sqrt[2]*c^(5/2)*ArcTan
h[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{5/2}\,dx$$
$$= -\int \frac{(1-ax)(c-acx)^{5/2}}{1+ax}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{7/2}}{1+ax}\,dx}{c}$$
$$= -\frac{2(c-acx)^{7/2}}{7ac} - 2\int \frac{(c-acx)^{5/2}}{1+ax}\,dx$$
$$= -\frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (4c)\int \frac{(c-acx)^{3/2}}{1+ax}\,dx$$
$$= -\frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (8c^2)\int \frac{\sqrt{c-acx}}{1+ax}\,d$$
$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} - (16$$
$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{(32}{}$$
$$= -\frac{16c^2\sqrt{c-acx}}{a} - \frac{8c(c-acx)^{3/2}}{3a} - \frac{4(c-acx)^{5/2}}{5a} - \frac{2(c-acx)^{7/2}}{7ac} + \frac{16}{}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.69

$$\frac{2c^2\left(\sqrt{c-acx}\,(-1037+269ax-87a^2x^2+15a^3x^3)+840\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*c^2*(Sqrt[c - a*c*x]*(-1037 + 269*a*x - 87*a^2*x^2 + 15*a^3*x^3) + 840*S
qrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(105*a)
```

Maple [A]
time = 0.20, size = 87, normalized size = 0.75

method	result

risch	$-\frac{2(15a^3x^3-87a^2x^2+269ax-1037)(ax-1)c^3}{105a\sqrt{-c(ax-1)}}+\frac{16c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{4c^2(-acx+c)^{\frac{3}{2}}}{3}+8c^3\sqrt{-acx+c}-8c^{\frac{7}{2}}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c(-acx+c)^{\frac{5}{2}}}{5}+\frac{4c^2(-acx+c)^{\frac{3}{2}}}{3}+8c^3\sqrt{-acx+c}-8c^{\frac{7}{2}}\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/7*(-a*c*x+c)^(7/2)+2/5*c*(-a*c*x+c)^(5/2)+4/3*c^2*(-a*c*x+c)^(3/2
)+8*c^3*(-a*c*x+c)^(1/2)-8*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(
1/2)/c^(1/2)))
```

Maxima [A]
time = 0.47, size = 109, normalized size = 0.94

$$-\frac{2\left(420\sqrt{2}\,c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+15(-acx+c)^{\frac{7}{2}}+42(-acx+c)^{\frac{5}{2}}c+140(-acx+c)^{\frac{3}{2}}c^2+840\sqrt{-acx+c}\,c^3\right)}{105\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/105*(420*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt
(2)*sqrt(c) + sqrt(-a*c*x + c))) + 15*(-a*c*x + c)^(7/2) + 42*(-a*c*x + c)^
(5/2)*c + 140*(-a*c*x + c)^(3/2)*c^2 + 840*sqrt(-a*c*x + c)*c^3)/(a*c)
```

Fricas [A]
time = 0.33, size = 182, normalized size = 1.57

$$\left[\frac{2\left(420\sqrt{2}\,c^{\frac{5}{2}}\log\left(\frac{acx-2\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3c}{ax+1}\right)+(15a^3c^2x^3-87a^2c^2x^2+269ac^2x-1037c^2)\sqrt{-acx+c}\right)}{105\,a},\ -\frac{2\left(840\sqrt{2}\,\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2c}\right)-(15a^3c^2x^3-87a^2c^2x^2+269ac^2x-1037c^2)\sqrt{-acx+c}\right)}{105\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/105*(420*sqrt(2)*c^(5/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*c^2*x - 1037*
c^2)*sqrt(-a*c*x + c))/a, -2/105*(840*sqrt(2)*sqrt(-c)*c^2*arctan(1/2*sqrt(
2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - (15*a^3*c^2*x^3 - 87*a^2*c^2*x^2 + 269*a*
c^2*x - 1037*c^2)*sqrt(-a*c*x + c))/a]
```

Sympy [A]
time = 32.16, size = 110, normalized size = 0.95

$$-\frac{16\sqrt{2}\,c^{3}\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{16c^{2}\sqrt{-acx+c}}{a}-\frac{8c(-acx+c)^{\frac{3}{2}}}{3a}-\frac{4(-acx+c)^{\frac{5}{2}}}{5a}-\frac{2(-acx+c)^{\frac{7}{2}}}{7ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -16*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
 16*c**2*sqrt(-a*c*x + c)/a - 8*c*(-a*c*x + c)**(3/2)/(3*a) - 4*(-a*c*x + c
)**(5/2)/(5*a) - 2*(-a*c*x + c)**(7/2)/(7*a*c)
```

Giac [A]
time = 0.40, size = 134, normalized size = 1.16

$$-\frac{16\,\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{2\left(15\,(acx-c)^{3}\sqrt{-acx+c}\,a^{6}c^{6}-42\,(acx-c)^{2}\sqrt{-acx+c}\,a^{6}c^{7}-140\,(-acx+c)^{\frac{3}{2}}a^{6}c^{8}-840\,\sqrt{-acx+c}\,a^{6}c^{9}\right)}{105\,a^{7}c^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -16*sqrt(2)*c^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c))
+ 2/105*(15*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^6*c^6 - 42*(a*c*x - c)^2*sqrt(
-a*c*x + c)*a^6*c^7 - 140*(-a*c*x + c)^(3/2)*a^6*c^8 - 840*sqrt(-a*c*x + c)
*a^6*c^9)/(a^7*c^7)
```

Mupad [B]
time = 0.07, size = 95, normalized size = 0.82

$$-\frac{4\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a}-\frac{8\,c\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a}-\frac{16\,c^2\,\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a\,c}-\frac{\sqrt{2}\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,16{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(5/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(5/2))/(5*a) - (8*c*(c - a*c*x)^(3/2))/(3*a) - (16*c^2*(c
- a*c*x)^(1/2))/a - (2*(c - a*c*x)^(7/2))/(7*a*c) - (2^(1/2)*c^(5/2)*atan((
2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*16i)/a
```

3.263 $\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{8c\sqrt{c-acx}}{a}-\frac{4(c-acx)^{3/2}}{3a}-\frac{2(c-acx)^{5/2}}{5ac}+\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

```
[Out] -4/3*(-a*c*x+c)^(3/2)/a-2/5*(-a*c*x+c)^(5/2)/a/c+8*c^(3/2)*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-8*c*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a}-\frac{2(c-acx)^{5/2}}{5ac}-\frac{4(c-acx)^{3/2}}{3a}-\frac{8c\sqrt{c-acx}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-8*c*Sqrt[c - a*c*x])/a - (4*(c - a*c*x)^(3/2))/(3*a) - (2*(c - a*c*x)^(5/
2))/(5*a*c) + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
)/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}(c-acx)^{3/2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}(c-acx)^{3/2}\,dx \\
&= -\int \frac{(1-ax)(c-acx)^{3/2}}{1+ax}\,dx \\
&= -\frac{\int \frac{(c-acx)^{5/2}}{1+ax}\,dx}{c} \\
&= -\frac{2(c-acx)^{5/2}}{5ac} - 2\int \frac{(c-acx)^{3/2}}{1+ax}\,dx \\
&= -\frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} - (4c)\int \frac{\sqrt{c-acx}}{1+ax}\,dx \\
&= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} - \left(8c^2\right)\int \frac{1}{(1+ax)\sqrt{c-
\end{aligned}
$$

$$
= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} + \frac{(16c)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx
}{a}
$$

$$
= -\frac{8c\sqrt{c-acx}}{a} - \frac{4(c-acx)^{3/2}}{3a} - \frac{2(c-acx)^{5/2}}{5ac} + \frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{2}}\right.}{a}
$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.75

$$\frac{-2c\sqrt{c-acx}\,(73-16ax+3a^2x^2)+120\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*c*Sqrt[c - a*c*x]*(73 - 16*a*x + 3*a^2*x^2) + 120*Sqrt[2]*c^(3/2)*ArcTa
nh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a)
```

Maple [A]
time = 0.20, size = 73, normalized size = 0.77

method	result	size
risch	$\frac{2(3a^2x^2-16ax+73)(ax-1)c^2}{15a\sqrt{-c(ax-1)}}+\frac{8c^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a}$	68
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^2\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	73
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^2\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/5*(-a*c*x+c)^(5/2)+2/3*c*(-a*c*x+c)^(3/2)+4*c^2*(-a*c*x+c)^(1/2)-
4*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.46, size = 95, normalized size = 1.00

$$-\frac{2\left(30\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{5}{2}}+10\,(-acx+c)^{\frac{3}{2}}c+60\,\sqrt{-acx+c}\,c^2\right)}{15\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/15*(30*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 10*(-a*c*x + c)^(3/
2)*c + 60*sqrt(-a*c*x + c)*c^2)/(a*c)
```

Fricas [A]
time = 0.34, size = 146, normalized size = 1.54

$$\left[\frac{2\left(30\sqrt{2}\,c^{\frac{3}{2}}\log\left(\frac{acx-2\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3c}{ax+1}\right)-\left(3a^2cx^2-16acx+73c\right)\sqrt{-acx+c}\right)}{15a},-\frac{2\left(60\sqrt{2}\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2c}\right)+\left(3a^2cx^2-16acx+73c\right)\sqrt{-acx+c}\right)}{15a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*c^(3/2)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) - (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a, -2/
15*(60*sqrt(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) +
 (3*a^2*c*x^2 - 16*a*c*x + 73*c)*sqrt(-a*c*x + c))/a]
```

Sympy [A]
time = 20.42, size = 92, normalized size = 0.97

$$-\frac{8\sqrt{2}\,c^2\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{8c\sqrt{-acx+c}}{a}-\frac{4(-acx+c)^{\frac{3}{2}}}{3a}-\frac{2(-acx+c)^{\frac{5}{2}}}{5ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -8*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*sqrt(-c)) -
8*c*sqrt(-a*c*x + c)/a - 4*(-a*c*x + c)**(3/2)/(3*a) - 2*(-a*c*x + c)**(5/2
)/(5*a*c)
```

Giac [A]
time = 0.40, size = 107, normalized size = 1.13

$$-\frac{8\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(3\left(acx-c\right)^2\sqrt{-acx+c}\,a^4c^4+10\left(-acx+c\right)^{\frac{3}{2}}a^4c^5+60\sqrt{-acx+c}\,a^4c^6\right)}{15a^5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -8*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) -
 2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^4 + 10*(-a*c*x + c)^(3/2)*a^4
*c^5 + 60*sqrt(-a*c*x + c)*a^4*c^6)/(a^5*c^5)
```

Mupad [B]
time = 1.25, size = 78, normalized size = 0.82

$$-\frac{4\left(c-a\,c\,x\right)^{3/2}}{3\,a}-\frac{8\,c\,\sqrt{c-a\,c\,x}}{a}-\frac{2\left(c-a\,c\,x\right)^{5/2}}{5\,a\,c}-\frac{\sqrt{2}\,c^{3/2}\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,8\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(3/2))/(3*a) - (8*c*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^
(5/2))/(5*a*c) - (2^(1/2)*c^(3/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^
(1/2)))*8i)/a
```

3.264 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$-\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {6302, 6265, 21, 52, 65, 212}

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{2(c-acx)^{3/2}}{3ac} - 2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} - (4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{a}\\
&= -\frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac} + \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 61, normalized size = 0.80

$$\frac{2(-7+ax)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A]
time = 0.20, size = 59, normalized size = 0.78

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c(ax-1)}}+\frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh
(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.46, size = 79, normalized size = 1.04

$$-\frac{2\left(3\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*
sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(
a*c)
```

Fricas [A]

time = 0.35, size = 119, normalized size = 1.57

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}\,(ax-7)\right)}{3a}, -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}\,(ax-7)\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

Sympy [A]

time = 2.28, size = 73, normalized size = 0.96

$$-\frac{2\cdot\left(\frac{2\sqrt{2}c^2\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A]

time = 0.41, size = 77, normalized size = 1.01

$$-\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left((-acx+c)^{\frac{3}{2}}a^2c^2+6\sqrt{-acx+c}\,a^2c^3\right)}{3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

Mupad [B]

time = 1.24, size = 61, normalized size = 0.80

$$\frac{4\sqrt{2}\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}-\frac{4\sqrt{c-a\,c\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*(
c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

3.265 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=58

$$-\frac{2\sqrt{c-acx}}{ac}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] 2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)-2*(-a*c*x
+c)^(1/2)/a/c
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-acx}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-acx}}\,dx \\
&= -\int \frac{1-ax}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{c} \\
&= -\frac{2\sqrt{c-acx}}{ac} - 2\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx \\
&= -\frac{2\sqrt{c-acx}}{ac} + \frac{4\mathrm{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac} \\
&= -\frac{2\sqrt{c-acx}}{ac} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 58, normalized size = 1.00

$$-\frac{2\sqrt{c-acx}}{ac}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (-2*Sqrt[c - a*c*x])/(a*c) + (2*Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/(a*Sqrt[c])
```

Maple [A]
time = 0.19, size = 45, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2\left(\sqrt{-acx+c}-\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}\right)}{ca}$	45
default	$-\frac{2\left(\sqrt{-acx+c}-\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}\right)}{ca}$	45
risch	$\frac{2ax-2}{a\sqrt{-c\left(ax-1\right)}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\sqrt{c}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*((-a*c*x+c)^(1/2)-arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1
/2)*c^(1/2))
```

Maxima [A]
time = 0.46, size = 68, normalized size = 1.17

$$-\frac{\sqrt{2}\,\sqrt{c}\,\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+2\,\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c))) + 2*sqrt(-a*c*x + c))/(a*c)
```

Fricas [A]
time = 0.33, size = 118, normalized size = 2.03

$$\left[\frac{\sqrt{2}\,\sqrt{c}\,\log\left(\frac{ax-\frac{2\,\sqrt{2}\,\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)-2\,\sqrt{-acx+c}}{ac},\ \frac{2\,\left(\sqrt{2}\,c\sqrt{-\frac{1}{c}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-\frac{1}{c}}}{ax-1}\right)-\sqrt{-acx+c}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*sqrt(c)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x +
 1)) - 2*sqrt(-a*c*x + c))/(a*c), 2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(2)*sq
rt(-a*c*x + c)*sqrt(-1/c)/(a*x - 1)) - sqrt(-a*c*x + c))/(a*c)]
```

Sympy [A]
time = 9.97, size = 60, normalized size = 1.03

$$-\frac{2\sqrt{-acx+c}}{ac}-\frac{2\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}}{\sqrt{-\frac{1}{c}}\,\sqrt{-acx+c}}\right)}{ac\sqrt{-\frac{1}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*sqrt(-a*c*x + c)/(a*c) - 2*sqrt(2)*atan(sqrt(2)/(sqrt(-1/c)*sqrt(-a*c*x
+ c)))/(a*c*sqrt(-1/c))
```

Giac [A]
time = 0.40, size = 51, normalized size = 0.88

$$-\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\,\sqrt{-acx+c}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2*s
qrt(-a*c*x + c)/(a*c)
```

Mupad [B]
time = 0.07, size = 47, normalized size = 0.81

$$\frac{2\sqrt{2}\ \mathrm{atanh}\left(\frac{\sqrt{2}\ \sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a\sqrt{c}}-\frac{2\sqrt{c-a\,c\,x}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(1/2)*(a*x + 1)),x)
[Out] (2*2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(1/2)) - (2
*(c - a*c*x)^(1/2))/(a*c)
```

3.266 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6265, 21, 65, 212}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{3/2}}\,dx\\ &= -\int \frac{1-ax}{(1+ax)(c-acx)^{3/2}}\,dx\\ &= -\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{c}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^2}\\ &= \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{ac^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
[Out] (Sqrt[2]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Maple [A]
time = 0.18, size = 29, normalized size = 0.78

method	result	size

derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$	29
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}}{a\,c^{\frac{3}{2}}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)
```

Maxima [A]
time = 0.47, size = 52, normalized size = 1.41

$$-\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)}{2\,ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + s
qrt(-a*c*x + c)))/(a*c^(3/2))
```

Fricas [A]
time = 0.34, size = 88, normalized size = 2.38

$$\left[\frac{\sqrt{2}\,\log\left(\frac{ax-\frac{2\sqrt{2}\,\sqrt{-acx+c}}{\sqrt{c}}-3}{ax+1}\right)}{2\,ac^{\frac{3}{2}}},\ \frac{\sqrt{2}\,\sqrt{-\frac{1}{c}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-\frac{1}{c}}}{ax-1}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log((a*x - 2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(c) - 3)/(a*x + 1))/
(a*c^(3/2)), sqrt(2)*sqrt(-1/c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(-1/c)/
(a*x - 1))/(a*c)]
```

Sympy [A]
time = 8.76, size = 41, normalized size = 1.11

$$-\frac{\sqrt{2}\ \operatorname{atan}\left(\frac{\sqrt{2}\ \sqrt{-acx+c}}{2\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(3/2),x)
[Out] -sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(a*c*sqrt(-c))
```

Giac [A]
time = 0.40, size = 36, normalized size = 0.97

$$-\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}\ c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(3/2),x, algorithm="giac")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

Mupad [B]
time = 0.08, size = 28, normalized size = 0.76

$$\frac{\sqrt{2}\ \operatorname{atanh}\left(\frac{\sqrt{2}\ \sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)}{a\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(3/2)*(a*x + 1)),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(a*c^(3/2))
```

3.267 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{ac^2\sqrt{c-acx}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

```
[Out] 1/2*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-1/a/c^2
/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 53, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}-\frac{1}{ac^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] -(1/(a*c^2*Sqrt[c - a*c*x])) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(
Sqrt[2]*a*c^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{5/2}}\,dx\\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{5/2}}\,dx\\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{c}\\
&= -\frac{1}{ac^2\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{2c^2}\\
&= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right)}{ac^3}\\
&= -\frac{1}{ac^2\sqrt{c-acx}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 37, normalized size = 0.65

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{1}{2}(1-ax)\right)}{ac^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 - a*x)/2]/(a*c^2*Sqrt[c - a*c*x]))
```

Maple [A]
time = 0.19, size = 50, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}+\frac{1}{2c\sqrt{-acx+c}}\right)}{ca}$	50
default	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}+\frac{1}{2c\sqrt{-acx+c}}\right)}{ca}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/c/a*(-1/4/c^(3/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))+
1/2/c/(-a*c*x+c)^(1/2))
```

Maxima [A]
time = 0.46, size = 71, normalized size = 1.25

$$-\frac{\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}+\frac{4}{\sqrt{-acx+c}\,c}}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
[Out] -1/4*(sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) +
sqrt(-a*c*x + c)))/c^(3/2) + 4/(sqrt(-a*c*x + c)*c))/(a*c)
```

Fricas [A]
time = 0.39, size = 146, normalized size = 2.56

$$\left[\frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(\frac{acx-2\,\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3\,c}{ax+1}\right)+4\,\sqrt{-acx+c}}{4\,(a^2c^3x-ac^3)},-\frac{\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2\,c}\right)-2\,\sqrt{-acx+c}}{2\,(a^2c^3x-ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3), -1/2*(sqr
t(2)*(a*x - 1)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2
*sqrt(-a*c*x + c))/(a^2*c^3*x - a*c^3)]
```

Sympy [A]
time = 8.76, size = 61, normalized size = 1.07

$$-\frac{1}{ac^2\sqrt{-acx+c}}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{2ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(5/2),x)
```

```
[Out] -1/(a*c**2*sqrt(-a*c*x + c)) - sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqr
t(-c)))/(2*a*c**2*sqrt(-c))
```

Giac [A]
time = 0.42, size = 54, normalized size = 0.95

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\,\sqrt{-c}}\right)}{2\,a\sqrt{-c}\,c^2}-\frac{1}{\sqrt{-acx+c}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
 - 1/(sqrt(-a*c*x + c)*a*c^2)
```

Mupad [B]
time = 1.23, size = 47, normalized size = 0.82

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{2\,a\,c^{5/2}}-\frac{1}{a\,c^2\,\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(5/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(2*a*c^(5/2)) - 1/
(a*c^2*(c - a*c*x)^(1/2))
```

3.268 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

```
[Out] -1/3/a/c^2/(-a*c*x+c)^(3/2)+1/4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2
))/a/c^(7/2)*2^(1/2)-1/2/a/c^3/(-a*c*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {6302, 6265, 21, 53, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{1}{3ac^2(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/3*1/(a*c^2*(c - a*c*x)^(3/2)) - 1/(2*a*c^3*Sqrt[c - a*c*x]) + ArcTanh[Sq
rt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{7/2}}\,dx\\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{7/2}}\,dx\\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{c}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{2c^2}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}-\frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{4c^3}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{2ac^4}\\
&= -\frac{1}{3ac^2(c-acx)^{3/2}}-\frac{1}{2ac^3\sqrt{c-acx}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 39, normalized size = 0.47

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{1}{2}(1-ax)\right)}{3ac^2(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(3/2)
)
```

Maple [A]
time = 0.19, size = 64, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{2\left(\frac{1}{4c^2\sqrt{-acx+c}}+\frac{1}{6c(-acx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}\right)}{ca}$	64
default	$-\frac{2\left(\frac{1}{4c^2\sqrt{-acx+c}}+\frac{1}{6c(-acx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}\right)}{ca}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/4/c^2/(-a*c*x+c)^(1/2)+1/6/c/(-a*c*x+c)^(3/2)-1/8/c^(5/2)*2^(1/2)
*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.47, size = 81, normalized size = 0.98

$$-\frac{\frac{3\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}-\frac{4\,(3\,acx-5\,c)}{(-acx+c)^{\frac{3}{2}}c^2}}{24\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c)
 + sqrt(-a*c*x + c)))/c^(5/2) - 4*(3*a*c*x - 5*c)/((-a*c*x + c)^(3/2)*c^2))
/(a*c)
```

Fricas [A]
time = 0.37, size = 196, normalized size = 2.36

$$\left[\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\,\sqrt{c}-3\,c}{ax+1}\right)+4\sqrt{-acx+c}\,(3\,ax-5)}{24\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)},-\frac{3\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\,\sqrt{-c}}{2\,c}\right)-2\sqrt{-acx+c}\,(3\,ax-5)}{12\left(a^3c^4x^2-2\,a^2c^4x+ac^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 5))/(a^
3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*sqrt(-a*c*x + c)
*(3*a*x - 5))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [A]
time = 16.03, size = 82, normalized size = 0.99

$$-\frac{1}{3ac^2\left(-acx+c\right)^{\frac{3}{2}}}-\frac{1}{2ac^3\sqrt{-acx+c}}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4ac^3\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(7/2),x)
```

```
[Out] -1/(3*a*c**2*(-a*c*x + c)**(3/2)) - 1/(2*a*c**3*sqrt(-a*c*x + c)) - sqrt(2)
*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/(4*a*c**3*sqrt(-c))
```

Giac [A]
time = 0.41, size = 73, normalized size = 0.88

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{4\,a\sqrt{-c}\,c^3}-\frac{3\,acx-5\,c}{6\left(acx-c\right)\sqrt{-acx+c}\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^3)
 - 1/6*(3*a*c*x - 5*c)/((a*c*x - c)*sqrt(-a*c*x + c)*a*c^3)
```

Mupad [B]
time = 0.10, size = 65, normalized size = 0.78

$$\frac{\sqrt{2}\ \mathrm{atanh}\left(\frac{\sqrt{2}\ \sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{4\,a\,c^{7/2}}-\frac{\frac{c-a\,c\,x}{2\,c^2}+\frac{1}{3\,c}}{a\,c\,{\left(c-a\,c\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(7/2)*(a*x + 1)),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(4*a*c^(7/2)) - ((
c - a*c*x)/(2*c^2) + 1/(3*c))/(a*c*(c - a*c*x)^(3/2))
```

3.269 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{5ac^2(c-acx)^{5/2}}-\frac{1}{6ac^3(c-acx)^{3/2}}-\frac{1}{4ac^4\sqrt{c-acx}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}$$

```
[Out] -1/5/a/c^2/(-a*c*x+c)^(5/2)-1/6/a/c^3/(-a*c*x+c)^(3/2)+1/8*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-1/4/a/c^4/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6302, 6265, 21, 53, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}-\frac{1}{4ac^4\sqrt{c-acx}}-\frac{1}{6ac^3(c-acx)^{3/2}}-\frac{1}{5ac^2(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -1/5*1/(a*c^2*(c - a*c*x)^(5/2)) - 1/(6*a*c^3*(c - a*c*x)^(3/2)) - 1/(4*a*c
^4*Sqrt[c - a*c*x]) + ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]/(4*Sqrt[2]
*a*c^(9/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-acx)^{9/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-acx)^{9/2}}\,dx \\
&= -\int \frac{1-ax}{(1+ax)(c-acx)^{9/2}}\,dx \\
&= -\frac{\int \frac{1}{(1+ax)(c-acx)^{7/2}}\,dx}{c} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{5/2}}\,dx}{2c^2} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{\int \frac{1}{(1+ax)(c-acx)^{3/2}}\,dx}{4c^3} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} - \frac{\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{8c^4} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-\right.}{4ac^5} \\
&= -\frac{1}{5ac^2(c-acx)^{5/2}} - \frac{1}{6ac^3(c-acx)^{3/2}} - \frac{1}{4ac^4\sqrt{c-acx}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 39, normalized size = 0.38

$$
-\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{1}{2}(1-ax)\right)}{5ac^2(c-acx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a*c*x)^(9/2)),x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 - a*x)/2]/(a*c^2*(c - a*c*x)^(5/2)
)
```

Maple [A]
time = 0.19, size = 78, normalized size = 0.75

method	result	size

derivativedivides	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}+\frac{1}{8c^{3}\sqrt{-acx+c}}+\frac{1}{12c^{2}(-acx+c)^{\frac{3}{2}}}+\frac{1}{10c(-acx+c)^{\frac{5}{2}}}\right)}{ca}$	78
default	$-\frac{2\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}+\frac{1}{8c^{3}\sqrt{-acx+c}}+\frac{1}{12c^{2}(-acx+c)^{\frac{3}{2}}}+\frac{1}{10c(-acx+c)^{\frac{5}{2}}}\right)}{ca}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(-1/16/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
+1/8/c^3/(-a*c*x+c)^(1/2)+1/12/c^2/(-a*c*x+c)^(3/2)+1/10/c/(-a*c*x+c)^(5/2)
)
```

Maxima [A]
time = 0.47, size = 101, normalized size = 0.97

$$-\frac{\frac{15\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{7}{2}}}+\frac{4\left(15\,(acx-c)^{2}-10\,(acx-c)c+12\,c^{2}\right)}{(-acx+c)^{\frac{5}{2}}c^{3}}}{240\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/240*(15*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(
c) + sqrt(-a*c*x + c)))/c^(7/2) + 4*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c +
12*c^2)/((-a*c*x + c)^(5/2)*c^3))/(a*c)
```

Fricas [A]
time = 0.34, size = 252, normalized size = 2.42

$$\left[\frac{15\sqrt{2}\,(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{c}\,\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3\,c}{ax+1}\right)+4\,(15\,a^{2}x^{2}-40\,ax+37)\sqrt{-acx+c}}{240\,(a^{4}c^{5}x^{3}-3\,a^{3}c^{5}x^{2}+3\,a^{2}c^{5}x-ac^{5})},-\frac{15\sqrt{2}\,(a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2\,c}\right)-2\,(15\,a^{2}x^{2}-40\,ax+37)\sqrt{-acx+c}}{120\,(a^{4}c^{5}x^{3}-3\,a^{3}c^{5}x^{2}+3\,a^{2}c^{5}x-ac^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log((a*c*x - 2
*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(a*x + 1)) + 4*(15*a^2*x^2 - 40*a*
x + 37)*sqrt(-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^
5), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/
```

```
2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 2*(15*a^2*x^2 - 40*a*x + 37)*sqrt(
-a*c*x + c))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [A]
time = 12.74, size = 100, normalized size = 0.96

$$-\frac{1}{5ac^2\left(-acx+c\right)^{\frac{5}{2}}}-\frac{1}{6ac^3\left(-acx+c\right)^{\frac{3}{2}}}-\frac{1}{4ac^4\sqrt{-acx+c}}-\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8ac^4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)**(9/2),x)
```

```
[Out] -1/(5*a*c**2*(-a*c*x + c)**(5/2)) - 1/(6*a*c**3*(-a*c*x + c)**(3/2)) - 1/(4
*a*c**4*sqrt(-a*c*x + c)) - sqrt(2)*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-
c)))/(8*a*c**4*sqrt(-c))
```

Giac [A]
time = 0.41, size = 93, normalized size = 0.89

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{8\,a\sqrt{-c}\,c^4}-\frac{15\,(acx-c)^2-10\,(acx-c)c+12\,c^2}{60\,(acx-c)^2\sqrt{-acx+c}\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a*c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)*c^4)
 - 1/60*(15*(a*c*x - c)^2 - 10*(a*c*x - c)*c + 12*c^2)/((a*c*x - c)^2*sqrt(
-a*c*x + c)*a*c^4)
```

Mupad [B]
time = 0.09, size = 79, normalized size = 0.76

$$\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{8\,a\,c^{9/2}}-\frac{\frac{c-a\,c\,x}{6\,c^2}+\frac{1}{5\,c}+\frac{(c-a\,c\,x)^2}{4\,c^3}}{a\,c\,(c-a\,c\,x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a*c*x)^(9/2)*(a*x + 1)),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/(8*a*c^(9/2)) - ((
c - a*c*x)/(6*c^2) + 1/(5*c) + (c - a*c*x)^2/(4*c^3))/(a*c*(c - a*c*x)^(5/2
))
```

3.270 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx$

Optimal. Leaf size=368

$$-\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{94208(c-acx)^{9/2}}{231a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^5}-\frac{40960(c-acx)^{9/2}}{231a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^4}+\frac{4096}{231a^4\left(1-\right.}$$

```
[Out] -16/33*(a-1/x)^5*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)-94208
/231*(-a*c*x+c)^(9/2)/a^6/(1-1/a/x)^(9/2)/x^5/(1+1/a/x)^(1/2)-40960/231*(-a
*c*x+c)^(9/2)/a^5/(1-1/a/x)^(9/2)/x^4/(1+1/a/x)^(1/2)+4096/231*(-a*c*x+c)^(
9/2)/a^4/(1-1/a/x)^(9/2)/x^3/(1+1/a/x)^(1/2)-1024/231*(a-1/x)^3*(-a*c*x+c)^
(9/2)/a^6/(1-1/a/x)^(9/2)/x^2/(1+1/a/x)^(1/2)+320/231*(a-1/x)^4*(-a*c*x+c)^
(9/2)/a^6/(1-1/a/x)^(9/2)/x/(1+1/a/x)^(1/2)+2/11*(a-1/x)^6*x*(-a*c*x+c)^(9/
2)/a^6/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$-\frac{94208(c-acx)^{9/2}}{231a^6x^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{1024\left(a-\frac{1}{x}\right)^3(c-acx)^{9/2}}{231a^6x^2\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^6(c-acx)^{9/2}}{11a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{16\left(a-\frac{1}{x}\right)^5(c-acx)^{9/2}}{33a^6\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{320\left(a-\frac{1}{x}\right)^4(c-acx)^{9/2}}{231a^6x\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}-\frac{40960(c-acx)^{9/2}}{231a^5x^4\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{4096(c-acx)^{9/2}}{231a^4x^3\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-16*(a - x^(-1))^5*(c - a*c*x)^(9/2))/(33*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 +
 1/(a*x)]) - (94208*(c - a*c*x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1
+ 1/(a*x)]*x^5) - (40960*(c - a*c*x)^(9/2))/(231*a^5*(1 - 1/(a*x))^(9/2)*Sq
rt[1 + 1/(a*x)]*x^4) + (4096*(c - a*c*x)^(9/2))/(231*a^4*(1 - 1/(a*x))^(9/2
)*Sqrt[1 + 1/(a*x)]*x^3) - (1024*(a - x^(-1))^3*(c - a*c*x)^(9/2))/(231*a^6
*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x^2) + (320*(a - x^(-1))^4*(c - a*c*
x)^(9/2))/(231*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1
))^6*x*(c - a*c*x)^(9/2))/(11*a^6*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{9/2}\,dx=\frac{(c-acx)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}x^{9/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{6}}{x^{13/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^{6}x(c-acx)^{9/2}}{11a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(24\left(\frac{1}{x}\right)^{9/2}(c-acx)^{9/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5}}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/}}\right.}{11a\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^{5}(c-acx)^{9/2}}{33a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{2\left(a-\frac{1}{x}\right)^{6}x(c-acx)^{9/2}}{11a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(160\left(\frac{1}{x}\right)^{9/2}\right.}{}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^{5}(c-acx)^{9/2}}{33a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{320\left(a-\frac{1}{x}\right)^{4}(c-acx)^{9/2}}{231a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right.}{11a^{6}\left(1-\right.}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^{5}(c-acx)^{9/2}}{33a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{1024\left(a-\frac{1}{x}\right)^{3}(c-acx)^{9/2}}{231a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^{2}}+\frac{320(a}{231a^{6}(1}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^{5}(c-acx)^{9/2}}{33a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{4096(c-acx)^{9/2}}{231a^{4}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^{3}}-\frac{1024(a}{231a^{6}(1}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^{5}(c-acx)^{9/2}}{33a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{40960(c-acx)^{9/2}}{231a^{5}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^{4}}+\frac{40}{231a^{4}(1}$$

$$=-\frac{16\left(a-\frac{1}{x}\right)^{5}(c-acx)^{9/2}}{33a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{94208(c-acx)^{9/2}}{231a^{6}\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}\,x^{5}}-\frac{40}{231a^{5}(1}$$

Mathematica [A]

time = 0.04, size = 84, normalized size = 0.23

$$\frac{2c^4\sqrt{c-acx}\,(-46355-23062ax+5419a^2x^2-2132a^3x^3+755a^4x^4-182a^5x^5+21a^6x^6)}{231a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*c^4*Sqrt[c - a*c*x]*(-46355 - 23062*a*x + 5419*a^2*x^2 - 2132*a^3*x^3 +
755*a^4*x^4 - 182*a^5*x^5 + 21*a^6*x^6))/(231*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 92, normalized size = 0.25

method	result	size
gosper	$\frac{2(ax+1)(21a^6x^6-182a^5x^5+755a^4x^4-2132a^3x^3+5419a^2x^2-23062ax-46355)(-acx+c)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{231a(ax-1)^6}$	88
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^4(21a^6x^6-182a^5x^5+755a^4x^4-2132a^3x^3+5419a^2x^2-23062ax-46355)}{231(ax-1)^2a}$	92
risch	$-\frac{2(21a^5x^5-203a^4x^4+958a^3x^3-3090a^2x^2+8509ax-31571)(ax+1)c^5\sqrt{\frac{ax-1}{ax+1}}}{231a\sqrt{-c(ax-1)}}+\frac{128c^5\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*c^4*(21*
a^6*x^6-182*a^5*x^5+755*a^4*x^4-2132*a^3*x^3+5419*a^2*x^2-23062*a*x-46355)/
a
```

Maxima [A]
time = 0.28, size = 152, normalized size = 0.41

$$\frac{2\,(21\,a^7\sqrt{-c}\,c^4x^7-161\,a^6\sqrt{-c}\,c^4x^6+573\,a^5\sqrt{-c}\,c^4x^5-1377\,a^4\sqrt{-c}\,c^4x^4+3287\,a^3\sqrt{-c}\,c^4x^3-17643\,a^2\sqrt{-c}\,c^4x^2-69417\,a\sqrt{-c}\,c^4x-46355\,\sqrt{-c}\,c^4)(ax-1)^2}{231\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*a^7*sqrt(-c)*c^4*x^7 - 161*a^6*sqrt(-c)*c^4*x^6 + 573*a^5*sqrt(-c
)*c^4*x^5 - 1377*a^4*sqrt(-c)*c^4*x^4 + 3287*a^3*sqrt(-c)*c^4*x^3 - 17643*a
^2*sqrt(-c)*c^4*x^2 - 69417*a*sqrt(-c)*c^4*x - 46355*sqrt(-c)*c^4)*(a*x - 1
)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A]
time = 0.35, size = 105, normalized size = 0.29

$$\frac{2\left(21\,a^{6}c^{4}x^{6}-182\,a^{5}c^{4}x^{5}+755\,a^{4}c^{4}x^{4}-2132\,a^{3}c^{4}x^{3}+5419\,a^{2}c^{4}x^{2}-23062\,ac^{4}x-46355\,c^{4}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{231\left(a^{2}x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*a^6*c^4*x^6 - 182*a^5*c^4*x^5 + 755*a^4*c^4*x^4 - 2132*a^3*c^4*x^
3 + 5419*a^2*c^4*x^2 - 23062*a*c^4*x - 46355*c^4)*sqrt(-a*c*x + c)*sqrt((a*
x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.45, size = 110, normalized size = 0.30

$$\frac{2\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(21\,a^5\,x^5-161\,a^4\,x^4+594\,a^3\,x^3-1538\,a^2\,x^2+3881\,a\,x-19181\right)}{231\,a}-\frac{131072\,c^4\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{231\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^4*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3881*a*x - 1538*a^2*x
^2 + 594*a^3*x^3 - 161*a^4*x^4 + 21*a^5*x^5 - 19181))/(231*a) - (131072*c^4
*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(231*a*(a*x - 1))
```

3.271 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx$

Optimal. Leaf size=311

$$-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{11776(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^4}+\frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{512(c\cdots}{63a^3\left(1-\frac{1}{a\cdots}\right.}$$

```
[Out] -40/63*(a-1/x)^4*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+11776
/63*(-a*c*x+c)^(7/2)/a^5/(1-1/a/x)^(7/2)/x^4/(1+1/a/x)^(1/2)+5120/63*(-a*c*
x+c)^(7/2)/a^4/(1-1/a/x)^(7/2)/x^3/(1+1/a/x)^(1/2)-512/63*(-a*c*x+c)^(7/2)/
a^3/(1-1/a/x)^(7/2)/x^2/(1+1/a/x)^(1/2)+128/63*(a-1/x)^3*(-a*c*x+c)^(7/2)/a
^5/(1-1/a/x)^(7/2)/x/(1+1/a/x)^(1/2)+2/9*(a-1/x)^5*x*(-a*c*x+c)^(7/2)/a^5/(
1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\frac{11776(c-acx)^{7/2}}{63a^5x^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^5(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}-\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5x\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{5120(c-acx)^{7/2}}{63a^4x^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}-\frac{512(c-acx)^{7/2}}{63a^3x^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-40*(a - x^(-1))^4*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]) + (11776*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 +
 1/(a*x)]*x^4) + (5120*(c - a*c*x)^(7/2))/(63*a^4*(1 - 1/(a*x))^(7/2)*Sqrt[
1 + 1/(a*x)]*x^3) - (512*(c - a*c*x)^(7/2))/(63*a^3*(1 - 1/(a*x))^(7/2)*Sqr
t[1 + 1/(a*x)]*x^2) + (128*(a - x^(-1))^3*(c - a*c*x)^(7/2))/(63*a^5*(1 - 1
/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x^(-1))^5*x*(c - a*c*x)^(7/2))
/(9*a^5*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{7/2}\,dx = \frac{(c-acx)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^5}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^5 x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(20\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\right.}{9a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^5 x(c-acx)^{7/2}}{9a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(320\left(\frac{1}{x}\right)^{7/2}\right.}{}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{128\left(a-\frac{1}{x}\right)^3(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2\left(a-\frac{1}{x}\right.}{9a^5\left(1-\right.}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{512(c-acx)^{7/2}}{63a^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^2} + \frac{128\left(a\right.}{63a^5\left(1-\right.}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{5120(c-acx)^{7/2}}{63a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{51}{63a^3\left(1-\right.}$$

$$= -\frac{40\left(a-\frac{1}{x}\right)^4(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{11776(c-acx)^{7/2}}{63a^5\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x^4} + \frac{51}{63a^4\left(1-\right.}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.24

$$-\frac{2c^3\sqrt{c-acx}\,(5797+2867ax-638a^2x^2+214a^3x^3-55a^4x^4+7a^5x^5)}{63a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*c^3*Sqrt[c - a*c*x]*(5797 + 2867*a*x - 638*a^2*x^2 + 214*a^3*x^3 - 55*a
^4*x^4 + 7*a^5*x^5))/(63*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 84, normalized size = 0.27

method	result	size
gosper	$\frac{2(ax+1)\left(7a^5x^5-55a^4x^4+214a^3x^3-638a^2x^2+2867ax+5797\right)(-acx+c)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{63a(ax-1)^5}$	80
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,c^3\left(7a^5x^5-55a^4x^4+214a^3x^3-638a^2x^2+2867ax+5797\right)}{63(ax-1)^2a}$	84
risch	$\frac{2\left(7a^4x^4-62a^3x^3+276a^2x^2-914ax+3781\right)(ax+1)c^4\sqrt{\frac{ax-1}{ax+1}}}{63a\sqrt{-c(ax-1)}}+\frac{64c^4\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/63*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*c^3*(7*a
^5*x^5-55*a^4*x^4+214*a^3*x^3-638*a^2*x^2+2867*a*x+5797)/a
```

Maxima [A]
time = 0.29, size = 136, normalized size = 0.44

$$-\frac{2\left(7\,a^6\sqrt{-c}\,c^3x^6-48\,a^5\sqrt{-c}\,c^3x^5+159\,a^4\sqrt{-c}\,c^3x^4-424\,a^3\sqrt{-c}\,c^3x^3+2229\,a^2\sqrt{-c}\,c^3x^2+8664\,a\sqrt{-c}\,c^3x+5797\sqrt{-c}\,c^3\right)(ax-1)^2}{63\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(7*a^6*sqrt(-c)*c^3*x^6 - 48*a^5*sqrt(-c)*c^3*x^5 + 159*a^4*sqrt(-c)*
c^3*x^4 - 424*a^3*sqrt(-c)*c^3*x^3 + 2229*a^2*sqrt(-c)*c^3*x^2 + 8664*a*sqr
t(-c)*c^3*x + 5797*sqrt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x
+ 1)^(3/2))
```

Fricas [A]
time = 0.36, size = 94, normalized size = 0.30

$$-\frac{2\left(7\,a^5c^3x^5-55\,a^4c^3x^4+214\,a^3c^3x^3-638\,a^2c^3x^2+2867\,ac^3x+5797\,c^3\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(7*a^5*c^3*x^5 - 55*a^4*c^3*x^4 + 214*a^3*c^3*x^3 - 638*a^2*c^3*x^2 +
 2867*a*c^3*x + 5797*c^3)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.41, size = 102, normalized size = 0.33

$$-\frac{2\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(7\,a^4\,x^4-48\,a^3\,x^3+166\,a^2\,x^2-472\,a\,x+2395\right)}{63\,a}-\frac{16384\,c^3\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{63\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(166*a^2*x^2 - 472*a
*x - 48*a^3*x^3 + 7*a^4*x^4 + 2395))/(63*a) - (16384*c^3*(c - a*c*x)^(1/2)*
((a*x - 1)/(a*x + 1))^(1/2))/(63*a*(a*x - 1))
```

3.272 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=254

$$-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{2944(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2}+\frac{128(c-}{35a^2\left(1-\frac{1}{ax}\right)^5}$$

```
[Out] -32/35*(a-1/x)^3*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-2944/
35*(-a*c*x+c)^(5/2)/a^4/(1-1/a/x)^(5/2)/x^3/(1+1/a/x)^(1/2)-256/7*(-a*c*x+c
)^(5/2)/a^3/(1-1/a/x)^(5/2)/x^2/(1+1/a/x)^(1/2)+128/35*(-a*c*x+c)^(5/2)/a^2
/(1-1/a/x)^(5/2)/x/(1+1/a/x)^(1/2)+2/7*(a-1/x)^4*x*(-a*c*x+c)^(5/2)/a^4/(1-
1/a/x)^(5/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$-\frac{2944(c-acx)^{5/2}}{35a^4x^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^4(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{256(c-acx)^{5/2}}{7a^3x^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{128(c-acx)^{5/2}}{35a^2x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-32*(a - x^(-1))^3*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]) - (2944*(c - a*c*x)^(5/2))/(35*a^4*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
1/(a*x)]*x^3) - (256*(c - a*c*x)^(5/2))/(7*a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 +
 1/(a*x)]*x^2) + (128*(c - a*c*x)^(5/2))/(35*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1
 + 1/(a*x)]*x) + (2*(a - x^(-1))^4*x*(c - a*c*x)^(5/2))/(7*a^4*(1 - 1/(a*x)
)^(5/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{\left(16\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,d\right.}{7a\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{2\left(a-\frac{1}{x}\right)^4x(c-acx)^{5/2}}{7a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{\left(192\left(\frac{1}{x}\right)^{5/2}(c\right.}{}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{128(c-acx)^{5/2}}{35a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2\left(a-\frac{1}{x}\right)^4}{7a^4\left(1-\frac{1}{ax}\right.}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{256(c-acx)^{5/2}}{7a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^2} + \frac{128(}{35a^2\left(1-\frac{1}{a}\right.}$$

$$= -\frac{32\left(a-\frac{1}{x}\right)^3(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{2944(c-acx)^{5/2}}{35a^4\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{256}{7a^3\left(1-\frac{1}{a}\right.}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.27

$$\frac{2c^2\sqrt{c-acx}\,(-1451-708ax+142a^2x^2-36a^3x^3+5a^4x^4)}{35a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*c^2*Sqrt[c - a*c*x]*(-1451 - 708*a*x + 142*a^2*x^2 - 36*a^3*x^3 + 5*a^4*
x^4))/(35*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.30

method	result	size
gosper	$\frac{2(ax+1)\left(5a^{4}x^{4}-36a^{3}x^{3}+142a^{2}x^{2}-708ax-1451\right)\left(-acx+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{35a\left(ax-1\right)^{4}}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\left(ax-1\right)}\,c^{2}\left(5a^{4}x^{4}-36a^{3}x^{3}+142a^{2}x^{2}-708ax-1451\right)}{35\left(ax-1\right)^{2}a}$	76
risch	$-\frac{2\left(5a^{3}x^{3}-41a^{2}x^{2}+183ax-891\right)(ax+1)c^{3}\sqrt{\frac{ax-1}{ax+1}}}{35a\sqrt{-c\left(ax-1\right)}}+\frac{32c^{3}\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c\left(ax-1\right)}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*c^2*(5*a^
4*x^4-36*a^3*x^3+142*a^2*x^2-708*a*x-1451)/a
```

Maxima [A]
time = 0.28, size = 120, normalized size = 0.47

$$\frac{2\left(5\,a^{5}\sqrt{-c}\,c^{2}x^{5}-31\,a^{4}\sqrt{-c}\,c^{2}x^{4}+106\,a^{3}\sqrt{-c}\,c^{2}x^{3}-566\,a^{2}\sqrt{-c}\,c^{2}x^{2}-2159\,a\sqrt{-c}\,c^{2}x-1451\,\sqrt{-c}\,c^{2}\right)(ax-1)^{2}}{35\,(a^{3}x^{2}-2\,a^{2}x+a)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*a^5*sqrt(-c)*c^2*x^5 - 31*a^4*sqrt(-c)*c^2*x^4 + 106*a^3*sqrt(-c)*c
^2*x^3 - 566*a^2*sqrt(-c)*c^2*x^2 - 2159*a*sqrt(-c)*c^2*x - 1451*sqrt(-c)*c
^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.33

$$\frac{2\left(5\,a^{4}c^{2}x^{4}-36\,a^{3}c^{2}x^{3}+142\,a^{2}c^{2}x^{2}-708\,ac^{2}x-1451\,c^{2}\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^{2}x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^4*c^2*x^4 - 36*a^3*c^2*x^3 + 142*a^2*c^2*x^2 - 708*a*c^2*x - 1451
*c^2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.38, size = 94, normalized size = 0.37

$$\frac{2\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^3\,x^3-31\,a^2\,x^2+111\,a\,x-597\right)}{35\,a}-\frac{4096\,c^2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{35\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(111*a*x - 31*a^2*x^2
+ 5*a^3*x^3 - 597))/(35*a) - (4096*c^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x +
1))^(1/2))/(35*a*(a*x - 1))
```

3.273 $\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=195

$$-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{184(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2}+\frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right)^3x(c-a\cdots}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1\cdots}}$$

```
[Out] -8/5*(-a*c*x+c)^(3/2)/a/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+184/5*(-a*c*x+c)^(3
/2)/a^3/(1-1/a/x)^(3/2)/x^2/(1+1/a/x)^(1/2)+16*(-a*c*x+c)^(3/2)/a^2/(1-1/a/
x)^(3/2)/x/(1+1/a/x)^(1/2)+2/5*(a-1/x)^3*x*(-a*c*x+c)^(3/2)/a^3/(1-1/a/x)^(
3/2)/(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6311, 6316, 96, 91, 79, 37}

$$\frac{184(c-acx)^{3/2}}{5a^3x^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{2x\left(a-\frac{1}{x}\right)^3(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{16(c-acx)^{3/2}}{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-8*(c - a*c*x)^(3/2))/(5*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + (184*(
c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x^2) + (16*(
c - a*c*x)^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x) + (2*(a - x
^(-1))^3*x*(c - a*c*x)^(3/2))/(5*a^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}(c-acx)^{3/2}\,dx=\frac{(c-acx)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$=-\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(12\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\right.}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{2\left(a-\frac{1}{x}\right)^3x(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(8\left(\frac{1}{x}\right)^{3/2}(c-\right.}{}$$

$$=-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{16(c-acx)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2\left(a-\frac{1}{x}\right)^3x(}{5a^3\left(1-\frac{1}{ax}\right)^{3/}}$$

$$=-\frac{8(c-acx)^{3/2}}{5a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{184(c-acx)^{3/2}}{5a^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2}+\frac{16(c-}{a^2\left(1-\frac{1}{ax}\right)^3}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.29

$$-\frac{2c\sqrt{c-acx}\left(91+43ax-7a^2x^2+a^3x^3\right)}{5a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a*c*x)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*c*Sqrt[c - a*c*x]*(91 + 43*a*x - 7*a^2*x^2 + a^3*x^3))/(5*a^2*Sqrt[1 -
1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.33

method	result	size

gosper	$\frac{2(ax+1)(a^3x^3-7a^2x^2+43ax+91)(-acx+c)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{5(ax-1)^3a}$	63
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,c(a^3x^3-7a^2x^2+43ax+91)}{5(ax-1)^2a}$	65
risch	$\frac{2(a^2x^2-8ax+51)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{5a\sqrt{-c\,(ax-1)}}+\frac{16c^2\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c\,(ax-1)}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*c*(a^3*x^
3-7*a^2*x^2+43*a*x+91)/a
```

Maxima [A]
time = 0.27, size = 93, normalized size = 0.48

$$-\frac{2\left(a^4\sqrt{-c}\,cx^4-6\,a^3\sqrt{-c}\,cx^3+36\,a^2\sqrt{-c}\,cx^2+134\,a\sqrt{-c}\,cx+91\,\sqrt{-c}\,c\right)(ax-1)^2}{5\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(a^4*sqrt(-c)*c*x^4 - 6*a^3*sqrt(-c)*c*x^3 + 36*a^2*sqrt(-c)*c*x^2 + 1
34*a*sqrt(-c)*c*x + 91*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1)^(3/2))
```

Fricas [A]
time = 0.33, size = 63, normalized size = 0.32

$$-\frac{2\,(a^3cx^3-7\,a^2cx^2+43\,acx+91\,c)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^2x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(a^3*c*x^3 - 7*a^2*c*x^2 + 43*a*c*x + 91*c)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1))/(a^2*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.36, size = 81, normalized size = 0.42

$$-\frac{2\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^2\,x^2-6\,a\,x+37\right)}{5\,a}-\frac{256\,c\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{5\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (2*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(a^2*x^2 - 6*a*x + 37)
)/(5*a) - (256*c*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(5*a*(a*x -
 1))
```

3.274 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=137

$$-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(1
/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6311, 6316, 91, 79, 37}

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{3a}$$

$$= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.35

$$\frac{2\sqrt{c-acx}\left(-23-10ax+a^2x^2\right)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

Maple [A]
time = 0.10, size = 56, normalized size = 0.41

method	result	size

gosper	$\frac{2(ax+1)(a^2x^2-10ax-23)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a(ax-1)^2}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,(a^2x^2-10ax-23)}{3(ax-1)^2a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c\,(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c\,(ax-1)}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(a^2*x^2-1
0*a*x-23)/a
```

Maxima [A]
time = 0.29, size = 75, normalized size = 0.55

$$\frac{2\left(a^3\sqrt{-c}\,x^3-9\,a^2\sqrt{-c}\,x^2-33\,a\sqrt{-c}\,x-23\,\sqrt{-c}\,\right)(ax-1)^2}{3\,(a^3x^2-2\,a^2x+a)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A]
time = 0.33, size = 50, normalized size = 0.36

$$\frac{2\,(a^2x^2-10\,ax-23)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2x-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.30, size = 71, normalized size = 0.52

$$\frac{2\sqrt{c-a\,c\,x}\,\left(a\,x-9\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{64\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.275 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx$

Optimal. Leaf size=85

$$\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}+\frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}$$

```
[Out] 6*(1-1/a/x)^(1/2)/a/(1+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)+2*x*(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6311, 6316, 79, 37}

$$\frac{2x\sqrt{1-\frac{1}{ax}}}{\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}+\frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]) + (2*Sqrt[1 - 1
/(a*x)]*x)/(Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx &= \frac{\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{x}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\,\sqrt{x}}\,dx}{\sqrt{c-acx}}\\
&= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}\\
&= \frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}} + \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{\frac{1}{x}}\,\sqrt{c-acx}}\\
&= \frac{6\sqrt{1-\frac{1}{ax}}}{a\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}} + \frac{2\sqrt{1-\frac{1}{ax}}\,x}{\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.56

$$\frac{2\sqrt{1-\frac{1}{ax}}\,(3+ax)}{a\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x]),x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)]*(3 + a*x))/(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.10, size = 51, normalized size = 0.60

method	result	size
gosper	$\frac{2(ax+1)(ax+3)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(ax-1)\sqrt{-acx+c}}$	47
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,(ax+3)}{(ax-1)^{2}ca}$	51
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{\sqrt{-c\,(ax-1)}\,a}+\frac{4\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c\,(ax-1)}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(a*x+3)/c/a
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.56

$$\frac{2\,(a^2x^2+4\,ax+3)(ax-1)}{\left(a^2\sqrt{-c}\,x-a\sqrt{-c}\,\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(a^2*x^2 + 4*a*x + 3)*(a*x - 1)/((a^2*sqrt(-c)*x - a*sqrt(-c))*(a*x + 1)^
(3/2))
```

Fricas [A]
time = 0.33, size = 44, normalized size = 0.52

$$-\frac{2\,\sqrt{-acx+c}\,(ax+3)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-a*c*x + c)*(a*x + 3)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(1/2),x)
[Out] Timed out
```

Giac [A]
time = 0.40, size = 42, normalized size = 0.49

$$2\left(\frac{\sqrt{-acx-c}}{ac^2}-\frac{2}{\sqrt{-acx-c}\,ac}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2*(sqrt(-a*c*x - c)/(a*c^2) - 2/(sqrt(-a*c*x - c)*a*c))*abs(c)
```

Mupad [B]
time = 1.40, size = 34, normalized size = 0.40

$$\frac{\left(2\,x+\frac{6}{a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(1/2),x)
[Out] ((2*x + 6/a)*((a*x - 1)/(a*x + 1))^(1/2))/(c - a*c*x)^(1/2)
```

3.276 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

```
[Out] -2*(a*x+1)/a*((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6309}

$$-\frac{2(ax+1)e^{-3\coth^{-1}(ax)}}{a(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 + a*x))/(a*E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2))
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = -\frac{2e^{-3\coth^{-1}(ax)}(1+ax)}{a(c-acx)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.41

$$-\frac{2\left(1-\frac{1}{ax}\right)^{3/2}x}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(3/2)),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(3/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(3/2))
```

Maple [A]
time = 0.09, size = 46, normalized size = 1.59

method	result	size
gosper	$-\frac{2(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{a(-acx+c)^{\frac{3}{2}}}$	35
default	$-\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}}{(ax-1)^{2}c^{2}a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)/c^2/a
```

Maxima [A]
time = 0.28, size = 45, normalized size = 1.55

$$-\frac{2\left(a\sqrt{-c}\,x+\sqrt{-c}\right)(ax-1)}{(a^{2}c^{2}x-ac^{2})(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="maxima")
[Out] -2*(a*sqrt(-c)*x + sqrt(-c))*(a*x - 1)/((a^2*c^2*x - a*c^2)*(a*x + 1)^(3/2)
)
```

Fricas [A]
time = 0.36, size = 43, normalized size = 1.48

$$-\frac{2\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{2}x-ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2*x - a*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 41, normalized size = 1.41

$$\frac{\left(\frac{\sqrt{2}}{a\sqrt{-c}} - \frac{2}{\sqrt{-acx-c}\,a}\right)|c|\mathrm{sgn}(ax+1)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2)/(a*sqrt(-c)) - 2/(sqrt(-a*c*x - c)*a))*abs(c)*sgn(a*x + 1)/c^2
```

Mupad [B]
time = 1.35, size = 32, normalized size = 1.10

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c\,\sqrt{c-a\,c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(3/2),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c*(c - a*c*x)^(1/2))
```

3.277 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

```
[Out] -1/2*a^(3/2)*(1-1/a/x)^(5/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^
(1/2))/(1/x)^(5/2)/(-a*c*x+c)^(5/2)*2^(1/2)+a*(1-1/a/x)^(5/2)*x^2/(-a*c*x+c
)^(5/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{ax^2\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\,(c-acx)^{5/2}}-\frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] (a*(1 - 1/(a*x))^(5/2)*x^2)/(Sqrt[1 + 1/(a*x)]*(c - a*c*x)^(5/2)) - (a^(3/2
)*(1 - 1/(a*x))^(5/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])])/(Sqrt[2]*(x^(-1))^(5/2)*(c - a*c*x)^(5/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{2x^2}{a}}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}x^2}{\sqrt{1+\frac{1}{ax}}\,(c-acx)^{5/2}} - \frac{a^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{2}\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}}$$

Mathematica [A]
time = 0.05, size = 122, normalized size = 1.02

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}-\sqrt{2}\sqrt{a}\sqrt{1+\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{2ac^2\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)] - Sqrt[2]*Sqrt[a]*Sqrt[1 + 1/(a*x)]*ArcT
anh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(2*a*c^2*Sqrt[1 +
 1/(a*x)]*Sqrt[x^(-1)]*Sqrt[c - a*c*x])
```

Maple [A]
time = 0.16, size = 85, normalized size = 0.71

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{-c\,(ax+1)}+2\sqrt{c}\right)}{2(ax-1)^{2}c^{\frac{7}{2}}a}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(7/2)*(
arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^(1/2)+2
*c^(1/2))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(5/2), x)
```

Fricas [A]
time = 0.38, size = 235, normalized size = 1.96

$$\left[-\frac{\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^{2}cx^{2}+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+4\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{4\,(a^{2}c^{3}x-ac^{3})},\ \frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2\,(a^{2}c^{3}x-ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
- 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3*x -
a*c^3), 1/2*(sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*sqrt((a*x -
 1)/(a*x + 1)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{{\left(c-a\,c\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(5/2), x)
```

3.278 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}-\frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}}+\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

```
[Out] 3/8*a^(5/2)*(1-1/a/x)^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(
1/2))/(1/x)^(7/2)/(-a*c*x+c)^(7/2)*2^(1/2)-1/2*a^2*(1-1/a/x)^(7/2)*x^2/(a-1
/x)/(-a*c*x+c)^(7/2)/(1+1/a/x)^(1/2)-3/4*a^2*(1-1/a/x)^(7/2)*x^3/(-a*c*x+c)
^(7/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}-\frac{3a^2x^3\left(1-\frac{1}{ax}\right)^{7/2}}{4\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}-\frac{a^2x^2\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\,(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a*x))^(7/2)*x^2)/((a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - a*c
*x)^(7/2)) - (3*a^2*(1 - 1/(a*x))^(7/2)*x^3)/(4*Sqrt[1 + 1/(a*x)]*(c - a*c*
x)^(7/2)) + (3*a^(5/2)*(1 - 1/(a*x))^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(
Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(4*Sqrt[2]*(x^(-1))^(7/2)*(c - a*c*x)^(7/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\operatorname{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right.}{4\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Su}}{8\,(}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)\operatorname{Su}}{4\left(\frac{1}{x}\right.}$$

$$= -\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}x^2}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} - \frac{3a^2\left(1-\frac{1}{ax}\right)^{7/2}x^3}{4\sqrt{1+\frac{1}{ax}}\,(c-acx)^{7/2}} + \frac{3a^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}\tanh}{4\sqrt{2}\left(\frac{1}{x}\right)^{7/}}$$

Mathematica [A]
time = 0.10, size = 140, normalized size = 0.76

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2\sqrt{\frac{1}{x}}\,(-1+3ax)-3\sqrt{2}\,\sqrt{a}\,\sqrt{1+\frac{1}{ax}}\,(-1+ax)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)\right)}{8ac^3\sqrt{1+\frac{1}{ax}}\,\sqrt{\frac{1}{x}}\,(-1+ax)\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a*c*x)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*Sqrt[x^(-1)]*(-1 + 3*a*x) - 3*Sqrt[2]*Sqrt[a]*Sqrt[1
+ 1/(a*x)]*(-1 + a*x)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a
*x)])]))/(8*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*(-1 + a*x)*Sqrt[c - a*c*x]
)
```

Maple [A]
time = 0.16, size = 129, normalized size = 0.70

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(3\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,ax\sqrt{-c\,(ax+1)}-3\arctan\left(\frac{\sqrt{-c\,(}\right.\right.}{8(ax-1)^{3}c^{\frac{9}{2}}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3*(a*x+1)*(-c*(a*x-1))^(1/2)/c^(9/2)*(
3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*a*x*(-c*(a*x+1))^(
1/2)-3*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*(-c*(a*x+1))^
(1/2)+6*a*x*c^(1/2)-2*c^(1/2))/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a*c*x + c)^(7/2), x)
```

Fricas [A]
time = 0.35, size = 285, normalized size = 1.55

$$\left[-\frac{3\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+4\,\sqrt{-acx+c}\,(3\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{16\,(a^3c^4x^2-2\,a^2c^4x+ac^4)},\ \frac{3\,\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-2\,\sqrt{-acx+c}\,(3\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{8\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 4*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1
```

```
)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1/8*(3*sqrt(2)*(a^2*x^2
- 2*a*x + 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))/(a*c*x - c)) - 2*sqrt(-a*c*x + c)*(3*a*x - 1)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a*c*x+c)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**A**]
time = 0.43, size = 90, normalized size = 0.49

$$-\frac{\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-acx-c}}{2\sqrt{c}}\right)}{ac^{\frac{5}{2}}}-\frac{2\,(3\,acx-c)}{\left((-acx-c)^{\frac{3}{2}}+2\sqrt{-acx-c}\,c\right)ac^2}\right)|c|\mathrm{sgn}(ax+1)}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a*c*x+c)^(7/2),x, algorithm="giac")
[Out] -1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c))/(a*c^(5/2)) -
2*(3*a*c*x - c)/(((-a*c*x - c)^(3/2) + 2*sqrt(-a*c*x - c)*c)*a*c^2))*abs(c)
*sgn(a*x + 1)/c^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{{\left(c-a\,c\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a*c*x)^(7/2), x)
```

3.279 $\int e^{\coth^{-1}(x)}x(1+x)\,dx$

Optimal. Leaf size=99

$$\sqrt{1+\frac{1}{x}}\,\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3+\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\,\sqrt{\frac{-1+x}{x}}\right.$$

```
[Out] arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+1/3*(1+1/x)^(3/2)*x^2*((-1+x)/x)^(1
/2)+1/3*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6310, 6315, 98, 96, 94, 212}

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{1}{3}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\sqrt{\frac{1}{x}+1}\,\sqrt{\frac{x-1}{x}}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\,\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x),x]
```

```
[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x + ((1 + x^(-1))^(3/2)*Sqrt[(-1 + x)/x]*
x^2)/3 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + ArcTanh[Sqrt[1 + x^(
-1)]*Sqrt[(-1 + x)/x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}x(1+x)\,dx=\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x^2\,dx$$
$$=-\text{Subst}\left(\int\frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^4}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3-\frac{2}{3}\text{Subst}\left(\int\frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^3}\,dx,x,\frac{1}{x}\right)$$
$$=\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3-\text{Subst}\left(\int\frac{\sqrt{1+x}}{\sqrt{1-x}\,x^2}\right.$$
$$=\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3$$
$$=\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3$$
$$=\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{3}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.41

$$\frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x\left(5+3x+x^2\right)+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x),x]
[Out] (Sqrt[1 - x^(-2)]*x*(5 + 3*x + x^2))/3 + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [A]
time = 0.10, size = 67, normalized size = 0.68

method	result	size
trager	$\frac{(1+x)(x^2+3x+5)\sqrt{-\frac{1-x}{1+x}}}{3}+\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$	62
risch	$\frac{(x^2+3x+5)(-1+x)}{3\sqrt{\frac{-1+x}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	62

default	$\frac{(-1+x)\left(((1+x)(-1+x))^{\frac{3}{2}}+3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)+6\sqrt{x^2-1}\right)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-1+x)*(((1+x)*(-1+x))^(3/2)+3*x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2))+6*
(x^2-1)^(1/2))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [A]
time = 0.26, size = 110, normalized size = 1.11

$$-\frac{2\left(3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+9\sqrt{\frac{x-1}{x+1}}\right)}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="maxima")
```

```
[Out] -2/3*(3*((x - 1)/(x + 1))^(5/2) - 8*((x - 1)/(x + 1))^(3/2) + 9*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.36, size = 57, normalized size = 0.58

$$\frac{1}{3}\left(x^3+4x^2+8x+5\right)\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 4*x^2 + 8*x + 5)*sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1
)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(x+1)}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x),x)
```

```
[Out] Integral(x*(x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Giac [A]
time = 0.41, size = 59, normalized size = 0.60

$$\frac{1}{3}\sqrt{x^2-1}\left(x\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{3}{\operatorname{sgn}(x+1)}\right)+\frac{5}{\operatorname{sgn}(x+1)}\right)-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x^2 - 1)*(x*(x/sgn(x + 1) + 3/sgn(x + 1)) + 5/sgn(x + 1)) - log(ab
s(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B]
time = 1.23, size = 94, normalized size = 0.95

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{6\sqrt{\frac{x-1}{x+1}}-\frac{16\left(\frac{x-1}{x+1}\right)^{3/2}}{3}+2\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - (6*((x - 1)/(x + 1))^(1/2) - (16*((x - 1
)/(x + 1))^(3/2))/3 + 2*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*
(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.280 $\int e^{\coth^{-1}(x)}(1+x)\,dx$

Optimal. Leaf size=79

$$\frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2+\frac{3}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] 3/2*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+1/2*(1+1/x)^(3/2)*x^2*((-1+x)/x
)^(1/2)+3/2*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6310, 6315, 96, 94, 212}

$$\frac{1}{2}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{3}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\frac{3}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x),x]
```

```
[Out] (3*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + ((1 + x^(-1))^(3/2)*Sqrt[(-1 +
x)/x]*x^2)/2 + (3*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}(1+x)\,dx &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)x\,dx \\
&= -\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2 - \frac{3}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}\,x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2 - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+\cdots}}\right. \\
&= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2 + \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cdots\right. \\
&= \frac{3}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{1}{2}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2 + \frac{3}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{-\cdots}\right.
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 0.51

$$\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x(4+x)+\frac{3}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 + x),x]
```

[Out] (Sqrt[1 - x^(-2)]*x*(4 + x))/2 + (3*Log[(1 + Sqrt[1 - x^(-2)])*x])/2

Maple [A]
time = 0.07, size = 57, normalized size = 0.72

method	result	size
default	$\frac{(-1+x)\left(x\sqrt{x^2-1}+4\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{2\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	57
risch	$\frac{(x+4)(-1+x)}{2\sqrt{\frac{-1+x}{1+x}}}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{2\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	58
trager	$\frac{(1+x)(x+4)\sqrt{-\frac{1-x}{1+x}}}{2}-\frac{3\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x),x,method=_RETURNVERBOSE)

[Out] 1/2*(-1+x)*(x*(x^2-1)^(1/2)+4*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)

Maxima [A]
time = 0.25, size = 87, normalized size = 1.10

$$\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-5\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1}-\frac{(x-1)^2}{(x+1)^2}-1}+\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="maxima")

[Out] (3*((x - 1)/(x + 1))^(3/2) - 5*sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) - (x - 1)^2/(x + 1)^2 - 1) + 3/2*log(sqrt((x - 1)/(x + 1)) + 1) - 3/2*log(sqrt((x - 1)/(x + 1)) - 1)

Fricas [A]
time = 0.36, size = 54, normalized size = 0.68

$$\frac{1}{2}\left(x^2+5x+4\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="fricas")

```
[Out] 1/2*(x^2 + 5*x + 4)*sqrt((x - 1)/(x + 1)) + 3/2*log(sqrt((x - 1)/(x + 1)) +
 1) - 3/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x),x)
```

```
[Out] Integral((x + 1)/sqrt((x - 1)/(x + 1)), x)
```

Giac [A]
time = 0.42, size = 48, normalized size = 0.61

$$\frac{1}{2}\sqrt{x^2-1}\left(\frac{x}{\operatorname{sgn}(x+1)}+\frac{4}{\operatorname{sgn}(x+1)}\right)-\frac{3\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 1)*(x/sgn(x + 1) + 4/sgn(x + 1)) - 3/2*log(abs(-x + sqrt(x^2
 - 1)))/sgn(x + 1)
```

Mupad [B]
time = 0.04, size = 68, normalized size = 0.86

$$3\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)+\frac{5\sqrt{\frac{x-1}{x+1}}-3\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2(x-1)}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 3*atanh(((x - 1)/(x + 1))^(1/2)) + (5*((x - 1)/(x + 1))^(1/2) - 3*((x - 1)/
(x + 1))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

3.281 $\int e^{\coth^{-1}(x)}(1-x)x\,dx$

Optimal. Leaf size=18

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

```
[Out] -1/3*(1-1/x^2)^(3/2)*x^3
```

Rubi [A]
time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6310, 6313, 270}

$$-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)*x,x]
```

```
[Out] -1/3*((1 - x^(-2))^(3/2)*x^3)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1-x)x\,dx = -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x^2\,dx$$
$$= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.17

$$-\frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x(-1+x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)*x,x]
```

```
[Out] -1/3*(Sqrt[1 - x^(-2)]*x*(-1 + x^2))
```

Maple [A]
time = 0.03, size = 22, normalized size = 1.22

method	result	size
gosper	$-\frac{(1+x)(-1+x)^2}{3\sqrt{\frac{-1+x}{1+x}}}$	22
default	$-\frac{(1+x)(-1+x)^2}{3\sqrt{\frac{-1+x}{1+x}}}$	22
risch	$-\frac{(x^2-1)(-1+x)}{3\sqrt{\frac{-1+x}{1+x}}}$	22
trager	$-\frac{(1+x)(x^2-1)\sqrt{-\frac{1-x}{1+x}}}{3}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(1+x)*(-1+x)^2/((-1+x)/(1+x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(14) = 28.

time = 0.27, size = 50, normalized size = 2.78

$$\frac{8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="maxima")
```

```
[Out] 8/3*((x - 1)/(x + 1))^(3/2)/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x
 - 1)^3/(x + 1)^3 - 1)
```

Fricas [A]
time = 0.38, size = 24, normalized size = 1.33

$$-\frac{1}{3}\left(x^3+x^2-x-1\right)\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + x^2 - x - 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx-\int\frac{x^2}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)*x,x)
```

```
[Out] -Integral(-x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(x**2/sqrt(x/(x + 1)
 - 1/(x + 1)), x)
```

Giac [A]
time = 0.43, size = 15, normalized size = 0.83

$$-\frac{\left(x^2-1\right)^{\frac{3}{2}}}{3\operatorname{sgn}\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)*x,x, algorithm="giac")
```

```
[Out] -1/3*(x^2 - 1)^(3/2)/sgn(x + 1)
```

Mupad [B]
time = 1.21, size = 18, normalized size = 1.00

$$-\frac{\left(\frac{x-1}{x+1}\right)^{3/2}(x+1)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1))/((x - 1)/(x + 1))^(1/2),x)
[Out] -(((x - 1)/(x + 1))^(3/2)*(x + 1)^3)/3
```

3.282 $\int e^{\coth^{-1}(x)}(1-x)\,dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] 1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6310, 6313, 272, 43, 65, 212}

$$\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x),x]
```

```
[Out] -1/2*(Sqrt[1 - x^(-2)]*x^2) + ArcTanh[Sqrt[1 - x^(-2)]]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)\,dx &= -\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)x\,dx\\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.11

$$
-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x),x]
```

```
[Out] -1/2*(Sqrt[1 - x^(-2)]*x^2) + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

Maple [A]
time = 0.07, size = 48, normalized size = 1.37

method	result	size
default	$-\frac{(-1+x)\left(x\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)\right)}{2\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	48
risch	$-\frac{x(-1+x)}{2\sqrt{\frac{-1+x}{1+x}}}+\frac{\ln\left(x+\sqrt{x^{2}-1}\right)\sqrt{(1+x)(-1+x)}}{2\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	56
trager	$-\frac{(1+x)\sqrt{-\frac{1-x}{1+x}}\,x}{2}+\frac{\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-1+x)*(x*(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+
x)*(-1+x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.
time = 0.26, size = 83, normalized size = 2.37

$$\frac{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+\sqrt{\frac{x-1}{x+1}}}{\frac{2(x-1)}{x+1}-\frac{(x-1)^{2}}{(x+1)^{2}}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="maxima")
```

```
[Out] (((x - 1)/(x + 1))^(3/2) + sqrt((x - 1)/(x + 1)))/(2*(x - 1)/(x + 1) - (x -
 1)^2/(x + 1)^2 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x
 - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.33, size = 51, normalized size = 1.46

$$-\frac{1}{2}\left(x^{2}+x\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + x)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) -
 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx-\int \left(-\frac{1}{\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x),x)
```

```
[Out] -Integral(x/sqrt(x/(x + 1) - 1/(x + 1)), x) - Integral(-1/sqrt(x/(x + 1) -
1/(x + 1)), x)
```

Giac [A]
time = 0.40, size = 38, normalized size = 1.09

$$-\frac{\sqrt{x^2-1}\,x}{2\,\mathrm{sgn}\left(x+1\right)}-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\,\mathrm{sgn}\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x^2 - 1)*x/sgn(x + 1) - 1/2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x +
1)
```

Mupad [B]
time = 0.03, size = 63, normalized size = 1.80

$$\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\sqrt{\frac{x-1}{x+1}}+\left(\frac{x-1}{x+1}\right)^{3/2}}{\frac{(x-1)^2}{(x+1)^2}-\frac{2\,(x-1)}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] atanh(((x - 1)/(x + 1))^(1/2)) - (((x - 1)/(x + 1))^(1/2) + ((x - 1)/(x + 1
))^(3/2))/((x - 1)^2/(x + 1)^2 - (2*(x - 1))/(x + 1) + 1)
```

3.283 $\int e^{\coth^{-1}(x)}x(1+x)^2\,dx$

Optimal. Leaf size=133

$$\frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2+\frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3+\frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}$$

```
[Out] 15/8*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+5/8*(1+1/x)^(3/2)*x^2*((-1+x)/
x)^(1/2)+1/4*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+1/4*(1+1/x)^(7/2)*x^4*((-1+
x)/x)^(1/2)+15/8*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {6310, 6315, 98, 96, 94, 212}

$$\frac{1}{4}\left(\frac{1}{x}+1\right)^{7/2}\sqrt{\frac{x-1}{x}}\,x^4+\frac{1}{4}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{5}{8}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{15}{8}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\frac{15}{8}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^2,x]
```

```
[Out] (15*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/8 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
 + x)/x]*x^2)/8 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/4 + ((1 + x^(-1
))^(7/2)*Sqrt[(-1 + x)/x]*x^4)/4 + (15*ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]])/8
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}x(1+x)^2\,dx = \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2 x^3\,dx$$
$$= -\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}\,x^5}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}\,x^4 - \frac{3}{4}\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}\,x^4}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}\,x^3 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{\frac{-1+x}{x}}\,x^4 - \frac{5}{4}\text{Subst}\left(\int \frac{(1+x}{\sqrt{1-}}\right.$$
$$= \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-x}{x}}\,x^3 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{7/2}\sqrt{-}$$
$$= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-}{x}}$$
$$= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-}{x}}$$
$$= \frac{15}{8}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{8}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{4}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{-\frac{1-}{x}}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.39

$$\frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x\left(24+15x+8x^2+2x^3\right) + \frac{15}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x*(24 + 15*x + 8*x^2 + 2*x^3))/8 + (15*Log[(1 + Sqrt[1 -
x^(-2)])*x])/8
```

Maple [A]
time = 0.11, size = 79, normalized size = 0.59

method	result	size
risch	$\frac{(2x^3+8x^2+15x+24)(-1+x)}{8\sqrt{\frac{-1+x}{1+x}}} + \frac{15\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{8\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	70

trager	$\frac{(1+x)(2x^3+8x^2+15x+24)\sqrt{-\frac{1-x}{1+x}}}{8}-\frac{15\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)}{8}$	74
default	$\frac{(-1+x)\left(2x(x^2-1)^{\frac{3}{2}}+8((1+x)(-1+x))^{\frac{3}{2}}+17x\sqrt{x^2-1}+32\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)}{8\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-1+x)*(2*x*(x^2-1)^(3/2)+8*((1+x)*(-1+x))^(3/2)+17*x*(x^2-1)^(1/2)+32*
(x^2-1)^(1/2)+15*ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(
1/2)
```

Maxima [A]
time = 0.27, size = 138, normalized size = 1.04

$$\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-55\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+73\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-49\sqrt{\frac{x-1}{x+1}}}{4\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(15*((x - 1)/(x + 1))^(7/2) - 55*((x - 1)/(x + 1))^(5/2) + 73*((x - 1)/
(x + 1))^(3/2) - 49*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 15/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.33, size = 66, normalized size = 0.50

$$\frac{1}{8}\left(2x^4+10x^3+23x^2+39x+24\right)\sqrt{\frac{x-1}{x+1}}+\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{15}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 + 10*x^3 + 23*x^2 + 39*x + 24)*sqrt((x - 1)/(x + 1)) + 15/8*log(
sqrt((x - 1)/(x + 1)) + 1) - 15/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**2,x)
```

```
[Out] Integral(x*(x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [A]
time = 0.41, size = 71, normalized size = 0.53

$$\frac{1}{8}\left(\left(2\,x\left(\frac{x}{\operatorname{sgn}\left(x+1\right)}+\frac{4}{\operatorname{sgn}\left(x+1\right)}\right)+\frac{15}{\operatorname{sgn}\left(x+1\right)}\right)x+\frac{24}{\operatorname{sgn}\left(x+1\right)}\right)\sqrt{x^2-1}-\frac{15\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\operatorname{sgn}\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/8*((2*x*(x/sgn(x + 1) + 4/sgn(x + 1)) + 15/sgn(x + 1))*x + 24/sgn(x + 1))
*sqrt(x^2 - 1) - 15/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B]
time = 0.05, size = 118, normalized size = 0.89

$$\frac{15\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}+\frac{\frac{49\sqrt{\frac{x-1}{x+1}}}{4}-\frac{73\,{\left(\frac{x-1}{x+1}\right)}^{3/2}}{4}+\frac{55\,{\left(\frac{x-1}{x+1}\right)}^{5/2}}{4}-\frac{15\,{\left(\frac{x-1}{x+1}\right)}^{7/2}}{4}}{\frac{6\,{\left(x-1\right)}^2}{{\left(x+1\right)}^2}-\frac{4\,\left(x-1\right)}{x+1}-\frac{4\,{\left(x-1\right)}^3}{{\left(x+1\right)}^3}+\frac{{\left(x-1\right)}^4}{{\left(x+1\right)}^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (15*atanh(((x - 1)/(x + 1))^(1/2)))/4 + ((49*((x - 1)/(x + 1))^(1/2))/4 - (
73*((x - 1)/(x + 1))^(3/2))/4 + (55*((x - 1)/(x + 1))^(5/2))/4 - (15*((x -
1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(
x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

3.284 $\int e^{\coth^{-1}(x)}(1+x)^2\,dx$

Optimal. Leaf size=106

$$\frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x+\frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{\frac{-1+x}{x}}\,x^2+\frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3+\frac{5}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{-\right.$$

```
[Out] 5/2*arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))+5/6*(1+1/x)^(3/2)*x^2*((-1+x)/x
)^(1/2)+1/3*(1+1/x)^(5/2)*x^3*((-1+x)/x)^(1/2)+5/2*x*(1+1/x)^(1/2)*((-1+x)/
x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6310, 6315, 96, 94, 212}

$$\frac{1}{3}\left(\frac{1}{x}+1\right)^{5/2}\sqrt{\frac{x-1}{x}}\,x^3+\frac{5}{6}\left(\frac{1}{x}+1\right)^{3/2}\sqrt{\frac{x-1}{x}}\,x^2+\frac{5}{2}\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x+\frac{5}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x)^2,x]
```

```
[Out] (5*Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x)/2 + (5*(1 + x^(-1))^(3/2)*Sqrt[(-1
+ x)/x]*x^2)/6 + ((1 + x^(-1))^(5/2)*Sqrt[(-1 + x)/x]*x^3)/3 + (5*ArcTanh[S
qrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]])/2
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}(1+x)^2\,dx &= \int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^2 x^2\,dx \\
&= -\text{Subst}\left(\int \frac{(1+x)^{5/2}}{\sqrt{1-x}\,x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3 - \frac{5}{3}\text{Subst}\left(\int \frac{(1+x)^{3/2}}{\sqrt{1-x}\,x^3}\,dx, x, \frac{1}{x}\right) \\
&= \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}\,x^3 - \frac{5}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\right. \\
&= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}} \\
&= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}} \\
&= \frac{5}{2}\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x + \frac{5}{6}\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2 + \frac{1}{3}\left(1+\frac{1}{x}\right)^{5/2}\sqrt{\frac{-1+x}{x}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.44

$$\frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x\left(22+9x+2x^2\right)+\frac{5}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(22 + 9*x + 2*x^2))/6 + (5*Log[(1 + Sqrt[1 - x^(-2)])*x
])/2
```

Maple [A]
time = 0.11, size = 69, normalized size = 0.65

method	result	size
risch	$\frac{(2x^2+9x+22)(-1+x)}{6\sqrt{\frac{-1+x}{1+x}}}+\frac{5\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{2\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	65
trager	$\frac{(1+x)(2x^2+9x+22)\sqrt{-\frac{1-x}{1+x}}}{6}+\frac{5\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	66
default	$\frac{(-1+x)\left(2((1+x)(-1+x))^{\frac{3}{2}}+9x\sqrt{x^2-1}+24\sqrt{x^2-1}+15\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-1+x)*(2*((1+x)*(-1+x))^(3/2)+9*x*(x^2-1)^(1/2)+24*(x^2-1)^(1/2)+15*ln
(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [A]
time = 0.25, size = 112, normalized size = 1.06

$$-\frac{15\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-40\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+33\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*((x - 1)/(x + 1))^(5/2) - 40*((x - 1)/(x + 1))^(3/2) + 33*sqrt((x
- 1)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x +
1)^3 - 1) + 5/2*log(sqrt((x - 1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x +
1)) - 1)
```

Fricas [A]
time = 0.42, size = 61, normalized size = 0.58

$$\frac{1}{6}\left(2x^3+11x^2+31x+22\right)\sqrt{\frac{x-1}{x+1}}+\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{5}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 + 11*x^2 + 31*x + 22)*sqrt((x - 1)/(x + 1)) + 5/2*log(sqrt((x -
1)/(x + 1)) + 1) - 5/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**2,x)
```

```
[Out] Integral((x + 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [A]
time = 0.41, size = 60, normalized size = 0.57

$$\frac{1}{6}\sqrt{x^2-1}\left(x\left(\frac{2x}{\operatorname{sgn}(x+1)}+\frac{9}{\operatorname{sgn}(x+1)}\right)+\frac{22}{\operatorname{sgn}(x+1)}\right)-\frac{5\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) + 9/sgn(x + 1)) + 22/sgn(x + 1)) - 5/2
*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B]
time = 0.05, size = 94, normalized size = 0.89

$$5\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{11\sqrt{\frac{x-1}{x+1}}-\frac{40\left(\frac{x-1}{x+1}\right)^{3/2}}{3}+5\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] 5*atanh(((x - 1)/(x + 1))^(1/2)) - (11*((x - 1)/(x + 1))^(1/2) - (40*((x -
1)/(x + 1))^(3/2))/3 + 5*((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3
*(x - 1)^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.285 $\int e^{\coth^{-1}(x)}(1-x)^2 x\,dx$

Optimal. Leaf size=71

$$\frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -1/3*(1-1/x^2)^(3/2)*x^3+1/4*(1-1/x^2)^(3/2)*x^4-1/8*arctanh((1-1/x^2)^(1/2
))+1/8*x^2*(1-1/x^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6310, 6313, 849, 821, 272, 43, 65, 212}

$$\frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2-\frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+\frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4-\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^2*x,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x^2)/8 - ((1 - x^(-2))^(3/2)*x^3)/3 + ((1 - x^(-2))^(3/2)
*x^4)/4 - ArcTanh[Sqrt[1 - x^(-2)]]/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1-x)^2x\,dx = \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2x^3\,dx$$
$$= -\text{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^5}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 + \frac{1}{4}\text{Subst}\left(\int \frac{(4-x)\sqrt{1-x^2}}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 - \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 - \frac{1}{8}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$
$$= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 + \frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\right.$$
$$= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,\right.$$
$$= \frac{1}{8}\sqrt{1-\frac{1}{x^2}}\,x^2 - \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3 + \frac{1}{4}\left(1-\frac{1}{x^2}\right)^{3/2}x^4 - \frac{1}{8}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.73

$$\frac{1}{24}\sqrt{1-\frac{1}{x^2}}\,x\left(8-3x-8x^2+6x^3\right) - \frac{1}{8}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2*x,x]
[Out] (Sqrt[1 - x^(-2)]*x*(8 - 3*x - 8*x^2 + 6*x^3))/24 - Log[(1 + Sqrt[1 - x^(-2
)])*x]/8
```

Maple [A]
time = 0.12, size = 70, normalized size = 0.99

method	result	size
default	$-\frac{(-1+x)\left(-6x(x^2-1)^{\frac{3}{2}}+8((1+x)(-1+x))^{\frac{3}{2}}-3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{24\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	70

risch	$\frac{(6x^3-8x^2-3x+8)(-1+x)}{24\sqrt{\frac{-1+x}{1+x}}}-\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{8\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	70
trager	$\frac{(1+x)(6x^3-8x^2-3x+8)\sqrt{-\frac{1-x}{1+x}}}{24}-\frac{\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{8}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-1+x)*(-6*x*(x^2-1)^(3/2)+8*((1+x)*(-1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3
*ln(x+(x^2-1)^(1/2)))/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(55) = 110$.
time = 0.26, size = 138, normalized size = 1.94

$$-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}+53\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}-11\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}+3\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}-\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="maxima")
```

```
[Out] -1/12*(3*((x - 1)/(x + 1))^(7/2) + 53*((x - 1)/(x + 1))^(5/2) - 11*((x - 1)
/(x + 1))^(3/2) + 3*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2
/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.34, size = 66, normalized size = 0.93

$$\frac{1}{24}\left(6x^4-2x^3-11x^2+5x+8\right)\sqrt{\frac{x-1}{x+1}}-\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\frac{1}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^4 - 2*x^3 - 11*x^2 + 5*x + 8)*sqrt((x - 1)/(x + 1)) - 1/8*log(sqr
t((x - 1)/(x + 1)) + 1) + 1/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2*x,x)
```

```
[Out] Integral(x*(x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [A]
time = 0.42, size = 72, normalized size = 1.01

$$\frac{1}{24}\left(\left(2\,x\left(\frac{3\,x}{\operatorname{sgn}(x+1)}-\frac{4}{\operatorname{sgn}(x+1)}\right)-\frac{3}{\operatorname{sgn}(x+1)}\right)x+\frac{8}{\operatorname{sgn}(x+1)}\right)\sqrt{x^2-1}+\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{8\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2*x,x, algorithm="giac")
```

```
[Out] 1/24*((2*x*(3*x/sgn(x + 1) - 4/sgn(x + 1)) - 3/sgn(x + 1))*x + 8/sgn(x + 1)
)*sqrt(x^2 - 1) + 1/8*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B]
time = 1.19, size = 118, normalized size = 1.66

$$\frac{\frac{\sqrt{\frac{x-1}{x+1}}}{4}-\frac{11\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{53\left(\frac{x-1}{x+1}\right)^{5/2}}{12}+\frac{\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6\,(x-1)^2}{(x+1)^2}-\frac{4\,(x-1)}{x+1}-\frac{4\,(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}-\frac{\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x - 1)^2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (((x - 1)/(x + 1))^(1/2)/4 - (11*((x - 1)/(x + 1))^(3/2))/12 + (53*((x - 1)
/(x + 1))^(5/2))/12 + ((x - 1)/(x + 1))^(7/2)/4)/((6*(x - 1)^2)/(x + 1)^2 -
 (4*(x - 1))/(x + 1) - (4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1) -
 atanh(((x - 1)/(x + 1))^(1/2))/4
```

3.286 $\int e^{\coth^{-1}(x)}(1-x)^2\,dx$

Optimal. Leaf size=53

$$-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] 1/3*(1-1/x^2)^(3/2)*x^3+1/2*arctanh((1-1/x^2)^(1/2))-1/2*x^2*(1-1/x^2)^(1/2
)
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6310, 6313, 821, 272, 43, 65, 212}

$$-\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^2,x]
```

```
[Out] -1/2*(Sqrt[1 - x^(-2)]*x^2) + ((1 - x^(-2))^(3/2)*x^3)/3 + ArcTanh[Sqrt[1 -
 x^(-2)]]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)^2\,dx &= \int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^2 x^2\,dx \\
&= -\text{Subst}\left(\int \frac{(1-x)\sqrt{1-x^2}}{x^4}\,dx,x,\frac{1}{x}\right) \\
&= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^3}\,dx,x,\frac{1}{x}\right) \\
&= \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,\frac{1}{x^2}\right) \\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,\frac{1}{x^2}\right) \\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{x^2}}\right) \\
&= -\frac{1}{2}\sqrt{1-\frac{1}{x^2}}\,x^2+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2}x^3+\frac{1}{2}\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.89

$$\frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x\left(-2-3x+2x^2\right)+\frac{1}{2}\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x*(-2 - 3*x + 2*x^2))/6 + Log[(1 + Sqrt[1 - x^(-2)])*x]/2
```

Maple [A]
time = 0.11, size = 60, normalized size = 1.13

method	result	size
default	$\frac{(-1+x)\left(2((1+x)(-1+x))^{\frac{3}{2}}-3x\sqrt{x^2-1}+3\ln\left(x+\sqrt{x^2-1}\right)\right)}{6\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	60
risch	$\frac{(2x^2-3x-2)(-1+x)}{6\sqrt{\frac{-1+x}{1+x}}}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{2\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	65

trager	$\frac{(1+x)(2x^2-3x-2)\sqrt{-\frac{1-x}{1+x}}}{6}+\frac{\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)}{2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-1+x)*(2*((1+x)*(-1+x))^(3/2)-3*x*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))
/((-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.26, size = 112, normalized size = 2.11

$$-\frac{3\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+8\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-3\sqrt{\frac{x-1}{x+1}}}{3\left(\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1\right)}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*((x - 1)/(x + 1))^(5/2) + 8*((x - 1)/(x + 1))^(3/2) - 3*sqrt((x - 1
)/(x + 1)))/(3*(x - 1)/(x + 1) - 3*(x - 1)^2/(x + 1)^2 + (x - 1)^3/(x + 1)^
3 - 1) + 1/2*log(sqrt((x - 1)/(x + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1))
 - 1)
```

Fricas [A]
time = 0.33, size = 61, normalized size = 1.15

$$\frac{1}{6}\left(2x^3-x^2-5x-2\right)\sqrt{\frac{x-1}{x+1}}+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3 - x^2 - 5*x - 2)*sqrt((x - 1)/(x + 1)) + 1/2*log(sqrt((x - 1)/(x
 + 1)) + 1) - 1/2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(x-1)^2}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**2,x)
```

```
[Out] Integral((x - 1)**2/sqrt((x - 1)/(x + 1)), x)
```

Giac [A]
time = 0.40, size = 60, normalized size = 1.13

$$\frac{1}{6}\sqrt{x^2-1}\left(x\left(\frac{2x}{\operatorname{sgn}(x+1)}-\frac{3}{\operatorname{sgn}(x+1)}\right)-\frac{2}{\operatorname{sgn}(x+1)}\right)-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{2\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(x^2 - 1)*(x*(2*x/sgn(x + 1) - 3/sgn(x + 1)) - 2/sgn(x + 1)) - 1/2*
log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B]
time = 1.18, size = 90, normalized size = 1.70

$$\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\frac{8\left(\frac{x-1}{x+1}\right)^{3/2}}{3}-\sqrt{\frac{x-1}{x+1}}+\left(\frac{x-1}{x+1}\right)^{5/2}}{\frac{3(x-1)}{x+1}-\frac{3(x-1)^2}{(x+1)^2}+\frac{(x-1)^3}{(x+1)^3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^2/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] atanh(((x - 1)/(x + 1))^(1/2)) - ((8*((x - 1)/(x + 1))^(3/2))/3 - ((x - 1)/
(x + 1))^(1/2) + ((x - 1)/(x + 1))^(5/2))/((3*(x - 1))/(x + 1) - (3*(x - 1)
^2)/(x + 1)^2 + (x - 1)^3/(x + 1)^3 - 1)
```

3.287 $\int \frac{e^{\coth^{-1}(x)}x}{1+x} \, dx$

Optimal. Leaf size=22

$$\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x$$

[Out] x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6310, 6314, 97}

$$\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\,x$$

Antiderivative was successfully verified.

[In] Int[(E^ArcCoth[x]*x)/(1 + x),x]

[Out] Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]*x

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{1+x}\,dx = \int \frac{e^{\coth^{-1}(x)}}{1+\frac{1}{x}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^2\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\,x$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.68

$$x\sqrt{\frac{-1+x^2}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x),x]
[Out] x*Sqrt[(-1 + x^2)/x^2]
```

Maple [A]
time = 0.08, size = 32, normalized size = 1.45

method	result	size
gosper	$\frac{-1+x}{\sqrt{\frac{-1+x}{1+x}}}$	16
risch	$\frac{-1+x}{\sqrt{\frac{-1+x}{1+x}}}$	16
trager	$(1+x)\sqrt{-\frac{1-x}{1+x}}$	19
default	$\frac{(-1+x)\sqrt{x^2-1}}{\sqrt{\frac{-1+x}{1+x}}\sqrt{(1+x)(-1+x)}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x,method=_RETURNVERBOSE)
[Out] 1/((-1+x)/(1+x))^(1/2)*(-1+x)/((1+x)*(-1+x))^(1/2)*(x^2-1)^(1/2)
```

Maxima [A]
time = 0.25, size = 26, normalized size = 1.18

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="maxima")
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Fricas [A]
time = 0.33, size = 15, normalized size = 0.68

$$(x+1)\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="fricas")
[Out] (x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x),x)
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)), x)
```

Giac [A]
time = 0.40, size = 14, normalized size = 0.64

$$\frac{\sqrt{x^2-1}}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x),x, algorithm="giac")
[Out] sqrt(x^2 - 1)/sgn(x + 1)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 1.18

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] -(2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x + 1) - 1)
```

3.288 $\int \frac{e^{\coth^{-1}(x)}}{1+x} \, dx$

Optimal. Leaf size=22

$$\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6310, 6315, 94, 212}

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 + x),x]
```

```
[Out] ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 + x)/x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] && !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
```

```
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{1+x}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)x}\,dx \\
&= -\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right) \\
&= \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.82

$$\log\left(x\left(1+\sqrt{\frac{-1+x^2}{x^2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x),x]
[Out] Log[x*(1 + Sqrt[(-1 + x^2)/x^2])]
```

Maple [A]
time = 0.10, size = 35, normalized size = 1.59

method	result	size
trager	$\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$	34
default	$\frac{(-1+x)\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x),x,method=_RETURNVERBOSE)
[Out] 1/((-1+x)/(1+x))^(1/2)*(-1+x)/((1+x)*(-1+x))^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A]
time = 0.26, size = 31, normalized size = 1.41

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.33, size = 31, normalized size = 1.41

$$\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [A]
time = 2.34, size = 29, normalized size = 1.32

$$-\log\left(-1+\frac{1}{\sqrt{1-\frac{2}{x+1}}}\right)+\log\left(1+\frac{1}{\sqrt{1-\frac{2}{x+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x),x)
```

```
[Out] -log(-1 + 1/sqrt(1 - 2/(x + 1))) + log(1 + 1/sqrt(1 - 2/(x + 1)))
```

Giac [A]
time = 0.41, size = 21, normalized size = 0.95

$$-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.64

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((((x - 1)/(x + 1))^(1/2)*(x + 1)),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2))
```

3.289 $\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx$

Optimal. Leaf size=47

$$\frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}-\sqrt{1-\frac{1}{x^2}}\,x-2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -2*arctanh((1-1/x^2)^(1/2))+2*(1+1/x)/(1-1/x^2)^(1/2)-x*(1-1/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6310, 6312, 866, 1819, 821, 272, 65, 212}

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\sqrt{1-\frac{1}{x^2}}\,x-2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x),x]
```

```
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - Sqrt[1 - x^(-2)]*x - 2*ArcTanh[Sqrt[1 -
 x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{1-x}\,dx &= -\int \frac{e^{\coth^{-1}(x)}}{1-\frac{1}{x}}\,dx \\
&= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2x^2}\,dx, x, \frac{1}{x}\right) \\
&= \text{Subst}\left(\int \frac{(1+x)^2}{x^2\,(1-x^2)^{3/2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{-1-2x}{x^2\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x + 2\text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x - 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \sqrt{1-\frac{1}{x^2}}\,x - 2\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.87

$$
-\frac{\sqrt{1-\frac{1}{x^2}}\,(-3+x)x}{-1+x} - 2\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x),x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*(-3 + x)*x)/(-1 + x)) - 2*Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(41) = 82$.
time = 0.12, size = 106, normalized size = 2.26

method	result
trager	$-\frac{(1+x)(-3+x)\sqrt{-\frac{1-x}{1+x}}}{-1+x}-2\ln\left(\sqrt{-\frac{1-x}{1+x}}\, x+\sqrt{-\frac{1-x}{1+x}}+x\right)$
risch	$-\frac{x^2-2x-3}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}-\frac{2\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$
default	$\frac{(x^2-1)^{\frac{3}{2}}-2\sqrt{x^2-1}\,x^2-2\ln\left(x+\sqrt{x^2-1}\right)x^2+4x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-2\sqrt{x^2-1}-2\ln\left(x+\sqrt{x^2-\right.}}{(-1+x)\sqrt{(1+x)(-1+x)}\,\sqrt{\frac{-1+x}{1+x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x,method=_RETURNVERBOSE)
```

```
[Out] ((x^2-1)^(3/2)-2*(x^2-1)^(1/2)*x^2-2*ln(x+(x^2-1)^(1/2))*x^2+4*x*(x^2-1)^(1
/2)+4*ln(x+(x^2-1)^(1/2))*x-2*(x^2-1)^(1/2)-2*ln(x+(x^2-1)^(1/2)))/(-1+x)/(
(1+x)*(-1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A]
time = 0.26, size = 74, normalized size = 1.57

$$\frac{2\left(\frac{2(x-1)}{x+1}-1\right)}{\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-\sqrt{\frac{x-1}{x+1}}}-2\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+2\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="maxima")
```

```
[Out] 2*(2*(x - 1)/(x + 1) - 1)/(((x - 1)/(x + 1))^(3/2) - sqrt((x - 1)/(x + 1)))
 - 2*log(sqrt((x - 1)/(x + 1)) + 1) + 2*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.40, size = 66, normalized size = 1.40

$$-\frac{2(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-2(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)+(x^2-2x-3)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="fricas")
```

```
[Out] -(2*(x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 2*(x - 1)*log(sqrt((x - 1)/(x
+ 1)) - 1) + (x^2 - 2*x - 3)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x),x)
[Out] -Integral(x/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

Giac [A]
time = 0.41, size = 65, normalized size = 1.38

$$\frac{2\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\mathrm{sgn}\,(x+1)}-\frac{\sqrt{x^2-1}}{\mathrm{sgn}\,(x+1)}-\frac{4}{\left(x-\sqrt{x^2-1}-1\right)\mathrm{sgn}\,(x+1)}-2\,\mathrm{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x),x, algorithm="giac")
[Out] 2*log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - sqrt(x^2 - 1)/sgn(x + 1) - 4/((
x - sqrt(x^2 - 1) - 1)*sgn(x + 1)) - 2*sgn(x + 1)
```

Mupad [B]
time = 1.19, size = 43, normalized size = 0.91

$$-\frac{2\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)\,\sqrt{\frac{x-1}{x+1}}-6}{2\,\sqrt{\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
[Out] -(2*x + 8*atanh(((x - 1)/(x + 1))^(1/2))*((x - 1)/(x + 1))^(1/2) - 6)/(2*((
x - 1)/(x + 1))^(1/2))
```

3.290 $\int \frac{e^{\coth^{-1}(x)}}{1-x} \, dx$

Optimal. Leaf size=33

$$\frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}}-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -arctanh((1-1/x^2)^(1/2))+2*(1+1/x)/(1-1/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6310, 6313, 866, 1819, 272, 65, 212}

$$\frac{2\left(\frac{1}{x}+1\right)}{\sqrt{1-\frac{1}{x^2}}}-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x),x]
[Out] (2*(1 + x^(-1)))/Sqrt[1 - x^(-2)] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(x)}}{1-x}\,dx &= -\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)x}\,dx\\ &= \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^2 x}\,dx, x, \frac{1}{x}\right)\\ &= \text{Subst}\left(\int \frac{(1+x)^2}{x\,(1-x^2)^{3/2}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} + \text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)\\ &= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\ &= \frac{2\left(1+\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x^2}}} - \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.15

$$\frac{2\sqrt{1-\frac{1}{x^2}}\,x}{-1+x} - \log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 - x),x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x)/(-1 + x) - Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(29) = 58$.
time = 0.12, size = 106, normalized size = 3.21

method	result

risch	$\frac{2}{\sqrt{\frac{-1+x}{1+x}}}-\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$
trager	$\frac{2\sqrt{-\frac{1-x}{1+x}}\,(1+x)}{-1+x}-\ln\left(\sqrt{-\frac{1-x}{1+x}}\,x+\sqrt{-\frac{1-x}{1+x}}+x\right)$
default	$\frac{(x^2-1)^{\frac{3}{2}}-\sqrt{x^2-1}\,x^2-\ln\left(x+\sqrt{x^2-1}\right)x^2+2x\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)x-\sqrt{x^2-1}-\ln\left(x+\sqrt{x^2-1}\right.}{(-1+x)\sqrt{(1+x)(-1+x)}\,\sqrt{\frac{-1+x}{1+x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x),x,method=_RETURNVERBOSE)
```

```
[Out] ((x^2-1)^(3/2)-(x^2-1)^(1/2)*x^2-ln(x+(x^2-1)^(1/2))*x^2+2*x*(x^2-1)^(1/2)+
2*ln(x+(x^2-1)^(1/2))*x-(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2)))/(-1+x)/((1+x)*(-
1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A]
time = 0.26, size = 44, normalized size = 1.33

$$\frac{2}{\sqrt{\frac{x-1}{x+1}}}-\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)+\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="maxima")
```

```
[Out] 2/sqrt((x - 1)/(x + 1)) - log(sqrt((x - 1)/(x + 1)) + 1) + log(sqrt((x - 1)
/(x + 1)) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.
time = 0.38, size = 61, normalized size = 1.85

$$-\frac{(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-(x-1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)-2\,(x+1)\sqrt{\frac{x-1}{x+1}}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="fricas")
```

```
[Out] -((x - 1)*log(sqrt((x - 1)/(x + 1)) + 1) - (x - 1)*log(sqrt((x - 1)/(x + 1)
) - 1) - 2*(x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}-\sqrt{\frac{x}{x+1}-\frac{1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x),x)
[Out] -Integral(1/(x*sqrt(x/(x + 1) - 1/(x + 1)) - sqrt(x/(x + 1) - 1/(x + 1))),
x)
```

Giac [A]
time = 0.41, size = 49, normalized size = 1.48

$$\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}-\frac{4}{\left(x-\sqrt{x^2-1}-1\right)\operatorname{sgn}(x+1)}-2\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x),x, algorithm="giac")
[Out] log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 4/((x - sqrt(x^2 - 1) - 1)*sgn(x
+ 1)) - 2*sgn(x + 1)
```

Mupad [B]
time = 1.19, size = 28, normalized size = 0.85

$$\frac{2}{\sqrt{\frac{x-1}{x+1}}}-2\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(((x - 1)/(x + 1))^(1/2)*(x - 1)),x)
[Out] 2/((x - 1)/(x + 1))^(1/2) - 2*atanh(((x - 1)/(x + 1))^(1/2))
```

3.291 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}+\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)$$

```
[Out] arctanh((1+1/x)^(1/2)*((-1+x)/x)^(1/2))-((-1+x)/x)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {6310, 6315, 98, 94, 212}

$$\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\sqrt{\frac{x-1}{x}}\right)-\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

```
[Out] -(Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[1 + x^(-1)]*Sqrt[(-1 +
x)/x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^2}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} - \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)\\
&= -\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}} + \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 36, normalized size = 0.80

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x}{1+x}+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^2,x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*x)/(1 + x)) + Log[(1 + Sqrt[1 - x^(-2)])*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(37) = 74$.
time = 0.12, size = 110, normalized size = 2.44

method	result
trager	$-\sqrt{-\frac{1-x}{1+x}}-\ln\left(-\sqrt{-\frac{1-x}{1+x}}\,x-\sqrt{-\frac{1-x}{1+x}}+x\right)$
risch	$-\frac{-1+x}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}+\frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$
default	$\frac{(-1+x)\left((x^2-1)^{\frac{3}{2}}-\sqrt{x^2-1}\,x^2+2\ln\left(x+\sqrt{x^2-1}\right)x^2-2x\sqrt{x^2-1}+4\ln\left(x+\sqrt{x^2-1}\right)x-\sqrt{x^2-1}+2\ln\left(x+\right.\right.}{2\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}\,(1+x)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1+x)*((x^2-1)^(3/2)-(x^2-1)^(1/2)*x^2+2*ln(x+(x^2-1)^(1/2))*x^2-2*x*(
x^2-1)^(1/2)+4*ln(x+(x^2-1)^(1/2))*x-(x^2-1)^(1/2)+2*ln(x+(x^2-1)^(1/2)))/(
(-1+x)/(1+x))^(1/2)/((1+x)*(-1+x))^(1/2)/(1+x)^2
```

Maxima [A]
time = 0.26, size = 44, normalized size = 0.98

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

Fricas [A]
time = 0.33, size = 44, normalized size = 0.98

$$-\sqrt{\frac{x-1}{x+1}}+\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="fricas")
[Out] -sqrt((x - 1)/(x + 1)) + log(sqrt((x - 1)/(x + 1)) + 1) - log(sqrt((x - 1)/
(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**2,x)
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**2), x)
```

Giac [A]
time = 0.41, size = 44, normalized size = 0.98

$$-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}-\frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^2,x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) - 2/((x - sqrt(x^2 - 1) + 1)*sgn(x
 + 1))
```

Mupad [B]
time = 0.03, size = 28, normalized size = 0.62

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - ((x - 1)/(x + 1))^(1/2)
```

3.292 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2} dx$

Optimal. Leaf size=21

$$\frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}$$

```
[Out] ((-1+x)/x)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6310, 6315, 37}

$$\frac{\sqrt{\frac{x-1}{x}}}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 + x)^2,x]
```

```
[Out] Sqrt[(-1 + x)/x]/Sqrt[1 + x^(-1)]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
```

m]

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{(1+x)^2}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^2 x^2}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{\frac{-1+x}{x}}}{\sqrt{1+\frac{1}{x}}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.86

$$\frac{\sqrt{1-\frac{1}{x^2}}\,x}{1+x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^2,x]
[Out] (Sqrt[1 - x^(-2)]*x)/(1 + x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 0.08, size = 37, normalized size = 1.76

method	result	size
trager	$\sqrt{-\frac{1-x}{1+x}}$	15
gosper	$\frac{-1+x}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	21
risch	$\frac{-1+x}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$	21
default	$\frac{\sqrt{x^2-1}\,(-1+x)}{(1+x)\sqrt{(1+x)(-1+x)}\,\sqrt{\frac{-1+x}{1+x}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (x^2-1)^(1/2)*(-1+x)/(1+x)/((1+x)*(-1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.52

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

Fricas [A]
time = 0.34, size = 11, normalized size = 0.52

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] sqrt((x - 1)/(x + 1))
```

Sympy [A]
time = 3.46, size = 8, normalized size = 0.38

$$\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**2,x)
```

```
[Out] sqrt((x - 1)/(x + 1))
```

Giac [A]
time = 0.40, size = 22, normalized size = 1.05

$$\frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 - 1) + 1)*sgn(x + 1))
```

Mupad [B]
time = 0.18, size = 11, normalized size = 0.52

$$\sqrt{1-\frac{2}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^2),x)
```

```
[Out] (1 - 2/(x + 1))^(1/2)
```

3.293 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] -4/3*(1+1/x)/(1-1/x^2)^(3/2)+arctanh((1-1/x^2)^(1/2))+1/3*(-3-5/x)/(1-1/x^2
)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6310, 6313, 866, 1819, 837, 12, 272, 65, 212}

$$-\frac{4\left(\frac{1}{x}+1\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{\frac{5}{x}+3}{3\sqrt{1-\frac{1}{x^2}}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

```
[Out] (-4*(1 + x^(-1)))/(3*(1 - x^(-2))^(3/2)) - (3 + 5/x)/(3*Sqrt[1 - x^(-2)]) +
 ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
```

```
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^2}\,dx &= \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3 x}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int \frac{(1+x)^3}{x\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}+\frac{1}{3}\text{Subst}\left(\int \frac{-3-5x}{x\left(1-x^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}}+\frac{1}{3}\text{Subst}\left(\int -\frac{3}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}}-\text{Subst}\left(\int \frac{1}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right)\\
&= -\frac{4\left(1+\frac{1}{x}\right)}{3\left(1-\frac{1}{x^2}\right)^{3/2}}-\frac{3+\frac{5}{x}}{3\sqrt{1-\frac{1}{x^2}}}+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 43, normalized size = 0.78

$$\frac{\sqrt{1-\frac{1}{x^2}}\,(5-7x)x}{3(-1+x)^2}+\log\left(\left(1+\sqrt{1-\frac{1}{x^2}}\right)x\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^2,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*(5 - 7*x)*x)/(3*(-1 + x)^2) + Log[(1 + Sqrt[1 - x^(-2)])*
x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(45) = 90$.
time = 0.12, size = 146, normalized size = 2.65

method	result
trager	$-\frac{(1+x)(7x-5)\sqrt{-\frac{1-x}{1+x}}}{3(-1+x)^2} - \ln\left(-\sqrt{-\frac{1-x}{1+x}}\, x - \sqrt{-\frac{1-x}{1+x}} + x\right)$
risch	$-\frac{7x^2+2x-5}{3(-1+x)\sqrt{\frac{-1+x}{1+x}}\,(1+x)} + \frac{\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)}$
default	$-\frac{3x(x^2-1)^{\frac{3}{2}} - 3\sqrt{x^2-1}\, x^3 - 3\ln\left(x+\sqrt{x^2-1}\right)x^3 - 2(x^2-1)^{\frac{3}{2}} + 9\sqrt{x^2-1}\, x^2 + 9\ln\left(x+\sqrt{x^2-1}\right)x^2 - 9x\sqrt{x^2 -}}{3(-1+x)^2\sqrt{(1+x)(-1+x)}\sqrt{\frac{-1+x}{1+x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*x*(x^2-1)^(3/2)-3*(x^2-1)^(1/2)*x^3-3*ln(x+(x^2-1)^(1/2))*x^3-2*(x^
2-1)^(3/2)+9*(x^2-1)^(1/2)*x^2+9*ln(x+(x^2-1)^(1/2))*x^2-9*x*(x^2-1)^(1/2)-
9*ln(x+(x^2-1)^(1/2))*x+3*(x^2-1)^(1/2)+3*ln(x+(x^2-1)^(1/2)))/(-1+x)^2/((1
+x)*(-1+x))^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A]
time = 0.26, size = 56, normalized size = 1.02

$$-\frac{\frac{6\,(x-1)}{x+1}+1}{3\,\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}} + \log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*(x - 1)/(x + 1) + 1)/((x - 1)/(x + 1))^(3/2) + log(sqrt((x - 1)/(x
+ 1)) + 1) - log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.35, size = 84, normalized size = 1.53

$$\frac{3\,(x^2-2\,x+1)\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - 3\,(x^2-2\,x+1)\log\left(\sqrt{\frac{x-1}{x+1}}-1\right) - (7\,x^2+2\,x-5)\sqrt{\frac{x-1}{x+1}}}{3\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(x^2 - 2*x + 1)*log(sqrt((x - 1)/(x + 1)) + 1) - 3*(x^2 - 2*x + 1)*l
og(sqrt((x - 1)/(x + 1)) - 1) - (7*x^2 + 2*x - 5)*sqrt((x - 1)/(x + 1)))/(x
^2 - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**2,x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Giac [A]
time = 0.41, size = 79, normalized size = 1.44

$$-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x+1)}+\frac{2\left(9\left(x-\sqrt{x^2-1}\right)^2-12\,x+12\,\sqrt{x^2-1}+7\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)}+\frac{7}{3}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^2,x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))/sgn(x + 1) + 2/3*(9*(x - sqrt(x^2 - 1))^2 - 1
2*x + 12*sqrt(x^2 - 1) + 7)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) + 7/3*sg
n(x + 1)
```

Mupad [B]
time = 0.04, size = 40, normalized size = 0.73

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{\frac{2\,(x-1)}{x+1}+\frac{1}{3}}{{\left(\frac{x-1}{x+1}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

```
[Out] 2*atanh(((x - 1)/(x + 1))^(1/2)) - ((2*(x - 1))/(x + 1) + 1/3)/((x - 1)/(x
+ 1))^(3/2)
```

3.294 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx$

Optimal. Leaf size=24

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

```
[Out] -1/3*(1-1/x^2)^(3/2)/(1-1/x)^3
```

Rubi [A]
time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6310, 6313, 665}

$$-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x)^2,x]
```

```
[Out] -1/3*(1 - x^(-2))^(3/2)/(1 - x^(-1))^3
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^2}\,dx = \int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^2 x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{3\left(1-\frac{1}{x}\right)^3}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$-\frac{\sqrt{1-\frac{1}{x^2}}\,x(1+x)}{3(-1+x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^2,x]
```

```
[Out] -1/3*(Sqrt[1 - x^(-2)]*x*(1 + x))/(-1 + x)^2
```

Maple [A]
time = 0.09, size = 35, normalized size = 1.46

method	result	size
gosper	$-\frac{1+x}{3(-1+x)\sqrt{\frac{-1+x}{1+x}}}$	22
trager	$-\frac{(1+x)^2\sqrt{-\frac{1-x}{1+x}}}{3(-1+x)^2}$	27
risch	$-\frac{x^2+2x+1}{3\sqrt{\frac{-1+x}{1+x}}\,(1+x)(-1+x)}$	32
default	$-\frac{(x^2-1)^{\frac{3}{2}}}{3\sqrt{\frac{-1+x}{1+x}}\,(-1+x)^2\sqrt{(1+x)(-1+x)}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/((-1+x)/(1+x))^(1/2)/(-1+x)^2/((1+x)*(-1+x))^(1/2)*(x^2-1)^(3/2)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.54

$$-\frac{1}{3\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="maxima")
```

```
[Out] -1/3/((x - 1)/(x + 1))^(3/2)
```

Fricas [A]
time = 0.36, size = 31, normalized size = 1.29

$$-\frac{(x^2+2x+1)\sqrt{\frac{x-1}{x+1}}}{3(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(x^2 + 2*x + 1)*sqrt((x - 1)/(x + 1))/(x^2 - 2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**2,x)
```

```
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*(x - 1)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.
time = 0.41, size = 46, normalized size = 1.92

$$\frac{2\left(3\left(x-\sqrt{x^2-1}\right)^2+1\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3\operatorname{sgn}(x+1)}+\frac{1}{3}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2 - 1) - 1)^3*sgn(x + 1)) +
1/3*sgn(x + 1)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.54

$$-\frac{1}{3\left(\frac{x-1}{x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x - 1)^2),x)
```

```
[Out] -1/(3*((x - 1)/(x + 1))^(3/2))
```

3.295 $\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx$

Optimal. Leaf size=65

$$\frac{2x^{1+m}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-\frac{3}{2}-m;-\frac{1}{2}-m;-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*x^(1+m)*hypergeom([-1/2, -3/2-m],[-1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/(3+2*m
)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6311, 6316, 66}

$$\frac{2x^{m+1}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{3}{2};-m-\frac{1}{2};-\frac{1}{ax}\right)}{(2m+3)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
```

```
[Out] (2*x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
```

```
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$
$$=-\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int x^{-\frac{5}{2}-m}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$=\frac{2x^{1+m}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-\frac{3}{2}-m;-\frac{1}{2}-m;-\frac{1}{ax}\right)}{(3+2m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.03

$$-\frac{x^{1+m}\sqrt{c-acx}\,{}_2F_1\left(-\frac{1}{2},-\frac{3}{2}-m;-\frac{1}{2}-m;-\frac{1}{ax}\right)}{\left(-\frac{3}{2}-m\right)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a*c*x],x]
[Out] -((x^(1 + m)*Sqrt[c - a*c*x]*Hypergeometric2F1[-1/2, -3/2 - m, -1/2 - m, -(
1/(a*x))])/((-3/2 - m)*Sqrt[1 - 1/(a*x)]))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-acx+c}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] integral(sqrt(-a*c*x + c)*(a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1)
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a*c*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \sqrt{c-a\,c\,x}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x^m*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.296 $\int e^{\coth^{-1}(ax)} x^2 \sqrt{c-acx}\, dx$

Optimal. Leaf size=140

$$\frac{16\left(1+\frac{1}{ax}\right)^{3/2} x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{8\left(1+\frac{1}{ax}\right)^{3/2} x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2} x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 16/105*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)-8/35*(1+1/a/x
)^(3/2)*x^2*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*(1+1/a/x)^(3/2)*x^3*(-a*
c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6311, 6316, 47, 37}

$$\frac{16x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{8x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (16*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (8
*(1 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*(1
+ 1/(a*x))^(3/2)*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\,\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(4\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\cdots}{\cdots}$$

$$= \frac{16\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}} - \frac{8\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^3\sqrt{c\cdots}}{7\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 0.46

$$\frac{2\sqrt{1+\frac{1}{ax}}\,(1+ax)\sqrt{c-acx}\,(8-12ax+15a^2x^2)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*Sqrt[c - a*c*x]*(8 - 12*a*x + 15*a^2*x^2))/(
105*a^3*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.12, size = 50, normalized size = 0.36

method	result	size
gosper	$\frac{2(ax+1)(15a^2x^2-12ax+8)\sqrt{-acx+c}}{105a^3\sqrt{\frac{ax-1}{ax+1}}}$	49
default	$\frac{2\sqrt{-c(ax-1)}\,(ax+1)(15a^2x^2-12ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,a^3}$	50
risch	$-\frac{2c(ax-1)(15a^3x^3+3a^2x^2-4ax+8)}{105\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax-1)}\,a^3}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(a*x+1)*(15*a^2*x^2-12*a*x
+8)/a^3
```

Maxima [A]
time = 0.26, size = 55, normalized size = 0.39

$$\frac{2\left(15\,a^3\sqrt{-c}\,x^3+3\,a^2\sqrt{-c}\,x^2-4\,a\sqrt{-c}\,x+8\,\sqrt{-c}\right)\sqrt{ax+1}}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2/105*(15*a^3*sqrt(-c)*x^3 + 3*a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x + 8*sqrt(-
c))*sqrt(a*x + 1)/a^3
```

Fricas [A]
time = 0.32, size = 69, normalized size = 0.49

$$\frac{2\left(15\,a^4x^4+18\,a^3x^3-a^2x^2+4\,ax+8\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105\left(a^4x-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/105*(15*a^4*x^4 + 18*a^3*x^3 - a^2*x^2 + 4*a*x + 8)*sqrt(-a*c*x + c)*sqrt
((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [A]
time = 36.32, size = 184, normalized size = 1.31

$$\frac{44\sqrt{-acx+c}}{105a^3\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}-\frac{94(-acx+c)^{\frac{3}{2}}}{105a^3c\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}+\frac{32(-acx+c)^{\frac{5}{2}}}{35a^3c^2\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}-\frac{2(-acx+c)^{\frac{7}{2}}}{7a^3c^3\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] 44*sqrt(-a*c*x + c)/(105*a**3*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))) -
 94*(-a*c*x + c)**(3/2)/(105*a**3*c*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x -
c))) + 32*(-a*c*x + c)**(5/2)/(35*a**3*c**2*sqrt(-a*c*x/(-a*c*x - c) + c/(-
a*c*x - c))) - 2*(-a*c*x + c)**(7/2)/(7*a**3*c**3*sqrt(-a*c*x/(-a*c*x - c)
+ c/(-a*c*x - c)))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.38, size = 57, normalized size = 0.41

$$\frac{2\sqrt{c-a\,c\,x}\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^2\,x^2-12\,a\,x+8\right)}{105\,a^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2)*(15*a^2*x^2 -
12*a*x + 8))/(105*a^3*(a*x - 1))
```

3.297 $\int e^{\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=92

$$-\frac{4\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -4/15*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(3
/2)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6311, 6316, 47, 37}

$$\frac{2x^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (-4*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]) + (2*(1
 + 1/(a*x))^(3/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)} x\sqrt{c-acx}\,dx &= \frac{\sqrt{c-acx}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5a\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{4\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}} + \frac{2\left(1+\frac{1}{ax}\right)^{3/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 56, normalized size = 0.61

$$\frac{2\sqrt{1+\frac{1}{ax}}\,(1+ax)(-2+3ax)\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*(1 + a*x)*(-2 + 3*a*x)*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1
 - 1/(a*x)])
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.46

method	result	size
gosper	$\frac{2(ax+1)(3ax-2)\sqrt{-acx+c}}{15a^2\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$\frac{2\sqrt{-c\,(ax-1)}\,(ax+1)(3ax-2)}{15\sqrt{\frac{ax-1}{ax+1}}\,a^2}$	42
risch	$-\frac{2c(ax-1)\left(3a^2x^2+ax-2\right)}{15\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\,a^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(a*x+1)*(3*a*x-2)/a^2
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.45

$$\frac{2\left(3\,a^2\sqrt{-c}\,x^2+a\sqrt{-c}\,x-2\,\sqrt{-c}\right)\sqrt{ax+1}}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/15*(3*a^2*sqrt(-c)*x^2 + a*sqrt(-c)*x - 2*sqrt(-c))*sqrt(a*x + 1)/a^2
```

Fricas [A]
time = 0.35, size = 61, normalized size = 0.66

$$\frac{2\,(3\,a^3x^3+4\,a^2x^2-ax-2)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3x-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(3*a^3*x^3 + 4*a^2*x^2 - a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1))/(a^3*x - a^2)
```

Sympy [A]
time = 20.07, size = 136, normalized size = 1.48

$$\frac{4\sqrt{-acx+c}}{15a^2\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}-\frac{14(-acx+c)^{\frac{3}{2}}}{15a^2c\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}+\frac{2(-acx+c)^{\frac{5}{2}}}{5a^2c^2\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] 4*sqrt(-a*c*x + c)/(15*a**2*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))) - 1
4*(-a*c*x + c)**(3/2)/(15*a**2*c*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))
) + 2*(-a*c*x + c)**(5/2)/(5*a**2*c**2*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x
 - c)))
```

Giac [A]
time = 0.42, size = 81, normalized size = 0.88

$$\frac{2\,c^2\left(\frac{2\sqrt{2}\,\sqrt{-c}}{ac}-\frac{3\,(acx+c)^2\sqrt{-acx-c}+5\,(-acx-c)^{\frac{3}{2}}c}{ac^3}\right)}{15\,a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*c^2*(2*sqrt(2)*sqrt(-c)/(a*c) - (3*(a*c*x + c)^2*sqrt(-a*c*x - c) + 5*
(-a*c*x - c)^(3/2)*c)/(a*c^3))/(a*abs(c)*sgn(a*x + 1))
```

Mupad [B]
time = 1.37, size = 49, normalized size = 0.53

$$\frac{2\sqrt{c-a\,c\,x}\,\left(a\,x+1\right)^2\left(3\,a\,x-2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,a^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*(3*a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(
15*a^2*(a*x - 1))
```

3.298 $\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=29

$$\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6309}

$$\frac{2(ax+1)\sqrt{c-acx}\,e^{\coth^{-1}(ax)}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
[Out] (2*E^ArcCoth[a*x]*(1 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{2e^{\coth^{-1}(ax)}(1+ax)\sqrt{c-acx}}{3a}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.48

$$\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a*c*x],x]
[Out] (2*(1 + 1/(a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.07, size = 36, normalized size = 1.24

method	result	size
gosper	$\frac{2(ax+1)\sqrt{-acx+c}}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$	35
default	$\frac{2\sqrt{-c\,(ax-1)}\,(ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\, a}$	36
risch	$-\frac{2c(ax+1)(ax-1)}{3\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\, a}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(a*x+1)/a
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.90

$$\frac{2\left(a\sqrt{-c}\,x+\sqrt{-c}\right)\sqrt{ax+1}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(a*sqrt(-c)*x + sqrt(-c))*sqrt(a*x + 1)/a
```

Fricas [A]
time = 0.36, size = 50, normalized size = 1.72

$$\frac{2\left(a^2x^2+2\,ax+1\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(a^2*x^2 + 2*a*x + 1)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [A]
time = 0.41, size = 49, normalized size = 1.69

$$\frac{2\,c^2\left(\frac{2\,\sqrt{2}\,\sqrt{-c}}{c}+\frac{(-acx-c)^{\frac{3}{2}}}{c^2}\right)}{3\,a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*c^2*(2*sqrt(2)*sqrt(-c)/c + (-a*c*x - c)^(3/2)/c^2)/(a*abs(c)*sgn(a*x +
 1))
```

Mupad [B]
time = 0.00, size = 43, normalized size = 1.48

$$\frac{2\,\sqrt{c-a\,c\,x}\,\,{\left(a\,x+1\right)}^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] (2*(c - a*c*x)^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1
))
```

3.299 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6311, 6316, 49, 56, 221}

$$\frac{2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] - (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\,\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} - \frac{2\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 0.80

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\,\sqrt{\frac{a+\frac{1}{x}}{a}} - \sqrt{\frac{1}{x}}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[(a + x^(-1))/a] - Sqrt[x^(-1)]*ArcSinh[Sqr
t[x^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.15, size = 70, normalized size = 0.74

method	result	size
default	$-\frac{2\sqrt{-c\,(a x-1)}\,\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c\,(a x+1)}}{\sqrt{c}}\right)-\sqrt{-c\,(a x+1)}\right)}{\sqrt{\frac{a x-1}{a x+1}}\,\sqrt{-c\,(a x+1)}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/((a*x-1)/(a*x+1))^(1/2)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))-(-c*(a*x+1))^(1/2))/(-c*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 207, normalized size = 2.20

$$\left[\frac{(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},\ -\frac{2\left((ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
```

```
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(a*x - 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A]
time = 0.42, size = 88, normalized size = 0.94

$$\frac{2\,c^3\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{c\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}}{\sqrt{c}}\right)-\sqrt{2}\,\sqrt{-c}\,\sqrt{c}}{c^{\frac{5}{2}}}-\frac{\sqrt{-acx-c}}{c^2}\right)}{|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
[Out] 2*c^3*(arctan(sqrt(-a*c*x - c)/sqrt(c))/c^(3/2) - (c*arctan(sqrt(2)*sqrt(-c
)/sqrt(c)) - sqrt(2)*sqrt(-c)*sqrt(c))/c^(5/2) - sqrt(-a*c*x - c)/c^2)/(abs
(c)*sgn(a*x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.300 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x}-\frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-arcsinh((1/x)^(1/2)/a^(
1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6311, 6316, 52, 56, 221}

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x)) - (Sqrt[a]*Sqr
t[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.03, size = 76, normalized size = 0.78

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] -((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.16, size = 78, normalized size = 0.80

method	result	size
default	$-\frac{\left(\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c\,(ax+1)}\,\sqrt{c}\right)\sqrt{-c\,(ax-1)}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax+1)}\,x\sqrt{c}}$	78
risch	$\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}-\frac{a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{-c\,(ax+1)}\,(ax-1)}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))*(-c*
(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 229, normalized size = 2.36

$$\left[\frac{(a^2x^2-ax)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^2-x)},\,-\frac{(a^2x^2-ax)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)
*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -((a^2*x^2 -
a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c
*x - c)) + sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [A]
time = 0.43, size = 101, normalized size = 1.04

$$\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-c}}{\sqrt{c}}\right)+\sqrt{2}\,a^2\sqrt{-c}\,\sqrt{c}}{c^{\frac{3}{2}}}+\frac{\sqrt{-acx-c}\,a}{cx}\right)c^2}{a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] (a^2*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - (a^2*c*arctan(sqrt(2)*sqrt(
-c)/sqrt(c)) + sqrt(2)*a^2*sqrt(-c)*sqrt(c))/c^(3/2) + sqrt(-a*c*x - c)*a/(
c*x))*c^2/(a*abs(c)*sgn(a*x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.301 $\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=101

$$\frac{4\sqrt{c-acx}}{a^4}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}$$

```
[Out] -14/3*(-a*c*x+c)^(3/2)/a^4/c+18/5*(-a*c*x+c)^(5/2)/a^4/c^2-10/7*(-a*c*x+c)^
(7/2)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4+4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A]
time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6302, 6265, 21, 78}

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^
(9/2))/(9*a^4*c^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx\\
&= -\int \frac{x^3(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\
&= -\left(c\int \frac{x^3(1+ax)}{\sqrt{c-acx}}\,dx\right)\\
&= -\left(c\int \left(\frac{2}{a^3\sqrt{c-acx}}-\frac{7\sqrt{c-acx}}{a^3c}+\frac{9(c-acx)^{3/2}}{a^3c^2}-\frac{5(c-acx)^{5/2}}{a^3c^3}+\frac{(c\cdots}{\cdots}\right.\right.\\
&= \frac{4\sqrt{c-acx}}{a^4}-\frac{14(c-acx)^{3/2}}{3a^4c}+\frac{18(c-acx)^{5/2}}{5a^4c^2}-\frac{10(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-a\cdots}{9a^4\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.48

$$\frac{2\sqrt{c-acx}\,(272+136ax+102a^2x^2+85a^3x^3+35a^4x^4)}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(272 + 136*a*x + 102*a^2*x^2 + 85*a^3*x^3 + 35*a^4*x^4))
/(315*a^4)
```

Maple [A]
time = 0.18, size = 75, normalized size = 0.74

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
trager	$\frac{2\sqrt{-acx+c}\,(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)}{315a^4}$	45
risch	$-\frac{2c(35a^4x^4+85a^3x^3+102a^2x^2+136ax+272)(ax-1)}{315a^4\sqrt{-c(ax-1)}}$	52
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}}{a^4c^4}$	75

default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{10c(-acx+c)^{\frac{7}{2}}}{7}+\frac{18c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{14c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}}{a^4c^4}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^4/a^4*(1/9*(-a*c*x+c)^(9/2)-5/7*c*(-a*c*x+c)^(7/2)+9/5*c^2*(-a*c*x+c)^(
5/2)-7/3*c^3*(-a*c*x+c)^(3/2)+2*c^4*(-a*c*x+c)^(1/2))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 0.73

$$\frac{2\left(35\,(-acx+c)^{\frac{9}{2}}-225\,(-acx+c)^{\frac{7}{2}}c+567\,(-acx+c)^{\frac{5}{2}}c^2-735\,(-acx+c)^{\frac{3}{2}}c^3+630\,\sqrt{-acx+c}\,c^4\right)}{315\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(-a*c*x + c)^(9/2) - 225*(-a*c*x + c)^(7/2)*c + 567*(-a*c*x + c)^
(5/2)*c^2 - 735*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(-a*c*x + c)*c^4)/(a^4*c^4
)
```

Fricas [A]
time = 0.33, size = 44, normalized size = 0.44

$$\frac{2\,(35\,a^4x^4+85\,a^3x^3+102\,a^2x^2+136\,ax+272)\sqrt{-acx+c}}{315\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^4*x^4 + 85*a^3*x^3 + 102*a^2*x^2 + 136*a*x + 272)*sqrt(-a*c*x +
 c)/a^4
```

Sympy [A]
time = 1.93, size = 83, normalized size = 0.82

$$\frac{2\cdot\left(2c^4\sqrt{-acx+c}-\frac{7c^3(-acx+c)^{\frac{3}{2}}}{3}+\frac{9c^2(-acx+c)^{\frac{5}{2}}}{5}-\frac{5c(-acx+c)^{\frac{7}{2}}}{7}+\frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(2*c**4*sqrt(-a*c*x + c) - 7*c**3*(-a*c*x + c)**(3/2)/3 + 9*c**2*(-a*c*x
+ c)**(5/2)/5 - 5*c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4*c*
*4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(83) = 166.
time = 0.40, size = 189, normalized size = 1.87

$$\frac{2\left(\frac{9\left(5\,(acx-c)^3\sqrt{-acx+c}+21\,(acx-c)^2\sqrt{-acx+c}\,c-35\,(-acx+c)^{\frac{3}{2}}c^2+35\,\sqrt{-acx+c}\,c^3\right)}{a^3c^3}+\frac{35\,(acx-c)^4\sqrt{-acx+c}+180\,(acx-c)^3\sqrt{-acx+c}\,c+378\,(acx-c)^2\sqrt{-acx+c}\,c^2-420\,(-acx+c)^{\frac{3}{2}}c^3+315\,\sqrt{-acx+c}\,c^4}{a^3c^4}\right)}{315\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(9*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(a*c*x - c)^2*sqrt(-a*c*x +
 c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*x + c)*c^3)/(a^3*c^3) + (3
5*(a*c*x - c)^4*sqrt(-a*c*x + c) + 180*(a*c*x - c)^3*sqrt(-a*c*x + c)*c + 3
78*(a*c*x - c)^2*sqrt(-a*c*x + c)*c^2 - 420*(-a*c*x + c)^(3/2)*c^3 + 315*sq
rt(-a*c*x + c)*c^4)/(a^3*c^4))/a
```

Mupad [B]
time = 0.04, size = 83, normalized size = 0.82

$$\frac{4\,\sqrt{c-a\,c\,x}}{a^4}-\frac{14\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^4\,c}+\frac{18\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^4\,c^2}-\frac{10\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^4\,c^3}+\frac{2\,{\left(c-a\,c\,x\right)}^{9/2}}{9\,a^4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^4 - (14*(c - a*c*x)^(3/2))/(3*a^4*c) + (18*(c - a*c
*x)^(5/2))/(5*a^4*c^2) - (10*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x
)^(9/2))/(9*a^4*c^4)
```

3.302 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=80

$$\frac{4\sqrt{c-acx}}{a^3}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{2(c-acx)^{7/2}}{7a^3c^3}$$

```
[Out] -10/3*(-a*c*x+c)^(3/2)/a^3/c+8/5*(-a*c*x+c)^(5/2)/a^3/c^2-2/7*(-a*c*x+c)^(7
/2)/a^3/c^3+4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A]
time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6302, 6265, 21, 78}

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{4\sqrt{c-acx}}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*x)
^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx\\ &= -\int \frac{x^2(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{x^2(1+ax)}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{a^2\sqrt{c-acx}}-\frac{5\sqrt{c-acx}}{a^2c}+\frac{4(c-acx)^{3/2}}{a^2c^2}-\frac{(c-acx)^{5/2}}{a^2c^3}\right)dx\right)\\ &= \frac{4\sqrt{c-acx}}{a^3}-\frac{10(c-acx)^{3/2}}{3a^3c}+\frac{8(c-acx)^{5/2}}{5a^3c^2}-\frac{2(c-acx)^{7/2}}{7a^3c^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.50

$$\frac{2\sqrt{c-acx}\,(104+52ax+39a^2x^2+15a^3x^3)}{105a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(104 + 52*a*x + 39*a^2*x^2 + 15*a^3*x^3))/(105*a^3)
```

Maple [A]
time = 0.18, size = 61, normalized size = 0.76

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
trager	$\frac{2\sqrt{-acx+c}\,(15a^3x^3+39a^2x^2+52ax+104)}{105a^3}$	37
risch	$-\frac{2c(15a^3x^3+39a^2x^2+52ax+104)(ax-1)}{105a^3\sqrt{-c(ax-1)}}$	44
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-2c^3\sqrt{-acx+c}\right)}{c^3a^3}$	61

default	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-2c^3\sqrt{-acx+c}\right)}{c^3a^3}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^3/a^3*(1/7*(-a*c*x+c)^(7/2)-4/5*c*(-a*c*x+c)^(5/2)+5/3*c^2*(-a*c*x+c)^
(3/2)-2*c^3*(-a*c*x+c)^(1/2))
```

Maxima [A]
time = 0.26, size = 60, normalized size = 0.75

$$-\frac{2\left(15\,(-acx+c)^{\frac{7}{2}}-84\,(-acx+c)^{\frac{5}{2}}c+175\,(-acx+c)^{\frac{3}{2}}c^2-210\,\sqrt{-acx+c}\,c^3\right)}{105\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/105*(15*(-a*c*x + c)^(7/2) - 84*(-a*c*x + c)^(5/2)*c + 175*(-a*c*x + c)^
(3/2)*c^2 - 210*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Fricas [A]
time = 0.33, size = 36, normalized size = 0.45

$$\frac{2\,(15\,a^3x^3+39\,a^2x^2+52\,ax+104)\sqrt{-acx+c}}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a^3*x^3 + 39*a^2*x^2 + 52*a*x + 104)*sqrt(-a*c*x + c)/a^3
```

Sympy [A]
time = 1.80, size = 68, normalized size = 0.85

$$-\frac{2\left(-2c^3\sqrt{-acx+c}+\frac{5c^2(-acx+c)^{\frac{3}{2}}}{3}-\frac{4c(-acx+c)^{\frac{5}{2}}}{5}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] -2*(-2*c**3*sqrt(-a*c*x + c) + 5*c**2*(-a*c*x + c)**(3/2)/3 - 4*c*(-a*c*x +
 c)**(5/2)/5 + (-a*c*x + c)**(7/2)/7)/(a**3*c**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(66) = 132.
time = 0.41, size = 142, normalized size = 1.78

$$\frac{2\left(\frac{7\left(3(acx-c)^{2}\sqrt{-acx+c}-10(-acx+c)^{\frac{3}{2}}c+15\sqrt{-acx+c}\,c^{2}\right)}{a^{2}c^{2}}+\frac{3\left(5(acx-c)^{3}\sqrt{-acx+c}+21(acx-c)^{2}\sqrt{-acx+c}\,c-35(-acx+c)^{\frac{3}{2}}c^{2}+35\sqrt{-acx+c}\,c^{3}\right)}{a^{2}c^{3}}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(a*c*x - c)^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*s
qrt(-a*c*x + c)*c^2)/(a^2*c^2) + 3*(5*(a*c*x - c)^3*sqrt(-a*c*x + c) + 21*(
a*c*x - c)^2*sqrt(-a*c*x + c)*c - 35*(-a*c*x + c)^(3/2)*c^2 + 35*sqrt(-a*c*
x + c)*c^3)/(a^2*c^3))/a
```

Mupad [B]
time = 0.05, size = 66, normalized size = 0.82

$$\frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{10\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^3\,c}+\frac{8\,{\left(c-a\,c\,x\right)}^{5/2}}{5\,a^3\,c^2}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^3 - (10*(c - a*c*x)^(3/2))/(3*a^3*c) + (8*(c - a*c*
x)^(5/2))/(5*a^3*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^3*c^3)
```

3.303 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=57

$$\frac{4\sqrt{c-acx}}{a^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}$$

```
[Out] -2*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2+4*(-a*c*x+c)^(1/2)/a
^2
```

Rubi [A]
time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6302, 6265, 21, 78}

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{4\sqrt{c-acx}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 - (2*(c - a*c*x)^(3/2))/(a^2*c) + (2*(c - a*c*x)^(5
/2))/(5*a^2*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx &= -\int e^{2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx\\ &= -\int \frac{x(1+ax)\sqrt{c-acx}}{1-ax}\,dx\\ &= -\left(c\int \frac{x(1+ax)}{\sqrt{c-acx}}\,dx\right)\\ &= -\left(c\int \left(\frac{2}{a\sqrt{c-acx}}-\frac{3\sqrt{c-acx}}{ac}+\frac{(c-acx)^{3/2}}{ac^2}\right)dx\right)\\ &= \frac{4\sqrt{c-acx}}{a^2}-\frac{2(c-acx)^{3/2}}{a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.54

$$\frac{2\sqrt{c-acx}\,(6+3ax+a^2x^2)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
[Out] (2*Sqrt[c - a*c*x]*(6 + 3*a*x + a^2*x^2))/(5*a^2)
```

Maple [A]
time = 0.18, size = 47, normalized size = 0.82

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(a^2x^2+3ax+6)}{5a^2}$	28
trager	$\frac{2\sqrt{-acx+c}\,(a^2x^2+3ax+6)}{5a^2}$	28
risch	$-\frac{2c(a^2x^2+3ax+6)(ax-1)}{5a^2\sqrt{-c(ax-1)}}$	35
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2c(-acx+c)^{\frac{3}{2}}+4c^2\sqrt{-acx+c}}{c^2a^2}$	47
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}-2c(-acx+c)^{\frac{3}{2}}+4c^2\sqrt{-acx+c}}{c^2a^2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^2/a^2*(1/5*(-a*c*x+c)^(5/2)-c*(-a*c*x+c)^(3/2)+2*c^2*(-a*c*x+c)^(1/2))
```

Maxima [A]
time = 0.25, size = 44, normalized size = 0.77

$$\frac{2\left((-acx+c)^{\frac{5}{2}}-5\,(-acx+c)^{\frac{3}{2}}c+10\,\sqrt{-acx+c}\,c^2\right)}{5\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*((-a*c*x + c)^(5/2) - 5*(-a*c*x + c)^(3/2)*c + 10*sqrt(-a*c*x + c)*c^2)
/(a^2*c^2)
```

Fricas [A]
time = 0.34, size = 27, normalized size = 0.47

$$\frac{2\,(a^2x^2+3\,ax+6)\sqrt{-acx+c}}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)/a^2
```

Sympy [A]
time = 2.18, size = 48, normalized size = 0.84

$$\frac{2\cdot\left(2c^2\sqrt{-acx+c}-c(-acx+c)^{\frac{3}{2}}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] 2*(2*c**2*sqrt(-a*c*x + c) - c*(-a*c*x + c)**(3/2) + (-a*c*x + c)**(5/2)/5)
/(a**2*c**2)
```

Giac [A]
time = 0.40, size = 92, normalized size = 1.61

$$-\frac{2\left(\frac{5\left((-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c\right)}{ac}-\frac{3\,(acx-c)^2\sqrt{-acx+c}-10\,(-acx+c)^{\frac{3}{2}}c+15\,\sqrt{-acx+c}\,c^2}{ac^2}\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/(a*c) - (3*(a*c*x - c)
^2*sqrt(-a*c*x + c) - 10*(-a*c*x + c)^(3/2)*c + 15*sqrt(-a*c*x + c)*c^2)/(a
*c^2))/a
```

Mupad [B]
time = 0.05, size = 46, normalized size = 0.81

$$\frac{2\,{\left(c-a\,c\,x\right)}^{5/2}-10\,c\,{\left(c-a\,c\,x\right)}^{3/2}+20\,c^2\,\sqrt{c-a\,c\,x}}{5\,a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*(c - a*c*x)^(5/2) - 10*c*(c - a*c*x)^(3/2) + 20*c^2*(c - a*c*x)^(1/2))/(
5*a^2*c^2)
```

3.304 $\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=38

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6265, 21, 45}

$$\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-acx}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx$$
$$= -\int \frac{(1+ax)\sqrt{c-acx}}{1-ax}\,dx$$
$$= -\left(c\int \frac{1+ax}{\sqrt{c-acx}}\,dx\right)$$
$$= -\left(c\int \left(\frac{2}{\sqrt{c-acx}} - \frac{\sqrt{c-acx}}{c}\right)dx\right)$$
$$= \frac{4\sqrt{c-acx}}{a} - \frac{2(c-acx)^{3/2}}{3ac}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.61

$$\frac{2(5+ax)\sqrt{c-acx}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*(5 + a*x)*Sqrt[c - a*c*x])/(3*a)
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.87

method	result	size
gosper	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
trager	$\frac{2\sqrt{-acx+c}\,(ax+5)}{3a}$	20
risch	$-\frac{2c(ax+5)(ax-1)}{3a\sqrt{-c(ax-1)}}$	27
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}-2c\sqrt{-acx+c}\right)}{ca}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)-2*c*(-a*c*x+c)^(1/2))

Maxima [A]
time = 0.25, size = 30, normalized size = 0.79

$$-\frac{2\left((-acx+c)^{\frac{3}{2}}-6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="maxima")

[Out] -2/3*((-a*c*x + c)^(3/2) - 6*sqrt(-a*c*x + c)*c)/(a*c)

Fricas [A]
time = 0.32, size = 19, normalized size = 0.50

$$\frac{2\sqrt{-acx+c}\,(ax+5)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(-a*c*x + c)*(a*x + 5)/a

Sympy [A]
time = 1.56, size = 31, normalized size = 0.82

$$-\frac{2\left(-2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2),x)

[Out] -2*(-2*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)

Giac [A]
time = 0.40, size = 44, normalized size = 1.16

$$\frac{2\left(3\sqrt{-acx+c}-\frac{(-acx+c)^{\frac{3}{2}}-3\sqrt{-acx+c}\,c}{c}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2),x, algorithm="giac")

[Out] 2/3*(3*sqrt(-a*c*x + c) - ((-a*c*x + c)^(3/2) - 3*sqrt(-a*c*x + c)*c)/c)/a

Mupad [B]

time = 0.00, size = 32, normalized size = 0.84

$$\frac{4\sqrt{c-a\,c\,x}}{a}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] (4*(c - a*c*x)^(1/2))/a - (2*(c - a*c*x)^(3/2))/(3*a*c)
```

3.305 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=39

$$2\sqrt{c-acx}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6302, 6265, 21, 81, 65, 214}

$$2\sqrt{c-acx}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx\\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x(1-ax)}\,dx\\
&= -\left(c\int \frac{1+ax}{x\sqrt{c-acx}}\,dx\right)\\
&= 2\sqrt{c-acx}-c\int \frac{1}{x\sqrt{c-acx}}\,dx\\
&= 2\sqrt{c-acx}+\frac{2\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= 2\sqrt{c-acx}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$
2\sqrt{c-acx}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A]
time = 0.19, size = 32, normalized size = 0.82

method	result	size
derivativedivides	$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}\;+2\sqrt{-acx+c}$	32
default	$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}\;+2\sqrt{-acx+c}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Maxima [A]
time = 0.46, size = 49, normalized size = 1.26

$$-\sqrt{c}\,\log\left(\frac{\sqrt{-acx+c}\,-\sqrt{c}}{\sqrt{-acx+c}\,+\sqrt{c}}\right)+2\,\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c))) + 2
*sqrt(-a*c*x + c)
```

Fricas [A]
time = 0.36, size = 82, normalized size = 2.10

$$\left[\sqrt{c}\,\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}\,-2\,c}{x}\right)+2\,\sqrt{-acx+c}\,,-2\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)+2\,\sqrt{-acx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x
+ c), -2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x + c)]
```

Sympy [A]
time = 2.21, size = 39, normalized size = 1.00

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x,x)
[Out] -2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

Giac [A]
time = 0.40, size = 40, normalized size = 1.03

$$-2\,c\left(\frac{\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
[Out] -2*c*(arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)/c)
```

Mupad [B]
time = 1.19, size = 31, normalized size = 0.79

$$2\,\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+2\,\sqrt{c-a\,c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2)
```

3.306 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 3*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

Rubi [A]
time = 0.13, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used = {6302, 6265, 21, 79, 65, 214}

$$\frac{\sqrt{c-acx}}{x}+3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^2(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x^2\sqrt{c-acx}}\,dx\right) \\
&= \frac{\sqrt{c-acx}}{x} - \frac{1}{2}(3ac)\int \frac{1}{x\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{x} + 3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) \\
&= \frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$\frac{\sqrt{c-acx}}{x} + 3a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] Sqrt[c - a*c*x]/x + 3*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]]
```

Maple [A]
time = 0.20, size = 45, normalized size = 1.07

method	result	size
risch	$-\frac{(ax-1)c}{x\sqrt{-c(ax-1)}}+3a\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}$	43
derivativedivides	$-2ca\left(-\frac{\sqrt{-acx+c}}{2acx}-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45
default	$-2ca\left(-\frac{\sqrt{-acx+c}}{2acx}-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*a*(-1/2*(-a*c*x+c)^(1/2)/a/c/x-3/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^
(1/2)))
```

Maxima [A]
time = 0.47, size = 62, normalized size = 1.48

$$-\frac{1}{2}\,ac\left(\frac{3\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\,\sqrt{-acx+c}}{acx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*c*(3*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/
sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

Fricas [A]
time = 0.37, size = 97, normalized size = 2.31

$$\left[\frac{3\,a\sqrt{c}\,x\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}}{2\,x},\,-\frac{3\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/2*(3*a*sqrt(c)*x*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*sqrt(-a*c*x + c))/x, -(3*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c))/x]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(36) = 72.
time = 4.18, size = 119, normalized size = 2.83

$$-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{2ac\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**2,x)

[Out] -a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + a*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 - 2*a*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x

Giac [A]
time = 0.41, size = 48, normalized size = 1.14

$$-\frac{\frac{3\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{-acx+c}\,a}{x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac")

[Out] -(3*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - sqrt(-a*c*x + c)*a/x)/a

Mupad [B]
time = 0.06, size = 34, normalized size = 0.81

$$\frac{\sqrt{c-a\,c\,x}}{x}+3\,a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)

[Out] (c - a*c*x)^(1/2)/x + 3*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2))

3.307 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{7}{4}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 7/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2+
7/4*a*(-a*c*x+c)^(1/2)/x
```

Rubi [A]
time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6302, 6265, 21, 79, 44, 65, 214}

$$\frac{7}{4}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] Sqrt[c - a*c*x]/(2*x^2) + (7*a*Sqrt[c - a*c*x])/(4*x) + (7*a^2*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx\\ &= -\int \frac{(1+ax)\sqrt{c-acx}}{x^3(1-ax)}\,dx\\ &= -\left(c\int \frac{1+ax}{x^3\sqrt{c-acx}}\,dx\right)\\ &= \frac{\sqrt{c-acx}}{2x^2}-\frac{1}{4}(7ac)\int \frac{1}{x^2\sqrt{c-acx}}\,dx\\ &= \frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}-\frac{1}{8}\left(7a^2c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx\\ &= \frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{1}{4}(7a)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,x,\sqrt{c-acx}\right)\\ &= \frac{\sqrt{c-acx}}{2x^2}+\frac{7a\sqrt{c-acx}}{4x}+\frac{7}{4}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.81

$$\frac{(2+7ax)\sqrt{c-acx}}{4x^2}+\frac{7}{4}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] ((2 + 7*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (7*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c
*x]/Sqrt[c]])/4
```

Maple [A]
time = 0.22, size = 65, normalized size = 0.96

method	result	size
risch	$-\frac{(7a^2x^2-5ax-2)c}{4x^2\sqrt{-c(ax-1)}}+\frac{7a^2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}}{4}$	54
derivativedivides	$2c^2a^2\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65

default	$2c^{2}a^{2}\left(\frac{-\frac{7(-acx+c)^{\frac{3}{2}}}{8c}+\frac{9\sqrt{-acx+c}}{8}}{a^{2}c^{2}x^{2}}+\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2*a^2*((-7/8/c*(-a*c*x+c)^(3/2)+9/8*(-a*c*x+c)^(1/2))/a^2/c^2/x^2+7/8/c
^(3/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.46, size = 103, normalized size = 1.51

$$-\frac{1}{8}\,a^{2}c^{2}\left(\frac{2\left(7\,(-acx+c)^{\frac{3}{2}}-9\,\sqrt{-acx+c}\,c\right)}{(acx-c)^{2}c+2\,(acx-c)c^{2}+c^{3}}+\frac{7\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*c^2*(2*(7*(-a*c*x + c)^(3/2) - 9*sqrt(-a*c*x + c)*c)/((a*c*x - c)^
2*c + 2*(a*c*x - c)*c^2 + c^3) + 7*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-
a*c*x + c) + sqrt(c)))/c^(3/2))
```

Fricas [A]
time = 0.37, size = 117, normalized size = 1.72

$$\left[\frac{7\,a^{2}\sqrt{c}\,x^{2}\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\,\sqrt{-acx+c}\,(7\,ax+2)}{8\,x^{2}},-\frac{7\,a^{2}\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-\sqrt{-acx+c}\,(7\,ax+2)}{4\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(7*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) +
 2*sqrt(-a*c*x + c)*(7*a*x + 2))/x^2, -1/4*(7*a^2*sqrt(-c)*x^2*arctan(sqrt(
-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(7*a*x + 2))/x^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(60) = 120$.
time = 8.77, size = 270, normalized size = 3.97

$$\frac{10a^{2}c^{4}\sqrt{-acx+c}}{16ac^{4}x-8c^{4}+8c^{2}\left(-acx+c\right)^{2}}-\frac{6a^{2}c^{3}(-acx+c)^{\frac{3}{2}}}{16ac^{4}x-8c^{4}+8c^{2}\left(-acx+c\right)^{2}}-\frac{3a^{2}c^{3}\sqrt{\frac{1}{c^{5}}}\log\left(-c^{3}\sqrt{\frac{1}{c^{5}}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^{2}c^{3}\sqrt{\frac{1}{c^{5}}}\log\left(c^{3}\sqrt{\frac{1}{c^{5}}}+\sqrt{-acx+c}\right)}{8}-\frac{a^{2}c^{2}\sqrt{\frac{1}{c^{3}}}\log\left(-c^{2}\sqrt{\frac{1}{c^{3}}}+\sqrt{-acx+c}\right)}{2}+\frac{a^{2}c^{2}\sqrt{\frac{1}{c^{3}}}\log\left(c^{2}\sqrt{\frac{1}{c^{3}}}+\sqrt{-acx+c}\right)}{2}+\frac{a\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] 10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**
2) - 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x
 + c)**2) - 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x
 + c))/8 + 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x +
 c))/8 - a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)
)/2 + a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2
+ a*sqrt(-a*c*x + c)/x
```

Giac [A]
time = 0.41, size = 76, normalized size = 1.12

$$-\frac{\frac{7\,a^{3}c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{7\,(-acx+c)^{\frac{3}{2}}a^{3}c-9\,\sqrt{-acx+c}\,a^{3}c^{2}}{a^{2}c^{2}x^{2}}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(7*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + (7*(-a*c*x + c)^
(3/2)*a^3*c - 9*sqrt(-a*c*x + c)*a^3*c^2)/(a^2*c^2*x^2))/a
```

Mupad [B]
time = 1.23, size = 54, normalized size = 0.79

$$\frac{9\,\sqrt{c-a\,c\,x}}{4\,x^2}+\frac{7\,a^2\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)}{4}-\frac{7\,{\left(c-a\,c\,x\right)}^{3/2}}{4\,c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] (9*(c - a*c*x)^(1/2))/(4*x^2) + (7*a^2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1
/2)))/4 - (7*(c - a*c*x)^(3/2))/(4*c*x^2)
```

3.308 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 11/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^3
+11/12*a*(-a*c*x+c)^(1/2)/x^2+11/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A]
time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.304$, Rules used = {6302, 6265, 21, 79, 44, 65, 214}

$$\frac{11}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{11a^2\sqrt{c-acx}}{8x}+\frac{\sqrt{c-acx}}{3x^3}+\frac{11a\sqrt{c-acx}}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] Sqrt[c - a*c*x]/(3*x^3) + (11*a*Sqrt[c - a*c*x])/(12*x^2) + (11*a^2*Sqrt[c
- a*c*x])/(8*x) + (11*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$$
$$= -\int \frac{(1+ax)\sqrt{c-acx}}{x^4(1-ax)}\,dx$$
$$= -\left(c\int \frac{1+ax}{x^4\sqrt{c-acx}}\,dx\right)$$
$$= \frac{\sqrt{c-acx}}{3x^3} - \frac{1}{6}(11ac)\int \frac{1}{x^3\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} - \frac{1}{8}\left(11a^2c\right)\int \frac{1}{x^2\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} + \frac{11a^2\sqrt{c-acx}}{8x} - \frac{1}{16}\left(11a^3c\right)\int \frac{1}{x\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} + \frac{11a^2\sqrt{c-acx}}{8x} + \frac{1}{8}\left(11a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,d\right.$$
$$= \frac{\sqrt{c-acx}}{3x^3} + \frac{11a\sqrt{c-acx}}{12x^2} + \frac{11a^2\sqrt{c-acx}}{8x} + \frac{11}{8}a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right.$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 0.71

$$\frac{\sqrt{c-acx}\,\left(8+22ax+33a^2x^2\right)}{24x^3} + \frac{11}{8}a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (Sqrt[c - a*c*x]*(8 + 22*a*x + 33*a^2*x^2))/(24*x^3) + (11*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8
```

Maple [A]
time = 0.20, size = 80, normalized size = 0.90

method	result	size
risch	$-\frac{\left(33a^3x^3-11a^2x^2-14ax-8\right)c}{24x^3\sqrt{-c\left(ax-1\right)}}+\frac{11a^3\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}}{8}$	62

derivativedivides	$-2c^3a^3\left(-\frac{\frac{11(-acx+c)^{\frac{5}{2}}}{16c^2}-\frac{11(-acx+c)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{a^3c^3x^3}-\frac{11\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$	80
default	$-2c^3a^3\left(-\frac{\frac{11(-acx+c)^{\frac{5}{2}}}{16c^2}-\frac{11(-acx+c)^{\frac{3}{2}}}{6c}+\frac{21\sqrt{-acx+c}}{16}}{a^3c^3x^3}-\frac{11\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{16c^{\frac{5}{2}}}\right)$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*c^3*a^3*(-(11/16/c^2*(-a*c*x+c)^(5/2)-11/6/c*(-a*c*x+c)^(3/2)+21/16*(-a*
c*x+c)^(1/2))/a^3/c^3/x^3-11/16/c^(5/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.48, size = 134, normalized size = 1.51

$$\frac{1}{48}a^3c^3\left(\frac{2\left(33\,(-acx+c)^{\frac{5}{2}}-88\,(-acx+c)^{\frac{3}{2}}c+63\,\sqrt{-acx+c}\,c^2\right)}{(acx-c)^3c^2+3\,(acx-c)^2c^3+3\,(acx-c)c^4+c^5}-\frac{33\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*a^3*c^3*(2*(33*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 63*sqrt(
-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*c
^4 + c^5) - 33*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)
))/c^(5/2))
```

Fricas [A]
time = 0.35, size = 133, normalized size = 1.49

$$\left[\frac{33\,a^3\sqrt{c}\,x^3\log\left(\frac{acx-2\,\sqrt{-acx+c}\,\sqrt{c}-2\,c}{x}\right)+2\,(33\,a^2x^2+22\,ax+8)\sqrt{-acx+c}}{48\,x^3},-\frac{33\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-(33\,a^2x^2+22\,ax+8)\sqrt{-acx+c}}{24\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(33*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x)
 + 2*(33*a^2*x^2 + 22*a*x + 8)*sqrt(-a*c*x + c))/x^3, -1/24*(33*a^3*sqrt(-c
)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (33*a^2*x^2 + 22*a*x + 8)*sqrt(
-a*c*x + c))/x^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(80) = 160$.
time = 9.16, size = 439, normalized size = 4.93

$$-\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}+\frac{80a^3c^5(-acx+c)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-\frac{30a^3c^4(-acx+c)^{\frac{5}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}+\frac{10a^3c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}-\frac{5a^3c^4\sqrt{\frac{1}{c^7}}\log\left(-c^4\sqrt{\frac{1}{c^7}}+\sqrt{-acx+c}\right)}{16}+\frac{5a^3c^4\sqrt{\frac{1}{c^7}}\log\left(c^4\sqrt{\frac{1}{c^7}}+\sqrt{-acx+c}\right)}{16}-\frac{6a^3c^3(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}-\frac{3a^3c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^3c^3\sqrt{\frac{1}{c^5}}\log\left(c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] -66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x
+ c)**2 + 48*c**3*(-a*c*x + c)**3) + 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144
*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) -
 30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c
*x + c)**2 + 48*c**3*(-a*c*x + c)**3) + 10*a**3*c**4*sqrt(-a*c*x + c)/(16*a
*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 5*a**3*c**4*sqrt(c**(-7))*log(
-c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 5*a**3*c**4*sqrt(c**(-7))*log(
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 - 6*a**3*c**3*(-a*c*x + c)**(3/2)
/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 3*a**3*c**3*sqrt(c**(-5)
)*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 + 3*a**3*c**3*sqrt(c**(-5))
*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8
```

Giac [A]
time = 0.42, size = 104, normalized size = 1.17

$$-\frac{\frac{33\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{33\,(acx-c)^2\sqrt{-acx+c}\,a^4c-88\,(-acx+c)^{\frac{3}{2}}a^4c^2+63\sqrt{-acx+c}\,a^4c^3}{a^3c^3x^3}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(33*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (33*(a*c*x - c
)^2*sqrt(-a*c*x + c)*a^4*c - 88*(-a*c*x + c)^(3/2)*a^4*c^2 + 63*sqrt(-a*c*x
 + c)*a^4*c^3)/(a^3*c^3*x^3))/a
```

Mupad [B]
time = 1.20, size = 74, normalized size = 0.83

$$\frac{21\,\sqrt{c-a\,c\,x}}{8\,x^3}-\frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3}+\frac{11\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3}-\frac{a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,11{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

```
[Out] (21*(c - a*c*x)^(1/2))/(8*x^3) - (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c
^(1/2))*11i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (11*(c - a*c*x)^(5/2))/
(8*c^2*x^3)
```

3.309 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)$$

```
[Out] 75/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x^
4+5/8*a*(-a*c*x+c)^(1/2)/x^3+25/32*a^2*(-a*c*x+c)^(1/2)/x^2+75/64*a^3*(-a*c
*x+c)^(1/2)/x
```

Rubi [A]
time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6302, 6265, 21, 79, 44, 65, 214}

$$\frac{75}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+\frac{75a^3\sqrt{c-acx}}{64x}+\frac{25a^2\sqrt{c-acx}}{32x^2}+\frac{\sqrt{c-acx}}{4x^4}+\frac{5a\sqrt{c-acx}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] Sqrt[c - a*c*x]/(4*x^4) + (5*a*Sqrt[c - a*c*x])/(8*x^3) + (25*a^2*Sqrt[c -
a*c*x])/(32*x^2) + (75*a^3*Sqrt[c - a*c*x])/(64*x) + (75*a^4*Sqrt[c]*ArcTan
h[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx \\
&= -\int \frac{(1+ax)\sqrt{c-acx}}{x^5(1-ax)}\,dx \\
&= -\left(c\int \frac{1+ax}{x^5\sqrt{c-acx}}\,dx\right) \\
&= \frac{\sqrt{c-acx}}{4x^4} - \frac{1}{8}(15ac)\int \frac{1}{x^4\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} - \frac{1}{16}(25a^2c)\int \frac{1}{x^3\sqrt{c-acx}}\,dx \\
&= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} - \frac{1}{64}(75a^3c)\int \frac{1}{x^2\sqrt{c-acx}} \\
&= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} + \frac{75a^3\sqrt{c-acx}}{64x} - \frac{1}{128}(75a^4 \\
&= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} + \frac{75a^3\sqrt{c-acx}}{64x} + \frac{1}{64}(75a^3) \\
&= \frac{\sqrt{c-acx}}{4x^4} + \frac{5a\sqrt{c-acx}}{8x^3} + \frac{25a^2\sqrt{c-acx}}{32x^2} + \frac{75a^3\sqrt{c-acx}}{64x} + \frac{75}{64}a^4\sqrt{c}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 71, normalized size = 0.65

$$
\frac{\sqrt{c-acx}\,(16+40ax+50a^2x^2+75a^3x^3)}{64x^4} + \frac{75}{64}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
[Out] (Sqrt[c - a*c*x]*(16 + 40*a*x + 50*a^2*x^2 + 75*a^3*x^3))/(64*x^4) + (75*a^
4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64
```

Maple [A]
time = 0.21, size = 93, normalized size = 0.85

method	result
risch	$-\frac{(75a^4x^4-25a^3x^3-10a^2x^2-24ax-16)c}{64x^4\sqrt{-c(ax-1)}} + \frac{75a^4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c}}{64}$

derivativedivides	$2c^4a^4\left(\frac{-\frac{75(-acx+c)^{\frac{7}{2}}}{128c^3}+\frac{275(-acx+c)^{\frac{5}{2}}}{128c^2}-\frac{365(-acx+c)^{\frac{3}{2}}}{128c}+\frac{181\sqrt{-acx+c}}{128}}{a^4c^4x^4}+\frac{75\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128c^{\frac{7}{2}}}\right)$
default	$2c^4a^4\left(\frac{-\frac{75(-acx+c)^{\frac{7}{2}}}{128c^3}+\frac{275(-acx+c)^{\frac{5}{2}}}{128c^2}-\frac{365(-acx+c)^{\frac{3}{2}}}{128c}+\frac{181\sqrt{-acx+c}}{128}}{a^4c^4x^4}+\frac{75\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{128c^{\frac{7}{2}}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^4*a^4*((-75/128/c^3*(-a*c*x+c)^(7/2)+275/128/c^2*(-a*c*x+c)^(5/2)-365/1
28/c*(-a*c*x+c)^(3/2)+181/128*(-a*c*x+c)^(1/2))/a^4/c^4/x^4+75/128/c^(7/2)*
arctanh((-a*c*x+c)^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.47, size = 163, normalized size = 1.48

$$-\frac{1}{128}a^4c^4\left(\frac{2\left(75\,(-acx+c)^{\frac{7}{2}}-275\,(-acx+c)^{\frac{5}{2}}c+365\,(-acx+c)^{\frac{3}{2}}c^2-181\,\sqrt{-acx+c}\,c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{75\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/128*a^4*c^4*(2*(75*(-a*c*x + c)^(7/2) - 275*(-a*c*x + c)^(5/2)*c + 365*(
-a*c*x + c)^(3/2)*c^2 - 181*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*(a
*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 75*log((
sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(7/2))
```

Fricas [A]
time = 0.37, size = 149, normalized size = 1.35

$$\left[\frac{75\,a^4\sqrt{c}\,x^4\log\left(\frac{acx-2\sqrt{-acx+c}\,\sqrt{c}-2c}{x}\right)+2\,(75\,a^3x^3+50\,a^2x^2+40\,ax+16)\sqrt{-acx+c}}{128\,x^4},\,-\frac{75\,a^4\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-(75\,a^3x^3+50\,a^2x^2+40\,ax+16)\sqrt{-acx+c}}{64\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(75*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x
) + 2*(75*a^3*x^3 + 50*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4, -1/64*
(75*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (75*a^3*x^3 + 50
*a^2*x^2 + 40*a*x + 16)*sqrt(-a*c*x + c))/x^4]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(100) = 200$.
time = 13.76, size = 639, normalized size = 5.81

$$\frac{558a^4c^8\sqrt{-acx+c}}{1536ac^8x-1152c^8+2304c^6\left(-acx+c\right)^2-1536c^5\left(-acx+c\right)^3+384c^4\left(-acx+c\right)^4}-\frac{1022a^4c^7\left(-acx+c\right)^{\frac{3}{2}}}{1536ac^8x-1152c^8+2304c^6\left(-acx+c\right)^2-1536c^5\left(-acx+c\right)^3+384c^4\left(-acx+c\right)^4}+\frac{770a^4c^6\left(-acx+c\right)^{\frac{5}{2}}}{1536ac^8x-1152c^8+2304c^6\left(-acx+c\right)^2-1536c^5\left(-acx+c\right)^3+384c^4\left(-acx+c\right)^4}-\frac{66a^4c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-\frac{210a^4c^5\left(-acx+c\right)^{\frac{7}{2}}}{1536ac^8x-1152c^8+2304c^6\left(-acx+c\right)^2-1536c^5\left(-acx+c\right)^3+384c^4\left(-acx+c\right)^4}+\frac{80a^4c^5\left(-acx+c\right)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-\frac{35a^4c^5\sqrt{\frac{1}{c^9}}\log\left(-c^5\sqrt{\frac{1}{c^9}}+\sqrt{-acx+c}\right)}{128}+\frac{35a^4c^5\sqrt{\frac{1}{c^9}}\log\left(c^5\sqrt{\frac{1}{c^9}}+\sqrt{-acx+c}\right)}{128}-\frac{30a^4c^4\left(-acx+c\right)^{\frac{5}{2}}}{-144ac^6x+96c^6-144c^4\left(-acx+c\right)^2+48c^3\left(-acx+c\right)^3}-\frac{5a^4c^4\sqrt{\frac{1}{c^7}}\log\left(-c^4\sqrt{\frac{1}{c^7}}+\sqrt{-acx+c}\right)}{16}+\frac{5a^4c^4\sqrt{\frac{1}{c^7}}\log\left(c^4\sqrt{\frac{1}{c^7}}+\sqrt{-acx+c}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] 558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c
*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 1022*a
**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x
 + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 770*a**4
*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 66*a**4*c**
6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48
*c**3*(-a*c*x + c)**3) - 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x -
 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c*
*4*(-a*c*x + c)**4) + 80*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 96*
c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 35*a**4*c**5*s
qrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 35*a**4*c**5
*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 - 30*a**4*c**
4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 +
 48*c**3*(-a*c*x + c)**3) - 5*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqrt(c**(-7
)) + sqrt(-a*c*x + c))/16 + 5*a**4*c**4*sqrt(c**(-7))*log(c**4*sqrt(c**(-7)
) + sqrt(-a*c*x + c))/16
```

Giac [**A**]
time = 0.42, size = 131, normalized size = 1.19

$$-\frac{\frac{75\,a^5c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{75\,(acx-c)^3\sqrt{-acx+c}\,a^5c+275\,(acx-c)^2\sqrt{-acx+c}\,a^5c^2-365\,(-acx+c)^{\frac{3}{2}}a^5c^3+181\sqrt{-acx+c}\,a^5c^4}{a^4c^4x^4}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/64*(75*a^5*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - (75*(a*c*x - c
)^3*sqrt(-a*c*x + c)*a^5*c + 275*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^5*c^2 - 3
65*(-a*c*x + c)^(3/2)*a^5*c^3 + 181*sqrt(-a*c*x + c)*a^5*c^4)/(a^4*c^4*x^4)
)/a
```

Mupad [**B**]
time = 0.07, size = 91, normalized size = 0.83

$$\frac{181\sqrt{c-a\,c\,x}}{64\,x^4}-\frac{365\,{\left(c-a\,c\,x\right)}^{3/2}}{64\,c\,x^4}+\frac{275\,{\left(c-a\,c\,x\right)}^{5/2}}{64\,c^2\,x^4}-\frac{75\,{\left(c-a\,c\,x\right)}^{7/2}}{64\,c^3\,x^4}-\frac{a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,75{}\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] (181*(c - a*c*x)^(1/2))/(64*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)
/c^(1/2))*75i)/64 - (365*(c - a*c*x)^(3/2))/(64*c*x^4) + (275*(c - a*c*x)^(
5/2))/(64*c^2*x^4) - (75*(c - a*c*x)^(7/2))/(64*c^3*x^4)
```

3.310 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=309

$$\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-a}}{63a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 1576/315*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(1-1/a/x)^(1/2)+472/315*x*(1+
1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+92/105*x^2*(1+1/a/x)^(1/2
)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+38/63*x^3*(1+1/a/x)^(1/2)*(-a*c*x+c)
^(1/2)/a/(1-1/a/x)^(1/2)+2/9*x^4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)
^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)
^(1/2)*(-a*c*x+c)^(1/2)/a^(9/2)/(1-1/a/x)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6311, 6316, 100, 157, 12, 95, 212}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{9/2}\sqrt{1-\frac{1}{ax}}}+\frac{1576\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}+\frac{38x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (1576*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(315*a^4*Sqrt[1 - 1/(a*x)]) + (472
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(315*a^3*Sqrt[1 - 1/(a*x)]) + (92*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) + (38*Sqrt[
1 + 1/(a*x)]*x^3*Sqrt[c - a*c*x])/(63*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/
(a*x)]*x^4*Sqrt[c - a*c*x])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]
*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])
])/(a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
```

```
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{11/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\,x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}+\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{19}{2a}-\frac{17x}{2a^2}}{x^{9/2}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\cdots\right.}{9\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{38\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.\cdots}{\cdots}$$

$$=\frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{63a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^4\sqrt{c}}{9\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{38\sqrt{1+\frac{1}{ax}}\,x^3}{63a\sqrt{1-}}$$

$$=\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}\,x^2}{105a^2\sqrt{1-}}$$

$$=\frac{1576\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{315a^4\sqrt{1-\frac{1}{ax}}}+\frac{472\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{315a^3\sqrt{1-\frac{1}{ax}}}+\frac{92\sqrt{1+\frac{1}{ax}}\,x^2}{105a^2\sqrt{1-}}$$

Mathematica [A]
time = 0.08, size = 130, normalized size = 0.42

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}\left(788+236ax+138a^2x^2+95a^3x^3+35a^4x^4\right)-630\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{315a^{9/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(788 + 236*a*x + 138*a^2*x^2
+ 95*a^3*x^3 + 35*a^4*x^4) - 630*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt
[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(315*a^(9/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.16, size = 161, normalized size = 0.52

method	result
risch	$-\frac{2(35a^4x^4+95a^3x^3+138a^2x^2+236ax+788)c(ax-1)}{315a^4\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-}{a^4\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{-c(ax-1)}}$
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\left(35a^4x^4\sqrt{-c(ax+1)}+95a^3x^3\sqrt{-c(ax+1)}+138a^2x^2\sqrt{-c(ax+1)}+236ax\sqrt{}\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(a*x-1)*(-c*(a*x-1))^(1/2)*(35*a^4*x^4*(-c*(a*x+1))^(1/2)+95*a^3*x^3*
(-c*(a*x+1))^(1/2)+138*a^2*x^2*(-c*(a*x+1))^(1/2)+236*a*x*(-c*(a*x+1))^(1/2
)-630*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))+788*(-
c*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 304, normalized size = 0.98

$$\left[\frac{2\left(315\sqrt{2}(ax-1)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+\left(35a^5x^5+130a^4x^4+233a^3x^3+374a^2x^2+1024ax+788\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{315(a^5x-a^4)}, -\frac{2\left(630\sqrt{2}(ax-1)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\left(35a^5x^5+130a^4x^4+233a^3x^3+374a^2x^2+1024ax+788\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}\right)}{315(a^5x-a^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/315*(315*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2
)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2
*x^2 - 2*a*x + 1)) + (35*a^5*x^5 + 130*a^4*x^4 + 233*a^3*x^3 + 374*a^2*x^2
+ 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*x - a^4)
, -2/315*(630*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqr
t(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (35*a^5*x^5 + 130*a^4*x^4 + 2
33*a^3*x^3 + 374*a^2*x^2 + 1024*a*x + 788)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/(a^5*x - a^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^3*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.311 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=261

$$\frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-$$

```
[Out] 104/21*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2)+32/21*x*(1+1/a/
x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)+6/7*x^2*(1+1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/7*x^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1
/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*
(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(7/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6311, 6316, 100, 157, 12, 95, 212}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{7/2}\sqrt{1-\frac{1}{ax}}}+\frac{104\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}}+\frac{32x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+\frac{6x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (104*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(21*a^3*Sqrt[1 - 1/(a*x)]) + (32*Sq
rt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(21*a^2*Sqrt[1 - 1/(a*x)]) + (6*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(7*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*x
)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(
a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
```

```
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{9/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{15}{2a}-\frac{13x}{2a^2}}{x^{7/2}\left(1-\frac{x}{a}\right)\sqrt{1+\cdots}}\right.}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\cdots\right.}{\cdots}$$

$$= \frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{7a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-\cdots}}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-\cdots}}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{ax}}} + \frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{ax}}} + \frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-\cdots}}{7a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{104\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{21a^3\sqrt{1-\frac{1}{\cdots}}} + \frac{32\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{21a^2\sqrt{1-\frac{1}{\cdots}}} + \frac{6\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-\cdots}}{7a\sqrt{1-\frac{1}{\cdots}}}$$

Mathematica [A]
time = 0.06, size = 122, normalized size = 0.47

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}\,(52+16ax+9a^2x^2+3a^3x^3)-42\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{21a^{7/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(52 + 16*a*x + 9*a^2*x^2 + 3*
a^3*x^3) - 42*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*
Sqrt[1 + 1/(a*x)])]))/(21*a^(7/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.16, size = 143, normalized size = 0.55

method	result
risch	$-\frac{2(3a^3x^3+9a^2x^2+16ax+52)c(ax-1)}{21a^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a^3\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{-c(ax-1)}}$
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\left(3a^3x^3\sqrt{-c(ax+1)}+9a^2x^2\sqrt{-c(ax+1)}+16ax\sqrt{-c(ax+1)}-42\sqrt{c}\sqrt{2}\,\mathrm{arc}\right.}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(a*x-1)*(-c*(a*x-1))^(1/2)*(3*a^3*x^3*(-c*(a*x+1))^(1/2)+9*a^2*x^2*(-c
*(a*x+1))^(1/2)+16*a*x*(-c*(a*x+1))^(1/2)-42*c^(1/2)*2^(1/2)*arctan(1/2*(-c
*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))+52*(-c*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(
3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 288, normalized size = 1.10

$$\left[\frac{2\left(21\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(3\,a^4x^4+12\,a^3x^3+25\,a^2x^2+68\,ax+52\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{21\,(a^4x-a^3)}, -\frac{2\left(42\,\sqrt{2}\,(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\left(3\,a^4x^4+12\,a^3x^3+25\,a^2x^2+68\,ax+52\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{21\,(a^4x-a^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [2/21*(21*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*sqrt
(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3), -2/21*(42*sqrt(2)*(a
*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x
 + 1))/(a*c*x - c)) - (3*a^4*x^4 + 12*a^3*x^3 + 25*a^2*x^2 + 68*a*x + 52)*s
qrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x - a^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [A]
time = 0.42, size = 145, normalized size = 0.56

$$-\frac{2\,c^2\left(\frac{2\,\sqrt{2}\left(21\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{c}}\right)-40\,\sqrt{-c}\right)}{a^2c}-\frac{42\,\sqrt{2}\,c^{\frac{7}{2}}\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx-c}}{2\,\sqrt{c}}\right)-3\,(acx+c)^3\sqrt{-acx-c}+7\,(-acx-c)^{\frac{3}{2}}c^2-42\,\sqrt{-acx-c}\,c^3}{a^2c^4}\right)}{21\,a|c|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a*c*x+c)^(1/2),x, algorithm="giac
")
```

```
[Out] -2/21*c^2*(2*sqrt(2)*(21*sqrt(c)*arctan(sqrt(-c)/sqrt(c)) - 40*sqrt(-c))/(a
^2*c) - (42*sqrt(2)*c^(7/2)*arctan(1/2*sqrt(2)*sqrt(-a*c*x - c)/sqrt(c)) -
```

```
3*(a*c*x + c)^3*sqrt(-a*c*x - c) + 7*(-a*c*x - c)^(3/2)*c^2 - 42*sqrt(-a*c*
x - c)*c^3)/(a^2*c^4))/(a*abs(c)*sgn(a*x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^2*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.312 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=211

$$\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-}}{a^{5/2}\sqrt{1-}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)+2/5*(1+1/a/x)^(5/2
)*x^2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a
^2/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2
^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(5/2)/(1-1/a/x)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6311, 6316, 98, 96, 95, 212}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{5/2}\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a^2*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(
a*x))^(3/2)*x*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*x))^(
5/2)*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])
/(a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{7/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.}{}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-}}{5\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-}}{5\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{5/2}x^2\sqrt{c-}}{5\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]

time = 0.05, size = 114, normalized size = 0.54

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}\left(38+11ax+3a^2x^2\right)-30\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{15a^{5/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(38 + 11*a*x + 3*a^2*x^2) - 3
0*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(
a*x)])]))/(15*a^(5/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.16, size = 125, normalized size = 0.59

method	result
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\left(3a^2x^2\sqrt{-c(ax+1)}+11ax\sqrt{-c(ax+1)}-30\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}}{2\sqrt{c}}\right.\right.}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a^2}$
risch	$-\frac{2(3a^2x^2+11ax+38)c(ax-1)}{15a^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a^2\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{-c(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(a*x-1)*(-c*(a*x-1))^(1/2)*(3*a^2*x^2*(-c*(a*x+1))^(1/2)+11*a*x*(-c*(a
*x+1))^(1/2)-30*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/
2))+38*(-c*(a*x+1))^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/
2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.36, size = 272, normalized size = 1.29

$$\left[\frac{2\left(15\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+\left(3\,a^3x^3+14\,a^2x^2+49\,ax+38\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{15\left(a^3x-a^2\right)}, -\frac{2\left(30\sqrt{2}\,(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\left(3\,a^3x^3+14\,a^2x^2+49\,ax+38\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{15\left(a^3x-a^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] [2/15*(15*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*
sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x
^2 - 2*a*x + 1)) + (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*
sqrt((a*x - 1)/(a*x + 1)))/(a^3*x - a^2), -2/15*(30*sqrt(2)*(a*x - 1)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - (3*a^3*x^3 + 14*a^2*x^2 + 49*a*x + 38)*sqrt(-a*c*x + c)*sqrt((a*x
- 1)/(a*x + 1)))/(a^3*x - a^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x*(c - a*c*x)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.313 $\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=163

$$\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*(1+1/a/x)^(3/2)*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+4*(1+1/a/x)^(1/2)*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(2^(1/2)*(1/x)^(1/2)/a^(1/2)/(1+1
/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(3/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{a^{3/2}\sqrt{1-\frac{1}{ax}}}+\frac{2x\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (4*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(a*Sqrt[1 - 1/(a*x)]) + (2*(1 + 1/(a*
x))^(3/2)*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[x^(-1)
]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)]
)])/(a^(3/2)*Sqrt[1 - 1/(a*x)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{5/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Su}}{}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Su}}{a}$$

$$=\frac{4\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}}+\frac{2\left(1+\frac{1}{ax}\right)^{3/2}x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{ta}}{a^{3/2}\sqrt{}}$$

Mathematica [A]

time = 0.04, size = 105, normalized size = 0.64

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}(7+ax)-6\sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)\right)}{3a^{3/2}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)]*(7 + a*x) - 6*Sqrt[2]*Sqrt[x^
(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/(3*a^(3
/2)*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.15, size = 106, normalized size = 0.65

method	result
default	$\frac{2(ax-1)\sqrt{-c(ax-1)}\left(ax\sqrt{-c(ax+1)}-6\sqrt{c}\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)+7\sqrt{-c(ax+1)}\right)}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax+1)}\,a}$
risch	$-\frac{2(ax+7)c(ax-1)}{3a\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c(ax-1)}}-\frac{4\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)\sqrt{-c(ax+1)}(ax-1)}{a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{-c(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(a*x-1)*(-c*(a*x-1))^(1/2)*(a*x*(-c*(a*x+1))^(1/2)-6*c^(1/2)*2^(1/2)*ar
ctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))+7*(-c*(a*x+1))^(1/2))/((a*x-1)
/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 250, normalized size = 1.53

$$\left[\frac{2\left(3\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(a^2x^2+8\,ax+7)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)},-\frac{2\left(6\,\sqrt{2}\,(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(a^2x^2+8\,ax+7)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\right)}{3\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sq
rt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2
 - 2*a*x + 1)) + (a^2*x^2 + 8*a*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
 + 1)))/(a^2*x - a), -2/3*(6*sqrt(2)*(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(
-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (a^2*x^2 + 8*a
*x + 7)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x - a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a*c*x)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.314 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=170

$$\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(1/
2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2
)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6311, 6316, 100, 163, 56, 221, 95, 212}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)]) -
 (4*Sqrt[2]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
```

```
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^{3/2}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,d\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{\frac{1}{x}}\sqrt{c}}{}$$

Mathematica [A]

time = 0.04, size = 120, normalized size = 0.71

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\,\sqrt{1+\frac{1}{ax}}+\sqrt{\frac{1}{x}}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-2\sqrt{2}\,\sqrt{\frac{1}{x}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{a}\,\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x,x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]] - 2*Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sq
rt[a]*Sqrt[1 + 1/(a*x)])]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.15, size = 107, normalized size = 0.63

method	result
default	$\frac{2(ax-1)\sqrt{-c\,(ax-1)}\,\left(-2\sqrt{c}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)+\sqrt{c}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c}$
	$\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*x-1)*(-c*(a*x-1))^(1/2)*(-2*c^(1/2)*2^(1/2)*arctan(1/2*(-c*(a*x+1))^(1
/2)*2^(1/2)/c^(1/2))+c^(1/2)*arctan((-c*(a*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1)
)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/(-c*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.36, size = 352, normalized size = 2.07

$$\left[\frac{2\sqrt{2}\,(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},\,-\frac{2\left(2\sqrt{2}\,(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="fricas
")
```

```
[Out] [(2*sqrt(2)*(a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt(2)*sqrt(-
a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a^2*x^2 - 2
*a*x + 1)) + (a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt
(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*(2*sqrt(2)*
(a*x - 1)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a
*x + 1))/(a*c*x - c)) - (a*x - 1)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*s
qrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*
x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.315 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x}+\frac{5\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{}}{\sqrt{a}\,\sqrt{1-}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+5*arcsinh((1/x)^(1/2)/a^
(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(2^(1/
2)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*a^(1/2)*(1/x)^(1/2)*(-a*c*x
+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6311, 6316, 104, 163, 56, 221, 95, 212}

$$\frac{\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (5*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)] -
 (4*Sqrt[2]*Sqrt[a]*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{5x}{2a^2}}{\sqrt{x}\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right.}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{5\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{a}\,\sqrt{}}{}$$

Mathematica [A]

time = 0.05, size = 120, normalized size = 0.70

$$\frac{\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\,\sqrt{\frac{1}{x}}+5\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)-4\sqrt{2}\,\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{a}\,\sqrt{1+\frac{1}{ax}}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^2,x]
```

```
[Out] (Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)] + 5*Sqrt[a]*A
rcSinh[Sqrt[x^(-1)]/Sqrt[a]] - 4*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[x^(-
1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])]))/Sqrt[1 - 1/(a*x)]
```

Maple [A]
time = 0.18, size = 117, normalized size = 0.68

method	result
default	$\frac{(ax-1)\sqrt{-c\,(ax-1)}\left(-4\sqrt{2}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)acx+5\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c\,(}\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax+1)}\,x\sqrt{c}}$
risch	$-\frac{c(ax-1)}{x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}-\frac{\left(\frac{4a\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{5a\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c\sqrt{-c\,(ax}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-4*2^(1/2)*ar
ctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a*c*x+5*arctan((-c*(a*x+1))^(1
/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))/(-c*(a*x+1))^(1/2)/x/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 390, normalized size = 2.27

$$\left[\frac{4\,\sqrt{2}\,(a^2x^2-ax)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^2x^2-2\,ax+1}\right)+5\,(a^2x^2-ax)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx-2\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^2-x)},-\frac{4\,\sqrt{2}\,(a^2x^2-ax)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-5\,(a^2x^2-ax)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2*sqrt
(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*c)/(a
^2*x^2 - 2*a*x + 1)) + 5*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x -
 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*
x^2 - x)) + 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
- x), -(4*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 5*(a^2*x^2 - a*x)*sqrt(c)*a
rctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^2,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a*c*x)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.316 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=224

$$\frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x}+\frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x}+\frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,a^{3/2}\sqrt{\frac{1}{x}}\ \cdots}{\cdots}$$

```
[Out] 1/2*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7/4*a*(1+1/a/x)^(1
/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+23/4*a^(3/2)*arcsinh((1/x)^(1/2)/a^(
1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(3/2)*arctanh(2^(1/2
)*(1/x)^(1/2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)
/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6311, 6316, 103, 159, 163, 56, 221, 95, 212}

$$\frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{2x\sqrt{1-\frac{1}{ax}}}+\frac{7a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (7*a*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (a*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x) + (23*a^(3/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(3/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}\left(\frac{1}{2}+\frac{7x}{2a}\right)}{\sqrt{x}\left(1-\frac{x}{a}\right)}\,dx\right.}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(23a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{}$$

$$= \frac{7a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x} + \frac{23a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}}{4\sqrt{1-}}$$

Mathematica [A]
time = 0.11, size = 132, normalized size = 0.59

$$\frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\,(2+9ax)+\frac{23a^{3/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}-\frac{16\sqrt{2}\,a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{4\sqrt{1-\frac{1}{ax}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^3,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(2 + 9*a*x) + (23*a^(3/2)*ArcSinh[Sqrt[
x^(-1)]/Sqrt[a]])/(x^(-1))^(3/2) - (16*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[2]*Sqr
t[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(3/2)))/(4*Sqrt[1 - 1/(a*
x)]*x^2)
```

Maple [A]
time = 0.16, size = 144, normalized size = 0.64

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-16\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^2cx^2+23c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^2x^2+9ax\right.}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\sqrt{-c(ax+1)}\,x^2}$
risch	$-\frac{(9a^2x^2+11ax+2)c(ax-1)}{4x^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^2\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{23a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{}}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-16*2^(1/2)
*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^2*c*x^2+23*c*arctan((-c*(
a*x+1))^(1/2)/c^(1/2))*a^2*x^2+9*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)+2*(-c*(a*x+
1))^(1/2)*c^(1/2))/c^(1/2)/(-c*(a*x+1))^(1/2)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.36, size = 428, normalized size = 1.91

$$\left[\frac{16\sqrt{2}\left(a^{3}x^{3}-a^{2}x^{2}\right)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+2\,acx+2\sqrt{2}\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3\,c}{a^{2}x^{2}-2\,ax+1}\right)+23\left(a^{3}x^{3}-a^{2}x^{2}\right)\sqrt{-c}\log\left(-\frac{a^{2}cx^{2}+acx-2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^{2}-x}\right)+2\left(9\,a^{2}x^{2}+11\,ax+2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{8\left(ax^{3}-x^{2}\right)},-\frac{16\sqrt{2}\left(a^{3}x^{3}-a^{2}x^{2}\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-23\left(a^{3}x^{3}-a^{2}x^{2}\right)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\left(9\,a^{2}x^{2}+11\,ax+2\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{4\left(ax^{3}-x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="fric
as")
```

```
[Out] [1/8*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x + 2
*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3*
c)/(a^2*x^2 - 2*a*x + 1)) + 23*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2
 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
- 2*c)/(a*x^2 - x)) + 2*(9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x + c)*sqrt((a*x
 - 1)/(a*x + 1)))/(a*x^3 - x^2), -1/4*(16*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(
c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x
 - c)) - 23*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqr
t((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (9*a^2*x^2 + 11*a*x + 2)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^3,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.317 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=274

$$\frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2}+\frac{13a^2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x}+\frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x}+\frac{45a^{5/2}\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sin}{8\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 1/3*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+3/4*a^2*(1+1/a/x
)^(3/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+13/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x
+c)^(1/2)/x/(1-1/a/x)^(1/2)+45/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)
^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(5/2)*arctanh(2^(1/2)*(1/x)^(1/
2)/a^(1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6311, 6316, 103, 159, 163, 56, 221, 95, 212}

$$\frac{45a^{5/2}\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,a^{5/2}\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{3a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}+\frac{13a^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}+\frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{3x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^2) + (13*a^2
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (3*a^2*(1 + 1
/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) + (45*a^(5/2)*Sqrt[x
^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
 - (4*Sqrt[2]*a^(5/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sqrt[x^
(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\,\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} - \frac{\left(a\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\,\sqrt{1+\frac{x}{a}}\,\left(\frac{3}{2}+\frac{9x}{2a}\right)}{1-\frac{x}{a}} \ldots\right.}{3\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(a^3\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right) \ldots}{\ldots}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-a \ldots}}{4\sqrt{1-\frac{1}{ax}}\,x} \ldots$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-a \ldots}}{4\sqrt{1-\frac{1}{ax}}\,x} \ldots$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-a \ldots}}{4\sqrt{1-\frac{1}{ax}}\,x} \ldots$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{3 \ldots} + \frac{13a^2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{8 \ldots} + \frac{3a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-a \ldots}}{4 \ldots} \ldots$$

Mathematica [A]
time = 0.14, size = 140, normalized size = 0.51

$$\frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\,(8+26ax+57a^2x^2)+\frac{135a^{5/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}-\frac{96\sqrt{2}\,a^{5/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{24\sqrt{1-\frac{1}{ax}}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^4,x]
```

```
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(8 + 26*a*x + 57*a^2*x^2) + (135*a^(5/2
)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2) - (96*Sqrt[2]*a^(5/2)*ArcTa
nh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/(x^(-1))^(5/2)))/(2
4*Sqrt[1 - 1/(a*x)]*x^3)
```

Maple [A]
time = 0.16, size = 165, normalized size = 0.60

method	result
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-96\sqrt{2}\arctan\left(\frac{\sqrt{-c(ax+1)}\sqrt{2}}{2\sqrt{c}}\right)a^3cx^3+135c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^3x^3+5\right.}{24\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{c}\sqrt{-c(ax+\cdots}}$
risch	$-\frac{(57a^3x^3+83a^2x^2+34ax+8)c(ax-1)}{24x^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^3\sqrt{2}\arctan\left(\frac{\sqrt{-acx-c}\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{45a^3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-96*2^(1/2
)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^3*c*x^3+135*c*arctan((-c
*(a*x+1))^(1/2)/c^(1/2))*a^3*x^3+57*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)+26*a
*x*(-c*(a*x+1))^(1/2)*c^(1/2)+8*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(-c*(a*
x+1))^(1/2)/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 444, normalized size = 1.62

$$\left[\frac{96\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+2acx+2\sqrt{2}\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-3c}{a^2x^2-2ax+1}\right)+135\left(a^4x^4-a^3x^3\right)\sqrt{-c}\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}(ax+1)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\left(57a^3x^3+83a^2x^2+34ax+8\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{48\left(ax^4-x^3\right)}, -\frac{96\sqrt{2}\left(a^4x^4-a^3x^3\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-135\left(a^4x^4-a^3x^3\right)\sqrt{c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\left(57a^3x^3+83a^2x^2+34ax+8\right)\sqrt{-acx+c}\sqrt{\frac{ax-1}{ax+1}}}{24\left(ax^4-x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="fric
as")
```

```
[Out] [1/48*(96*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x +
2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 3
*c)/(a^2*x^2 - 2*a*x + 1)) + 135*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x
^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)
) - 2*c)/(a*x^2 - x)) + 2*(57*a^3*x^3 + 83*a^2*x^2 + 34*a*x + 8)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^3), -1/24*(96*sqrt(2)*(a^4*x^4
 - a^3*x^3)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/
(a*x + 1))/(a*c*x - c)) - 135*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(sqrt(-a*c*
x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (57*a^3*x^3 + 83*a^
2*x^2 + 34*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^4 - x^
3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^4,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.318 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=322

$$\frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3}+\frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2}+\frac{107a^3\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{64\sqrt{1-\frac{1}{ax}}\,x}+\frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-ac}}{32\sqrt{1-\frac{1}{ax}}\,x}$$

```
[Out] 1/4*a*(1+1/a/x)^(3/2)*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+11/24*a^2*(1+1/a
/x)^(3/2)*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+21/32*a^3*(1+1/a/x)^(3/2)*(-
a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+107/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2
)/x/(1-1/a/x)^(1/2)+363/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)
*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)-4*a^(7/2)*arctanh(2^(1/2)*(1/x)^(1/2)/a^(
1/2)/(1+1/a/x)^(1/2))*2^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6311, 6316, 103, 159, 163, 56, 221, 95, 212}

$$\frac{363a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\,a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{\frac{1}{ax}+1}}\right)}{\sqrt{1-\frac{1}{ax}}}+\frac{21a^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{32x\sqrt{1-\frac{1}{ax}}}+\frac{107a^3\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}+\frac{11a^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{24x^2\sqrt{1-\frac{1}{ax}}}+\frac{a\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-acx}}{4x^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
```

```
[Out] (a*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^3) + (11*a^2
*(1 + 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^2) + (107*a^3
*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) + (21*a^3*(1 +
 1/(a*x))^(3/2)*Sqrt[c - a*c*x])/(32*Sqrt[1 - 1/(a*x)]*x) + (363*a^(7/2)*Sq
rt[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a
*x)]) - (4*Sqrt[2]*a^(7/2)*Sqrt[x^(-1)]*Sqrt[c - a*c*x]*ArcTanh[(Sqrt[2]*Sq
rt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} - \frac{\left(a\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\sqrt{1+\frac{x}{a}}\left(\frac{5}{2}+\frac{11x}{2a}\right.}{1-\frac{x}{a}}\right.}{4\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{\left(a^3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.}{}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{21a^3\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c}}{32\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c}}{64\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c}}{64\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{a\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{11a^2\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{107a^3\sqrt{1+\frac{1}{ax}}\sqrt{c}}{64\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.18, size = 148, normalized size = 0.46

$$\frac{\sqrt{c-acx}\left(\sqrt{1+\frac{1}{ax}}\,(48+136ax+214a^2x^2+447a^3x^3)+\frac{1089a^{7/2}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}-\frac{768\sqrt{2}\,a^{7/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{a}\sqrt{1+\frac{1}{ax}}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\right)}{192\sqrt{1-\frac{1}{ax}}\,x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a*c*x])/x^5,x]
[Out] (Sqrt[c - a*c*x]*(Sqrt[1 + 1/(a*x)]*(48 + 136*a*x + 214*a^2*x^2 + 447*a^3*x
^3) + (1089*a^(7/2)*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2) - (768*Sq
rt[2]*a^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/(Sqrt[a]*Sqrt[1 + 1/(a*x)])])/
(x^(-1))^(7/2)))/(192*Sqrt[1 - 1/(a*x)]*x^4)
```

Maple [A]
time = 0.17, size = 186, normalized size = 0.58

method	result
risch	$-\frac{(447a^4x^4+661a^3x^3+350a^2x^2+184ax+48)c(ax-1)}{192x^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}-\frac{\left(\frac{4a^4\sqrt{2}\,\arctan\left(\frac{\sqrt{-acx-c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{363a^4\arctan\left(\frac{\sqrt{-acx-}}{\sqrt{c}}\right.}{64\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c(ax-1)}}$
default	$\frac{(ax-1)\sqrt{-c(ax-1)}\left(-768\sqrt{2}\,\arctan\left(\frac{\sqrt{-c(ax+1)}\,\sqrt{2}}{2\sqrt{c}}\right)a^4cx^4+1089c\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4+\right.}{192\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
[Out] 1/192/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(-c*(a*x-1))^(1/2)*(-768*2^(1
/2)*arctan(1/2*(-c*(a*x+1))^(1/2)*2^(1/2)/c^(1/2))*a^4*c*x^4+1089*c*arctan(
(-c*(a*x+1))^(1/2)/c^(1/2))*a^4*x^4+447*a^3*x^3*(-c*(a*x+1))^(1/2)*c^(1/2)+
214*a^2*x^2*(-c*(a*x+1))^(1/2)*c^(1/2)+136*a*x*(-c*(a*x+1))^(1/2)*c^(1/2)+4
8*(-c*(a*x+1))^(1/2)*c^(1/2))/c^(1/2)/(-c*(a*x+1))^(1/2)/x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a*c*x + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 460, normalized size = 1.43

$$\left[\frac{768\,\sqrt{2}\,(a^5x^5 - a^4x^4)\sqrt{-c}\,\log\left(-\frac{a^2cx^2 + 2\,acx + 2\,\sqrt{2}\,\sqrt{-acx + c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}} - 3\,c}{a^2x^2 - 2\,ax + 1}\right) + 1089\,(a^5x^5 - a^4x^4)\sqrt{-c}\,\log\left(-\frac{a^2cx^2 + acx - 2\,\sqrt{-acx + c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}} - 2\,c}{ax^2 - x}\right) + 2\,(447\,a^4x^4 + 661\,a^3x^3 + 350\,a^2x^2 + 184\,ax + 48)\sqrt{-acx + c}\,\sqrt{\frac{ax-1}{ax+1}}}{384\,(ax^5 - x^4)}, -\frac{768\,\sqrt{2}\,(a^5x^5 - a^4x^4)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx + c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx - c}\right) - 1089\,(a^5x^5 - a^4x^4)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx + c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx - c}\right) - (447\,a^4x^4 + 661\,a^3x^3 + 350\,a^2x^2 + 184\,ax + 48)\sqrt{-acx + c}\,\sqrt{\frac{ax-1}{ax+1}}}{192\,(ax^5 - x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="fric
as")
```

```
[Out] [1/384*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + 2*a*c*x
+ 2*sqrt(2)*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) -
 3*c)/(a^2*x^2 - 2*a*x + 1)) + 1089*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*
c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x +
 1)) - 2*c)/(a*x^2 - x)) + 2*(447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184
*a*x + 48)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/19
2*(768*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*arctan(sqrt(2)*sqrt(-a*c*x + c)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - 1089*(a^5*x^5 - a^4*x^4)*s
qrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c
)) - (447*a^4*x^4 + 661*a^3*x^3 + 350*a^2*x^2 + 184*a*x + 48)*sqrt(-a*c*x +
 c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a*c*x+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a*c*x+c)^(1/2)/x^5,x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a*c*x)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.319 $\int e^{\coth^{-1}(x)}x(1+x)^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{46\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{8\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

```
[Out] 46/21*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+92/21*(1+x)^(3/2)*((-1+x)/
x)^(1/2)/(1+1/x)^(3/2)/x+8/7*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+2
/7*x^2*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {6311, 6316, 91, 79, 47, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}x^2}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{8\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}x}{7\left(\frac{1}{x}+1\right)^{3/2}}+\frac{46\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}}+\frac{92\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{21\left(\frac{1}{x}+1\right)^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (46*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)) + (92*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(21*(1 + x^(-1))^(3/2)*x) + (8*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(7*(1 + x^(-1))^(3/2)) + (2*Sqrt[-((1 - x)/x)]*x^2*(1 +
x)^(3/2))/(7*(1 + x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)} x(1+x)^{3/2}\,dx = \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2} x^{5/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2} x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{(1+x)^2}{\sqrt{1-x}\,x^{9/2}}\,dx, x, \frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{10+\frac{7x}{2}}{\sqrt{1-x}\,x^{7/2}}\,dx, x, \ldots\right)}{7\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{8\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(23\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\ldots}{7\ldots}$$

$$= \frac{46\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}} + \frac{8\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x^2(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}\ldots$$

$$= \frac{46\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}} + \frac{92\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{21\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{8\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{7\left(1+\frac{1}{x}\right)^{3/2}}\ldots$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.32

$$\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}\left(46+23x+12x^2+3x^3\right)}{21\sqrt{1+\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*x*(1 + x)^(3/2),x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(46 + 23*x + 12*x^2 + 3*x^3))/(21*Sqrt[1 +
x^(-1)])
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.26

method	result	size

gosper	$\frac{2(-1+x)(3x^3+12x^2+23x+46)}{21\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$	37
default	$\frac{2(-1+x)(3x^3+12x^2+23x+46)}{21\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$	37
risch	$\frac{2(-1+x)(3x^3+12x^2+23x+46)}{21\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(-1+x)*(3*x^3+12*x^2+23*x+46)/(1+x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A]
time = 0.26, size = 27, normalized size = 0.19

$$\frac{2\,(3\,x^4+9\,x^3+11\,x^2+23\,x-46)}{21\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^4 + 9*x^3 + 11*x^2 + 23*x - 46)/sqrt(x - 1)
```

Fricas [A]
time = 0.33, size = 33, normalized size = 0.23

$$\frac{2}{21}\left(3\,x^3+12\,x^2+23\,x+46\right)\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*x^3 + 12*x^2 + 23*x + 46)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 71.45, size = 194, normalized size = 1.35

$$-2\left(\begin{cases}\frac{8x\sqrt{x-1}}{15}+\frac{\sqrt{x-1}\,(x+1)^2}{5}+\frac{8\sqrt{x-1}}{3} & \text{for } |x+1|>2\\ \frac{8ix\sqrt{1-x}}{15}+\frac{i\sqrt{1-x}\,(x+1)^2}{5}+\frac{8i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)+2\left(\begin{cases}\frac{32x\sqrt{x-1}}{35}+\frac{\sqrt{x-1}\,(x+1)^3}{7}+\frac{12\sqrt{x-1}\,(x+1)^2}{35}+\frac{32\sqrt{x-1}}{7} & \text{for } |x+1|>2\\ \frac{32ix\sqrt{1-x}}{35}+\frac{i\sqrt{1-x}\,(x+1)^3}{7}+\frac{12i\sqrt{1-x}\,(x+1)^2}{35}+\frac{32i\sqrt{1-x}}{7} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(3/2),x)
```

```
[Out] -2*Piecewise((8*x*sqrt(x - 1)/15 + sqrt(x - 1)*(x + 1)**2/5 + 8*sqrt(x - 1)
/3, Abs(x + 1) > 2), (8*I*x*sqrt(1 - x)/15 + I*sqrt(1 - x)*(x + 1)**2/5 + 8
*I*sqrt(1 - x)/3, True)) + 2*Piecewise((32*x*sqrt(x - 1)/35 + sqrt(x - 1)*(
x + 1)**3/7 + 12*sqrt(x - 1)*(x + 1)**2/35 + 32*sqrt(x - 1)/7, Abs(x + 1) >
 2), (32*I*x*sqrt(1 - x)/35 + I*sqrt(1 - x)*(x + 1)**3/7 + 12*I*sqrt(1 - x)
*(x + 1)**2/35 + 32*I*sqrt(1 - x)/7, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.33, size = 48, normalized size = 0.33

$$\sqrt{\frac{x-1}{x+1}}\left(\frac{46\,x\,\sqrt{x+1}}{21}+\frac{92\,\sqrt{x+1}}{21}+\frac{8\,x^2\,\sqrt{x+1}}{7}+\frac{2\,x^3\,\sqrt{x+1}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((x - 1)/(x + 1))^(1/2)*((46*x*(x + 1)^(1/2))/21 + (92*(x + 1)^(1/2))/21 +
(8*x^2*(x + 1)^(1/2))/7 + (2*x^3*(x + 1)^(1/2))/7)
```

3.320 $\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{28\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x}+\frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

```
[Out] 28/15*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)+86/15*(1+x)^(3/2)*((-1+x)/
x)^(1/2)/(1+1/x)^(3/2)/x+2/5*x*(1+x)^(3/2)*((-1+x)/x)^(1/2)/(1+1/x)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6311, 6316, 91, 79, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\,x(x+1)^{3/2}}{5\left(\frac{1}{x}+1\right)^{3/2}}+\frac{28\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}}+\frac{86\sqrt{-\frac{1-x}{x}}\,(x+1)^{3/2}}{15\left(\frac{1}{x}+1\right)^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 + x)^(3/2),x]
```

```
[Out] (28*Sqrt[-((1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)) + (86*Sqrt[-(
(1 - x)/x)]*(1 + x)^(3/2))/(15*(1 + x^(-1))^(3/2)*x) + (2*Sqrt[-((1 - x)/x)
]*x*(1 + x)^(3/2))/(5*(1 + x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}(1+x)^{3/2}\,dx = \frac{(1+x)^{3/2}\int e^{\coth^{-1}(x)}\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\,dx}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{(1+x)^2}{\sqrt{1-x}\,x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(2\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Subst}\left(\int\frac{7+\frac{5x}{2}}{\sqrt{1-x}\,x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

$$= \frac{28\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}} - \frac{\left(43\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}\right)\text{Sub}}{15\,(1}$$

$$= \frac{28\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}} + \frac{86\sqrt{-\frac{1-x}{x}}\,(1+x)^{3/2}}{15\left(1+\frac{1}{x}\right)^{3/2}x} + \frac{2\sqrt{-\frac{1-x}{x}}\,x(1+x)^{3/2}}{5\left(1+\frac{1}{x}\right)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.38

$$\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}\,(43+14x+3x^2)}{15\sqrt{1+\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 + x)^(3/2),x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(43 + 14*x + 3*x^2))/(15*Sqrt[1 + x^(-1)])
```

Maple [A]
time = 0.07, size = 32, normalized size = 0.30

method	result	size
gosper	$\frac{2(-1+x)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$	32
default	$\frac{2(-1+x)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$	32
risch	$\frac{2(-1+x)(3x^2+14x+43)}{15\sqrt{1+x}\,\sqrt{\frac{-1+x}{1+x}}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(-1+x)*(3*x^2+14*x+43)/(1+x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [A]
time = 0.26, size = 22, normalized size = 0.21

$$\frac{2\,(3\,x^3+11\,x^2+29\,x-43)}{15\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
[Out] 2/15*(3*x^3 + 11*x^2 + 29*x - 43)/sqrt(x - 1)
```

Fricas [A]
time = 0.33, size = 28, normalized size = 0.26

$$\frac{2}{15}\,(3\,x^2+14\,x+43)\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + 14*x + 43)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Integral((x + 1)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.26, size = 38, normalized size = 0.36

$$\sqrt{\frac{x-1}{x+1}}\,\left(\frac{28\,x\,\sqrt{x+1}}{15}+\frac{86\,\sqrt{x+1}}{15}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((x - 1)/(x + 1))^(1/2)*((28*x*(x + 1)^(1/2))/15 + (86*(x + 1)^(1/2))/15 +
(2*x^2*(x + 1)^(1/2))/5)
```

3.321 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx$

Optimal. Leaf size=104

$$\frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] 44/105*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)-22/35*(1+1/x)^(3/2)*(1-x)^(3
/2)*x/(1-1/x)^(3/2)+2/7*(1+1/x)^(3/2)*(1-x)^(3/2)*x^2/(1-1/x)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6311, 6316, 79, 47, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{44\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
```

```
[Out] (44*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(105*(1 - x^(-1))^(3/2)) - (22*(1 + x
^(-1))^(3/2)*(1 - x)^(3/2)*x)/(35*(1 - x^(-1))^(3/2)) + (2*(1 + x^(-1))^(3/
2)*(1 - x)^(3/2)*x^2)/(7*(1 - x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(x)}(1-x)^{3/2}x\,dx &= \frac{(1-x)^{3/2}\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^{3/2}x^{5/2}\,dx}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}\\
&= -\frac{\left((1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\mathrm{Subst}\left(\int\frac{(1-x)\sqrt{1+x}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}+\frac{\left(11(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\mathrm{Subst}\left(\int\frac{\sqrt{1+x}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{7\left(1-\frac{1}{x}\right)^{3/2}}\\
&= -\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}-\frac{\left(22(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right.}{35}\\
&= \frac{44\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{105\left(1-\frac{1}{x}\right)^{3/2}}-\frac{22\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{35\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x^2}{7\left(1-\frac{1}{x}\right)^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 46, normalized size = 0.44

$$-\frac{2\sqrt{1+\frac{1}{x}}\,\sqrt{1-x}\,(1+x)\,(22-33x+15x^2)}{105\sqrt{\frac{-1+x}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2)*x,x]
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(22 - 33*x + 15*x^2))/(105*Sqrt[(-
1 + x)/x])
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.33

method	result	size
gosper	$-\frac{2(1+x)(15x^2-33x+22)\sqrt{1-x}}{105\sqrt{\frac{-1+x}{1+x}}}$	34
default	$-\frac{2(1+x)(15x^2-33x+22)\sqrt{1-x}}{105\sqrt{\frac{-1+x}{1+x}}}$	34
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)(15x^3-18x^2-11x+22)}{105\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x,method=_RETURNVERBOSE)
[Out] -2/105*(1+x)*(15*x^2-33*x+22)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.26, size = 22, normalized size = 0.21

$$\frac{2}{105}\left(-15i\,x^3+18i\,x^2+11i\,x-22i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="maxima")
[Out] 2/105*(-15*I*x^3 + 18*I*x^2 + 11*I*x - 22*I)*sqrt(x + 1)
```

Fricas [A]
time = 0.32, size = 45, normalized size = 0.43

$$-\frac{2\,(15\,x^4-3\,x^3-29\,x^2+11\,x+22)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] -2/105*(15*x^4 - 3*x^3 - 29*x^2 + 11*x + 22)*sqrt(-x + 1)*sqrt((x - 1)/(x +
 1))/(x - 1)
```

Sympy [A]
time = 175.10, size = 107, normalized size = 1.03

$$\frac{2(1-x)^{\frac{7}{2}}}{7\sqrt{-\frac{x}{-x-1}+\frac{1}{-x-1}}}-\frac{18(1-x)^{\frac{5}{2}}}{35\sqrt{-\frac{x}{-x-1}+\frac{1}{-x-1}}}-\frac{4(1-x)^{\frac{3}{2}}}{105\sqrt{-\frac{x}{-x-1}+\frac{1}{-x-1}}}-\frac{16\sqrt{1-x}}{105\sqrt{-\frac{x}{-x-1}+\frac{1}{-x-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2)*x,x)
```

```
[Out] 2*(1 - x)**(7/2)/(7*sqrt(-x/(-x - 1) + 1/(-x - 1))) - 18*(1 - x)**(5/2)/(35
*sqrt(-x/(-x - 1) + 1/(-x - 1))) - 4*(1 - x)**(3/2)/(105*sqrt(-x/(-x - 1) +
 1/(-x - 1))) - 16*sqrt(1 - x)/(105*sqrt(-x/(-x - 1) + 1/(-x - 1)))
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 63, normalized size = 0.61

$$\frac{16}{105}i\sqrt{2}\,\mathrm{sgn}(x+1)-\frac{2\left(15\,(x+1)^3\sqrt{-x-1}-63\,(x+1)^2\sqrt{-x-1}-70\,(-x-1)^{\frac{3}{2}}-8i\sqrt{2}\right)\mathrm{sgn}(x)}{105\,\mathrm{sgn}\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 16/105*I*sqrt(2)*sgn(x + 1) - 2/105*(15*(x + 1)^3*sqrt(-x - 1) - 63*(x + 1)
^2*sqrt(-x - 1) - 70*(-x - 1)^(3/2) - 8*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

Mupad [B]
time = 1.31, size = 35, normalized size = 0.34

$$\frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2\,(15\,x^2-33\,x+22)}{105\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x)^(3/2))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2*(15*x^2 - 33*x + 22))/(105*(1 - x)^(1/
2))
```

3.322 $\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx$

Optimal. Leaf size=68

$$-\frac{14\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}$$

```
[Out] -14/15*(1+1/x)^(3/2)*(1-x)^(3/2)/(1-1/x)^(3/2)+2/5*(1+1/x)^(3/2)*(1-x)^(3/2
)*x/(1-1/x)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6311, 6316, 79, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}-\frac{14\left(\frac{1}{x}+1\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*(1 - x)^(3/2),x]
```

```
[Out] (-14*(1 + x^(-1))^(3/2)*(1 - x)^(3/2))/(15*(1 - x^(-1))^(3/2)) + (2*(1 + x^
(-1))^(3/2)*(1 - x)^(3/2)*x)/(5*(1 - x^(-1))^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(x)}(1-x)^{3/2}\,dx &= \frac{(1-x)^{3/2}\int e^{\coth^{-1}(x)}\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}\\
&= -\frac{\left((1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{(1-x)\sqrt{1+x}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{x}\right)^{3/2}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}+\frac{\left(7(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\left(1-\frac{1}{x}\right)^{3/2}}\\
&= -\frac{14\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}}{15\left(1-\frac{1}{x}\right)^{3/2}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}(1-x)^{3/2}x}{5\left(1-\frac{1}{x}\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.60

$$-\frac{2\sqrt{1+\frac{1}{x}}\,\sqrt{1-x}\,(1+x)(-7+3x)}{15\sqrt{\frac{-1+x}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*(1 - x)^(3/2),x]
[Out] (-2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(-7 + 3*x))/(15*Sqrt[(-1 + x)/x])
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.43

method	result	size
gosper	$-\frac{2(1+x)(3x-7)\sqrt{1-x}}{15\sqrt{\frac{-1+x}{1+x}}}$	29
default	$-\frac{2(1+x)(3x-7)\sqrt{1-x}}{15\sqrt{\frac{-1+x}{1+x}}}$	29
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)(3x^2-4x-7)}{15\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(1+x)*(3*x-7)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.26, size = 17, normalized size = 0.25

$$\frac{2}{15}\left(-3i\,x^2+4i\,x+7i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(-3*I*x^2 + 4*I*x + 7*I)*sqrt(x + 1)
```

Fricas [A]
time = 0.33, size = 40, normalized size = 0.59

$$-\frac{2\left(3\,x^3-x^2-11\,x-7\right)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*x^3 - x^2 - 11*x - 7)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(1-x)^{\frac{3}{2}}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(3/2),x)
```

```
[Out] Integral((1 - x)**(3/2)/sqrt((x - 1)/(x + 1)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.42, size = 49, normalized size = 0.72

$$-\frac{16}{15}i\sqrt{2}\,\mathrm{sgn}(x+1)-\frac{2\left(3\,(x+1)^2\sqrt{-x-1}+10\,(-x-1)^{\frac{3}{2}}+8i\sqrt{2}\right)\mathrm{sgn}(x)}{15\,\mathrm{sgn}\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*I*sqrt(2)*sgn(x + 1) - 2/15*(3*(x + 1)^2*sqrt(-x - 1) + 10*(-x - 1)^
(3/2) + 8*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

Mupad [B]
time = 1.25, size = 30, normalized size = 0.44

$$\frac{2\,(3\,x-7)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{15\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] (2*(3*x - 7)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

3.323 $\int e^{\coth^{-1}(x)} x\sqrt{1+x}\, dx$

Optimal. Leaf size=107

$$\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{6\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

```
[Out] 12/5*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+6/5*x*((-1+x)/x)^(1/2)*(1+x
)^(1/2)/(1+1/x)^(1/2)+2/5*x^2*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6311, 6316, 79, 47, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}\,x^2}{5\sqrt{\frac{1}{x}+1}}+\frac{6\sqrt{-\frac{1-x}{x}}\sqrt{x+1}\,x}{5\sqrt{\frac{1}{x}+1}}+\frac{12\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{5\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*x*Sqrt[1 + x],x]
```

```
[Out] (12*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (6*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(5*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x)/x)]*x^2*Sqrt[
1 + x])/(5*Sqrt[1 + x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)} x\sqrt{1+x}\,dx = \frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}\,x^{3/2}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1+x}{\sqrt{1-x}\,x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}\,x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} - \frac{\left(9\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x}\,x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1+\frac{1}{x}}}$$

$$= \frac{6\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} - \frac{\left(6\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\text{Subst}\left(\int\right.}{5\sqrt{1+\frac{1}{x}}}$$

$$= \frac{12\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{6\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x^2\sqrt{1+x}}{5\sqrt{1+\frac{1}{x}}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.36

$$\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}\,(6+3x+x^2)}{5\sqrt{1+\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*x*Sqrt[1 + x],x]
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(6 + 3*x + x^2))/(5*Sqrt[1 + x^(-1)])
```

Maple [A]
time = 0.11, size = 30, normalized size = 0.28

method	result	size

gosper	$\frac{2(-1+x)(x^2+3x+6)}{5\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	30
default	$\frac{2(-1+x)(x^2+3x+6)}{5\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	30
risch	$\frac{2(-1+x)(x^2+3x+6)}{5\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(-1+x)*(x^2+3*x+6)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.19

$$\frac{2\left(x^3+2\,x^2+3\,x-6\right)}{5\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/5*(x^3 + 2*x^2 + 3*x - 6)/sqrt(x - 1)
```

Fricas [A]
time = 0.33, size = 26, normalized size = 0.24

$$\frac{2}{5}\left(x^2+3\,x+6\right)\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(x^2 + 3*x + 6)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 5.96, size = 129, normalized size = 1.21

$$-2\left(\begin{cases}\frac{x\sqrt{x-1}}{3}+\frac{5\sqrt{x-1}}{3} & \text{for } |x+1|>2\\ \frac{ix\sqrt{1-x}}{3}+\frac{5i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)+2\left(\begin{cases}\frac{8x\sqrt{x-1}}{15}+\frac{\sqrt{x-1}\,(x+1)^2}{5}+\frac{8\sqrt{x-1}}{3} & \text{for } |x+1|>2\\ \frac{8ix\sqrt{1-x}}{15}+\frac{i\sqrt{1-x}\,(x+1)^2}{5}+\frac{8i\sqrt{1-x}}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1+x)**(1/2),x)
```

```
[Out] -2*Piecewise((x*sqrt(x - 1)/3 + 5*sqrt(x - 1)/3, Abs(x + 1) > 2), (I*x*sqrt
(1 - x)/3 + 5*I*sqrt(1 - x)/3, True)) + 2*Piecewise((8*x*sqrt(x - 1)/15 + s
qrt(x - 1)*(x + 1)**2/5 + 8*sqrt(x - 1)/3, Abs(x + 1) > 2), (8*I*x*sqrt(1 -
 x)/15 + I*sqrt(1 - x)*(x + 1)**2/5 + 8*I*sqrt(1 - x)/3, True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.25, size = 38, normalized size = 0.36

$$\sqrt{\frac{x-1}{x+1}}\,\left(\frac{6\,x\,\sqrt{x+1}}{5}+\frac{12\,\sqrt{x+1}}{5}+\frac{2\,x^2\,\sqrt{x+1}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 1)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] ((x - 1)/(x + 1))^(1/2)*((6*x*(x + 1)^(1/2))/5 + (12*(x + 1)^(1/2))/5 + (2*
x^2*(x + 1)^(1/2))/5)
```

3.324 $\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx$

Optimal. Leaf size=70

$$\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}+\frac{2\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}$$

```
[Out] 10/3*((-1+x)/x)^(1/2)*(1+x)^(1/2)/(1+1/x)^(1/2)+2/3*x*((-1+x)/x)^(1/2)*(1+x
)^(1/2)/(1+1/x)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6311, 6316, 79, 37}

$$\frac{2\sqrt{-\frac{1-x}{x}}\sqrt{x+1}\,x}{3\sqrt{\frac{1}{x}+1}}+\frac{10\sqrt{-\frac{1-x}{x}}\sqrt{x+1}}{3\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 + x],x]
```

```
[Out] (10*Sqrt[-((1 - x)/x)]*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)]) + (2*Sqrt[-((1 - x
)/x)]*x*Sqrt[1 + x])/(3*Sqrt[1 + x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}\sqrt{1+x}\,dx = \frac{\sqrt{1+x}\int e^{\coth^{-1}(x)}\sqrt{1+\frac{1}{x}}\sqrt{x}\,dx}{\sqrt{1+\frac{1}{x}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\operatorname{Subst}\left(\int \frac{1+x}{\sqrt{1-x}\,x^{5/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1+\frac{1}{x}}}$$

$$= \frac{2\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}} - \frac{\left(5\sqrt{\frac{1}{x}}\sqrt{1+x}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{1+\frac{1}{x}}}$$

$$= \frac{10\sqrt{-\frac{1-x}{x}}\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}} + \frac{2\sqrt{-\frac{1-x}{x}}\,x\sqrt{1+x}}{3\sqrt{1+\frac{1}{x}}}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.49

$$\frac{2\sqrt{\frac{-1+x}{x}}\sqrt{1+x}\,(5+x)}{3\sqrt{1+\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*Sqrt[1 + x]*(5 + x))/(3*Sqrt[1 + x^(-1)])
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.36

method	result	size
gosper	$\frac{2(-1+x)(x+5)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(-1+x)(x+5)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(-1+x)(x+5)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-1+x)*(x+5)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.26, size = 15, normalized size = 0.21

$$\frac{2\,(x^2+4\,x-5)}{3\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x^2 + 4*x - 5)/sqrt(x - 1)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.30

$$\frac{2}{3}\,(x+5)\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 5)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [A]
time = 3.31, size = 56, normalized size = 0.80

$$2\left(\begin{cases}2\sqrt{2}\left(\frac{\sqrt{2}\,(x-1)^{\frac{3}{2}}}{12}+\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right) & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1+x)**(1/2),x)
[Out] 2*Piecewise((2*sqrt(2)*(sqrt(2)*(x - 1)**(3/2)/12 + sqrt(2)*sqrt(x - 1)/2),
 (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2))))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1+x)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.24, size = 21, normalized size = 0.30

$$\frac{2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+5)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 5))/3
```

3.325 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,x\,dx$

Optimal. Leaf size=71

$$-\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x}{15\sqrt{1-\frac{1}{x}}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}}$$

```
[Out] -4/15*(1+1/x)^(3/2)*x*(1-x)^(1/2)/(1-1/x)^(1/2)+2/5*(1+1/x)^(3/2)*x^2*(1-x)
^(1/2)/(1-1/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6311, 6316, 47, 37}

$$\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}}-\frac{4\left(\frac{1}{x}+1\right)^{3/2}\sqrt{1-x}\,x}{15\sqrt{1-\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
```

```
[Out] (-4*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(15*Sqrt[1 - x^(-1)]) + (2*(1 + x^(-1
))^(3/2)*Sqrt[1 - x]*x^2)/(5*Sqrt[1 - x^(-1)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(x)}\sqrt{1-x}\,x\,dx &= \frac{\sqrt{1-x}\int e^{\coth^{-1}(x)}\sqrt{1-\frac{1}{x}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{x}}\,\sqrt{x}}\\
&= -\frac{\left(\sqrt{1-x}\,\sqrt{\frac{1}{x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{x}}}\\
&= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}}+\frac{\left(2\sqrt{1-x}\,\sqrt{\frac{1}{x}}\right)\text{Subst}\left(\int\frac{\sqrt{1+x}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{5\sqrt{1-\frac{1}{x}}}\\
&= -\frac{4\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x}{15\sqrt{1-\frac{1}{x}}}+\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x^2}{5\sqrt{1-\frac{1}{x}}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.58

$$\frac{2\sqrt{1+\frac{1}{x}}\,\sqrt{1-x}\,(1+x)(-2+3x)}{15\sqrt{\frac{-1+x}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x]*x,x]
[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[1 - x]*(1 + x)*(-2 + 3*x))/(15*Sqrt[(-1 + x)/x])
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.41

method	result	size
gosper	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{-1+x}{1+x}}}$	29
default	$\frac{2(1+x)(3x-2)\sqrt{1-x}}{15\sqrt{\frac{-1+x}{1+x}}}$	29
risch	$-\frac{2\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)(3x^2+x-2)}{15\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(1+x)*(3*x-2)*(1-x)^(1/2)/((-1+x)/(1+x))^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.26, size = 17, normalized size = 0.24

$$-\frac{2}{15}\left(-3i\,x^2-i\,x+2i\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="maxima")
[Out] -2/15*(-3*I*x^2 - I*x + 2*I)*sqrt(x + 1)
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.56

$$\frac{2\left(3\,x^3+4\,x^2-x-2\right)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^3 + 4*x^2 - x - 2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [A]
time = 12.49, size = 80, normalized size = 1.13

$$\frac{2(1-x)^{\frac{5}{2}}}{5\sqrt{-\frac{x}{-x-1}+\frac{1}{-x-1}}}-\frac{14(1-x)^{\frac{3}{2}}}{15\sqrt{-\frac{x}{-x-1}+\frac{1}{-x-1}}}+\frac{4\sqrt{1-x}}{15\sqrt{-\frac{x}{-x-1}+\frac{1}{-x-1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x*(1-x)**(1/2),x)
```

```
[Out] 2*(1 - x)**(5/2)/(5*sqrt(-x/(-x - 1) + 1/(-x - 1))) - 14*(1 - x)**(3/2)/(15
*sqrt(-x/(-x - 1) + 1/(-x - 1))) + 4*sqrt(1 - x)/(15*sqrt(-x/(-x - 1) + 1/(
-x - 1)))
```

Giac [C] Result contains complex when optimal does not.
time = 0.40, size = 49, normalized size = 0.69

$$-\frac{4}{15}i\sqrt{2}\,\mathrm{sgn}(x+1)+\frac{2\left(3\,(x+1)^{2}\sqrt{-x-1}+5\,(-x-1)^{\frac{3}{2}}-2i\sqrt{2}\right)\mathrm{sgn}(x)}{15\,\mathrm{sgn}\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x*(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -4/15*I*sqrt(2)*sgn(x + 1) + 2/15*(3*(x + 1)^2*sqrt(-x - 1) + 5*(-x - 1)^(3
/2) - 2*I*sqrt(2))*sgn(x)/sgn(x + 1)
```

Mupad [B]
time = 1.27, size = 30, normalized size = 0.42

$$-\frac{2\,(3\,x-2)\,\sqrt{\frac{x-1}{x+1}}\,(x+1)^{2}}{15\,\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x)^(1/2))/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] -(2*(3*x - 2)*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(15*(1 - x)^(1/2))
```

3.326 $\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx$

Optimal. Leaf size=20

$$\frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}\,(1+x)$$

```
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6309}

$$\frac{2}{3}\sqrt{1-x}\,(x+1)e^{\coth^{-1}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]*Sqrt[1 - x],x]
[Out] (2*E^ArcCoth[x]*Sqrt[1 - x]*(1 + x))/3
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(x)}\sqrt{1-x}\,dx = \frac{2}{3}e^{\coth^{-1}(x)}\sqrt{1-x}\,(1+x)$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.70

$$\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{1-x}\,x}{3\sqrt{1-\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]*Sqrt[1 - x],x]
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[1 - x]*x)/(3*Sqrt[1 - x^(-1)])
```

Maple [A]
time = 0.08, size = 24, normalized size = 1.20

method	result	size
gosper	$\frac{2(1+x)\sqrt{1-x}}{3\sqrt{\frac{-1+x}{1+x}}}$	24
default	$\frac{2(1+x)\sqrt{1-x}}{3\sqrt{\frac{-1+x}{1+x}}}$	24
risch	$-\frac{2(1+x)\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1-x}\,\sqrt{-1-x}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1+x)*(1-x)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.26, size = 12, normalized size = 0.60

$$-\frac{2}{3}\sqrt{x+1}\,(-i\,x-i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="maxima")
[Out] -2/3*sqrt(x + 1)*(-I*x - I)
```

Fricas [A]
time = 0.34, size = 33, normalized size = 1.65

$$\frac{2\left(x^2+2\,x+1\right)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^2 + 2*x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{1-x}}{\sqrt{\frac{x-1}{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*(1-x)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)/sqrt((x - 1)/(x + 1)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 33, normalized size = 1.65

$$-\frac{4}{3}i\sqrt{2}\,\mathrm{sgn}(x+1)-\frac{2\left((-x-1)^{\frac{3}{2}}+2i\sqrt{2}\right)\mathrm{sgn}(x)}{3\,\mathrm{sgn}\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -4/3*I*sqrt(2)*sgn(x + 1) - 2/3*((-x - 1)^(3/2) + 2*I*sqrt(2))*sgn(x)/sgn(x
 + 1)
```

Mupad [B]
time = 1.27, size = 25, normalized size = 1.25

$$-\frac{2\sqrt{\frac{x-1}{x+1}}\,(x+1)^2}{3\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/((x - 1)/(x + 1))^(1/2),x)
```

```
[Out] -(2*((x - 1)/(x + 1))^(1/2)*(x + 1)^2)/(3*(1 - x)^(1/2))
```

3.327 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=73

$$\frac{4\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{1+x}}+\frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{1+x}}$$

```
[Out] 4/3*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)+2/3*x^2*(1+1/x)^(1/2)*((-1
+x)/x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6311, 6316, 47, 37}

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{x+1}}+\frac{4\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

```
[Out] (4*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/(3*Sqrt[1 + x]) + (2*Sqrt[1 + x^(
-1)]*Sqrt[-((1 - x)/x)]*x^2)/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(x)} x}{\sqrt{1+x}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1+\frac{1}{x}}}\,dx}{\sqrt{1+x}}\\
&= -\frac{\sqrt{1+\frac{1}{x}}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{5/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\,\sqrt{1+x}}\\
&= \frac{2\sqrt{1+\frac{1}{x}}\,\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{1+x}} - \frac{\left(2\sqrt{1+\frac{1}{x}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx, x, \frac{1}{x}\right)}{3\sqrt{\frac{1}{x}}\,\sqrt{1+x}}\\
&= \frac{4\sqrt{1+\frac{1}{x}}\,\sqrt{-\frac{1-x}{x}}\,x}{3\sqrt{1+x}} + \frac{2\sqrt{1+\frac{1}{x}}\,\sqrt{-\frac{1-x}{x}}\,x^2}{3\sqrt{1+x}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.36

$$
\frac{2\sqrt{1-\frac{1}{x^2}}\,x(2+x)}{3\sqrt{1+x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 + x],x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x*(2 + x))/(3*Sqrt[1 + x])
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.34

method	result	size
gosper	$\frac{2(-1+x)(x+2)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	25
default	$\frac{2(-1+x)(x+2)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	25
risch	$\frac{2(-1+x)(x+2)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(-1+x)*(x+2)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.25, size = 13, normalized size = 0.18

$$\frac{2\,(x^2+x-2)}{3\,\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^2 + x - 2)/sqrt(x - 1)
```

Fricas [A]
time = 0.32, size = 21, normalized size = 0.29

$$\frac{2}{3}\,(x+2)\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x + 2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 6.28, size = 48, normalized size = 0.66

$$\begin{cases} \frac{2x\sqrt{x-1}}{3}+\frac{4\sqrt{x-1}}{3} & \text{for } |x|>1 \\ \frac{2ix\sqrt{1-x}}{3}+\frac{4i\sqrt{1-x}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*x*sqrt(x - 1)/3 + 4*sqrt(x - 1)/3, Abs(x) > 1), (2*I*x*sqrt(1
- x)/3 + 4*I*sqrt(1 - x)/3, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.24, size = 21, normalized size = 0.29

$$\frac{2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}\,(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] (2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)*(x + 2))/3
```

3.328 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x}{\sqrt{1+x}}$$

[Out] 2*x*(1+1/x)^(1/2)*((-1+x)/x)^(1/2)/(1+x)^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 37}

$$\frac{2\sqrt{\frac{1}{x}+1}\sqrt{-\frac{1-x}{x}}\,x}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[x]/Sqrt[1 + x],x]

[Out] (2*Sqrt[1 + x^(-1)]*Sqrt[-((1 - x)/x)]*x)/Sqrt[1 + x]

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+x}}\,dx = \frac{\left(\sqrt{1+\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1+\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1+x}}$$
$$= -\frac{\sqrt{1+\frac{1}{x}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{1+x}}$$
$$= \frac{2\sqrt{1+\frac{1}{x}}\sqrt{-\frac{1-x}{x}}\,x}{\sqrt{1+x}}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.64

$$\frac{2\sqrt{1-\frac{1}{x^2}}\,x}{\sqrt{1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 + x],x]
[Out] (2*Sqrt[1 - x^(-2)]*x)/Sqrt[1 + x]
```

Maple [A]
time = 0.07, size = 22, normalized size = 0.67

method	result	size
gosper	$\frac{-2+2x}{\sqrt{\frac{-1+x}{1+x}}\sqrt{1+x}}$	22
default	$\frac{-2+2x}{\sqrt{\frac{-1+x}{1+x}}\sqrt{1+x}}$	22
risch	$\frac{-2+2x}{\sqrt{\frac{-1+x}{1+x}}\sqrt{1+x}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1+x)/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.26, size = 7, normalized size = 0.21

$$2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x - 1)
```

Fricas [A]
time = 0.33, size = 18, normalized size = 0.55

$$2\sqrt{x+1}\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + 1)*sqrt((x - 1)/(x + 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 6.51, size = 19, normalized size = 0.58

$$\begin{cases} 2\sqrt{x-1} & \text{for } |x| > 1 \\ 2i\sqrt{1-x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((2*sqrt(x - 1), Abs(x) > 1), (2*I*sqrt(1 - x), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.41, size = 23, normalized size = 0.70

$$-\frac{2\left(i\sqrt{2}-\sqrt{x-1}\right)\operatorname{sgn}(x)}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*(I*sqrt(2) - sqrt(x - 1))*sgn(x)/sgn(x + 1)
```

Mupad [B]
time = 1.22, size = 18, normalized size = 0.55

$$2\sqrt{\frac{x-1}{x+1}}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)),x)
[Out] 2*((x - 1)/(x + 1))^(1/2)*(x + 1)^(1/2)
```

3.329 $\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=126

$$\frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

```
[Out] 2/3*(1+1/x)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(1/2)+2*x*(1-1/x)^(1/2)*(1+1/x)^(
1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))*2^(1/2)*(1-1/
x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6311, 6316, 98, 96, 95, 212}

$$\frac{2\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{3\sqrt{1-x}}+\frac{2\sqrt{1-\frac{1}{x}}\sqrt{\frac{1}{x}+1}\,x}{\sqrt{1-x}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] + (2*Sqrt[1 - x^(-1)]*(
1 + x^(-1))^(3/2)*x^2)/(3*Sqrt[1 - x]) - (2*Sqrt[2]*Sqrt[1 - x^(-1)]*ArcTan
h[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[x^(-1)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{\sqrt{1-x}}\,dx = \frac{\left(\sqrt{1-\frac{1}{x}}\,\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}\sqrt{x}}{\sqrt{1-\frac{1}{x}}}\,dx}{\sqrt{1-x}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{5/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\,\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\,\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\,\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\,\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\,\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{\left(2\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\,\sqrt{1 \cdots}}\right.}{\sqrt{1-x}\,\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\,\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\,\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{\left(4\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \cdots\right.}{\sqrt{1-x}\,\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\,\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} + \frac{2\sqrt{1-\frac{1}{x}}\,\left(1+\frac{1}{x}\right)^{3/2}x^2}{3\sqrt{1-x}} - \frac{2\sqrt{2}\,\sqrt{1-\frac{1}{x}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\,\sqrt{\frac{1}{x}}}$$

<u>Mathematica</u> [A]
time = 0.03, size = 69, normalized size = 0.55

$$\frac{2\sqrt{\frac{-1+x}{x}}\,x\left(\sqrt{1+\frac{1}{x}}\,(4+x) - 3\sqrt{2}\,\sqrt{\frac{1}{x}}\,\tanh^{-1}\left(\sqrt{2}\,\sqrt{\frac{1}{1+x}}\right)\right)}{3\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)]*(4 + x) - 3*Sqrt[2]*Sqrt[x^(-1)]*Ar
cTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(3*Sqrt[1 - x])
```

Maple [A]
time = 0.09, size = 66, normalized size = 0.52

method	result	size
default	$\frac{2\sqrt{1-x}\left(3\sqrt{2}\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-x\sqrt{-1-x}-4\sqrt{-1-x}\right)}{3\sqrt{\frac{-1+x}{1+x}}\,\sqrt{-1-x}}$	66
risch	$\frac{2(x+4)\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)}{3\sqrt{-1-x}\,\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)\sqrt{1-x}}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2)*(3*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(
1/2))-x*(-1-x)^(1/2)-4*(-1-x)^(1/2))/(-1-x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A]
time = 0.33, size = 72, normalized size = 0.57

$$\frac{2\left(3\sqrt{2}\,(x-1)\arctan\left(\frac{\sqrt{2}\,\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x^2+5\,x+4)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}\right)}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x
 - 1)) - (x^2 + 5*x + 4)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-12*
atan(i)+20*i)*1/3/sqrt(2)*sign(sageVARx+1)-(-2/3*sqrt(-sageVARx-1)*(-sageVA
Rx-1)+2*sqrt(-sageVARx-1)+1/3*(12*atan(i)-20*i)/sqrt(2)-4*atan(sqrt(-sageVA
Rx-1)/sqrt(2))/s
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

3.330 $\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=90

$$\frac{2\sqrt{1-\frac{1}{x}}\,\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}}-\frac{2\sqrt{2}\,\sqrt{1-\frac{1}{x}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\,\sqrt{\frac{1}{x}}}$$

```
[Out] 2*x*(1-1/x)^(1/2)*(1+1/x)^(1/2)/(1-x)^(1/2)-2*arctanh(2^(1/2)*(1/x)^(1/2)/(
1+1/x)^(1/2))*2^(1/2)*(1-1/x)^(1/2)/(1-x)^(1/2)/(1/x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6311, 6316, 96, 95, 212}

$$\frac{2\sqrt{1-\frac{1}{x}}\,\sqrt{\frac{1}{x}+1}\,x}{\sqrt{1-x}}-\frac{2\sqrt{2}\,\sqrt{1-\frac{1}{x}}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{1-x}\,\sqrt{\frac{1}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/Sqrt[1 - x] - (2*Sqrt[2]*Sqrt[1 - x
^(-1)]*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[1 - x]*Sqrt[
x^(-1)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-x}}\,dx = \frac{\left(\sqrt{1-\frac{1}{x}}\sqrt{x}\right)\int \frac{e^{\coth^{-1}(x)}}{\sqrt{1-\frac{1}{x}}\sqrt{x}}\,dx}{\sqrt{1-x}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\,\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} - \frac{\left(2\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} - \frac{\left(4\sqrt{1-\frac{1}{x}}\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

$$= \frac{2\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{\sqrt{1-x}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{x}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{1-x}\sqrt{\frac{1}{x}}}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 0.70

$$\frac{2\sqrt{\frac{-1+x}{x}}\,x\left(\sqrt{1+\frac{1}{x}} - \sqrt{2}\sqrt{\frac{1}{x}}\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[x]/Sqrt[1 - x],x]
```

```
[Out] (2*Sqrt[(-1 + x)/x]*x*(Sqrt[1 + x^(-1)] - Sqrt[2]*Sqrt[x^(-1)]*ArcTanh[Sqrt
[2]*Sqrt[(1 + x)^(-1)]]))/Sqrt[1 - x]
```

Maple [A]
time = 0.09, size = 55, normalized size = 0.61

method	result	size
default	$\frac{2\sqrt{1-x}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-\sqrt{-1-x}\right)}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{-1-x}}$	55
risch	$\frac{2\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)}{\sqrt{-1-x}\,\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1-x}}+\frac{2\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)}{\sqrt{\frac{-1+x}{1+x}}\,(1+x)\sqrt{1-x}}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/((-1+x)/(1+x))^(1/2)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2)
)-(-1-x)^(1/2))/(-1-x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x + 1)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A]
time = 0.34, size = 66, normalized size = 0.73

$$\frac{2\left(\sqrt{2}\,(x-1)\arctan\left(\frac{\sqrt{2}\,\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-(x+1)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*(x - 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1))/(x - 1
)) - (x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*sqrt(1 - x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-4*a
tan(i)+4*i)/sqrt(2)*sign(sageVARx+1)-(2*sqrt(-sageVARx-1)+(4*atan(i)-4*i)/s
qrt(2)-4*atan(sqrt(-sageVARx-1)/sqrt(2))/sqrt(2))*sign(sageVARx)/sign(sageV
ARx+1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(1/2)), x)
```

3.331 $\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}}+\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

```
[Out] (1+1/x)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^(3
/2)/(1+x)^(3/2)+2*(1+1/x)^(3/2)*x^2*((-1+x)/x)^(1/2)/(1+x)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6311, 6316, 98, 95, 209}

$$\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}+\frac{2\left(\frac{1}{x}+1\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

```
[Out] (2*(1 + x^(-1))^(3/2)*Sqrt[-((1 - x)/x)]*x^2)/(1 + x)^(3/2) + (Sqrt[2]*(1 +
 x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/x)]])/((x^(-1))
^(3/2)*(1 + x)^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{(1+x)^{3/2}}\,dx = \frac{\left(\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2}\sqrt{x}}\,dx}{(1+x)^{3/2}}$$

$$= -\frac{\left(1+\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x^{3/2}(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}} + \frac{\left(1+\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{x}\,(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}} + \frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

$$= \frac{2\left(1+\frac{1}{x}\right)^{3/2}\sqrt{-\frac{1-x}{x}}\,x^2}{(1+x)^{3/2}} + \frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.70

$$\frac{\sqrt{1+\frac{1}{x}}\,x\left(2\sqrt{\frac{-1+x}{x}}-\sqrt{2}\sqrt{\frac{1}{x}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{-1+x}{x^2}}\,x}{\sqrt{2}}\right)\right)}{\sqrt{1+x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 + x^(-1)]*x*(2*Sqrt[(-1 + x)/x] - Sqrt[2]*Sqrt[x^(-1)]*ArcTan[(Sqrt
[(-1 + x)/x^2]*x)/Sqrt[2]]))/Sqrt[1 + x]
```

Maple [A]
time = 0.09, size = 47, normalized size = 0.51

method	result	size

default	$-\frac{\sqrt{-1+x}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)-2\sqrt{-1+x}\right)}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	47
risch	$\frac{-2+2x}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{-1+x}}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1+x)^(1/2)*(2^(1/2)*arctan(1/2*(-1+x)^(1/2)*2^(1/2))-2*(-1+x)^(1/2))/((-
1+x)/(1+x))^(1/2)/(1+x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.49

$$-\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}\right)+2\,\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1))) + 2*sqrt(x +
 1)*sqrt((x - 1)/(x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1+x)**(3/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)/(x + 1))*(x + 1)**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(sq
rt(sageVARx-1)+(atan(i)-2*i)/sqrt(2)-atan(sqrt(sageVARx-1)/sqrt(2))/sqrt(2)
)*sign(sageVARx)/sign(sageVARx+1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

3.332 $\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}$$

```
[Out] -(1+1/x)^(3/2)*arctan(2^(1/2)*(1/x)^(1/2)/((-1+x)/x)^(1/2))*2^(1/2)/(1/x)^(
3/2)/(1+x)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6311, 6316, 95, 209}

$$-\frac{\sqrt{2}\left(\frac{1}{x}+1\right)^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] -((Sqrt[2]*(1 + x^(-1))^(3/2)*ArcTan[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[-((1 - x)/
x)]])/((x^(-1))^(3/2)*(1 + x)^(3/2)))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(x)}}{(1+x)^{3/2}}\,dx &= \frac{\left(\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1+\frac{1}{x}\right)^{3/2}x^{3/2}}\,dx}{(1+x)^{3/2}}\\
&= -\frac{\left(1+\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{x}\,(1+x)}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= -\frac{\left(2\left(1+\frac{1}{x}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{-1+x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}\\
&= -\frac{\sqrt{2}\left(1+\frac{1}{x}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{-\frac{1-x}{x}}}\right)}{\left(\frac{1}{x}\right)^{3/2}(1+x)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.71

$$\sqrt{2}\sqrt{\frac{1}{1+x}}\sqrt{1+x}\,\text{ArcTan}\left(\frac{\sqrt{\frac{-1+x}{x^2}}\,x}{\sqrt{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 + x)^(3/2),x]
```

```
[Out] Sqrt[2]*Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcTan[(Sqrt[(-1 + x)/x^2]*x)/Sqrt[2
]]
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.64

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{-1+x}}{\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1+x}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*arctan(1/2*(-1+x)^(1/2)*2^(1/2))/((-1+x)/(1+x))^(1/2)/(1+x)^(1/2)*(
-1+x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A]
time = 0.35, size = 26, normalized size = 0.45

$$\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\,\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x + 1)*sqrt((x - 1)/(x + 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)/(x + 1))*(x + 1)**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(-1
/2*sqrt(2)*atan(i)+1/2*sqrt(2)*atan(sqrt(sageVARx-1)/sqrt(2)))*sign(sageVAR
x)/sign(sageVARx+1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{x-1}{x+1}}\,(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(x + 1)^(3/2)), x)
```

3.333 $\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}\,x^2}{2(1-x)^{3/2}}-\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}}-\frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] -5/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(3/2)/(
1/x)^(3/2)*2^(1/2)-1/2*(1+1/x)^(3/2)*x^2*(1-1/x)^(1/2)/(1-x)^(3/2)+5/2*(1-1
/x)^(3/2)*x^2*(1+1/x)^(1/2)/(1-x)^(3/2)
```

Rubi [A]

time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6311, 6316, 98, 96, 95, 212}

$$-\frac{\sqrt{1-\frac{1}{x}}\left(\frac{1}{x}+1\right)^{3/2}x^2}{2(1-x)^{3/2}}+\frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{\frac{1}{x}+1}\,x^2}{2(1-x)^{3/2}}-\frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] (5*(1 - x^(-1))^(3/2)*Sqrt[1 + x^(-1)]*x^2)/(2*(1 - x)^(3/2)) - (Sqrt[1 - x
^(-1)]*(1 + x^(-1))^(3/2)*x^2)/(2*(1 - x)^(3/2)) - (5*(1 - x^(-1))^(3/2)*Ar
cTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*(x^(
-1))^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}x}{(1-x)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{x}\right)^{3/2} x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2}\sqrt{x}}\,dx}{(1-x)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{x}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2x^{3/2}}\,dx, x, \frac{1}{x}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)x^{3/2}}\,dx, x, \frac{1}{x}\right)}{4(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}\,x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\,\sqrt{\cdots}}\right.}{2(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}\,x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{\left(5\left(1-\frac{1}{x}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x\cdots\right.}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= \frac{5\left(1-\frac{1}{x}\right)^{3/2}\sqrt{1+\frac{1}{x}}\,x^2}{2(1-x)^{3/2}} - \frac{\sqrt{1-\frac{1}{x}}\left(1+\frac{1}{x}\right)^{3/2}x^2}{2(1-x)^{3/2}} - \frac{5\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Mathematica [A]
time = 0.05, size = 75, normalized size = 0.58

$$-\frac{\sqrt{\frac{-1+x}{x}}\,x\left(2\sqrt{1+\frac{1}{x}}\,(3-2x)+5\sqrt{2}\,(-1+x)\sqrt{\frac{1}{x}}\,\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)\right)}{2(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[x]*x)/(1 - x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[(-1 + x)/x]*x*(2*Sqrt[1 + x^(-1)]*(3 - 2*x) + 5*Sqrt[2]*(-1 + x)
*Sqrt[x^(-1)]*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]]))/(1 - x)^(3/2)
```

Maple [A]
time = 0.10, size = 90, normalized size = 0.69

method	result
default	$-\frac{\sqrt{1-x}\,\left(5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)x-5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)-4x\sqrt{-1-x}+6\sqrt{-1-x}\right)}{2\sqrt{\frac{-1+x}{1+x}}\,(-1+x)\sqrt{-1-x}}$
risch	$-\frac{(2x^2-x-3)\sqrt{\frac{(1+x)(1-x)}{-1+x}}}{\sqrt{-1-x}\,\sqrt{\frac{-1+x}{1+x}}\,(1+x)\sqrt{1-x}}-\frac{5\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)}{2\sqrt{\frac{-1+x}{1+x}}\,(1+x)\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((-1+x)/(1+x))^(1/2)/(-1+x)*(1-x)^(1/2)*(5*2^(1/2)*arctan(1/2*(-1-x)^(
1/2)*2^(1/2))*x-5*2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))-4*x*(-1-x)^(1/2)
+6*(-1-x)^(1/2))/(-1-x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A]
time = 0.34, size = 84, normalized size = 0.65

$$-\frac{5\sqrt{2}\,(x^2-2x+1)\arctan\left(\frac{\sqrt{2}\,\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{x-1}\right)-2\,(2x^2-x-3)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{2\,(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(5*sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x
 + 1))/(x - 1)) - 2*(2*x^2 - x - 3)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^
2 - 2*x + 1)
```

Sympy [A]
time = 79.46, size = 126, normalized size = 0.97

$$2\left(\begin{cases}\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-x-1}}{2}-\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)\right) & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)-2\left(\begin{cases}\frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)*x/(1-x)**(3/2),x)
```

```
[Out] 2*Piecewise((sqrt(2)*(sqrt(2)*sqrt(-x - 1)/2 - acos(sqrt(2)/sqrt(1 - x))),
(sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2)))) - 2*Piecewise((sqrt(2)
*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1 - x))/(2*sqrt(1 - x))
)/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

Giac [A]
time = 0.43, size = 41, normalized size = 0.32

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)-2\sqrt{-x-1}+\frac{\sqrt{-x-1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)*x/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) - 2*sqrt(-x - 1) + sqrt(-x - 1
)/(x - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{x-1}{x+1}}\,{(1-x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

```
[Out] int(x/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

3.334 $\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{\sqrt{1-\frac{1}{x}}\,\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}}-\frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

```
[Out] -1/2*(1-1/x)^(3/2)*arctanh(2^(1/2)*(1/x)^(1/2)/(1+1/x)^(1/2))/(1-x)^(3/2)/(
1/x)^(3/2)*2^(1/2)-x*(1-1/x)^(1/2)*(1+1/x)^(1/2)/(1-x)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6311, 6316, 96, 95, 212}

$$-\frac{\sqrt{\frac{1}{x}+1}\,x\sqrt{1-\frac{1}{x}}}{(1-x)^{3/2}}-\frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}+1}}\right)}{\sqrt{2}\,(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[x]/(1 - x)^(3/2),x]
```

```
[Out] -((Sqrt[1 - x^(-1)]*Sqrt[1 + x^(-1)]*x)/(1 - x)^(3/2)) - ((1 - x^(-1))^(3/2
)*ArcTanh[(Sqrt[2]*Sqrt[x^(-1)])/Sqrt[1 + x^(-1)]])/(Sqrt[2]*(1 - x)^(3/2)*
(x^(-1))^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(x)}}{(1-x)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}\right)\int \frac{e^{\coth^{-1}(x)}}{\left(1-\frac{1}{x}\right)^{3/2}x^{3/2}}\,dx}{(1-x)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{(1-x)^2\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{x}\sqrt{1+x}}\,dx, x, \frac{1}{x}\right)}{2(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

$$= -\frac{\sqrt{1-\frac{1}{x}}\sqrt{1+\frac{1}{x}}\,x}{(1-x)^{3/2}} - \frac{\left(1-\frac{1}{x}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{1}{x}}}{\sqrt{1+\frac{1}{x}}}\right)}{\sqrt{2}(1-x)^{3/2}\left(\frac{1}{x}\right)^{3/2}}$$

Mathematica [**A**]
time = 0.06, size = 58, normalized size = 0.64

$$\frac{\frac{2}{\sqrt{\frac{1}{1+x}}} + \sqrt{2}(-1+x)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{1}{1+x}}\right)}{2\sqrt{-\frac{(-1+x)^2}{x^2}}\,x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[x]/(1 - x)^(3/2),x]
[Out] (2/Sqrt[(1 + x)^(-1)] + Sqrt[2]*(-1 + x)*ArcTanh[Sqrt[2]*Sqrt[(1 + x)^(-1)]
])/(2*Sqrt[-((-1 + x)^2/x^2)]*x)
```

Maple [**A**]
time = 0.10, size = 79, normalized size = 0.88

method	result	size
default	$-\frac{\sqrt{1-x}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)x-\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)+2\sqrt{-1-x}\right)}{2\sqrt{\frac{-1+x}{1+x}}\,(-1+x)\sqrt{-1-x}}$	79
risch	$\frac{\sqrt{\frac{(1+x)(1-x)}{-1+x}}}{\sqrt{-1-x}\,\sqrt{\frac{-1+x}{1+x}}\,\sqrt{1-x}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{-1-x}\,\sqrt{2}}{2}\right)\sqrt{\frac{(1+x)(1-x)}{-1+x}}\,(-1+x)}{2\sqrt{\frac{-1+x}{1+x}}\,(1+x)\sqrt{1-x}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((-1+x)/(1+x))^(1/2)/(-1+x)*(1-x)^(1/2)*(2^(1/2)*arctan(1/2*(-1-x)^(1/
2)*2^(1/2))*x-2^(1/2)*arctan(1/2*(-1-x)^(1/2)*2^(1/2))+2*(-1-x)^(1/2))/(-1-
x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x + 1)^(3/2)*sqrt((x - 1)/(x + 1))), x)
```

Fricas [A]
time = 0.33, size = 76, normalized size = 0.84

$$-\frac{\sqrt{2}\,(x^2-2\,x+1)\arctan\left(\frac{\sqrt{2}\,\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{x-1}\right)+2\,(x+1)\sqrt{-x+1}\,\sqrt{\frac{x-1}{x+1}}}{2\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x^2 - 2*x + 1)*arctan(sqrt(2)*sqrt(-x + 1)*sqrt((x - 1)/(x +
 1))/(x - 1)) + 2*(x + 1)*sqrt(-x + 1)*sqrt((x - 1)/(x + 1)))/(x^2 - 2*x +
1)
```

Sympy [A]
time = 76.94, size = 70, normalized size = 0.78

$$-2\left(\begin{cases}\frac{\sqrt{2}\left(\frac{\operatorname{acos}\left(\frac{\sqrt{2}}{\sqrt{1-x}}\right)}{2}-\frac{\sqrt{2}\sqrt{1-\frac{2}{1-x}}}{2\sqrt{1-x}}\right)}{2} & \text{for}\ \sqrt{1-x}>-\sqrt{2}\wedge\sqrt{1-x}<\sqrt{2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))**(1/2)/(1-x)**(3/2),x)
```

```
[Out] -2*Piecewise((sqrt(2)*(acos(sqrt(2)/sqrt(1 - x))/2 - sqrt(2)*sqrt(1 - 2/(1
- x))/(2*sqrt(1 - x)))/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2)
)))
```

Giac [A]
time = 0.38, size = 32, normalized size = 0.36

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-x-1}\right)+\frac{\sqrt{-x-1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)/(1+x))^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x - 1)) + sqrt(-x - 1)/(x - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{\frac{x-1}{x+1}}\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)),x)
```

```
[Out] int(1/(((x - 1)/(x + 1))^(1/2)*(1 - x)^(3/2)), x)
```

3.335 $\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{1+\frac{1}{ax}}\,x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}}-\frac{2(5+4m)x^m\sqrt{c-acx}\;{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2}-m;-\frac{1}{ax}\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -2*(5+4*m)*x^m*hypergeom([1/2, -1/2-m],[1/2-m],-1/a/x)*(-a*c*x+c)^(1/2)/a/(
4*m^2+8*m+3)/(1-1/a/x)^(1/2)+2*x^(1+m)*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(3+
2*m)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6311, 6316, 80, 66}

$$\frac{2\sqrt{\frac{1}{ax}+1}\,x^{m+1}\sqrt{c-acx}}{(2m+3)\sqrt{1-\frac{1}{ax}}}-\frac{2(4m+5)x^m\sqrt{c-acx}\;{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{1}{2}-m;-\frac{1}{ax}\right)}{a(2m+1)(2m+3)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*x^(1 + m)*Sqrt[c - a*c*x])/((3 + 2*m)*Sqrt[1 - 1/(a*x)
]) - (2*(5 + 4*m)*x^m*Sqrt[c - a*c*x]*Hypergeometric2F1[1/2, -1/2 - m, 1/2
- m, -(1/(a*x))])/(a*(1 + 2*m)*(3 + 2*m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{\frac{1}{2}+m}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{x^{-\frac{5}{2}-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} + \frac{\left((5+4m)\left(\frac{1}{x}\right)^{\frac{1}{2}+m}x^m\sqrt{c-acx}\right)\text{Subst}\left(\int\right.}{a(3+2m)\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\,x^{1+m}\sqrt{c-acx}}{(3+2m)\sqrt{1-\frac{1}{ax}}} - \frac{2(5+4m)x^m\sqrt{c-acx}\;{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2}-\right.}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [**A**]

time = 0.08, size = 102, normalized size = 0.78

$$\frac{2x^m\sqrt{c-acx}\left(a(1+2m)\sqrt{1+\frac{1}{ax}}\,x-(5+4m)\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2}-m;-\frac{1}{ax}\right)\right)}{a(1+2m)(3+2m)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*x^m*Sqrt[c - a*c*x]*(a*(1 + 2*m)*Sqrt[1 + 1/(a*x)]*x - (5 + 4*m)*Hyperge
ometric2F1[1/2, -1/2 - m, 1/2 - m, -(1/(a*x))]))/(a*(1 + 2*m)*(3 + 2*m)*Sqr
t[1 - 1/(a*x)])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*c*x + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.336 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=142

$$\frac{152c\sqrt{1-\frac{1}{a^2x^2}}\,x}{105a^2\sqrt{c-acx}}+\frac{38\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}}{105a^2}+\frac{6\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}}{35a^2c}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2(c-acx)^{3/2}}{7ac}$$

```
[Out] 6/35*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a^2/c-2/7*x^2*(-a*c*x+c)^(3/2)*
(1-1/a^2/x^2)^(1/2)/a/c+152/105*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(-a*c*x+c)^(1/2
)+38/105*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a^2
```

Rubi [A]
time = 0.16, antiderivative size = 185, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6311, 6316, 79, 47, 37}

$$-\frac{208\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{104x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^3\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{26x^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (-208*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) + (104
*Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (26*Sqr
t[1 + 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 +
1/(a*x)]*x^3*Sqrt[c - a*c*x])/(7*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{9/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}+\frac{\left(13\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{7/2}\sqrt{1+\frac{x}{a}}}\,d\right.}{7a\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{26\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}}-\frac{\left(52\sqrt{\frac{1}{x}}\sqrt{c-\right.}{}$$

$$=\frac{104\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{26\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x^3\sqrt{}}{7\sqrt{1-}}$$

$$=-\frac{208\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{104\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{26\sqrt{1+\frac{1}{ax}}\,x^2}{35a\sqrt{1-}}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.47

$$\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}\,(-104+52ax-39a^2x^2+15a^3x^3)}{105a^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(-104 + 52*a*x - 39*a^2*x^2 + 15*a^3*x
^3))/(105*a^3*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.14, size = 65, normalized size = 0.46

method	result	size
risch	$-\frac{2 c \sqrt{\frac{a x -1}{a x +1}}\, \left(15 a^{3} x^{3}-39 a^{2} x^{2}+52 a x -104\right) \left(a x +1\right)}{105 \sqrt{-c \left(a x -1\right)}\, a^{3}}$	59
gosper	$\frac{2 \left(a x +1\right) \left(15 a^{3} x^{3}-39 a^{2} x^{2}+52 a x -104\right) \sqrt{-a c x +c}\, \sqrt{\frac{a x -1}{a x +1}}}{105 a^{3} \left(a x -1\right)}$	64
default	$\frac{2 \sqrt{\frac{a x -1}{a x +1}}\, \left(a x +1\right) \sqrt{-c \left(a x -1\right)}\, \left(15 a^{3} x^{3}-39 a^{2} x^{2}+52 a x -104\right)}{105 \left(a x -1\right) a^{3}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(15*a^3*x^3-39*a^2
*x^2+52*a*x-104)/(a*x-1)/a^3
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.58

$$\frac{2 \left(15 \, a^{4} \sqrt{-c} x^{4} - 24 \, a^{3} \sqrt{-c} x^{3} + 13 \, a^{2} \sqrt{-c} x^{2} - 52 \, a \sqrt{-c} x - 104 \, \sqrt{-c}\right)}{105 \, {\left(a^{4} x - a^{3}\right)} \sqrt{a x + 1}} (a x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^4*sqrt(-c)*x^4 - 24*a^3*sqrt(-c)*x^3 + 13*a^2*sqrt(-c)*x^2 - 52
*a*sqrt(-c)*x - 104*sqrt(-c))*(a*x - 1)/((a^4*x - a^3)*sqrt(a*x + 1))
```

Fricas [A]
time = 0.35, size = 69, normalized size = 0.49

$$\frac{2 \, {\left(15 \, a^{4} x^{4} - 24 \, a^{3} x^{3} + 13 \, a^{2} x^{2} - 52 \, a x - 104\right)} \sqrt{-a c x + c} \sqrt{\frac{a x - 1}{a x + 1}}}{105 \, {\left(a^{4} x - a^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(15*a^4*x^4 - 24*a^3*x^3 + 13*a^2*x^2 - 52*a*x - 104)*sqrt(-a*c*x + c
)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [A]
time = 106.24, size = 230, normalized size = 1.62

$$\frac{304c\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{105a^3\sqrt{-acx+c}}-\frac{76\sqrt{-acx+c}\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{105a^3}-\frac{62(-acx+c)^{\frac{3}{2}}\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{105a^3c}+\frac{24(-acx+c)^{\frac{5}{2}}\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{35a^3c^2}-\frac{2(-acx+c)^{\frac{7}{2}}\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{7a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] 304*c*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))/(105*a**3*sqrt(-a*c*x + c)
) - 76*sqrt(-a*c*x + c)*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))/(105*a**
3) - 62*(-a*c*x + c)**(3/2)*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))/(105
*a**3*c) + 24*(-a*c*x + c)**(5/2)*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c)
)/(35*a**3*c**2) - 2*(-a*c*x + c)**(7/2)*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c
*x - c))/(7*a**3*c**3)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.29, size = 88, normalized size = 0.62

$$\frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-9\,a^2\,x^2+4\,a\,x-48\right)}{105\,a^3}-\frac{304\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x - 9*a^2*x^2 + 15*a^
3*x^3 - 48))/(105*a^3) - (304*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)
)/(105*a^3*(a*x - 1))
```

3.337 $\int e^{-\coth^{-1}(ax)} x \sqrt{c - acx} \, dx$

Optimal. Leaf size=104

$$-\frac{8c\sqrt{1-\frac{1}{a^2x^2}}\,x}{5a\sqrt{c-acx}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}}{5a}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x(c-acx)^{3/2}}{5ac}$$

```
[Out] -2/5*x*(-a*c*x+c)^(3/2)*(1-1/a^2/x^2)^(1/2)/a/c-8/5*c*x*(1-1/a^2/x^2)^(1/2)
/a/(-a*c*x+c)^(1/2)-2/5*x*(1-1/a^2/x^2)^(1/2)*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6311, 6316, 79, 47, 37}

$$\frac{12\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}}+\frac{2x^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}}-\frac{6x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
```

```
[Out] (12*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(5*a^2*Sqrt[1 - 1/(a*x)]) - (6*Sqrt[
1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(5*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1 + 1/(a*
x)]*x^2*Sqrt[c - a*c*x])/(5*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)} x\sqrt{c-acx}\, dx = \frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\, x^{3/2}\, dx}{\sqrt{1-\frac{1}{ax}}\,\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{7/2}\sqrt{1+\frac{x}{a}}}\, dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\, x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} + \frac{\left(9\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{x^{5/2}\sqrt{1+\frac{x}{a}}}\, dx, x\right.}{5a\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}\, x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\, x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}} - \frac{\left(6\sqrt{\frac{1}{x}}\,\sqrt{c-acx}\right)}{}$$

$$= \frac{12\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{5a^2\sqrt{1-\frac{1}{ax}}} - \frac{6\sqrt{1+\frac{1}{ax}}\, x\sqrt{c-acx}}{5a\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{1+\frac{1}{ax}}\, x^2\sqrt{c-ac}}{5\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.56

$$\frac{2\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}\,(6-3ax+a^2x^2)}{5a^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^ArcCoth[a*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x]*(6 - 3*a*x + a^2*x^2))/(5*a^2*Sqrt[1 -
 1/(a*x)])
```

Maple [A]
time = 0.14, size = 56, normalized size = 0.54

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-3ax+6)(ax+1)}{5\sqrt{-c\left(ax-1\right)}\,a^2}$	50
gosper	$\frac{2(ax+1)(a^2x^2-3ax+6)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5a^2(ax-1)}$	55
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\left(ax-1\right)}\,(a^2x^2-3ax+6)}{5(ax-1)a^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(a^2*x^2-3*a*x+6)/(a
*x-1)/a^2
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.66

$$\frac{2\left(a^3\sqrt{-c}\,x^3-2\,a^2\sqrt{-c}\,x^2+3\,a\sqrt{-c}\,x+6\,\sqrt{-c}\,\right)(ax-1)}{5\,(a^3x-a^2)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(a^3*sqrt(-c)*x^3 - 2*a^2*sqrt(-c)*x^2 + 3*a*sqrt(-c)*x + 6*sqrt(-c))*(
a*x - 1)/((a^3*x - a^2)*sqrt(a*x + 1))
```

Fricas [A]
time = 0.35, size = 60, normalized size = 0.58

$$\frac{2\,(a^3x^3-2\,a^2x^2+3\,ax+6)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^3x-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^3*x^3 - 2*a^2*x^2 + 3*a*x + 6)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x
+ 1))/(a^3*x - a^2)
```

Sympy [A]
time = 51.33, size = 182, normalized size = 1.75

$$-\frac{16c\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{5a^2\sqrt{-acx+c}}+\frac{4\sqrt{-acx+c}\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{5a^2}-\frac{2(-acx+c)^{\frac{3}{2}}\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{5a^2c}+\frac{2(-acx+c)^{\frac{5}{2}}\sqrt{-\frac{acx}{-acx-c}+\frac{c}{-acx-c}}}{5a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -16*c*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))/(5*a**2*sqrt(-a*c*x + c))
+ 4*sqrt(-a*c*x + c)*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))/(5*a**2) -
2*(-a*c*x + c)**(3/2)*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))/(5*a**2*c)
 + 2*(-a*c*x + c)**(5/2)*sqrt(-a*c*x/(-a*c*x - c) + c/(-a*c*x - c))/(5*a**2
*c**2)
```

Giac [A]
time = 0.40, size = 69, normalized size = 0.66

$$-\frac{4\sqrt{-acx-c}\,|c|}{a^2c}-\frac{2\left((acx+c)^2\sqrt{-acx-c}\,|c|+5\,(-acx-c)^{\frac{3}{2}}c|c|\right)}{5\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-a*c*x - c)*abs(c)/(a^2*c) - 2/5*((a*c*x + c)^2*sqrt(-a*c*x - c)*ab
s(c) + 5*(-a*c*x - c)^(3/2)*c*abs(c))/(a^2*c^3)
```

Mupad [B]
time = 1.29, size = 57, normalized size = 0.55

$$\frac{2\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(a^3\,x^3-2\,a^2\,x^2+3\,a\,x+6\right)}{5\,a^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a*x - 2*a^2*x^2 + a^3*x
^3 + 6))/(5*a^2*(a*x - 1))
```

3.338 $\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=62

$$\frac{8c\sqrt{1-\frac{1}{a^2x^2}}\,x}{3\sqrt{c-acx}}+\frac{2}{3}\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-acx}$$

```
[Out] 8/3*c*x*(1-1/a^2/x^2)^(1/2)/(-a*c*x+c)^(1/2)+2/3*x*(1-1/a^2/x^2)^(1/2)*(-a*
c*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6311, 6316, 79, 37}

$$\frac{2x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}-\frac{10\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
```

```
[Out] (-10*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]) + (2*Sqrt[1
 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1-\frac{x}{a}}{x^{5/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}+\frac{\left(5\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx,x,\right.}{3a\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{10\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [**A**]

time = 0.02, size = 50, normalized size = 0.81

$$\frac{2\sqrt{1+\frac{1}{ax}}\,(-5+ax)\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^ArcCoth[a*x],x]
[Out] (2*Sqrt[1 + 1/(a*x)]*(-5 + a*x)*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.08, size = 48, normalized size = 0.77

method	result	size
risch	$-\frac{2c\sqrt{\frac{ax-1}{ax+1}}\,(ax-5)(ax+1)}{3\sqrt{-c\,(ax-1)}\;a}$	42
gosper	$\frac{2(ax+1)(ax-5)\sqrt{-acx+c}\;\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)a}$	47
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,(ax-5)}{3(ax-1)a}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(a*x-5)/(a*x-1)/a
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.87

$$\frac{2\left(a^2\sqrt{-c}\,x^2-4\,a\sqrt{-c}\,x-5\,\sqrt{-c}\,\right)(ax-1)}{3\left(a^2x-a\right)\sqrt{ax+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] 2/3*(a^2*sqrt(-c)*x^2 - 4*a*sqrt(-c)*x - 5*sqrt(-c))*(a*x - 1)/((a^2*x - a)
*sqrt(a*x + 1))
```

Fricas [A]
time = 0.34, size = 50, normalized size = 0.81

$$\frac{2\left(a^2x^2-4\,ax-5\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 4*a*x - 5)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2*x
 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)), x)
```

Giac [A]
time = 0.41, size = 43, normalized size = 0.69

$$\frac{2\,(-acx-c)^{\frac{3}{2}}|c|}{3\,ac^2}+\frac{4\,\sqrt{-acx-c}\,|c|}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(-a*c*x - c)^(3/2)*abs(c)/(a*c^2) + 4*sqrt(-a*c*x - c)*abs(c)/(a*c)
```

Mupad [B]
time = 0.00, size = 71, normalized size = 1.15

$$\frac{2\,\sqrt{c-a\,c\,x}\,\left(a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{16\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (16*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.339 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)+2*arcsinh((1/x)^(1/2)/a^
(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6311, 6316, 79, 56, 221}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[x^(-1)]*S
qrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{x^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x,\right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{}\right.}{a\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.79

$$\frac{2\sqrt{c-acx}\left(\sqrt{a}\sqrt{1+\frac{1}{ax}}+\sqrt{\frac{1}{x}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(Sqrt[a]*Sqrt[1 + 1/(a*x)] + Sqrt[x^(-1)]*ArcSinh[Sqrt[x
^(-1)]/Sqrt[a]]))/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.15, size = 80, normalized size = 0.85

method	result	size
default	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\,\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)+\sqrt{-c\,(ax+1)}\right)}{(ax-1)\sqrt{-c\,(ax+1)}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arctan((-c*(a
*x+1))^(1/2)/c^(1/2))+(-c*(a*x+1))^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [A]
time = 0.36, size = 206, normalized size = 2.19

$$\left[\frac{(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},\ \frac{2\left((ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)+\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), 2*((a*x - 1)*sqrt(c)*arct
an(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) + sqrt(-
a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x, x)
```

Giac [A]
time = 0.41, size = 40, normalized size = 0.43

$$-2\left(\frac{\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{-acx-c}}{c}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
[Out] -2*(arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) + sqrt(-a*c*x - c)/c)*abs(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.340 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x}-\frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] (1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)-3*arcsinh((1/x)^(1/2)/a^
(1/2))*a^(1/2)*(1/x)^(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6311, 6316, 81, 56, 221}

$$\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) - (3*Sqrt[a]*Sqrt
[x^(-1)]*Sqrt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1-\frac{x}{a}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right.}{2\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{\left(3\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \sqrt{\frac{1}{x}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} - \frac{3\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.81

$$-\frac{\sqrt{\frac{1}{x}}\sqrt{c-acx}\left(-\sqrt{1+\frac{1}{ax}}\sqrt{\frac{1}{x}}+3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] -((Sqrt[x^(-1)]*Sqrt[c - a*c*x]*(-(Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]) + 3*Sqrt
[a]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]]))/Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.16, size = 90, normalized size = 0.94

method	result	size
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{-c\,(ax-1)}\left(-3\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)acx+\sqrt{-c\,(ax+1)}\,\sqrt{c}\right)}{(ax-1)\sqrt{-c\,(ax+1)}\,x\sqrt{c}}$	90
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\sqrt{-c\,(ax-1)}}-\frac{3a\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax+1)}}{\sqrt{-c\,(ax-1)}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-c*(a*x-1))^(1/2)*(-3*arctan((-c*(a*x+1))^
(1/2)/c^(1/2))*a*c*x+(-c*(a*x+1))^(1/2)*c^(1/2))/(a*x-1)/(-c*(a*x+1))^(1/2)
/x/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A]
time = 0.39, size = 232, normalized size = 2.42

$$\left[\frac{3\,(a^2x^2-ax)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\,(ax^2-x)},\,-\frac{3\,(a^2x^2-ax)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^2-x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(3*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqr
t(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), -(3*(a^2*x^
2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/
(a*c*x - c)) - sqrt(-a*c*x + c)*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2
 - x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1))/x**2, x)
```

Giac [A]
time = 0.42, size = 48, normalized size = 0.50

$$a\left(\frac{3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{-acx-c}}{acx}\right)|c|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] a*(3*arctan(sqrt(-a*c*x - c)/sqrt(c))/sqrt(c) - sqrt(-a*c*x - c)/(a*c*x))*a
bs(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

3.341 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=139

$$\frac{4\sqrt{c-acx}}{a^4}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{2(c-acx)^{5/2}}{5a^4c^2}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a^4/c+2/5*(-a*c*x+c)^(5/2)/a^4/c^2-2/7*(-a*c*x+c)^(7/2
)/a^4/c^3+2/9*(-a*c*x+c)^(9/2)/a^4/c^4-4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/
2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4+4*(-a*c*x+c)^(1/2)/a^4
```

Rubi [A]
time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6302, 6265, 21, 90, 52, 65, 212}

$$\frac{2(c-acx)^{9/2}}{9a^4c^4}-\frac{2(c-acx)^{7/2}}{7a^4c^3}+\frac{2(c-acx)^{5/2}}{5a^4c^2}+\frac{2(c-acx)^{3/2}}{3a^4c}+\frac{4\sqrt{c-acx}}{a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x)^
(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(9/
2))/(9*a^4*c^4) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[
c])])/a^4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-acx}\,dx$$
$$= -\int \frac{x^3(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$= -\frac{\int \frac{x^3(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$= -\frac{\int \left(\frac{(c-acx)^{3/2}}{a^3} - \frac{(c-acx)^{3/2}}{a^3(1+ax)} - \frac{(c-acx)^{5/2}}{a^3c} + \frac{(c-acx)^{7/2}}{a^3c^2}\right)dx}{c}$$
$$= \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^3c}$$
$$= \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-acx)^{9/2}}{9a^4c^4} + \frac{2\int \frac{\sqrt{c}}{}}{}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-ac}{9a^4c}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-ac}{9a^4c}$$
$$= \frac{4\sqrt{c-acx}}{a^4} + \frac{2(c-acx)^{3/2}}{3a^4c} + \frac{2(c-acx)^{5/2}}{5a^4c^2} - \frac{2(c-acx)^{7/2}}{7a^4c^3} + \frac{2(c-ac}{9a^4c}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.61

$$\frac{2\left(\sqrt{c-acx}\,(788-236ax+138a^2x^2-95a^3x^3+35a^4x^4)-630\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)\right)}{315a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(Sqrt[c - a*c*x]*(788 - 236*a*x + 138*a^2*x^2 - 95*a^3*x^3 + 35*a^4*x^4)
 - 630*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]))/(315*a^
4)
```

Maple [A]
time = 0.22, size = 101, normalized size = 0.73

method	result

risch	$-\frac{2(35a^4x^4-95a^3x^3+138a^2x^2-236ax+788)(ax-1)c}{315a^4\sqrt{-c\,(ax-1)}}-\frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^4}$
derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}-4c^{\frac{9}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{2\sqrt{c}}\right.}{a^4c^4}$
default	$\frac{\frac{2(-acx+c)^{\frac{9}{2}}}{9}-\frac{2c(-acx+c)^{\frac{7}{2}}}{7}+\frac{2c^2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c^3(-acx+c)^{\frac{3}{2}}}{3}+4c^4\sqrt{-acx+c}-4c^{\frac{9}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{2\sqrt{c}}\right.}{a^4c^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^4/a^4*(1/9*(-a*c*x+c)^(9/2)-1/7*c*(-a*c*x+c)^(7/2)+1/5*c^2*(-a*c*x+c)^(
5/2)+1/3*c^3*(-a*c*x+c)^(3/2)+2*c^4*(-a*c*x+c)^(1/2)-2*c^(9/2)*2^(1/2)*arct
anh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.46, size = 123, normalized size = 0.88

$$\frac{2\left(315\,\sqrt{2}\,c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+35\,(-acx+c)^{\frac{9}{2}}-45\,(-acx+c)^{\frac{7}{2}}c+63\,(-acx+c)^{\frac{5}{2}}c^2+105\,(-acx+c)^{\frac{3}{2}}c^3+630\,\sqrt{-acx+c}\,c^4\right)}{315\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(2)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(
2)*sqrt(c) + sqrt(-a*c*x + c))) + 35*(-a*c*x + c)^(9/2) - 45*(-a*c*x + c)^(
7/2)*c + 63*(-a*c*x + c)^(5/2)*c^2 + 105*(-a*c*x + c)^(3/2)*c^3 + 630*sqrt(
-a*c*x + c)*c^4)/(a^4*c^4)
```

Fricas [A]
time = 0.34, size = 168, normalized size = 1.21

$$\left[\frac{2\left(315\,\sqrt{2}\,\sqrt{c}\,\log\left(\frac{acx+2\,\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3c}{ax+1}\right)+(35\,a^4x^4-95\,a^3x^3+138\,a^2x^2-236\,ax+788)\sqrt{-acx+c}\right)}{315\,a^4},\frac{2\left(630\,\sqrt{2}\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2\,c}\right)+(35\,a^4x^4-95\,a^3x^3+138\,a^2x^2-236\,ax+788)\sqrt{-acx+c}\right)}{315\,a^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*x + 788
)*sqrt(-a*c*x + c))/a^4, 2/315*(630*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqr
t(-a*c*x + c)*sqrt(-c)/c) + (35*a^4*x^4 - 95*a^3*x^3 + 138*a^2*x^2 - 236*a*
x + 788)*sqrt(-a*c*x + c))/a^4]
```

Sympy [A]
time = 5.15, size = 126, normalized size = 0.91

$$2 \cdot \frac{\left(\frac{2\sqrt{2}\, c^5 \operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}} + 2c^4\sqrt{-acx+c} + \frac{c^3(-acx+c)^{\frac{3}{2}}}{3} + \frac{c^2(-acx+c)^{\frac{5}{2}}}{5} - \frac{c(-acx+c)^{\frac{7}{2}}}{7} + \frac{(-acx+c)^{\frac{9}{2}}}{9}\right)}{a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] 2*(2*sqrt(2)*c**5*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*
c**4*sqrt(-a*c*x + c) + c**3*(-a*c*x + c)**(3/2)/3 + c**2*(-a*c*x + c)**(5/
2)/5 - c*(-a*c*x + c)**(7/2)/7 + (-a*c*x + c)**(9/2)/9)/(a**4*c**4)
```

Giac [A]
time = 0.42, size = 159, normalized size = 1.14

$$\frac{4\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^4\sqrt{-c}} + \frac{2\left(35\,(acx-c)^4\sqrt{-acx+c}\,a^{32}c^{32} + 45\,(acx-c)^3\sqrt{-acx+c}\,a^{32}c^{33} + 63\,(acx-c)^2\sqrt{-acx+c}\,a^{32}c^{34} + 105\,(-acx+c)^{\frac{3}{2}}a^{32}c^{35} + 630\sqrt{-acx+c}\,a^{32}c^{36}\right)}{315\,a^{36}c^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^4*sqrt(-c)) +
2/315*(35*(a*c*x - c)^4*sqrt(-a*c*x + c)*a^32*c^32 + 45*(a*c*x - c)^3*sqrt(
-a*c*x + c)*a^32*c^33 + 63*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^32*c^34 + 105*(
-a*c*x + c)^(3/2)*a^32*c^35 + 630*sqrt(-a*c*x + c)*a^32*c^36)/(a^36*c^36)
```

Mupad [B]
time = 0.08, size = 114, normalized size = 0.82

$$\frac{4\sqrt{c-a\,c\,x}}{a^4} + \frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^4\,c} + \frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^4\,c^2} - \frac{2\,(c-a\,c\,x)^{7/2}}{7\,a^4\,c^3} + \frac{2\,(c-a\,c\,x)^{9/2}}{9\,a^4\,c^4} + \frac{\sqrt{2}\,\sqrt{c}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\sqrt{c}}\right)4\mathrm{i}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^4 + (2*(c - a*c*x)^(3/2))/(3*a^4*c) + (2*(c - a*c*x
)^(5/2))/(5*a^4*c^2) - (2*(c - a*c*x)^(7/2))/(7*a^4*c^3) + (2*(c - a*c*x)^(
9/2))/(9*a^4*c^4) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2
*c^(1/2)))*4i)/a^4
```

3.342 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$-\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a^3/c-2/7*(-a*c*x+c)^(7/2)/a^3/c^3+4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3-4*(-a*c*x+c)^(1/2)/a^3
```

Rubi [A]
time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6302, 6265, 21, 90, 52, 65, 212}

$$-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{4\sqrt{c-acx}}{a^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c*x)
^(7/2))/(7*a^3*c^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])])/a^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=-\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-acx}\,dx$$
$$=-\int\frac{x^2(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$=-\frac{\int\frac{x^2(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$=-\frac{\int\left(\frac{(c-acx)^{3/2}}{a^2(1+ax)}-\frac{(c-acx)^{5/2}}{a^2c}\right)dx}{c}$$
$$=-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{\int\frac{(c-acx)^{3/2}}{1+ax}\,dx}{a^2c}$$
$$=-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{2\int\frac{\sqrt{c-acx}}{1+ax}\,dx}{a^2}$$
$$=-\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}-\frac{(4c)\int\frac{1}{(1+ax)\sqrt{c-acx}}\,d}{a^2}$$
$$=-\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{8\text{Subst}\left(\int\frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{}\right.}{a^3}$$
$$=-\frac{4\sqrt{c-acx}}{a^3}-\frac{2(c-acx)^{3/2}}{3a^3c}-\frac{2(c-acx)^{7/2}}{7a^3c^3}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{2}}\right.}{a^3}$$

Mathematica [A]
time = 0.06, size = 78, normalized size = 0.80

$$\frac{2\sqrt{c-acx}\,(-52+16ax-9a^2x^2+3a^3x^3)+84\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{21a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-52 + 16*a*x - 9*a^2*x^2 + 3*a^3*x^3) + 84*Sqrt[2]*Sqrt
[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(21*a^3)
```

Maple [A]
time = 0.22, size = 75, normalized size = 0.77

method	result	size

risch	$-\frac{2(3a^3x^3-9a^2x^2+16ax-52)(ax-1)c}{21a^3\sqrt{-c(ax-1)}}+\frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^3}$	74
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+2c^3\sqrt{-acx+c}-2c^{\frac{7}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{c^3a^3}$	75
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{7}{2}}}{7}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+2c^3\sqrt{-acx+c}-2c^{\frac{7}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{c^3a^3}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c^3/a^3*(1/7*(-a*c*x+c)^(7/2)+1/3*c^2*(-a*c*x+c)^(3/2)+2*c^3*(-a*c*x+c)^
(1/2)-2*c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.46, size = 97, normalized size = 1.00

$$-\frac{2\left(21\sqrt{2}\,c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{7}{2}}+7\,(-acx+c)^{\frac{3}{2}}c^2+42\,\sqrt{-acx+c}\,c^3\right)}{21\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/21*(21*sqrt(2)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2
)*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(7/2) + 7*(-a*c*x + c)^(3/2
)*c^2 + 42*sqrt(-a*c*x + c)*c^3)/(a^3*c^3)
```

Fricas [A]
time = 0.35, size = 153, normalized size = 1.58

$$\left[\frac{2\left(21\sqrt{2}\,\sqrt{c}\,\log\left(\frac{acx-2\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3c}{ax+1}\right)+(3\,a^3x^3-9\,a^2x^2+16\,ax-52)\sqrt{-acx+c}\right)}{21\,a^3},-\frac{2\left(42\sqrt{2}\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2\,c}\right)-(3\,a^3x^3-9\,a^2x^2+16\,ax-52)\sqrt{-acx+c}\right)}{21\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/
a^3, -2/21*(42*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c
)/c) - (3*a^3*x^3 - 9*a^2*x^2 + 16*a*x - 52)*sqrt(-a*c*x + c))/a^3]
```

Sympy [A]
time = 3.63, size = 95, normalized size = 0.98

$$-\frac{2\cdot\left(\frac{2\sqrt{2}\,c^4\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^3\sqrt{-acx+c}+\frac{c^2(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{7}{2}}}{7}\right)}{a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**4*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c**3*sqrt(-a*c*x + c) + c**2*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(7/2)/7
)/(a**3*c**3)
```

Giac [A]
time = 0.41, size = 105, normalized size = 1.08

$$-\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^3\sqrt{-c}}+\frac{2\left(3\,(acx-c)^3\sqrt{-acx+c}\,a^{18}c^{18}-7\,(-acx+c)^{\frac{3}{2}}a^{18}c^{20}-42\sqrt{-acx+c}\,a^{18}c^{21}\right)}{21\,a^{21}c^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^3*sqrt(-c)) +
 2/21*(3*(a*c*x - c)^3*sqrt(-a*c*x + c)*a^18*c^18 - 7*(-a*c*x + c)^(3/2)*a^
18*c^20 - 42*sqrt(-a*c*x + c)*a^18*c^21)/(a^21*c^21)
```

Mupad [B]
time = 1.26, size = 80, normalized size = 0.82

$$-\frac{4\sqrt{c-a\,c\,x}}{a^3}-\frac{2\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,a^3\,c}-\frac{2\,{\left(c-a\,c\,x\right)}^{7/2}}{7\,a^3\,c^3}-\frac{\sqrt{2}\,\sqrt{c}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\sqrt{c}}\right)\,4\mathrm{i}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] - (4*(c - a*c*x)^(1/2))/a^3 - (2*(c - a*c*x)^(3/2))/(3*a^3*c) - (2*(c - a*c
*x)^(7/2))/(7*a^3*c^3) - (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1
i)/(2*c^(1/2)))*4i)/a^3
```

3.343 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx$

Optimal. Leaf size=97

$$\frac{4\sqrt{c-acx}}{a^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{2(c-acx)^{5/2}}{5a^2c^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

```
[Out] 2/3*(-a*c*x+c)^(3/2)/a^2/c+2/5*(-a*c*x+c)^(5/2)/a^2/c^2-4*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2+4*(-a*c*x+c)^(1/2)/a^2
```

Rubi [A]
time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6265, 21, 81, 52, 65, 212}

$$\frac{2(c-acx)^{5/2}}{5a^2c^2}+\frac{2(c-acx)^{3/2}}{3a^2c}+\frac{4\sqrt{c-acx}}{a^2}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a*c*x])/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x)^
(5/2))/(5*a^2*c^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sq
rt[c])])/a^2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = -\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-acx}\,dx$$
$$= -\int \frac{x(1-ax)\sqrt{c-acx}}{1+ax}\,dx$$
$$= -\frac{\int \frac{x(c-acx)^{3/2}}{1+ax}\,dx}{c}$$
$$= \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{ac}$$
$$= \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{2\int \frac{\sqrt{c-acx}}{1+ax}\,dx}{a}$$
$$= \frac{4\sqrt{c-acx}}{a^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} + \frac{(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx}{a}$$
$$= \frac{4\sqrt{c-acx}}{a^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c}\right.}{a^2}$$
$$= \frac{4\sqrt{c-acx}}{a^2} + \frac{2(c-acx)^{3/2}}{3a^2c} + \frac{2(c-acx)^{5/2}}{5a^2c^2} - \frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-c}}{\sqrt{2}\sqrt{}}\right.}{a^2}$$

Mathematica [A]
time = 0.06, size = 70, normalized size = 0.72

$$\frac{2\sqrt{c-acx}\,(38-11ax+3a^2x^2)-60\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{15a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(38 - 11*a*x + 3*a^2*x^2) - 60*Sqrt[2]*Sqrt[c]*ArcTanh[S
qrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/(15*a^2)
```

Maple [A]
time = 0.22, size = 73, normalized size = 0.75

method	result	size
risch	$-\frac{2(3a^2x^2-11ax+38)(ax-1)c}{15a^2\sqrt{-c(ax-1)}}-\frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a^2}$	66

derivativedivides	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{2}a^{2}}$	73
default	$\frac{\frac{2(-acx+c)^{\frac{5}{2}}}{5}+\frac{2c(-acx+c)^{\frac{3}{2}}}{3}+4c^{2}\sqrt{-acx+c}-4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{2}a^{2}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c^2/a^2*(1/5*(-a*c*x+c)^(5/2)+1/3*c*(-a*c*x+c)^(3/2)+2*c^2*(-a*c*x+c)^(1/
2)-2*c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.47, size = 95, normalized size = 0.98

$$\frac{2\left(15\sqrt{2}\,c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)+3\,(-acx+c)^{\frac{5}{2}}+5\,(-acx+c)^{\frac{3}{2}}c+30\,\sqrt{-acx+c}\,c^{2}\right)}{15\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)
*sqrt(c) + sqrt(-a*c*x + c))) + 3*(-a*c*x + c)^(5/2) + 5*(-a*c*x + c)^(3/2)
*c + 30*sqrt(-a*c*x + c)*c^2)/(a^2*c^2)
```

Fricas [A]
time = 0.35, size = 136, normalized size = 1.40

$$\left[\frac{2\left(15\sqrt{2}\,\sqrt{c}\,\log\left(\frac{acx+2\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3c}{ax+1}\right)+\left(3\,a^{2}x^{2}-11\,ax+38\right)\sqrt{-acx+c}\right)}{15\,a^{2}},\ \frac{2\left(30\sqrt{2}\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2c}\right)+\left(3\,a^{2}x^{2}-11\,ax+38\right)\sqrt{-acx+c}\right)}{15\,a^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) -
 3*c)/(a*x + 1)) + (3*a^2*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2, 2/15*(3
0*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) + (3*a^2
*x^2 - 11*a*x + 38)*sqrt(-a*c*x + c))/a^2]
```

Sympy [A]
time = 3.10, size = 92, normalized size = 0.95

$$\frac{2\cdot\left(\frac{2\sqrt{2}\,c^{3}\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c^{2}\sqrt{-acx+c}+\frac{c(-acx+c)^{\frac{3}{2}}}{3}+\frac{(-acx+c)^{\frac{5}{2}}}{5}\right)}{a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] 2*(2*sqrt(2)*c**3*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*
c**2*sqrt(-a*c*x + c) + c*(-a*c*x + c)**(3/2)/3 + (-a*c*x + c)**(5/2)/5)/(a
**2*c**2)
```

Giac [A]
time = 0.40, size = 105, normalized size = 1.08

$$\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a^2\sqrt{-c}}+\frac{2\left(3\,(acx-c)^2\sqrt{-acx+c}\,a^8c^8+5\,(-acx+c)^{\frac{3}{2}}a^8c^9+30\,\sqrt{-acx+c}\,a^8c^{10}\right)}{15\,a^{10}c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a^2*sqrt(-c)) +
2/15*(3*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^8*c^8 + 5*(-a*c*x + c)^(3/2)*a^8*c
^9 + 30*sqrt(-a*c*x + c)*a^8*c^10)/(a^10*c^10)
```

Mupad [B]
time = 0.09, size = 80, normalized size = 0.82

$$\frac{4\sqrt{c-a\,c\,x}}{a^2}+\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a^2\,c}+\frac{2\,(c-a\,c\,x)^{5/2}}{5\,a^2\,c^2}+\frac{\sqrt{2}\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*(c - a*c*x)^(1/2))/a^2 + (2*(c - a*c*x)^(3/2))/(3*a^2*c) + (2*(c - a*c*x
)^(5/2))/(5*a^2*c^2) + (2^(1/2)*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)
/(2*c^(1/2)))*4i)/a^2
```

3.344 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=76

$$-\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -2/3*(-a*c*x+c)^(3/2)/a/c+4*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)/a-4*(-a*c*x+c)^(1/2)/a
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6302, 6265, 21, 52, 65, 212}

$$-\frac{2(c-acx)^{3/2}}{3ac}-\frac{4\sqrt{c-acx}}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a*c*x])/a - (2*(c - a*c*x)^(3/2))/(3*a*c) + (4*Sqrt[2]*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}\,dx\\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{\int \frac{(c-acx)^{3/2}}{1+ax}\,dx}{c}\\
&= -\frac{2(c-acx)^{3/2}}{3ac}-2\int \frac{\sqrt{c-acx}}{1+ax}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}-(4c)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx,x,\sqrt{c-acx}\right)}{a}\\
&= -\frac{4\sqrt{c-acx}}{a}-\frac{2(c-acx)^{3/2}}{3ac}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 61, normalized size = 0.80

$$\frac{2(-7+ax)\sqrt{c-acx}+12\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (2*(-7 + a*x)*Sqrt[c - a*c*x] + 12*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/
(Sqrt[2]*Sqrt[c])])/(3*a)
```

Maple [A]
time = 0.19, size = 59, normalized size = 0.78

method	result	size
risch	$-\frac{2(ax-7)(ax-1)c}{3a\sqrt{-c\,(ax-1)}}+\frac{4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c}}{a}$	57
derivativedivides	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59
default	$-\frac{2\left(\frac{(-acx+c)^{\frac{3}{2}}}{3}+2c\sqrt{-acx+c}-2c^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{ca}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/a*(1/3*(-a*c*x+c)^(3/2)+2*c*(-a*c*x+c)^(1/2)-2*c^(3/2)*2^(1/2)*arctanh
(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.47, size = 79, normalized size = 1.04

$$-\frac{2\left(3\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)+(-acx+c)^{\frac{3}{2}}+6\sqrt{-acx+c}\,c\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*(3*sqrt(2)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*
sqrt(c) + sqrt(-a*c*x + c))) + (-a*c*x + c)^(3/2) + 6*sqrt(-a*c*x + c)*c)/(
a*c)
```

Fricas [A]
time = 0.36, size = 119, normalized size = 1.57

$$\left[\frac{2\left(3\sqrt{2}\sqrt{c}\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{-acx+c}\,(ax-7)\right)}{3a}, -\frac{2\left(6\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-\sqrt{-acx+c}\,(ax-7)\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*sqrt(c)*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3
*c)/(a*x + 1)) + sqrt(-a*c*x + c)*(a*x - 7))/a, -2/3*(6*sqrt(2)*sqrt(-c)*ar
ctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(a*x - 7))
/a]
```

Sympy [A]
time = 2.16, size = 73, normalized size = 0.96

$$-\frac{2\cdot\left(\frac{2\sqrt{2}c^{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2c\sqrt{-acx+c}+\frac{(-acx+c)^{\frac{3}{2}}}{3}\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -2*(2*sqrt(2)*c**2*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2
*c*sqrt(-a*c*x + c) + (-a*c*x + c)**(3/2)/3)/(a*c)
```

Giac [A]
time = 0.43, size = 77, normalized size = 1.01

$$-\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left((-acx+c)^{\frac{3}{2}}a^{2}c^{2}+6\sqrt{-acx+c}\,a^{2}c^{3}\right)}{3\,a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/(a*sqrt(-c)) - 2
/3*((-a*c*x + c)^(3/2)*a^2*c^2 + 6*sqrt(-a*c*x + c)*a^2*c^3)/(a^3*c^3)
```

Mupad [B]
time = 0.00, size = 61, normalized size = 0.80

$$\frac{4\sqrt{2}\,\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)}{a}-\frac{2\,(c-a\,c\,x)^{3/2}}{3\,a\,c}-\frac{4\,\sqrt{c-a\,c\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (4*2^(1/2)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2))))/a - (2*(
c - a*c*x)^(3/2))/(3*a*c) - (4*(c - a*c*x)^(1/2))/a
```

3.345 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=74

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6302, 6265, 21, 86, 162, 65, 214, 212}

$$2\sqrt{c-acx}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x(1+ax)}\,dx}{c}$$
$$= 2\sqrt{c-acx} - \frac{\int \frac{ac^2-3a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{ac}$$
$$= 2\sqrt{c-acx} - c\int \frac{1}{x\sqrt{c-acx}}\,dx + (4ac)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$
$$= 2\sqrt{c-acx} - 8\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\,dx, x, \sqrt{c-acx}\right) + \frac{2\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx,\right.}{a}$$
$$= 2\sqrt{c-acx} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right.$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.00

$$2\sqrt{c-acx} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - a*c*x] + 2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] - 4*Sqrt[2]*
Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A]
time = 0.20, size = 58, normalized size = 0.78

method	result
derivativedivides	$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c} - 4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c} + 2\sqrt{-acx+}$
default	$2\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)\sqrt{c} - 4\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sqrt{c} + 2\sqrt{-acx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(-a*c*x+c)^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(-a*c*x+c)^(1/2)
```

Maxima [**A**]
time = 0.48, size = 98, normalized size = 1.32

$$2\sqrt{2}\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)-\sqrt{c}\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c
) + sqrt(-a*c*x + c))) - sqrt(c)*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*
c*x + c) + sqrt(c))) + 2*sqrt(-a*c*x + c)
```

Fricas [**A**]
time = 0.35, size = 157, normalized size = 2.12

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{acx+2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+\sqrt{c}\log\left(\frac{acx-2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c},4\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)+2\sqrt{-acx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c) - 3*c)/(
a*x + 1)) + sqrt(c)*log((a*c*x - 2*sqrt(-a*c*x + c)*sqrt(c) - 2*c)/x) + 2*s
qrt(-a*c*x + c), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqr
t(-c)/c) - 2*sqrt(-c)*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) + 2*sqrt(-a*c*x +
 c)]
```

Sympy [**A**]
time = 3.18, size = 80, normalized size = 1.08

$$-\frac{2c\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}\,c\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] -2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*atan(sqrt(2)*sq
rt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

Giac [A]
time = 0.41, size = 67, normalized size = 0.91

$$\frac{4\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\,c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{-acx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 2*c*ar
ctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 2*sqrt(-a*c*x + c)
```

Mupad [B]
time = 0.08, size = 57, normalized size = 0.77

$$2\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+2\sqrt{c-a\,c\,x}-4\sqrt{2}\,\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

```
[Out] 2*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 2*(c - a*c*x)^(1/2) - 4*2^(1/2
)*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

3.346 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)$$

```
[Out] -5*a*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*arctanh(1/2*(-a*c*x+c)^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+(-a*c*x+c)^(1/2)/x
```

Rubi [A]
time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6302, 6265, 21, 100, 162, 65, 214, 212}

$$\frac{\sqrt{c-acx}}{x}-5a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x^2(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x^2(1+ax)}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{x} + \frac{\int \frac{\frac{5ac^2}{2}-\frac{3}{2}a^2c^2x}{x(1+ax)\sqrt{c-acx}}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{x} + \frac{1}{2}(5ac)\int \frac{1}{x\sqrt{c-acx}}\,dx - \left(4a^2c\right)\int \frac{1}{(1+ax)\sqrt{c-acx}}\,dx$$
$$= \frac{\sqrt{c-acx}}{x} - 5\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) + (8a)\text{Subst}\left(\int \frac{1}{2-\frac{x^2}{c}}\right.$$
$$= \frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right.$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 1.00

$$\frac{\sqrt{c-acx}}{x} - 5a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) + 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a*c*x]/x - 5*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]] + 4*Sqrt[2
]*a*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A]
time = 0.21, size = 71, normalized size = 0.91

method	result	s
derivativedivides	$-2ca\left(-\frac{\sqrt{-acx+c}}{2acx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{2\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)$	7

default	$-2ca\left(-\frac{\sqrt{-acx+c}}{2acx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{2\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}\right)$
risch	$-\frac{(ax-1)c}{x\sqrt{-c\,(ax-1)}}+\frac{a\left(\frac{8\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{10\,\mathrm{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*a*(-1/2*(-a*c*x+c)^(1/2)/a/c/x+5/2/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^
(1/2))-2*2^(1/2)/c^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.48, size = 111, normalized size = 1.42

$$-\frac{1}{2}\,ac\left(\frac{4\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{\sqrt{c}}-\frac{5\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\sqrt{-acx+c}}{acx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*c*(4*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt
(c) + sqrt(-a*c*x + c)))/sqrt(c) - 5*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt
(-a*c*x + c) + sqrt(c)))/sqrt(c) - 2*sqrt(-a*c*x + c)/(a*c*x))
```

Fricas [A]
time = 0.40, size = 176, normalized size = 2.26

$$\left[\frac{4\sqrt{2}\,a\sqrt{c}\,x\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+5\,a\sqrt{c}\,x\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\sqrt{-acx+c}}{2x},\ -\frac{4\sqrt{2}\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-5\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\sqrt{-acx+c}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(2)*a*sqrt(c)*x*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(c)
 - 3*c)/(a*x + 1)) + 5*a*sqrt(c)*x*log((a*c*x + 2*sqrt(-a*c*x + c)*sqrt(c)
- 2*c)/x) + 2*sqrt(-a*c*x + c))/x, -(4*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt
(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 5*a*sqrt(-c)*x*arctan(sqrt(-a*c*x + c)*s
qrt(-c)/c) - sqrt(-a*c*x + c))/x]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(73) = 146.
time = 4.50, size = 162, normalized size = 2.08

$$-\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{ac^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}+\frac{6ac\operatorname{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{2}\,ac\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] -a*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + a*c**
2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c))/2 + 6*a*c*atan(s
qrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 4*sqrt(2)*a*c*atan(sqrt(2)*sqrt(-a*c*x
 + c)/(2*sqrt(-c)))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

Giac [A]
time = 0.41, size = 71, normalized size = 0.91

$$-\frac{4\sqrt{2}\,ac\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{5\,ac\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 5*a
*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + sqrt(-a*c*x + c)/x
```

Mupad [B]
time = 1.24, size = 61, normalized size = 0.78

$$\frac{\sqrt{c-a\,c\,x}}{x}-5\,a\,\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{c-a\,c\,x}}{\sqrt{c}}\right)+4\,\sqrt{2}\,a\,\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}}{2\,\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

```
[Out] (c - a*c*x)^(1/2)/x - 5*a*c^(1/2)*atanh((c - a*c*x)^(1/2)/c^(1/2)) + 4*2^(1
/2)*a*c^(1/2)*atanh((2^(1/2)*(c - a*c*x)^(1/2))/(2*c^(1/2)))
```

3.347 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}+\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 23/4*a^2*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^2*arctanh(1/2*(-a*c*
x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/2*(-a*c*x+c)^(1/2)/x^2-9/4*a*
(-a*c*x+c)^(1/2)/x
```

Rubi [A]
time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6302, 6265, 21, 100, 156, 162, 65, 214, 212}

$$\frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{\sqrt{c-acx}}{2x^2}-\frac{9a\sqrt{c-acx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] Sqrt[c - a*c*x]/(2*x^2) - (9*a*Sqrt[c - a*c*x])/(4*x) + (23*a^2*Sqrt[c]*Arc
Tanh[Sqrt[c - a*c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a
*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x^3(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x^3(1+ax)}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{2x^2} + \frac{\int \frac{\frac{9ac^2}{2}-\frac{7}{2}a^2c^2x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{2c}$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} - \frac{\int \frac{\frac{23a^2c^3}{4}-\frac{9}{4}a^3c^3x}{x(1+ax)\sqrt{c-acx}}\,dx}{2c^2}$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} - \frac{1}{8}(23a^2c)\int \frac{1}{x\sqrt{c-acx}}\,dx + (4a^3c)\int \frac{}{(1+}$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} + \frac{1}{4}(23a)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{ac}}\,dx, x, \sqrt{c-acx}\right) -$$
$$= \frac{\sqrt{c-acx}}{2x^2} - \frac{9a\sqrt{c-acx}}{4x} + \frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.88

$$\frac{(2-9ax)\sqrt{c-acx}}{4x^2} + \frac{23}{4}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] ((2 - 9*a*x)*Sqrt[c - a*c*x])/(4*x^2) + (23*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/Sqrt[c]])/4 - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*S
qrt[c])]
```

Maple [A]
time = 0.22, size = 94, normalized size = 0.89

method	result

risch	$\frac{(9a^2x^2-11ax+2)c}{4x^2\sqrt{-c(ax-1)}}-\frac{a^2\left(\frac{32\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{46\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c}{8}$
derivativedivides	$2c^2a^2\left(\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7c\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right.}{c^{\frac{3}{2}}}\right.$
default	$2c^2a^2\left(\frac{\frac{\frac{9(-acx+c)^{\frac{3}{2}}}{8}-\frac{7c\sqrt{-acx+c}}{8}}{a^2c^2x^2}+\frac{23\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{8\sqrt{c}}}{c}-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right.}{c^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^2*a^2*(1/c*((9/8*(-a*c*x+c)^(3/2)-7/8*c*(-a*c*x+c)^(1/2))/a^2/c^2/x^2+2
3/8/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2)))-2/c^(3/2)*2^(1/2)*arctanh(1/
2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.47, size = 152, normalized size = 1.43

$$\frac{1}{8}a^2c^2\left(\frac{2\left(9(-acx+c)^{\frac{3}{2}}-7\sqrt{-acx+c}\,c\right)}{(acx-c)^2c+2(acx-c)c^2+c^3}+\frac{16\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{3}{2}}}-\frac{23\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*c^2*(2*(9*(-a*c*x + c)^(3/2) - 7*sqrt(-a*c*x + c)*c)/((a*c*x - c)^2
*c + 2*(a*c*x - c)*c^2 + c^3) + 16*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*
c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*c*x + c)))/c^(3/2) - 23*log((sqrt(-a*c
*x + c) - sqrt(c))/(sqrt(-a*c*x + c) + sqrt(c)))/c^(3/2))
```

Fricas [A]
time = 0.38, size = 204, normalized size = 1.92

$$\left[\frac{16\sqrt{2}\,a^2\sqrt{c}\,x^2\log\left(\frac{acx+2\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{c}-3c}{ax+1}\right)+23\,a^2\sqrt{c}\,x^2\log\left(\frac{acx-2\sqrt{-acx+c}\,\sqrt{c}-2c}{x}\right)-2\sqrt{-acx+c}\,(9\,ax-2)}{8\,x^2},\ \frac{16\sqrt{2}\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}\,\sqrt{-c}}{2\,c}\right)-23\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{-c}}{c}\right)-\sqrt{-acx+c}\,(9\,ax-2)}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*a^2*sqrt(c)*x^2*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)*sq
rt(c) - 3*c)/(a*x + 1)) + 23*a^2*sqrt(c)*x^2*log((a*c*x - 2*sqrt(-a*c*x + c
)*sqrt(c) - 2*c)/x) - 2*sqrt(-a*c*x + c)*(9*a*x - 2))/x^2, 1/4*(16*sqrt(2)*
a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt(-c)/c) - 23*a^2*s
qrt(-c)*x^2*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - sqrt(-a*c*x + c)*(9*a*x -
 2))/x^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(99) = 198$.
time = 8.54, size = 352, normalized size = 3.32

$$\frac{10a^2c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}-\frac{6a^2c^3(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2\left(-acx+c\right)^2}-\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^2c^3\sqrt{\frac{1}{c^5}}\log\left(c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}+\frac{3a^2c^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{3a^2c^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)}{2}-\frac{8a^2c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}\,a^2c\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{3a\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] 10*a**2*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**
2) - 6*a**2*c**3*(-a*c*x + c)**(3/2)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x
 + c)**2) - 3*a**2*c**3*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x
 + c))/8 + 3*a**2*c**3*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x +
 c))/8 + 3*a**2*c**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x +
c))/2 - 3*a**2*c**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)
)/2 - 8*a**2*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*a**2*c*
atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqrt(-c) - 3*a*sqrt(-a*c*x + c)
/x
```

Giac [A]
time = 0.41, size = 106, normalized size = 1.00

$$\frac{4\sqrt{2}\,a^2c\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{23\,a^2c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{9\,(-acx+c)^{\frac{3}{2}}a^2c-7\sqrt{-acx+c}\,a^2c^2}{4\,a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^2*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 23
/4*a^2*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/4*(9*(-a*c*x + c)^(
3/2)*a^2*c - 7*sqrt(-a*c*x + c)*a^2*c^2)/(a^2*c^2*x^2)
```

Mupad [B]
time = 1.24, size = 88, normalized size = 0.83

$$\frac{9\,{\left(c-a\,c\,x\right)}^{3/2}}{4\,c\,x^2}-\frac{a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,23{}\mathrm{i}}{4}-\frac{7\,\sqrt{c-a\,c\,x}}{4\,x^2}+\sqrt{2}\,a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,4{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] (9*(c - a*c*x)^(3/2))/(4*c*x^2) - (a^2*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/
c^(1/2))*23i)/4 - (7*(c - a*c*x)^(1/2))/(4*x^2) + 2^(1/2)*a^2*c^(1/2)*atan(
(2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

3.348 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{2}}\right.$$

```
[Out] -45/8*a^3*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a^3*arctanh(1/2*(-a*c
*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/3*(-a*c*x+c)^(1/2)/x^3-13/12
*a*(-a*c*x+c)^(1/2)/x^2+19/8*a^2*(-a*c*x+c)^(1/2)/x
```

Rubi [A]
time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6302, 6265, 21, 100, 156, 162, 65, 214, 212}

$$-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}\,a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)+\frac{19a^2\sqrt{c-acx}}{8x}+\frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] Sqrt[c - a*c*x]/(3*x^3) - (13*a*Sqrt[c - a*c*x])/(12*x^2) + (19*a^2*Sqrt[c
- a*c*x])/(8*x) - (45*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*S
qrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx \\
&= -\int \frac{(1-ax)\sqrt{c-acx}}{x^4(1+ax)}\,dx \\
&= -\frac{\int \frac{(c-acx)^{3/2}}{x^4(1+ax)}\,dx}{c} \\
&= \frac{\sqrt{c-acx}}{3x^3}+\frac{\int \frac{\frac{13ac^2}{2}-\frac{11}{2}a^2c^2x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{3c} \\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}-\frac{\int \frac{\frac{57a^2c^3}{4}-\frac{39}{4}a^3c^3x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{6c^2} \\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}+\frac{\int \frac{\frac{135a^3c^4}{8}-\frac{57}{8}a^4c^4x}{x(1+ax)\sqrt{c-acx}}\,dx}{6c^3} \\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}+\frac{1}{16}\left(45a^3c\right)\int \frac{1}{x\sqrt{c-acx}} \\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}-\frac{1}{8}\left(45a^2\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x}{a}} \right. \\
&= \frac{\sqrt{c-acx}}{3x^3}-\frac{13a\sqrt{c-acx}}{12x^2}+\frac{19a^2\sqrt{c-acx}}{8x}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{c}}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 101, normalized size = 0.80

$$
\frac{\sqrt{c-acx}\left(8-26ax+57a^2x^2\right)}{24x^3}-\frac{45}{8}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)+4\sqrt{2}a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(8 - 26*a*x + 57*a^2*x^2))/(24*x^3) - (45*a^3*Sqrt[c]*ArcT
anh[Sqrt[c - a*c*x]/Sqrt[c]])/8 + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - a*
c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A]
time = 0.22, size = 108, normalized size = 0.85

method	result

risch	$-\frac{(57a^3x^3-83a^2x^2+34ax-8)c}{24x^3\sqrt{-c(ax-1)}}+\frac{a^3\left(\frac{64\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{90\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c}{16}$
derivativedivides	$-2c^3a^3\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13c^2\sqrt{-acx+c}}{16}}{a^3c^3x^3}+\cdots}{c^2}\right.$
default	$-2c^3a^3\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\frac{-\frac{19(-acx+c)^{\frac{5}{2}}}{16}+\frac{11c(-acx+c)^{\frac{3}{2}}}{6}-\frac{13c^2\sqrt{-acx+c}}{16}}{a^3c^3x^3}+\cdots}{c^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*c^3*a^3*(-2/c^(5/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2)
)+1/c^2*((-19/16*(-a*c*x+c)^(5/2)+11/6*c*(-a*c*x+c)^(3/2)-13/16*c^2*(-a*c*x
+c)^(1/2))/a^3/c^3/x^3+45/16/c^(1/2)*arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

Maxima [A]

time = 0.47, size = 183, normalized size = 1.44

$$\frac{1}{48}a^3c^3\left(\frac{2\left(57(-acx+c)^{\frac{5}{2}}-88(-acx+c)^{\frac{3}{2}}c+39\sqrt{-acx+c}\,c^2\right)}{(acx-c)^3c^2+3(acx-c)^2c^3+3(acx-c)c^4+c^5}-\frac{96\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{5}{2}}}+\frac{135\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] 1/48*a^3*c^3*(2*(57*(-a*c*x + c)^(5/2) - 88*(-a*c*x + c)^(3/2)*c + 39*sqrt(
-a*c*x + c)*c^2)/((a*c*x - c)^3*c^2 + 3*(a*c*x - c)^2*c^3 + 3*(a*c*x - c)*c
^4 + c^5) - 96*sqrt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*s
qrt(c) + sqrt(-a*c*x + c)))/c^(5/2) + 135*log((sqrt(-a*c*x + c) - sqrt(c))/
(sqrt(-a*c*x + c) + sqrt(c)))/c^(5/2))
```

Fricas [A]

time = 0.42, size = 220, normalized size = 1.73

$$\left[\frac{96\sqrt{2}\,a^3\sqrt{c}\,x^3\log\left(\frac{acx-2\sqrt{2}\sqrt{-acx+c}\sqrt{c}-3c}{ax+1}\right)+135\,a^3\sqrt{c}\,x^3\log\left(\frac{acx+2\sqrt{-acx+c}\sqrt{c}-2c}{x}\right)+2\left(57\,a^2x^2-26\,ax+8\right)\sqrt{-acx+c}}{48\,x^3},\ -\frac{96\sqrt{2}\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{2}\sqrt{-acx+c}\sqrt{-c}}{2c}\right)-135\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-acx+c}\sqrt{-c}}{c}\right)-\left(57\,a^2x^2-26\,ax+8\right)\sqrt{-acx+c}}{24\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*a^3*sqrt(c)*x^3*log((a*c*x - 2*sqrt(2)*sqrt(-a*c*x + c)*s
qrt(c) - 3*c)/(a*x + 1)) + 135*a^3*sqrt(c)*x^3*log((a*c*x + 2*sqrt(-a*c*x +
 c)*sqrt(c) - 2*c)/x) + 2*(57*a^2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3,
-1/24*(96*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)*sqrt
(-c)/c) - 135*a^3*sqrt(-c)*x^3*arctan(sqrt(-a*c*x + c)*sqrt(-c)/c) - (57*a^
2*x^2 - 26*a*x + 8)*sqrt(-a*c*x + c))/x^3]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(119) = 238$.
time = 9.38, size = 614, normalized size = 4.83

$$-\frac{66a^3c^6\sqrt{-acx+c}}{-144ac^6x+96c^6-144c^4(-acx+c)^2+48c^3(-acx+c)^3}+\frac{80a^3c^5(-acx+c)^{\frac{3}{2}}}{-144ac^6x+96c^6-144c^4(-acx+c)^2+48c^3(-acx+c)^3}-\frac{30a^3c^4(-acx+c)^{\frac{5}{2}}}{-144ac^6x+96c^6-144c^4(-acx+c)^2+48c^3(-acx+c)^3}-\frac{30a^3c^4\sqrt{-acx+c}}{16ac^4x-8c^4+8c^2(-acx+c)^2}-\frac{5a^3c^4\sqrt{\frac{1}{c^7}}\log\left(-c^4\sqrt{\frac{1}{c^7}}+\sqrt{-acx+c}\right)}{16}+\frac{5a^3c^4\sqrt{\frac{1}{c^7}}\log\left(c^4\sqrt{\frac{1}{c^7}}+\sqrt{-acx+c}\right)}{16}+\frac{18a^3c^3(-acx+c)^{\frac{3}{2}}}{16ac^4x-8c^4+8c^2(-acx+c)^2}+\frac{9a^3c^3\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}-\frac{9a^3c^3\sqrt{\frac{1}{c^5}}\log\left(c^3\sqrt{\frac{1}{c^5}}+\sqrt{-acx+c}\right)}{8}-2a^3c^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)+2a^3c^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{-acx+c}\right)+\frac{8a^3c\,\mathrm{atan}\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{4\sqrt{2}\,a^3c\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4a^2\sqrt{-acx+c}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] -66*a**3*c**6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x
 + c)**2 + 48*c**3*(-a*c*x + c)**3) + 80*a**3*c**5*(-a*c*x + c)**(3/2)/(-144
*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) -
 30*a**3*c**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c
*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 30*a**3*c**4*sqrt(-a*c*x + c)/(16*a
*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 5*a**3*c**4*sqrt(c**(-7))*log(
-c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 5*a**3*c**4*sqrt(c**(-7))*log(
c**4*sqrt(c**(-7)) + sqrt(-a*c*x + c))/16 + 18*a**3*c**3*(-a*c*x + c)**(3/2
)/(16*a*c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) + 9*a**3*c**3*sqrt(c**(-5
))*log(-c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 9*a**3*c**3*sqrt(c**(-5)
)*log(c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/8 - 2*a**3*c**2*sqrt(c**(-3))*
log(-c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) + 2*a**3*c**2*sqrt(c**(-3))*log
(c**2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) + 8*a**3*c*atan(sqrt(-a*c*x + c)/sq
rt(-c))/sqrt(-c) - 4*sqrt(2)*a**3*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-
c)))/sqrt(-c) + 4*a**2*sqrt(-a*c*x + c)/x
```

Giac [**A**]
time = 0.41, size = 133, normalized size = 1.05

$$-\frac{4\sqrt{2}\,a^3c\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{45\,a^3c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{57\,(acx-c)^2\sqrt{-acx+c}\,a^3c-88\,(-acx+c)^{\frac{3}{2}}a^3c^2+39\,\sqrt{-acx+c}\,a^3c^3}{24\,a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 4
5/8*a^3*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) + 1/24*(57*(a*c*x - c)
```

```
^2*sqrt(-a*c*x + c)*a^3*c - 88*(-a*c*x + c)^(3/2)*a^3*c^2 + 39*sqrt(-a*c*x
+ c)*a^3*c^3)/(a^3*c^3*x^3)
```

Mupad [B]
time = 0.13, size = 105, normalized size = 0.83

$$\frac{13\sqrt{c-a\,c\,x}}{8\,x^3}+\frac{a^3\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,\,1\mathrm{i}}{\sqrt{c}}\right)\,45\mathrm{i}}{8}-\frac{11\,{\left(c-a\,c\,x\right)}^{3/2}}{3\,c\,x^3}+\frac{19\,{\left(c-a\,c\,x\right)}^{5/2}}{8\,c^2\,x^3}-\sqrt{2}\,a^3\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,4\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

```
[Out] (13*(c - a*c*x)^(1/2))/(8*x^3) + (a^3*c^(1/2)*atan(((c - a*c*x)^(1/2)*1i)/c
^(1/2))*45i)/8 - (11*(c - a*c*x)^(3/2))/(3*c*x^3) + (19*(c - a*c*x)^(5/2))/
(8*c^2*x^3) - 2^(1/2)*a^3*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^
(1/2)))*4i
```

3.349 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}+\frac{107a^2\sqrt{c-acx}}{96x^2}-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a$$

```
[Out] 363/64*a^4*arctanh((-a*c*x+c)^(1/2)/c^(1/2))*c^(1/2)-4*a^4*arctanh(1/2*(-a*
c*x+c)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+1/4*(-a*c*x+c)^(1/2)/x^4-17/2
4*a*(-a*c*x+c)^(1/2)/x^3+107/96*a^2*(-a*c*x+c)^(1/2)/x^2-149/64*a^3*(-a*c*x
+c)^(1/2)/x
```

Rubi [A]
time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6302, 6265, 21, 100, 156, 162, 65, 214, 212}

$$\frac{363}{64}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right)-4\sqrt{2}\,a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\sqrt{c}}\right)-\frac{149a^3\sqrt{c-acx}}{64x}+\frac{107a^2\sqrt{c-acx}}{96x^2}+\frac{\sqrt{c-acx}}{4x^4}-\frac{17a\sqrt{c-acx}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] Sqrt[c - a*c*x]/(4*x^4) - (17*a*Sqrt[c - a*c*x])/(24*x^3) + (107*a^2*Sqrt[c
 - a*c*x])/(96*x^2) - (149*a^3*Sqrt[c - a*c*x])/(64*x) + (363*a^4*Sqrt[c]*A
rcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c
- a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6265

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Int[u*(c + d*x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c
, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$$
$$= -\int \frac{(1-ax)\sqrt{c-acx}}{x^5(1+ax)}\,dx$$
$$= -\frac{\int \frac{(c-acx)^{3/2}}{x^5(1+ax)}\,dx}{c}$$
$$= \frac{\sqrt{c-acx}}{4x^4} + \frac{\int \frac{\frac{17ac^2}{2}-\frac{15}{2}a^2c^2x}{x^4(1+ax)\sqrt{c-acx}}\,dx}{4c}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} - \frac{\int \frac{\frac{107a^2c^3}{4}-\frac{85}{4}a^3c^3x}{x^3(1+ax)\sqrt{c-acx}}\,dx}{12c^2}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} + \frac{\int \frac{\frac{447a^3c^4}{8}-\frac{321}{8}a^4c^4x}{x^2(1+ax)\sqrt{c-acx}}\,dx}{24c^3}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} - \frac{\int \frac{1}{x(}}{}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} - \frac{1}{128}$$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{1}{64}($$
$$= \frac{\sqrt{c-acx}}{4x^4} - \frac{17a\sqrt{c-acx}}{24x^3} + \frac{107a^2\sqrt{c-acx}}{96x^2} - \frac{149a^3\sqrt{c-acx}}{64x} + \frac{363}{64}$$

Mathematica [A]
time = 0.08, size = 109, normalized size = 0.74

$$\frac{\sqrt{c-acx}\,(48-136ax+214a^2x^2-447a^3x^3)}{192x^4} + \frac{363}{64}a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{c}}\right) - 4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-acx}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a*c*x]*(48 - 136*a*x + 214*a^2*x^2 - 447*a^3*x^3))/(192*x^4) + (3
63*a^4*Sqrt[c]*ArcTanh[Sqrt[c - a*c*x]/Sqrt[c]])/64 - 4*Sqrt[2]*a^4*Sqrt[c]
*ArcTanh[Sqrt[c - a*c*x]/(Sqrt[2]*Sqrt[c])]
```

Maple [A]
time = 0.23, size = 122, normalized size = 0.82

method	result
risch	$\frac{(447a^4x^4-661a^3x^3+350a^2x^2-184ax+48)c}{192x^4\sqrt{-c(ax-1)}}-\frac{a^4\left(\frac{512\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}}-\frac{726\operatorname{arctanh}\left(\frac{\sqrt{-acx}}{\sqrt{c}}\right.}{\sqrt{c}}\right.}{128}$
derivativedivides	$2c^4a^4\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107c^3\sqrt{}}{}}{a^4c^4x^4}}{c^3}\right.$
default	$2c^4a^4\left(-\frac{2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-acx+c}\,\sqrt{2}}{2\sqrt{c}}\right)}{c^{\frac{7}{2}}}+\frac{\frac{\frac{149(-acx+c)^{\frac{7}{2}}}{128}-\frac{1127c(-acx+c)^{\frac{5}{2}}}{384}+\frac{1049c^2(-acx+c)^{\frac{3}{2}}}{384}-\frac{107c^3\sqrt{}}{}}{a^4c^4x^4}}{c^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*c^4*a^4*(-2/c^(7/2)*2^(1/2)*arctanh(1/2*(-a*c*x+c)^(1/2)*2^(1/2)/c^(1/2))
+1/c^3*((149/128*(-a*c*x+c)^(7/2)-1127/384*c*(-a*c*x+c)^(5/2)+1049/384*c^2*
(-a*c*x+c)^(3/2)-107/128*c^3*(-a*c*x+c)^(1/2))/a^4/c^4/x^4+363/128/c^(1/2)*
arctanh((-a*c*x+c)^(1/2)/c^(1/2))))
```

Maxima [A]

time = 0.47, size = 212, normalized size = 1.43

$$\frac{1}{384}a^4c^4\left(\frac{2\left(447\,(-acx+c)^{\frac{7}{2}}-1127\,(-acx+c)^{\frac{5}{2}}c+1049\,(-acx+c)^{\frac{3}{2}}c^2-321\,\sqrt{-acx+c}\,c^3\right)}{(acx-c)^4c^3+4\,(acx-c)^3c^4+6\,(acx-c)^2c^5+4\,(acx-c)c^6+c^7}+\frac{768\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,\sqrt{c}-\sqrt{-acx+c}}{\sqrt{2}\,\sqrt{c}+\sqrt{-acx+c}}\right)}{c^{\frac{7}{2}}}-\frac{1089\,\log\left(\frac{\sqrt{-acx+c}-\sqrt{c}}{\sqrt{-acx+c}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] 1/384*a^4*c^4*(2*(447*(-a*c*x + c)^(7/2) - 1127*(-a*c*x + c)^(5/2)*c + 1049
*(-a*c*x + c)^(3/2)*c^2 - 321*sqrt(-a*c*x + c)*c^3)/((a*c*x - c)^4*c^3 + 4*
(a*c*x - c)^3*c^4 + 6*(a*c*x - c)^2*c^5 + 4*(a*c*x - c)*c^6 + c^7) + 768*sq
rt(2)*log(-(sqrt(2)*sqrt(c) - sqrt(-a*c*x + c))/(sqrt(2)*sqrt(c) + sqrt(-a*
c*x + c)))/c^(7/2) - 1089*log((sqrt(-a*c*x + c) - sqrt(c))/(sqrt(-a*c*x + c
) + sqrt(c)))/c^(7/2))
```

Fricas [A]
time = 0.37, size = 236, normalized size = 1.59

$$\left[\frac{768 \sqrt{2} a^{4} \sqrt{c} x^{4} \log \left(\frac{a c x+2 \sqrt{2} \sqrt{-a c x+c} \sqrt{c}-3 c}{a x+1}\right)+1089 a^{4} \sqrt{c} x^{4} \log \left(\frac{a c x-2 \sqrt{-a c x+c} \sqrt{c}-2 c}{x}\right)-2\left(447 a^{3} x^{3}-214 a^{2} x^{2}+136 a x-48\right) \sqrt{-a c x+c}}{384 x^{4}}, \frac{768 \sqrt{2} a^{4} \sqrt{-c} x^{4} \arctan \left(\frac{\sqrt{2} \sqrt{-a c x+c} \sqrt{-c}}{2 c}\right)-1089 a^{4} \sqrt{-c} x^{4} \arctan \left(\frac{\sqrt{-a c x+c} \sqrt{-c}}{c}\right)-\left(447 a^{3} x^{3}-214 a^{2} x^{2}+136 a x-48\right) \sqrt{-a c x+c}}{192 x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(768*sqrt(2)*a^4*sqrt(c)*x^4*log((a*c*x + 2*sqrt(2)*sqrt(-a*c*x + c)
*sqrt(c) - 3*c)/(a*x + 1)) + 1089*a^4*sqrt(c)*x^4*log((a*c*x - 2*sqrt(-a*c*
x + c)*sqrt(c) - 2*c)/x) - 2*(447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqr
t(-a*c*x + c))/x^4, 1/192*(768*sqrt(2)*a^4*sqrt(-c)*x^4*arctan(1/2*sqrt(2)*
sqrt(-a*c*x + c)*sqrt(-c)/c) - 1089*a^4*sqrt(-c)*x^4*arctan(sqrt(-a*c*x + c
)*sqrt(-c)/c) - (447*a^3*x^3 - 214*a^2*x^2 + 136*a*x - 48)*sqrt(-a*c*x + c)
)/x^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(139) = 278$.
time = 16.56, size = 991, normalized size = 6.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] 558*a**4*c**8*sqrt(-a*c*x + c)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c
*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) - 1022*a
**4*c**7*(-a*c*x + c)**(3/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x
 + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 770*a**4
*c**6*(-a*c*x + c)**(5/2)/(1536*a*c**8*x - 1152*c**8 + 2304*c**6*(-a*c*x +
c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c**4*(-a*c*x + c)**4) + 198*a**4*c*
*6*sqrt(-a*c*x + c)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2 + 4
8*c**3*(-a*c*x + c)**3) - 210*a**4*c**5*(-a*c*x + c)**(7/2)/(1536*a*c**8*x
- 1152*c**8 + 2304*c**6*(-a*c*x + c)**2 - 1536*c**5*(-a*c*x + c)**3 + 384*c
**4*(-a*c*x + c)**4) - 240*a**4*c**5*(-a*c*x + c)**(3/2)/(-144*a*c**6*x + 9
6*c**6 - 144*c**4*(-a*c*x + c)**2 + 48*c**3*(-a*c*x + c)**3) - 35*a**4*c**5
*sqrt(c**(-9))*log(-c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 35*a**4*c*
*5*sqrt(c**(-9))*log(c**5*sqrt(c**(-9)) + sqrt(-a*c*x + c))/128 + 90*a**4*c
**4*(-a*c*x + c)**(5/2)/(-144*a*c**6*x + 96*c**6 - 144*c**4*(-a*c*x + c)**2
 + 48*c**3*(-a*c*x + c)**3) + 40*a**4*c**4*sqrt(-a*c*x + c)/(16*a*c**4*x -
8*c**4 + 8*c**2*(-a*c*x + c)**2) + 15*a**4*c**4*sqrt(c**(-7))*log(-c**4*sqr
t(c**(-7)) + sqrt(-a*c*x + c))/16 - 15*a**4*c**4*sqrt(c**(-7))*log(c**4*sqr
t(c**(-7)) + sqrt(-a*c*x + c))/16 - 24*a**4*c**3*(-a*c*x + c)**(3/2)/(16*a*
c**4*x - 8*c**4 + 8*c**2*(-a*c*x + c)**2) - 3*a**4*c**3*sqrt(c**(-5))*log(-
c**3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 + 3*a**4*c**3*sqrt(c**(-5))*log(c*
```

```
*3*sqrt(c**(-5)) + sqrt(-a*c*x + c))/2 + 2*a**4*c**2*sqrt(c**(-3))*log(-c**
2*sqrt(c**(-3)) + sqrt(-a*c*x + c)) - 2*a**4*c**2*sqrt(c**(-3))*log(c**2*sq
rt(c**(-3)) + sqrt(-a*c*x + c)) - 8*a**4*c*atan(sqrt(-a*c*x + c)/sqrt(-c))/
sqrt(-c) + 4*sqrt(2)*a**4*c*atan(sqrt(2)*sqrt(-a*c*x + c)/(2*sqrt(-c)))/sqr
t(-c) - 4*a**3*sqrt(-a*c*x + c)/x
```

Giac [A]
time = 0.42, size = 160, normalized size = 1.08

$$\frac{4\sqrt{2}\,a^4c\arctan\left(\frac{\sqrt{2}\,\sqrt{-acx+c}}{2\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{363\,a^4c\arctan\left(\frac{\sqrt{-acx+c}}{\sqrt{-c}}\right)}{64\sqrt{-c}}-\frac{447\,(acx-c)^3\sqrt{-acx+c}\,a^4c+1127\,(acx-c)^2\sqrt{-acx+c}\,a^4c^2-1049\,(-acx+c)^{\frac{3}{2}}a^4c^3+321\sqrt{-acx+c}\,a^4c^4}{192\,a^4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 36
3/64*a^4*c*arctan(sqrt(-a*c*x + c)/sqrt(-c))/sqrt(-c) - 1/192*(447*(a*c*x -
 c)^3*sqrt(-a*c*x + c)*a^4*c + 1127*(a*c*x - c)^2*sqrt(-a*c*x + c)*a^4*c^2
- 1049*(-a*c*x + c)^(3/2)*a^4*c^3 + 321*sqrt(-a*c*x + c)*a^4*c^4)/(a^4*c^4*
x^4)
```

Mupad [B]
time = 1.26, size = 122, normalized size = 0.82

$$\frac{1049\,(c-a\,c\,x)^{3/2}}{192\,c\,x^4}-\frac{a^4\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c-a\,c\,x}\,1\mathrm{i}}{\sqrt{c}}\right)363\mathrm{i}}{64}-\frac{107\sqrt{c-a\,c\,x}}{64\,x^4}-\frac{1127\,(c-a\,c\,x)^{5/2}}{192\,c^2\,x^4}+\frac{149\,(c-a\,c\,x)^{7/2}}{64\,c^3\,x^4}+\sqrt{2}\,a^4\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-a\,c\,x}\,1\mathrm{i}}{2\sqrt{c}}\right)4\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

```
[Out] (1049*(c - a*c*x)^(3/2))/(192*c*x^4) - (a^4*c^(1/2)*atan(((c - a*c*x)^(1/2)
*1i)/c^(1/2))*363i)/64 - (107*(c - a*c*x)^(1/2))/(64*x^4) - (1127*(c - a*c*
x)^(5/2))/(192*c^2*x^4) + (149*(c - a*c*x)^(7/2))/(64*c^3*x^4) + 2^(1/2)*a^
4*c^(1/2)*atan((2^(1/2)*(c - a*c*x)^(1/2)*1i)/(2*c^(1/2)))*4i
```

3.350 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx$

Optimal. Leaf size=281

$$\frac{1312\sqrt{1+\frac{1}{ax}}\,\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-a}}{15a^2\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -82/9*x^2*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-8/9*x^3*(-a*
c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/9*x^4*(-a*c*x+c)^(1/2)/(1-
1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1312/45*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^4/(
1-1/a/x)^(1/2)-656/45*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^3/(1-1/a/x)^(1/2
)+164/15*x^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6311, 6316, 91, 79, 47, 37}

$$\frac{1312\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656x\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}+\frac{164x^2\sqrt{\frac{1}{ax}+1}\,\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (1312*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(45*a^4*Sqrt[1 - 1/(a*x)]) - (656*
Sqrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(45*a^3*Sqrt[1 - 1/(a*x)]) - (82*x^2*S
qrt[c - a*c*x])/(9*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (164*Sqrt[1 +
 1/(a*x)]*x^2*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (8*x^3*Sqrt[c -
 a*c*x])/(9*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^4*Sqrt[c - a*c*x]
)/(9*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
```

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{7/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{11/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{14}{a}+\frac{9x}{2a^2}}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x\right.}{9\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(41\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.}{}$$

$$=-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{8x^3\sqrt{c-acx}}{9a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^4\sqrt{c-acx}}{9\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

$$=-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}\,x^2\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{8x^3\sqrt{c-}}{9a\sqrt{1-\frac{1}{}}}$$

$$=-\frac{656\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-acx}}{9a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{164\sqrt{1+\frac{1}{ax}}}{15a^2\sqrt{1}}$$

$$=\frac{1312\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{45a^4\sqrt{1-\frac{1}{ax}}}-\frac{656\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{45a^3\sqrt{1-\frac{1}{ax}}}-\frac{82x^2\sqrt{c-}}{9a^2\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 0.26

$$\frac{2\sqrt{c-acx}\,(656+328ax-82a^2x^2+41a^3x^3-20a^4x^4+5a^5x^5)}{45a^5\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(656 + 328*a*x - 82*a^2*x^2 + 41*a^3*x^3 - 20*a^4*x^4 +
5*a^5*x^5))/(45*a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 81, normalized size = 0.29

method	result	size
gosper	$\frac{2(ax+1)(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45a^4(ax-1)^2}$	80
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,(5a^5x^5-20a^4x^4+41a^3x^3-82a^2x^2+328ax+656)}{45(ax-1)^2a^4}$	81
risch	$-\frac{2(5a^4x^4-25a^3x^3+66a^2x^2-148ax+476)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{45a^4\sqrt{-c\,(ax-1)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^4\sqrt{-c\,(ax-1)}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(5*a^5*x^
5-20*a^4*x^4+41*a^3*x^3-82*a^2*x^2+328*a*x+656)/a^4
```

Maxima [A]
time = 0.30, size = 117, normalized size = 0.42

$$\frac{2\,(5\,a^6\sqrt{-c}\,x^6-15\,a^5\sqrt{-c}\,x^5+21\,a^4\sqrt{-c}\,x^4-41\,a^3\sqrt{-c}\,x^3+246\,a^2\sqrt{-c}\,x^2+984\,a\sqrt{-c}\,x+656\,\sqrt{-c}\,)(ax-1)^2}{45\,(a^6x^2-2\,a^5x+a^4)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 2/45*(5*a^6*sqrt(-c)*x^6 - 15*a^5*sqrt(-c)*x^5 + 21*a^4*sqrt(-c)*x^4 - 41*a
^3*sqrt(-c)*x^3 + 246*a^2*sqrt(-c)*x^2 + 984*a*sqrt(-c)*x + 656*sqrt(-c))*(
a*x - 1)^2/((a^6*x^2 - 2*a^5*x + a^4)*(a*x + 1)^(3/2))
```

Fricas [**A**]
time = 0.34, size = 77, normalized size = 0.27

$$\frac{2\left(5\,a^{5}x^{5}-20\,a^{4}x^{4}+41\,a^{3}x^{3}-82\,a^{2}x^{2}+328\,ax+656\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{45\left(a^{5}x-a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(5*a^5*x^5 - 20*a^4*x^4 + 41*a^3*x^3 - 82*a^2*x^2 + 328*a*x + 656)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^5*x - a^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.41, size = 74, normalized size = 0.26

$$\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^5\,x^5-20\,a^4\,x^4+41\,a^3\,x^3-82\,a^2\,x^2+328\,a\,x+656\right)}{45\,a^4\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(328*a*x - 82*a^2*x^2 + 41
*a^3*x^3 - 20*a^4*x^4 + 5*a^5*x^5 + 656))/(45*a^4*(a*x - 1))
```

3.351 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx$

Optimal. Leaf size=231

$$-\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1336\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

```
[Out] -334/35*x*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-44/35*x^2*(-
a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/7*x^3*(-a*c*x+c)^(1/2)/(
1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2672/105*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^
3/(1-1/a/x)^(1/2)+1336/105*x*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)
^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6311, 6316, 91, 79, 47, 37}

$$-\frac{2672\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}+\frac{1336x\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2672*Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(105*a^3*Sqrt[1 - 1/(a*x)]) - (33
4*x*Sqrt[c - a*c*x])/(35*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1336*S
qrt[1 + 1/(a*x)]*x*Sqrt[c - a*c*x])/(105*a^2*Sqrt[1 - 1/(a*x)]) - (44*x^2*S
qrt[c - a*c*x])/(35*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^3*Sqrt[c
- a*c*x])/(7*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
```

```
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{5/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{11}{a}+\frac{7x}{2a^2}}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\right.}{7\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^3\sqrt{c-acx}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(167\sqrt{\frac{1}{x}}\sqrt{c-ac\right.}}{}$$

$$=-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{44x^2\sqrt{c-acx}}{35a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^3\sqrt{c-ac}}{7\sqrt{1-\frac{1}{ax}}\sqrt{1}}$$

$$=-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1336\sqrt{1+\frac{1}{ax}}\,x\sqrt{c-acx}}{105a^2\sqrt{1-\frac{1}{ax}}}-\frac{44x^2\sqrt{c}}{35a\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{2672\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{105a^3\sqrt{1-\frac{1}{ax}}}-\frac{334x\sqrt{c-acx}}{35a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1336\sqrt{1+\frac{1}{ax}}}{105a^2\sqrt{1}}$$

Mathematica [A]

time = 0.03, size = 65, normalized size = 0.28

$$\frac{2\sqrt{c-acx}\,(-1336-668ax+167a^2x^2-66a^3x^3+15a^4x^4)}{105a^4\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-1336 - 668*a*x + 167*a^2*x^2 - 66*a^3*x^3 + 15*a^4*x^4
))/(105*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 73, normalized size = 0.32

method	result	size
gosper	$\frac{2(ax+1)(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105a^3(ax-1)^2}$	72
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,(15a^4x^4-66a^3x^3+167a^2x^2-668ax-1336)}{105(ax-1)^2a^3}$	73
risch	$-\frac{2(15a^3x^3-81a^2x^2+248ax-916)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{105a^3\sqrt{-c\,(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^3\sqrt{-c\,(ax-1)}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(15*a^4*
x^4-66*a^3*x^3+167*a^2*x^2-668*a*x-1336)/a^3
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.45

$$\frac{2\,\left(15\,a^5\sqrt{-c}\,x^5-51\,a^4\sqrt{-c}\,x^4+101\,a^3\sqrt{-c}\,x^3-501\,a^2\sqrt{-c}\,x^2-2004\,a\sqrt{-c}\,x-1336\,\sqrt{-c}\,\right)(ax-1)^2}{105\,(a^5x^2-2\,a^4x+a^3)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 2/105*(15*a^5*sqrt(-c)*x^5 - 51*a^4*sqrt(-c)*x^4 + 101*a^3*sqrt(-c)*x^3 - 5
01*a^2*sqrt(-c)*x^2 - 2004*a*sqrt(-c)*x - 1336*sqrt(-c))*(a*x - 1)^2/((a^5*
x^2 - 2*a^4*x + a^3)*(a*x + 1)^(3/2))
```

Fricas [A]
time = 0.33, size = 69, normalized size = 0.30

$$\frac{2\,(15\,a^4x^4-66\,a^3x^3+167\,a^2x^2-668\,ax-1336)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^4x-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(15*a^4*x^4 - 66*a^3*x^3 + 167*a^2*x^2 - 668*a*x - 1336)*sqrt(-a*c*x
+ c)*sqrt((a*x - 1)/(a*x + 1))/(a^4*x - a^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.34, size = 88, normalized size = 0.38

$$\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(15\,a^3\,x^3-51\,a^2\,x^2+116\,a\,x-552\right)}{105\,a^3}-\frac{3776\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{105\,a^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(116*a*x - 51*a^2*x^2 + 15
*a^3*x^3 - 552))/(105*a^3) - (3776*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^
(1/2))/(105*a^3*(a*x - 1))
```

3.352 $\int e^{-3\coth^{-1}(ax)} x\sqrt{c-acx}\, dx$

Optimal. Leaf size=182

$$-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -158/15*(-a*c*x+c)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32/15*x*(-a*c*
x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/5*x^2*(-a*c*x+c)^(1/2)/(1-1/
a/x)^(1/2)/(1+1/a/x)^(1/2)+316/15*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/a^2/(1-1
/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6311, 6316, 91, 79, 47, 37}

$$\frac{316\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}}-\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-158*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (316*
Sqrt[1 + 1/(a*x)]*Sqrt[c - a*c*x])/(15*a^2*Sqrt[1 - 1/(a*x)]) - (32*x*Sqrt[
c - a*c*x])/(15*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2*x^2*Sqrt[c - a*
c*x])/(5*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x\sqrt{c-acx}\,dx = \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^{3/2}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{-\frac{8}{a}+\frac{5x}{2a^2}}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x,\right.}{5\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-acx}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(79\sqrt{\frac{1}{x}}\sqrt{c-acx}\right.}{}$$

$$= -\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{32x\sqrt{c-acx}}{15a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x^2\sqrt{c-a}}{5\sqrt{1-\frac{1}{ax}}\sqrt{1}}$$

$$= -\frac{158\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{316\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{15a^2\sqrt{1-\frac{1}{ax}}} - \frac{32x\sqrt{c-}}{15a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.31

$$\frac{2\sqrt{c-acx}\,(158+79ax-16a^2x^2+3a^3x^3)}{15a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a*c*x])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(158 + 79*a*x - 16*a^2*x^2 + 3*a^3*x^3))/(15*a^3*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.36

method	result	size
gosper	$\frac{2(ax+1)(3a^3x^3-16a^2x^2+79ax+158)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15a^2(ax-1)^2}$	64
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,(3a^3x^3-16a^2x^2+79ax+158)}{15(ax-1)^2a^2}$	65
risch	$-\frac{2(3a^2x^2-19ax+98)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{15a^2\sqrt{-c\,(ax-1)}}-\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a^2\sqrt{-c\,(ax-1)}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(3*a^3*x^
3-16*a^2*x^2+79*a*x+158)/a^2
```

Maxima [A]
time = 0.30, size = 91, normalized size = 0.50

$$\frac{2\left(3\,a^4\sqrt{-c}\,x^4-13\,a^3\sqrt{-c}\,x^3+63\,a^2\sqrt{-c}\,x^2+237\,a\sqrt{-c}\,x+158\,\sqrt{-c}\,\right)(ax-1)^2}{15\,(a^4x^2-2\,a^3x+a^2)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*a^4*sqrt(-c)*x^4 - 13*a^3*sqrt(-c)*x^3 + 63*a^2*sqrt(-c)*x^2 + 237*
a*sqrt(-c)*x + 158*sqrt(-c))*(a*x - 1)^2/((a^4*x^2 - 2*a^3*x + a^2)*(a*x +
1)^(3/2))
```

Fricas [A]
time = 0.33, size = 61, normalized size = 0.34

$$\frac{2\,(3\,a^3x^3-16\,a^2x^2+79\,ax+158)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3x-a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^3*x^3 - 16*a^2*x^2 + 79*a*x + 158)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1))/(a^3*x - a^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.33, size = 58, normalized size = 0.32

$$\frac{2\,\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(3\,a^3\,x^3-16\,a^2\,x^2+79\,a\,x+158\right)}{15\,a^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a*x - 16*a^2*x^2 + 3*a
^3*x^3 + 158))/(15*a^2*(a*x - 1))
```

3.353 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=137

$$-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}$$

```
[Out] -20/3*(-a*c*x+c)^(1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-46/3*(-a*c*x+c)^(1
/2)/a^2/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+2/3*x*(-a*c*x+c)^(1/2)/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 91, 79, 37}

$$-\frac{46\sqrt{c-acx}}{3a^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-20*Sqrt[c - a*c*x])/(3*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (46*Sqrt[
c - a*c*x])/(3*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (2*x*Sqrt[c - a
*c*x])/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-acx}\,dx &= \frac{\sqrt{c-acx}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}} \\
&= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} \\
&= \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{-\frac{5}{a}+\frac{3x}{2a^2}}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}} \\
&= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(23\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\ldots}{3a\ldots} \\
&= -\frac{20\sqrt{c-acx}}{3a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{46\sqrt{c-acx}}{3a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{2x\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\ldots}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.35

$$\frac{2\sqrt{c-acx}\,(-23-10ax+a^2x^2)}{3a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(-23 - 10*a*x + a^2*x^2))/(3*a^2*Sqrt[1 - 1/(a^2*x^2)]*x
)
```

Maple [A]
time = 0.09, size = 56, normalized size = 0.41

method	result	size

gosper	$\frac{2(ax+1)(a^2x^2-10ax-23)\sqrt{-acx+c}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3a(ax-1)^2}$	55
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\,(a^2x^2-10ax-23)}{3(ax-1)^2a}$	56
risch	$-\frac{2(ax-11)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{3a\sqrt{-c(ax-1)}}+\frac{8c\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{-c(ax-1)}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(a^2*x^2-1
0*a*x-23)/a
```

Maxima [A]
time = 0.29, size = 75, normalized size = 0.55

$$\frac{2\left(a^3\sqrt{-c}\,x^3-9\,a^2\sqrt{-c}\,x^2-33\,a\sqrt{-c}\,x-23\,\sqrt{-c}\right)(ax-1)^2}{3\left(a^3x^2-2\,a^2x+a\right)(ax+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(a^3*sqrt(-c)*x^3 - 9*a^2*sqrt(-c)*x^2 - 33*a*sqrt(-c)*x - 23*sqrt(-c))
*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1)^(3/2))
```

Fricas [A]
time = 0.35, size = 50, normalized size = 0.36

$$\frac{2\left(a^2x^2-10\,ax-23\right)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{3\left(a^2x-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a^2*x^2 - 10*a*x - 23)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1))/(a^2
*x - a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 0.00, size = 71, normalized size = 0.52

$$\frac{2\sqrt{c-a\,c\,x}\,\left(a\,x-9\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a}-\frac{64\sqrt{c-a\,c\,x}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,a\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*(c - a*c*x)^(1/2)*(a*x - 9)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a) - (64*(c
- a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/(3*a*(a*x - 1))
```

3.354 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx$

Optimal. Leaf size=140

$$\frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+10*(-a*c*x+c)^(1/2)/a/x/
(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*
(-a*c*x+c)^(1/2)/a^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6311, 6316, 91, 79, 56, 221}

$$\frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{10\sqrt{c-acx}}{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (10*Sqrt[c - a*
c*x])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (2*Sqrt[x^(-1)]*Sqrt[c -
a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(Sqrt[a]*Sqrt[1 - 1/(a*x)])
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
```

```
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{\sqrt{x}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Subst}\left(\int \frac{-\frac{2}{a}+\frac{x}{2a^2}}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Sub}}{a\sqrt{}}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\left(2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\mathrm{Su}}{a}$$

$$= \frac{2\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{10\sqrt{c-acx}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{2\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}}{\sqrt{a}\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]

time = 0.08, size = 78, normalized size = 0.56

$$\frac{2\sqrt{c-acx}\left(a+\frac{5}{x}-\sqrt{a}\,\sqrt{1+\frac{1}{ax}}\,\sqrt{\frac{1}{x}}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (2*Sqrt[c - a*c*x]*(a + 5/x - Sqrt[a]*Sqrt[1 + 1/(a*x)]*Sqrt[x^(-1)]*ArcSin
h[Sqrt[x^(-1)]/Sqrt[a]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.15, size = 80, normalized size = 0.57

method	result	size
default	$\frac{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(\sqrt{c}\,\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)\sqrt{-c\,(ax+1)}+acx+5c\right)}{(ax-1)^2c}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(c^(1/2)*arc
tan((-c*(a*x+1))^(1/2)/c^(1/2))*(-c*(a*x+1))^(1/2)+a*c*x+5*c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Fricas [A]
time = 0.35, size = 207, normalized size = 1.48

$$\left[\frac{(ax-1)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx+2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2c}{ax^2-x}\right)+2\sqrt{-acx+c}\,(ax+5)\sqrt{\frac{ax-1}{ax+1}}}{ax-1},\ -\frac{2\left((ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}\,(ax+5)\sqrt{\frac{ax-1}{ax+1}}\right)}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c*x + c)*(a*x + 1)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) + 2*sqrt(-a*c*x + c
)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1), -2*((a*x - 1)*sqrt(c)*arc
tan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - sqrt(
-a*c*x + c)*(a*x + 5)*sqrt((a*x - 1)/(a*x + 1)))/(a*x - 1)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

```
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.355 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx$

Optimal. Leaf size=140

$$-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-(1+1/a/x)^(1/2)*(-a*c
*x+c)^(1/2)/x/(1-1/a/x)^(1/2)+7*arcsinh((1/x)^(1/2)/a^(1/2))*a^(1/2)*(1/x)^
(1/2)*(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6311, 6316, 91, 81, 56, 221}

$$-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) - (Sqrt[1 + 1/
(a*x)]*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*x) + (7*Sqrt[a]*Sqrt[x^(-1)]*Sqr
t[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/Sqrt[1 - 1/(a*x)]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^2}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{3/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{\sqrt{x}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\frac{3}{2a^2}-\frac{x}{2a^3}}{\sqrt{x}\sqrt{1+\frac{x}{a}}}\,d\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Sul}}{2\sqrt{}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{\left(7\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Sul}}{\sqrt{}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\,x} + \frac{7\sqrt{a}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\text{sin}}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]

time = 0.06, size = 79, normalized size = 0.56

$$\frac{\sqrt{c-acx}\left(-1-9ax+\frac{7a^{3/2}\sqrt{1+\frac{1}{ax}}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{3/2}}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^2),x]
[Out] (Sqrt[c - a*c*x]*(-1 - 9*a*x + (7*a^(3/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^
(-1)]/Sqrt[a]])/(x^(-1))^(3/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A]
time = 0.17, size = 86, normalized size = 0.61

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(7\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)ax\sqrt{-c(ax+1)}+9ax\sqrt{c}+\sqrt{c}\right)\sqrt{-c(ax-1)}}{(ax-1)^2\sqrt{c}\,x}$	86
risch	$\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\sqrt{-c(ax-1)}}-\frac{\left(-\frac{8a}{\sqrt{-acx-c}}-\frac{7a\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c(ax+1)}}{\sqrt{-c(ax-1)}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(7*arctan((-c*(a*x+1))^(1/2)/c^(1/2))*a*x*
(-c*(a*x+1))^(1/2)+9*a*x*c^(1/2)+c^(1/2))*(-c*(a*x-1))^(1/2)/(a*x-1)^2/c^(1
/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A]
time = 0.36, size = 233, normalized size = 1.66

$$\left[\frac{7\left(a^{2}x^{2}-ax\right)\sqrt{-c}\,\log\left(-\frac{a^{2}cx^{2}+acx-2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^{2}-x}\right)-2\sqrt{-acx+c}\,(9\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{2\left(ax^{2}-x\right)},\ \frac{7\left(a^{2}x^{2}-ax\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-\sqrt{-acx+c}\,(9\,ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ax^{2}-x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(7*(a^2*x^2 - a*x)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a*c*x +
c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) - 2*sqr
t(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^2 - x), (7*(a^2*x
^2 - a*x)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))
/(a*c*x - c)) - sqrt(-a*c*x + c)*(9*a*x + 1)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^2 - x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.356 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx$

Optimal. Leaf size=190

$$-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2}+\frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x}-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}(}{4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/2*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^2/(1-1/a/x)^(1/2)+47/4*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/
2)/x/(1-1/a/x)^(1/2)-47/4*a^(3/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*
(-a*c*x+c)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6311, 6316, 91, 81, 52, 56, 221}

$$-\frac{47a^{3/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{2x^2\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{47a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^2) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(2*Sqrt[1 - 1/(a*x)]*x^2) + (47*a*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x) - (47*a^(3/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(4*Sqrt[1 - 1/(a*x)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^3}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{5/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(\frac{11}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{\left(47\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{S}}{4\sqrt{}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^2} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{2\sqrt{1-\frac{1}{ax}}\,x^2} + \frac{47a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x}$$

Mathematica [A]
time = 0.06, size = 90, normalized size = 0.47

$$-\frac{\sqrt{c-acx}\left(2-13ax-47a^2x^2+\frac{47a^{5/2}\sqrt{1+\frac{1}{ax}}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{5/2}}\right)}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -1/4*(Sqrt[c - a*c*x]*(2 - 13*a*x - 47*a^2*x^2 + (47*a^(5/2)*Sqrt[1 + 1/(a*
x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(5/2)))/(a*Sqrt[1 - 1/(a^2*x^2)
]*x^3)
```

Maple [A]
time = 0.18, size = 103, normalized size = 0.54

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(47\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)a^2x^2\sqrt{-c\,(ax+1)}+47a^2x^2\sqrt{c}+13ax\sqrt{c}-2\right.}{4(ax-1)^2\sqrt{c}\,x^2}$
risch	$-\frac{(15a^2x^2+13ax-2)c\sqrt{\frac{ax-1}{ax+1}}}{4x^2\sqrt{-c\,(ax-1)}}-\frac{\left(\frac{8a^2}{\sqrt{-acx-c}}+\frac{47a^2\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{4\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax+1)}}{\sqrt{-c\,(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(47*arctan
((-c*(a*x+1))^(1/2)/c^(1/2))*a^2*x^2*(-c*(a*x+1))^(1/2)+47*a^2*x^2*c^(1/2)+
13*a*x*c^(1/2)-2*c^(1/2))/c^(1/2)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A]
time = 0.38, size = 262, normalized size = 1.38

$$\left[\frac{47\,(a^3x^3 - a^2x^2)\sqrt{-c}\,\log\left(-\frac{a^2cx^2 + acx + 2\,\sqrt{-acx + c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}} - 2\,c}{ax^2 - x}\right) + 2\,(47\,a^2x^2 + 13\,ax - 2)\sqrt{-acx + c}\,\sqrt{\frac{ax-1}{ax+1}}}{8\,(ax^3 - x^2)}, -\frac{47\,(a^3x^3 - a^2x^2)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx + c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx - c}\right) - (47\,a^2x^2 + 13\,ax - 2)\sqrt{-acx + c}\,\sqrt{\frac{ax-1}{ax+1}}}{4\,(ax^3 - x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas
")
```

```
[Out] [1/8*(47*(a^3*x^3 - a^2*x^2)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(-a*c
*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)) +
2*(47*a^2*x^2 + 13*a*x - 2)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*
x^3 - x^2), -1/4*(47*(a^3*x^3 - a^2*x^2)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (47*a^2*x^2 + 13*a*x - 2)*sq
rt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^3 - x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.357 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx$

Optimal. Leaf size=238

$$-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3}+\frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{12\sqrt{1-\frac{1}{ax}}\,x^2}-\frac{119a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{8\sqrt{1-\frac{1}{ax}}\,x}+$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/3*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^3/(1-1/a/x)^(1/2)+119/12*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x^2/(1-1/a/x)^(1/2)-119/8*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/
a/x)^(1/2)+119/8*a^(5/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c*x+c
)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6311, 6316, 91, 81, 52, 56, 221}

$$\frac{119a^{5/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{8\sqrt{1-\frac{1}{ax}}}-\frac{119a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{8x\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{3x^3\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{119a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{12x^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^3) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(3*Sqrt[1 - 1/(a*x)]*x^3) + (119*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(12*Sqrt[1 - 1/(a*x)]*x^2) - (119*a^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(8*Sqrt[1 - 1/(a*x)]*x) + (119*a^(5/2)*Sqrt[x^(-1)]*Sqrt
[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(8*Sqrt[1 - 1/(a*x)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^4}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{7/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{3/2}\left(\frac{19}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,d\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{\left(119\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{6\sqrt{}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{12\sqrt{1-\frac{1}{ax}}\,x^2}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{12\sqrt{1-\frac{1}{ax}}\,x^2}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{12\sqrt{1-\frac{1}{ax}}\,x^2}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^3} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{3\sqrt{1-\frac{1}{ax}}\,x^3} + \frac{119a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{12\sqrt{1-\frac{1}{ax}}\,x^2}$$

Mathematica [**A**]
time = 0.06, size = 98, normalized size = 0.41

$$\frac{\sqrt{c-acx}\left(-8+38ax-119a^2x^2-357a^3x^3+\frac{357a^{7/2}\sqrt{1+\frac{1}{ax}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{7/2}}\right)}{24a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a*c*x]*(-8 + 38*a*x - 119*a^2*x^2 - 357*a^3*x^3 + (357*a^(7/2)*Sq
rt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^(-1))^(7/2)))/(24*a*Sqrt[
1 - 1/(a^2*x^2)]*x^4)
```

Maple [**A**]
time = 0.17, size = 114, normalized size = 0.48

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c\,(ax-1)}\,\left(357\arctan\left(\frac{\sqrt{-c\,(ax+1)}}{\sqrt{c}}\right)a^3x^3\sqrt{-c\,(ax+1)}+357a^3x^3\sqrt{c}+119a^2x^2\right.}{24(ax-1)^2\sqrt{c}\,x^3}$
risch	$\frac{(165a^3x^3+119a^2x^2-38ax+8)c\sqrt{\frac{ax-1}{ax+1}}}{24x^3\sqrt{-c\,(ax-1)}}-\frac{\left(-\frac{8a^3}{\sqrt{-acx-c}}-\frac{119a^3\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{8\sqrt{c}}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax}}{\sqrt{-c\,(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(357*arc
tan((-c*(a*x+1))^(1/2)/c^(1/2))*a^3*x^3*(-c*(a*x+1))^(1/2)+357*a^3*x^3*c^(1
/2)+119*a^2*x^2*c^(1/2)-38*a*x*c^(1/2)+8*c^(1/2))/c^(1/2)/x^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A]
time = 0.34, size = 278, normalized size = 1.17

$$\left[\frac{357\,(a^4x^4-a^3x^3)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx-2\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)-2\,(357\,a^3x^3+119\,a^2x^2-38\,ax+8)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{48\,(ax^4-x^3)}, \frac{357\,(a^4x^4-a^3x^3)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(357\,a^3x^3+119\,a^2x^2-38\,ax+8)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{24\,(ax^4-x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas
")
```

```
[Out] [1/48*(357*(a^4*x^4 - a^3*x^3)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x - 2*sqrt(-a
*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x))
- 2*(357*a^3*x^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1
)/(a*x + 1)))/(a*x^4 - x^3), 1/24*(357*(a^4*x^4 - a^3*x^3)*sqrt(c)*arctan(s
qrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a*c*x - c)) - (357*a^3*x
^3 + 119*a^2*x^2 - 38*a*x + 8)*sqrt(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/
(a*x^4 - x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c - a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.358 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx$

Optimal. Leaf size=286

$$-\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4}-\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4}+\frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{24\sqrt{1-\frac{1}{ax}}\,x^3}-\frac{1115a^2\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{96\sqrt{1-\frac{1}{ax}}\,x^2}+$$

```
[Out] -8*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1/4*(1+1/a/x)^(1/2)
*(-a*c*x+c)^(1/2)/x^4/(1-1/a/x)^(1/2)+223/24*a*(1+1/a/x)^(1/2)*(-a*c*x+c)^(
1/2)/x^3/(1-1/a/x)^(1/2)-1115/96*a^2*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x^2/(
1-1/a/x)^(1/2)+1115/64*a^3*(1+1/a/x)^(1/2)*(-a*c*x+c)^(1/2)/x/(1-1/a/x)^(1/
2)-1115/64*a^(7/2)*arcsinh((1/x)^(1/2)/a^(1/2))*(1/x)^(1/2)*(-a*c*x+c)^(1/2
)/(1-1/a/x)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6311, 6316, 91, 81, 52, 56, 221}

$$-\frac{1115a^{7/2}\sqrt{\frac{1}{x}}\sqrt{c-acx}\,\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{64\sqrt{1-\frac{1}{ax}}}+\frac{1115a^3\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{64x\sqrt{1-\frac{1}{ax}}}-\frac{1115a^2\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{96x^2\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{4x^4\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-acx}}{x^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{223a\sqrt{\frac{1}{ax}+1}\sqrt{c-acx}}{24x^3\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-8*Sqrt[c - a*c*x])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x^4) - (Sqrt[1 +
1/(a*x)]*Sqrt[c - a*c*x])/(4*Sqrt[1 - 1/(a*x)]*x^4) + (223*a*Sqrt[1 + 1/(a*
x)]*Sqrt[c - a*c*x])/(24*Sqrt[1 - 1/(a*x)]*x^3) - (1115*a^2*Sqrt[1 + 1/(a*x
)]*Sqrt[c - a*c*x])/(96*Sqrt[1 - 1/(a*x)]*x^2) + (1115*a^3*Sqrt[1 + 1/(a*x)
]*Sqrt[c - a*c*x])/(64*Sqrt[1 - 1/(a*x)]*x) - (1115*a^(7/2)*Sqrt[x^(-1)]*Sq
rt[c - a*c*x]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(64*Sqrt[1 - 1/(a*x)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-acx}}{x^5}\,dx = \frac{\sqrt{c-acx}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^{9/2}}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$= -\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} + \frac{\left(2a^2\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int \frac{x^{5/2}\left(\frac{27}{2a^2}-\frac{x}{2a^3}\right)}{\sqrt{1+\frac{x}{a}}}\,d\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{\left(223\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)}{8\sqrt{}}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{24\sqrt{1-\frac{1}{ax}}\,x^3}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{24\sqrt{1-\frac{1}{ax}}\,x^3}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{24\sqrt{1-\frac{1}{ax}}\,x^3}$$

$$= -\frac{8\sqrt{c-acx}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x^4} - \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-acx}}{4\sqrt{1-\frac{1}{ax}}\,x^4} + \frac{223a\sqrt{1+\frac{1}{ax}}\sqrt{c-ac}}{24\sqrt{1-\frac{1}{ax}}\,x^3}$$

Mathematica [A]
time = 0.06, size = 106, normalized size = 0.37

$$-\frac{\sqrt{c-acx}\left(48-200ax+446a^2x^2-1115a^3x^3-3345a^4x^4+\frac{3345a^{9/2}\sqrt{1+\frac{1}{ax}}\sinh^{-1}\left(\frac{\sqrt{\frac{1}{x}}}{\sqrt{a}}\right)}{\left(\frac{1}{x}\right)^{9/2}}\right)}{192a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a*c*x]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/192*(Sqrt[c - a*c*x]*(48 - 200*a*x + 446*a^2*x^2 - 1115*a^3*x^3 - 3345*a
^4*x^4 + (3345*a^(9/2)*Sqrt[1 + 1/(a*x)]*ArcSinh[Sqrt[x^(-1)]/Sqrt[a]])/(x^
(-1))^(9/2)))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5)
```

Maple [A]
time = 0.18, size = 125, normalized size = 0.44

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{-c(ax-1)}\left(3345\arctan\left(\frac{\sqrt{-c(ax+1)}}{\sqrt{c}}\right)a^4x^4\sqrt{-c(ax+1)}+3345a^4x^4\sqrt{c}+1115a^3x^3\right.}{192(ax-1)^2\sqrt{c}\,x^4}$
risch	$-\frac{(1809a^4x^4+1115a^3x^3-446a^2x^2+200ax-48)c\sqrt{\frac{ax-1}{ax+1}}}{192x^4\sqrt{-c(ax-1)}}-\frac{\left(\frac{8a^4}{\sqrt{-acx-c}}+\frac{1115a^4\arctan\left(\frac{\sqrt{-acx-c}}{\sqrt{c}}\right)}{64\sqrt{c}}\right)c\sqrt{\frac{ax-}{ax+}}}{\sqrt{-c(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(-c*(a*x-1))^(1/2)*(3345*ar
ctan((-c*(a*x+1))^(1/2)/c^(1/2))*a^4*x^4*(-c*(a*x+1))^(1/2)+3345*a^4*x^4*c^
(1/2)+1115*a^3*x^3*c^(1/2)-446*a^2*x^2*c^(1/2)+200*a*x*c^(1/2)-48*c^(1/2))/
c^(1/2)/x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a*c*x + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Fricas [A]
time = 0.37, size = 294, normalized size = 1.03

$$\left[\frac{3345\,(a^5x^5-a^4x^4)\sqrt{-c}\,\log\left(-\frac{a^2cx^2+acx+2\,\sqrt{-acx+c}\,(ax+1)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}-2\,c}{ax^2-x}\right)+2\,(3345\,a^4x^4+1115\,a^3x^3-446\,a^2x^2+200\,ax-48)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{384\,(ax^5-x^4)}, -\frac{3345\,(a^5x^5-a^4x^4)\sqrt{c}\,\arctan\left(\frac{\sqrt{-acx+c}\,\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}}{acx-c}\right)-(3345\,a^4x^4+1115\,a^3x^3-446\,a^2x^2+200\,ax-48)\sqrt{-acx+c}\,\sqrt{\frac{ax-1}{ax+1}}}{192\,(ax^5-x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas
")
```

```
[Out] [1/384*(3345*(a^5*x^5 - a^4*x^4)*sqrt(-c)*log(-(a^2*c*x^2 + a*c*x + 2*sqrt(
-a*c*x + c)*(a*x + 1)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1)) - 2*c)/(a*x^2 - x)
) + 2*(3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqrt(-a*c*
x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4), -1/192*(3345*(a^5*x^5 - a^
4*x^4)*sqrt(c)*arctan(sqrt(-a*c*x + c)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))/(a
*c*x - c)) - (3345*a^4*x^4 + 1115*a^3*x^3 - 446*a^2*x^2 + 200*a*x - 48)*sqr
t(-a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a*x^5 - x^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*c*x+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a\,c\,x}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
[Out] int(((c - a*c*x)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.359 $\int e^{n \coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx$

Optimal. Leaf size=278

$$-\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)}+\frac{2(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-}{a^2(6+n)\left(8+6n+n^2\right)x}$$

```
[Out] -(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(2+1/2*n
)/a/(4+n)/(6+n)+2*(n^2+14*n+56)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-
a*c*x+c)^(2+1/2*n)/a^2/(6+n)/(n^2+6*n+8)/x+(8+n)*(1-1/a/x)^(-2-1/2*n)*(1+1/
a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/(6+n)-(a-1/x)*(1-1/a/x)^(-2-1/2*n)*(1
+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(2+1/2*n)/a
```

Rubi [A]

time = 0.18, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6311, 6316, 92, 80, 47, 37}

$$\frac{2(n^2+14n+56)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a^2(n+6)\left(n^2+6n+8\right)x}-\frac{(n^2+14n+56)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a(n+4)(n+6)}+\frac{(n+8)x\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{n+6}-\frac{x\left(a-\frac{1}{x}\right)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}(c-acx)^{\frac{n+4}{2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

```
[Out] -(((56 + 14*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c
- a*c*x)^((4 + n)/2))/(a*(4 + n)*(6 + n))) + (2*(56 + 14*n + n^2)*(1 - 1/(a
*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^((4 + n)/2))/(a^2*(6
+ n)*(8 + 6*n + n^2)*x) + ((8 + n)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^(
(2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/(6 + n) - ((a - x^(-1))*(1 - 1/(a*x))
^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^((4 + n)/2))/a
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{2+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}x^{-2-\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{2+\frac{n}{2}}x^{2+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\frac{1}{x}\right)^{2+\frac{n}{2}}(c-acx)^{2+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-4-\frac{n}{2}}\left(1-\frac{x}{a}\right)^{2}\left(1\right.\right.\right.$$

$$= -\frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{a} + \left(a\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\right.\right.$$

$$= \frac{(8+n)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{4+n}{2}}}{6+n} - \frac{\left(a-\frac{1}{x}\right)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(\right.}{}$$

$$= -\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{(8+n)\left(1-\frac{1}{ax}\right.}{}$$

$$= -\frac{(56+14n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{4+n}{2}}}{a(4+n)(6+n)} + \frac{2(56+14n+n}{}$$

Mathematica [A]
time = 0.05, size = 116, normalized size = 0.42

$$\frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}\left(n^2(-1+ax)^2+8(7-4ax+a^2x^2)+2n(7-10ax+3a^2x^2)\right)}{a(2+n)(4+n)(6+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(2 + n/2),x]
```

```
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(n^2*(-1 + a*x)^2 +
8*(7 - 4*a*x + a^2*x^2) + 2*n*(7 - 10*a*x + 3*a^2*x^2)))/(a*(2 + n)*(4 + n)
*(6 + n)*(1 - 1/(a*x))^(n/2))
```

Maple [A]
time = 0.12, size = 104, normalized size = 0.37

method	result	size
gosper	$\frac{2(ax+1)(a^2n^2x^2+6a^2n\,x^2+8a^2x^2-2a\,n^2x-20anx-32ax+n^2+14n+56)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{2+\frac{n}{2}}}{(ax-1)^2a(n^3+12n^2+44n+48)}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*x+1)*(a^2*n^2*x^2+6*a^2*n*x^2+8*a^2*x^2-2*a*n^2*x-20*a*n*x-32*a*x+n^2+
14*n+56)*exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n)/(a*x-1)^2/a/(n^3+12*n^2+4
4*n+48)
```

Maxima [A]
time = 0.27, size = 122, normalized size = 0.44

$$\frac{2\left((n^2+6\,n+8)a^3(-c)^{\frac{1}{2}\,n}\,c^2x^3-(n^2+14\,n+24)a^2(-c)^{\frac{1}{2}\,n}\,c^2x^2-(n^2+6\,n-24)a(-c)^{\frac{1}{2}\,n}\,c^2x+(n^2+14\,n+56)(-c)^{\frac{1}{2}\,n}\,c^2\right)(ax+1)^{\frac{1}{2}\,n}}{(n^3+12\,n^2+44\,n+48)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="maxima")
```

```
[Out] 2*((n^2 + 6*n + 8)*a^3*(-c)^(1/2*n)*c^2*x^3 - (n^2 + 14*n + 24)*a^2*(-c)^(1
/2*n)*c^2*x^2 - (n^2 + 6*n - 24)*a*(-c)^(1/2*n)*c^2*x + (n^2 + 14*n + 56)*(
-c)^(1/2*n)*c^2)*(a*x + 1)^(1/2*n)/((n^3 + 12*n^2 + 44*n + 48)*a)
```

Fricas [A]
time = 0.34, size = 185, normalized size = 0.67

$$\frac{2\,((a^3n^2+6\,a^3n+8\,a^3)x^3-(a^2n^2+14\,a^2n+24\,a^2)x^2+n^2-(an^2+6\,an-24\,a)x+14\,n+56)(-acx+c)^{\frac{1}{2}\,n+2}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an^3+12\,an^2+(a^3n^3+12\,a^3n^2+44\,a^3n+48\,a^3)x^2+44\,an-2\,(a^2n^3+12\,a^2n^2+44\,a^2n+48\,a^2)x+48\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="fricas")
```

```
[Out] 2*((a^3*n^2 + 6*a^3*n + 8*a^3)*x^3 - (a^2*n^2 + 14*a^2*n + 24*a^2)*x^2 + n^
2 - (a*n^2 + 6*a*n - 24*a)*x + 14*n + 56)*(-a*c*x + c)^(1/2*n + 2)*((a*x +
1)/(a*x - 1))^(1/2*n)/(a*n^3 + 12*a*n^2 + (a^3*n^3 + 12*a^3*n^2 + 44*a^3*n
+ 48*a^3)*x^2 + 44*a*n - 2*(a^2*n^3 + 12*a^2*n^2 + 44*a^2*n + 48*a^2)*x + 4
8*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} c^{\frac{n}{2}+2}xe^{\frac{i\pi n}{2}} & \text{for}\: a=0 \\ -\frac{\int \frac{1}{axe^{6\operatorname{acoth}(ax)}-e^{6\operatorname{acoth}(ax)}}\,dx}{c} & \text{for}\: n=-6 \\ \int e^{-4\operatorname{acoth}(ax)}\,dx & \text{for}\: n=-4 \\ -c\left(\int axe^{-2\operatorname{acoth}(ax)}\,dx+\int\left(-e^{-2\operatorname{acoth}(ax)}\right)dx\right) & \text{for}\: n=-2 \\ \frac{2a^3c^2n^2x^3(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}+\frac{12a^3c^2nx^3(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}+\frac{16a^3c^2x^3(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}-\frac{2a^2c^2n^2x^2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}-\frac{28a^2c^2nx^2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}-\frac{48a^2c^2x^2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}-\frac{2ac^2n^2x(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}-\frac{12ac^2nx(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}+\frac{48ac^2x(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}+\frac{2c^2n^2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}+\frac{28c^2n(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a}+\frac{112c^2(-acx+c)^{\frac{n}{2}}e^{n\operatorname{acoth}(ax)}}{an^3+12an^2+44an+48a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(2+1/2*n),x)
```

```
[Out] Piecewise((c**(n/2 + 2)*x*exp(I*pi*n/2), Eq(a, 0)), (-Integral(1/(a*x*exp(6
*acoth(a*x)) - exp(6*acoth(a*x))), x)/c, Eq(n, -6)), (Integral(exp(-4*acoth
(a*x)), x), Eq(n, -4)), (-c*(Integral(a*x*exp(-2*acoth(a*x)), x) + Integral
(-exp(-2*acoth(a*x)), x)), Eq(n, -2)), (2*a**3*c**2*n**2*x**3*(-a*c*x + c)*
*(n/2)*exp(n*acoth(a*x))/(a*n**3 + 12*a*n**2 + 44*a*n + 48*a) + 12*a**3*c**
2*n*x**3*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3 + 12*a*n**2 + 44*a*n
 + 48*a) + 16*a**3*c**2*x**3*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3
+ 12*a*n**2 + 44*a*n + 48*a) - 2*a**2*c**2*n**2*x**2*(-a*c*x + c)**(n/2)*ex
p(n*acoth(a*x))/(a*n**3 + 12*a*n**2 + 44*a*n + 48*a) - 28*a**2*c**2*n*x**2*
```

```
(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3 + 12*a*n**2 + 44*a*n + 48*a)
- 48*a**2*c**2*x**2*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3 + 12*a*n*
*2 + 44*a*n + 48*a) - 2*a*c**2*n**2*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))
/(a*n**3 + 12*a*n**2 + 44*a*n + 48*a) - 12*a*c**2*n*x*(-a*c*x + c)**(n/2)*e
xp(n*acoth(a*x))/(a*n**3 + 12*a*n**2 + 44*a*n + 48*a) + 48*a*c**2*x*(-a*c*x
 + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3 + 12*a*n**2 + 44*a*n + 48*a) + 2*c**
2*n**2*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3 + 12*a*n**2 + 44*a*n +
 48*a) + 28*c**2*n*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3 + 12*a*n**
2 + 44*a*n + 48*a) + 112*c**2*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**3
 + 12*a*n**2 + 44*a*n + 48*a), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(2+1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n + 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B]
time = 1.79, size = 223, normalized size = 0.80

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{x^3\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}\,\left(2\,n^2+12\,n+16\right)}{n^3+12\,n^2+44\,n+48}+\frac{{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}\,\left(2\,n^2+28\,n+112\right)}{a^3\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{2\,x\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}\,\left(n^2+6\,n-24\right)}{a^2\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{x^2\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}+2}\,\left(2\,n^2+28\,n+48\right)}{a\,\left(n^3+12\,n^2+44\,n+48\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 2),x)
[Out] (((a*x + 1)/(a*x))^(n/2)*((x^3*(c - a*c*x)^(n/2 + 2)*(12*n + 2*n^2 + 16))/(
44*n + 12*n^2 + n^3 + 48) + ((c - a*c*x)^(n/2 + 2)*(28*n + 2*n^2 + 112))/(a
^3*(44*n + 12*n^2 + n^3 + 48)) - (2*x*(c - a*c*x)^(n/2 + 2)*(6*n + n^2 - 24
))/(a^2*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(c - a*c*x)^(n/2 + 2)*(28*n + 2*
n^2 + 48))/(a*(44*n + 12*n^2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*(1/a^2
 - (2*x)/a + x^2))
```

3.360 $\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)}+\frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n}$$

```
[Out] -2*(6+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*(-a*c*x+c)^(1+1/2*n)/a/(n
^2+6*n+8)+2*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(1+1/2*n)
/(4+n)
```

Rubi [A]
time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6311, 6316, 80, 37}

$$\frac{2x\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{n+4}-\frac{2(n+6)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^{\frac{n+2}{2}}}{a(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*(6 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - a*c*x)^
((2 + n)/2))/(a*(2 + n)*(4 + n)) + (2*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x)
)^((2 + n)/2)*x*(c - a*c*x)^((2 + n)/2))/(4 + n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
```

```
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{1+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}x^{-1-\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{1+\frac{n}{2}}x^{1+\frac{n}{2}}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}(c-acx)^{1+\frac{n}{2}}\right)\text{Subst}\left(\int x^{-3-\frac{n}{2}}\left(1-\frac{x}{a}\right)\left(1\right.\right.\right.$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{\frac{2+n}{2}}}{4+n} + \frac{\left((6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(\frac{1}{x}\right)^{1+\frac{n}{2}}\right.}{}$$

$$= -\frac{2(6+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}(c-acx)^{\frac{2+n}{2}}}{a(2+n)(4+n)} + \frac{2\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right.}{4+}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.61

$$-\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(1+ax)(c-acx)^{n/2}(-6+2ax+n(-1+ax))}{a(2+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(1 + n/2),x]
```

```
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*(1 + a*x)*(c - a*c*x)^(n/2)*(-6 + 2*a*x + n*(-1 +
 a*x)))/(a*(2 + n)*(4 + n)*(1 - 1/(a*x))^(n/2))
```

Maple [A]
time = 0.10, size = 61, normalized size = 0.48

method	result	size

gosper	$\frac{2(-acx+c)^{1+\frac{n}{2}} e^{n \operatorname{arccoth}(ax)}(anx+2ax-n-6)(ax+1)}{(ax-1)a(n^2+6n+8)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-a*c*x+c)^(1+1/2*n)*exp(n*arccoth(a*x))*(a*n*x+2*a*x-n-6)*(a*x+1)/(a*x-1
)/a/(n^2+6*n+8)
```

Maxima [A]
time = 0.28, size = 68, normalized size = 0.54

$$-\frac{2\left(a^2(-c)^{\frac{1}{2}n}c(n+2)x^2-4a(-c)^{\frac{1}{2}n}cx-(-c)^{\frac{1}{2}n}c(n+6)\right)(ax+1)^{\frac{1}{2}n}}{(n^2+6n+8)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="maxima")
```

```
[Out] -2*(a^2*(-c)^(1/2*n)*c*(n + 2)*x^2 - 4*a*(-c)^(1/2*n)*c*x - (-c)^(1/2*n)*c*
(n + 6))*(a*x + 1)^(1/2*n)/((n^2 + 6*n + 8)*a)
```

Fricas [A]
time = 0.35, size = 93, normalized size = 0.73

$$-\frac{2\left((a^2n+2a^2)x^2-4ax-n-6\right)(-acx+c)^{\frac{1}{2}n+1}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{an^2+6an-(a^2n^2+6a^2n+8a^2)x+8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="fricas")
```

```
[Out] -2*((a^2*n + 2*a^2)*x^2 - 4*a*x - n - 6)*(-a*c*x + c)^(1/2*n + 1)*((a*x + 1
)/(a*x - 1))^(1/2*n)/(a*n^2 + 6*a*n - (a^2*n^2 + 6*a^2*n + 8*a^2)*x + 8*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} c^{\frac{n}{2}+1} x e^{\frac{i\pi n}{2}} & \text{for}\ a = 0 \\ -\frac{\int \frac{1}{axe^{4\operatorname{acoth}(ax)} - e^{4\operatorname{acoth}(ax)}}\, dx}{c} & \text{for}\ n = -4 \\ \int e^{-2\operatorname{acoth}(ax)}\, dx & \text{for}\ n = -2 \\ -\frac{2a^2cnx^2(-acx+c)^{\frac{n}{2}} e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a} - \frac{4a^2cx^2(-acx+c)^{\frac{n}{2}} e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a} + \frac{8acx(-acx+c)^{\frac{n}{2}} e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a} + \frac{2cn(-acx+c)^{\frac{n}{2}} e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a} + \frac{12c(-acx+c)^{\frac{n}{2}} e^{n\operatorname{acoth}(ax)}}{an^2+6an+8a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1+1/2*n),x)
```

```
[Out] Piecewise((c**(n/2 + 1)*x*exp(I*pi*n/2), Eq(a, 0)), (-Integral(1/(a*x*exp(4
*acoth(a*x)) - exp(4*acoth(a*x))), x)/c, Eq(n, -4)), (Integral(exp(-2*acoth
(a*x)), x), Eq(n, -2)), (-2*a**2*c*n*x**2*(-a*c*x + c)**(n/2)*exp(n*acoth(a
*x))/(a*n**2 + 6*a*n + 8*a) - 4*a**2*c*x**2*(-a*c*x + c)**(n/2)*exp(n*acoth
(a*x))/(a*n**2 + 6*a*n + 8*a) + 8*a*c*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x
))/(a*n**2 + 6*a*n + 8*a) + 2*c*n*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*
n**2 + 6*a*n + 8*a) + 12*c*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n**2 +
6*a*n + 8*a), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1+1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n + 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B]
time = 1.34, size = 140, normalized size = 1.10

$$-\frac{\left(\frac{(2\,n+12)\,{(c-a\,c\,x)}^{\frac{n}{2}+1}}{a^2\,(n^2+6\,n+8)}-\frac{x^2\,(2\,n+4)\,{(c-a\,c\,x)}^{\frac{n}{2}+1}}{n^2+6\,n+8}+\frac{8\,x\,{(c-a\,c\,x)}^{\frac{n}{2}+1}}{a\,(n^2+6\,n+8)}\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^{n/2}}{\left(x-\frac{1}{a}\right)\,{\left(\frac{a\,x-1}{a\,x}\right)}^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 + 1),x)
```

```
[Out] -((((2*n + 12)*(c - a*c*x)^(n/2 + 1))/(a^2*(6*n + n^2 + 8)) - (x^2*(2*n + 4
)*(c - a*c*x)^(n/2 + 1))/(6*n + n^2 + 8) + (8*x*(c - a*c*x)^(n/2 + 1))/(a*(
6*n + n^2 + 8)))*((a*x + 1)/(a*x))^(n/2))/((x - 1/a)*((a*x - 1)/(a*x))^(n/2
))
```

3.361 $\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx$

Optimal. Leaf size=36

$$\frac{2e^{n\coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

```
[Out] 2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{6309\}$

$$\frac{2(ax+1)(c-acx)^{n/2}e^{n\coth^{-1}(ax)}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

```
[Out] (2*E^(n*ArcCoth[a*x])*(1 + a*x)*(c - a*c*x)^(n/2))/(a*(2 + n))
```

Rule 6309

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_))^(p_.), x_Symbol] :> S
imp[(1 + a*x)*(c + d*x)^p*(E^(n*ArcCoth[a*x])/(a*(p + 1))), x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a*c + d, 0] && EqQ[p, n/2] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{n/2}\,dx=\frac{2e^{n\coth^{-1}(ax)}(1+ax)(c-acx)^{n/2}}{a(2+n)}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.61

$$-\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}}x(c-acx)^{n/2}}{-1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(n/2),x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + n/2)*x*(c - a*c*x)^(n/2))/((-1 - n/2)*(1 - 1/(a*x))^(
n/2)))
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.94

method	result
gosper	$\frac{2\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}(ax+1)(-acx+c)^{\frac{n}{2}}}{a(2+n)}$
risch	$\frac{2(ax+1)(ax-1)^{-\frac{n}{2}}(ax+1)^{\frac{n}{2}}\,\mathrm{e}^{\frac{n\left(i\pi\mathrm{csgn}(ic(ax-1))^{3}+i\pi\mathrm{csgn}(ic(ax-1))^{2}\mathrm{csgn}(ic)+i\pi\mathrm{csgn}(ic(ax-1))^{2}\mathrm{csgn}(i(ax-1))-i\pi\,\mathrm{csgn}(ic(ax-1))\mathrm{csgn}(ic)\mathrm{csg}}{4}}}{a(2+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x,method=_RETURNVERBOSE)
[Out] 2*exp(n*arccoth(a*x))*(a*x+1)*(-a*c*x+c)^(1/2*n)/a/(2+n)
```

Maxima [A]
time = 0.28, size = 37, normalized size = 1.03

$$\frac{2\left(a(-c)^{\frac{1}{2}\,n}\,x+(-c)^{\frac{1}{2}\,n}\right)(ax+1)^{\frac{1}{2}\,n}}{a(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="maxima")
[Out] 2*(a*(-c)^(1/2*n)*x + (-c)^(1/2*n))*(a*x + 1)^(1/2*n)/(a*(n + 2))
```

Fricas [A]
time = 0.35, size = 44, normalized size = 1.22

$$\frac{2\,(ax+1)(-acx+c)^{\frac{1}{2}\,n}\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{an+2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="fricas")
[Out] 2*(a*x + 1)*(-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*n + 2*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{x}{c} & \text{for}\: a = 0 \wedge n = -2 \\ c^{\frac{n}{2}} x e^{\frac{i\pi n}{2}} & \text{for}\: a = 0 \\ -\frac{\int \frac{1}{axe^{2\,\mathrm{acoth}\,(ax)} - e^{2\,\mathrm{acoth}\,(ax)}}\, dx}{c} & \text{for}\: n = -2 \\ \frac{2ax(-acx+c)^{\frac{n}{2}} e^{n\,\mathrm{acoth}\,(ax)}}{an+2a} + \frac{2(-acx+c)^{\frac{n}{2}} e^{n\,\mathrm{acoth}\,(ax)}}{an+2a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2*n),x)
```

```
[Out] Piecewise((-x/c, Eq(a, 0) & Eq(n, -2)), (c**(n/2)*x*exp(I*pi*n/2), Eq(a, 0)
), (-Integral(1/(a*x*exp(2*acoth(a*x)) - exp(2*acoth(a*x))), x)/c, Eq(n, -2
)), (2*a*x*(-a*c*x + c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a) + 2*(-a*c*x +
c)**(n/2)*exp(n*acoth(a*x))/(a*n + 2*a), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2*n),x, algorithm="giac")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [B]
time = 1.26, size = 55, normalized size = 1.53

$$\frac{2\left(\frac{1}{a\,x}+1\right)^{n/2}\,(c-a\,c\,x)^{n/2}\,(a\,x+1)}{a\left(1-\frac{1}{a\,x}\right)^{n/2}\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2),x)
```

```
[Out] (2*(1/(a*x) + 1)^(n/2)*(c - a*c*x)^(n/2)*(a*x + 1))/(a*(1 - 1/(a*x))^(n/2)*
(n + 2))
```

3.362 $\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx$

Optimal. Leaf size=80

$$\frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

```
[Out] 2*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x*(-a*c*x+c)^(-1+1/2*n)*hypergeom([
1, -1/2*n],[1-1/2*n],2/(a+1/x)/x)/n
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6311, 6316, 133}

$$\frac{2x\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{n/2}(c-acx)^{\frac{n-2}{2}}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
```

```
[Out] (2*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x*(c - a*c*x)^((-2 + n)/2)*H
ypergeometric2F1[1, -1/2*n, 1 - n/2, 2/((a + x^(-1))*x)])/n
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
```

```
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-1+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}} x^{1-\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-1+\frac{n}{2}} x^{-1+}$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{x}\right)^{-1+\frac{n}{2}}(c-acx)^{-1+\frac{n}{2}}\right)\text{Subst}\left(\int\frac{x^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)}{1-\frac{x}{a}}\right.\right.$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x(c-acx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{n}$$

Mathematica [A]
time = 0.02, size = 78, normalized size = 0.98

$$-\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{2}{1+ax}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-1 + n/2),x]
[Out] (-2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 -
n/2, 2/(1 + a*x)])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{-1+\frac{n}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="fricas")
[Out] integral((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-c(ax-1))^{\frac{n}{2}-1} e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-1+1/2*n),x)
[Out] Integral((-c*(a*x - 1))**(n/2 - 1)*exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-1+1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n - 1)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 1), x)
```

3.363 $\int e^{n \coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x(c-acx)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

```
[Out] -2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-1+1/2*n)*x*(-a*c*x+c)^(-2+1/2*n)*hyperge
om([2, 1-1/2*n],[2-1/2*n],2/(a+1/x)/x)/(2-n)
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6311, 6316, 133}

$$-\frac{2x\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}(c-acx)^{\frac{n-4}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*x*(c - a*c*x)^((-4 +
 n)/2)*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, 2/((a + x^(-1))*x)])/(2 - n)
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
```

```
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{-2+\frac{n}{2}}\,dx = \left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}} x^{2-\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{-2+\frac{n}{2}} x^{-2+\frac{n}{2}}$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{x}\right)^{-2+\frac{n}{2}}(c-acx)^{-2+\frac{n}{2}}\right)\text{Subst}\left(\int \frac{x^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{\left(1-\frac{x}{a}\right)^2}\right.\right.$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x(c-acx)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{(a+}\right.}{2-n}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 1.01

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}(c-acx)^{n/2}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{2}{1+ax}\right)}{ac^2(-2+n)(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(-2 + n/2),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*(c - a*c*x)^(n/2)*Hypergeometric2F1[2, 1 - n/2, 2 -
n/2, 2/(1 + a*x)])/(a*c^2*(-2 + n)*(1 - 1/(a*x))^(n/2)*(1 + a*x))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{-2+\frac{n}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="fricas")
[Out] integral((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(-2+1/2*n),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(-2+1/2*n),x, algorithm="giac")
[Out] integrate((-a*c*x + c)^(1/2*n - 2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^{\frac{n}{2}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(n/2 - 2), x)
```

3.364 $\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx$

Optimal. Leaf size=104

$$\frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^p\,{}_2F_1\left(\frac{1}{2}(n-2p),-1-p;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+p}$$

[Out] ((a-1/x)/(a+1/x))^(1/2*n-p)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^p*hypergeom([-1-p, 1/2*n-p],[-p],2/(a+1/x)/x)/(1+p)/((1-1/a/x)^(1/2*n))

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6311, 6316, 134}

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}(c-acx)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}{}_2F_1\left(\frac{1}{2}(n-2p),-p-1;-p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{p+1}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]

[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c - a*c*x)^p*Hypergeometric2F1[(n - 2*p)/2, -1 - p, -p, 2/((a + x^(-1))*x)])/((1 + p)*(1 - 1/(a*x))^(n/2))

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^p\,dx = \left(\left(1-\frac{1}{ax}\right)^{-p}x^{-p}(c-acx)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p x^p\,dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{x}\right)^p(c-acx)^p\right)\text{Subst}\left(\int x^{-2-p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)\right.\right.$$
$$= \frac{\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^p\,{}_2F_1\left(\frac{1}{2}(n-2p),-1-p;-\right.}{1+p}$$

Mathematica [A]
time = 0.03, size = 104, normalized size = 1.00

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(n-2p)}(1+ax)(c-acx)^p\,{}_2F_1\left(-1-p,\frac{n}{2}-p;-p;\frac{2}{1+ax}\right)}{a(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^p,x]
[Out] ((1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((n - 2*p)/2)*(1 + a*x)*(c - a*
c*x)^p*Hypergeometric2F1[-1 - p, n/2 - p, -p, 2/(1 + a*x)])/(a*(1 + p)*(1 -
 1/(a*x))^(n/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="maxima")
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="fricas")
[Out] integral((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c(ax-1)\right)^p e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**p,x)
[Out] Integral((-c*(a*x - 1))**p*exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^p,x, algorithm="giac")
[Out] integrate((-a*c*x + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^p,x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^p, x)
```

3.365 $\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx$

Optimal. Leaf size=81

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

[Out] -32*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([5, 4-1/2*n],[5-1/2*n],(a-1/x)/(a+1/x))/a/(8-n)

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6315, 133}

$$-\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Antiderivative was successfully verified.

[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]

[Out] (-32*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric2F1[5, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)^3\,dx &= -\left(\left(a^3c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^3x^3\,dx\right)\\ &= \left(a^3c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^5}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{32c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}{}_2F_1\left(5,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(81) = 162.
time = 1.83, size = 190, normalized size = 2.35

$$-\frac{c^3e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(-48+44n-12n^2+n^3)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(an^3x+n^2(-1-12ax+a^2x^2)+2n(6+21ax-6a^2x^2+a^3x^3)+6(-7-4ax+6a^2x^2-4a^3x^3+a^4x^4)+(-48+44n-12n^2+n^3)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{24a(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^3,x]
```

```
[Out] -1/24*(c^3*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(-48 + 44*n - 12*n^2 +
n^3)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(
a*n^3*x + n^2*(-1 - 12*a*x + a^2*x^2) + 2*n*(6 + 21*a*x - 6*a^2*x^2 + a^3*x
^3) + 6*(-7 - 4*a*x + 6*a^2*x^2 - 4*a^3*x^3 + a^4*x^4) + (-48 + 44*n - 12*n
^2 + n^3)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*(2 +
 n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}(-acx+c)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^3*c^3*x^3 - 3*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3axe^{n\operatorname{acoth}(ax)}\,dx+\int\left(-3a^2x^2e^{n\operatorname{acoth}(ax)}\right)\,dx+\int a^3x^3e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-e^{n\operatorname{acoth}(ax)}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a*x*exp(n*acoth(a*x)), x) + Integral(-3*a**2*x**2*exp(n*a
coth(a*x)), x) + Integral(a**3*x**3*exp(n*acoth(a*x)), x) + Integral(-exp(n
*acoth(a*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a*c*x - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^3,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^3, x)
```

3.366 $\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx$

Optimal. Leaf size=81

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\,{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] 16*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([4, 3-1/2*n],[4-1
/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6315, 133}

$$\frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\,{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (16*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[4, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\int e^{n\coth^{-1}(ax)}(c-acx)^2\,dx &= \left(a^2c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^2x^2\,dx \\
&= -\left(\left(a^2c^2\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^4}\,dx,x,\frac{1}{x}\right)\right) \\
&= \frac{16c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\,{}_2F_1\left(4,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}
\end{aligned}
$$

Mathematica [A]
time = 0.73, size = 144, normalized size = 1.78

$$
\frac{c^2e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(8-6n+n^2)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(6+6ax+an^2x-6a^2x^2+2a^3x^3+n(-1-6ax+a^2x^2)+(8-6n+n^2)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{6a(2+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(8 - 6*n + n^2)*Hypergeometri
c2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(6 + 6*a*x + a*n^2*
x - 6*a^2*x^2 + 2*a^3*x^3 + n*(-1 - 6*a*x + a^2*x^2) + (8 - 6*n + n^2)*Hype
rgeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(6*a*(2 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$
\int e^{n\,\mathrm{arccoth}(ax)}\left(-acx+c\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{2}\left(\int\left(-2axe^{n\operatorname{acoth}(ax)}\right)dx+\int a^{2}x^{2}e^{n\operatorname{acoth}(ax)}\,dx+\int e^{n\operatorname{acoth}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a*x*exp(n*acoth(a*x)), x) + Integral(a**2*x**2*exp(n*acot
h(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*c*x - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^2,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^2, x)
```

3.367 $\int e^{n\coth^{-1}(ax)}(c-acx)\,dx$

Optimal. Leaf size=79

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] -8*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([3, 2-1/2*n],[3-1/2
*n],(a-1/x)/(a+1/x))/a/(4-n)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6310, 6315, 133}

$$-\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\,{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] (-8*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1[
3, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}(c-acx)\,dx &= -\left((ac)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)x\,dx\right)\\ &= (ac)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^3}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{8c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(3,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 104, normalized size = 1.32

$$-\frac{ce^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}(-2+n)n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+a(-2+n)x+a^2x^2+(-2+n)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{2a(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x),x]
```

```
[Out] -1/2*(c*E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1
[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*(-2 + n)*x + a^
2*x^2 + (-2 + n)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/
(a*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="maxima")
[Out] -integrate((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="fricas")
[Out] integral(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int axe^{n\operatorname{acoth}(ax)}\,dx+\int\left(-e^{n\operatorname{acoth}(ax)}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c),x)
[Out] -c*(Integral(a*x*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c),x, algorithm="giac")
[Out] integrate(-(a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,(c-a\,c\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x), x)
```

3.368 $\int \frac{e^{n \coth^{-1}(ax)}}{c-acx}\,dx$

Optimal. Leaf size=71

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] 2*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/
((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6310, 6315, 133}

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(
a + x^(-1))])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-acx}\,dx = -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)x}\,dx}{ac}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx, x, \frac{1}{x}\right)}{ac}$$
$$= \frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Mathematica [A]
time = 0.13, size = 87, normalized size = 1.23

$$-\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{acn(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + Hypergeometric2F1[1, n/2, 1 + n
/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{-acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="fricas")
[Out] integral(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c),x)
[Out] -Integral(exp(n*acoth(a*x))/(a*x - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c),x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x), x)
```

3.369 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}$$

```
[Out] -(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)
```

Rubi [A]
time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6310, 6315, 37}

$$-\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
```

```
[Out] -(((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^2}\,dx = \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^2 x^2}\,dx}{a^2c^2}$$
$$= -\frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^2c^2}$$
$$= -\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}$$

Mathematica [A]
time = 0.10, size = 33, normalized size = 0.69

$$-\frac{e^{n\coth^{-1}(ax)}(1+ax)}{ac^2(2+n)(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^2,x]
[Out] -((E^(n*ArcCoth[a*x])*(1 + a*x))/(a*c^2*(2 + n)*(-1 + a*x)))
```

Maple [A]
time = 0.08, size = 33, normalized size = 0.69

method	result	size
gosper	$-\frac{e^{n\,\mathrm{arccoth}(ax)}(ax+1)}{(ax-1)c^2(2+n)a}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -exp(n*arccoth(a*x))*(a*x+1)/(a*x-1)/c^2/(2+n)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Fricas [A]
time = 0.37, size = 58, normalized size = 1.21

$$\frac{(ax+1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n+2\,ac^2-(a^2c^2n+2\,a^2c^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*x + 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n + 2*a*c^2 - (a^2*c^2*n + 2
*a^2*c^2)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.68, size = 187, normalized size = 3.90

$$\begin{cases} \frac{xe^{\frac{i\pi n}{2}}}{c^2} & \text{for } a=0 \\ \tilde{\infty}xe^{\infty n} & \text{for } a=\frac{1}{x} \\ -\frac{ax\operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)}-ac^2e^{2\operatorname{acoth}(ax)}}-\frac{\operatorname{acoth}(ax)}{a^2c^2xe^{2\operatorname{acoth}(ax)}-ac^2e^{2\operatorname{acoth}(ax)}} & \text{for } n=-2 \\ -\frac{axe^{n\operatorname{acoth}(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2}-\frac{e^{n\operatorname{acoth}(ax)}}{a^2c^2nx+2a^2c^2x-ac^2n-2ac^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**2,x)
```

```
[Out] Piecewise((x*exp(I*pi*n/2)/c**2, Eq(a, 0)), (zoo*x*exp(oo*n), Eq(a, 1/x)),
(-a*x*acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x)))
 - acoth(a*x)/(a**2*c**2*x*exp(2*acoth(a*x)) - a*c**2*exp(2*acoth(a*x))), E
q(n, -2)), (-a*x*exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**2*
n - 2*a*c**2) - exp(n*acoth(a*x))/(a**2*c**2*n*x + 2*a**2*c**2*x - a*c**2*n
 - 2*a*c**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^2, x)
```

Mupad [B]
time = 1.53, size = 32, normalized size = 0.67

$$-\frac{\mathrm{e}^{n\operatorname{acoth}(a\,x)}\,(a\,x+1)}{a\,c^2\,(a\,x-1)\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^2,x)
[Out] -(exp(n*acoth(a*x))*(a*x + 1))/(a*c^2*(a*x - 1)*(n + 2))
```

3.370 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx$

Optimal. Leaf size=104

$$\frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)}-\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(n^2+6*n+8)+(1-1/a/x)
^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^3/(4+n)
```

Rubi [A]

time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6310, 6315, 80, 37}

$$\frac{\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+4)}-\frac{(n+3)\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{ac^3(n+2)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] ((1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(4 + n)) - ((3
+ n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^3*(2 + n)*(4
+ n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
```

```
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^3}\,dx &= -\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^3x^3}\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int x\left(1-\frac{x}{a}\right)^{-3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)}{a^3c^3}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)}-\frac{(3+n)\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)}{a^2c^3(4+n)}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(4+n)}-\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^3(2+n)(4+n)}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 64, normalized size = 0.62

$$
\frac{e^{n\coth^{-1}(ax)}(3+n-ax)\left(\cosh\left(3\coth^{-1}(ax)\right)+\sinh\left(3\coth^{-1}(ax)\right)\right)}{a^2c^3(2+n)(4+n)\sqrt{1-\frac{1}{a^2x^2}}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 + n - a*x)*(Cosh[3*ArcCoth[a*x]] + Sinh[3*ArcCoth[a*
x]]))/(a^2*c^3*(2 + n)*(4 + n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.08, size = 46, normalized size = 0.44

method	result	size

gosper	$-\frac{e^{n\,\mathrm{arccoth}(ax)}(ax-n-3)(ax+1)}{(ax-1)^2c^3(n^2+6n+8)a}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(n*arccoth(a*x))*(a*x-n-3)*(a*x+1)/(a*x-1)^2/c^3/(n^2+6*n+8)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Fricas [A]
time = 0.35, size = 128, normalized size = 1.23

$$-\frac{\left(a^2x^2-(an+2\,a)x-n-3\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^2+6\,ac^3n+8\,ac^3+(a^3c^3n^2+6\,a^3c^3n+8\,a^3c^3)x^2-2\,(a^2c^3n^2+6\,a^2c^3n+8\,a^2c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*x^2 - (a*n + 2*a)*x - n - 3)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^2
 + 6*a*c^3*n + 8*a*c^3 + (a^3*c^3*n^2 + 6*a^3*c^3*n + 8*a^3*c^3)*x^2 - 2*(a
^2*c^3*n^2 + 6*a^2*c^3*n + 8*a^2*c^3)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 52.64, size = 1112, normalized size = 10.69

$$\begin{cases} \frac{xe^{\frac{i\pi n}{2}}}{c^3} & \text{for } a=0 \\ \frac{a^2x^2\operatorname{acoth}(ax)}{2a^3c^3x^2e^{4\operatorname{acoth}(ax)}-4a^2c^3xe^{4\operatorname{acoth}(ax)}+2ac^3e^{4\operatorname{acoth}(ax)}}+\frac{2ax\operatorname{acoth}(ax)}{2a^3c^3x^2e^{4\operatorname{acoth}(ax)}-4a^2c^3xe^{4\operatorname{acoth}(ax)}+2ac^3e^{4\operatorname{acoth}(ax)}}-\frac{ax}{2a^3c^3x^2e^{4\operatorname{acoth}(ax)}-4a^2c^3xe^{4\operatorname{acoth}(ax)}+2ac^3e^{4\operatorname{acoth}(ax)}}+\frac{\operatorname{acoth}(ax)}{2a^3c^3x^2e^{4\operatorname{acoth}(ax)}-4a^2c^3xe^{4\operatorname{acoth}(ax)}+2ac^3e^{4\operatorname{acoth}(ax)}}-\frac{1}{2a^3c^3x^2e^{4\operatorname{acoth}(ax)}-4a^2c^3xe^{4\operatorname{acoth}(ax)}+2ac^3e^{4\operatorname{acoth}(ax)}} & \text{for } n=-4 \\ -\frac{a^2x^2\operatorname{acoth}(ax)}{2a^3c^3x^2e^{2\operatorname{acoth}(ax)}-4a^2c^3xe^{2\operatorname{acoth}(ax)}+2ac^3e^{2\operatorname{acoth}(ax)}}+\frac{ax}{2a^3c^3x^2e^{2\operatorname{acoth}(ax)}-4a^2c^3xe^{2\operatorname{acoth}(ax)}+2ac^3e^{2\operatorname{acoth}(ax)}}+\frac{\operatorname{acoth}(ax)}{2a^3c^3x^2e^{2\operatorname{acoth}(ax)}-4a^2c^3xe^{2\operatorname{acoth}(ax)}+2ac^3e^{2\operatorname{acoth}(ax)}}+\frac{1}{2a^3c^3x^2e^{2\operatorname{acoth}(ax)}-4a^2c^3xe^{2\operatorname{acoth}(ax)}+2ac^3e^{2\operatorname{acoth}(ax)}} & \text{for } n=-2 \\ -\frac{a^2x^2e^{n\operatorname{acoth}(ax)}}{a^3c^3n^2x^2+6a^3c^3nx^2+8a^3c^3x^2-2a^2c^3n^2x-12a^2c^3nx-16a^2c^3x+ac^3n^2+6ac^3n+8ac^3}+\frac{anxe^{n\operatorname{acoth}(ax)}}{a^3c^3n^2x^2+6a^3c^3nx^2+8a^3c^3x^2-2a^2c^3n^2x-12a^2c^3nx-16a^2c^3x+ac^3n^2+6ac^3n+8ac^3}+\frac{2axe^{n\operatorname{acoth}(ax)}}{a^3c^3n^2x^2+6a^3c^3nx^2+8a^3c^3x^2-2a^2c^3n^2x-12a^2c^3nx-16a^2c^3x+ac^3n^2+6ac^3n+8ac^3}+\frac{ne^{n\operatorname{acoth}(ax)}}{a^3c^3n^2x^2+6a^3c^3nx^2+8a^3c^3x^2-2a^2c^3n^2x-12a^2c^3nx-16a^2c^3x+ac^3n^2+6ac^3n+8ac^3}+\frac{3e^{n\operatorname{acoth}(ax)}}{a^3c^3n^2x^2+6a^3c^3nx^2+8a^3c^3x^2-2a^2c^3n^2x-12a^2c^3nx-16a^2c^3x+ac^3n^2+6ac^3n+8ac^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**3,x)
```

```
[Out] Piecewise((x*exp(I*pi*n/2)/c**3, Eq(a, 0)), (a**2*x**2*acoth(a*x)/(2*a**3*c
**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp
(4*acoth(a*x))) + 2*a*x*acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*
a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - a*x/(2*a**3*c
**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp
```

```
(4*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(4*acoth(a*x)) - 4*a**2*c
**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth(a*x))) - 1/(2*a**3*c**3*x**2
*exp(4*acoth(a*x)) - 4*a**2*c**3*x*exp(4*acoth(a*x)) + 2*a*c**3*exp(4*acoth
(a*x))), Eq(n, -4)), (-a**2*x**2*acoth(a*x)/(2*a**3*c**3*x**2*exp(2*acoth(a
*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x))) + a*x/
(2*a**3*c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a
*c**3*exp(2*acoth(a*x))) + acoth(a*x)/(2*a**3*c**3*x**2*exp(2*acoth(a*x)) -
 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*exp(2*acoth(a*x))) + 1/(2*a**3*
c**3*x**2*exp(2*acoth(a*x)) - 4*a**2*c**3*x*exp(2*acoth(a*x)) + 2*a*c**3*ex
p(2*acoth(a*x))), Eq(n, -2)), (-a**2*x**2*exp(n*acoth(a*x))/(a**3*c**3*n**2
*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**
2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + a*n*x*
exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x
**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2
+ 6*a*c**3*n + 8*a*c**3) + 2*a*x*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6
*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*
x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3) + n*exp(n*acoth(a
*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2 + 8*a**3*c**3*x**2 - 2*a**2*
c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*x + a*c**3*n**2 + 6*a*c**3*n
+ 8*a*c**3) + 3*exp(n*acoth(a*x))/(a**3*c**3*n**2*x**2 + 6*a**3*c**3*n*x**2
 + 8*a**3*c**3*x**2 - 2*a**2*c**3*n**2*x - 12*a**2*c**3*n*x - 16*a**2*c**3*
x + a*c**3*n**2 + 6*a*c**3*n + 8*a*c**3), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^3,x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^3, x)
```

Mupad [**B**]
time = 1.65, size = 113, normalized size = 1.09

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{n+3}{a^3\,c^3\,\left(n^2+6\,n+8\right)}-\frac{x^2}{a\,c^3\,\left(n^2+6\,n+8\right)}+\frac{x\,\left(n+2\right)}{a^2\,c^3\,\left(n^2+6\,n+8\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^2}-\frac{2\,x}{a}+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^3,x)
[Out] (((a*x + 1)/(a*x))^(n/2)*((n + 3)/(a^3*c^3*(6*n + n^2 + 8)) - x^2/(a*c^3*(6
*n + n^2 + 8)) + (x*(n + 2))/(a^2*c^3*(6*n + n^2 + 8))))/(((a*x - 1)/(a*x))
^(n/2)*(1/a^2 - (2*x)/a + x^2))
```

3.371 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx$

Optimal. Leaf size=224

$$\frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)}-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)}-\frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-1-}}{ac^4(6+n)\left(8+6n\right.}$$

```
[Out] -(n^2+8*n+14)*(1-1/a/x)^(-2-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^2+10*n+24)-
(n^2+8*n+14)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(n^3+12*n^2+44*
n+48)+(5+n)*(1-1/a/x)^(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^4/(6+n)-(1-1/a/x)^
(-3-1/2*n)*(1+1/a/x)^(1+1/2*n)/a^2/c^4/x
```

Rubi [A]
time = 0.18, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6310, 6315, 92, 80, 47, 37}

$$-\frac{\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{a^2c^4x}-\frac{(n^2+8n+14)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-2}}{ac^4(n+4)(n+6)}-\frac{(n^2+8n+14)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^4(n+6)\left(n^2+6n+8\right)}+\frac{(n+5)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-3}}{ac^4(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] ((5 + n)*(1 - 1/(a*x))^(-3 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^4*(6 + n)
) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-2 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(
a*c^4*(4 + n)*(6 + n)) - ((14 + 8*n + n^2)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/
(a*x))^((2 + n)/2))/(a*c^4*(6 + n)*(8 + 6*n + n^2)) - ((1 - 1/(a*x))^(-3 -
n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a^2*c^4*x)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 6310

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*x^p*(1 + c/(d*x))^p*E^(n*ArcCoth[a*x]), x], x] /; Free
Q[{a, c, d, n}, x] && EqQ[a^2*c^2 - d^2, 0] &&  !IntegerQ[n/2] && IntegerQ[
p]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^4}\,dx = \frac{\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^4 x^4}\,dx}{a^4c^4}$$
$$= -\frac{\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{a^4c^4}$$
$$= -\frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x} - \frac{\text{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-4-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\left(-1-\frac{(4+n)x}{a}\right)\,dx, x,\right.}{a^2c^4}$$
$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{a^2c^4x} - \frac{(14+8n+n^2)\,\text{Subs}}{}$$
$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} - \frac{(1-}{}$$
$$= \frac{(5+n)\left(1-\frac{1}{ax}\right)^{-3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(6+n)} - \frac{(14+8n+n^2)\left(1-\frac{1}{ax}\right)^{-2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^4(4+n)(6+n)} - \frac{(14-}{}$$

Mathematica [A]
time = 0.20, size = 83, normalized size = 0.37

$$-\frac{e^{n\coth^{-1}(ax)}\left(-12-8n-n^2+(4+n)^2\cosh\left(2\coth^{-1}(ax)\right)-2(4+n)\sinh\left(2\coth^{-1}(ax)\right)\right)\left(\cosh\left(4\coth^{-1}(ax)\right)+\sinh\left(4\coth^{-1}(ax)\right)\right)}{2ac^4(2+n)(4+n)(6+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^4,x]
```

```
[Out] -1/2*(E^(n*ArcCoth[a*x])*(-12 - 8*n - n^2 + (4 + n)^2*Cosh[2*ArcCoth[a*x]]
- 2*(4 + n)*Sinh[2*ArcCoth[a*x]])*(Cosh[4*ArcCoth[a*x]] + Sinh[4*ArcCoth[a*
x]]))/(a*c^4*(2 + n)*(4 + n)*(6 + n))
```

Maple [A]
time = 0.08, size = 68, normalized size = 0.30

method	result	size
gosper	$-\frac{(ax+1)\left(2a^2x^2-2anx-8ax+n^2+8n+14\right)e^{n\,\text{arccoth}(ax)}}{(ax-1)^3c^4a(n^2+8n+12)(4+n)}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x+1)*(2*a^2*x^2-2*a*n*x-8*a*x+n^2+8*n+14)*exp(n*arccoth(a*x))/(a*x-1)^3
/c^4/a/(n^2+8*n+12)/(4+n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

Fricas [A]
time = 0.35, size = 228, normalized size = 1.02

$$\frac{\left(2\,a^{3}x^{3}-2\,(a^{2}n+3\,a^{2})x^{2}+n^{2}+(an^{2}+6\,an+6\,a)x+8\,n+14\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{4}n^{3}+12\,ac^{4}n^{2}+44\,ac^{4}n+48\,ac^{4}-(a^{4}c^{4}n^{3}+12\,a^{4}c^{4}n^{2}+44\,a^{4}c^{4}n+48\,a^{4}c^{4})x^{3}+3\,(a^{3}c^{4}n^{3}+12\,a^{3}c^{4}n^{2}+44\,a^{3}c^{4}n+48\,a^{3}c^{4})x^{2}-3\,(a^{2}c^{4}n^{3}+12\,a^{2}c^{4}n^{2}+44\,a^{2}c^{4}n+48\,a^{2}c^{4})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="fricas")
```

```
[Out] (2*a^3*x^3 - 2*(a^2*n + 3*a^2)*x^2 + n^2 + (a*n^2 + 6*a*n + 6*a)*x + 8*n +
14)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^3 + 12*a*c^4*n^2 + 44*a*c^4*n +
48*a*c^4 - (a^4*c^4*n^3 + 12*a^4*c^4*n^2 + 44*a^4*c^4*n + 48*a^4*c^4)*x^3 +
 3*(a^3*c^4*n^3 + 12*a^3*c^4*n^2 + 44*a^3*c^4*n + 48*a^3*c^4)*x^2 - 3*(a^2*
c^4*n^3 + 12*a^2*c^4*n^2 + 44*a^2*c^4*n + 48*a^2*c^4)*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 171.54, size = 3534, normalized size = 15.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**4,x)
```

```
[Out] Piecewise((x*exp(I*pi*n/2)/c**4, Eq(a, 0)), (-a**3*x**3*acoth(a*x)/(4*a**4*
c**4*x**3*exp(6*acoth(a*x)) - 12*a**3*c**4*x**2*exp(6*acoth(a*x)) + 12*a**2
*c**4*x*exp(6*acoth(a*x)) - 4*a*c**4*exp(6*acoth(a*x))) - 3*a**2*x**2*acoth
(a*x)/(4*a**4*c**4*x**3*exp(6*acoth(a*x)) - 12*a**3*c**4*x**2*exp(6*acoth(a
*x)) + 12*a**2*c**4*x*exp(6*acoth(a*x)) - 4*a*c**4*exp(6*acoth(a*x))) + a**
2*x**2/(4*a**4*c**4*x**3*exp(6*acoth(a*x)) - 12*a**3*c**4*x**2*exp(6*acoth(
a*x)) + 12*a**2*c**4*x*exp(6*acoth(a*x)) - 4*a*c**4*exp(6*acoth(a*x))) - 3*
a*x*acoth(a*x)/(4*a**4*c**4*x**3*exp(6*acoth(a*x)) - 12*a**3*c**4*x**2*exp(
6*acoth(a*x)) + 12*a**2*c**4*x*exp(6*acoth(a*x)) - 4*a*c**4*exp(6*acoth(a*x
))) + 3*a*x/(4*a**4*c**4*x**3*exp(6*acoth(a*x)) - 12*a**3*c**4*x**2*exp(6*a
coth(a*x)) + 12*a**2*c**4*x*exp(6*acoth(a*x)) - 4*a*c**4*exp(6*acoth(a*x)))
 - acoth(a*x)/(4*a**4*c**4*x**3*exp(6*acoth(a*x)) - 12*a**3*c**4*x**2*exp(6
*acoth(a*x)) + 12*a**2*c**4*x*exp(6*acoth(a*x)) - 4*a*c**4*exp(6*acoth(a*x)
```

```
)) + 2/(4*a**4*c**4*x**3*exp(6*acoth(a*x)) - 12*a**3*c**4*x**2*exp(6*acoth(
a*x)) + 12*a**2*c**4*x*exp(6*acoth(a*x)) - 4*a*c**4*exp(6*acoth(a*x))), Eq(
n, -6)), (a**3*x**3*acoth(a*x)/(2*a**4*c**4*x**3*exp(4*acoth(a*x)) - 6*a**3
*c**4*x**2*exp(4*acoth(a*x)) + 6*a**2*c**4*x*exp(4*acoth(a*x)) - 2*a*c**4*e
xp(4*acoth(a*x))) + a**2*x**2*acoth(a*x)/(2*a**4*c**4*x**3*exp(4*acoth(a*x)
) - 6*a**3*c**4*x**2*exp(4*acoth(a*x)) + 6*a**2*c**4*x*exp(4*acoth(a*x)) -
2*a*c**4*exp(4*acoth(a*x))) - a**2*x**2/(2*a**4*c**4*x**3*exp(4*acoth(a*x))
 - 6*a**3*c**4*x**2*exp(4*acoth(a*x)) + 6*a**2*c**4*x*exp(4*acoth(a*x)) - 2
*a*c**4*exp(4*acoth(a*x))) - a*x*acoth(a*x)/(2*a**4*c**4*x**3*exp(4*acoth(a
*x)) - 6*a**3*c**4*x**2*exp(4*acoth(a*x)) + 6*a**2*c**4*x*exp(4*acoth(a*x))
 - 2*a*c**4*exp(4*acoth(a*x))) - a*x/(2*a**4*c**4*x**3*exp(4*acoth(a*x)) -
6*a**3*c**4*x**2*exp(4*acoth(a*x)) + 6*a**2*c**4*x*exp(4*acoth(a*x)) - 2*a*
c**4*exp(4*acoth(a*x))) - acoth(a*x)/(2*a**4*c**4*x**3*exp(4*acoth(a*x)) -
6*a**3*c**4*x**2*exp(4*acoth(a*x)) + 6*a**2*c**4*x*exp(4*acoth(a*x)) - 2*a*
c**4*exp(4*acoth(a*x))), Eq(n, -4)), (-a**3*x**3*acoth(a*x)/(4*a**4*c**4*x*
*3*exp(2*acoth(a*x)) - 12*a**3*c**4*x**2*exp(2*acoth(a*x)) + 12*a**2*c**4*x
*exp(2*acoth(a*x)) - 4*a*c**4*exp(2*acoth(a*x))) + a**2*x**2*acoth(a*x)/(4*
a**4*c**4*x**3*exp(2*acoth(a*x)) - 12*a**3*c**4*x**2*exp(2*acoth(a*x)) + 12
*a**2*c**4*x*exp(2*acoth(a*x)) - 4*a*c**4*exp(2*acoth(a*x))) + a**2*x**2/(4
*a**4*c**4*x**3*exp(2*acoth(a*x)) - 12*a**3*c**4*x**2*exp(2*acoth(a*x)) + 1
2*a**2*c**4*x*exp(2*acoth(a*x)) - 4*a*c**4*exp(2*acoth(a*x))) + a*x*acoth(a
*x)/(4*a**4*c**4*x**3*exp(2*acoth(a*x)) - 12*a**3*c**4*x**2*exp(2*acoth(a*x
)) + 12*a**2*c**4*x*exp(2*acoth(a*x)) - 4*a*c**4*exp(2*acoth(a*x))) - a*x/(
4*a**4*c**4*x**3*exp(2*acoth(a*x)) - 12*a**3*c**4*x**2*exp(2*acoth(a*x)) +
12*a**2*c**4*x*exp(2*acoth(a*x)) - 4*a*c**4*exp(2*acoth(a*x))) - acoth(a*x)
/(4*a**4*c**4*x**3*exp(2*acoth(a*x)) - 12*a**3*c**4*x**2*exp(2*acoth(a*x))
+ 12*a**2*c**4*x*exp(2*acoth(a*x)) - 4*a*c**4*exp(2*acoth(a*x))) - 2/(4*a**
4*c**4*x**3*exp(2*acoth(a*x)) - 12*a**3*c**4*x**2*exp(2*acoth(a*x)) + 12*a*
*2*c**4*x*exp(2*acoth(a*x)) - 4*a*c**4*exp(2*acoth(a*x))), Eq(n, -2)), (-2*
a**3*x**3*exp(n*acoth(a*x))/(a**4*c**4*n**3*x**3 + 12*a**4*c**4*n**2*x**3 +
 44*a**4*c**4*n*x**3 + 48*a**4*c**4*x**3 - 3*a**3*c**4*n**3*x**2 - 36*a**3*
c**4*n**2*x**2 - 132*a**3*c**4*n*x**2 - 144*a**3*c**4*x**2 + 3*a**2*c**4*n*
*3*x + 36*a**2*c**4*n**2*x + 132*a**2*c**4*n*x + 144*a**2*c**4*x - a*c**4*n
**3 - 12*a*c**4*n**2 - 44*a*c**4*n - 48*a*c**4) + 2*a**2*n*x**2*exp(n*acoth
(a*x))/(a**4*c**4*n**3*x**3 + 12*a**4*c**4*n**2*x**3 + 44*a**4*c**4*n*x**3
+ 48*a**4*c**4*x**3 - 3*a**3*c**4*n**3*x**2 - 36*a**3*c**4*n**2*x**2 - 132*
a**3*c**4*n*x**2 - 144*a**3*c**4*x**2 + 3*a**2*c**4*n**3*x + 36*a**2*c**4*n
**2*x + 132*a**2*c**4*n*x + 144*a**2*c**4*x - a*c**4*n**3 - 12*a*c**4*n**2
- 44*a*c**4*n - 48*a*c**4) + 6*a**2*x**2*exp(n*acoth(a*x))/(a**4*c**4*n**3*
x**3 + 12*a**4*c**4*n**2*x**3 + 44*a**4*c**4*n*x**3 + 48*a**4*c**4*x**3 - 3
*a**3*c**4*n**3*x**2 - 36*a**3*c**4*n**2*x**2 - 132*a**3*c**4*n*x**2 - 144*
a**3*c**4*x**2 + 3*a**2*c**4*n**3*x + 36*a**2*c**4*n**2*x + 132*a**2*c**4*n
*x + 144*a**2*c**4*x - a*c**4*n**3 - 12*a*c**4*n**2 - 44*a*c**4*n - 48*a*c*
*4) - a*n**2*x*exp(n*acoth(a*x))/(a**4*c**4*n**3*x**3 + 12*a**4*c**4*n**2*x
**3 + 44*a**4*c**4*n*x**3 + 48*a**4*c**4*x**3 - 3*a**3*c**4*n**3*x**2 - 36*
```

```
a**3*c**4*n**2*x**2 - 132*a**3*c**4*n*x**2 - 144*a**3*c**4*x**2 + 3*a**2*c*
*4*n**3*x + 36*a**2*c**4*n**2*x + 132*a**2*c**4*n*x + 144*a**2*c**4*x - a*c
**4*n**3 - 12*a*c**4*n**2 - 44*a*c**4*n - 48*a*c**4) - 6*a*n*x*exp(n*acoth(
a*x))/(a**4*c**4*n**3*x**3 + 12*a**4*c**4*n**2*x**3 + 44*a**4*c**4*n*x**3 +
 48*a**4*c**4*x**3 - 3*a**3*c**4*n**3*x**2 - 36*a**3*c**4*n**2*x**2 - 132*a
**3*c**4*n*x**2 - 144*a**3*c**4*x**2 + 3*a**2*c**4*n**3*x + 36*a**2*c**4*n*
*2*x + 132*a**2*c**4*n*x + 144*a**2*c**4*x - a*c**4*n**3 - 12*a*c**4*n**2 -
 44*a*c**4*n - 48*a*c**4) - 6*a*x*exp(n*acoth(a*x))/(a**4*c**4*n**3*x**3 +
12*a**4*c**4*n**2*x**3 + 44*a**4*c**4*n*x**3 + ...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c)^4, x)
```

Mupad [B]
time = 1.81, size = 180, normalized size = 0.80

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{2\,x^3}{a\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}+\frac{n^2+8\,n+14}{a^4\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}-\frac{x^2\,\left(2\,n+6\right)}{a^2\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}+\frac{x\,\left(n^2+6\,n+6\right)}{a^3\,c^4\,\left(n^3+12\,n^2+44\,n+48\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{3\,x}{a^2}-\frac{1}{a^3}+x^3-\frac{3\,x^2}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^4,x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*((2*x^3)/(a*c^4*(44*n + 12*n^2 + n^3 + 48)) + (8*
n + n^2 + 14)/(a^4*c^4*(44*n + 12*n^2 + n^3 + 48)) - (x^2*(2*n + 6))/(a^2*c
^4*(44*n + 12*n^2 + n^3 + 48)) + (x*(6*n + n^2 + 6))/(a^3*c^4*(44*n + 12*n^
2 + n^3 + 48))))/(((a*x - 1)/(a*x))^(n/2)*((3*x)/a^2 - 1/a^3 + x^3 - (3*x^2
)/a))
```

3.372 $\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,{}_2F_1\left(-\frac{7}{2},\frac{1}{2}(-5+n);-\frac{5}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/7*((a-1/x)/(a+1/x))^(-5/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(5/2)*h
ypergeom([-7/2, -5/2+1/2*n],[-5/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.14, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6311, 6316, 134}

$$\frac{2}{7}x(c-acx)^{5/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-5}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\,{}_2F_1\left(-\frac{7}{2},\frac{n-5}{2};-\frac{5}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-5 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(5/2)*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/((a + x^(-1))*x)
])/(7*(1 - 1/(a*x))^(n/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{5/2}\,dx = \frac{(c-acx)^{5/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{9/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$= \frac{2}{7}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-5+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{5/2}\,{}_2F_1\left(-\frac{7}{2},\frac{1}{2}(-\right.$$

Mathematica [A]
time = 0.07, size = 103, normalized size = 1.05

$$\frac{2c^2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^3\sqrt{c-acx}\,{}_2F_1\left(-\frac{7}{2},\frac{1}{2}(-5+n);-\frac{5}{2};\frac{2}{1+ax}\right)}{7a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(5/2),x]
[Out] (2*c^2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^3*
Sqrt[c - a*c*x]*Hypergeometric2F1[-7/2, (-5 + n)/2, -5/2, 2/(1 + a*x)])/(7*
a*(1 - 1/(a*x))^(n/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(5/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*x^2 - 2*a*c^2*x + c^2)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x -
 1))^(1/2*n), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(5/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,6,1,0,0]%%%}+%%%{3,[0,4,1,1,0]%%%}+%%%{-3,[0,2,1,2,0]%%
%}+%%%
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(5/2), x)
```

3.373 $\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-3+n);-\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/5*((a-1/x)/(a+1/x))^(-3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*(-a*c*x+c)^(3/2)*h
ypergeom([-5/2, -3/2+1/2*n],[-3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6311, 6316, 134}

$$\frac{2}{5}x(c-acx)^{3/2}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\,{}_2F_1\left(-\frac{5}{2},\frac{n-3}{2};-\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*(c
- a*c*x)^(3/2)*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/((a + x^(-1))*x)
])/(5*(1 - 1/(a*x))^(n/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}(c-acx)^{3/2}\,dx = \frac{(c-acx)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\,dx}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}$$

$$= -\frac{\left(\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{7/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{2}{5}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-3+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x(c-acx)^{3/2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(\right.$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 1.03

$$-\frac{2c\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)^2\sqrt{c-acx}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-3+n);-\frac{3}{2};\frac{2}{1+ax}\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a*c*x)^(3/2),x]
[Out] (-2*c*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)^2*S
qrt[c - a*c*x]*Hypergeometric2F1[-5/2, (-3 + n)/2, -3/2, 2/(1 + a*x)])/(5*a
*(1 - 1/(a*x))^(n/2))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}(-acx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*c*x + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*c*x - c)*sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,4,1,0,0]%%%}+%%%{-2,[0,2,1,1,0]%%%}+%%%{1,[0,0,1,2,0]%%%
} / %%
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a\,c\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(3/2), x)
```

3.374 $\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx$

Optimal. Leaf size=98

$$\frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\sqrt{c-acx}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-1+n);-\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

```
[Out] 2/3*((a-1/x)/(a+1/x))^(-1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-3/2, -
1/2+1/2*n],[-1/2],2/(a+1/x)/x)*(-a*c*x+c)^(1/2)/((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6311, 6316, 134}

$$\frac{2}{3}x\sqrt{c-acx}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(-\frac{3}{2},\frac{n-1}{2};-\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((-1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Sqr
t[c - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/((a + x^(-1))*x)])
/(3*(1 - 1/(a*x))^(n/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-acx}\,dx=\frac{\sqrt{c-acx}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\sqrt{x}\,dx}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}$$

$$=-\frac{\left(\sqrt{\frac{1}{x}}\sqrt{c-acx}\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{5/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{2}{3}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(-1+n)}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\sqrt{c-acx}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-1+\right.$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.00

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1}{2}(-1+n)}(1+ax)\sqrt{c-acx}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-1+n);-\frac{1}{2};\frac{2}{1+ax}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a*c*x],x]
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((-1 + n)/2)*(1 + a*x)*Sqrt[c
 - a*c*x]*Hypergeometric2F1[-3/2, (-1 + n)/2, -1/2, 2/(1 + a*x)])/(3*a*(1 -
 1/(a*x))^(n/2))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{-acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a*c*x+c)**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a\,c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a*c*x)^(1/2), x)
```

3.375 $\int \frac{e^{n \coth^{-1}(ax)}}{\sqrt{c-acx}} \, dx$

Optimal. Leaf size=96

$$\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\, {}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

```
[Out] 2*((a-1/x)/(a+1/x))^(1/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([-1/2, 1/2+
1/2*n],[1/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6311, 6316, 134}

$$\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}} {}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2*((a - x^(-1))/(a + x^(-1)))^((1 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hype
rgeometric2F1[-1/2, (1 + n)/2, 1/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*Sqrt[c - a*c*x])
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-acx}}\,dx = \frac{\left(\sqrt{1-\frac{1}{ax}}\sqrt{x}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}\sqrt{x}}\,dx}{\sqrt{c-acx}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{\frac{1}{x}}\sqrt{c-acx}}$$

$$= \frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} x\, {}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{1}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{\sqrt{c-acx}}$$

Mathematica [A]
time = 0.04, size = 96, normalized size = 1.00

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}(1+ax)\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{1}{2};\frac{2}{1+ax}\right)}{a\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a*c*x],x]
```

```
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*(1 + a*x)*Hyperge
ometric2F1[-1/2, (1 + n)/2, 1/2, 2/(1 + a*x)])/(a*(1 - 1/(a*x))^(n/2)*Sqrt[
c - a*c*x])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\text{arccoth}(ax)}}{\sqrt{-acx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}{(ax)}}}{\sqrt{-c(ax-1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a*c*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a\,c\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(1/2), x)
```

3.376 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

```
[Out] -2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1+1/a/x)^(1+1/2*n)*x*hypergeom([1/2, 3/2+
1/2*n],[3/2],2/(a+1/x)/x)/((1-1/a/x)^(1/2*n))/(-a*c*x+c)^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6311, 6316, 134}

$$-\frac{2x\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
```

```
[Out] (-2*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x*Hyp
ergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/((1 - 1/(a*x))^(n/
2)*(c - a*c*x)^(3/2))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
```

```
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}x^{3/2}}\,dx}{(c-acx)^{3/2}}$$
$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{3/2}(c-acx)^{3/2}}$$
$$= -\frac{2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(c-acx)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 94, normalized size = 0.98

$$\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{1+ax}\right)}{ac\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(3/2),x]
[Out] (2*(1 + 1/(a*x))^(n/2)*((-1 + a*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1
[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)])/(a*c*(1 - 1/(a*x))^(n/2)*Sqrt[c - a*c*x
])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{(-acx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*
c^2*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{(-c(ax-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n \operatorname{acoth}(a\,x)}}{{(c-a\,c\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(3/2), x)
```

3.377 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}}+\frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(3+n)(c-acx)^{5/2}}$$

```
[Out] -a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(3+n)/(-a*c*x+c)^(5/2)+a*((a
-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2*hype
rgeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(3+n)/(-a*c*x+c)^(5/2)
```

Rubi [A]
time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6311, 6316, 96, 134}

$$\frac{ax^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{(n+3)(c-acx)^{5/2}}-\frac{ax^2\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{(n+3)(c-acx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((3 + n)*(c -
 a*c*x)^(5/2))) + (a*((a - x^(-1))/(a + x^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^
((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2*Hypergeometric2F1[1/2, (3 + n)/2,
 3/2, 2/((a + x^(-1))*x)])/((3 + n)*(c - a*c*x)^(5/2))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] && !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}x^{5/2}}\,dx}{(c-acx)^{5/2}} \\
&= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx,x,\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}} \\
&= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{\sqrt{x}}\,dx,x,\frac{1}{x}\right.}{2(3+n)\left(\frac{1}{x}\right)^{5/2}(c-acx)^{5/2}} \\
&= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(3+n)(c-acx)^{5/2}} + \frac{a\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\right.}{(3+n)(c-acx)^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 117, normalized size = 0.70

$$
\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(-1-ax+(-1+ax)\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{1+ax}\right)\right)}{ac^2(3+n)(-1+ax)\sqrt{c-acx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(5/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(-1 - a*x + (-1 + a*x)*((-1 + a*x)/(1 + a*x))^((1 + n)
/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*x)]))/(a*c^2*(3 + n)*(1
 - 1/(a*x))^(n/2)*(-1 + a*x)*Sqrt[c - a*c*x])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{(-acx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*x^3 - 3*a
^2*c^3*x^2 + 3*a*c^3*x - c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}{\left(ax \right)}}}{\left(- c \left(ax - 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(5/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{(c-a\,c\,x)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(5/2), x)
```

3.378 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx$

Optimal. Leaf size=245

$$-\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}}+\frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}}-\frac{3a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3\,{}_2}{2\left(15+8n+n^2\right)(c-acx}$$

```
[Out] -a*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^2/(5+n)/(-a*c*x+c)^(7/2)+3/2*a
^2*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/2*n)*x^3/(n^2+8*n+15)/(-a*c*x+c)^(7/2
)-3/2*a^2*((a-1/x)/(a+1/x))^(3/2+1/2*n)*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(1+1/
2*n)*x^3*hypergeom([1/2, 3/2+1/2*n],[3/2],2/(a+1/x)/x)/(n^2+8*n+15)/(-a*c*x
+c)^(7/2)
```

Rubi [A]

time = 0.18, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6311, 6316, 96, 134}

$$-\frac{3a^2x^3\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{n+3}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{3}{2};\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}+\frac{3a^2x^3\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}}{2\left(n^2+8n+15\right)(c-acx)^{7/2}}-\frac{ax^2\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}}{(n+5)(c-acx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] -((a*(1 - 1/(a*x))^((2 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^2)/((5 + n)*(c -
 a*c*x)^(7/2))) + (3*a^2*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2
)*x^3)/(2*(15 + 8*n + n^2)*(c - a*c*x)^(7/2)) - (3*a^2*((a - x^(-1))/(a + x
^(-1)))^((3 + n)/2)*(1 - 1/(a*x))^((4 - n)/2)*(1 + 1/(a*x))^((2 + n)/2)*x^3
*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/((a + x^(-1))*x)])/(2*(15 + 8*n +
 n^2)*(c - a*c*x)^(7/2))
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1))))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
```

```
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6311

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_), x_Symbol]
 :> Dist[(c + d*x)^p/(x^p*(1 + c/(d*x))^p), Int[u*x^p*(1 + c/(d*x))^p*E^(n*
ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0]
 &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-acx)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}x^{7/2}}\,dx}{(c-acx)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int x^{3/2}\left(1-\frac{x}{a}\right)^{-\frac{7}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}\,dx, x, \frac{1}{x}\right)}{\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{\left(3a\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)\right.}{2(5+n)\left(\frac{1}{x}\right)^{7/2}(c-acx)^{7/2}}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{\left(3a^2\left(1-\frac{1}{ax}\right)^{7/2}\right)}{4(3+n}$$

$$= -\frac{a\left(1-\frac{1}{ax}\right)^{\frac{2-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^2}{(5+n)(c-acx)^{7/2}} + \frac{3a^2\left(1-\frac{1}{ax}\right)^{\frac{4-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x^3}{2\left(15+8n+n^2\right)(c-acx)^{7/2}} - \frac{3a^2\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3+n}{2}}(1-}{2}$$

Mathematica [A]
time = 0.11, size = 138, normalized size = 0.56

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left((9+2n-3ax)(1+ax)+3(-1+ax)^2\left(\frac{-1+ax}{1+ax}\right)^{\frac{1+n}{2}}{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{3}{2};\frac{2}{1+ax}\right)\right)}{2ac^3(3+n)(5+n)(-1+ax)^2\sqrt{c-acx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a*c*x)^(7/2),x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*((9 + 2*n - 3*a*x)*(1 + a*x) + 3*(-1 + a*x)^2*((-1 + a
*x)/(1 + a*x))^((1 + n)/2)*Hypergeometric2F1[1/2, (3 + n)/2, 3/2, 2/(1 + a*
x)]))/(2*a*c^3*(3 + n)*(5 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x)^2*Sqrt[c - a*
c*x])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n \operatorname{arccoth}(ax)}}{(-acx+c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*c*x + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^4*x^4 - 4*a^
3*c^4*x^3 + 6*a^2*c^4*x^2 - 4*a*c^4*x + c^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a*c*x+c)**(7/2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a*c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a*c*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{(c-a\,c\,x)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - a*c*x)^(7/2), x)
```

3.379 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=114

$$-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x-\frac{c^4\csc^{-1}(ax)}{2a}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -1/3*c^4*(1-1/a^2/x^2)^(3/2)/a+c^4*(1-1/a^2/x^2)^(3/2)*x-1/2*c^4*arccsc(a*x
)/a-3*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(6*a-1/x)*(1-1/a^2/x^2)^(1
/2)/a^2
```

Rubi [A]
time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.450$, Rules used = {6312, 1821, 1823, 829, 858, 222, 272, 65, 214}

$$-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2}+c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] -1/3*(c^4*(1 - 1/(a^2*x^2))^(3/2))/a + (c^4*Sqrt[1 - 1/(a^2*x^2)]*(6*a - x^
(-1)))/(2*a^2) + c^4*(1 - 1/(a^2*x^2))^(3/2)*x - (c^4*ArcCsc[a*x])/(2*a) -
(3*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\left(c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2} x + c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}\left(\frac{3c^3}{a}-\frac{c^3x}{a^2}+\frac{c^3x^2}{a^3}\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2} x - \frac{1}{3}\left(a^2c\right)\text{Subst}\left(\int \frac{\left(-\frac{9c^3}{a^3}+\frac{3c^3x}{a^4}\right)\sqrt{1}}{x}\right.$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2} x + \frac{1}{6}\left(a^4c\right)\text{Su}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2} x - \frac{c^4\text{Subst}\left(\right.}{}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2} x - \frac{c^4\csc^{-1}(a}{2a}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2} x - \frac{c^4\csc^{-1}(a}{2a}$$

$$= -\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(6a-\frac{1}{x}\right)}{2a^2} + c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2} x - \frac{c^4\csc^{-1}(a}{2a}$$

Mathematica [A]
time = 0.12, size = 175, normalized size = 1.54

$$\frac{c^4\left(-2+9ax-14a^2x^2-15a^3x^3+16a^4x^4+6a^5x^5+24a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+9a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\text{ArcSin}\left(\frac{1}{ax}\right)-18a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{6a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*(-2 + 9*a*x - 14*a^2*x^2 - 15*a^3*x^3 + 16*a^4*x^4 + 6*a^5*x^5 + 24*a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 9*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 18*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(100) = 200$.
time = 0.07, size = 224, normalized size = 1.96

method	result
risch	$\frac{(ax-1)(16a^2x^2-9ax+2)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^3\sqrt{(ax+1)(ax-1)}-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right.}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^4\left(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+18(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+18\ln\left(\frac{a^2x+\sqrt{a^2x^2-}}{\sqrt{a^2}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)*c^4*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+18*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2)*a^2*x^2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+18*ln((a^
2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+3*a^3*x^3*(a^2)^(1/
2)*arctan(1/(a^2*x^2-1)^(1/2))-9*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+2*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a^
4/x^3/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(100) = 200$.
time = 0.49, size = 224, normalized size = 1.96

$$\frac{1}{3}\left(\frac{3\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-37\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-15\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^4*log(sqrt((a*x - 1)
/(a*x + 1)) + 1)/a^2 + 9*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21*c
^4*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 37*c^
4*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x +
1)^4 + a^2))*a
```

Fricas [A]
time = 0.35, size = 156, normalized size = 1.37

$$\frac{6\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^4x^4+22\,a^3c^4x^3+7\,a^2c^4x^2-7\,ac^4x+2\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^4*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^4*x^4 + 22*a^3*c^4*x^3 + 7*a^2*c^4*x^2 - 7*a*c^4*x + 2*c^
4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^3}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a/(x**3*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(6*a**2/(x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))), x) + Integral(-4*a**3/(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)
)/a**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(100) = 200.
time = 0.42, size = 248, normalized size = 2.18

$$\frac{c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\,\mathrm{sgn}\left(ax+1\right)}+\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^4}{a\,\mathrm{sgn}\left(ax+1\right)}+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4+36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-9\left(x|a|-\sqrt{a^2x^2-1}\right)c^4|a|+16\,ac^4}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 3*c^4*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
^4/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) +
12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 + 36*(x*abs(a) - sqrt(a^2*x^2 - 1
))^2*a*c^4 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) + 16*a*c^4)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.32, size = 183, normalized size = 1.61

$$\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (17*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^4*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^4
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.380 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=88

$$\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{c^3\csc^{-1}(ax)}{2a}-\frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x+1/2*c^3*arccsc(a*x)/a-2*c^3*arctanh((1-1/a^2/x^2)
^(1/2))/a+1/2*c^3*(4*a+1/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

Rubi [A]
time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {6312, 1821, 829, 858, 222, 272, 65, 214}

$$\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2}+c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}-\frac{2c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*Sqrt[1 - 1/(a^2*x^2)]*(4*a + x^(-1)))/(2*a^2) + c^3*(1 - 1/(a^2*x^2))^
(3/2)*x + (c^3*ArcCsc[a*x])/(2*a) - (2*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/
a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\left(c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + c\text{Subst}\left(\int \frac{\left(\frac{2c^2}{a}+\frac{c^2x}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x - \frac{1}{2}\left(a^2c\right)\text{Subst}\left(\int \frac{-\frac{4c^2}{a^3}-\frac{c^2x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} + \frac{c^3\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} - \left(2ac^3\right)\text{Subst}\left(\right.$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}\left(4a+\frac{1}{x}\right)}{2a^2} + c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x + \frac{c^3\csc^{-1}(ax)}{2a} - \frac{2c^3\tanh^{-1}\left(\sqrt{}\right.}{a}$$

<u>Mathematica</u> [A]
time = 0.10, size = 167, normalized size = 1.90

$$\frac{c^3\left(1-4ax-3a^2x^2+4a^3x^3+2a^4x^4+2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\text{ArcSin}\left(\frac{1}{ax}\right)-4a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{2a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(1 - 4*a*x - 3*a^2*x^2 + 4*a^3*x^3 + 2*a^4*x^4 + 2*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 2*a^3*Sqrt[1 - 1/(a^2*x^2)]*
x^3*ArcSin[1/(a*x)] - 4*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(78) = 156$.
time = 0.06, size = 200, normalized size = 2.27

method	result
risch	$\frac{(ax-1)(2a^2x^2+4ax-1)c^3}{2x^2a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(-\frac{2a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)c^3\sqrt{(ax+1)(ax-1)}}{a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^3\left(-4\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+4\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+4\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{}}{\sqrt{a^2}}\right)\right.}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,a^3x^2\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x-1)*c^3*(-4*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+4*(a^2)^(1/2)*(a
^2*x^2-1)^(3/2)*a*x-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+4*ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2)^(1/2)*arctan(1/(a^2*x^
2-1)^(1/2))*a^2*x^2-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/
((a*x+1)*(a*x-1))^(1/2)/a^3/x^2/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(78) = 156$.
time = 0.50, size = 201, normalized size = 2.28

$$-\left(\frac{c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c^3*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1)/a^2 - 2*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (3*c^3*((a
*x - 1)/(a*x + 1))^(5/2) - 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqrt((
a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2
 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Fricas [A]
time = 0.36, size = 146, normalized size = 1.66

$$-\frac{2\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^3x^3+6\,a^2c^3x^2+3\,ac^3x-c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*a^2*c^3*x^2*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 4*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) - (2*a^3*c^3*x^3 + 6*a^2*c^3*x^2 + 3*a*c^3*x - c^3)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{a^3}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^2}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(a**3/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a/(x**2*sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**2/(x*sqrt(a*x/(a*x + 1) - 1/(a*
x + 1))), x))/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(78) = 156.
time = 0.42, size = 221, normalized size = 2.51

$$-\frac{c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\,\mathrm{sgn}\left(ax+1\right)}+\frac{2\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^3}{a\,\mathrm{sgn}\left(ax+1\right)}+\frac{\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|+4\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3-\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|+4\,ac^3}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^2a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + 2*c^3*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^3/(a*sgn(a*x + 1)) + ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a) + 4*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - (x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*ab
s(a) + 4*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a)*sgn(a*x
+ 1))
```

Mupad [B]
time = 0.11, size = 163, normalized size = 1.85

$$\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-3\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 3*
c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x - 1
)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c^3*atan(((a*x - 1)/(a*x
 + 1))^(1/2)))/a - (4*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.381 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=62

$$\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a+1/x)*x*(1-1/a^2
/x^2)^(1/2)/a
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.350$, Rules used = {6312, 827, 858, 222, 272, 65, 214}

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)}{a}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a + x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a - (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{1}{2}c\text{Subst}\left(\int \frac{\frac{2c}{a}+\frac{2cx}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a^2} + \frac{c^2\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - (ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}dx, x, \sqrt{1\right.$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a+\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - \frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(62) = 124.
time = 0.11, size = 158, normalized size = 2.55

$$\frac{c^2\left(-2-2ax+2a^2x^2+2a^3x^3-2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2\text{ArcSin}\left(\frac{1}{ax}\right)-2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*(-2 - 2*a*x + 2*a^2*x^2 + 2*a^3*x^3 - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*
ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/
```

```
(a*x)] - 2*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(
2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(58) = 116$.
time = 0.06, size = 168, normalized size = 2.71

method	result
risch	$\frac{(ax-1)c^2}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a\sqrt{(ax+1)(ax-1)}-\frac{a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{(ax+}}{a^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^{}}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,a^2x\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x-1)*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)
^(1/2))/(a^2)^(1/2))*a^2*x-(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x)/((a
*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(58) = 116$.
time = 0.49, size = 125, normalized size = 2.02

$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)
) + 1)/a^2 - c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(58) = 116$.

time = 0.36, size = 119, normalized size = 1.92

$$-\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2+2\,ac^2x+c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 + 2*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a/(x*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x))/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.
time = 0.41, size = 137, normalized size = 2.21

$$-\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\,(ax+1)}+\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}\,c^2}{a\mathrm{sgn}\,(ax+1)}+\frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^2/(a*sgn(a*x + 1)) + 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
*sgn(a*x + 1))
```

Mupad [B]
time = 1.22, size = 90, normalized size = 1.45

$$\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

3.382 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=27

$$c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\csc^{-1}(ax)}{a}$$

[Out] c*arccsc(a*x)/a+c*x*(1-1/a^2/x^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6312, 283, 222}

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]*(c - c/(a*x)),x]

[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x + (c*ArcCsc[a*x])/a

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx=-\left(c\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{a^2}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\csc^{-1}(ax)}{a}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 1.15

$$\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x+\text{ArcSin}\left(\frac{1}{ax}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x)),x]
```

```
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x + ArcSin[1/(a*x)]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25)=50$.
time = 0.03, size = 63, normalized size = 2.33

method	result	size
default	$\frac{(ax-1)c\left(\sqrt{a^2x^2-1}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,a}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/((a*x+1)*(a*x-1))^(1/2)*c/a*((a^2*x^2-1)^
(1/2)+arctan(1/(a^2*x^2-1)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.
time = 0.49, size = 66, normalized size = 2.44

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2)
```

Fricas [A]
time = 0.35, size = 48, normalized size = 1.78

$$-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - (a*c*x + c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{a}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))), x))/a
```

Giac [A]
time = 0.40, size = 42, normalized size = 1.56

$$-\frac{c\arctan\left(\sqrt{a^2x^2-1}\right)-\sqrt{a^2x^2-1}\,c}{a\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] -(c*arctan(sqrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)*c)/(a*sgn(a*x + 1))
```

Mupad [B]
time = 0.06, size = 60, normalized size = 2.22

$$\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*atan
(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.383 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}} \, dx$

Optimal. Leaf size=70

$$-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] 2*arctanh((1-1/a^2/x^2)^(1/2))/a/c-2*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(1
-1/a^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (-2*(a + x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)
/c + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^2-\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{(2a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A]

time = 0.06, size = 63, normalized size = 0.90

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-3+ax)+2(-1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac(-1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x)),x]
```

```
[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3 + a*x) + 2*(-1 + a*x)*Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c*(-1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(64) = 128.
time = 0.10, size = 250, normalized size = 3.57

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a\sqrt{a^2}}-\frac{2\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{a^3\left(x-\frac{1}{a}\right)}\right)a\sqrt{(ax+1)(ax-1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{2\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+2\sqrt{(ax+1)(ax-1)}\sqrt{a^2}\,a^2x^2-4\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1}}{\sqrt{a^2}}\right.}{a\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] (2*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+2*((
a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-4*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-4*((
a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a
*x-1))^(1/2))/(a^2)^(1/2))+2*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/a/(a^2)^(
1/2)/(a*x-1)/c/((a*x+1)*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.27, size = 116, normalized size = 1.66

$$-2\,a\left(\frac{\frac{2\,(ax-1)}{ax+1}-1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*((2*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Fricas [A]
time = 0.34, size = 94, normalized size = 1.34

$$\frac{2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-2\,ax-3)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*(a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2*(a*x - 1)*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) + (a^2*x^2 - 2*a*x - 3)*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c*x - a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int\frac{x}{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.22, size = 62, normalized size = 0.89

$$\frac{2\,a\,x+8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-6}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (2*a*x + 8*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 6)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.384 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -4/3*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c
^2+1/3*(-9*a-11/x)/a^2/c^2/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (-4*(a + x^(-1)))/(3*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (9*a + 11/x)/(3*a^2
*c^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (3*ArcTanh[Sq
rt[1 - 1/(a^2*x^2)]])/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^3}\,dx,x,\frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^3-\frac{9c^3x}{a}-\frac{8c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^3+\frac{9c^3x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x\right.}{ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x\right.}{2ac^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(3a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,x,\right.}{c^2}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{3a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{9a+\frac{11}{x}}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A]
time = 0.04, size = 94, normalized size = 0.90

$$\frac{14-5ax-16a^2x^2+3a^3x^3+9a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^2,x]
```

```
[Out] (14 - 5*a*x - 16*a^2*x^2 + 3*a^3*x^3 + 9*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(93) = 186$.
time = 0.11, size = 339, normalized size = 3.23

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{13\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{3a^4\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{3a^5\left(x-\frac{1}{a}\right)^2}\right.}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-9\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3+6\sqrt{a^2}\,((ax+1)(ax-1))^{\frac{3}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-9*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-9*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+6*(a^2)^(1/2)*((a*x+1)*(a*x
-1))^(3/2)*a*x+27*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+27*ln((a^2*x+
(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-5*((a*x+1)*(a*x-1
))^(3/2)*(a^2)^(1/2)-27*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-27*ln((a^2*
x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+9*(a^2)^(1/2)*((a
*x+1)*(a*x-1))^(1/2)+9*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^
2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^2/c^2/((a*x+1)*(a*x-1))^(1/2)/((a*x-1)/(a*
x+1))^(1/2)
```

Maxima [A]
time = 0.27, size = 137, normalized size = 1.30

$$\frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c^2))
```

Fricas [A]
time = 0.35, size = 134, normalized size = 1.28

$$\frac{9\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-16\,a^2x^2-5\,ax+14)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a*x*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**
2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.10, size = 104, normalized size = 0.99

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}-\frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)}-\frac{6\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((11*(a*x - 1))/(3*(a*x +
1)) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c^2*((a*x - 1)/(a*x + 1))^(3/2)
 - a*c^2*((a*x - 1)/(a*x + 1))^(5/2))
```

3.385 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] -8/5*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+8/x)/a^2/c^3/(1-1/a^2/x^
2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c^3+1/15*(-60*a-79/x)/a^2/c^3/(1-
1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

Rubi [A]
time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-8*(a + x^(-1)))/(5*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 8/x))/(15
*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (60*a + 79/x)/(15*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]])/(a*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^4-\frac{20c^4x}{a}-\frac{27c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^4+\frac{60c^4x}{a}+\frac{64c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^4-\frac{60c^4}{a}\cdots}{x^2\sqrt{1-\frac{x}{a}\cdots}}\cdots\right.}{15c^7}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{4\text{S}\cdots}{\cdots}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} - \frac{2\text{S}\cdots}{\cdots}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{(4\cdots}{\cdots}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{5a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{8}{x}\right)}{15a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{60a+\frac{79}{x}}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3} + \frac{4\,\text{t}\cdots}{\cdots}$$

Mathematica [A]
time = 0.05, size = 104, normalized size = 0.75

$$\frac{-94+128ax+73a^2x^2-134a^3x^3+15a^4x^4+60a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^3,x]
```

```
[Out] (-94 + 128*a*x + 73*a^2*x^2 - 134*a^3*x^3 + 15*a^4*x^4 + 60*a*Sqrt[1 - 1/(a
^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^3*Sqrt[1
- 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(122) = 244$.
time = 0.12, size = 431, normalized size = 3.12

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{104\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{15a^5\left(x-\frac{1}{a}\right)}-\frac{31\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{15a^6\left(x-\frac{1}{a}\right)^2}\right.}{c^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-60\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^4x^4-60\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^5x^4+45\sqrt{a^2}\,((ax+1)(ax-1)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-60*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-60*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+45*(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(3/2)*a^2*x^2+240*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+240*
ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-76*(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-360*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)
*a^2*x^2-360*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^
3*x^2+34*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+240*((a*x+1)*(a*x-1))^(1/2)*(a
^2)^(1/2)*a*x+240*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2
))*a^2*x-60*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-60*a*ln((a^2*x+(a^2)^(1/2)*
((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^3/c^3/((a*x+1)
*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.26, size = 153, normalized size = 1.11

$$\frac{1}{30}\,a\left(\frac{\frac{22\,(ax-1)}{ax+1}+\frac{155\,(ax-1)^2}{(ax+1)^2}-\frac{240\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{120\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*a*((22*(a*x - 1)/(a*x + 1) + 155*(a*x - 1)^2/(a*x + 1)^2 - 240*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
- 1)/(a*x + 1))^(5/2)) + 120*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) -
 120*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A]
time = 0.34, size = 170, normalized size = 1.23

$$\frac{60\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-60\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a^4x^4-134\,a^3x^3+73\,a^2x^2+128\,ax-94)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 60*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 134*a^3*x^3 + 73*a^2*x^2 + 128*a*x - 94)*sqrt((a*x - 1)/(
a*x + 1)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\int\frac{x^3}{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x +
1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Giac [A]
time = 0.42, size = 63, normalized size = 0.46

$$-\frac{4\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^3|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac^3\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^3*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^3*sgn(a*x + 1))
```

Mupad [B]
time = 1.22, size = 121, normalized size = 0.88

$$\frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{31\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{16\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{22\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((31*(a*x - 1)^2)/(3*(a*x
+ 1)^2) - (16*(a*x - 1)^3)/(a*x + 1)^3 + (22*(a*x - 1))/(15*(a*x + 1)) + 1/
5)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(7/
2))
```

3.386 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=171

$$-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{5}{}$$

```
[Out] -16/7*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-4/35*(7*a+17/x)/a^2/c^4/(1-1/a^2/
x^2)^(5/2)+1/105*(-175*a-307/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+5*arctanh((1-1/
a^2/x^2)^(1/2))/a/c^4+1/105*(-525*a-719/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x*(1
-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A]
time = 0.35, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (-16*(a + x^(-1)))/(7*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (4*(7*a + 17/x))/(
35*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (175*a + 307/x)/(105*a^2*c^4*(1 - 1/(
a^2*x^2))^(3/2)) - (525*a + 719/x)/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (S
qrt[1 - 1/(a^2*x^2)]*x)/c^4 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^5-\frac{35c^5x}{a}-\frac{61c^5x^2}{a^2}+\frac{7c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{35c^5+\frac{175c^5x}{a}+\frac{272c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^9}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-105c^5-5\ldots}{x^2(1\ldots}\right.}{1\ldots}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x\ldots}}}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x\ldots}}}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x\ldots}}}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{105a^2c^4\sqrt{1-\frac{1}{a^2x\ldots}}}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{7a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{4\left(7a+\frac{17}{x}\right)}{35a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{175a+\frac{307}{x}}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{525a+\frac{719}{x}}{\ldots}$$

Mathematica [A]
time = 0.06, size = 112, normalized size = 0.65

$$\frac{824-1947ax+485a^2x^2+1812a^3x^3-1339a^4x^4+105a^5x^5+525a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{105a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^4,x]
```

```
[Out] (824 - 1947*a*x + 485*a^2*x^2 + 1812*a^3*x^3 - 1339*a^4*x^4 + 105*a^5*x^5 +
 525*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])
/(105*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(151) = 302$.
time = 0.12, size = 523, normalized size = 3.06

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{1024\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{105a^6\left(x-\frac{1}{a}\right)}-\frac{2\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{7a^9\left(x-\frac{1}{a}\right)^4}\right.}{c}$
default	$-\frac{-525\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^5x^5-525\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^6x^5+420((ax+1)(ax-1))^{\frac{3}{2}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(-525*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5-525*ln((a^2*x+(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+420*((a*x+1)*(a*x-1))
^(3/2)*(a^2)^(1/2)*a^3*x^3+2625*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4
+2625*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4-1
076*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2-5250*(a^2)^(1/2)*((a*x+1)*(
a*x-1))^(1/2)*a^3*x^3-5250*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(
a^2)^(1/2))*a^4*x^3+970*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+5250*((a*x+
1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+5250*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(
a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-299*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)
-2625*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-2625*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+525*(a^2)^(1/2)*((a*x+1)*(a*x-1))
```

```
^(1/2)+525*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a
/(a^2)^(1/2)/(a*x-1)^4/c^4/((a*x+1)*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.26, size = 169, normalized size = 0.99

$$\frac{1}{420}\,a\left(\frac{\frac{111\,(ax-1)}{ax+1}+\frac{469\,(ax-1)^2}{(ax+1)^2}+\frac{2765\,(ax-1)^3}{(ax+1)^3}-\frac{4200\,(ax-1)^4}{(ax+1)^4}+15}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{2100\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{2100\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/420*a*((111*(a*x - 1)/(a*x + 1) + 469*(a*x - 1)^2/(a*x + 1)^2 + 2765*(a*x
 - 1)^3/(a*x + 1)^3 - 4200*(a*x - 1)^4/(a*x + 1)^4 + 15)/(a^2*c^4*((a*x - 1
)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 2100*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 2100*log(sqrt((a*x - 1)/(a*x + 1)) - 1
)/(a^2*c^4))
```

Fricas [A]
time = 0.37, size = 204, normalized size = 1.19

$$\frac{525\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-525\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(105\,a^5x^5-1339\,a^4x^4+1812\,a^3x^3+485\,a^2x^2-1947\,ax+824)\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) - 525*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sq
rt((a*x - 1)/(a*x + 1)) - 1) + (105*a^5*x^5 - 1339*a^4*x^4 + 1812*a^3*x^3 +
 485*a^2*x^2 - 1947*a*x + 824)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^4*x^4 - 4*
a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**3*x**
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
```

```
a*x + 1)) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x)/c**4
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.11, size = 137, normalized size = 0.80

$$\frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{67\,{\left(a\,x-1\right)}^2}{15\,{\left(a\,x+1\right)}^2}+\frac{79\,{\left(a\,x-1\right)}^3}{3\,{\left(a\,x+1\right)}^3}-\frac{40\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}+\frac{37\,\left(a\,x-1\right)}{35\,\left(a\,x+1\right)}+\frac{1}{7}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}-4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((67*(a*x - 1)^2)/(15*(a*
x + 1)^2) + (79*(a*x - 1)^3)/(3*(a*x + 1)^3) - (40*(a*x - 1)^4)/(a*x + 1)^4
 + (37*(a*x - 1))/(35*(a*x + 1)) + 1/7)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2
) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2))
```

3.387 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=61

$$-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}$$

```
[Out] -1/4*c^5/a^5/x^4+c^5/a^4/x^3-c^5/a^3/x^2-2*c^5/a^2/x+c^5*x-3*c^5*ln(x)/a
```

Rubi [A]

time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 76}

$$-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}-\frac{3c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] -1/4*c^5/(a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*
x - (3*c^5*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx \\
&= \frac{c^5\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^5}{x^5}\,dx}{a^5} \\
&= \frac{c^5\int\frac{(1-ax)^4(1+ax)}{x^5}\,dx}{a^5} \\
&= \frac{c^5\int\left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5} \\
&= -\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(x)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 1.03

$$
-\frac{c^5}{4a^5x^4}+\frac{c^5}{a^4x^3}-\frac{c^5}{a^3x^2}-\frac{2c^5}{a^2x}+c^5x-\frac{3c^5\log(ax)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] -1/4*c^5/(a^5*x^4) + c^5/(a^4*x^3) - c^5/(a^3*x^2) - (2*c^5)/(a^2*x) + c^5*
x - (3*c^5*Log[a*x])/a
```

Maple [A]
time = 0.18, size = 47, normalized size = 0.77

method	result
default	$\frac{c^5\left(a^5x-\frac{1}{4x^4}-\frac{2a^3}{x}-\frac{a^2}{x^2}-3a^4\ln(x)+\frac{a}{x^3}\right)}{a^5}$
risch	$c^5x+\frac{-2a^3c^5x^3-a^2c^5x^2+a\,c^5x-\frac{1}{4}c^5}{a^5x^4}-\frac{3c^5\ln(x)}{a}$
norman	$\frac{c^5x+a^4c^5x^5-\frac{c^5}{4a}-c^5a\,x^2-2c^5a^2x^3}{a^4x^4}-\frac{3c^5\ln(x)}{a}$
meijerg	$-\frac{c^5(-ax-\ln(-ax+1))}{a}-\frac{4c^5\ln(-ax+1)}{a}-\frac{5c^5(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{5c^5\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^5/a^5*(a^5*x-1/4/x^4-2*a^3/x-a^2/x^2-3*a^4*ln(x)+a/x^3)
```

Maxima [A]
time = 0.26, size = 57, normalized size = 0.93

$$c^5x-\frac{3\,c^5\log\left(x\right)}{a}-\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - 3*c^5*log(x)/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*x + c
^5)/(a^5*x^4)
```

Fricas [A]
time = 0.34, size = 67, normalized size = 1.10

$$\frac{4\,a^5c^5x^5-12\,a^4c^5x^4\log\left(x\right)-8\,a^3c^5x^3-4\,a^2c^5x^2+4\,ac^5x-c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) - 8*a^3*c^5*x^3 - 4*a^2*c^5*x^2
+ 4*a*c^5*x - c^5)/(a^5*x^4)
```

Sympy [A]
time = 0.13, size = 63, normalized size = 1.03

$$\frac{a^5c^5x-3a^4c^5\log\left(x\right)+\frac{-8a^3c^5x^3-4a^2c^5x^2+4ac^5x-c^5}{4x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - 3*a**4*c**5*log(x) + (-8*a**3*c**5*x**3 - 4*a**2*c**5*x**2 +
 4*a*c**5*x - c**5)/(4*x**4))/a**5
```

Giac [A]
time = 0.41, size = 58, normalized size = 0.95

$$c^5x-\frac{3\,c^5\log\left(|x|\right)}{a}-\frac{8\,a^3c^5x^3+4\,a^2c^5x^2-4\,ac^5x+c^5}{4\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*x - 3*c^5*log(abs(x))/a - 1/4*(8*a^3*c^5*x^3 + 4*a^2*c^5*x^2 - 4*a*c^5*
x + c^5)/(a^5*x^4)
```

Mupad [B]
time = 0.07, size = 51, normalized size = 0.84

$$-\frac{c^5\,\left(4\,a^2\,x^2-4\,a\,x+8\,a^3\,x^3-4\,a^5\,x^5+12\,a^4\,x^4\,\ln\left(x\right)+1\right)}{4\,a^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^5*(a*x + 1))/(a*x - 1),x)
[Out] -(c^5*(4*a^2*x^2 - 4*a*x + 8*a^3*x^3 - 4*a^5*x^5 + 12*a^4*x^4*log(x) + 1))/
(4*a^5*x^4)
```

3.388 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=40

$$\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}$$

[Out] 1/3*c^4/a^4/x^3-c^4/a^3/x^2+c^4*x-2*c^4*ln(x)/a

Rubi [A]
time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6266, 6264, 76}

$$\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]

[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[x])/a

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx$$
$$= -\frac{c^4\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^4}{x^4}dx}{a^4}$$
$$= -\frac{c^4\int \frac{(1-ax)^3(1+ax)}{x^4}dx}{a^4}$$
$$= -\frac{c^4\int \left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}$$
$$= \frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(x)}{a}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 1.05

$$\frac{c^4}{3a^4x^3}-\frac{c^4}{a^3x^2}+c^4x-\frac{2c^4\log(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^4,x]
[Out] c^4/(3*a^4*x^3) - c^4/(a^3*x^2) + c^4*x - (2*c^4*Log[a*x])/a
```

Maple [A]
time = 0.17, size = 32, normalized size = 0.80

method	result
default	$\frac{c^4\left(a^4x-\frac{a}{x^2}-2a^3\ln(x)+\frac{1}{3x^3}\right)}{a^4}$
risch	$c^4x+\frac{-a\,c^4x+\frac{1}{3}c^4}{a^4x^3}-\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^3c^4x^4+\frac{c^4}{3a}-c^4x}{a^3x^3}-\frac{2c^4\ln(x)}{a}$
meijerg	$-\frac{c^4(-ax-\ln(-ax+1))}{a}-\frac{3c^4\ln(-ax+1)}{a}-\frac{2c^4(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{2c^4\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{3c^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
[Out] c^4/a^4*(a^4*x-a/x^2-2*a^3*ln(x)+1/3/x^3)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.92

$$c^4x-\frac{2\,c^4\log(x)}{a}-\frac{3\,ac^4x-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x - 2*c^4*log(x)/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Fricas [A]
time = 0.34, size = 43, normalized size = 1.08

$$\frac{3\,a^4c^4x^4 - 6\,a^3c^4x^3\log\left(x\right) - 3\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 - 6*a^3*c^4*x^3*log(x) - 3*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A]
time = 0.09, size = 39, normalized size = 0.98

$$\frac{a^4c^4x - 2a^3c^4\log\left(x\right) + \frac{-3ac^4x+c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x - 2*a**3*c**4*log(x) + (-3*a*c**4*x + c**4)/(3*x**3))/a**4
```

Giac [A]
time = 0.43, size = 38, normalized size = 0.95

$$c^4x - \frac{2\,c^4\log\left(|x|\right)}{a} - \frac{3\,ac^4x - c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] c^4*x - 2*c^4*log(abs(x))/a - 1/3*(3*a*c^4*x - c^4)/(a^4*x^3)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.88

$$-\frac{c^4\,\left(3\,a\,x - 3\,a^4\,x^4 + 6\,a^3\,x^3\,\ln\left(x\right) - 1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] -(c^4*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

3.389 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=39

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}$$

```
[Out] -1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x-c^3*ln(x)/a
```

Rubi [A]
time = 0.09, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 76}

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}-\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] -1/2*c^3/(a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx \\
&= \frac{c^3\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3} \\
&= \frac{c^3\int \frac{(1-ax)^2(1+ax)}{x^3}dx}{a^3} \\
&= \frac{c^3\int \left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3} \\
&= -\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(x)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 1.05

$$-\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{c^3\log(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] -1/2*c^3/(a^3*x^2) + c^3/(a^2*x) + c^3*x - (c^3*Log[a*x])/a
```

Maple [A]
time = 0.14, size = 31, normalized size = 0.79

method	result
default	$\frac{c^3\left(a^3x+\frac{a}{x}-\frac{1}{2x^2}-a^2\ln(x)\right)}{a^3}$
risch	$c^3x+\frac{a\,c^3x-\frac{1}{2}c^3}{a^3x^2}-\frac{c^3\ln(x)}{a}$
norman	$\frac{c^3x+a^2c^3x^3-\frac{c^3}{2a}}{a^2x^2}-\frac{c^3\ln(x)}{a}$
meijerg	$-\frac{c^3(-ax-\ln(-ax+1))}{a}-\frac{2c^3\ln(-ax+1)}{a}+\frac{2c^3\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{c^3\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^3*(a^3*x+a/x-1/2/x^2-a^2*ln(x))
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.95

$$c^3x-\frac{c^3\log(x)}{a}+\frac{2\,ac^3x-c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] c^3*x - c^3*log(x)/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Fricas [A]
time = 0.35, size = 45, normalized size = 1.15

$$\frac{2\,a^3c^3x^3 - 2\,a^2c^3x^2\log\left(x\right) + 2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 - 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A]
time = 0.08, size = 37, normalized size = 0.95

$$\frac{a^3c^3x - a^2c^3\log\left(x\right) + \frac{2ac^3x-c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**3,x)
```

```
[Out] (a**3*c**3*x - a**2*c**3*log(x) + (2*a*c**3*x - c**3)/(2*x**2))/a**3
```

Giac [A]
time = 0.41, size = 38, normalized size = 0.97

$$c^3x - \frac{c^3\log\left(|x|\right)}{a} + \frac{2\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] c^3*x - c^3*log(abs(x))/a + 1/2*(2*a*c^3*x - c^3)/(a^3*x^2)
```

Mupad [B]
time = 1.18, size = 35, normalized size = 0.90

$$\frac{c^3\,\left(2\,a\,x + 2\,a^3\,x^3 - 2\,a^2\,x^2\,\ln\left(x\right) - 1\right)}{2\,a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^3*(2*a*x + 2*a^3*x^3 - 2*a^2*x^2*log(x) - 1))/(2*a^3*x^2)
```

3.390 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=16

$$\frac{c^2}{a^2x}+c^2x$$

[Out] c^2/a^2/x+c^2*x

Rubi [A]
time = 0.08, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6302, 6266, 6264, 74, 14}

$$\frac{c^2}{a^2x}+c^2x$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]

[Out] c^2/(a^2*x) + c^2*x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx\\ &= -\frac{c^2\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^2}{x^2}\,dx}{a^2}\\ &= -\frac{c^2\int \frac{(1-ax)(1+ax)}{x^2}\,dx}{a^2}\\ &= -\frac{c^2\int \frac{1-a^2x^2}{x^2}\,dx}{a^2}\\ &= -\frac{c^2\int \left(-a^2+\frac{1}{x^2}\right)\,dx}{a^2}\\ &= \frac{c^2}{a^2x}+c^2x\end{aligned}$$

Mathematica [A]
time = 0.04, size = 16, normalized size = 1.00

$$\frac{c^2}{a^2x}+c^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^2,x]
[Out] c^2/(a^2*x) + c^2*x
```

Maple [A]
time = 0.13, size = 17, normalized size = 1.06

method	result	size
default	$\frac{c^2\left(a^2x+\frac{1}{x}\right)}{a^2}$	17
risch	$\frac{c^2}{a^2x}+c^2x$	17
gosper	$\frac{c^2\left(a^2x^2+1\right)}{x\,a^2}$	20
norman	$\frac{\frac{c^2}{a}+a\,c^2x^2}{ax}$	24

meijerg	$-\frac{c^2(-ax-\ln(-ax+1))}{a}-\frac{c^2\ln(-ax+1)}{a}+\frac{c^2(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{c^2\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2/a^2*(a^2*x+1/x)
```

Maxima [A]
time = 0.26, size = 16, normalized size = 1.00

$$c^2x+\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + c^2/(a^2*x)
```

Fricas [A]
time = 0.32, size = 21, normalized size = 1.31

$$\frac{a^2c^2x^2+c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^2*c^2*x^2 + c^2)/(a^2*x)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.94

$$\frac{a^2c^2x+\frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**2,x)
```

```
[Out] (a**2*c**2*x + c**2/x)/a**2
```

Giac [A]
time = 0.40, size = 16, normalized size = 1.00

$$c^2x+\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] c^2*x + c^2/(a^2*x)
```

Mupad [B]
time = 0.03, size = 19, normalized size = 1.19

$$\frac{c^2\,\left(a^2\,x^2+1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^2*(a^2*x^2 + 1))/(a^2*x)
```

3.391 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=11

$$cx+\frac{c\log(x)}{a}$$

```
[Out] c*x+c*ln(x)/a
```

Rubi [A]
time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6302, 6266, 6264, 45}

$$\frac{c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x + (c*Log[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=\frac{c\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$=\frac{c\int\frac{1+ax}{x}\,dx}{a}$$
$$=\frac{c\int\left(a+\frac{1}{x}\right)\,dx}{a}$$
$$=cx+\frac{c\log(x)}{a}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$cx+\frac{c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] c*x + (c*Log[x])/a
```

Maple [A]
time = 0.11, size = 12, normalized size = 1.09

method	result	size
default	$\frac{c(ax+\ln(x))}{a}$	12
norman	$cx+\frac{c\ln(x)}{a}$	12
risch	$cx+\frac{c\ln(x)}{a}$	12
meijerg	$-\frac{c(-ax-\ln(-ax+1))}{a}+\frac{c(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] c/a*(a*x+ln(x))
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.00

$$cx+\frac{c\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*x + c*log(x)/a
```

Fricas [A]
time = 0.33, size = 13, normalized size = 1.18

$$\frac{acx + c\log{(x)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*c*x + c*log(x))/a
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.91

$$\frac{acx + c\log{(x)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x)
```

```
[Out] (a*c*x + c*log(x))/a
```

Giac [A]
time = 0.42, size = 12, normalized size = 1.09

$$cx + \frac{c\log{(|x|)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] c*x + c*log(abs(x))/a
```

Mupad [B]
time = 1.17, size = 11, normalized size = 1.00

$$\frac{c\,(\ln{(x)} + a\,x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c*(log(x) + a*x))/a
```

3.392 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=37

$$\frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}$$

```
[Out] x/c+2/a/c/(-a*x+1)+3*ln(-a*x+1)/a/c
```

Rubi [A]

time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 78}

$$\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c + 2/(a*c*(1 - a*x)) + (3*Log[1 - a*x])/(a*c)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx\\ &= \frac{a\int \frac{e^{2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\ &= \frac{a\int \frac{x(1+ax)}{(1-ax)^2}\,dx}{c}\\ &= \frac{a\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{c}\\ &= \frac{x}{c}+\frac{2}{ac(1-ax)}+\frac{3\log(1-ax)}{ac}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.81

$$\frac{ax+\frac{2}{1-ax}+3\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x)),x]
[Out] (a*x + 2/(1 - a*x) + 3*Log[1 - a*x])/(a*c)
```

Maple [A]
time = 0.09, size = 35, normalized size = 0.95

method	result	size
default	$\frac{a\left(\frac{x}{a}-\frac{2}{a^2(ax-1)}+\frac{3\ln(ax-1)}{a^2}\right)}{c}$	35
risch	$\frac{x}{c}-\frac{2}{ac(ax-1)}+\frac{3\ln(ax-1)}{ac}$	36
norman	$\frac{\frac{a\,x^2}{c}-\frac{3x}{c}}{ax-1}+\frac{3\ln(ax-1)}{ac}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] a/c*(x/a-2/a^2/(a*x-1)+3/a^2*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.95

$$\frac{x}{c} - \frac{2}{a^2cx - ac} + \frac{3\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="maxima")
[Out] x/c - 2/(a^2*c*x - a*c) + 3*log(a*x - 1)/(a*c)
```

Fricas [A]
time = 0.34, size = 40, normalized size = 1.08

$$\frac{a^2x^2 - ax + 3(ax-1)\log(ax-1) - 2}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)/(a^2*c*x - a*c)
```

Sympy [A]
time = 0.07, size = 26, normalized size = 0.70

$$-\frac{2}{a^2cx - ac} + \frac{x}{c} + \frac{3\log(ax-1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x)
[Out] -2/(a**2*c*x - a*c) + x/c + 3*log(a*x - 1)/(a*c)
```

Giac [A]
time = 0.41, size = 36, normalized size = 0.97

$$\frac{x}{c} + \frac{3\log(|ax-1|)}{ac} - \frac{2}{(ax-1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x),x, algorithm="giac")
[Out] x/c + 3*log(abs(a*x - 1))/(a*c) - 2/((a*x - 1)*a*c)
```

Mupad [B]
time = 0.06, size = 34, normalized size = 0.92

$$\frac{x}{c}+\frac{2}{a\,\left(c-a\,c\,x\right)}+\frac{3\,\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))*(a*x - 1)),x)
[Out] x/c + 2/(a*(c - a*c*x)) + (3*log(a*x - 1))/(a*c)
```

3.393 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=53

$$\frac{x}{c^2}-\frac{1}{ac^2(1-ax)^2}+\frac{5}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

```
[Out] x/c^2-1/a/c^2/(-a*x+1)^2+5/a/c^2/(-a*x+1)+4*ln(-a*x+1)/a/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 78}

$$\frac{5}{ac^2(1-ax)}-\frac{1}{ac^2(1-ax)^2}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 - 1/(a*c^2*(1 - a*x)^2) + 5/(a*c^2*(1 - a*x)) + (4*Log[1 - a*x])/(a*c
^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx\\ &= -\frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\ &= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c^2}\\ &= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c^2}\\ &= \frac{x}{c^2}-\frac{1}{ac^2(1-ax)^2}+\frac{5}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.96

$$\frac{x}{c^2}-\frac{1}{ac^2(-1+ax)^2}-\frac{5}{ac^2(-1+ax)}+\frac{4\log(1-ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^2,x]
[Out] x/c^2 - 1/(a*c^2*(-1 + a*x)^2) - 5/(a*c^2*(-1 + a*x)) + (4*Log[1 - a*x])/(a
*c^2)
```

Maple [A]
time = 0.10, size = 49, normalized size = 0.92

method	result	size
risch	$\frac{x}{c^2}+\frac{-5c^2x+\frac{4c^2}{a}}{c^4(ax-1)^2}+\frac{4\ln(ax-1)}{a\,c^2}$	47
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{5}{a^3(ax-1)}-\frac{1}{a^3(ax-1)^2}+\frac{4\ln(ax-1)}{a^3}\right)}{c^2}$	49
norman	$\frac{\frac{a^2x^3}{c}-\frac{6a\,x^2}{c}+\frac{4x}{c}}{(ax-1)^2c}+\frac{4\ln(ax-1)}{a\,c^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c^2*(x/a^2-5/a^3/(a*x-1)-1/a^3/(a*x-1)^2+4/a^3*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.04

$$-\frac{5\,ax-4}{a^3c^2x^2-2\,a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\,\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 + 4*log(a*x - 1)/(
a*c^2)
```

Fricas [A]
time = 0.34, size = 70, normalized size = 1.32

$$\frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log\left(ax-1\right)+4}{a^3c^2x^2-2\,a^2c^2x+ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A]
time = 0.14, size = 49, normalized size = 0.92

$$\frac{-5ax+4}{a^3c^2x^2-2a^2c^2x+ac^2}+\frac{x}{c^2}+\frac{4\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**2,x)
```

```
[Out] (-5*a*x + 4)/(a**3*c**2*x**2 - 2*a**2*c**2*x + a*c**2) + x/c**2 + 4*log(a*x
 - 1)/(a*c**2)
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.79

$$\frac{x}{c^2}+\frac{4\,\log\left(|ax-1|\right)}{ac^2}-\frac{5\,ax-4}{\left(ax-1\right)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 + 4*log(abs(a*x - 1))/(a*c^2) - (5*a*x - 4)/((a*x - 1)^2*a*c^2)
```

Mupad [B]
time = 1.21, size = 54, normalized size = 1.02

$$\frac{x}{c^2} - \frac{5\,x - \frac{4}{a}}{a^2\,c^2\,x^2 - 2\,a\,c^2\,x + c^2} + \frac{4\,\ln\left(a\,x - 1\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^2*(a*x - 1)),x)
```

```
[Out] x/c^2 - (5*x - 4/a)/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (4*log(a*x - 1))/(a*c
^2)
```

3.394 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=73

$$\frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}$$

```
[Out] x/c^3+2/3/a/c^3/(-a*x+1)^3-7/2/a/c^3/(-a*x+1)^2+9/a/c^3/(-a*x+1)+5*ln(-a*x+
1)/a/c^3
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 78}

$$\frac{9}{ac^3(1-ax)}-\frac{7}{2ac^3(1-ax)^2}+\frac{2}{3ac^3(1-ax)^3}+\frac{5\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 + 2/(3*a*c^3*(1 - a*x)^3) - 7/(2*a*c^3*(1 - a*x)^2) + 9/(a*c^3*(1 - a
*x)) + (5*Log[1 - a*x])/(a*c^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx\\ &= \frac{a^3\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}\\ &= \frac{a^3\int \frac{x^3(1+ax)}{(1-ax)^4}\,dx}{c^3}\\ &= \frac{a^3\int \left(\frac{1}{a^3}+\frac{2}{a^3(-1+ax)^4}+\frac{7}{a^3(-1+ax)^3}+\frac{9}{a^3(-1+ax)^2}+\frac{5}{a^3(-1+ax)}\right)dx}{c^3}\\ &= \frac{x}{c^3}+\frac{2}{3ac^3(1-ax)^3}-\frac{7}{2ac^3(1-ax)^2}+\frac{9}{ac^3(1-ax)}+\frac{5\log(1-ax)}{ac^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.86

$$\frac{-37+81ax-36a^2x^2-18a^3x^3+6a^4x^4+30(-1+ax)^3\log(1-ax)}{6ac^3(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^3,x]
[Out] (-37 + 81*a*x - 36*a^2*x^2 - 18*a^3*x^3 + 6*a^4*x^4 + 30*(-1 + a*x)^3*Log[1
 - a*x])/(6*a*c^3*(-1 + a*x)^3)
```

Maple [A]
time = 0.09, size = 61, normalized size = 0.84

method	result	size
risch	$\frac{x}{c^3}+\frac{-9c^3a\,x^2+\frac{29c^3x}{2}-\frac{37c^3}{6a}}{c^6(ax-1)^3}+\frac{5\ln(ax-1)}{c^3a}$	56
default	$\frac{a^3\left(\frac{x}{a^3}-\frac{9}{a^4(ax-1)}-\frac{2}{3a^4(ax-1)^3}-\frac{7}{2a^4(ax-1)^2}+\frac{5\ln(ax-1)}{a^4}\right)}{c^3}$	61
norman	$\frac{\frac{a^3x^4}{c}-\frac{5x}{c}+\frac{25a\,x^2}{2c}-\frac{55a^2x^3}{6c}}{(ax-1)^3c^2}+\frac{5\ln(ax-1)}{c^3a}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/c^3*(x/a^3-9/a^4/(a*x-1)-2/3/a^4/(a*x-1)^3-7/2/a^4/(a*x-1)^2+5/a^4*ln(a
*x-1))
```

Maxima [A]
time = 0.27, size = 75, normalized size = 1.03

$$-\frac{54\,a^2x^2-87\,ax+37}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}+\frac{x}{c^3}+\frac{5\,\log\left(ax-1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(54*a^2*x^2 - 87*a*x + 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
- a*c^3) + x/c^3 + 5*log(a*x - 1)/(a*c^3)
```

Fricas [A]
time = 0.39, size = 100, normalized size = 1.37

$$\frac{6\,a^4x^4-18\,a^3x^3-36\,a^2x^2+81\,ax+30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(ax-1\right)-37}{6\,(a^4c^3x^3-3\,a^3c^3x^2+3\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^4*x^4 - 18*a^3*x^3 - 36*a^2*x^2 + 81*a*x + 30*(a^3*x^3 - 3*a^2*x^2
 + 3*a*x - 1)*log(a*x - 1) - 37)/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x
 - a*c^3)
```

Sympy [A]
time = 0.19, size = 73, normalized size = 1.00

$$\frac{-54a^2x^2+87ax-37}{6a^4c^3x^3-18a^3c^3x^2+18a^2c^3x-6ac^3}+\frac{x}{c^3}+\frac{5\log\left(ax-1\right)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**3,x)
```

```
[Out] (-54*a**2*x**2 + 87*a*x - 37)/(6*a**4*c**3*x**3 - 18*a**3*c**3*x**2 + 18*a*
*2*c**3*x - 6*a*c**3) + x/c**3 + 5*log(a*x - 1)/(a*c**3)
```

Giac [A]
time = 0.41, size = 50, normalized size = 0.68

$$\frac{x}{c^3} + \frac{5 \log (|ax - 1|)}{ac^3} - \frac{54 a^2x^2 - 87 ax + 37}{6 (ax - 1)^3 ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 + 5*log(abs(a*x - 1))/(a*c^3) - 1/6*(54*a^2*x^2 - 87*a*x + 37)/((a*x
- 1)^3*a*c^3)
```

Mupad [B]
time = 0.08, size = 71, normalized size = 0.97

$$\frac{9 a x^2 - \frac{29 x}{2} + \frac{37}{6 a}}{-a^3 c^3 x^3 + 3 a^2 c^3 x^2 - 3 a c^3 x + c^3} + \frac{x}{c^3} + \frac{5 \ln (a x - 1)}{a c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^3*(a*x - 1)),x)
```

```
[Out] (9*a*x^2 - (29*x)/2 + 37/(6*a))/(c^3 + 3*a^2*c^3*x^2 - a^3*c^3*x^3 - 3*a*c^
3*x) + x/c^3 + (5*log(a*x - 1))/(a*c^3)
```

3.395 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=87

$$\frac{x}{c^4}-\frac{1}{2ac^4(1-ax)^4}+\frac{3}{ac^4(1-ax)^3}-\frac{8}{ac^4(1-ax)^2}+\frac{14}{ac^4(1-ax)}+\frac{6\log(1-ax)}{ac^4}$$

```
[Out] x/c^4-1/2/a/c^4/(-a*x+1)^4+3/a/c^4/(-a*x+1)^3-8/a/c^4/(-a*x+1)^2+14/a/c^4/(
-a*x+1)+6*ln(-a*x+1)/a/c^4
```

Rubi [A]
time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 78}

$$\frac{14}{ac^4(1-ax)}-\frac{8}{ac^4(1-ax)^2}+\frac{3}{ac^4(1-ax)^3}-\frac{1}{2ac^4(1-ax)^4}+\frac{6\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 - 1/(2*a*c^4*(1 - a*x)^4) + 3/(a*c^4*(1 - a*x)^3) - 8/(a*c^4*(1 - a*x
)^2) + 14/(a*c^4*(1 - a*x)) + (6*Log[1 - a*x])/(a*c^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx\\
&= -\frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\
&= -\frac{a^4\int \frac{x^4(1+ax)}{(1-ax)^5}\,dx}{c^4}\\
&= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{2}{a^4(-1+ax)^5}-\frac{9}{a^4(-1+ax)^4}-\frac{16}{a^4(-1+ax)^3}-\frac{14}{a^4(-1+ax)^2}-\frac{6}{a^4(-1+ax)}\right)dx}{c^4}\\
&= \frac{x}{c^4}-\frac{1}{2ac^4(1-ax)^4}+\frac{3}{ac^4(1-ax)^3}-\frac{8}{ac^4(1-ax)^2}+\frac{14}{ac^4(1-ax)}+\frac{6\log(1-ax)}{ac^4}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 71, normalized size = 0.82

$$\frac{17-56ax+60a^2x^2-16a^3x^3-8a^4x^4+2a^5x^5+12(-1+ax)^4\log(1-ax)}{2ac^4(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^4,x]
[Out] (17 - 56*a*x + 60*a^2*x^2 - 16*a^3*x^3 - 8*a^4*x^4 + 2*a^5*x^5 + 12*(-1 + a
*x)^4*Log[1 - a*x])/(2*a*c^4*(-1 + a*x)^4)
```

Maple [A]
time = 0.09, size = 73, normalized size = 0.84

method	result	size
risch	$\frac{x}{c^4}+\frac{-14a^2c^4x^3+34c^4a\,x^2-29c^4x+\frac{17c^4}{2a}}{c^8(ax-1)^4}+\frac{6\ln(ax-1)}{c^4a}$	67
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{14}{a^5(ax-1)}-\frac{1}{2a^5(ax-1)^4}-\frac{3}{a^5(ax-1)^3}-\frac{8}{a^5(ax-1)^2}+\frac{6\ln(ax-1)}{a^5}\right)}{c^4}$	73
norman	$\frac{\frac{a^4x^5}{c}+\frac{6x}{c}-\frac{21a\,x^2}{c}+\frac{26a^2x^3}{c}-\frac{25a^3x^4}{2c}}{(ax-1)^4c^3}+\frac{6\ln(ax-1)}{c^4a}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^4*(x/a^4-14/a^5/(a*x-1)-1/2/a^5/(a*x-1)^4-3/a^5/(a*x-1)^3-8/a^5/(a*x-
1)^2+6/a^5*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 93, normalized size = 1.07

$$-\frac{28\,a^3x^3-68\,a^2x^2+58\,ax-17}{2\,\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}+\frac{x}{c^4}+\frac{6\,\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x - 17)/(a^5*c^4*x^4 - 4*a^4*c^4*x^3 +
 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4) + x/c^4 + 6*log(a*x - 1)/(a*c^4)
```

Fricas [A]
time = 0.35, size = 126, normalized size = 1.45

$$\frac{2\,a^5x^5-8\,a^4x^4-16\,a^3x^3+60\,a^2x^2-56\,ax+12\,\left(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1\right)\log\left(ax-1\right)+17}{2\,\left(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5*x^5 - 8*a^4*x^4 - 16*a^3*x^3 + 60*a^2*x^2 - 56*a*x + 12*(a^4*x^4
 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 17)/(a^5*c^4*x^4 - 4*a
^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)
```

Sympy [A]
time = 0.25, size = 94, normalized size = 1.08

$$\frac{-28a^3x^3+68a^2x^2-58ax+17}{2a^5c^4x^4-8a^4c^4x^3+12a^3c^4x^2-8a^2c^4x+2ac^4}+\frac{x}{c^4}+\frac{6\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] (-28*a**3*x**3 + 68*a**2*x**2 - 58*a*x + 17)/(2*a**5*c**4*x**4 - 8*a**4*c**
4*x**3 + 12*a**3*c**4*x**2 - 8*a**2*c**4*x + 2*a*c**4) + x/c**4 + 6*log(a*x
 - 1)/(a*c**4)
```

Giac [A]
time = 0.41, size = 58, normalized size = 0.67

$$\frac{x}{c^4} + \frac{6 \log(|ax - 1|)}{ac^4} - \frac{28 a^3 x^3 - 68 a^2 x^2 + 58 ax - 17}{2 (ax - 1)^4 ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 + 6*log(abs(a*x - 1))/(a*c^4) - 1/2*(28*a^3*x^3 - 68*a^2*x^2 + 58*a*x
 - 17)/((a*x - 1)^4*a*c^4)
```

Mupad [B]
time = 0.09, size = 90, normalized size = 1.03

$$\frac{x}{c^4} - \frac{29\,x - 34\,a\,x^2 - \frac{17}{2\,a} + 14\,a^2\,x^3}{a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 + 6\,a^2\,c^4\,x^2 - 4\,a\,c^4\,x + c^4} + \frac{6\,\ln(a\,x - 1)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^4*(a*x - 1)),x)
```

```
[Out] x/c^4 - (29*x - 34*a*x^2 - 17/(2*a) + 14*a^2*x^3)/(c^4 + 6*a^2*c^4*x^2 - 4*
a^3*c^4*x^3 + a^4*c^4*x^4 - 4*a*c^4*x) + (6*log(a*x - 1))/(a*c^4)
```

3.396 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=103

$$\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a}+\frac{3c^4\csc^{-1}(ax)}{2a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 1/3*c^4*(1-1/a^2/x^2)^(3/2)*(3*a+1/x)*x/a+3/2*c^4*arccsc(a*x)/a-c^4*arctanh
((1-1/a^2/x^2)^(1/2))/a+1/2*c^4*(2*a+3/x)*(1-1/a^2/x^2)^(1/2)/a^2
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 827, 829, 858, 222, 272, 65, 214}

$$\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2}+\frac{c^4x\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)}{3a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{3c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] (c^4*Sqrt[1 - 1/(a^2*x^2)]*(2*a + 3/x))/(2*a^2) + (c^4*(1 - 1/(a^2*x^2))^(3
/2)*(3*a + x^(-1))*x)/(3*a) + (3*c^4*ArcCsc[a*x])/(2*a) - (c^4*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\left(c^3\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{1}{2}c^3\text{Subst}\left(\int \frac{\left(\frac{2c}{a}+\frac{6cx}{a^2}\right)\sqrt{1-\frac{x^2}{a^2}}}{x}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} - \frac{1}{4}\left(a^2c^3\right)\text{Subst}\left(\int \cdots\right.$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{\left(3c^4\right)\text{Subst}\left(\int \cdots\right.}{2a^2 \cdots}$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} + \frac{c^4 \text{S}\cdots}{\cdots}$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} - (ac\cdots$$

$$= \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}\left(2a+\frac{3}{x}\right)}{2a^2} + \frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(3a+\frac{1}{x}\right)x}{3a} + \frac{3c^4\csc^{-1}(ax)}{2a} - \frac{c^4\,\text{t}\cdots}{\cdots}$$

Mathematica [A]
time = 0.15, size = 175, normalized size = 1.70

$$-\frac{c^4\left(-8+12ax+40a^2x^2+12a^3x^3-32a^4x^4-24a^5x^5+42a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-15a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\text{ArcSin}\left(\frac{1}{ax}\right)+24a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{24a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

[Out] -1/24*(c^4*(-8 + 12*a*x + 40*a^2*x^2 + 12*a^3*x^3 - 32*a^4*x^4 - 24*a^5*x^5 + 42*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 15*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] + 24*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(91) = 182.
time = 0.06, size = 233, normalized size = 2.26

method	result
risch	$\frac{(ax-1)(8a^2x^2+3ax-2)c^4}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^3\sqrt{(ax+1)(ax-1)}-\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right)c}{a^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c^4\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x,method=_RETURNVERBOSE)

[Out] -1/6*(a*x-1)^2*c^4*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3-9*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+3*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/a^4/x^3/(a^2)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(91) = 182.
time = 0.47, size = 223, normalized size = 2.17

$$-\frac{1}{3}\left(\frac{9\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+29\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="maxima")

[Out] -1/3*(9*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3*c^4*((a*x - 1)/(a*x + 1))^(7/2) + c^4*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)

```
*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^
4 + a^2))*a
```

Fricas [A]
time = 0.35, size = 156, normalized size = 1.51

$$-\frac{18\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^4x^4+14\,a^3c^4x^3+11\,a^2c^4x^2+ac^4x-2\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^4*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^4*x^4 + 14*a^3*c^4*x^3 + 11*a^2*c^4*x^2 + a*c^4*x - 2*c^4
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\left(-\frac{4a}{\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^3}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**4,x)
```

```
[Out] c**4*(Integral(-4*a/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**2/(a*x
**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**3/(a*x**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)),
x) + Integral(a**4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/
(a*x + 1)), x))/a**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(91) = 182.
time = 0.42, size = 248, normalized size = 2.41

$$-\frac{3\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}+\frac{c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^4}{a\mathrm{sgn}\left(ax+1\right)}-\frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4-12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4-3\left(x|a|-\sqrt{a^2x^2-1}\right)c^4|a|-8\,ac^4}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) + c^4*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^4/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) -
 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4 - 12*(x*abs(a) - sqrt(a^2*x^2 -
1))^2*a*c^4 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(a) - 8*a*c^4)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 0.13, size = 183, normalized size = 1.78

$$\frac{5\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^4*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^4*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.397 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=61

$$\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{2a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x+3/2*c^3*arccsc(a*x)/a+3/2*c^3*(1-1/a^2/x^2)^(1/2)
/a^2/x
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6312, 283, 201, 222}

$$c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+\frac{3c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (3*c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*(1 - 1/(a^2*x^2))^(3/2)*x + (
3*c^3*ArcCsc[a*x])/(2*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{(3c^3)\,\text{Subst}\left(\int\sqrt{1-\frac{x^2}{a^2}}\,dx,x,\frac{1}{x}\right)}{a^2}$$

$$= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{(3c^3)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a^2}$$

$$= \frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x+\frac{3c^3\csc^{-1}(ax)}{2a}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.84

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\,(1+2a^2x^2)+3ax\text{ArcSin}\left(\frac{1}{ax}\right)\right)}{2a^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^3,x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(1 + 2*a^2*x^2) + 3*a*x*ArcSin[1/(a*x)]))/(2*a^
2*x)
```

Maple [A]
time = 0.06, size = 105, normalized size = 1.72

method	result	size
default	$-\frac{(ax-1)^2 c^3 \left(-3a^2x^2\sqrt{a^2x^2-1}-3a^2x^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)+(a^2x^2-1)^{\frac{3}{2}}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,a^3x^2}$	105
risch	$\frac{(ax-1)c^3}{2x^2a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}+a^2\sqrt{(ax+1)(ax-1)}\right)c^3\sqrt{(ax+1)(ax-1)}}{a^3(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x-1)^2*c^3*(-3*a^2*x^2*(a^2*x^2-1)^(1/2)-3*a^2*x^2*arctan(1/(a^2*x^
2-1)^(1/2))+(a^2*x^2-1)^(3/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*
x-1))^(1/2)/a^3/x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(53) = 106$.
time = 0.47, size = 151, normalized size = 2.48

$$-\left(\frac{3\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -(3*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - (3*c^3*((a*x - 1)/(a*x + 1)
)^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^3*sqrt((a*x - 1)/(a*x + 1
)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^
2/(a*x + 1)^3 + a^2))*a
```

Fricas [A]
time = 0.33, size = 85, normalized size = 1.39

$$-\frac{6\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-(2\,a^3c^3x^3+2\,a^2c^3x^2+ac^3x+c^3)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - (2*a^3*c^3*x^3 + 2*
a^2*c^3*x^2 + a*c^3*x + c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{3}\left(\int \frac{3a}{\frac{ax^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^{2}}{\frac{ax^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int \frac{a^{3}}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**3,x)
```

```
[Out] c**3*(Integral(3*a/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**2/(a*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1)), x) + Integral(a**3/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integr
al(-1/(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**3*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**3
```

Giac [A]
time = 0.40, size = 75, normalized size = 1.23

$$-\frac{3\,a^{4}c^{3}\arctan\left(\sqrt{a^{2}x^{2}-1}\right)-2\,\sqrt{a^{2}x^{2}-1}\,a^{4}c^{3}-\frac{\sqrt{a^{2}x^{2}-1}\,a^{2}c^{3}}{x^{2}}}{2\,a^{5}\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*a^4*c^3*arctan(sqrt(a^2*x^2 - 1)) - 2*sqrt(a^2*x^2 - 1)*a^4*c^3 - s
qrt(a^2*x^2 - 1)*a^2*c^3/x^2)/(a^5*sgn(a*x + 1))
```

Mupad [B]
time = 1.23, size = 119, normalized size = 1.95

$$\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+c^3\,x\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,x}+\frac{c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (c^3*((a*x - 1)/(a*x + 1))^(1/2))/a - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/a + c^3*x*((a*x - 1)/(a*x + 1))^(1/2) + (c^3*((a*x - 1)/(a*x + 1))^(1/
2))/(2*a^2*x) + (c^3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a^3*x^2)
```

3.398 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=63

$$\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a}+\frac{c^2\csc^{-1}(ax)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c^2*arccsc(a*x)/a+c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+c^2*(a-1/x)*x*(1-1/a^2
/x^2)^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 864, 827, 858, 222, 272, 65, 214}

$$\frac{c^2x\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}{a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)]*(a - x^(-1))*x)/a + (c^2*ArcCsc[a*x])/a + (c^2*A
rcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 864

```
Int[((x_)^(n_.)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x_)), x_Symbol]
:> Int[x^n*(a/d + c*(x/e))*(a + c*x^2)^(p - 1), x] /; FreeQ[{a, c, d, e, n,
 p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ( !IntegerQ[n] ||  !In
tegerQ[2*p] || IGtQ[n, 2] || (GtQ[p, 0] && NeQ[n, 2]))
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)}dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(c^3\text{Subst}\left(\int \frac{\left(\frac{1}{c}+\frac{x}{ac}\right)\sqrt{1-\frac{x^2}{a^2}}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{1}{2}c^3\text{Subst}\left(\int \frac{-\frac{2}{ac}+\frac{2x}{a^2c}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a^2} - \frac{c^2\text{Subst}\left(\int -\right.}{}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} - \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}dx, x, \frac{1}{x^2}\right)}{2a}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + (ac^2)\,\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}dx, x,\right.$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)x}{a} + \frac{c^2\csc^{-1}(ax)}{a} + \frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(63) = 126.

time = 0.10, size = 154, normalized size = 2.44

$$-\frac{c^2\left(-1+ax+a^2x^2-a^3x^3+4a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2\text{ArcSin}\left(\frac{1}{ax}\right)-a^2\sqrt{1-\frac{1}{a^2x^2}}\,x^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

[Out] -((c^2*(-1 + a*x + a^2*x^2 - a^3*x^3 + 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcS
in[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcSin[1/(a*x
)] - a^2*Sqrt[1 - 1/(a^2*x^2)]*x^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/(a^3*Sq
rt[1 - 1/(a^2*x^2)]*x^2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(59) = 118$.
time = 0.06, size = 174, normalized size = 2.76

method	result
risch	$-\frac{(ax-1)c^2}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a\sqrt{(ax+1)(ax-1)}+\frac{a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{(ax-}}{a^2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,a^2x\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x,method=_RETURNVERBOSE)

[Out] (a*x-1)^2*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+(a^2*x^2-1)^(3/2)*(a^
2)^(1/2)+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2
)^(1/2))/(a^2)^(1/2))*a^2*x+(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x)/((
a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/a^2/x/(a^2)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(59) = 118$.
time = 0.47, size = 125, normalized size = 1.98

$$-\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="maxima")

[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c^2*log(sqrt((a*x - 1)/(a*x +
1)) + 1)/a^2 + c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a

Fricas [A]
time = 0.35, size = 114, normalized size = 1.81

$$-\frac{2\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^2c^2x^2-c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c^2*x*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) + a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^2*x^2
 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{2a}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**2,x)
```

```
[Out] c**2*(Integral(-2*a/(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**2/(a*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1)), x) + Integral(1/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
 + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(59) = 118.
time = 0.42, size = 138, normalized size = 2.19

$$-\frac{2\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^2}{a\mathrm{sgn}\left(ax+1\right)}-\frac{2\,c^2}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -2*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^2/(a*sgn(a*x + 1)) - 2*c^2/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
*sgn(a*x + 1))
```

Mupad [B]
time = 1.21, size = 90, normalized size = 1.43

$$\frac{4\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

3.399 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=49

$$c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -c*arccsc(a*x)/a+2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {6312, 866, 1821, 858, 222, 272, 65, 214}

$$cx\sqrt{1-\frac{1}{a^2x^2}}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a + (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\left(c^3\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=-\frac{\text{Subst}\left(\int\frac{\left(c+\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\text{Subst}\left(\int\frac{-\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}-\frac{(2c)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right.}{a}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}-\frac{c\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{a}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}+(2ac)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right)$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}+\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 1.49

$$\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x-2\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\text{ArcSin}\left(\frac{1}{ax}\right)+2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x)),x]

[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*ArcSin[1/(a*x)] + 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(45) = 90$.
time = 0.08, size = 145, normalized size = 2.96

method	result
default	$-\frac{(ax-1)^2 c\left(\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}-2a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)-2\sqrt{a^2}\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x,method=_RETURNVERBOSE)

[Out] -(a*x-1)^2*c*((a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)^(1/2))*(a^2)^(1/2)-2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))-2*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/a/(a^2)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.
time = 0.47, size = 114, normalized size = 2.33

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="maxima")

[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)

Fricas [A]
time = 0.34, size = 88, normalized size = 1.80

$$\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{a}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x),x)
```

```
[Out] c*(Integral(a/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**2*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) - x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
1)), x))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.
time = 0.43, size = 91, normalized size = 1.86

$$\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c}{a\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x),x, algorithm="giac")
```

```
[Out] 2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 2*c*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/(
a*sgn(a*x + 1))
```

Mupad [B]
time = 0.09, size = 82, normalized size = 1.67

$$\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (4*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1
))
```

3.400 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=105

$$-\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -8/3*(a+1/x)/a^2/c/(1-1/a^2/x^2)^(3/2)+4*arctanh((1-1/a^2/x^2)^(1/2))/a/c-4
/3*(3*a+4/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}+\frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] (-8*(a + x^(-1)))/(3*a^2*c*(1 - 1/(a^2*x^2))^(3/2)) - (4*(3*a + 4/x))/(3*a^
2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c + (4*ArcTanh[Sqrt[
1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3c^4-\frac{12c^4x}{a}-\frac{13c^4x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{3c^4+\frac{12c^4x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{4\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x\right.}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x\right.}{ac}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{(4a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x,\right.}{c}$$

$$= -\frac{8\left(a+\frac{1}{x}\right)}{3a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{4\left(3a+\frac{4}{x}\right)}{3a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A]
time = 0.07, size = 70, normalized size = 0.67

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(19-26ax+3a^2x^2)}{(-1+ax)^2}+12\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{3ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 26*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + 12*Lo
g[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(3*a*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(93) = 186$.
time = 0.10, size = 346, normalized size = 3.30

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a\sqrt{a^2}}-\frac{20\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{3a^3\left(x-\frac{1}{a}\right)}-\frac{4\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\right.}}{3a^4\left(x-\frac{1}{a}\right)^2}\right.}{c(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{12\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3+12\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3-36\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax-}}{\sqrt{a^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(12*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3
+12*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-36*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-9*(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(3/2)*a*x-36*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+36*ln((a^2*x+(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+7*((a*x+1)*(a*x-1))^(3/
2)*(a^2)^(1/2)+36*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-12*a*ln((a^2*x+(a
^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))-12*(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(1/2))/a/(a^2)^(1/2)/(a*x-1)/c/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-
1)/(a*x+1))^(3/2)
```

Maxima [A]

time = 0.27, size = 133, normalized size = 1.27

$$\frac{2}{3}\,a\left(\frac{\frac{8\,(ax-1)}{ax+1}-\frac{12\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{6\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] 2/3*a*((8*(a*x - 1)/(a*x + 1) - 12*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a*
x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 6*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 6*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^
2*c))
```

Fricas [A]
time = 0.34, size = 128, normalized size = 1.22

$$\frac{12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-23\,a^2x^2-7\,ax+19)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3cx^2-2\,a^2cx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] 1/3*(12*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*(a^2*
x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 23*a^2*x
^2 - 7*a*x + 19)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int\frac{x}{\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a*x
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x)/c
```

Giac [A]
time = 0.41, size = 63, normalized size = 0.60

$$-\frac{4\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c|a|\mathrm{sgn}\,(ax+1)}+\frac{\sqrt{a^2x^2-1}}{ac\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] -4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c*abs(a)*sgn(a*x + 1)) + sqrt(a
^2*x^2 - 1)/(a*c*sgn(a*x + 1))
```

Mupad [B]
time = 1.24, size = 100, normalized size = 0.95

$$\frac{8\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{16\,(a\,x-1)}{3\,(a\,x+1)}-\frac{8\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{2}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (8*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((16*(a*x - 1))/(3*(a*x + 1)
) - (8*(a*x - 1)^2)/(a*x + 1)^2 + 2/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) - a
*c*((a*x - 1)/(a*x + 1))^(5/2))
```

3.401 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}+\frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -16/5*(a+1/x)/a^2/c^2/(1-1/a^2/x^2)^(5/2)-4/15*(5*a+11/x)/a^2/c^2/(1-1/a^2/
x^2)^(3/2)+5*arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+1/15*(-75*a-103/x)/a^2/c^2/
(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}+\frac{5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-16*(a + x^(-1)))/(5*a^2*c^2*(1 - 1/(a^2*x^2))^(5/2)) - (4*(5*a + 11/x))/(
15*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)) - (75*a + 103/x)/(15*a^2*c^2*Sqrt[1 - 1
/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 + (5*ArcTanh[Sqrt[1 - 1/(a^2*x
^2)]])/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^5}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^5-\frac{25c^5x}{a}-\frac{39c^5x^2}{a^2}+\frac{5c^5x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^5+\frac{75c^5x}{a}+\frac{88c^5x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^5-75}{x^2\sqrt{1-}}\right.}{15c^7}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5}{}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{5}{}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{(5}{}$$

$$= -\frac{16\left(a+\frac{1}{x}\right)}{5a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{4\left(5a+\frac{11}{x}\right)}{15a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{75a+\frac{103}{x}}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} + \frac{5}{}$$

Mathematica [A]
time = 0.05, size = 104, normalized size = 0.75

$$\frac{-118+161ax+91a^2x^2-173a^3x^3+15a^4x^4+75a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] (-118 + 161*a*x + 91*a^2*x^2 - 173*a^3*x^3 + 15*a^4*x^4 + 75*a*Sqrt[1 - 1/(
a^2*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^2*Sqrt[1
 - 1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(122) = 244$.
time = 0.12, size = 438, normalized size = 3.17

method	result
risch	$\frac{ax-1}{a c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{143\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{15a^4\left(x-\frac{1}{a}\right)}-\frac{52\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{15a^5\left(x-\frac{1}{a}\right)^2}\right.}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-75\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^4x^4-75\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^5x^4+60\sqrt{a^2}\,((ax+1)(ax-1)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-75*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-75*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+60*(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(3/2)*a^2*x^2+300*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+300*
ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-97*(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-450*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)
*a^2*x^2-450*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^
3*x^2+43*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+300*((a*x+1)*(a*x-1))^(1/2)*(a
^2)^(1/2)*a*x+300*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2
))*a^2*x-75*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-75*a*ln((a^2*x+(a^2)^(1/2)*
((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^2/c^2/((a*x+1)
*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.26, size = 153, normalized size = 1.11

$$\frac{1}{15}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}+\frac{100\,(ax-1)^2}{(ax+1)^2}-\frac{150\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{75\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{75\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] 1/15*a*((17*(a*x - 1)/(a*x + 1) + 100*(a*x - 1)^2/(a*x + 1)^2 - 150*(a*x -
1)^3/(a*x + 1)^3 + 3)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x
- 1)/(a*x + 1))^(5/2)) + 75*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) -
75*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A]
time = 0.34, size = 170, normalized size = 1.23

$$\frac{75\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-75\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a^4x^4-173\,a^3x^3+91\,a^2x^2+161\,ax-118)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 75*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 173*a^3*x^3 + 91*a^2*x^2 + 161*a*x - 118)*sqrt((a*x - 1)/
(a*x + 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a*x*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x
+ 1)), x)/c**2
```

Giac [A]
time = 0.43, size = 63, normalized size = 0.46

$$-\frac{5 \log \left(\left|-x|a| + \sqrt{a^2x^2 - 1}\right|\right)}{c^2|a|\mathrm{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2-1}}{ac^2\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -5*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

Mupad [B]
time = 0.09, size = 120, normalized size = 0.87

$$\frac{10\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{20\,{(a\,x-1)}^2}{3\,{(a\,x+1)}^2} - \frac{10\,{(a\,x-1)}^3}{{(a\,x+1)}^3} + \frac{17\,(a\,x-1)}{15\,(a\,x+1)} + \frac{1}{5}}{a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2} - a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (10*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((20*(a*x - 1)^2)/(3*(a*x
 + 1)^2) - (10*(a*x - 1)^3)/(a*x + 1)^3 + (17*(a*x - 1))/(15*(a*x + 1)) + 1
/5)/(a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

3.402 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=165

$$-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}+\frac{6\tanh$$

```
[Out] -32/7*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(7/2)-2/7*(7*a+13/x)/a^2/c^3/(1-1/a^2/x
^2)^(3/2)-16/7/a^2/c^3/(1-1/a^2/x^2)^(5/2)/x+6*arctanh((1-1/a^2/x^2)^(1/2))
/a/c^3+1/7*(-42*a-59/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c
^3
```

Rubi [A]
time = 0.35, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}-\frac{16}{7a^2c^3x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{6\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (-32*(a + x^(-1)))/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(7/2)) - (2*(7*a + 13/x))/(
7*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (42*a + 59/x)/(7*a^2*c^3*Sqrt[1 - 1/(a
^2*x^2)]) - 16/(7*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2)*x) + (Sqrt[1 - 1/(a^2*x^2
)]*x)/c^3 + (6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{-7c^6-\frac{42c^6x}{a}-\frac{80c^6x^2}{a^2}+\frac{42c^6x^3}{a^3}+\frac{7c^6x^4}{a^4}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{7c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} - \frac{\text{Subst}\left(\int \frac{35c^6+\frac{210c^6x}{a}+\frac{355c^6x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{35c^9}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x} + \frac{\text{Subst}\left(\int \frac{-105c^6-63}{x^2\left(1-\right.}\right.}{10}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}$$

$$= -\frac{32\left(a+\frac{1}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{2\left(7a+\frac{13}{x}\right)}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{42a+\frac{59}{x}}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16}{7a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}$$

Mathematica [A]
time = 0.06, size = 112, normalized size = 0.68

$$\frac{66-156ax+39a^2x^2+145a^3x^3-109a^4x^4+7a^5x^5+42a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{7a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] (66 - 156*a*x + 39*a^2*x^2 + 145*a^3*x^3 - 109*a^4*x^4 + 7*a^5*x^5 + 42*a*S
qrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(7*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(145) = 290$.
time = 0.12, size = 530, normalized size = 3.21

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{88\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{7a^5\left(x-\frac{1}{a}\right)}-\frac{4\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{7a^8\left(x-\frac{1}{a}\right)^4}\right.}{c^3(}$
default	$-\frac{-42\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^5x^5-42\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^6x^5+35((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*(-42*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5-42*ln((a^2*x+(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+35*((a*x+1)*(a*x-1))^(3/2
)*(a^2)^(1/2)*a^3*x^3+210*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+210*l
n((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4-87*(a^2)
^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2-420*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1
/2)*a^3*x^3-420*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))
*a^4*x^3+78*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+420*((a*x+1)*(a*x-1))^(
1/2)*(a^2)^(1/2)*a^2*x^2+420*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))
/(a^2)^(1/2))*a^3*x^2-24*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-210*((a*x+1)*(
a*x-1))^(1/2)*(a^2)^(1/2)*a*x-210*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2))/(a^2)^(1/2))*a^2*x+42*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+42*a*ln((a^
```

```
2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1
)^3/c^3/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.26, size = 169, normalized size = 1.02

$$\frac{1}{14}\,a\left(\frac{\frac{6\,(ax-1)}{ax+1}+\frac{21\,(ax-1)^2}{(ax+1)^2}+\frac{112\,(ax-1)^3}{(ax+1)^3}-\frac{168\,(ax-1)^4}{(ax+1)^4}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{84\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/14*a*((6*(a*x - 1)/(a*x + 1) + 21*(a*x - 1)^2/(a*x + 1)^2 + 112*(a*x - 1)
^3/(a*x + 1)^3 - 168*(a*x - 1)^4/(a*x + 1)^4 + 1)/(a^2*c^3*((a*x - 1)/(a*x
+ 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 84*log(sqrt((a*x - 1)/
(a*x + 1)) + 1)/(a^2*c^3) - 84*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3)
)
```

Fricas [A]
time = 0.33, size = 204, normalized size = 1.24

$$\frac{42\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(7\,a^5x^5-109\,a^4x^4+145\,a^3x^3+39\,a^2x^2-156\,ax+66)\sqrt{\frac{ax-1}{ax+1}}}{7\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*(42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/(a
*x + 1)) + 1) - 42*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((
a*x - 1)/(a*x + 1)) - 1) + (7*a^5*x^5 - 109*a^4*x^4 + 145*a^3*x^3 + 39*a^2*
x^2 - 156*a*x + 66)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3
 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\int\frac{x^3}{\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 4*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 6*a**2*x**2*sqrt
```

```
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**3
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.11, size = 137, normalized size = 0.83

$$\frac{12\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{3\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{16\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{24\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}+\frac{6\,\left(a\,x-1\right)}{7\,\left(a\,x+1\right)}+\frac{1}{7}}{2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}-2\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (12*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((3*(a*x - 1)^2)/(a*x + 1
)^2 + (16*(a*x - 1)^3)/(a*x + 1)^3 - (24*(a*x - 1)^4)/(a*x + 1)^4 + (6*(a*x
 - 1))/(7*(a*x + 1)) + 1/7)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 2*a*c^3*
((a*x - 1)/(a*x + 1))^(9/2))
```

3.403 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=204

$$\frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}$$

[Out] 16/63*(9*a-5/x)/a^2/c^4/(1-1/a^2/x^2)^(7/2)-64/9*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(9/2)-8/105*(21*a+41/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/315*(-735*a-1417/x)/a^2/c^4/(1-1/a^2/x^2)^(3/2)+7*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/315*(-2205*a-3149/x)/a^2/c^4/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4

Rubi [A]
time = 0.44, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$,
Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$\frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2205a+\frac{3149}{x}}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{7\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]

[Out] (16*(9*a - 5/x))/(63*a^2*c^4*(1 - 1/(a^2*x^2))^(7/2)) - (64*(a + x^(-1)))/(9*a^2*c^4*(1 - 1/(a^2*x^2))^(9/2)) - (8*(21*a + 41/x))/(105*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (735*a + 1417/x)/(315*a^2*c^4*(1 - 1/(a^2*x^2))^(3/2)) - (2205*a + 3149/x)/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^4 + (7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c^4)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^7}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^{11}}$$

$$= -\frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{-9c^7-\frac{63c^7x}{a}-\frac{134c^7x^2}{a^2}+\frac{198c^7x^3}{a^3}+\frac{63c^7x^4}{a^4}+\frac{9c^7x^5}{a^5}}{x^2\left(1-\frac{x^2}{a^2}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{9c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{\text{Subst}\left(\int \frac{63c^7+\frac{441c^7x}{a}+\frac{921c^7x^2}{a^2}+\frac{63c^7x^3}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{63c^{11}}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-315c^7-22}{x^2(1}\right.}{3}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)}$$

$$= \frac{16\left(9a-\frac{5}{x}\right)}{63a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{7/2}} - \frac{64\left(a+\frac{1}{x}\right)}{9a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{9/2}} - \frac{8\left(21a+\frac{41}{x}\right)}{105a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{735a+\frac{1417}{x}}{315a^2c^4\left(1-\frac{1}{a^2x^2}\right)}$$

Mathematica [**A**]
time = 0.06, size = 120, normalized size = 0.59

$$\frac{-3464+11651ax-10232a^2x^2-5567a^3x^3+13241a^4x^4-6224a^5x^5+315a^6x^6+2205a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{315a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-3464 + 11651*a*x - 10232*a^2*x^2 - 5567*a^3*x^3 + 13241*a^4*x^4 - 6224*a^
5*x^5 + 315*a^6*x^6 + 2205*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/(315*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(180) = 360.
time = 0.12, size = 622, normalized size = 3.05

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{4964\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{315a^6\left(x-\frac{1}{a}\right)}-\frac{164\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a \cdots}}{63a^9\left(x-\frac{1}{a}\right)^4}\cdots\right.}{\cdots}$
default	$-\frac{-2205\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^6x^6-2205\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^7x^6+1890((ax+1)(ax-1)\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(-2205*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^6*x^6-2205*ln((a^2*x+(a
^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^7*x^6+1890*((a*x+1)*(a*x-
1))^(3/2)*(a^2)^(1/2)*a^4*x^4+13230*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5
*x^5+13230*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*
x^5-6376*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3-33075*(a^2)^(1/2)*((a*
x+1)*(a*x-1))^(1/2)*a^4*x^4-33075*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2))/(a^2)^(1/2))*a^5*x^4+8646*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2
+44100*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+44100*ln((a^2*x+(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-5349*(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(3/2)*a*x-33075*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-33075
*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+1259*(
(a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+13230*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2
```

```
)*a*x+13230*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2
*x-2205*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-2205*a*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^4/c^4/((a*x+1)*(
a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.27, size = 185, normalized size = 0.91

$$\frac{1}{1260}\,a\left(\frac{\frac{235\,(ax-1)}{ax+1}+\frac{801\,(ax-1)^2}{(ax+1)^2}+\frac{2289\,(ax-1)^3}{(ax+1)^3}+\frac{11760\,(ax-1)^4}{(ax+1)^4}-\frac{17640\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{8820\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{8820\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/1260*a*((235*(a*x - 1)/(a*x + 1) + 801*(a*x - 1)^2/(a*x + 1)^2 + 2289*(a*
x - 1)^3/(a*x + 1)^3 + 11760*(a*x - 1)^4/(a*x + 1)^4 - 17640*(a*x - 1)^5/(a
*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)) + 8820*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 882
0*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A]
time = 0.34, size = 240, normalized size = 1.18

$$\frac{2205\,(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-2205\,(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(315\,a^6x^6-6224\,a^5x^5+13241\,a^4x^4-5567\,a^3x^3-10232\,a^2x^2+11651\,ax-3464)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 2205*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 -
10*a^2*x^2 + 5*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^6*x^6 -
 6224*a^5*x^5 + 13241*a^4*x^4 - 5567*a^3*x^3 - 10232*a^2*x^2 + 11651*a*x -
3464)*sqrt((a*x - 1)/(a*x + 1)))/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*
x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{5a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{10a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{10a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{5ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 5*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 10*a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 10*a**2*x**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/(a*x + 1) + 5*a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Giac [A]
time = 0.49, size = 63, normalized size = 0.31

$$-\frac{7\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{c^4|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}}{ac^4\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -7*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^4*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^4*sgn(a*x + 1))
```

Mupad [B]
time = 1.25, size = 153, normalized size = 0.75

$$\frac{14\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{89\,{\left(a\,x-1\right)}^2}{35\,{\left(a\,x+1\right)}^2}+\frac{109\,{\left(a\,x-1\right)}^3}{15\,{\left(a\,x+1\right)}^3}+\frac{112\,{\left(a\,x-1\right)}^4}{3\,{\left(a\,x+1\right)}^4}-\frac{56\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{47\,\left(a\,x-1\right)}{63\,\left(a\,x+1\right)}+\frac{1}{9}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}-4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (14*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((89*(a*x - 1)^2)/(35*(a*
x + 1)^2) + (109*(a*x - 1)^3)/(15*(a*x + 1)^3) + (112*(a*x - 1)^4)/(3*(a*x
+ 1)^4) - (56*(a*x - 1)^5)/(a*x + 1)^5 + (47*(a*x - 1))/(63*(a*x + 1)) + 1/
9)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(11
/2))
```

3.404 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5\,dx$

Optimal. Leaf size=64

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}$$

```
[Out] 1/4*c^5/a^5/x^4-1/3*c^5/a^4/x^3-c^5/a^3/x^2+2*c^5/a^2/x+c^5*x-c^5*ln(x)/a
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6266, 6264, 90}

$$\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}-\frac{c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
```

```
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^5 dx \\
&= -\frac{c^5\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^5}{x^5}\,dx}{a^5} \\
&= -\frac{c^5\int \frac{(1-ax)^3(1+ax)^2}{x^5}\,dx}{a^5} \\
&= -\frac{c^5\int \left(-a^5+\frac{1}{x^5}-\frac{a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5} \\
&= \frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(x)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 66, normalized size = 1.03

$$
\frac{c^5}{4a^5x^4}-\frac{c^5}{3a^4x^3}-\frac{c^5}{a^3x^2}+\frac{2c^5}{a^2x}+c^5x-\frac{c^5\log(ax)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^5,x]
[Out] c^5/(4*a^5*x^4) - c^5/(3*a^4*x^3) - c^5/(a^3*x^2) + (2*c^5)/(a^2*x) + c^5*x
 - (c^5*Log[a*x])/a
```

Maple [A]
time = 0.25, size = 48, normalized size = 0.75

method	result
default	$\frac{c^5\left(a^5x+\frac{1}{4x^4}+\frac{2a^3}{x}-\frac{a^2}{x^2}-a^4\ln(x)-\frac{a}{3x^3}\right)}{a^5}$
risch	$c^5x+\frac{2a^3c^5x^3-a^2c^5x^2-\frac{1}{3}a\,c^5x+\frac{1}{4}c^5}{a^5x^4}-\frac{c^5\ln(x)}{a}$
norman	$\frac{a^4c^5x^5+a^5c^5x^6-\frac{c^5}{4a}+\frac{7c^5x}{12}+\frac{2c^5a\,x^2}{3}-3c^5a^2x^3}{(ax-1)a^4x^4}-\frac{c^5\ln(x)}{a}$
meijerg	$-\frac{c^5\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{3c^5\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}+\frac{c^5x}{-ax+1}+\frac{5c^5\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)}{a}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x,method=_RETURNVERBOSE)
[Out] c^5/a^5*(a^5*x+1/4/x^4+2*a^3/x-a^2/x^2-a^4*ln(x)-1/3*a/x^3)
```

Maxima [A]
time = 0.30, size = 59, normalized size = 0.92

$$c^5x - \frac{c^5 \log(x)}{a} + \frac{24\,a^3c^5x^3 - 12\,a^2c^5x^2 - 4\,ac^5x + 3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] c^5*x - c^5*log(x)/a + 1/12*(24*a^3*c^5*x^3 - 12*a^2*c^5*x^2 - 4*a*c^5*x +
3*c^5)/(a^5*x^4)
```

Fricas [A]
time = 0.33, size = 67, normalized size = 1.05

$$\frac{12\,a^5c^5x^5 - 12\,a^4c^5x^4\log(x) + 24\,a^3c^5x^3 - 12\,a^2c^5x^2 - 4\,ac^5x + 3\,c^5}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*c^5*x^5 - 12*a^4*c^5*x^4*log(x) + 24*a^3*c^5*x^3 - 12*a^2*c^5*
x^2 - 4*a*c^5*x + 3*c^5)/(a^5*x^4)
```

Sympy [A]
time = 0.14, size = 63, normalized size = 0.98

$$\frac{a^5c^5x - a^4c^5\log(x) + \frac{24a^3c^5x^3 - 12a^2c^5x^2 - 4ac^5x + 3c^5}{12x^4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**5,x)
```

```
[Out] (a**5*c**5*x - a**4*c**5*log(x) + (24*a**3*c**5*x**3 - 12*a**2*c**5*x**2 -
4*a*c**5*x + 3*c**5)/(12*x**4))/a**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(60) = 120.
time = 0.41, size = 123, normalized size = 1.92

$$\frac{c^5\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{c^5\log\left(\left|-\frac{1}{ax-1} - 1\right|\right)}{a} + \frac{\left(12\,c^5 + \frac{37\,c^5}{ax-1} + \frac{52\,c^5}{(ax-1)^2} + \frac{42\,c^5}{(ax-1)^3} + \frac{12\,c^5}{(ax-1)^4}\right)(ax-1)}{12\,a\left(\frac{1}{ax-1} + 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c^5*log(abs(-1/(a*x - 1) - 1
))/a + 1/12*(12*c^5 + 37*c^5/(a*x - 1) + 52*c^5/(a*x - 1)^2 + 42*c^5/(a*x -
 1)^3 + 12*c^5/(a*x - 1)^4)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^4)
```

Mupad [B]
time = 0.06, size = 51, normalized size = 0.80

$$-\frac{c^5\,\left(4\,a\,x+12\,a^2\,x^2-24\,a^3\,x^3-12\,a^5\,x^5+12\,a^4\,x^4\,\ln\left(x\right)-3\right)}{12\,a^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] -(c^5*(4*a*x + 12*a^2*x^2 - 24*a^3*x^3 - 12*a^5*x^5 + 12*a^4*x^4*log(x) - 3
))/(12*a^5*x^4)
```

3.405 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=30

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

```
[Out] -1/3*c^4/a^4/x^3+2*c^4/a^2/x+c^4*x
```

Rubi [A]
time = 0.08, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {6302, 6266, 6264, 74, 276}

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
```

```
[Out] -1/3*c^4/(a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Rule 74

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
&& (NeQ[m, -1] || (EqQ[e, 0] && (EqQ[p, 1] ||  !IntegerQ[p])))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx\\ &= \frac{c^4\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\ &= \frac{c^4\int \frac{(1-ax)^2(1+ax)^2}{x^4}\,dx}{a^4}\\ &= \frac{c^4\int \frac{(1-a^2x^2)^2}{x^4}\,dx}{a^4}\\ &= \frac{c^4\int \left(a^4+\frac{1}{x^4}-\frac{2a^2}{x^2}\right)dx}{a^4}\\ &= -\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{c^4}{3a^4x^3}+\frac{2c^4}{a^2x}+c^4x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^4,x]
[Out] -1/3*c^4/(a^4*x^3) + (2*c^4)/(a^2*x) + c^4*x
```

Maple [A]
time = 0.21, size = 27, normalized size = 0.90

method	result
default	$\frac{c^4\left(a^4x+\frac{2a^2}{x}-\frac{1}{3x^3}\right)}{a^4}$
gosper	$\frac{c^4(3a^4x^4+6a^2x^2-1)}{3x^3a^4}$
risch	$c^4x+\frac{2a^2c^4x^2-\frac{1}{3}c^4}{a^4x^3}$

norman	$\frac{a^3c^4x^4+a^4c^4x^5+\frac{c^4}{3a}-\frac{c^4x}{3}-2c^4a\,x^2}{(ax-1)a^3x^3}$
meijerg	$-\frac{c^4\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{2c^4\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{c^4x}{-ax+1}+\frac{4c^4\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)}{a}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^4/a^4*(a^4*x+2*a^2/x-1/3/x^3)
```

Maxima [A]
time = 0.26, size = 31, normalized size = 1.03

$$c^4x+\frac{6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 1/3*(6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Fricas [A]
time = 0.33, size = 36, normalized size = 1.20

$$\frac{3\,a^4c^4x^4+6\,a^2c^4x^2-c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 + 6*a^2*c^4*x^2 - c^4)/(a^4*x^3)
```

Sympy [A]
time = 0.07, size = 31, normalized size = 1.03

$$\frac{a^4c^4x+\frac{6a^2c^4x^2-c^4}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**4,x)
```

```
[Out] (a**4*c**4*x + (6*a**2*c**4*x**2 - c**4)/(3*x**3))/a**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.
time = 0.40, size = 59, normalized size = 1.97

$$\frac{(ax-1)c^4}{a}-\frac{5\,c^4+\frac{9\,c^4}{ax-1}+\frac{3\,c^4}{(ax-1)^2}}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] (a*x - 1)*c^4/a - 1/3*(5*c^4 + 9*c^4/(a*x - 1) + 3*c^4/(a*x - 1)^2)/(a*(1/(
a*x - 1) + 1)^3)
```

Mupad [B]
time = 0.05, size = 27, normalized size = 0.90

$$\frac{c^4\,\left(a^4\,x^4+2\,a^2\,x^2-\frac{1}{3}\right)}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^4*(2*a^2*x^2 + a^4*x^4 - 1/3))/(a^4*x^3)
```

3.406 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=38

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}$$

[Out] 1/2*c^3/a^3/x^2+c^3/a^2/x+c^3*x+c^3*ln(x)/a

Rubi [A]
time = 0.09, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6266, 6264, 76}

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+\frac{c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]

[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[x])/a

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx$$
$$= -\frac{c^3\int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^3}{x^3}dx}{a^3}$$
$$= -\frac{c^3\int \frac{(1-ax)(1+ax)^2}{x^3}dx}{a^3}$$
$$= -\frac{c^3\int \left(-a^3+\frac{1}{x^3}+\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3}$$
$$= \frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(x)}{a}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 1.05

$$\frac{c^3}{2a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{c^3\log(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^3,x]
[Out] c^3/(2*a^3*x^2) + c^3/(a^2*x) + c^3*x + (c^3*Log[a*x])/a
```

Maple [A]
time = 0.19, size = 30, normalized size = 0.79

method	result
default	$\frac{c^3\left(a^3x+\frac{a}{x}+\frac{1}{2x^2}+a^2\ln(x)\right)}{a^3}$
risch	$c^3x+\frac{a\,c^3x+\frac{1}{2}c^3}{a^3x^2}+\frac{c^3\ln(x)}{a}$
norman	$\frac{a^3c^3x^4-\frac{c^3}{2a}-\frac{c^3x}{2}}{(ax-1)a^2x^2}+\frac{c^3\ln(x)}{a}$
meijerg	$-\frac{c^3\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{c^3\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{2c^3x}{-ax+1}+\frac{2c^3\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)}{a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x,method=_RETURNVERBOSE)
[Out] c^3/a^3*(a^3*x+a/x+1/2/x^2+a^2*ln(x))
```

Maxima [A]
time = 0.26, size = 34, normalized size = 0.89

$$c^3x+\frac{c^3\log\left(x\right)}{a}+\frac{2\,ac^3x+c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + c^3*log(x)/a + 1/2*(2*a*c^3*x + c^3)/(a^3*x^2)
```

Fricas [A]
time = 0.38, size = 43, normalized size = 1.13

$$\frac{2\,a^3c^3x^3+2\,a^2c^3x^2\log\left(x\right)+2\,ac^3x+c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 + 2*a^2*c^3*x^2*log(x) + 2*a*c^3*x + c^3)/(a^3*x^2)
```

Sympy [A]
time = 0.09, size = 37, normalized size = 0.97

$$\frac{a^3c^3x+a^2c^3\log\left(x\right)+\frac{2ac^3x+c^3}{2x^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**3,x)
```

```
[Out] (a**3*c**3*x + a**2*c**3*log(x) + (2*a*c**3*x + c**3)/(2*x**2))/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(36) = 72.
time = 0.43, size = 98, normalized size = 2.58

$$-\frac{c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{c^3\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(2\,c^3+\frac{c^3}{ax-1}-\frac{2\,c^3}{(ax-1)^2}\right)(ax-1)}{2\,a\left(\frac{1}{ax-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] -c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + c^3*log(abs(-1/(a*x - 1) -
1))/a + 1/2*(2*c^3 + c^3/(a*x - 1) - 2*c^3/(a*x - 1)^2)*(a*x - 1)/(a*(1/(a*
x - 1) + 1)^2)
```

Mupad [B]
time = 1.18, size = 31, normalized size = 0.82

$$\frac{c^3\,\left(a\,x+a^3\,x^3+a^2\,x^2\,\ln\left(x\right)+\frac{1}{2}\right)}{a^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^3*(a*x + a^3*x^3 + a^2*x^2*log(x) + 1/2))/(a^3*x^2)
```

3.407 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=27

$$-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}$$

```
[Out] -c^2/a^2/x+c^2*x+2*c^2*ln(x)/a
```

Rubi [A]
time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6266, 6264, 45}

$$-\frac{c^2}{a^2x}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
```

```
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx$$
$$= \frac{c^2 \int \frac{e^{4\tanh^{-1}(ax)}(1-ax)^2}{x^2} dx}{a^2}$$
$$= \frac{c^2 \int \frac{(1+ax)^2}{x^2} dx}{a^2}$$
$$= \frac{c^2 \int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right) dx}{a^2}$$
$$= -\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(x)}{a}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 1.07

$$-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\log(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x))^2,x]
[Out] -(c^2/(a^2*x)) + c^2*x + (2*c^2*Log[a*x])/a
```

Maple [A]
time = 0.16, size = 24, normalized size = 0.89

method	result	size
default	$\frac{c^2\left(a^2x-\frac{1}{x}+2a\ln(x)\right)}{a^2}$	24
risch	$-\frac{c^2}{a^2x}+c^2x+\frac{2c^2\ln(x)}{a}$	28
norman	$\frac{\frac{c^2}{a}-2a\,c^2x^2+a^2c^2x^3}{(ax-1)ax}+\frac{2c^2\ln(x)}{a}$	53
meijerg	$-\frac{c^2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{2c^2x}{-ax+1}-\frac{c^2\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x,method=_RETURNVERBOSE)
[Out] c^2/a^2*(a^2*x-1/x+2*a*ln(x))
```

Maxima [A]
time = 0.30, size = 27, normalized size = 1.00

$$c^2x+\frac{2\,c^2\log(x)}{a}-\frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a - c^2/(a^2*x)
```

Fricas [A]
time = 0.34, size = 32, normalized size = 1.19

$$\frac{a^2c^2x^2 + 2\,ac^2x\log\left(x\right) - c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] (a^2*c^2*x^2 + 2*a*c^2*x*log(x) - c^2)/(a^2*x)
```

Sympy [A]
time = 0.05, size = 26, normalized size = 0.96

$$\frac{a^2c^2x + 2ac^2\log\left(x\right) - \frac{c^2}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x)**2,x)
```

```
[Out] (a**2*c**2*x + 2*a*c**2*log(x) - c**2/x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(27) = 54.
time = 0.40, size = 94, normalized size = 3.48

$$-\frac{2\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} + \frac{2\,c^2\log\left(\left|-\frac{1}{ax-1} - 1\right|\right)}{a} + \frac{c^2 + \frac{2\,c^2}{ax-1}}{a^2\left(\frac{1}{(ax-1)a} + \frac{1}{(ax-1)^2a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] -2*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 2*c^2*log(abs(-1/(a*x - 1
) - 1))/a + (c^2 + 2*c^2/(a*x - 1))/(a^2*(1/((a*x - 1)*a) + 1/((a*x - 1)^2*
a)))
```

Mupad [B]
time = 1.18, size = 25, normalized size = 0.93

$$\frac{c^2\left(a^2\,x^2 + 2\,a\,x\,\ln\left(x\right) - 1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^2*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

3.408 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=25

$$cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}$$

```
[Out] c*x-c*ln(x)/a+4*c*ln(-a*x+1)/a
```

Rubi [A]
time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6266, 6264, 84}

$$-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$=-\frac{c\int\frac{e^{4\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$=-\frac{c\int\frac{(1+ax)^2}{x(1-ax)}\,dx}{a}$$
$$=-\frac{c\int\left(-a+\frac{1}{x}-\frac{4a}{-1+ax}\right)\,dx}{a}$$
$$=cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$cx-\frac{c\log(x)}{a}+\frac{4c\log(1-ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a*x)),x]
[Out] c*x - (c*Log[x])/a + (4*c*Log[1 - a*x])/a
```

Maple [A]
time = 0.15, size = 22, normalized size = 0.88

method	result	siz
default	$\frac{c(ax-\ln(x)+4\ln(ax-1))}{a}$	22
risch	$cx-\frac{c\ln(x)}{a}+\frac{4c\ln(-ax+1)}{a}$	26
norman	$\frac{ac\,x^2-cx}{ax-1}-\frac{c\ln(x)}{a}+\frac{4c\ln(ax-1)}{a}$	41
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{cx}{-ax+1}-\frac{c\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)}{a}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] c/a*(a*x-ln(x)+4*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.96

$$cx+\frac{4\,c\log\left(ax-1\right)}{a}-\frac{c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="maxima")
```

```
[Out] c*x + 4*c*log(a*x - 1)/a - c*log(x)/a
```

Fricas [A]
time = 0.35, size = 23, normalized size = 0.92

$$\frac{acx + 4\,c\log\left(ax - 1\right) - c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*c*x + 4*c*log(a*x - 1) - c*log(x))/a
```

Sympy [A]
time = 0.12, size = 17, normalized size = 0.68

$$cx + \frac{c\left(-\log\left(x\right) + 4\log\left(x - \frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a/x),x)
```

```
[Out] c*x + c*(-log(x) + 4*log(x - 1/a))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.
time = 0.39, size = 55, normalized size = 2.20

$$\frac{(ax-1)c}{a} - \frac{3\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{c\log\left(\left|-\frac{1}{ax-1} - 1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a/x),x, algorithm="giac")
```

```
[Out] (a*x - 1)*c/a - 3*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - c*log(abs(-1
/(a*x - 1) - 1))/a
```

Mupad [B]
time = 0.07, size = 24, normalized size = 0.96

$$c\,x - \frac{c\,\ln\left(x\right)}{a} + \frac{4\,c\,\ln\left(a\,x - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c*x - (c*log(x))/a + (4*c*log(a*x - 1))/a
```

3.409 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=53

$$\frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}$$

```
[Out] x/c-2/a/c/(-a*x+1)^2+8/a/c/(-a*x+1)+5*ln(-a*x+1)/a/c
```

Rubi [A]
time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 78}

$$\frac{8}{ac(1-ax)}-\frac{2}{ac(1-ax)^2}+\frac{5\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] x/c - 2/(a*c*(1 - a*x)^2) + 8/(a*c*(1 - a*x)) + (5*Log[1 - a*x])/(a*c)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx\\ &= -\frac{a\int \frac{e^{4\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}\\ &= -\frac{a\int \frac{x(1+ax)^2}{(1-ax)^3}\,dx}{c}\\ &= -\frac{a\int \left(-\frac{1}{a}-\frac{4}{a(-1+ax)^3}-\frac{8}{a(-1+ax)^2}-\frac{5}{a(-1+ax)}\right)dx}{c}\\ &= \frac{x}{c}-\frac{2}{ac(1-ax)^2}+\frac{8}{ac(1-ax)}+\frac{5\log(1-ax)}{ac}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.96

$$-\frac{a\left(-\frac{x}{a}+\frac{2}{a^2(1-ax)^2}-\frac{8}{a^2(1-ax)}-\frac{5\log(1-ax)}{a^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x)),x]
[Out] -((a*(-(x/a) + 2/(a^2*(1 - a*x)^2) - 8/(a^2*(1 - a*x)) - (5*Log[1 - a*x])/a
^2))/c)
```

Maple [A]
time = 0.13, size = 47, normalized size = 0.89

method	result	size
risch	$\frac{x}{c}+\frac{-8cx+\frac{6c}{a}}{c^2(ax-1)^2}+\frac{5\ln(ax-1)}{ac}$	43
default	$\frac{a\left(\frac{x}{a}-\frac{8}{a^2(ax-1)}-\frac{2}{a^2(ax-1)^2}+\frac{5\ln(ax-1)}{a^2}\right)}{c}$	47
norman	$\frac{\frac{a^2x^3}{c}-\frac{8a\,x^2}{c}+\frac{5x}{c}}{(ax-1)^2}+\frac{5\ln(ax-1)}{ac}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] a/c*(x/a-8/a^2/(a*x-1)-2/a^2/(a*x-1)^2+5/a^2*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.92

$$-\frac{2\,(4\,ax-3)}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{5\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*(4*a*x - 3)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Fricas [A]
time = 0.33, size = 64, normalized size = 1.21

$$\frac{a^3x^3-2\,a^2x^2-7\,ax+5\,(a^2x^2-2\,ax+1)\log\left(ax-1\right)+6}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 7*a*x + 5*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 6)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A]
time = 0.13, size = 41, normalized size = 0.77

$$\frac{-8ax+6}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{5\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x),x)
```

```
[Out] (-8*a*x + 6)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 5*log(a*x - 1)/(a*c)
```

Giac [A]
time = 0.39, size = 74, normalized size = 1.40

$$\frac{ax-1}{ac}-\frac{5\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{2\left(\frac{4\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}\right)}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x),x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c) - 5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - 2*(4*a^3
*c/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

Mupad [B]
time = 0.06, size = 48, normalized size = 0.91

$$\frac{x}{c}-\frac{8\,x-\frac{6}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{5\,\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))*(a*x - 1)^2),x)
[Out] x/c - (8*x - 6/a)/(c + a^2*c*x^2 - 2*a*c*x) + (5*log(a*x - 1))/(a*c)
```

3.410 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}$$

```
[Out] x/c^2+4/3/a/c^2/(-a*x+1)^3-6/a/c^2/(-a*x+1)^2+13/a/c^2/(-a*x+1)+6*ln(-a*x+1
)/a/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\frac{13}{ac^2(1-ax)}-\frac{6}{ac^2(1-ax)^2}+\frac{4}{3ac^2(1-ax)^3}+\frac{6\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] x/c^2 + 4/(3*a*c^2*(1 - a*x)^3) - 6/(a*c^2*(1 - a*x)^2) + 13/(a*c^2*(1 - a*
x)) + (6*Log[1 - a*x])/(a*c^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx\\
&= \frac{a^2 \int \frac{e^{4\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2}\\
&= \frac{a^2 \int \frac{x^2(1+ax)^2}{(1-ax)^4}\,dx}{c^2}\\
&= \frac{a^2 \int \left(\frac{1}{a^2}+\frac{4}{a^2(-1+ax)^4}+\frac{12}{a^2(-1+ax)^3}+\frac{13}{a^2(-1+ax)^2}+\frac{6}{a^2(-1+ax)}\right)dx}{c^2}\\
&= \frac{x}{c^2}+\frac{4}{3ac^2(1-ax)^3}-\frac{6}{ac^2(1-ax)^2}+\frac{13}{ac^2(1-ax)}+\frac{6\log(1-ax)}{ac^2}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.89

$$\frac{-25+57ax-30a^2x^2-9a^3x^3+3a^4x^4+18(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^2,x]
[Out] (-25 + 57*a*x - 30*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 18*(-1 + a*x)^3*Log[1
- a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A]
time = 0.14, size = 61, normalized size = 0.86

method	result	size
risch	$\frac{x}{c^2}+\frac{-13a\,c^2x^2+20c^2x-\frac{25c^2}{3a}}{c^4(ax-1)^3}+\frac{6\ln(ax-1)}{a\,c^2}$	56
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{13}{a^3(ax-1)}-\frac{4}{3a^3(ax-1)^3}-\frac{6}{a^3(ax-1)^2}+\frac{6\ln(ax-1)}{a^3}\right)}{c^2}$	61
norman	$\frac{\frac{a^3x^4}{c}-\frac{6x}{c}+\frac{15a\,x^2}{c}-\frac{34a^2x^3}{3c}}{(ax-1)^3c}+\frac{6\ln(ax-1)}{a\,c^2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c^2*(x/a^2-13/a^3/(a*x-1)-4/3/a^3/(a*x-1)^3-6/a^3/(a*x-1)^2+6/a^3*ln(a*
x-1))
```

Maxima [A]
time = 0.27, size = 75, normalized size = 1.06

$$-\frac{39\,a^2x^2-60\,ax+25}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{6\,\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(39*a^2*x^2 - 60*a*x + 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 6*log(a*x - 1)/(a*c^2)
```

Fricas [A]
time = 0.33, size = 100, normalized size = 1.41

$$\frac{3\,a^4x^4-9\,a^3x^3-30\,a^2x^2+57\,ax+18\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(ax-1\right)-25}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 30*a^2*x^2 + 57*a*x + 18*(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)*log(a*x - 1) - 25)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2)
```

Sympy [A]
time = 0.19, size = 73, normalized size = 1.03

$$\frac{-39a^2x^2+60ax-25}{3a^4c^2x^3-9a^3c^2x^2+9a^2c^2x-3ac^2}+\frac{x}{c^2}+\frac{6\log\left(ax-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**2,x)
```

```
[Out] (-39*a**2*x**2 + 60*a*x - 25)/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2
*c**2*x - 3*a*c**2) + x/c**2 + 6*log(a*x - 1)/(a*c**2)
```

Giac [A]
time = 0.41, size = 94, normalized size = 1.32

$$\frac{ax-1}{ac^2}-\frac{6\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{39\,a^5c^4}{ax-1}+\frac{18\,a^5c^4}{(ax-1)^2}+\frac{4\,a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 6*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(39*a^5*c^4/(a*x - 1) + 18*a^5*c^4/(a*x - 1)^2 + 4*a^5*c^4/(a*x - 1)^3)/(a^
6*c^6)
```

Mupad [B]
time = 0.07, size = 71, normalized size = 1.00

$$\frac{13\,a\,x^2-20\,x+\frac{25}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{6\,\ln\left(a\,x-1\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))^2*(a*x - 1)^2),x)
```

```
[Out] (13*a*x^2 - 20*x + 25/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x
) + x/c^2 + (6*log(a*x - 1))/(a*c^2)
```

3.411 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}$$

```
[Out] x/c^3-1/a/c^3/(-a*x+1)^4+16/3/a/c^3/(-a*x+1)^3-25/2/a/c^3/(-a*x+1)^2+19/a/c
^3/(-a*x+1)+7*ln(-a*x+1)/a/c^3
```

Rubi [A]
time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 90}

$$\frac{19}{ac^3(1-ax)}-\frac{25}{2ac^3(1-ax)^2}+\frac{16}{3ac^3(1-ax)^3}-\frac{1}{ac^3(1-ax)^4}+\frac{7\log(1-ax)}{ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
```

```
[Out] x/c^3 - 1/(a*c^3*(1 - a*x)^4) + 16/(3*a*c^3*(1 - a*x)^3) - 25/(2*a*c^3*(1 -
 a*x)^2) + 19/(a*c^3*(1 - a*x)) + (7*Log[1 - a*x])/(a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx\\
&= -\frac{a^3\int \frac{e^{4\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3}\\
&= -\frac{a^3\int \frac{x^3(1+ax)^2}{(1-ax)^5}\,dx}{c^3}\\
&= -\frac{a^3\int \left(-\frac{1}{a^3}-\frac{4}{a^3(-1+ax)^5}-\frac{16}{a^3(-1+ax)^4}-\frac{25}{a^3(-1+ax)^3}-\frac{19}{a^3(-1+ax)^2}-\frac{7}{a^3(-1+ax)}\right)dx}{c^3}\\
&= \frac{x}{c^3}-\frac{1}{ac^3(1-ax)^4}+\frac{16}{3ac^3(1-ax)^3}-\frac{25}{2ac^3(1-ax)^2}+\frac{19}{ac^3(1-ax)}+\frac{7\log(1-ax)}{ac^3}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 71, normalized size = 0.80

$$\frac{65-218ax+243a^2x^2-78a^3x^3-24a^4x^4+6a^5x^5+42(-1+ax)^4\log(1-ax)}{6ac^3(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^3,x]
[Out] (65 - 218*a*x + 243*a^2*x^2 - 78*a^3*x^3 - 24*a^4*x^4 + 6*a^5*x^5 + 42*(-1
+ a*x)^4*Log[1 - a*x])/(6*a*c^3*(-1 + a*x)^4)
```

Maple [A]
time = 0.14, size = 73, normalized size = 0.82

method	result	size
risch	$\frac{x}{c^3}+\frac{-19a^2c^3x^3+\frac{89c^3a\,x^2}{2}-\frac{112c^3x}{3}+\frac{65c^3}{6a}}{c^6(ax-1)^4}+\frac{7\ln(ax-1)}{c^3a}$	67
default	$\frac{a^3\left(\frac{x}{a^3}-\frac{19}{a^4(ax-1)}-\frac{16}{3a^4(ax-1)^3}-\frac{25}{2a^4(ax-1)^2}-\frac{1}{a^4(ax-1)^4}+\frac{7\ln(ax-1)}{a^4}\right)}{c^3}$	73
norman	$\frac{\frac{a^4x^5}{c}+\frac{7x}{c}-\frac{49a\,x^2}{2c}+\frac{91a^2x^3}{3c}-\frac{89a^3x^4}{6c}}{(ax-1)^4c^2}+\frac{7\ln(ax-1)}{c^3a}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/c^3*(x/a^3-19/a^4/(a*x-1)-16/3/a^4/(a*x-1)^3-25/2/a^4/(a*x-1)^2-1/a^4/(
a*x-1)^4+7/a^4*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 93, normalized size = 1.04

$$-\frac{114\,a^3x^3-267\,a^2x^2+224\,ax-65}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}+\frac{x}{c^3}+\frac{7\,\log(ax-1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(114*a^3*x^3 - 267*a^2*x^2 + 224*a*x - 65)/(a^5*c^3*x^4 - 4*a^4*c^3*x^
3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 + 7*log(a*x - 1)/(a*c^3)
```

Fricas [A]
time = 0.40, size = 126, normalized size = 1.42

$$\frac{6\,a^5x^5-24\,a^4x^4-78\,a^3x^3+243\,a^2x^2-218\,ax+42\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log(ax-1)+65}{6\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^5*x^5 - 24*a^4*x^4 - 78*a^3*x^3 + 243*a^2*x^2 - 218*a*x + 42*(a^4*
x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 65)/(a^5*c^3*x^4 -
4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A]
time = 0.26, size = 94, normalized size = 1.06

$$\frac{-114a^3x^3+267a^2x^2-224ax+65}{6a^5c^3x^4-24a^4c^3x^3+36a^3c^3x^2-24a^2c^3x+6ac^3}+\frac{x}{c^3}+\frac{7\log(ax-1)}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**3,x)
```

```
[Out] (-114*a**3*x**3 + 267*a**2*x**2 - 224*a*x + 65)/(6*a**5*c**3*x**4 - 24*a**4
*c**3*x**3 + 36*a**3*c**3*x**2 - 24*a**2*c**3*x + 6*a*c**3) + x/c**3 + 7*lo
g(a*x - 1)/(a*c**3)
```

Giac [A]
time = 0.40, size = 109, normalized size = 1.22

$$\frac{ax-1}{ac^3}-\frac{7\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3}-\frac{\frac{114\,a^7c^9}{ax-1}+\frac{75\,a^7c^9}{(ax-1)^2}+\frac{32\,a^7c^9}{(ax-1)^3}+\frac{6\,a^7c^9}{(ax-1)^4}}{6\,a^8c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^3) - 7*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/6*
(114*a^7*c^9/(a*x - 1) + 75*a^7*c^9/(a*x - 1)^2 + 32*a^7*c^9/(a*x - 1)^3 +
6*a^7*c^9/(a*x - 1)^4)/(a^8*c^12)
```

Mupad [B]
time = 1.23, size = 90, normalized size = 1.01

$$\frac{x}{c^3}-\frac{\frac{112\,x}{3}-\frac{89\,a\,x^2}{2}-\frac{65}{6\,a}+19\,a^2\,x^3}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}+\frac{7\,\ln\left(a\,x-1\right)}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))^3*(a*x - 1)^2),x)
```

```
[Out] x/c^3 - ((112*x)/3 - (89*a*x^2)/2 - 65/(6*a) + 19*a^2*x^3)/(c^3 + 6*a^2*c^3
*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (7*log(a*x - 1))/(a*c^3)
```

3.412 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=105

$$\frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8\log(1-ax)}{ac^4}$$

```
[Out] x/c^4+4/5/a/c^4/(-a*x+1)^5-5/a/c^4/(-a*x+1)^4+41/3/a/c^4/(-a*x+1)^3-22/a/c^
4/(-a*x+1)^2+26/a/c^4/(-a*x+1)+8*ln(-a*x+1)/a/c^4
```

Rubi [A]
time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$\frac{26}{ac^4(1-ax)}-\frac{22}{ac^4(1-ax)^2}+\frac{41}{3ac^4(1-ax)^3}-\frac{5}{ac^4(1-ax)^4}+\frac{4}{5ac^4(1-ax)^5}+\frac{8\log(1-ax)}{ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] x/c^4 + 4/(5*a*c^4*(1 - a*x)^5) - 5/(a*c^4*(1 - a*x)^4) + 41/(3*a*c^4*(1 -
a*x)^3) - 22/(a*c^4*(1 - a*x)^2) + 26/(a*c^4*(1 - a*x)) + (8*Log[1 - a*x])/
(a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx \\
&= \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4} \\
&= \frac{a^4\int \frac{x^4(1+ax)^2}{(1-ax)^6}\,dx}{c^4} \\
&= \frac{a^4\int \left(\frac{1}{a^4}+\frac{4}{a^4(-1+ax)^6}+\frac{20}{a^4(-1+ax)^5}+\frac{41}{a^4(-1+ax)^4}+\frac{44}{a^4(-1+ax)^3}+\frac{26}{a^4(-1+ax)^2}+\frac{8}{a^4(-1+ax)}\right)dx}{c^4} \\
&= \frac{x}{c^4}+\frac{4}{5ac^4(1-ax)^5}-\frac{5}{ac^4(1-ax)^4}+\frac{41}{3ac^4(1-ax)^3}-\frac{22}{ac^4(1-ax)^2}+\frac{26}{ac^4(1-ax)}+\frac{8
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.75

$$\frac{-202+890ax-1480a^2x^2+1080a^3x^3-240a^4x^4-75a^5x^5+15a^6x^6+120(-1+ax)^5\log(1-ax)}{15ac^4(-1+ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a*x))^4,x]
```

```
[Out] (-202 + 890*a*x - 1480*a^2*x^2 + 1080*a^3*x^3 - 240*a^4*x^4 - 75*a^5*x^5 +
15*a^6*x^6 + 120*(-1 + a*x)^5*Log[1 - a*x])/(15*a*c^4*(-1 + a*x)^5)
```

Maple [A]
time = 0.13, size = 85, normalized size = 0.81

method	result	size
risch	$\frac{x}{c^4}+\frac{-26a^3c^4x^4+82a^2c^4x^3-\frac{311c^4a\,x^2}{3}+\frac{181c^4x}{3}-\frac{202c^4}{15a}}{c^8(ax-1)^5}+\frac{8\ln(ax-1)}{c^4a}$	78
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{26}{a^5(ax-1)}-\frac{4}{5a^5(ax-1)^5}-\frac{41}{3a^5(ax-1)^3}-\frac{22}{a^5(ax-1)^2}-\frac{5}{a^5(ax-1)^4}+\frac{8\ln(ax-1)}{a^5}\right)}{c^4}$	85
norman	$\frac{\frac{a^5x^6}{c}-\frac{8x}{c}+\frac{36a\,x^2}{c}-\frac{188a^2x^3}{3c}+\frac{154a^3x^4}{3c}-\frac{277a^4x^5}{15c}}{(ax-1)^5c^3}+\frac{8\ln(ax-1)}{c^4a}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^4*(x/a^4-26/a^5/(a*x-1)-4/5/a^5/(a*x-1)^5-41/3/a^5/(a*x-1)^3-22/a^5/(
a*x-1)^2-5/a^5/(a*x-1)^4+8/a^5*ln(a*x-1))
```

Maxima [A]
time = 0.28, size = 113, normalized size = 1.08

$$-\frac{390\,a^4x^4-1230\,a^3x^3+1555\,a^2x^2-905\,ax+202}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}+\frac{x}{c^4}+\frac{8\,\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/15*(390*a^4*x^4 - 1230*a^3*x^3 + 1555*a^2*x^2 - 905*a*x + 202)/(a^6*c^4*
x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*c^4*x^2 + 5*a^2*c^4*x - a*c^4
) + x/c^4 + 8*log(a*x - 1)/(a*c^4)
```

Fricas [A]
time = 0.33, size = 154, normalized size = 1.47

$$\frac{15\,a^6x^6-75\,a^5x^5-240\,a^4x^4+1080\,a^3x^3-1480\,a^2x^2+890\,ax+120\,(a^5x^5-5\,a^4x^4+10\,a^3x^3-10\,a^2x^2+5\,ax-1)\log\left(ax-1\right)-202}{15\,(a^6c^4x^5-5\,a^5c^4x^4+10\,a^4c^4x^3-10\,a^3c^4x^2+5\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*x^6 - 75*a^5*x^5 - 240*a^4*x^4 + 1080*a^3*x^3 - 1480*a^2*x^2 +
 890*a*x + 120*(a^5*x^5 - 5*a^4*x^4 + 10*a^3*x^3 - 10*a^2*x^2 + 5*a*x - 1)*
log(a*x - 1) - 202)/(a^6*c^4*x^5 - 5*a^5*c^4*x^4 + 10*a^4*c^4*x^3 - 10*a^3*
c^4*x^2 + 5*a^2*c^4*x - a*c^4)
```

Sympy [A]
time = 0.31, size = 114, normalized size = 1.09

$$\frac{-390a^4x^4+1230a^3x^3-1555a^2x^2+905ax-202}{15a^6c^4x^5-75a^5c^4x^4+150a^4c^4x^3-150a^3c^4x^2+75a^2c^4x-15ac^4}+\frac{x}{c^4}+\frac{8\log\left(ax-1\right)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a/x)**4,x)
```

```
[Out] (-390*a**4*x**4 + 1230*a**3*x**3 - 1555*a**2*x**2 + 905*a*x - 202)/(15*a**6
*c**4*x**5 - 75*a**5*c**4*x**4 + 150*a**4*c**4*x**3 - 150*a**3*c**4*x**2 +
75*a**2*c**4*x - 15*a*c**4) + x/c**4 + 8*log(a*x - 1)/(a*c**4)
```

Giac [A]
time = 0.40, size = 124, normalized size = 1.18

$$\frac{ax-1}{ac^4}-\frac{8\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4}-\frac{\frac{390\,a^9c^{16}}{ax-1}+\frac{330\,a^9c^{16}}{(ax-1)^2}+\frac{205\,a^9c^{16}}{(ax-1)^3}+\frac{75\,a^9c^{16}}{(ax-1)^4}+\frac{12\,a^9c^{16}}{(ax-1)^5}}{15\,a^{10}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^4) - 8*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 1/15
*(390*a^9*c^16/(a*x - 1) + 330*a^9*c^16/(a*x - 1)^2 + 205*a^9*c^16/(a*x - 1
)^3 + 75*a^9*c^16/(a*x - 1)^4 + 12*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

Mupad [B]
time = 1.25, size = 109, normalized size = 1.04

$$\frac{x}{c^4}+\frac{\frac{311\,a\,x^2}{3}-\frac{181\,x}{3}+\frac{202}{15\,a}-82\,a^2\,x^3+26\,a^3\,x^4}{-a^5\,c^4\,x^5+5\,a^4\,c^4\,x^4-10\,a^3\,c^4\,x^3+10\,a^2\,c^4\,x^2-5\,a\,c^4\,x+c^4}+\frac{8\,\ln\left(a\,x-1\right)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a*x))^4*(a*x - 1)^2),x)
```

```
[Out] x/c^4 + ((311*a*x^2)/3 - (181*x)/3 + 202/(15*a) - 82*a^2*x^3 + 26*a^3*x^4)/
(c^4 + 10*a^2*c^4*x^2 - 10*a^3*c^4*x^3 + 5*a^4*c^4*x^4 - a^5*c^4*x^5 - 5*a*
c^4*x) + (8*log(a*x - 1))/(a*c^4)
```

3.413 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=135

$$-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{25c^4\csc^{-1}(ax)}{2a}-\frac{5c^4\tanh^{-1}\left(\sqrt{1\right.}}{a}$$

```
[Out] -25/2*c^4*arccsc(a*x)/a-5*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a-32/3*c^4*(1-1/
a^2/x^2)^(1/2)/a-1/3*c^4*(1-1/a^2/x^2)^(1/2)/a^3/x^2+5/2*c^4*(1-1/a^2/x^2)^
(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {6312, 1821, 1823, 858, 222, 272, 65, 214}

$$c^4x\sqrt{1-\frac{1}{a^2x^2}}-\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{5c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{25c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

```
[Out] (-32*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) - (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*
x^2) + (5*c^4*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*
x - (25*c^4*ArcCsc[a*x])/(2*a) - (5*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c^4\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{5c^5}{a}-\frac{10c^5x}{a^2}+\frac{10c^5x^2}{a^3}-\frac{5c^5x^3}{a^4}+\frac{c^5x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\text{Subst}\left(\int \frac{-\frac{15c^5}{a^3}+\frac{30c^5x}{a^4}-\frac{32c^5x^2}{a^5}+\frac{15c^5x^3}{a^6}}{x\sqrt{1-\frac{x^2}{a^2}}} d\right.}{3c}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^4\text{Subst}\left(\int \frac{\frac{30c^5}{a^5}-\frac{7}{}}{x\sqrt{}}\right.}{c}$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x$$

$$= -\frac{32c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} - \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + \frac{5c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x$$

Mathematica [A]
time = 0.10, size = 175, normalized size = 1.30

$$\frac{c^4\left(2-15ax+62a^2x^2+9a^3x^3-64a^4x^4+6a^5x^5+90a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-30a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\text{ArcSin}\left(\frac{1}{ax}\right)-30a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{6a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(2 - 15*a*x + 62*a^2*x^2 + 9*a^3*x^3 - 64*a^4*x^4 + 6*a^5*x^5 + 90*a^4
*Sqrt[1 - 1/(a^2*x^2)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 30*a^4*Sqrt[
1 - 1/(a^2*x^2)]*x^4*ArcSin[1/(a*x)] - 30*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^4*Arc
Tanh[Sqrt[1 - 1/(a^2*x^2)]]))/(6*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(117) = 234$.
time = 0.11, size = 290, normalized size = 2.15

method	result
risch	$-\frac{(ax+1)\left(64a^2x^2-15ax+2\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(a^3\sqrt{(ax+1)(ax-1)}-\frac{5a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{25a^3\arctan\left(\right.}{2}\right.}{a^4(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-66\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+96\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3-96\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4*(-66*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
a^4*x^4+96*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-96*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+66*(a^2*x^2-1)^(3/2)*(a^
2)^(1/2)*a^2*x^2-75*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-75*a^3*x^3*(a^2)^
(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+66*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*a^4*x^3-15*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+2*(a^2*x^2-1)^
(3/2)*(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1/2)/a^4/(a^2)^(1/2)/x^3
```

Maxima [A]
time = 0.47, size = 223, normalized size = 1.65

$$\frac{1}{3}\left(\frac{75\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{87\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+61\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-55\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-45\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(75*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 15*c^4*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 15*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (8
7*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 61*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 55
*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 45*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*
x - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
 + 1)^4 + a^2))*a
```

Fricas [A]
time = 0.36, size = 156, normalized size = 1.16

$$\frac{150\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+30\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(6\,a^4c^4x^4-58\,a^3c^4x^3-49\,a^2c^4x^2+13\,ac^4x-2\,c^4)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(150*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 30*a^3*c^4*x^3*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) + 30*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
1)) - 1) + (6*a^4*c^4*x^4 - 58*a^3*c^4*x^3 - 49*a^2*c^4*x^2 + 13*a*c^4*x -
2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^3}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-4*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x**3, x) + Integral(6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x*
*2, x) + Integral(-4*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x, x))/a**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(117) = 234.
time = 0.43, size = 265, normalized size = 1.96

$$\frac{25\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{a}+\frac{5\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^4\mathrm{sgn}(ax+1)}{a}-\frac{15\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^4|a|\mathrm{sgn}(ax+1)+60\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^4\mathrm{sgn}(ax+1)+132\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^4\mathrm{sgn}(ax+1)-15\left(x|a|-\sqrt{a^2x^2-1}\right)c^4|a|\mathrm{sgn}(ax+1)+64\,ac^4\mathrm{sgn}(ax+1)}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 25*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 5*c^4*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
4*sgn(a*x + 1)/a - 1/3*(15*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(
a*x + 1) + 60*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 132*(x*
abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 15*(x*abs(a) - sqrt(a^2*
x^2 - 1))*c^4*abs(a)*sgn(a*x + 1) + 64*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sq
rt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B]
time = 0.12, size = 185, normalized size = 1.37

$$\frac{25\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{15\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{55\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{61\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-29\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{10\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (25*c^4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (15*c^4*((a*x - 1)/(a*x + 1)
)^(1/2) + (55*c^4*((a*x - 1)/(a*x + 1))^(3/2))/3 - (61*c^4*((a*x - 1)/(a*x
+ 1))^(5/2))/3 - 29*c^4*((a*x - 1)/(a*x + 1))^(7/2))/(a + (2*a*(a*x - 1))/(
a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1)^4)/(a*x + 1)^4) - (
10*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.414 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=106

$$-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{13c^3\csc^{-1}(ax)}{2a}-\frac{4c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -13/2*c^3*arccsc(a*x)/a-4*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a-4*c^3*(1-1/a^2
/x^2)^(1/2)/a+1/2*c^3*(1-1/a^2/x^2)^(1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6312, 1821, 1823, 858, 222, 272, 65, 214}

$$c^3x\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{4c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{13c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

```
[Out] (-4*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) +
c^3*Sqrt[1 - 1/(a^2*x^2)]*x - (13*c^3*ArcCsc[a*x])/(2*a) - (4*c^3*ArcTanh[S
qrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^4}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{4c^4}{a}-\frac{6c^4x}{a^2}+\frac{4c^4x^2}{a^3}-\frac{c^4x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\text{Subst}\left(\int \frac{-\frac{8c^4}{a^3}+\frac{13c^4x}{a^4}-\frac{8c^4x^2}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^4\text{Subst}\left(\int \frac{\frac{8c^4}{a^5}-\cdots}{x\sqrt{1-\cdots}}\right.}{2c}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{(13c^3)\,\text{Subst}\left(\int -\cdots\right.}{2\cdots}$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} + \cdots$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} - \cdots$$

$$= -\frac{4c^3\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^3\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{13c^3\csc^{-1}(ax)}{2a} - \cdots$$

<u>**Mathematica**</u> **[A]**

time = 0.08, size = 167, normalized size = 1.58

$$\frac{c^3\left(-1+8ax-a^2x^2-8a^3x^3+2a^4x^4+10a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-8a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\text{ArcSin}\left(\frac{1}{ax}\right)-8a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{2a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(-1 + 8*a*x - a^2*x^2 - 8*a^3*x^3 + 2*a^4*x^4 + 10*a^3*Sqrt[1 - 1/(a^2
*x^2)]*x^3*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*
x^3*ArcSin[1/(a*x)] - 8*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^3*ArcTanh[Sqrt[1 - 1/(a
^2*x^2)]]))/(2*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(94) = 188.
time = 0.10, size = 266, normalized size = 2.51

method	result
risch	$-\frac{(ax+1)(8ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(a^2\sqrt{(ax+1)(ax-1)}-\frac{4a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{13a^2\arctan\left(\frac{1}{\sqrt{a^2x^2-\cdots}}\right.}{2}\cdots\right.}{a^3(ax-1)\cdots}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(-8\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+16\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2-16\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+\cdots}}{\sqrt{a^2}}\right.\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3*(-8*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a
^3*x^3+16*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-16*ln((a^2*x+(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+8*(a^2)^(1/2)*(a^2*x^2-1)
^(3/2)*a*x-13*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-13*(a^2)^(1/2)*arctan(1
/(a^2*x^2-1)^(1/2))*a^2*x^2+8*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2
)^(1/2))*a^3*x^2-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1/2)/a^3
/(a^2)^(1/2)/x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(94) = 188.
time = 0.46, size = 201, normalized size = 1.90

$$\left(\frac{13\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{4\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-5\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] (13*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 4*c^3*log(sqrt((a*x - 1)/(a
*x + 1)) + 1)/a^2 + 4*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (11*c^3*
((a*x - 1)/(a*x + 1))^(5/2) + 2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*sqr
t((a*x - 1)/(a*x + 1)))/((a*x - 1)*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1
)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3 + a^2))*a
```

Fricas [A]
time = 0.36, size = 143, normalized size = 1.35

$$\frac{26\,a^{2}c^{3}x^{2}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-8\,a^{2}c^{3}x^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+8\,a^{2}c^{3}x^{2}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^{3}c^{3}x^{3}-6\,a^{2}c^{3}x^{2}-7\,ac^{3}x+c^{3}\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(26*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) - 8*a^2*c^3*x^2*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 8*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x + 1))
 - 1) + (2*a^3*c^3*x^3 - 6*a^2*c^3*x^2 - 7*a*c^3*x + c^3)*sqrt((a*x - 1)/(a
*x + 1)))/(a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{3}\left(\int a^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{3}}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{2}}\,dx+\int\left(-\frac{3a^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**3*(Integral(a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**3, x) + Integral(3*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x**2, x) + Integral(-3*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x
, x))/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(94) = 188.
time = 0.42, size = 232, normalized size = 2.19

$$\frac{13\,c^{3}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)\mathrm{sgn}(ax+1)}{a}+\frac{4\,c^{3}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^{2}x^{2}-1}\,c^{3}\mathrm{sgn}(ax+1)}{a}-\frac{\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{3}c^{3}|a|\mathrm{sgn}(ax+1)+8\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}ac^{3}\mathrm{sgn}(ax+1)-\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)c^{3}|a|\mathrm{sgn}(ax+1)+8\,ac^{3}\mathrm{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}+1\right)^{2}a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 13*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 4*c^3*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^
3*sgn(a*x + 1)/a - ((x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn(a*x + 1
) + 8*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - (x*abs(a) - sqr
t(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 8*a*c^3*sgn(a*x + 1))/(((x*abs(a)
 - sqrt(a^2*x^2 - 1))^2 + 1)^2*a*abs(a))
```

Mupad [B]
time = 1.25, size = 163, normalized size = 1.54

$$\frac{2\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{13\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{8\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 5*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 11
*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (13*c^3*atan(((a*x - 1)/
(a*x + 1))^(1/2)))/a - (8*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.415 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=77

$$-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+c^2\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{3c^2\csc^{-1}(ax)}{a}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -3*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a-c^2*(1-1/a^2/x^2)
^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {6312, 1821, 1823, 858, 222, 272, 65, 214}

$$c^2x\sqrt{1-\frac{1}{a^2x^2}}-\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{3c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

```
[Out] -((c^2*Sqrt[1 - 1/(a^2*x^2)])/a) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x - (3*c^2*Arc
Csc[a*x])/a - (3*c^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^3}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{3c^3}{a}-\frac{3c^3x}{a^2}+\frac{c^3x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^2\text{Subst}\left(\int \frac{-\frac{3c^3}{a^3}+\frac{3c^3x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{a^2}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{}{x\sqrt{1}}\right.}{2a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} - (3ac^2)\,\text{Subst}\left(\int \frac{}{a^2}\right.$$

$$= -\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{3c^2\csc^{-1}(ax)}{a} - \frac{3c^2\tanh^{-1}\left(\sqrt{1-\right.}{a}$$

Mathematica [A]

time = 0.13, size = 55, normalized size = 0.71

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\,(-1+ax) - 3\text{ArcSin}\left(\frac{1}{ax}\right) - 3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*ArcTanh[Sqrt
[1 - 1/(a^2*x^2)]]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(71) = 142$.
time = 0.10, size = 227, normalized size = 2.95

method	result
risch	$-\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(a\sqrt{(ax+1)(ax-1)}-\frac{3a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c^2\sqrt{\cdots}}{a^2(ax-1)\cdots}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+4\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,ax-4\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(a\cdots}}{\sqrt{a^2}}\right.\cdots\right.}{\sqrt{(ax+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2
+4*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-4*ln((a^2*x+(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*(a^2)^(
1/2)*(a^2*x^2-1)^(1/2)*a*x-3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x+ln
((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x)/((a*x+1)*(a*x-1)
)^(1/2)/a^2/(a^2)^(1/2)/x
```

Maxima [A]
time = 0.46, size = 126, normalized size = 1.64

$$-\left(\frac{4\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6
*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x
+ 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [A]

time = 0.33, size = 113, normalized size = 1.47

$$\frac{6\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2c^2x^2-c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] (6*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c^2*x*log(sqrt((a*x - 1)
/(a*x + 1)) + 1) + 3*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c^2*
x^2 - c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(1/2),x)
[Out] c**2*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**2, x) + Integral(-2*a*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/x, x))/a**2
```

Giac [A]

time = 0.41, size = 130, normalized size = 1.69

$$\frac{6\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^2\operatorname{sgn}(ax+1)}{a}-\frac{2\,c^2\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] 6*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2
*sgn(a*x + 1)/a - 2*c^2*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1
)*abs(a))
```

Mupad [B]

time = 1.22, size = 90, normalized size = 1.17

$$\frac{4\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}+\frac{6\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c^2*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) + (6*
c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2*atanh(((a*x - 1)/(a*x + 1
))^(1/2)))/a
```

3.416 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=49

$$c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] -c*arccsc(a*x)/a-2*c*arctanh((1-1/a^2/x^2)^(1/2))/a+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.350$, Rules used = {6312, 1821, 858, 222, 272, 65, 214}

$$cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}-\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^ArcCoth[a*x],x]
```

```
[Out] c*Sqrt[1 - 1/(a^2*x^2)]*x - (c*ArcCsc[a*x])/a - (2*c*ArcTanh[Sqrt[1 - 1/(a^
2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx=-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^2}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{\text{Subst}\left(\int\frac{\frac{2c^2}{a}-\frac{c^2x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}+\frac{(2c)\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right.}{a}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}+\frac{c\text{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,x,\frac{1}{x^2}\right)}{a}$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}-(2ac)\text{Subst}\left(\int\frac{1}{a^2-a^2x^2}\,dx,x,\sqrt{1-\frac{1}{a^2x^2}}\right.$$

$$=c\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{c\csc^{-1}(ax)}{a}-\frac{2c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 1.49

$$\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x-2\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2\text{ArcSin}\left(\frac{1}{ax}\right)-2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^ArcCoth[a*x],x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x - 2*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*Arc
Sin[1/(a*x)] - 2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(45) = 90$.
time = 0.08, size = 136, normalized size = 2.78

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(2a\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)+\sqrt{a^2x^2-1}\,\sqrt{a^2}+\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\sqrt{a^2}}{\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x
-1))^(1/2))/(a^2)^(1/2))+(a^2*x^2-1)^(1/2)*(a^2)^(1/2)+arctan(1/(a^2*x^2-1)
^(1/2))*(a^2)^(1/2)-2*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/((a*x+1)*(a*x-1)
)^(1/2)/a/(a^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(45) = 90$.
time = 0.47, size = 114, normalized size = 2.33

$$-2\,a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}-\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) - c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2
 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.35, size = 88, normalized size = 1.80

$$\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+2\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) - 2*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a*c*x + c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x}\right)dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/x, x))/a
```

Giac [A]
time = 0.42, size = 85, normalized size = 1.73

$$\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{2\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c\operatorname{sgn}(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 2*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a
```

Mupad [B]
time = 0.07, size = 82, normalized size = 1.67

$$\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{4\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,(a\,x-1)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (4*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1
))
```

3.417 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=19

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6312, 270}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/c
```

Maple [A]
time = 0.11, size = 28, normalized size = 1.47

method	result	size
gosper	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}$	28
trager	$\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{ac}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.
time = 0.26, size = 44, normalized size = 2.32

$$-\frac{2\,a\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] -2*a*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c)
```

Fricas [A]
time = 0.33, size = 27, normalized size = 1.42

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\int\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x),x)
```

```
[Out] a*Integral(x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x - 1), x)/c
```

Giac [A]
time = 0.41, size = 24, normalized size = 1.26

$$\frac{\sqrt{a^2x^2-1}\,\mathrm{sgn}(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] sqrt(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

Mupad [B]
time = 0.05, size = 39, normalized size = 2.05

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,\left(a\,x-1\right)}{a\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x)),x)
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1))
```

3.418 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2} dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a/c^2+2*x*(1-1/a^2/x^2)^(1/2)/c^2-a*x*(1-1/a^2
/x^2)^(1/2)/c^2/(a-1/x)
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6312, 871, 821, 272, 65, 214}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{ax\sqrt{1-\frac{1}{a^2x^2}}}{c^2\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - (a*Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*(a - x^(
-1))) + ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\left(a-\frac{1}{x}\right)} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 0.95

$$\frac{-2-ax+a^2x^2+a\sqrt{1-\frac{1}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^2),x]
```

```
[Out] (-2 - a*x + a^2*x^2 + a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(67) = 134$.
time = 0.15, size = 255, normalized size = 3.49

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{a^4\left(x-\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)\,(a\cdots}}{c^2(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-3\sqrt{(ax+1)\,(ax-1)}\,\sqrt{a^2}\,a^2x^2-2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)\,(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+((ax+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(-3*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1
/2)*a^2*x^2-2*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a
^3*x^2+((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+6*((a*x+1)*(a*x-1))^(1/2)*(a^2)^
(1/2)*a*x+4*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2
*x-3*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x+1)*(a*x-1))^(1/2)/c^2/(a*x-1)^2/(a^2)^
(1/2)
```

Maxima [A]
time = 0.25, size = 120, normalized size = 1.64

$$-a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2) - a^2*
c^2*sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^
2) + log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A]
time = 0.34, size = 97, normalized size = 1.33

$$\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2x-ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^2
*x - a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 2*a*x + 1
), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.23, size = 62, normalized size = 0.85

$$\frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^2,x)
```

```
[Out] (2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 4)/(2*a*c^2*((a*x - 1)/(a*x + 1))^(1/2))
```

3.419 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3} dx$

Optimal. Leaf size=105

$$-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

```
[Out] -2/3*(a+1/x)/a^2/c^3/(1-1/a^2/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c
^3+1/3*(-6*a-7/x)/a^2/c^3/(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^3
```

Rubi [A]
time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (-2*(a + x^(-1)))/(3*a^2*c^3*(1 - 1/(a^2*x^2))^(3/2)) - (6*a + 7/x)/(3*a^2*
c^3*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^3 + (2*ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]])/(a*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx=-\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{c^5}$$

$$=-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}+\frac{\text{Subst}\left(\int \frac{-3c^2-\frac{6c^2x}{a}-\frac{4c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$=-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\text{Subst}\left(\int \frac{3c^2+\frac{6c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{3c^5}$$

$$=-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}-\frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx\right.}{ac^3}$$

$$=-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}-\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx,\right.}{ac^3}$$

$$=-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}+\frac{(2a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx,\right.}{c^3}$$

$$=-\frac{2\left(a+\frac{1}{x}\right)}{3a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{6a+\frac{7}{x}}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^3}$$

Mathematica [A]
time = 0.04, size = 94, normalized size = 0.90

$$\frac{10-4ax-11a^2x^2+3a^3x^3+6a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^3),x]
```

```
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*
x)*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 +
 a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(93) = 186.
time = 0.14, size = 344, normalized size = 3.28

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\Bigg(\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^3\sqrt{a^2}}-\frac{8\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{3a^5\left(x-\frac{1}{a}\right)}-\frac{\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\,(x}}{3a^6\left(x-\frac{1}{a}\right)^2}}{c^3(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\Bigg(-27\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3+15\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(-27*(a^2)^(1/2)*((a*x+1)*(a*x-1))^
(1/2)*a^3*x^3-24*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)
)*a^4*x^3+15*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+81*((a*x+1)*(a*x-1))^(
1/2)*(a^2)^(1/2)*a^2*x^2+72*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/
(a^2)^(1/2))*a^3*x^2-13*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-81*((a*x+1)*(a*
x-1))^(1/2)*(a^2)^(1/2)*a*x-72*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
))/(a^2)^(1/2))*a^2*x+27*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+24*a*ln((a^2*x
+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x+1)*(a*x-1))^(1/2)
/c^3/(a*x-1)^3/(a^2)^(1/2)
```

Maxima [A]

time = 0.26, size = 137, normalized size = 1.30

$$\frac{1}{6}\,a\left(\frac{\frac{14\,(ax-1)}{ax+1}-\frac{27\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] 1/6*a*((14*(a*x - 1)/(a*x + 1) - 27*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^3*(
(a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log(
sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 12*log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^3))
```

Fricas [A]
time = 0.35, size = 134, normalized size = 1.28

$$\frac{6\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-11\,a^2x^2-4\,ax+10)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^3x^2-2\,a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 11*a^2*x^2
 - 4*a*x + 10)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^
3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\int\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-3a^2x^2+3ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*Integral(x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - 3*a**2*x*
*2 + 3*a*x - 1), x)/c**3
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.25, size = 105, normalized size = 1.00

$$\frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}-\frac{\frac{14\,(a\,x-1)}{3\,(a\,x+1)}-\frac{9\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-2\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^3,x)
```

```
[Out] (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^3) - ((14*(a*x - 1))/(3*(a*x +
1)) - (9*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(2*a*c^3*((a*x - 1)/(a*x + 1))^(3/
2) - 2*a*c^3*((a*x - 1)/(a*x + 1))^(5/2))
```

3.420 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=138

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] -4/5*(a+1/x)/a^2/c^4/(1-1/a^2/x^2)^(5/2)+1/5*(-5*a-7/x)/a^2/c^4/(1-1/a^2/x^
2)^(3/2)+3*arctanh((1-1/a^2/x^2)^(1/2))/a/c^4+1/5*(-15*a-19/x)/a^2/c^4/(1-1
/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A]
time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^4}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (-4*(a + x^(-1)))/(5*a^2*c^4*(1 - 1/(a^2*x^2))^(5/2)) - (5*a + 7/x)/(5*a^2*
c^4*(1 - 1/(a^2*x^2))^(3/2)) - (15*a + 19/x)/(5*a^2*c^4*Sqrt[1 - 1/(a^2*x^2
)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^4 + (3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a
*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^3}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^3-\frac{15c^3x}{a}-\frac{16c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^3+\frac{45c^3x}{a}+\frac{42c^3x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^3-\frac{45c^3}{a}\cdots}{x^2\sqrt{1-\frac{x^2}{a^2}}}\cdots\right)}{15c^7}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{3\text{Su}\cdots}{\cdots}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} - \frac{3\text{Su}\cdots}{\cdots}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{(3a\cdots}{\cdots}$$

$$= -\frac{4\left(a+\frac{1}{x}\right)}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{5a+\frac{7}{x}}{5a^2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{15a+\frac{19}{x}}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^4} + \frac{3\,\text{ta}\cdots}{\cdots}$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 0.75

$$\frac{-24+33ax+18a^2x^2-34a^3x^3+5a^4x^4+15a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{5a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^4),x]
```

```
[Out] (-24 + 33*a*x + 18*a^2*x^2 - 34*a^3*x^3 + 5*a^4*x^4 + 15*a*Sqrt[1 - 1/(a^2*
x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(5*a^2*c^4*Sqrt[1 - 1/
(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(122) = 244$.
time = 0.14, size = 436, normalized size = 3.16

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{24\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{5a^6\left(x-\frac{1}{a}\right)}-\frac{6\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(\right.}}{5a^7\left(x-\frac{1}{a}\right)^2}\right.}{c^4(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-125\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^4x^4-120\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^5x^4+85\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)/a*(-125*(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(1/2)*a^4*x^4-120*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/
2))*a^5*x^4+85*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2+500*(a^2)^(1/2)*
((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+480*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(1/2))/(a^2)^(1/2))*a^4*x^3-148*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-75
0*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-720*ln((a^2*x+(a^2)^(1/2)*((a
*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+67*((a*x+1)*(a*x-1))^(3/2)*(a^2)
^(1/2)+500*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+480*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-125*(a^2)^(1/2)*((a*x+1)*(a*x
-1))^(1/2)-120*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)
))/((a*x+1)*(a*x-1))^(1/2)/c^4/(a*x-1)^4/(a^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 153, normalized size = 1.11

$$\frac{1}{20}\,a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A]
time = 0.34, size = 170, normalized size = 1.23

$$\frac{15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(5\,a^4x^4-34\,a^3x^3+18\,a^2x^2+33\,ax-24)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^4x^3-3\,a^3c^4x^2+3\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-4a^3x^3+6a^2x^2-4ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 4*a**3*x*
*3 + 6*a**2*x**2 - 4*a*x + 1), x)/c**4
```

Giac [A]
time = 0.45, size = 59, normalized size = 0.43

$$-\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{c^4|a|}+\frac{\sqrt{a^2x^2-1}\,\operatorname{sgn}(ax+1)}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] -3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^4*abs(a)) + sqrt
(a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^4)
```

Mupad [B]
time = 1.24, size = 121, normalized size = 0.88

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}-\frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{25\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{9\,(a\,x-1)}{5\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}-4\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^4,x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^4) - ((15*(a*x - 1)^2)/(a*x + 1
)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(4*
a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

3.421 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=65

$$\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}$$

```
[Out] 1/3*c^4/a^4/x^3-3*c^4/a^3/x^2+16*c^4/a^2/x+c^4*x+26*c^4*ln(x)/a-32*c^4*ln(a
*x+1)/a
```

Rubi [A]
time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6266, 6264, 90}

$$\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(ax+1)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[x])/a - (32*c^4*Log[1 + a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx\\ &= -\frac{c^4\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^4}{x^4}\,dx}{a^4}\\ &= -\frac{c^4\int \frac{(1-ax)^5}{x^4(1+ax)}\,dx}{a^4}\\ &= -\frac{c^4\int \left(-a^4+\frac{1}{x^4}-\frac{6a}{x^3}+\frac{16a^2}{x^2}-\frac{26a^3}{x}+\frac{32a^4}{1+ax}\right) dx}{a^4}\\ &= \frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(x)}{a}-\frac{32c^4\log(1+ax)}{a}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 67, normalized size = 1.03

$$\frac{c^4}{3a^4x^3}-\frac{3c^4}{a^3x^2}+\frac{16c^4}{a^2x}+c^4x+\frac{26c^4\log(ax)}{a}-\frac{32c^4\log(1+ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(3*a^4*x^3) - (3*c^4)/(a^3*x^2) + (16*c^4)/(a^2*x) + c^4*x + (26*c^4*Lo
g[a*x])/a - (32*c^4*Log[1 + a*x])/a
```

Maple [A]
time = 0.18, size = 51, normalized size = 0.78

method	result
default	$\frac{c^4\left(a^4x-32a^3\ln(ax+1)+\frac{1}{3x^3}-\frac{3a}{x^2}+\frac{16a^2}{x}+26a^3\ln(x)\right)}{a^4}$
risch	$c^4x+\frac{16a^2c^4x^2-3a\,c^4x+\frac{1}{3}c^4}{a^4x^3}+\frac{26c^4\ln(-x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
norman	$\frac{a^3c^4x^4+\frac{c^4}{3a}-3c^4x+16c^4a\,x^2}{a^3x^3}+\frac{26c^4\ln(x)}{a}-\frac{32c^4\ln(ax+1)}{a}$
meijerg	$\frac{c^4(ax-\ln(ax+1))}{a}-\frac{5c^4\ln(ax+1)}{a}+\frac{10c^4(-\ln(ax+1)+\ln(x)+\ln(a))}{a}-\frac{10c^4\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\frac{5c^4\left(-\ln(ax+1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^4/a^4*(a^4*x-32*a^3*ln(a*x+1)+1/3/x^3-3*a/x^2+16*a^2/x+26*a^3*ln(x))
```

Maxima [A]
time = 0.26, size = 60, normalized size = 0.92

$$c^4x - \frac{32\,c^4\log\left(ax+1\right)}{a} + \frac{26\,c^4\log\left(x\right)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 32*c^4*log(a*x + 1)/a + 26*c^4*log(x)/a + 1/3*(48*a^2*c^4*x^2 - 9*a
*c^4*x + c^4)/(a^4*x^3)
```

Fricas [A]
time = 0.36, size = 71, normalized size = 1.09

$$\frac{3\,a^4c^4x^4 - 96\,a^3c^4x^3\log\left(ax+1\right) + 78\,a^3c^4x^3\log\left(x\right) + 48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^4*x^4 - 96*a^3*c^4*x^3*log(a*x + 1) + 78*a^3*c^4*x^3*log(x) +
48*a^2*c^4*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Sympy [A]
time = 0.23, size = 56, normalized size = 0.86

$$c^4x + \frac{2c^4\cdot\left(13\log\left(x\right) - 16\log\left(x+\frac{1}{a}\right)\right)}{a} + \frac{48a^2c^4x^2 - 9ac^4x + c^4}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] c**4*x + 2*c**4*(13*log(x) - 16*log(x + 1/a))/a + (48*a**2*c**4*x**2 - 9*a*
c**4*x + c**4)/(3*a**4*x**3)
```

Giac [A]
time = 0.41, size = 62, normalized size = 0.95

$$c^4x - \frac{32\,c^4\log\left(|ax+1|\right)}{a} + \frac{26\,c^4\log\left(|x|\right)}{a} + \frac{48\,a^2c^4x^2 - 9\,ac^4x + c^4}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 32*c^4*log(abs(a*x + 1))/a + 26*c^4*log(abs(x))/a + 1/3*(48*a^2*c^4
*x^2 - 9*a*c^4*x + c^4)/(a^4*x^3)
```

Mupad [B]
time = 0.10, size = 61, normalized size = 0.94

$$c^4\,x+\frac{16\,a^2\,c^4\,x^2-3\,a\,c^4\,x+\frac{c^4}{3}}{a^4\,x^3}+\frac{26\,c^4\,\ln\left(x\right)}{a}-\frac{32\,c^4\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] c^4*x + (c^4/3 + 16*a^2*c^4*x^2 - 3*a*c^4*x)/(a^4*x^3) + (26*c^4*log(x))/a
- (32*c^4*log(a*x + 1))/a
```

3.422 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=54

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}$$

```
[Out] -1/2*c^3/a^3/x^2+5*c^3/a^2/x+c^3*x+11*c^3*ln(x)/a-16*c^3*ln(a*x+1)/a
```

Rubi [A]
time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 90}

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(ax+1)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/2*c^3/(a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[x])/a - (16*c^3*
Log[1 + a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx\\
&= \frac{c^3\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^3}{x^3}\,dx}{a^3}\\
&= \frac{c^3\int\frac{(1-ax)^4}{x^3(1+ax)}\,dx}{a^3}\\
&= \frac{c^3\int\left(a^3+\frac{1}{x^3}-\frac{5a}{x^2}+\frac{11a^2}{x}-\frac{16a^3}{1+ax}\right)\,dx}{a^3}\\
&= -\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(x)}{a}-\frac{16c^3\log(1+ax)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 1.04

$$-\frac{c^3}{2a^3x^2}+\frac{5c^3}{a^2x}+c^3x+\frac{11c^3\log(ax)}{a}-\frac{16c^3\log(1+ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/2*c^3/(a^3*x^2) + (5*c^3)/(a^2*x) + c^3*x + (11*c^3*Log[a*x])/a - (16*c^
3*Log[1 + a*x])/a
```

Maple [A]
time = 0.16, size = 43, normalized size = 0.80

method	result
default	$\frac{c^3\left(a^3x-16a^2\ln(ax+1)-\frac{1}{2x^2}+\frac{5a}{x}+11a^2\ln(x)\right)}{a^3}$
risch	$c^3x+\frac{5a\,c^3x-\frac{1}{2}c^3}{a^3x^2}-\frac{16c^3\ln(ax+1)}{a}+\frac{11c^3\ln(-x)}{a}$
norman	$\frac{a^2c^3x^3-\frac{c^3}{2a}+5c^3x}{a^2x^2}+\frac{11c^3\ln(x)}{a}-\frac{16c^3\ln(ax+1)}{a}$
meijerg	$\frac{c^3(ax-\ln(ax+1))}{a}-\frac{4c^3\ln(ax+1)}{a}+\frac{6c^3(-\ln(ax+1)+\ln(x)+\ln(a))}{a}-\frac{4c^3\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\frac{c^3\left(-\ln(ax+1)+\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^3*(a^3*x-16*a^2*ln(a*x+1)-1/2/x^2+5*a/x+11*a^2*ln(x))
```

Maxima [A]
time = 0.26, size = 51, normalized size = 0.94

$$c^3x - \frac{16\,c^3\log\left(ax+1\right)}{a} + \frac{11\,c^3\log\left(x\right)}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 16*c^3*log(a*x + 1)/a + 11*c^3*log(x)/a + 1/2*(10*a*c^3*x - c^3)/(a
^3*x^2)
```

Fricas [A]
time = 0.35, size = 62, normalized size = 1.15

$$\frac{2\,a^3c^3x^3 - 32\,a^2c^3x^2\log\left(ax+1\right) + 22\,a^2c^3x^2\log\left(x\right) + 10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*c^3*x^3 - 32*a^2*c^3*x^2*log(a*x + 1) + 22*a^2*c^3*x^2*log(x) +
10*a*c^3*x - c^3)/(a^3*x^2)
```

Sympy [A]
time = 0.19, size = 42, normalized size = 0.78

$$c^3x + \frac{c^3\cdot\left(11\log\left(x\right) - 16\log\left(x+\frac{1}{a}\right)\right)}{a} + \frac{10ac^3x - c^3}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*x + c**3*(11*log(x) - 16*log(x + 1/a))/a + (10*a*c**3*x - c**3)/(2*a**
3*x**2)
```

Giac [A]
time = 0.40, size = 53, normalized size = 0.98

$$c^3x - \frac{16\,c^3\log\left(|ax+1|\right)}{a} + \frac{11\,c^3\log\left(|x|\right)}{a} + \frac{10\,ac^3x - c^3}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 16*c^3*log(abs(a*x + 1))/a + 11*c^3*log(abs(x))/a + 1/2*(10*a*c^3*x
 - c^3)/(a^3*x^2)
```

Mupad [B]
time = 0.08, size = 51, normalized size = 0.94

$$c^3\,x-\frac{\frac{c^3}{2}-5\,a\,c^3\,x}{a^3\,x^2}+\frac{11\,c^3\,\ln\left(x\right)}{a}-\frac{16\,c^3\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^3*(a*x - 1))/(a*x + 1),x)
[Out] c^3*x - (c^3/2 - 5*a*c^3*x)/(a^3*x^2) + (11*c^3*log(x))/a - (16*c^3*log(a*x
 + 1))/a
```

3.423 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=40

$$\frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}$$

```
[Out] c^2/a^2/x+c^2*x+4*c^2*ln(x)/a-8*c^2*ln(a*x+1)/a
```

Rubi [A]
time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6266, 6264, 90}

$$\frac{c^2}{a^2x}+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(ax+1)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[x])/a - (8*c^2*Log[1 + a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx\\ &= -\frac{c^2\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^2}{x^2}dx}{a^2}\\ &= -\frac{c^2\int \frac{(1-ax)^3}{x^2(1+ax)}dx}{a^2}\\ &= -\frac{c^2\int\left(-a^2+\frac{1}{x^2}-\frac{4a}{x}+\frac{8a^2}{1+ax}\right)dx}{a^2}\\ &= \frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}-\frac{8c^2\log(1+ax)}{a}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.05

$$\frac{c^2}{a^2x}+c^2x+\frac{4c^2\log(ax)}{a}-\frac{8c^2\log(1+ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^2/E^(2*ArcCoth[a*x]),x]
[Out] c^2/(a^2*x) + c^2*x + (4*c^2*Log[a*x])/a - (8*c^2*Log[1 + a*x])/a
```

Maple [A]
time = 0.13, size = 31, normalized size = 0.78

method	result	size
default	$\frac{c^2\left(a^2x-8a\ln(ax+1)+\frac{1}{x}+4a\ln(x)\right)}{a^2}$	31
risch	$\frac{c^2}{a^2x}+c^2x+\frac{4c^2\ln(-x)}{a}-\frac{8c^2\ln(ax+1)}{a}$	43
norman	$\frac{\frac{c^2}{a}+a\,c^2x^2}{ax}+\frac{4c^2\ln(x)}{a}-\frac{8c^2\ln(ax+1)}{a}$	49
meijerg	$\frac{c^2(ax-\ln(ax+1))}{a}-\frac{3c^2\ln(ax+1)}{a}+\frac{3c^2(-\ln(ax+1)+\ln(x)+\ln(a))}{a}-\frac{c^2\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^2/a^2*(a^2*x-8*a*ln(a*x+1)+1/x+4*a*ln(x))
```

Maxima [A]
time = 0.26, size = 40, normalized size = 1.00

$$c^2x - \frac{8\,c^2\log\left(ax+1\right)}{a} + \frac{4\,c^2\log\left(x\right)}{a} + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c^2*x - 8*c^2*log(a*x + 1)/a + 4*c^2*log(x)/a + c^2/(a^2*x)
```

Fricas [A]
time = 0.37, size = 43, normalized size = 1.08

$$\frac{a^2c^2x^2 - 8\,ac^2x\log\left(ax+1\right) + 4\,ac^2x\log\left(x\right) + c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a^2*c^2*x^2 - 8*a*c^2*x*log(a*x + 1) + 4*a*c^2*x*log(x) + c^2)/(a^2*x)
```

Sympy [A]
time = 0.14, size = 31, normalized size = 0.78

$$c^2x + \frac{4c^2\left(\log\left(x\right) - 2\log\left(x+\frac{1}{a}\right)\right)}{a} + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*(a*x-1)/(a*x+1),x)
[Out] c**2*x + 4*c**2*(log(x) - 2*log(x + 1/a))/a + c**2/(a**2*x)
```

Giac [A]
time = 0.40, size = 42, normalized size = 1.05

$$c^2x - \frac{8\,c^2\log\left(|ax+1|\right)}{a} + \frac{4\,c^2\log\left(|x|\right)}{a} + \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c^2*x - 8*c^2*log(abs(a*x + 1))/a + 4*c^2*log(abs(x))/a + c^2/(a^2*x)
```

Mupad [B]
time = 1.22, size = 40, normalized size = 1.00

$$c^2\,x + \frac{c^2}{a^2\,x} + \frac{4\,c^2\,\ln\left(x\right)}{a} - \frac{8\,c^2\,\ln\left(a\,x+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^2*(a*x - 1))/(a*x + 1),x)
[Out] c^2*x + c^2/(a^2*x) + (4*c^2*log(x))/a - (8*c^2*log(a*x + 1))/a
```

3.424 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=23

$$cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

[Out] c*x+c*ln(x)/a-4*c*ln(a*x+1)/a

Rubi [A]

time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6302, 6266, 6264, 84}

$$\frac{c\log(x)}{a}-\frac{4c\log(ax+1)}{a}+cx$$

Antiderivative was successfully verified.

[In] Int[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]

[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$
$$= \frac{c\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)}{x}\,dx}{a}$$
$$= \frac{c\int \frac{(1-ax)^2}{x(1+ax)}\,dx}{a}$$
$$= \frac{c\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)\,dx}{a}$$
$$= cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$cx+\frac{c\log(x)}{a}-\frac{4c\log(1+ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))/E^(2*ArcCoth[a*x]),x]
[Out] c*x + (c*Log[x])/a - (4*c*Log[1 + a*x])/a
```

Maple [A]
time = 0.11, size = 20, normalized size = 0.87

method	result	size
default	$\frac{c(ax-4\ln(ax+1)+\ln(x))}{a}$	20
norman	$cx+\frac{c\ln(x)}{a}-\frac{4c\ln(ax+1)}{a}$	24
risch	$cx-\frac{4c\ln(ax+1)}{a}+\frac{c\ln(-x)}{a}$	26
meijerg	$\frac{c(ax-\ln(ax+1))}{a}-\frac{2c\ln(ax+1)}{a}+\frac{c(-\ln(ax+1)+\ln(x)+\ln(a))}{a}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c/a*(a*x-4*ln(a*x+1)+ln(x))
```

Maxima [A]
time = 0.25, size = 23, normalized size = 1.00

$$cx-\frac{4\,c\log\left(ax+1\right)}{a}+\frac{c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c*x - 4*c*log(a*x + 1)/a + c*log(x)/a
```

Fricas [A]
time = 0.36, size = 22, normalized size = 0.96

$$\frac{acx - 4\,c\log\left(ax + 1\right) + c\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a*c*x - 4*c*log(a*x + 1) + c*log(x))/a
```

Sympy [A]
time = 0.11, size = 17, normalized size = 0.74

$$cx + \frac{c\left(\log\left(x\right) - 4\log\left(x + \frac{1}{a}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x)
[Out] c*x + c*(log(x) - 4*log(x + 1/a))/a
```

Giac [A]
time = 0.41, size = 25, normalized size = 1.09

$$cx - \frac{4\,c\log\left(|ax + 1|\right)}{a} + \frac{c\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c*x - 4*c*log(abs(a*x + 1))/a + c*log(abs(x))/a
```

Mupad [B]
time = 0.06, size = 23, normalized size = 1.00

$$c\,x + \frac{c\,\ln\left(x\right)}{a} - \frac{4\,c\,\ln\left(a\,x + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))*(a*x - 1))/(a*x + 1),x)
[Out] c*x + (c*log(x))/a - (4*c*log(a*x + 1))/a
```

3.425 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=20

$$\frac{x}{c}-\frac{\log(1+ax)}{ac}$$

```
[Out] x/c-ln(a*x+1)/a/c
```

Rubi [A]
time = 0.08, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6266, 6264, 45}

$$\frac{x}{c}-\frac{\log(ax+1)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] x/c - Log[1 + a*x]/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$$
$$= \frac{a\int \frac{e^{-2\tanh^{-1}(ax)}x}{1-ax}\,dx}{c}$$
$$= \frac{a\int \frac{x}{1+ax}\,dx}{c}$$
$$= \frac{a\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)\,dx}{c}$$
$$= \frac{x}{c}-\frac{\log(1+ax)}{ac}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.10

$$\frac{a\left(\frac{x}{a}-\frac{\log(1+ax)}{a^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))),x]
[Out] (a*(x/a - Log[1 + a*x]/a^2))/c
```

Maple [A]
time = 0.09, size = 23, normalized size = 1.15

method	result	size
norman	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
risch	$\frac{x}{c}-\frac{\ln(ax+1)}{ac}$	21
default	$\frac{a\left(\frac{x}{a}-\frac{\ln(ax+1)}{a^2}\right)}{c}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x),x,method=_RETURNVERBOSE)
[Out] a/c*(x/a-ln(a*x+1)/a^2)
```

Maxima [A]
time = 0.26, size = 20, normalized size = 1.00

$$\frac{x}{c}-\frac{\log\left(ax+1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] x/c - log(a*x + 1)/(a*c)
```

Fricas [A]
time = 0.34, size = 19, normalized size = 0.95

$$\frac{ax - \log(ax + 1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] (a*x - log(a*x + 1))/(a*c)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.85

$$a\left(\frac{x}{ac} - \frac{\log(ax + 1)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x)
```

```
[Out] a*(x/(a*c) - log(a*x + 1)/(a**2*c))
```

Giac [A]
time = 0.40, size = 21, normalized size = 1.05

$$\frac{x}{c} - \frac{\log(|ax + 1|)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] x/c - log(abs(a*x + 1))/(a*c)
```

Mupad [B]
time = 1.20, size = 19, normalized size = 0.95

$$-\frac{\ln(a\,x + 1) - a\,x}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))*(a*x + 1)),x)
```

```
[Out] -(log(a*x + 1) - a*x)/(a*c)
```

3.426 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}$$

```
[Out] x/c^2-arctanh(a*x)/a/c^2
```

Rubi [A]
time = 0.09, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6302, 6266, 6264, 84, 213}

$$\frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] x/c^2 - ArcTanh[a*x]/(a*c^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx \\
&= -\frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{(1-ax)^2}\,dx}{c^2} \\
&= -\frac{a^2\int \frac{x^2}{(1-ax)(1+ax)}\,dx}{c^2} \\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{1}{a^2(-1+a^2x^2)}\right)dx}{c^2} \\
&= \frac{x}{c^2}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2} \\
&= \frac{x}{c^2}-\frac{\tanh^{-1}(ax)}{ac^2}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.
time = 0.05, size = 39, normalized size = 2.17

$$\frac{x}{c^2}+\frac{\log(1-ax)}{2ac^2}-\frac{\log(1+ax)}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^2),x]
[Out] x/c^2 + Log[1 - a*x]/(2*a*c^2) - Log[1 + a*x]/(2*a*c^2)
```

Maple [A]
time = 0.09, size = 36, normalized size = 2.00

method	result	size
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{\ln(ax+1)}{2a^3}+\frac{\ln(ax-1)}{2a^3}\right)}{c^2}$	36
risch	$\frac{x}{c^2}-\frac{\ln(ax+1)}{2a\,c^2}+\frac{\ln(-ax+1)}{2a\,c^2}$	36

norman	$\frac{\frac{a\,x^2}{c}-\frac{x}{c}}{c(ax-1)}+\frac{\ln(ax-1)}{2a\,c^2}-\frac{\ln(ax+1)}{2a\,c^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c^2*(x/a^2-1/2/a^3*ln(a*x+1)+1/2/a^3*ln(a*x-1))
```

Maxima [A]
time = 0.25, size = 34, normalized size = 1.89

$$\frac{x}{c^2}-\frac{\log\left(ax+1\right)}{2\,ac^2}+\frac{\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] x/c^2 - 1/2*log(a*x + 1)/(a*c^2) + 1/2*log(a*x - 1)/(a*c^2)
```

Fricas [A]
time = 0.35, size = 27, normalized size = 1.50

$$\frac{2\,ax-\log\left(ax+1\right)+\log\left(ax-1\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x - log(a*x + 1) + log(a*x - 1))/(a*c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.07, size = 34, normalized size = 1.89

$$a^2\left(\frac{x}{a^2c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{2}-\frac{\log\left(x+\frac{1}{a}\right)}{2}}{a^3c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(x/(a**2*c**2) + (log(x - 1/a)/2 - log(x + 1/a)/2)/(a**3*c**2))
```

Giac [A]
time = 0.41, size = 36, normalized size = 2.00

$$\frac{x}{c^2}-\frac{\log\left(|ax+1|\right)}{2\,ac^2}+\frac{\log\left(|ax-1|\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 1/2*log(abs(a*x + 1))/(a*c^2) + 1/2*log(abs(a*x - 1))/(a*c^2)
```

Mupad [B]
time = 1.24, size = 17, normalized size = 0.94

$$-\frac{\operatorname{atanh}(a\,x)-a\,x}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^2*(a*x + 1)),x)
```

```
[Out] -(atanh(a*x) - a*x)/(a*c^2)
```

3.427 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=57

$$\frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

```
[Out] x/c^3+1/2/a/c^3/(-a*x+1)+5/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 90}

$$\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] x/c^3 + 1/(2*a*c^3*(1 - a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/(
4*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx \\
&= \frac{a^3\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^3}\,dx}{c^3} \\
&= \frac{a^3\int \frac{x^3}{(1-ax)^2(1+ax)}\,dx}{c^3} \\
&= \frac{a^3\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c^3} \\
&= \frac{x}{c^3}+\frac{1}{2ac^3(1-ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.98

$$\frac{x}{c^3}-\frac{1}{2ac^3(-1+ax)}+\frac{5\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^3),x]
[Out] x/c^3 - 1/(2*a*c^3*(-1 + a*x)) + (5*Log[1 - a*x])/(4*a*c^3) - Log[1 + a*x]/
(4*a*c^3)
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.84

method	result	size
default	$\frac{a^3\left(\frac{x}{a^3}-\frac{\ln(ax+1)}{4a^4}-\frac{1}{2a^4(ax-1)}+\frac{5\ln(ax-1)}{4a^4}\right)}{c^3}$	48
risch	$\frac{x}{c^3}-\frac{1}{2a(ax-1)c^3}+\frac{5\ln(-ax+1)}{4a\,c^3}-\frac{\ln(ax+1)}{4a\,c^3}$	51
norman	$\frac{\frac{a^2x^3}{c}+\frac{3x}{2c}-\frac{5a\,x^2}{2c}}{c^2(ax-1)^2}+\frac{5\ln(ax-1)}{4c^3a}-\frac{\ln(ax+1)}{4a\,c^3}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/c^3*(x/a^3-1/4*ln(a*x+1)/a^4-1/2/a^4/(a*x-1)+5/4/a^4*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 53, normalized size = 0.93

$$-\frac{1}{2\,(a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{\log{(ax+1)}}{4\,ac^3}+\frac{5\,\log{(ax-1)}}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 5/4*log(a*x -
 1)/(a*c^3)
```

Fricas [A]
time = 0.33, size = 59, normalized size = 1.04

$$\frac{4\,a^2x^2-4\,ax-(ax-1)\log{(ax+1)}+5\,(ax-1)\log{(ax-1)}-2}{4\,(a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
- 2)/(a^2*c^3*x - a*c^3)
```

Sympy [A]
time = 0.17, size = 56, normalized size = 0.98

$$a^3\left(-\frac{1}{2a^5c^3x-2a^4c^3}+\frac{x}{a^3c^3}+\frac{\frac{5\log{\left(x-\frac{1}{a}\right)}}{4}-\frac{\log{\left(x+\frac{1}{a}\right)}}{4}}{a^4c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(-1/(2*a**5*c**3*x - 2*a**4*c**3) + x/(a**3*c**3) + (5*log(x - 1/a)/4
- log(x + 1/a)/4)/(a**4*c**3))
```

Giac [A]
time = 0.40, size = 51, normalized size = 0.89

$$\frac{x}{c^3}-\frac{\log{(|ax+1|)}}{4\,ac^3}+\frac{5\,\log{(|ax-1|)}}{4\,ac^3}-\frac{1}{2\,(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 5/4*log(abs(a*x - 1))/(a*c^3) - 1/2
/((a*x - 1)*a*c^3)
```

Mupad [B]
time = 0.09, size = 52, normalized size = 0.91

$$\frac{x}{c^3}+\frac{1}{2\,a\,\left(c^3-a\,c^3\,x\right)}+\frac{5\,\ln\left(a\,x-1\right)}{4\,a\,c^3}-\frac{\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^3*(a*x + 1)),x)
```

```
[Out] x/c^3 + 1/(2*a*(c^3 - a*c^3*x)) + (5*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)
/(4*a*c^3)
```

3.428 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=75

$$\frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}$$

```
[Out] x/c^4-1/4/a/c^4/(-a*x+1)^2+7/4/a/c^4/(-a*x+1)+17/8*ln(-a*x+1)/a/c^4-1/8*ln(
a*x+1)/a/c^4
```

Rubi [A]
time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6266, 6264, 90}

$$\frac{7}{4ac^4(1-ax)}-\frac{1}{4ac^4(1-ax)^2}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(ax+1)}{8ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] x/c^4 - 1/(4*a*c^4*(1 - a*x)^2) + 7/(4*a*c^4*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6266

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_.), x_Symbol
] :> Dist[d^p, Int[u*(1 + c*(x/d))^p*(E^(n*ArcTanh[a*x])/x^p), x], x] /; Fr
eeQ[{a, c, d, n}, x] && EqQ[c^2 - a^2*d^2, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx\\ &= -\frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-ax)^4}\,dx}{c^4}\\ &= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^4}\\ &= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^4}\\ &= \frac{x}{c^4}-\frac{1}{4ac^4(1-ax)^2}+\frac{7}{4ac^4(1-ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 0.97

$$\frac{x}{c^4}-\frac{1}{4ac^4(-1+ax)^2}-\frac{7}{4ac^4(-1+ax)}+\frac{17\log(1-ax)}{8ac^4}-\frac{\log(1+ax)}{8ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^4),x]
[Out] x/c^4 - 1/(4*a*c^4*(-1 + a*x)^2) - 7/(4*a*c^4*(-1 + a*x)) + (17*Log[1 - a*x
])/(8*a*c^4) - Log[1 + a*x]/(8*a*c^4)
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.80

method	result	size
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{\ln(ax+1)}{8a^5}-\frac{1}{4a^5(ax-1)^2}-\frac{7}{4a^5(ax-1)}+\frac{17\ln(ax-1)}{8a^5}\right)}{c^4}$	60
risch	$\frac{x}{c^4}+\frac{-\frac{7c^4x}{4}+\frac{3c^4}{2a}}{c^8(ax-1)^2}+\frac{17\ln(-ax+1)}{8a\,c^4}-\frac{\ln(ax+1)}{8a\,c^4}$	62
norman	$\frac{\frac{a^3x^4}{c}-\frac{9x}{4c}+\frac{23a\,x^2}{4c}-\frac{9a^2x^3}{2c}}{c^3(ax-1)^3}+\frac{17\ln(ax-1)}{8c^4a}-\frac{\ln(ax+1)}{8a\,c^4}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^4*(x/a^4-1/8/a^5*ln(a*x+1)-1/4/a^5/(a*x-1)^2-7/4/a^5/(a*x-1)+17/8/a^5
*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 69, normalized size = 0.92

$$-\frac{7\,ax-6}{4\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}+\frac{x}{c^4}-\frac{\log{(ax+1)}}{8\,ac^4}+\frac{17\,\log{(ax-1)}}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 1/8*log(a*x
+ 1)/(a*c^4) + 17/8*log(a*x - 1)/(a*c^4)
```

Fricas [A]
time = 0.33, size = 93, normalized size = 1.24

$$\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-(a^2x^2-2\,ax+1)\log{(ax+1)}+17\,(a^2x^2-2\,ax+1)\log{(ax-1)}+12}{8\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*
c^4)
```

Sympy [A]
time = 0.22, size = 73, normalized size = 0.97

$$a^4\left(\frac{-7ax+6}{4a^7c^4x^2-8a^6c^4x+4a^5c^4}+\frac{x}{a^4c^4}+\frac{\frac{17\log{\left(x-\frac{1}{a}\right)}}{8}-\frac{\log{\left(x+\frac{1}{a}\right)}}{8}}{a^5c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**4,x)
```

```
[Out] a**4*((-7*a*x + 6)/(4*a**7*c**4*x**2 - 8*a**6*c**4*x + 4*a**5*c**4) + x/(a*
*4*c**4) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**4))
```

Giac [A]
time = 0.41, size = 57, normalized size = 0.76

$$\frac{x}{c^4}-\frac{\log{(|ax+1|)}}{8\,ac^4}+\frac{17\,\log{(|ax-1|)}}{8\,ac^4}-\frac{7\,ax-6}{4\,(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 1/8*log(abs(a*x + 1))/(a*c^4) + 17/8*log(abs(a*x - 1))/(a*c^4) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^4)
```

Mupad [B]
time = 0.10, size = 68, normalized size = 0.91

$$\frac{x}{c^4}-\frac{\frac{7\,x}{4}-\frac{3}{2\,a}}{a^2\,c^4\,x^2-2\,a\,c^4\,x+c^4}+\frac{17\,\ln\left(a\,x-1\right)}{8\,a\,c^4}-\frac{\ln\left(a\,x+1\right)}{8\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^4*(a*x + 1)),x)
```

```
[Out] x/c^4 - ((7*x)/4 - 3/(2*a))/(c^4 + a^2*c^4*x^2 - 2*a*c^4*x) + (17*log(a*x -
 1))/(8*a*c^4) - log(a*x + 1)/(8*a*c^4)
```

3.429 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4\,dx$

Optimal. Leaf size=164

$$\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}+\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^4\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{91c^4\csc^{-1}(ax)}{2a}-\frac{7c}{}$$

```
[Out] 91/2*c^4*arccsc(a*x)/a-7*c^4*arctanh((1-1/a^2/x^2)^(1/2))/a+64*c^4*(a-1/x)/
a^2/(1-1/a^2/x^2)^(1/2)+68/3*c^4*(1-1/a^2/x^2)^(1/2)/a+1/3*c^4*(1-1/a^2/x^2
)^(1/2)/a^3/x^2-7/2*c^4*(1-1/a^2/x^2)^(1/2)/a^2/x+c^4*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.409$, Rules used = {6312, 1819, 1821, 1823, 858, 222, 272, 65, 214}

$$\frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^4x\sqrt{1-\frac{1}{a^2x^2}}+\frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a}-\frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{7c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2}+\frac{91c^4\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (68*c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a) + (64*c^4*(a - x^(-1)))/(a^2*Sqrt[1 -
1/(a^2*x^2)]) + (c^4*Sqrt[1 - 1/(a^2*x^2)])/(3*a^3*x^2) - (7*c^4*Sqrt[1 - 1
/(a^2*x^2)])/(2*a^2*x) + c^4*Sqrt[1 - 1/(a^2*x^2)]*x + (91*c^4*ArcCsc[a*x])
/(2*a) - (7*c^4*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^4 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^7}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^7+\frac{7c^7x}{a}+\frac{42c^7x^2}{a^2}-\frac{22c^7x^3}{a^3}+\frac{7c^7x^4}{a^4}-\frac{c^7x^5}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{7c^7}{a}-\frac{42c^7x}{a^2}+\frac{22c^7x^2}{a^3}-\frac{7c^7x^3}{a^4}+\frac{c^7x^4}{a^5}}{x\sqrt{1-\frac{x^2}{a^2}}}dx \ldots}{c^3}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\text{Subst}\left(\int \frac{\frac{21c^7}{a^3}+\frac{126c^7x}{a^4}\ldots}{x\sqrt{\ldots}}\ldots\right.}{3\ldots}$$

$$= \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{a^4 9\ldots}{\ldots}$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\ldots$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\ldots$$

$$= \frac{68c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a} + \frac{64c^4\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{\ldots}}} + \frac{c^4\sqrt{1-\frac{1}{a^2x^2}}}{3a^3x^2} - \frac{7c^4\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x} + c^4\ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.72, size = 567, normalized size = 3.46

$$\frac{c^4\left(2772\sqrt{2}\,a^3x^3(-1+ax)^3(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+1980\sqrt{2}\,a^2x^2(-1+ax)^4(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+35\left(-198a^2\sqrt{1+\frac{1}{ax}}\,x^2+1716a^3\sqrt{1+\frac{1}{ax}}\,x^3-7425a^4\sqrt{1+\frac{1}{ax}}\,x^4+26268a^5\sqrt{1+\frac{1}{ax}}\,x^5+29403a^6\sqrt{1+\frac{1}{ax}}\,x^6-50160a^7\sqrt{1+\frac{1}{ax}}\,x^7+396a^8\sqrt{1+\frac{1}{ax}}\,x^8+66726a^6\sqrt{1-\frac{1}{ax}}\,x^6\operatorname{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+66726a^7\sqrt{1-\frac{1}{ax}}\,x^7\operatorname{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-1980a^6\sqrt{1-\frac{1}{ax}}\,x^6\operatorname{ArcSin}\left(\frac{1}{ax}\right)-1980a^7\sqrt{1-\frac{1}{ax}}\,x^7\operatorname{ArcSin}\left(\frac{1}{ax}\right)-2772a^7\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1+\frac{1}{ax}}\,x^7\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+44\sqrt{2}\,ax(-1+ax)^5(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+36\sqrt{2}\,(-1+ax)^6(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{11}{2};\frac{13}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)\right)\right)}{13860a^7\sqrt{1-\frac{1}{ax}}\,x^6(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(2772*Sqrt[2]*a^3*x^3*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/
2, 7/2, (1 - 1/(a*x))/2] + 1980*Sqrt[2]*a^2*x^2*(-1 + a*x)^4*(1 + a*x)*Hype
rgeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] + 35*(-198*a^2*Sqrt[1 + 1/(a*
x)]*x^2 + 1716*a^3*Sqrt[1 + 1/(a*x)]*x^3 - 7425*a^4*Sqrt[1 + 1/(a*x)]*x^4 +
 26268*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 29403*a^6*Sqrt[1 + 1/(a*x)]*x^6 - 50160*
a^7*Sqrt[1 + 1/(a*x)]*x^7 + 396*a^8*Sqrt[1 + 1/(a*x)]*x^8 + 66726*a^6*Sqrt[
1 - 1/(a*x)]*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 66726*a^7*Sqrt[1 - 1/(
a*x)]*x^7*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 1980*a^6*Sqrt[1 - 1/(a*x)]*x^
6*ArcSin[1/(a*x)] - 1980*a^7*Sqrt[1 - 1/(a*x)]*x^7*ArcSin[1/(a*x)] - 2772*a
^7*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^7*ArcTanh[Sqrt[1 - 1/(a^2*x^2)
]] + 44*Sqrt[2]*a*x*(-1 + a*x)^5*(1 + a*x)*Hypergeometric2F1[3/2, 9/2, 11/2
, (1 - 1/(a*x))/2] + 36*Sqrt[2]*(-1 + a*x)^6*(1 + a*x)*Hypergeometric2F1[3/
2, 11/2, 13/2, (1 - 1/(a*x))/2])))/(13860*a^7*Sqrt[1 - 1/(a*x)]*x^6*(1 + a*
x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(144) = 288$.
time = 0.12, size = 672, normalized size = 4.10

method	result
risch	$\frac{(ax+1)\left(136a^2x^2-21ax+2\right)c^4\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(a^3\sqrt{(ax+1)(ax-1)}-\frac{7a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{64a^2\sqrt{a^2\left(x\right.}}{}\right.}{}$
default	$-\frac{\left(-138\sqrt{a^2}\sqrt{a^2x^2-1}\,a^6x^6+96\sqrt{(ax+1)(ax-1)}\sqrt{a^2}\,a^5x^5-96\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1}}{\sqrt{a^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-138*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a^6*x^6+96*((a*x+1)*(a*x-1))^(1/2)
*(a^2)^(1/2)*a^5*x^5-96*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2
)^(1/2))*a^6*x^5+138*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-549*(a^2*x^2-1)^
(1/2)*(a^2)^(1/2)*a^5*x^5-273*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x
```

```
^5+138*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5+192*((
a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3+192*(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(1/2)*a^4*x^4-192*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/
2))*a^5*x^4+255*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-684*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*a^4*x^4-546*a^4*x^4*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+27
6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^5*x^4+96*(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-96*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(1/2))/(a^2)^(1/2))*a^4*x^3+98*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-27
3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-273*a^3*x^3*(a^2)^(1/2)*arctan(1/(a
^2*x^2-1)^(1/2))+138*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*
a^4*x^3-17*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
))/a^4*c^4*((a*x-1)/(a*x+1))^(3/2)/x^3/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/
(a*x-1)
```

Maxima [A]

time = 0.48, size = 246, normalized size = 1.50

$$-\frac{1}{3}\left(\frac{273\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{21\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{192\,c^4\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{153\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+91\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-169\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-123\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(273*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 21*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 21*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
192*c^4*sqrt((a*x - 1)/(a*x + 1))/a^2 + (153*c^4*((a*x - 1)/(a*x + 1))^(7/2
) + 91*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 169*c^4*((a*x - 1)/(a*x + 1))^(3/2
) - 123*c^4*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x - 1)*a^2/(a*x + 1) - 2*(a*x
- 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1)^4 + a^2))*a
```

Fricas [A]

time = 0.34, size = 157, normalized size = 0.96

$$-\frac{546\,a^3c^4x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-42\,a^3c^4x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^4x^4+526\,a^3c^4x^3+115\,a^2c^4x^2-19\,ac^4x+2\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(546*a^3*c^4*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 42*a^3*c^4*x^3*lo
g(sqrt((a*x - 1)/(a*x + 1)) + 1) - 42*a^3*c^4*x^3*log(sqrt((a*x - 1)/(a*x +
 1)) - 1) - (6*a^4*c^4*x^4 + 526*a^3*c^4*x^3 + 115*a^2*c^4*x^2 - 19*a*c^4*x
 + 2*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{5a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{10a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{10a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{5a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^5x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Int
egral(5*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(
-10*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(1
0*a**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-5*a**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.14, size = 211, normalized size = 1.29

$$\frac{41\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{169\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-\frac{91\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-51\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{64\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{91\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,14{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (41*c^4*((a*x - 1)/(a*x + 1))^(1/2) + (169*c^4*((a*x - 1)/(a*x + 1))^(3/2))
/3 - (91*c^4*((a*x - 1)/(a*x + 1))^(5/2))/3 - 51*c^4*((a*x - 1)/(a*x + 1))^
(7/2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*
(a*x - 1)^4)/(a*x + 1)^4) + (64*c^4*((a*x - 1)/(a*x + 1))^(1/2))/a - (91*c^
4*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^4*atan(((a*x - 1)/(a*x + 1))^(1
/2)*1i)*14i)/a
```

3.430 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3\,dx$

Optimal. Leaf size=135

$$\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{33c^3\csc^{-1}(ax)}{2a}-\frac{6c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 33/2*c^3*arccsc(a*x)/a-6*c^3*arctanh((1-1/a^2/x^2)^(1/2))/a+32*c^3*(a-1/x)/
a^2/(1-1/a^2/x^2)^(1/2)+6*c^3*(1-1/a^2/x^2)^(1/2)/a-1/2*c^3*(1-1/a^2/x^2)^(
1/2)/a^2/x+c^3*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6312, 1819, 1821, 1823, 858, 222, 272, 65, 214}

$$\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^3x\sqrt{1-\frac{1}{a^2x^2}}+\frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}-\frac{6c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{33c^3\csc^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (6*c^3*Sqrt[1 - 1/(a^2*x^2)])/a + (32*c^3*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^
2*x^2)]) - (c^3*Sqrt[1 - 1/(a^2*x^2)])/(2*a^2*x) + c^3*Sqrt[1 - 1/(a^2*x^2)
]*x + (33*c^3*ArcCsc[a*x])/(2*a) - (6*c^3*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^3 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^6}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\text{Subst}\left(\int \frac{-c^6+\frac{6c^6x}{a}+\frac{16c^6x^2}{a^2}-\frac{6c^6x^3}{a^3}+\frac{c^6x^4}{a^4}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x-\frac{\text{Subst}\left(\int \frac{-\frac{6c^6}{a}-\frac{16c^6x}{a^2}+\frac{6c^6x^2}{a^3}-\frac{c^6x^3}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= \frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{a^2\text{Subst}\left(\int \frac{\frac{12c^6}{a^3}+\frac{33c^6}{a^4}\cdots}{x\sqrt{1-\cdots}}\cdots\right)}{2c^3\cdots}$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x-\cdots$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x+\cdots$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x+3\cdots$$

$$= \frac{6c^3\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{32c^3\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{2a^2x}+c^3\sqrt{1-\frac{1}{a^2x^2}}\,x+3\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.29, size = 663, normalized size = 4.91

$$\frac{c^3\left(420a^2\sqrt{1+\frac{1}{ax}}x^2-3465a^3\sqrt{1+\frac{1}{ax}}x^3+16800a^4\sqrt{1+\frac{1}{ax}}x^4+17955a^5\sqrt{1+\frac{1}{ax}}x^5-32340a^6\sqrt{1+\frac{1}{ax}}x^6+630a^7\sqrt{1+\frac{1}{ax}}x^7+44730a^5\sqrt{1-\frac{1}{ax}}x^5\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+44730a^6\sqrt{1-\frac{1}{ax}}x^6\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2520a^5\sqrt{1-\frac{1}{ax}}x^5\text{ArcSin}\left(\frac{1}{ax}\right)-2520a^6\sqrt{1-\frac{1}{ax}}x^6\text{ArcSin}\left(\frac{1}{ax}\right)-3780a^6\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1+\frac{1}{ax}}x^6\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+126\sqrt{2}a^2x^2(-1+ax)^3(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+90\sqrt{2}ax(-1+ax)^4(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)-70\sqrt{2}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+280\sqrt{2}ax\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)-350\sqrt{2}a^2x^2\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+350\sqrt{2}a^4x^4\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)-280\sqrt{2}a^5x^5\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+70\sqrt{2}a^6x^6\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)\right)}{630a^6\sqrt{1-\frac{1}{ax}}x^5(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(420*a^2*Sqrt[1 + 1/(a*x)]*x^2 - 3465*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 1680
0*a^4*Sqrt[1 + 1/(a*x)]*x^4 + 17955*a^5*Sqrt[1 + 1/(a*x)]*x^5 - 32340*a^6*S
qrt[1 + 1/(a*x)]*x^6 + 630*a^7*Sqrt[1 + 1/(a*x)]*x^7 + 44730*a^5*Sqrt[1 - 1
/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] + 44730*a^6*Sqrt[1 - 1/(a*x)]
*x^6*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2520*a^5*Sqrt[1 - 1/(a*x)]*x^5*Arc
Sin[1/(a*x)] - 2520*a^6*Sqrt[1 - 1/(a*x)]*x^6*ArcSin[1/(a*x)] - 3780*a^6*Sq
rt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^6*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]] +
126*Sqrt[2]*a^2*x^2*(-1 + a*x)^3*(1 + a*x)*Hypergeometric2F1[3/2, 5/2, 7/2,
 (1 - 1/(a*x))/2] + 90*Sqrt[2]*a*x*(-1 + a*x)^4*(1 + a*x)*Hypergeometric2F1
[3/2, 7/2, 9/2, (1 - 1/(a*x))/2] - 70*Sqrt[2]*Hypergeometric2F1[3/2, 9/2, 1
1/2, (1 - 1/(a*x))/2] + 280*Sqrt[2]*a*x*Hypergeometric2F1[3/2, 9/2, 11/2, (
1 - 1/(a*x))/2] - 350*Sqrt[2]*a^2*x^2*Hypergeometric2F1[3/2, 9/2, 11/2, (1
- 1/(a*x))/2] + 350*Sqrt[2]*a^4*x^4*Hypergeometric2F1[3/2, 9/2, 11/2, (1 -
1/(a*x))/2] - 280*Sqrt[2]*a^5*x^5*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/
(a*x))/2] + 70*Sqrt[2]*a^6*x^6*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - 1/(a*
x))/2]))/(630*a^6*Sqrt[1 - 1/(a*x)]*x^5*(1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(121) = 242$.
time = 0.12, size = 450, normalized size = 3.33

method	result
risch	$\frac{(ax+1)(12ax-1)c^3\sqrt{\frac{ax-1}{ax+1}}}{2x^2a^3}+\frac{\left(a^2\sqrt{(ax+1)(ax-1)}-\frac{6a^3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{32a\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2}}{x+\frac{1}{a}}\right.}{a^3(ax-1}$
default	$-\frac{\left(-12\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+12(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-57\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4-33a^4x^4\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-12*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^5*x^5+12*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2)*a^3*x^3-57*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4-33*a^4*x^4*(a^2)^(1/
```

```
2)*arctan(1/(a^2*x^2-1)^(1/2))+12*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/
(a^2)^(1/2))*a^5*x^4+32*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2+23*(a^2
*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-78*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-
66*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+24*ln((a^2*x+(a^2*x^2-1)
^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+10*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a
*x-33*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-33*(a^2)^(1/2)*arctan(1/(a^2*x^
2-1)^(1/2))*a^2*x^2+12*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2)
)*a^3*x^2-(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/a^3*c^3*((a*x-1)/(a*x+1))^(3/2)/x^
2/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]
time = 0.48, size = 225, normalized size = 1.67

$$-\left(\frac{33\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{6\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{32\,c^3\sqrt{\frac{ax-1}{ax+1}}}{a^2}+\frac{11\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-6\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-13\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-\frac{(ax-1)^2a^2}{(ax+1)^2}-\frac{(ax-1)^3a^2}{(ax+1)^3}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(33*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 6*c^3*log(sqrt((a*x - 1)/(
a*x + 1)) + 1)/a^2 - 6*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 32*c^3*
sqrt((a*x - 1)/(a*x + 1))/a^2 + (11*c^3*((a*x - 1)/(a*x + 1))^(5/2) - 6*c^3
*((a*x - 1)/(a*x + 1))^(3/2) - 13*c^3*sqrt((a*x - 1)/(a*x + 1)))/((a*x - 1)
*a^2/(a*x + 1) - (a*x - 1)^2*a^2/(a*x + 1)^2 - (a*x - 1)^3*a^2/(a*x + 1)^3
+ a^2))*a
```

Fricas [A]
time = 0.35, size = 146, normalized size = 1.08

$$-\frac{66\,a^2c^3x^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-12\,a^2c^3x^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3c^3x^3+78\,a^2c^3x^2+11\,ac^3x-c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(66*a^2*c^3*x^2*arctan(sqrt((a*x - 1)/(a*x + 1))) + 12*a^2*c^3*x^2*log
(sqrt((a*x - 1)/(a*x + 1)) + 1) - 12*a^2*c^3*x^2*log(sqrt((a*x - 1)/(a*x +
1)) - 1) - (2*a^3*c^3*x^3 + 78*a^2*c^3*x^2 + 11*a*c^3*x - c^3)*sqrt((a*x -
1)/(a*x + 1)))/(a^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\left(-\frac{4a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{6a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{4a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^4x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Inte
gral(-4*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(
6*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-4*a**
3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**4*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.27, size = 190, normalized size = 1.41

$$\frac{13\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}+6\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-11\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a+\frac{a\,\left(a\,x-1\right)}{a\,x+1}-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}+\frac{32\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}-\frac{33\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,12{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (13*c^3*((a*x - 1)/(a*x + 1))^(1/2) + 6*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 1
1*c^3*((a*x - 1)/(a*x + 1))^(5/2))/(a + (a*(a*x - 1))/(a*x + 1) - (a*(a*x -
 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) + (32*c^3*((a*x - 1)/(a*x
 + 1))^(1/2))/a - (33*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c^3*atan(
((a*x - 1)/(a*x + 1))^(1/2)*1i)*12i)/a
```

3.431 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2\,dx$

Optimal. Leaf size=105

$$\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}+\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{5c^2\csc^{-1}(ax)}{a}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] 5*c^2*arccsc(a*x)/a-5*c^2*arctanh((1-1/a^2/x^2)^(1/2))/a+16*c^2*(a-1/x)/a^2
/(1-1/a^2/x^2)^(1/2)+c^2*(1-1/a^2/x^2)^(1/2)/a+c^2*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.409$, Rules used = {6312, 1819, 1821, 1823, 858, 222, 272, 65, 214}

$$\frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c^2x\sqrt{1-\frac{1}{a^2x^2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a}-\frac{5c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{5c^2\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/a + (16*c^2*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*
x^2)]) + c^2*Sqrt[1 - 1/(a^2*x^2)]*x + (5*c^2*ArcCsc[a*x])/a - (5*c^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
```

```
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^5}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^5+\frac{5c^5x}{a}+\frac{5c^5x^2}{a^2}-\frac{c^5x^3}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{5c^5}{a}-\frac{5c^5x}{a^2}+\frac{c^5x^2}{a^3}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{a^2\text{Subst}\left(\int \frac{\frac{5c^5}{a^3}+\frac{5c^5x}{a^4}}{x\sqrt{1-\frac{x^2}{a^2}}} \cdots\right.}{c^3}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{(5c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\cdots}} \cdots\right.}{a^2}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} + \frac{(5c^2)\cdots}{\cdots}$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} - \left(5ac^2\cdots\right.$$

$$= \frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a} + \frac{16c^2\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c^2\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{5c^2\csc^{-1}(ax)}{a} - \frac{5c^2\,\text{ta}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.26, size = 424, normalized size = 4.04

$$\frac{c^2\left(-35a^2\sqrt{1+\frac{1}{ax}}x^2+315a^3\sqrt{1+\frac{1}{ax}}x^3+280a^4\sqrt{1+\frac{1}{ax}}x^4-595a^5\sqrt{1+\frac{1}{ax}}x^5+35a^6\sqrt{1+\frac{1}{ax}}x^6+910a^4\sqrt{1-\frac{1}{ax}}x^4\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)+910a^5\sqrt{1-\frac{1}{ax}}x^5\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-105a^4\sqrt{1-\frac{1}{ax}}x^4\text{ArcSin}\left(\frac{1}{ax}\right)-105a^5\sqrt{1-\frac{1}{ax}}x^5\text{ArcSin}\left(\frac{1}{ax}\right)-175a^5\sqrt{1-\frac{1}{a^2x^2}}\sqrt{1+\frac{1}{ax}}x^5\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)+7\sqrt{2}ax(-1+ax)^3(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)+5\sqrt{2}(-1+ax)^4(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)\right)}{35a^5\sqrt{1-\frac{1}{ax}}x^4(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-35*a^2*Sqrt[1 + 1/(a*x)]*x^2 + 315*a^3*Sqrt[1 + 1/(a*x)]*x^3 + 280*a
^4*Sqrt[1 + 1/(a*x)]*x^4 - 595*a^5*Sqrt[1 + 1/(a*x)]*x^5 + 35*a^6*Sqrt[1 +
1/(a*x)]*x^6 + 910*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[
2]] + 910*a^5*Sqrt[1 - 1/(a*x)]*x^5*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 105
*a^4*Sqrt[1 - 1/(a*x)]*x^4*ArcSin[1/(a*x)] - 105*a^5*Sqrt[1 - 1/(a*x)]*x^5*
ArcSin[1/(a*x)] - 175*a^5*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^5*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]] + 7*Sqrt[2]*a*x*(-1 + a*x)^3*(1 + a*x)*Hypergeome
tric2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2] + 5*Sqrt[2]*(-1 + a*x)^4*(1 + a*x)*
Hypergeometric2F1[3/2, 7/2, 9/2, (1 - 1/(a*x))/2]))/(35*a^5*Sqrt[1 - 1/(a*x
)]*x^4*(1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(97) = 194$.
time = 0.12, size = 600, normalized size = 5.71

method	result
risch	$\frac{(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(a\sqrt{(ax+1)(ax-1)}-\frac{5a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{16\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\right.}}{x+\frac{1}{a}} \ldots\right.}{a^2(ax-1)}$ …
default	$-\frac{\left(-\sqrt{a^2x^2-1}\sqrt{a^2}\,a^4x^4-4\sqrt{a^2}\sqrt{(ax+1)(ax-1)}\,a^3x^3+4\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a \ldots\right.}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4-4*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1
/2)*a^3*x^3+4*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a
^4*x^3+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-7*(a^2*x^2-1)^(1/2)*(a^2)^(1/2
)*a^3*x^3-5*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+ln((a^2*x+(a^2*
x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+8*(a^2)^(1/2)*((a*x+1)*(a*x-
1))^(3/2)*a*x-8*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+8*ln((a^2*x+(a^
2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+2*(a^2)^(1/2)*(a^2*x
```

```
^2-1)^(3/2)*a*x-11*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2-10*(a^2)^(1/2)*arc
tan(1/(a^2*x^2-1)^(1/2))*a^2*x^2+2*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))
/(a^2)^(1/2))*a^3*x^2-4*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+4*ln((a^2*x
+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+(a^2*x^2-1)^(3/2)*
(a^2)^(1/2)-5*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-5*(a^2)^(1/2)*arctan(1/(a^2
*x^2-1)^(1/2))*a*x+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^
2*x)/a^2*c^2*((a*x-1)/(a*x+1))^(3/2)/x/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/
(a*x-1)
```

Maxima [A]
time = 0.48, size = 149, normalized size = 1.42

$$-\left(\frac{4\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{10\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{5\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{16\,c^2\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c^2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 10*
c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 5*c^2*log(sqrt((a*x - 1)/(a*x +
 1)) + 1)/a^2 - 5*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 16*c^2*sqrt(
(a*x - 1)/(a*x + 1))/a^2)*a
```

Fricas [A]
time = 0.35, size = 120, normalized size = 1.14

$$-\frac{10\,ac^2x\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-5\,ac^2x\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2c^2x^2+18\,ac^2x+c^2)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(10*a*c^2*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 5*a*c^2*x*log(sqrt((a*x -
1)/(a*x + 1)) + 1) - 5*a*c^2*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c^
2*x^2 + 18*a*c^2*x + c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\right)dx+\int\frac{3a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Int
egral(3*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-3*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.10, size = 117, normalized size = 1.11

$$\frac{16\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{4\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{10\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,10{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (16*c^2*((a*x - 1)/(a*x + 1))^(1/2))/a + (4*c^2*((a*x - 1)/(a*x + 1))^(1/2)
)/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (10*c^2*atan(((a*x - 1)/(a*x + 1))^(1
/2)))/a + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*10i)/a
```

3.432 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=75

$$\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+c\sqrt{1-\frac{1}{a^2x^2}}\,x+\frac{c\csc^{-1}(ax)}{a}-\frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a-4*c*arctanh((1-1/a^2/x^2)^(1/2))/a+8*c*(a-1/x)/a^2/(1-1/a^2
/x^2)^(1/2)+c*x*(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6312, 1819, 1821, 858, 222, 272, 65, 214}

$$\frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+cx\sqrt{1-\frac{1}{a^2x^2}}-\frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}+\frac{c\csc^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (8*c*(a - x^(-1)))/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + c*Sqrt[1 - 1/(a^2*x^2)]*x
+ (c*ArcCsc[a*x])/a - (4*c*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/a
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^4}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^4+\frac{4c^4x}{a}+\frac{c^4x^2}{a^2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x - \frac{\text{Subst}\left(\int \frac{-\frac{4c^4}{a}-\frac{c^4x}{a^2}}{x\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}dx, x, \frac{1}{x}\right)}{a^2} + \frac{(4c)\text{Su}}{}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} + \frac{(2c)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}d\right.}{a}$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} - (4ac)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}d\right.$$

$$= \frac{8c\left(a-\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}} + c\sqrt{1-\frac{1}{a^2x^2}}\,x + \frac{c\csc^{-1}(ax)}{a} - \frac{4c\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.32, size = 234, normalized size = 3.12

$$\frac{5a^2cx^2\left((1+ax)\left(\sqrt{1+\frac{1}{ax}}\,(2-3ax+a^2x^2)+6a\sqrt{1-\frac{1}{ax}}\,x\text{ArcSin}\left(\frac{\sqrt{1-\frac{1}{ax}}}{\sqrt{2}}\right)-2a\sqrt{1-\frac{1}{ax}}\,x\text{ArcSin}\left(\frac{1}{ax}\right)\right)-4a^2\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{1+\frac{1}{ax}}\,x^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)\right)+\sqrt{2}\,c(-1+ax)^3(1+ax)\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1}{2}\left(1-\frac{1}{ax}\right)\right)}{5a^4\sqrt{1-\frac{1}{ax}}\,x^3(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (5*a^2*c*x^2*((1 + a*x)*(Sqrt[1 + 1/(a*x)]*(2 - 3*a*x + a^2*x^2) + 6*a*Sqrt
[1 - 1/(a*x)]*x*ArcSin[Sqrt[1 - 1/(a*x)]/Sqrt[2]] - 2*a*Sqrt[1 - 1/(a*x)]*x
*ArcSin[1/(a*x)]) - 4*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[1 + 1/(a*x)]*x^2*ArcTa
nh[Sqrt[1 - 1/(a^2*x^2)]]) + Sqrt[2]*c*(-1 + a*x)^3*(1 + a*x)*Hypergeometri
c2F1[3/2, 5/2, 7/2, (1 - 1/(a*x))/2])/(5*a^4*Sqrt[1 - 1/(a*x)]*x^3*(1 + a*x
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(69) = 138$.
time = 0.08, size = 376, normalized size = 5.01

method	result
default	$-\frac{\left(-4\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+4\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-4*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+4*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a
^2*x^2-(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^2*x^2+4*((a*x+1)*(a*x-1))^
(3/2)*(a^2)^(1/2)-8*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+8*ln((a^2*x+(a^
2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-2*(a^2)^(1/2)*(a^2*x^2
-1)^(1/2)*a*x-2*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x-4*(a^2)^(1/2)*(
(a*x+1)*(a*x-1))^(1/2)+4*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(
a^2)^(1/2))-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)-arctan(1/(a^2*x^2-1)^(1/2))*(a^2)
^(1/2))/a*c*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a*
x-1)
```

Maxima [A]
time = 0.49, size = 135, normalized size = 1.80

$$-2a\left(\frac{c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2}{ax+1}-a^2}+\frac{c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{2c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{2c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{4c\sqrt{\frac{ax-1}{ax+1}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*a*(c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2/(a*x + 1) - a^2) + c*arcta
n(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 2*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a
^2 - 2*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - 4*c*sqrt((a*x - 1)/(a*x +
 1))/a^2)
```

Fricas [A]
time = 0.35, size = 92, normalized size = 1.23

$$-\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-4\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(acx+9\,c)\sqrt{\frac{ax-1}{ax+1}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*c*arctan(sqrt((a*x - 1)/(a*x + 1))) + 4*c*log(sqrt((a*x - 1)/(a*x + 1))
 + 1) - 4*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a*c*x + 9*c)*sqrt((a*x -
1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\,dx+\int\left(-\frac{2a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^2x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-
2*a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**2*x*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

[Out] undef

<u>**Mupad**</u> [B]
time = 0.09, size = 107, normalized size = 1.43

$$\frac{2\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,\left(a\,x-1\right)}{a\,x+1}}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{8\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a}+\frac{c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,8{}\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (2*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1))/(a*x + 1)) - (2*c*atan
(((a*x - 1)/(a*x + 1))^(1/2)))/a + (c*atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*
8i)/a + (8*c*((a*x - 1)/(a*x + 1))^(1/2))/a
```

3.433 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=72

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c}-\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

```
[Out] -2*arctanh((1-1/a^2/x^2)^(1/2))/a/c+2*(a-1/x)/a^2/c/(1-1/a^2/x^2)^(1/2)+x*(
1-1/a^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6312, 1819, 821, 272, 65, 214}

$$\frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c}-\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]
```

```
[Out] (2*(a - x^(-1)))/(a^2*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/
c - (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(a*c)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-c^2+\frac{2c^2x}{a}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{2\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{ac}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c}$$

$$= \frac{2\left(a-\frac{1}{x}\right)}{a^2c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c} - \frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.85

$$\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(3+ax)-2(1+ax)\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a(c+acx)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))),x]

[Out] (a*Sqrt[1 - 1/(a^2*x^2)]*x*(3 + a*x) - 2*(1 + a*x)*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*(c + a*c*x))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(66) = 132$.
time = 0.10, size = 250, normalized size = 3.47

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a\sqrt{a^2}}+\frac{2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^3\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)}\cdots}{c(ax-1)}$
default	$-\frac{\left(-2\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x,method=_RETURNVERBOSE)

[Out] -(-2*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+2*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-4*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+4*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-2*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)

Maxima [A]
time = 0.26, size = 120, normalized size = 1.67

$$-2\,a\left(\frac{\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="maxima")

[Out] -2*a*(sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))

Fricas [A]
time = 0.34, size = 69, normalized size = 0.96

$$\frac{(ax+3)\sqrt{\frac{ax-1}{ax+1}} - 2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + 2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] ((a*x + 3)*sqrt((a*x - 1)/(a*x + 1)) - 2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
 + 2*log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int\left(-\frac{x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\right)dx + \int\frac{ax^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x),x)
```

```
[Out] a*(Integral(-x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x) + Inte
gral(a*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x))/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.06, size = 87, normalized size = 1.21

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}} + \frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c} - \frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (2*((a*
x - 1)/(a*x + 1))^(1/2))/(a*c) - (4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*
c)
```

3.434 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2}-\frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

```
[Out] -arctanh((1-1/a^2/x^2)^(1/2))/a/c^2-(a-1/x)*x/a/c^2/(1-1/a^2/x^2)^(1/2)+2*x
*(1-1/a^2/x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6312, 837, 821, 272, 65, 214}

$$-\frac{x\left(a-\frac{1}{x}\right)}{ac^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{c^2}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/c^2 - ((a - x^(-1))*x)/(a*c^2*Sqrt[1 - 1/(a^2*x
^2)]) - ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]/(a*c^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{c-\frac{cx}{a}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a^2\text{Subst}\left(\int \frac{\frac{2c}{a^2}-\frac{cx}{a^3}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\,dx, x, \frac{1}{x^2}\right)}{2ac^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx, x, \sqrt{1-\frac{1}{a^2x^2}}\right)}{c^2}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2} - \frac{\left(a-\frac{1}{x}\right)x}{ac^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^2}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.93

$$\frac{-2+ax+a^2x^2-a\sqrt{1-\frac{1}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{a^2c^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^2),x]
```

```
[Out] (-2 + a*x + a^2*x^2 - a*Sqrt[1 - 1/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - 1/(a^2*x^2
)]])/(a^2*c^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(68) = 136$.
time = 0.10, size = 250, normalized size = 3.38

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2} + \frac{\left(-\frac{\ln\left(\frac{a^2 x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}} + \frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(}}{c^2(ax-1)}$
default	$-\frac{\left(-3\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-3*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+2*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x+1)*(a*x-1))^(3/2)*(a^
2)^(1/2)-6*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+4*ln((a^2*x+(a^2)^(1/2)*
((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-3*(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(1/2)+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*(
(a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c^2/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]
time = 0.25, size = 125, normalized size = 1.69

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2} + \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2} - \frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2} - \frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^2) + l
og(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - log(sqrt((a*x - 1)/(a*x + 1))
 - 1)/(a^2*c^2) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Fricas [A]
time = 0.35, size = 67, normalized size = 0.91

$$\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\right)dx + \int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3-a^2x^2-ax+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**2,x)
```

```
[Out] a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 - a**2*x*
*2 - a*x + 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3
*x**3 - a**2*x**2 - a*x + 1), x))/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 1.19, size = 90, normalized size = 1.22

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}} + \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2} - \frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^2,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + ((a
*x - 1)/(a*x + 1))^(1/2)/(a*c^2) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(
a*c^2)
```

3.435 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx$

Optimal. Leaf size=45

$$-\frac{2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -2/a^2/c^3/x/(1-1/a^2/x^2)^(1/2)+x/c^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6312, 277, 197}

$$\frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2}{a^2c^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
```

```
[Out] -2/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x) + x/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2\text{Subst}\left(\int \frac{1}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a^2c^3}$$

$$= -\frac{2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{x}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.73

$$\frac{-2+a^2x^2}{a^2c^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^3),x]
[Out] (-2 + a^2*x^2)/(a^2*c^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.14, size = 50, normalized size = 1.11

method	result	size
trager	$\frac{(a^2x^2-2)\sqrt{-\frac{-ax+1}{ax+1}}}{a\,c^3(ax-1)}$	41
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(a^2x^2-2)(ax+1)}{a(ax-1)^2c^3}$	44
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(a^3x^3+a^2x^2-2ax-2)}{a(ax-1)^2c^3}$	50
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3} - \frac{\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3(ax-1)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a^3*x^3+a^2*x^2-2*a*x-2)/a/(a*x-1)^2/c^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.
time = 0.27, size = 92, normalized size = 2.04

$$-\frac{1}{2}\,a\left(\frac{\frac{5\,(ax-1)}{ax+1}-1}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*((5*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2) -
a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Fricas [A]
time = 0.35, size = 42, normalized size = 0.93

$$\frac{(a^2x^2-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3x-ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="fricas")
```

```
[Out] (a^2*x^2 - 2)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int\left(-\frac{x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\right)dx+\int\frac{ax^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^3x^3+2ax-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**3,x)
```

```
[Out] a**3*(Integral(-x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**3*
x**3 + 2*a*x - 1), x) + Integral(a*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a**4*x**4 - 2*a**3*x**3 + 2*a*x - 1), x))/c**3
```

Giac [A]
time = 0.40, size = 42, normalized size = 0.93

$$\frac{\left(\frac{\sqrt{a^2x^2-1}}{c^3}-\frac{1}{\sqrt{a^2x^2-1}\,c^3}\right)\operatorname{sgn}(ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^3,x, algorithm="giac")
[Out] (sqrt(a^2*x^2 - 1)/c^3 - 1/(sqrt(a^2*x^2 - 1)*c^3))*sgn(a*x + 1)/a
```

Mupad [B]
time = 0.07, size = 41, normalized size = 0.91

$$\frac{a^2\,x^2-2}{\left(x\,a^2\,c^3+a\,c^3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^3,x)
[Out] (a^2*x^2 - 2)/((a*c^3 + a^2*c^3*x)*((a*x - 1)/(a*x + 1))^(1/2))
```

3.436 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx$

Optimal. Leaf size=111

$$\frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}-\frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

```
[Out] arctanh((1-1/a^2/x^2)^(1/2))/a/c^4-1/3*a*x/c^4/(a-1/x)/(1-1/a^2/x^2)^(1/2)-
1/3*(4*a+3/x)*x/a/c^4/(1-1/a^2/x^2)^(1/2)+8/3*x*(1-1/a^2/x^2)^(1/2)/c^4
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6312, 871, 837, 821, 272, 65, 214}

$$-\frac{x\left(4a+\frac{3}{x}\right)}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{8x\sqrt{1-\frac{1}{a^2x^2}}}{3c^4}-\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8*Sqrt[1 - 1/(a^2*x^2)]*x)/(3*c^4) - (a*x)/(3*c^4*Sqrt[1 - 1/(a^2*x^2)]*(a
 - x^(-1))) - ((4*a + 3/x)*x)/(3*a*c^4*Sqrt[1 - 1/(a^2*x^2)]) + ArcTanh[Sqr
t[1 - 1/(a^2*x^2)]]/(a*c^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 871

```
Int[(((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_))/((d_) + (e_.)*(x
_)), x_Symbol] :> Simp[d*(f + g*x)^(n + 1)*((a + c*x^2)^(p + 1)/(2*a*p*(e*f
 - d*g)*(d + e*x))), x] + Dist[1/(p*(2*c*d)*(e*f - d*g)), Int[(f + g*x)^n*(
a + c*x^2)^p*(c*e*f*(2*p + 1) - c*d*g*(n + 2*p + 1) + c*e*g*(n + 2*p + 2)*x
), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[n, 0] && ILtQ[n + 2*p, 0] &&  !IGtQ[n
, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} + \frac{a^2\text{Subst}\left(\int \frac{-\frac{4c}{a^2}-\frac{3cx}{a^3}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= -\frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a^4\text{Subst}\left(\int \frac{-\frac{8c}{a^4}-\frac{3cx}{a^5}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{3c^5}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{a^2}}}\right.}{ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a^2}}}\right.}{2ac^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{a\text{Subst}\left(\int \frac{1}{a^2-a^2x^2}\,dx\right.}{c^4}$$

$$= \frac{8\sqrt{1-\frac{1}{a^2x^2}}\,x}{3c^4} - \frac{ax}{3c^4\sqrt{1-\frac{1}{a^2x^2}}\left(a-\frac{1}{x}\right)} - \frac{\left(4a+\frac{3}{x}\right)x}{3ac^4\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right.}{ac^4}$$

Mathematica [A]

time = 0.05, size = 94, normalized size = 0.85

$$\frac{8-5ax-7a^2x^2+3a^3x^3+3a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{3a^2c^4\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^4),x]
```

```
[Out] (8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3 + 3*a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*x)
*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(3*a^2*c^4*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a
*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(97) = 194$.
time = 0.15, size = 523, normalized size = 4.71

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{19\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{12a^6\left(x-\frac{1}{a}\right)}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x\right.}}{4a^6\left(x+\frac{1}{a}\right)}\right.}{c^4(ax-1)}$
default	$-\frac{\left(-45\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^5x^5-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^6x^5+21((ax+1)(ax-1))^{\frac{3}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-45*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5-24*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+21*((a*x+1)*(a*x-1))^(3/
2)*(a^2)^(1/2)*a^3*x^3+45*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+24*ln
((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+11*(a^2)^
(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2+90*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
)*a^3*x^3+48*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^
4*x^3-5*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-90*((a*x+1)*(a*x-1))^(1/2)*
(a^2)^(1/2)*a^2*x^2-48*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)
^(1/2))*a^3*x^2-19*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-45*((a*x+1)*(a*x-1))
^(1/2)*(a^2)^(1/2)*a*x-24*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a
^2)^(1/2))*a^2*x+45*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+24*a*ln((a^2*x+(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a
^2)^(1/2)/c^4/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)^4
```

Maxima [A]
time = 0.26, size = 160, normalized size = 1.44

$$\frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^4*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^4) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^4))
```

Fricas [A]
time = 0.33, size = 134, normalized size = 1.21

$$\frac{3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-7\,a^2x^2-5\,ax+8)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5-3a^4x^4+2a^3x^3+2a^2x^2-3ax+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**4,x)
```

```
[Out] a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*
x**4 + 2*a**3*x**3 + 2*a**2*x**2 - 3*a*x + 1), x) + Integral(a*x**5*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 - 3*a**4*x**4 + 2*a**3*x**3 + 2*a**2*
x**2 - 3*a*x + 1), x))/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^4,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4, x)
```

Mupad [B]
time = 1.24, size = 128, normalized size = 1.15

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^4}-\frac{\frac{17\,\left(a\,x-1\right)}{3\,\left(a\,x+1\right)}-\frac{14\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{1}{3}}{4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-4\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^4,x)
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^4) - ((17*(a*x - 1))/(3*(a*x + 1)) - (14
*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 4*a
*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^4)
```

3.437 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx$

Optimal. Leaf size=138

$$-\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

```
[Out] -2/5*(a+1/x)/a^2/c^5/(1-1/a^2/x^2)^(5/2)+1/15*(-10*a-13/x)/a^2/c^5/(1-1/a^2
/x^2)^(3/2)+2*arctanh((1-1/a^2/x^2)^(1/2))/a/c^5+1/15*(-30*a-41/x)/a^2/c^5/
(1-1/a^2/x^2)^(1/2)+x*(1-1/a^2/x^2)^(1/2)/c^5
```

Rubi [A]
time = 0.27, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$,
Rules used = {6312, 866, 1819, 821, 272, 65, 214}

$$-\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^5}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{ac^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] (-2*(a + x^(-1)))/(5*a^2*c^5*(1 - 1/(a^2*x^2))^(5/2)) - (10*a + 13/x)/(15*a
^2*c^5*(1 - 1/(a^2*x^2))^(3/2)) - (30*a + 41/x)/(15*a^2*c^5*Sqrt[1 - 1/(a^2
*x^2)]) + (Sqrt[1 - 1/(a^2*x^2)]*x)/c^5 + (2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]]
)/(a*c^5)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 866

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[d^(2*m)/a^m, Int[(f + g*x)^n*((a + c*x^2)^(m + p)
/(d - e*x)^m), x], x] /; FreeQ[{a, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*
g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[f, 0] && ILtQ[m, -1]
 &&  !(IGtQ[n, 0] && ILtQ[m + n, 0] &&  !GtQ[p, 1])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^5}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(c-\frac{cx}{a}\right)^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{\text{Subst}\left(\int \frac{\left(c+\frac{cx}{a}\right)^2}{x^2\left(1-\frac{x^2}{a^2}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-5c^2-\frac{10c^2x}{a}-\frac{8c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{15c^2+\frac{30c^2x}{a}+\frac{26c^2x^2}{a^2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{15c^7}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{-15c^2-}{x^2\sqrt{1-}}\right)}{15c}$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} -$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} -$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} +$$

$$= -\frac{2\left(a+\frac{1}{x}\right)}{5a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}} - \frac{10a+\frac{13}{x}}{15a^2c^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}} - \frac{30a+\frac{41}{x}}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^5} +$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 0.75

$$\frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2\tanh^{-1}\left(\sqrt{1-\frac{1}{a^2x^2}}\right)}{15a^2c^5\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^5),x]
```

```
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*a*Sqrt[1 - 1/(a^2
*x^2)]*x*(-1 + a*x)^2*ArcTanh[Sqrt[1 - 1/(a^2*x^2)]])/(15*a^2*c^5*Sqrt[1 -
1/(a^2*x^2)]*x*(-1 + a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(122) = 244.
time = 0.16, size = 615, normalized size = 4.46

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^5}+\frac{\left(\frac{2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^5\sqrt{a^2}}-\frac{383\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{120a^7\left(x-\frac{1}{a}\right)}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(\cdots\right.}}{8a^7\left(x+\frac{1}{a}\right)}\cdots\right.}{\cdots}$
default	$-\frac{\left(-75\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^6x^6-60\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^7x^6+45((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(-75*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^6*x^6-60*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^7*x^6+45*((a*x+1)*(a*x-1))^(3/
2)*(a^2)^(1/2)*a^4*x^4+150*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5+120*
ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+2*((a*x
+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3+75*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/
2)*a^4*x^4+60*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a
^5*x^4-64*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2-300*(a^2)^(1/2)*((a*x
+1)*(a*x-1))^(1/2)*a^3*x^3-240*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
))/(a^2)^(1/2))*a^4*x^3-14*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+75*((a*x
+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+60*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a
*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+37*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+1
50*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+120*ln((a^2*x+(a^2)^(1/2)*((a*x+
```

```
1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-75*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
)-60*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x
-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/c^5/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)^5
```

Maxima [A]
time = 0.27, size = 176, normalized size = 1.28

$$\frac{1}{120}\,a\left(\frac{\frac{32\,(ax-1)}{ax+1}+\frac{310\,(ax-1)^2}{(ax+1)^2}-\frac{585\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^5\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^5}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^5}+\frac{15\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="maxima")
```

```
[Out] 1/120*a*((32*(a*x - 1)/(a*x + 1) + 310*(a*x - 1)^2/(a*x + 1)^2 - 585*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^5*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^5*((a*x
 - 1)/(a*x + 1))^(5/2)) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^5)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^5) + 15*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*c^5))
```

Fricas [A]
time = 0.34, size = 170, normalized size = 1.23

$$\frac{30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-30\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a^4x^4-76\,a^3x^3+32\,a^2x^2+82\,ax-56)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^5x^3-3\,a^3c^5x^2+3\,a^2c^5x-ac^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 30*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^4*x^4 - 76*a^3*x^3 + 32*a^2*x^2 + 82*a*x - 56)*sqrt((a*x - 1)/(a*
x + 1)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^5\left(\int\left(-\frac{x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\right)dx+\int\frac{ax^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-4a^5x^5+5a^4x^4-5a^2x^2+4ax-1}\,dx\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**5,x)
```

```
[Out] a**5*(Integral(-x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*
x**5 + 5*a**4*x**4 - 5*a**2*x**2 + 4*a*x - 1), x) + Integral(a*x**6*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 4*a**5*x**5 + 5*a**4*x**4 - 5*a**2*
x**2 + 4*a*x - 1), x))/c**5
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.09, size = 144, normalized size = 1.04

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^5}-\frac{\frac{62\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{39\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{32\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-8\,a\,c^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}+\frac{4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^5,x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(8*a*c^5) - ((62*(a*x - 1)^2)/(3*(a*x + 1)^2) -
 (39*(a*x - 1)^3)/(a*x + 1)^3 + (32*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(8*a*c
^5*((a*x - 1)/(a*x + 1))^(5/2) - 8*a*c^5*((a*x - 1)/(a*x + 1))^(7/2)) + (4*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^5)
```

3.438 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=235

$$\frac{173c^5\sqrt{1-\frac{1}{a^2x^2}}}{105a\sqrt{c-\frac{c}{ax}}}+\frac{227c^4\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{105a}+\frac{59c^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35a}+\frac{9c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7a}$$

```
[Out] -7*c^(9/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+59/35*c^3
*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+9/7*c^2*(c-c/a/x)^(5/2)*(1-1/a^2/x^2
)^(1/2)/a+c*(c-c/a/x)^(7/2)*x*(1-1/a^2/x^2)^(1/2)+173/105*c^5*(1-1/a^2/x^2)
^(1/2)/a/(c-c/a/x)^(1/2)+227/105*c^4*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)/a
```

Rubi [A]
time = 0.12, antiderivative size = 279, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.318$, Rules used = {6317, 6314, 99, 158, 152, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^4\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(400a-\frac{227}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{9/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] ((400*a - 227/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(105*a^2*(1 - 1/(a*
x))^(9/2)) + (59*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(35*
a^3*(1 - 1/(a*x))^(9/2)) + (9*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x)
)^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^4*Sqrt[1 + 1/(a*x)]*(c
 - c/(a*x))^(9/2)*x)/(a^4*(1 - 1/(a*x))^(9/2)) - (7*(c - c/(a*x))^(9/2)*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(-\frac{7}{2a}-\frac{9x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\sqrt{1+\frac{x}{a}}}dx\right.}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^4\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^4\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{(2a}{}$$

$$=\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{(a}{}$$

$$=\frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+$$

$$=\frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+$$

$$=\frac{\left(400a-\frac{227}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{105a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{59\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^3\left(1-\frac{1}{ax}\right)^{9/2}}+$$

Mathematica [A]

time = 0.08, size = 109, normalized size = 0.46

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,(-30+162ax-356a^2x^2+292a^3x^3+105a^4x^4)-735a^3x^3\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{105a^4\sqrt{1-\frac{1}{ax}}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-30 + 162*a*x - 356*a^2*x^2 + 29
2*a^3*x^3 + 105*a^4*x^4) - 735*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(105*a^
4*Sqrt[1 - 1/(a*x)]*x^3)
```

Maple [A]
time = 0.08, size = 166, normalized size = 0.71

method	result
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(-210a^{\frac{9}{2}}\sqrt{x(ax+1)}\,x^4+735\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4-584a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}+7}{210\sqrt{\frac{ax-1}{ax+1}}\,x^3a^{\frac{9}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(105a^5x^5+397a^4x^4-64a^3x^3-194a^2x^2+132ax-30)c^4\sqrt{\frac{c(ax-1)}{ax}}}{105x^3a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{7\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^4\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/210/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)/x^3*c^4/a^(9/2)*(-210*
a^(9/2)*(x*(a*x+1))^(1/2)*x^4+735*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*a^4*x^4-584*a^(7/2)*x^3*(x*(a*x+1))^(1/2)+712*a^(5/2)*x^2*(x*(
a*x+1))^(1/2)-324*a^(3/2)*x*(x*(a*x+1))^(1/2)+60*(x*(a*x+1))^(1/2)*a^(1/2))
/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.38, size = 437, normalized size = 1.86

$$\left[\frac{735\,(a^4c^4x^4 - a^3c^4x^3)\sqrt{c}\log\left(-\frac{8\,a^3cx^3 - 7\,acx - 4\,(2\,a^3x^3 + 3\,a^2x^2 + ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}} - c}{ax-1}\right) + 4\,(105\,a^5c^4x^5 + 397\,a^4c^4x^4 - 64\,a^3c^4x^3 - 194\,a^2c^4x^2 + 132\,ac^4x - 30\,c^4)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{420\,(a^5x^4 - a^4x^3)}, \frac{735\,(a^4c^4x^4 - a^3c^4x^3)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2 - acx - c}\right) + 2\,(105\,a^5c^4x^5 + 397\,a^4c^4x^4 - 64\,a^3c^4x^3 - 194\,a^2c^4x^2 + 132\,ac^4x - 30\,c^4)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{210\,(a^5x^4 - a^4x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(735*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(105*a^5*c^4*x^5 + 397*a^4*c^4*x^4 -
64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x - 30*c^4)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/210*(735*(a^4*c^4*x
^4 - a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(105*a^5
*c^4*x^5 + 397*a^4*c^4*x^4 - 64*a^3*c^4*x^3 - 194*a^2*c^4*x^2 + 132*a*c^4*x
 - 30*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^
4*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.439 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2} dx$

Optimal. Leaf size=196

$$\frac{49c^4\sqrt{1-\frac{1}{a^2x^2}}}{15a\sqrt{c-\frac{c}{ax}}}+\frac{31c^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}}{15a}+\frac{7c^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5a}+c\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}x-$$

```
[Out] -5*c^(7/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a+7/5*c^2*(
c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/a+c*(c-c/a/x)^(5/2)*x*(1-1/a^2/x^2)^(1/2
)+49/15*c^4*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+31/15*c^3*(1-1/a^2/x^2)^(
1/2)*(c-c/a/x)^(1/2)/a
```

Rubi [A]
time = 0.10, antiderivative size = 221, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6317, 6314, 99, 158, 152, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(80a-\frac{31}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]
```

```
[Out] ((80*a - 31/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2*(1 - 1/(a*x))
^(7/2)) + (7*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*(
1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)
*x)/(a^3*(1 - 1/(a*x))^(7/2)) - (5*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(-\frac{5}{2a}-\frac{7x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}dx\right.}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{(2a}{}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{(a-}{}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{(a-}{}$$

$$=\frac{\left(80a-\frac{31}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{7\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{(a-}{}$$

Mathematica [A]

time = 0.05, size = 101, normalized size = 0.52

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,(6-28ax+56a^2x^2+15a^3x^3)-75a^2x^2\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}\,x^2}$$

Antiderivative was successfully verified.

[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(7/2),x]

[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(6 - 28*a*x + 56*a^2*x^2 + 15*a^3
*x^3) - 75*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)

Maple [A]
time = 0.06, size = 149, normalized size = 0.76

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(30a^{\frac{7}{2}}x^3\sqrt{x\,(ax+1)}+112a^{\frac{5}{2}}x^2\sqrt{x\,(ax+1)}-75\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^3x^3-56a^{\frac{3}{2}}x\,\sqrt{}\right.}{30\sqrt{\frac{ax-1}{ax+1}}\,x^2a^{\frac{7}{2}}\sqrt{x\,(ax+1)}}$
risch	$\frac{(15a^4x^4+71a^3x^3+28a^2x^2-22ax+6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/30/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*a^(7/2)*x^3*(x*(
a*x+1))^(1/2)+112*a^(5/2)*x^2*(x*(a*x+1))^(1/2)-75*ln(1/2*(2*(x*(a*x+1))^(1
/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3-56*a^(3/2)*x*(x*(a*x+1))^(1/2)+12*(x*
(a*x+1))^(1/2)*a^(1/2))/x^2/a^(7/2)/(x*(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")

[Out] integrate((c - c/(a*x))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)

Fricas [A]
time = 0.38, size = 415, normalized size = 2.12

$$\left[\frac{75\left(a^{3} c^{3} x^{3}-a^{2} c^{3} x^{2}\right) \sqrt{c} \log \left(-\frac{8 a^{3} c x^{3}-7 a c x-4\left(2 a^{3} x^{3}+3 a^{2} x^{2}+a x\right) \sqrt{c} \sqrt{\frac{a x-1}{a x+1}} \sqrt{\frac{a c x-c}{a x}}-c}{a x-1}\right)+4\left(15 a^{4} c^{3} x^{4}+71 a^{3} c^{3} x^{3}+28 a^{2} c^{3} x^{2}-22 a c^{3} x+6 c^{3}\right) \sqrt{\frac{a x-1}{a x+1}} \sqrt{\frac{a c x-c}{a x}}}{60\left(a^{4} x^{3}-a^{3} x^{2}\right)}, \frac{75\left(a^{3} c^{3} x^{3}-a^{2} c^{3} x^{2}\right) \sqrt{-c} \arctan \left(\frac{2\left(a^{2} x^{2}+a x\right) \sqrt{-c} \sqrt{\frac{a x-1}{a x+1}} \sqrt{\frac{a c x-c}{a x}}}{2 a^{2} c x^{2}-a c x-c}\right)+2\left(15 a^{4} c^{3} x^{4}+71 a^{3} c^{3} x^{3}+28 a^{2} c^{3} x^{2}-22 a c^{3} x+6 c^{3}\right) \sqrt{\frac{a x-1}{a x+1}} \sqrt{\frac{a c x-c}{a x}}}{30\left(a^{4} x^{3}-a^{3} x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 + 28*a
^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(75*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)
*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 71*a^3*c^3*x^3 +
 28*a^2*c^3*x^2 - 22*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.440 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=157

$$-\frac{2c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -2/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^4*(1-1/a^2/x^2)^(3/2)*x/(c
-c/a/x)^(3/2)-3*c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
)/a+3*c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {6317, 6314, 91, 81, 52, 65, 214}

$$-\frac{2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{3\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(a*(1 - 1/(a*x))^(5/2)) - (2*(1 +
 1/(a*x))^(3/2)*(c - c/(a*x))^(5/2))/(3*a*(1 - 1/(a*x))^(5/2)) + ((1 + 1/(a
*x))^(3/2)*(c - c/(a*x))^(5/2)*x)/(1 - 1/(a*x))^(5/2) - (3*(c - c/(a*x))^(5
/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\operatorname{Subst}\left(\int\frac{\left(-\frac{3}{2a}+\frac{x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x}dx,x,\frac{1}{x}\right.}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}x}{\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(3\left(c-\frac{c}{ax}\right)^{5/2}\right)\operatorname{Su}}{2a}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{a\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/2}}{3a\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{5/}}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\left(-2+10ax+3a^2x^2\right)-9ax\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 10*a*x + 3*a^2*x^2) - 9*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A]
time = 0.06, size = 132, normalized size = 0.84

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c^{2} \left(6a^{\frac{5}{2}} x^{2} \sqrt{x\left(ax+1\right)}+20a^{\frac{3}{2}} x \sqrt{x\left(ax+1\right)}-9\ln\left(\frac{2\sqrt{x\left(ax+1\right)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right) a^{2} x^{2}-4\sqrt{x\left(ax\right.}\right.}{6\sqrt{\frac{ax-1}{ax+1}}\, x\, a^{\frac{5}{2}} \sqrt{x\left(ax+1\right)}}$
risch	$\frac{\left(3a^{3}x^{3}+13a^{2}x^{2}+8ax-2\right)c^{2}\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^{2}\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)}-\frac{3\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c^{2}\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\left(ax+1\right)}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,\left(ax+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x^2*(x*(a*
x+1))^(1/2)+20*a^(3/2)*x*(x*(a*x+1))^(1/2)-9*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^
(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-4*(x*(a*x+1))^(1/2)*a^(1/2))/x/a^(5/2)/(x*(
a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.39, size = 381, normalized size = 2.43

$$\left[\frac{9\left(a^{2}c^{2}x^{2}-ac^{2}x\right)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3\,a^{3}c^{2}x^{3}+13\,a^{2}c^{2}x^{2}+8\,ac^{2}x-2\,c^{2}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{12\left(a^{3}x^{2}-a^{2}x\right)},\ \frac{9\left(a^{2}c^{2}x^{2}-ac^{2}x\right)\sqrt{-c}\,\arctan\left(\frac{2\left(a^{2}x^{2}+ax\right)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\left(3\,a^{3}c^{2}x^{3}+13\,a^{2}c^{2}x^{2}+8\,ac^{2}x-2\,c^{2}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{6\left(a^{3}x^{2}-a^{2}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x
- 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*
x), 1/6*(9*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) + 2*(3*a^3*c^2*x^3 + 13*a^2*c^2*x^2 + 8*a*c^2*x - 2*c^2)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.441 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)-c^(3/2)*arctanh(c^(1/2)*(1-1/a^2/
x^2)^(1/2)/(c-c/a/x)^(1/2))/a+c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {6312, 893, 879, 889, 214}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2))^
(3/2)*x)/(c - c/(a*x))^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^
2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

```
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=-\left(c\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^2\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^3\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=\frac{c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.60

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(2+ax)-\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + a*x) - ArcTanh[Sqrt[1 + 1/(a*x
)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.05, size = 106, normalized size = 0.91

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\, c\left(2a^{\frac{3}{2}}x\sqrt{x\,(ax+1)}-\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+4\sqrt{x\,(ax+1)}\,\sqrt{a}\right)}{2\sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{3}{2}}\sqrt{x\,(ax+1)}}$	106
risch	$\frac{(a^2x^2+3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1)}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^(3/2)*x*(x*(a*x+1)
)^(1/2)-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+4*(x*(a*x
+1))^(1/2)*a^(1/2))/a^(3/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.40, size = 313, normalized size = 2.68

$$\left[\frac{(acx-c)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2+3\,acx+2\,c)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)}, \frac{(acx-c)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(a^2cx^2+3\,acx+2\,c)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
```

```
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*((a*c*x - c)*sqrt(-c)*arctan(2*(a^2*x^
2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*
c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 3*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.442 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=78

$$\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6312, 877, 889, 214}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 66, normalized size = 0.85

$$\frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 87, normalized size = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x\,(ax+1)}\,\sqrt{a}+\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{x\,(ax+1)}\,\sqrt{a}}$	87
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1)}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^(1/2)*a^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

time = 0.41, size = 295, normalized size = 3.78

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4(a^2x-a)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)-2(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.443 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

```
[Out] 3*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-2*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)+x*(1-1/a
/x)^(1/2)*(1+1/a/x)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6317, 6314, 101, 162, 65, 214, 212}

$$\frac{x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\sqrt{c-\frac{c}{ax}}}+\frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/Sqrt[c - c/(a*x)] + (3*Sqrt[1 - 1/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) - (2*Sqrt[2]*Sqrt[1
 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c - c/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\frac{3}{2a}+\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(2\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\right.}{}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(3\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{c-\frac{c}{ax}}} - \frac{\left(4\sqrt{1-\right.}{}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{3\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\right.}{a\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A]

time = 0.04, size = 93, normalized size = 0.61

$$\frac{\sqrt{1-\frac{1}{ax}}\left(a\sqrt{1+\frac{1}{ax}}\,x+3\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 3*ArcTanh[Sqrt[1 + 1/(a*x)]] -
2*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[c - c/(a*x)])
```

Maple [A]
time = 0.14, size = 151, normalized size = 0.99

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)\sqrt{a}+3\ln\left(\frac{2\sqrt{x(ax+\cdots}{2\cdots}\right.\right.\cdots}{2\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{3}{2}}c\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a\sqrt{a^2c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)\cdots}}{x-\frac{1}{a}}\right.\cdots}{a^2\sqrt{c}\cdots}\right.\cdots}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^(1/2)*a^
(3/2)*(1/a)^(1/2)-2*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3
*a*x+1)/(a*x-1))*a^(1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(
1/2))*a*(1/a)^(1/2))/a^(3/2)/c/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.43, size = 517, normalized size = 3.40

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+\frac{2\sqrt{2}\,(acx-c)\log\left(-\frac{17\,a^3x^3-3\,a^2x^2-13\,ax-\frac{4\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}-1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)}{\sqrt{c}}}{4\,(a^2cx-ac)},\ \frac{2\sqrt{2}\,(acx-c)\sqrt{-\frac{1}{c}}\arctan\left(\frac{2\sqrt{2}\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}}{3\,a^2x^2-2\,ax-1}\right)-3\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2cx-ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)) + 2*sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sq
rt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*c*
x - a*c), 1/2*(2*sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 +
 a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x
^2 - 2*a*x - 1)) - 3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c))
 + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*c*x - a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.444 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=215

$$-\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\tanh}{\sqrt{2}\,a\,(c}$$

```
[Out] 5*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-7/2*(1-1/a/x)^
(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*(1-1
/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a*(1-1/a/x)^(3/2)*x*(1+
1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6317, 6314, 101, 156, 162, 65, 214, 212}

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*(c - c/(a*x))^(3/2
)) + (a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*x)
)^(3/2)) + (5*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*
x))^(3/2)) - (7*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sq
rt[2]*a*(c - c/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\frac{5}{2a}+\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{-}{x\left(1-\frac{x}{a}\right.}\right.}{2\left(c-\frac{c}{ax}\right)^{3/}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(7\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{}{\left(1-\frac{x}{a}\right)}\right.}{2a^2\left(c-\frac{c}{ax}\right)^{3}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-a+ax}\right.}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{5\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right.}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [A]

time = 0.06, size = 122, normalized size = 0.57

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}\,x(-2+ax)+10(-1+ax)\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-7\sqrt{2}\,(-1+ax)\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{2ac\sqrt{c-\frac{c}{ax}}\,(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-2 + a*x) + 10*(-1 + a*x)*ArcT
anh[Sqrt[1 + 1/(a*x)]] - 7*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqr
t[2]]))/(2*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A]
time = 0.16, size = 259, normalized size = 1.20

method	result
default	$\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{x\,(ax+1)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-7a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x-8\sqrt{x\,(ax+1)}$
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^2\sqrt{a^2c}}-\frac{7\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\,(x-}}{x-\frac{1}{a}}\right)}{4a^3\sqrt{c}}\right.}{c\sqrt{\frac{ax}{ax}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*(x*(a*x+1))^
(1/2)*a^(5/2)*(1/a)^(1/2)*x-7*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*
(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x-8*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2
)+10*a^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*
x+7*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))
*a^(1/2)-10*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(
1/2))/a^(3/2)/c^2/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.44, size = 594, normalized size = 2.76

$$\left[\frac{7\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\sqrt{2}\left(3\,a^3x^3+4\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+10\left(a^2x^2-2\,ax+1\right)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\left(2\,a^3x^3+3\,a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\left(a^3x^3-a^2x^2-2\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)},\ \frac{7\sqrt{2}\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\arctan\left(\frac{2\sqrt{2}\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\,a^2cx^2-2\,acx-c}\right)-10\left(a^2x^2-2\,ax+1\right)\sqrt{-c}\,\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+4\left(a^3x^3-a^2x^2-2\,ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\left(a^3c^2x^2-2\,a^2c^2x+ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
[Out] [1/8*(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 10*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*
a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x)) - c)/(a*x - 1)) + 8*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), 1/4*
(7*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*
sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2
*a*c*x - c)) - 10*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 4*(a^3*x^3 - a^2*x^2 - 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.445 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\right.}}{a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 7*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-79/16*(1-1/a/x
)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/2*
a*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-23/8*(1-1/a/x)^
(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^2*(1-1/a/x)^(5/2)*x*(1+1/a/
x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6317, 6314, 101, 156, 162, 65, 214, 212}

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{23\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{79\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{8\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-3*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))^2*(c - c/(a*x)
)^(5/2)) - (23*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(8*(a - x^(-1))*(c -
c/(a*x))^(5/2)) + (a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1
))^2*(c - c/(a*x))^(5/2)) + (7*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)
]])/(a*(c - c/(a*x))^(5/2)) - (79*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a
*x)]/Sqrt[2]])/(8*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !Inte
gerQ[n/2] && !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\frac{7}{2a}+\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int -\right.}{4\left(c-\frac{c}{a\right.}}$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} -$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} -$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} -$$

$$= -\frac{3a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{23\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{8\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} +$$

Mathematica [A]
time = 0.08, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}\,x(23-35ax+8a^2x^2)+112(-1+ax)^2\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-79\sqrt{2}\,(-1+ax)^2\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{16ac^2\sqrt{c-\frac{c}{ax}}\,(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(23 - 35*a*x + 8*a^2*x^2) + 112
*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 79*Sqrt[2]*(-1 + a*x)^2*ArcTanh[
Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(16*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A]
time = 0.16, size = 366, normalized size = 1.32

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{79\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\,(x-}}{x-\frac{1}{a}}\right.}{32a^4\sqrt{c}}\right.}{}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32\sqrt{x\,(ax+1)}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-79a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x^2-140\sqrt{x\,(ax+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(c*(a*x-1)/a/x)^(1/2)*x*(32*(x*(a*x+
1))^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2-79*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1
/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-140*(x*(a*x+1))^(1/2)*a^(5/2)
*(1/a)^(1/2)*x+112*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^
3*(1/a)^(1/2)*x^2+158*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))
^(1/2)*a+3*a*x+1)/(a*x-1))*x+92*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-224*a
^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x+112*
ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-79*2^(1
```

```
/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)
)/a^(3/2)/c^3/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.45, size = 668, normalized size = 2.41

$$\left[\frac{79\sqrt{2}\,(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\,(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+112\,(a^3x^3-3a^2x^2+3ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\,(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\,(8a^4x^4-27a^3x^3-12a^2x^2+23ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{64\,(a^4c^3x^3-3a^3c^3x^2+3a^2c^3x-ac^3)},\ \frac{79\sqrt{2}\,(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)-112\,(a^3x^3-3a^2x^2+3ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+4\,(8a^4x^4-27a^3x^3-12a^2x^2+23ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{32\,(a^4c^3x^3-3a^3c^3x^2+3a^2c^3x-ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(79*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c*
x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)) + 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(8*
a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(8*a^4*x^4 - 27*a^
3*x^3 - 12*a^2*x^2 + 23*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3*x - a*c^3), 1/32*(79*sqrt(2)*
(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
2*a*c*x - c)) - 112*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*(a^
2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*
a^2*c*x^2 - a*c*x - c)) + 4*(8*a^4*x^4 - 27*a^3*x^3 - 12*a^2*x^2 + 23*a*x)*
sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3
*x^2 + 3*a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.446 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=143

$$\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 5/3*c^3*(c-c/a/x)^(3/2)/a+c^2*(c-c/a/x)^(5/2)/a+5/7*c*(c-c/a/x)^(7/2)/a+(c-
c/a/x)^(9/2)*x-5*c^(9/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+5*c^4*(c-c/a/x)
^(1/2)/a
```

Rubi [A]
time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+x\left(c-\frac{c}{ax}\right)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (5*c^4*Sqrt[c - c/(a*x)])/a + (5*c^3*(c - c/(a*x))^(3/2))/(3*a) + (c^2*(c -
 c/(a*x))^(5/2))/a + (5*c*(c - c/(a*x))^(7/2))/(7*a) + (c - c/(a*x))^(9/2)*
x - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$$

$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}{1-ax}\,dx$$

$$=\frac{c\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{x}\,dx}{a}$$

$$=\frac{c\int\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}\,dx}{a}$$

$$=-\frac{c\text{Subst}\left(\int\frac{(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{(5c)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{(5c^2)\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{(5c^3)\,\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}\right.}{2a}$$

$$=\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{ax}\right)^{9/2}x+\frac{(5c^4)\,\text{S}}{}$$

$$=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{a}\right.$$

$$=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{a}\right.$$

$$=\frac{5c^4\sqrt{c-\frac{c}{ax}}}{a}+\frac{5c^3\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{5/2}}{a}+\frac{5c\left(c-\frac{c}{ax}\right)^{7/2}}{7a}+\left(c-\frac{c}{a}\right.$$

Mathematica [A]
time = 0.10, size = 91, normalized size = 0.64

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\,(6-18ax+4a^2x^2+92a^3x^3+21a^4x^4)}{21a^4x^3}-\frac{5c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(6 - 18*a*x + 4*a^2*x^2 + 92*a^3*x^3 + 21*a^4*x^4))/
(21*a^4*x^3) - (5*c^(9/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A]
time = 0.12, size = 163, normalized size = 1.14

method	result
risch	$\frac{(21a^5x^5+71a^4x^4-88a^3x^3-22a^2x^2+24ax-6)c^4\sqrt{\frac{c(ax-1)}{ax}}}{21x^3a^4(ax-1)}-\frac{5\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)c^4\sqrt{\frac{c(ax-1)}{ax}}\ \sqrt{c\,(a\cdots}}{2\sqrt{a^2c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(-210a^{\frac{9}{2}}\sqrt{a\,x^2-x}\,x^5+105\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^5+168a^{\frac{7}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^3-16a^{\frac{5}{2}}\left(a\,x^2-x\right)\cdots\right.}{42x^4\sqrt{(ax-1)\,x}\,a^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/42*(c*(a*x-1)/a/x)^(1/2)/x^4*c^4*(-210*a^(9/2)*(a*x^2-x)^(1/2)*x^5+105*l
n(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^4*x^5+168*a^(7/2)*(a*x
^2-x)^(3/2)*x^3-16*a^(5/2)*(a*x^2-x)^(3/2)*x^2-24*a^(3/2)*(a*x^2-x)^(3/2)*x
+12*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)*x)^(1/2)/a^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(9/2)/(a*x - 1), x)
```

Fricas [A]
time = 0.39, size = 234, normalized size = 1.64

$$\left[\frac{105\,a^{3}c^{\frac{9}{2}}x^{3}\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(21\,a^{4}c^{4}x^{4}+92\,a^{3}c^{4}x^{3}+4\,a^{2}c^{4}x^{2}-18\,ac^{4}x+6\,c^{4})\sqrt{\frac{acx-c}{ax}}}{42\,a^{4}x^{3}}, \frac{105\,a^{3}\sqrt{-c}\,c^{4}x^{3}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+(21\,a^{4}c^{4}x^{4}+92\,a^{3}c^{4}x^{3}+4\,a^{2}c^{4}x^{2}-18\,ac^{4}x+6\,c^{4})\sqrt{\frac{acx-c}{ax}}}{21\,a^{4}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/42*(105*a^3*c^(9/2)*x^3*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + c) + 2*(21*a^4*c^4*x^4 + 92*a^3*c^4*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x
 + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^3), 1/21*(105*a^3*sqrt(-c)*c^4*x^
3*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (21*a^4*c^4*x^4 + 92*a^3*c^4
*x^3 + 4*a^2*c^4*x^2 - 18*a*c^4*x + 6*c^4)*sqrt((a*c*x - c)/(a*x)))/(a^4*x^
3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(9/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{9/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(9/2)*(a*x + 1))/(a*x - 1), x)
```

3.447 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=118

$$\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-3*c^(7/2)*a
rctanh((c-c/a/x)^(1/2)/c^(1/2))/a+3*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A]
time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (3*c^3*Sqrt[c - c/(a*x)])/a + (c^2*(c - c/(a*x))^(3/2))/a + (3*c*(c - c/(a*
x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (3*c^(7/2)*ArcTanh[Sqrt[c - c/(a
*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}{1-ax}dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{x}dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}{x^2}dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x}dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c^2)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c^3)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\right.}{2a}\\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{(3c^4)\text{Sul}}{}\\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-(3c^3)\text{Sul}\\
&= \frac{3c^3\sqrt{c-\frac{c}{ax}}}{a}+\frac{c^2\left(c-\frac{c}{ax}\right)^{3/2}}{a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{3c^{7/2}\tan}{}
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 83, normalized size = 0.70

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\,(-2+4ax+8a^2x^2+5a^3x^3)}{5a^3x^2}-\frac{3c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(-2 + 4*a*x + 8*a^2*x^2 + 5*a^3*x^3))/(5*a^3*x^2) -
(3*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [A]
time = 0.11, size = 144, normalized size = 1.22

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(30a^{\frac{7}{2}}\sqrt{a\,x^2-x}\,x^4-20a^{\frac{5}{2}}(a\,x^2-x)^{\frac{3}{2}}x^2-15\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^4-4a^{\frac{3}{2}}(a\,x^2-x)^{\frac{3}{2}}x+4(\cdots\right.}{10x^3\sqrt{(ax-1)\,x}\,a^{\frac{7}{2}}}$
risch	$\frac{(5a^4x^4+3a^3x^3-4a^2x^2-6ax+2)c^3\sqrt{\frac{c(ax-1)}{ax}}}{5x^2a^3(ax-1)}-\frac{3\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)c^3\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,a\cdots}}{2\sqrt{a^2c}\,(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*a^(7/2)*(a*x^2-x)^(1/2)*x^4-20*a^(5/2)*(
a*x^2-x)^(3/2)*x^2-15*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a
^3*x^4-4*a^(3/2)*(a*x^2-x)^(3/2)*x+4*(a*x^2-x)^(3/2)*a^(1/2))/x^3/((a*x-1)*
x)^(1/2)/a^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a*x))^(7/2)/(a*x - 1), x)
```

Fricas [A]
time = 0.36, size = 212, normalized size = 1.80

$$\left[\frac{15 \, a^{2} c^{\frac{7}{2}} x^{2} \log\left(-2 \, a c x + 2 \, a \sqrt{c} x \sqrt{\frac{a c x - c}{a x}} + c\right) + 2 \, {\left(5 \, a^{3} c^{3} x^{3} + 8 \, a^{2} c^{3} x^{2} + 4 \, a c^{3} x - 2 \, c^{3}\right)} \sqrt{\frac{a c x - c}{a x}}}{10 \, a^{3} x^{2}}, \frac{15 \, a^{2} \sqrt{-c} c^{3} x^{2} \arctan\left(\frac{\sqrt{-c} \sqrt{\frac{a c x - c}{a x}}}{c}\right) + {\left(5 \, a^{3} c^{3} x^{3} + 8 \, a^{2} c^{3} x^{2} + 4 \, a c^{3} x - 2 \, c^{3}\right)} \sqrt{\frac{a c x - c}{a x}}}{5 \, a^{3} x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/10*(15*a^2*c^(7/2)*x^2*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3)*sqrt((a*c*
x - c)/(a*x)))/(a^3*x^2), 1/5*(15*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + (5*a^3*c^3*x^3 + 8*a^2*c^3*x^2 + 4*a*c^3*x - 2*c^3
)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 8.10, size = 731, normalized size = 6.19

$$c^{3} \left(\begin{cases} - \frac{\sqrt{c} \operatorname{acosh}{\left(\sqrt{a} \sqrt{x} \right)}}{a} + \frac{\sqrt{c} \sqrt{x} \sqrt{a x - 1}}{\sqrt{a}} & \text{for}\: \left|{a x}\right| > 1 \\- \frac{i \sqrt{a} \sqrt{c} x^{\frac{3}{2}}}{\sqrt{- a x + 1}} + \frac{i \sqrt{c} \operatorname{asin}{\left(\sqrt{a} \sqrt{x} \right)}}{a} + \frac{i \sqrt{c} \sqrt{x}}{\sqrt{a} \sqrt{- a x + 1}} & \text{otherwise} \end{cases}\right) + \frac{2 c^{4} \operatorname{atan}{\left(\frac{\sqrt{c - \frac{c}{a x}}}{\sqrt{- c}} \right)}}{a \sqrt{- c}} + \frac{2 c^{3} \sqrt{c - \frac{c}{a x}}}{a} - \frac{c^{3} \left(\begin{cases} 0 & \text{for}\: c = 0 \\\frac{2 a \left(c - \frac{c}{a x}\right)^{\frac{3}{2}}}{3 c} & \text{otherwise} \end{cases}\right)}{a^{2}} + \frac{c^{3} \left(\begin{cases} - \frac{4 a^{\frac{11}{2}} \sqrt{c} x^{\frac{7}{2}}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} + \frac{4 a^{\frac{9}{2}} \sqrt{c} x^{\frac{5}{2}}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} + \frac{4 a^{5} \sqrt{c} x^{3} \sqrt{a x - 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} - \frac{2 a^{4} \sqrt{c} x^{2} \sqrt{a x - 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} - \frac{8 a^{3} \sqrt{c} x \sqrt{a x - 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} + \frac{6 a^{2} \sqrt{c} \sqrt{a x - 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} & \text{for}\: \left|{a x}\right| > 1 \\- \frac{4 a^{\frac{11}{2}} \sqrt{c} x^{\frac{7}{2}}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} + \frac{4 a^{\frac{9}{2}} \sqrt{c} x^{\frac{5}{2}}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} + \frac{4 i a^{5} \sqrt{c} x^{3} \sqrt{- a x + 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} - \frac{2 i a^{4} \sqrt{c} x^{2} \sqrt{- a x + 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} - \frac{8 i a^{3} \sqrt{c} x \sqrt{- a x + 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} + \frac{6 i a^{2} \sqrt{c} \sqrt{- a x + 1}}{15 a^{\frac{7}{2}} x^{\frac{7}{2}} - 15 a^{\frac{5}{2}} x^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(7/2),x)
```

```
[Out] c**3*Piecewise((-sqrt(c)*acosh(sqrt(a)*sqrt(x))/a + sqrt(c)*sqrt(x)*sqrt(a*
x - 1)/sqrt(a), Abs(a*x) > 1), (-I*sqrt(a)*sqrt(c)*x**(3/2)/sqrt(-a*x + 1)
+ I*sqrt(c)*asin(sqrt(a)*sqrt(x))/a + I*sqrt(c)*sqrt(x)/(sqrt(a)*sqrt(-a*x
+ 1)), True)) + 2*c**4*atan(sqrt(c - c/(a*x))/sqrt(-c))/(a*sqrt(-c)) + 2*c*
*3*sqrt(c - c/(a*x))/a - c**3*Piecewise((0, Eq(c, 0)), (2*a*(c - c/(a*x))**
(3/2)/(3*c), True))/a**2 + c**3*Piecewise((-4*a**(11/2)*sqrt(c)*x**(7/2)/(1
5*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(
15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 4*a**5*sqrt(c)*x**3*sqrt(a*x
 - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 2*a**4*sqrt(c)*x**2*s
qrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) - 8*a**3*sqrt(c)
*x*sqrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*a**2*sqr
t(c)*sqrt(a*x - 1)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), Abs(a*x)
> 1), (-4*a**(11/2)*sqrt(c)*x**(7/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x*
*(5/2)) + 4*a**(9/2)*sqrt(c)*x**(5/2)/(15*a**(7/2)*x**(7/2) - 15*a**(5/2)*x
**(5/2)) + 4*I*a**5*sqrt(c)*x**3*sqrt(-a*x + 1)/(15*a**(7/2)*x**(7/2) - 15*
a**(5/2)*x**(5/2)) - 2*I*a**4*sqrt(c)*x**2*sqrt(-a*x + 1)/(15*a**(7/2)*x**(
7/2) - 15*a**(5/2)*x**(5/2)) - 8*I*a**3*sqrt(c)*x*sqrt(-a*x + 1)/(15*a**(7/
2)*x**(7/2) - 15*a**(5/2)*x**(5/2)) + 6*I*a**2*sqrt(c)*sqrt(-a*x + 1)/(15*a
**(7/2)*x**(7/2) - 15*a**(5/2)*x**(5/2)), True))/a**3
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.448 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-c^(5/2)*arctanh((c-c/a/x)^(1/2)/c
^(1/2))/a+c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A]
time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x))^(
5/2)*x - (c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c^2\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{c^3\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,\right.}{2a}\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-c^2\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\right.\\
&= \frac{c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A]

time = 0.04, size = 75, normalized size = 0.79

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\,(2-2ax+3a^2x^2)-3ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 2*a*x + 3*a^2*x^2) - 3*a*c^(5/2)*x*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/(3*a^2*x)
```

Maple [A]
time = 0.10, size = 108, normalized size = 1.14

method	result	siz
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(-6\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x^3+3\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x^3+4(a\,x^2-x)^{\frac{3}{2}}\sqrt{a}\right)}{6x^2\sqrt{(ax-1)\,x}\,a^{\frac{5}{2}}}$	10
risch	$\frac{(3a^3x^3-5a^2x^2+4ax-2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}-\frac{\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)c^2\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{2\sqrt{a^2c}\,(ax-1)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3+3*ln(1/2
*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^3+4*(a*x^2-x)^(3/2)*a^(
1/2))/((a*x-1)*x)^(1/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^(5/2)/(a*x - 1), x)
```

Fricas [A]
time = 0.35, size = 182, normalized size = 1.92

$$\left[\frac{3\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(3\,a^{2}c^{2}x^{2}-2\,ac^{2}x+2\,c^{2})\sqrt{\frac{acx-c}{ax}}}{6\,a^{2}x},\ \frac{3\,a\sqrt{-c}\,c^{2}x\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)+(3\,a^{2}c^{2}x^{2}-2\,ac^{2}x+2\,c^{2})\sqrt{\frac{acx-c}{ax}}}{3\,a^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c) + 2*(3*a^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)
, 1/3*(3*a*sqrt(-c)*c^2*x*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (3*a
^2*c^2*x^2 - 2*a*c^2*x + 2*c^2)*sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.449 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=70

$$-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] (c-c/a/x)^(3/2)*x+c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a-c*(c-c/a/x)^(1
/2)/a
```

Rubi [A]
time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {6302, 6268, 25, 528, 382, 79, 52, 65, 214}

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x + (c^(3/2)*ArcTanh[Sqrt[
c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{(a+x)\sqrt{c-\frac{cx}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+c\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)\\
&= -\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 55, normalized size = 0.79

$$\frac{c\sqrt{c-\frac{c}{ax}}\,(-2+ax)+c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) + c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]
])/a
```

Maple [A]
time = 0.10, size = 103, normalized size = 1.47

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c\left(-2\sqrt{a\,x^{2}-x}\,a^{\frac{3}{2}}x^{2}+4\left(a\,x^{2}-x\right)^{\frac{3}{2}}\sqrt{a}+\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^{2}\right)}{2x\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}}$	103
risch	$\frac{\left(a^{2}x^{2}-3ax+2\right)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\ln\left(\frac{-\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{2\sqrt{a^{2}c}\,(ax-1)}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)/x*c*(-2*(a*x^2-x)^(1/2)*a^(3/2)*x^2+4*(a*x^2-x)^(
3/2)*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*x^2)/((a
*x-1)*x)^(1/2)/a^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^(3/2)/(a*x - 1), x)
```

Fricas [A]
time = 0.35, size = 137, normalized size = 1.96

$$\left[\frac{c^{\frac{3}{2}}\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a}, -\frac{\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c^(3/2)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2
*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(sqrt(-c)*c*arctan(sqrt(-c)*sqr
t((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a*x))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.450 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=50

$$\sqrt{c-\frac{c}{ax}}\,x+\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 382, 79, 65, 214}

$$x\sqrt{c-\frac{c}{ax}}+\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \sqrt{c-\frac{c}{ax}}\,x - \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \sqrt{c-\frac{c}{ax}}\,x + 3\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.00

$$\sqrt{c-\frac{c}{ax}}\,x + \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]

[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(42) = 84$.
time = 0.14, size = 120, normalized size = 2.40

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{2\sqrt{a^2c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^2-x}\,\sqrt{a}-4\sqrt{(ax-1)\,x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1}}{2}\right.\right.}{2\sqrt{(ax-1)\,x}\,\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(1/2)-4*((a*x-1)*x)^(1/2)*a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)

Fricas [A]
time = 0.35, size = 124, normalized size = 2.48

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")

[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)

[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.
time = 0.43, size = 96, normalized size = 1.92

$$\frac{3\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{2\,a}-\frac{3\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{2\,a\mathrm{sgn}\left(x\right)}+\frac{\sqrt{a^2cx^2-acx}\,|a|}{a^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")

[Out] 3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)

[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)

3.451 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=70

$$-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{x}{\sqrt{c-\frac{c}{ax}}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] 5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)-5/a/(c-c/a/x)^(1/2)+x/(c-c/a/x
)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\frac{x}{\sqrt{c-\frac{c}{ax}}}-\frac{5}{a\sqrt{c-\frac{c}{ax}}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] -5/(a*Sqrt[c - c/(a*x)]) + x/Sqrt[c - c/(a*x)] + (5*ArcTanh[Sqrt[c - c/(a*x
)]/Sqrt[c]])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

$$= -\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{(5c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} - \frac{5\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c}$$

$$= -\frac{5}{a\sqrt{c-\frac{c}{ax}}} + \frac{x}{\sqrt{c-\frac{c}{ax}}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 43, normalized size = 0.61

$$\frac{ax - 5\,{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1 - \frac{1}{ax}\right)}{a\sqrt{c - \frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (a*x - 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*Sqrt[c - c/(a*x)]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(60) = 120.
time = 0.15, size = 194, normalized size = 2.77

method	result
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a\sqrt{a^2c}}-\frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{a^3c\left(x-\frac{1}{a}\right)}\right)\sqrt{c\,(ax-1)\,ax}}{\sqrt{\frac{c(ax-1)}{ax}}\;x}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\;x\left(10a^{\frac{5}{2}}\sqrt{(ax-1)\,x}\;x^2+5\ln\left(\frac{2\sqrt{(ax-1)\,x}\;\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x^2-8a^{\frac{3}{2}}((ax-1)x)^{\frac{3}{2}}-20a^{\frac{3}{2}}\sqrt{(ax-1}}{2\sqrt{(ax-1)\,x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(10*a^(5/2)*((a*x-1)*x)^(1/2)*x^2+5*ln(1/2*(2*(
(a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^2-8*a^(3/2)*((a*x-1)*x)^(3
/2)-20*a^(3/2)*((a*x-1)*x)^(1/2)*x-10*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*a*x+10*((a*x-1)*x)^(1/2)*a^(1/2)+5*ln(1/2*(2*((a*x-1)*x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c/(a*x-1)^2/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a*x))), x)
```

Fricas [A]
time = 0.33, size = 176, normalized size = 2.51

$$\left[\frac{5\,(ax-1)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(a^2x^2-5\,ax)\sqrt{\frac{acx-c}{ax}}}{2\,(a^2cx-ac)},-\frac{5\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(a^2x^2-5\,ax)\sqrt{\frac{acx-c}{ax}}}{a^2cx-ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*
x)) + c) + 2*(a^2*x^2 - 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c), -(
5*(a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a^2*x^2
- 5*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(60) = 120.
time = 0.47, size = 167, normalized size = 2.39

$$\frac{5\,\log\left(|a||c|^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{6\,a\sqrt{c}}-\frac{5\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^3|a|+5\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^2a\sqrt{c}-4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)c|a|+ac^{\frac{3}{2}}\right|\right)\operatorname{sgn}(x)}{6\,a\sqrt{c}}+\frac{\sqrt{a^2cx^2-acx}\,|a|\operatorname{sgn}(x)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 5/6*log(abs(a)*abs(c)^(3/2))*sgn(x)/(a*sqrt(c)) - 5/6*log(abs(-2*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*abs(a) + 5*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - a*c*x))^2*a*sqrt(c) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c*abs
(a) + a*c^(3/2)))*sgn(x)/(a*sqrt(c)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x
)/(a^2*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(1/2)*(a*x - 1)), x)
```

3.452 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -7/3/a/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(3/2)+7*arctanh((c-c/a/x)^(1/2)/c^(1/2))
/a/c^(3/2)-7/a/c/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{ac\sqrt{c-\frac{c}{ax}}}-\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -7/(3*a*(c - c/(a*x))^(3/2)) - 7/(a*c*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^
(3/2) + (7*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(3/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx \\
&= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}\,dx \\
&= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}x}\,dx}{a} \\
&= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{a} \\
&= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(7c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} \\
&= -\frac{7}{3a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{ac\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}
\end{aligned}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.02, size = 55, normalized size = 0.58

$$\frac{x\left(3ax-7\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{1}{ax}\right)\right)}{3c\sqrt{c-\frac{c}{ax}}\,(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (x*(3*a*x - 7*Hypergeometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c*Sqrt[c -
 c/(a*x)]*(-1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(81) = 162$.
time = 0.16, size = 260, normalized size = 2.74

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^2\sqrt{a^2c}}-\frac{22\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{3a^4c\left(x-\frac{1}{a}\right)}-\frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2}}{3a^5c\left(x-\right.}\right.}{cx\sqrt{\frac{c(ax-1)}{ax}}}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-42\sqrt{(ax-1)\,x}\,a^{\frac{7}{2}}x^3+36((ax-1)x)^{\frac{3}{2}}a^{\frac{5}{2}}x-21\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3x^3+126a^{\frac{5}{2}}\sqrt{(a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*(a*x-1)/a/x)^(1/2)*x*(-42*((a*x-1)*x)^(1/2)*a^(7/2)*x^3+36*((a*x-1)
*x)^(3/2)*a^(5/2)*x-21*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*a^3*x^3+126*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-28*a^(3/2)*((a*x-1)*x)^(3/2)+63
*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^2-126*a^(3/2)*
((a*x-1)*x)^(1/2)*x-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*a*x+42*((a*x-1)*x)^(1/2)*a^(1/2)+21*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^2/(a*x-1)^3/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 238, normalized size = 2.51

$$\left[\frac{21\,(a^2x^2-2\,ax+1)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(3\,a^3x^3-28\,a^2x^2+21\,ax)\sqrt{\frac{acx-c}{ax}}}{6\,(a^3c^2x^2-2\,a^2c^2x+ac^2)},\,-\frac{21\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(3\,a^3x^3-28\,a^2x^2+21\,ax)\sqrt{\frac{acx-c}{ax}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(21*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) + 2*(3*a^3*x^3 - 28*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c
)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(21*(a^2*x^2 - 2*a*x +
1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^3*x^3 - 28*a^
2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(81) = 162.
time = 0.52, size = 243, normalized size = 2.56

$$\frac{7\log\left(c^2|a|\sqrt{|c|}\right)\mathrm{sgn}(x)}{10\,ac^{\frac{3}{2}}}-\frac{7\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^5|a|+9\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^4a\sqrt{c}-16\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^3c|a|+14\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^2ac^{\frac{3}{2}}-6\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)c^2|a|+ac^{\frac{5}{2}}\right|\right)\mathrm{sgn}(x)}{10\,ac^{\frac{3}{2}}}+\frac{\sqrt{a^2cx^2-acx}\,|a|\mathrm{sgn}(x)}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] 7/10*log(c^2*abs(a)*sqrt(abs(c)))*sgn(x)/(a*c^(3/2)) - 7/10*log(abs(-2*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*abs(a) + 9*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - a*c*x))^4*a*sqrt(c) - 16*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x)
)^3*c*abs(a) + 14*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^(3/2) - 6
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^2*abs(a) + a*c^(5/2)))*sgn(x)/
(a*c^(3/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a^2*c^2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(3/2)*(a*x - 1)), x)
```

3.453 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}$$

```
[Out] -9/5/a/(c-c/a/x)^(5/2)-3/a/c/(c-c/a/x)^(3/2)+x/(c-c/a/x)^(5/2)+9*arctanh((c
-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)-9/a/c^2/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}-\frac{9}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] -9/(5*a*(c - c/(a*x))^(5/2)) - 3/(a*c*(c - c/(a*x))^(3/2)) - 9/(a*c^2*Sqrt[
c - c/(a*x)]) + x/(c - c/(a*x))^(5/2) + (9*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c
]])/(a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{(9c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{9\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{c}{}}}\right.}{2ac^2}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,\right.}{c^3}$$

$$= -\frac{9}{5a\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{3}{ac\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{9}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{9\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right.}{ac^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 58, normalized size = 0.49

$$\frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{1}{ax}\right)}{5a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] x/(c - c/(a*x))^(5/2) - (9*Hypergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)])/(
5*a*(c - c/(a*x))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(102) = 204.
time = 0.17, size = 328, normalized size = 2.78

method	result
risch	$\frac{ax-1}{a c^2\sqrt{\frac{c(ax-1)}{ax}}}+\Bigg(\frac{9\ln\left(\frac{-\frac{1}{2}ac+c a^2 x}{\sqrt{a^2c}}+\sqrt{a^2 c x^2-acx}\right)}{2a^3\sqrt{a^2c}}-\frac{54\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{5a^5c\left(x-\frac{1}{a}\right)}-\frac{18\sqrt{a^2c\left(x-\right.}}{5a^6}\ldots\Bigg)\Big/\left(c^2x\sqrt{\frac{c(ax-1)}{ax}}\ldots\right)$
default	$\sqrt{\frac{c(ax-1)}{ax}}\,x\left(90a^{\frac{9}{2}}\sqrt{(ax-1)x}\,x^4+45\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^4x^4-80a^{\frac{7}{2}}((ax-1)x)^{\frac{3}{2}}x^2-360\sqrt{(ax-}\ldots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(c*(a*x-1)/a/x)^(1/2)*x*(90*a^(9/2)*((a*x-1)*x)^(1/2)*x^4+45*ln(1/2*(2
*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^4*x^4-80*a^(7/2)*((a*x-1)*x)
^(3/2)*x^2-360*((a*x-1)*x)^(1/2)*a^(7/2)*x^3-180*ln(1/2*(2*((a*x-1)*x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*a^3*x^3+132*((a*x-1)*x)^(3/2)*a^(5/2)*x+540*a^(
5/2)*((a*x-1)*x)^(1/2)*x^2+270*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*a^2*x^2-60*a^(3/2)*((a*x-1)*x)^(3/2)-360*a^(3/2)*((a*x-1)*x)^(1/2
)*x-180*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*x+90*((a*x-
1)*x)^(1/2)*a^(1/2)+45*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
))/((a*x-1)*x)^(1/2)/c^3/(a*x-1)^4/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(5/2)), x)
```

Fricas [A]
time = 0.34, size = 294, normalized size = 2.49

$$\left[\frac{45\,(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\sqrt{c}\,\log\left(-2\,acx - 2\,a\sqrt{c}\,x\sqrt{\frac{acx - c}{ax}} + c\right) + 2\,(5\,a^4x^4 - 69\,a^3x^3 + 105\,a^2x^2 - 45\,ax)\sqrt{\frac{acx - c}{ax}}}{10\,(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3)}, -\frac{45\,(a^3x^3 - 3\,a^2x^2 + 3\,ax - 1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx - c}{ax}}}{c}\right) - (5\,a^4x^4 - 69\,a^3x^3 + 105\,a^2x^2 - 45\,ax)\sqrt{\frac{acx - c}{ax}}}{5\,(a^4c^3x^3 - 3\,a^3c^3x^2 + 3\,a^2c^3x - ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/10*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt
(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^
2 - 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c
^3*x - a*c^3), -1/5*(45*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (5*a^4*x^4 - 69*a^3*x^3 + 105*a^2*x^2
- 45*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^3*x^3 - 3*a^3*c^3*x^2 + 3*a^2*c^3
*x - a*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax + 1}{\left(-c\left(-1 + \frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(102) = 204.
time = 0.57, size = 316, normalized size = 2.68

$$\frac{9\,\log\left(c^2|a||c|^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{14\,ac^{\frac{5}{2}}} - \frac{9\,\log\left(\left|-2\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)^7|a| + 13\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)^6 a\sqrt{c} - 36\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)^5 c|a| + 55\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)^4 ac^{\frac{3}{2}} - 50\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)^3 c^2|a| + 27\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)^2 ac^{\frac{5}{2}} - 8\left(\sqrt{a^2c}\,x - \sqrt{a^2cx^2 - acx}\right)c^3|a| + ac^{\frac{7}{2}}\right|\right)\operatorname{sgn}(x)}{14\,ac^{\frac{5}{2}}} + \frac{\sqrt{a^2cx^2 - acx}\,|a|\operatorname{sgn}(x)}{a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] 9/14*log(c^2*abs(a)*abs(c)^(3/2))*sgn(x)/(a*c^(5/2)) - 9/14*log(abs(-2*(sqr
t(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*abs(a) + 13*(sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - a*c*x))^6*a*sqrt(c) - 36*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
```

```
))^5*c*abs(a) + 55*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^(3/2) -
50*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*c^2*abs(a) + 27*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - a*c*x))^2*a*c^(5/2) - 8*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - a*c*x))*c^3*abs(a) + a*c^(7/2)))*sgn(x)/(a*c^(5/2)) + sqrt(a^2*c*x^2 -
 a*c*x)*abs(a)*sgn(x)/(a^2*c^3)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(5/2)*(a*x - 1)), x)
```

3.454 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}$$

```
[Out] -11/7/a/(c-c/a/x)^(7/2)-11/5/a/c/(c-c/a/x)^(5/2)-11/3/a/c^2/(c-c/a/x)^(3/2)
+x/(c-c/a/x)^(7/2)+11*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(7/2)-11/a/c^3/(
c-c/a/x)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6302, 6268, 25, 528, 382, 79, 53, 65, 214}

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}-\frac{11}{ac^3\sqrt{c-\frac{c}{ax}}}-\frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] -11/(7*a*(c - c/(a*x))^(7/2)) - 11/(5*a*c*(c - c/(a*x))^(5/2)) - 11/(3*a*c^
2*(c - c/(a*x))^(3/2)) - 11/(a*c^3*Sqrt[c - c/(a*x)]) + x/(c - c/(a*x))^(7/
2) + (11*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$$

$$= -\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\left(c-\frac{c}{ax}\right)^{9/2}x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{(11c)\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\text{Subst}\left(\int \frac{1}{x\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11\text{Subst}\left(\int \frac{}{x(c}\right.}{2a}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{11}{7a\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{11}{5ac\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{11}{ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 46, normalized size = 0.32

$$\frac{7x - \frac{11\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};1-\frac{1}{ax}\right)}{a}}{7\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a*x))^(7/2),x]
```

```
[Out] (7*x - (11*Hypergeometric2F1[-7/2, 1, -5/2, 1 - 1/(a*x)])/a)/(7*(c - c/(a*x
))^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(123) = 246.
time = 0.18, size = 396, normalized size = 2.73

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}} + \frac{\left(\frac{11\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^4\sqrt{a^2c}} - \frac{4\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{7a^9c\left(x-\frac{1}{a}\right)^4} - \frac{102\sqrt{a^2c\left(x-\frac{1}{a}\right.}}{35a^8c(}\right.}{}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2310\sqrt{(ax-1)\,x}\,a^{\frac{11}{2}}x^5+2100((ax-1)x)^{\frac{3}{2}}a^{\frac{9}{2}}x^3-1155\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^5x^5+115\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/210*(c*(a*x-1)/a/x)^(1/2)*x*(-2310*((a*x-1)*x)^(1/2)*a^(11/2)*x^5+2100*(
(a*x-1)*x)^(3/2)*a^(9/2)*x^3-1155*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x
-1)/a^(1/2))*a^5*x^5+11550*a^(9/2)*((a*x-1)*x)^(1/2)*x^4-5368*a^(7/2)*((a*x
-1)*x)^(3/2)*x^2+5775*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*a^4*x^4-23100*((a*x-1)*x)^(1/2)*a^(7/2)*x^3+4928*((a*x-1)*x)^(3/2)*a^(5/2)
*x-11550*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^3*x^3+2310
0*a^(5/2)*((a*x-1)*x)^(1/2)*x^2-1540*a^(3/2)*((a*x-1)*x)^(3/2)+11550*ln(1/2
*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^2-11550*a^(3/2)*((a*x
-1)*x)^(1/2)*x-5775*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a
*x+2310*((a*x-1)*x)^(1/2)*a^(1/2)+1155*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^5/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a*x))^(7/2)), x)
```

Fricas [A]
time = 0.36, size = 346, normalized size = 2.39

$$\left[\frac{1155\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(105\,a^5x^5-1936\,a^4x^4+4466\,a^3x^3-3850\,a^2x^2+1155\,ax)\sqrt{\frac{acx-c}{ax}}}{210\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)},\ -\frac{1155\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-(105\,a^5x^5-1936\,a^4x^4+4466\,a^3x^3-3850\,a^2x^2+1155\,ax)\sqrt{\frac{acx-c}{ax}}}{105\,(a^5c^4x^4-4\,a^4c^4x^3+6\,a^3c^4x^2-4\,a^2c^4x+ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/210*(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*log(-2*a
*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(105*a^5*x^5 - 1936*a
^4*x^4 + 4466*a^3*x^3 - 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(
a^5*c^4*x^4 - 4*a^4*c^4*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4), -1/105*
(1155*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(sqrt(-c
)*sqrt((a*c*x - c)/(a*x))/c) - (105*a^5*x^5 - 1936*a^4*x^4 + 4466*a^3*x^3 -
 3850*a^2*x^2 + 1155*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^5*c^4*x^4 - 4*a^4*c^4
*x^3 + 6*a^3*c^4*x^2 - 4*a^2*c^4*x + a*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(123) = 246.
time = 0.68, size = 389, normalized size = 2.68

$$\frac{11\,\log\left(c^4|a|\sqrt{|c|}\right)\mathrm{sgn}(x)}{18\,ac^{\frac{7}{2}}}-\frac{11\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^9|a|+17\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^8a\sqrt{c}-64\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^7c|a|+140\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^6ac^{\frac{3}{2}}-196\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^5c^2|a|+182\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^4ac^{\frac{5}{2}}-112\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^3c^3|a|+44\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^2ac^{\frac{7}{2}}-10\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)c^4|a|+ac^{\frac{9}{2}}\right|\right)\mathrm{sgn}(x)}{18\,ac^{\frac{7}{2}}}+\frac{\sqrt{a^2cx^2-acx}\,|a|\mathrm{sgn}(x)}{a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] 11/18*log(c^4*abs(a)*sqrt(abs(c)))*sgn(x)/(a*c^(7/2)) - 11/18*log(abs(-2*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*abs(a) + 17*(sqrt(a^2*c)*x - sqrt
```

```
(a^2*c*x^2 - a*c*x))^8*a*sqrt(c) - 64*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))^7*c*abs(a) + 140*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a*c^(3/2)
 - 196*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*c^2*abs(a) + 182*(sqrt(a
^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a*c^(5/2) - 112*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^3*c^3*abs(a) + 44*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c
*x))^2*a*c^(7/2) - 10*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*c^4*abs(a)
+ a*c^(9/2)))*sgn(x)/(a*c^(7/2)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)*sgn(x)/(a
^2*c^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a\,x}\right)}^{7/2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a*x))^(7/2)*(a*x - 1)), x)
```

3.455 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=268

$$\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^2\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^3}{}$$

```
[Out] 3/35*(28*a-17/x)*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^2/(1-1/a/x)^(9/2)+9/7*(a
-1/x)^2*(1+1/a/x)^(3/2)*(c-c/a/x)^(9/2)/a^3/(1-1/a/x)^(9/2)+(a-1/x)^3*(1+1/
a/x)^(3/2)*(c-c/a/x)^(9/2)*x/a^3/(1-1/a/x)^(9/2)-3*(c-c/a/x)^(9/2)*arctanh(
(1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+3*(c-c/a/x)^(9/2)*(1+1/a/x)^(1/2)/a/(1-1
/a/x)^(9/2)
```

Rubi [A]
time = 0.11, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6317, 6314, 99, 158, 152, 52, 65, 214}

$$\frac{9\left(a-\frac{1}{x}\right)^2\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{x\left(a-\frac{1}{x}\right)^3\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{3\left(c-\frac{c}{ax}\right)^{9/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(a*(1 - 1/(a*x))^(9/2)) + (3*(28*
a - 17/x)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(35*a^2*(1 - 1/(a*x))^(9
/2)) + (9*(a - x^(-1))^2*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2))/(7*a^3*(1
 - 1/(a*x))^(9/2)) + ((a - x^(-1))^3*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^(9/2
)*x)/(a^3*(1 - 1/(a*x))^(9/2)) - (3*(c - c/(a*x))^(9/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
```

```
!IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3}\left(1+\frac{x}{a}\right)^{3/2}}{x^{2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\operatorname{Subst}\left(\int\frac{\left(-\frac{3}{2a}-\frac{9x}{2a^{2}}\right)\left(1-\frac{x}{a}\right)^{2}\sqrt{\cdots}}{x}\cdots\right.}{\left(1-\frac{1}{ax}\right)^{9/2}\cdots}$$

$$=\frac{9\left(a-\frac{1}{x}\right)^{2}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^{3}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}x}{a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}-\cdots$$

$$=\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^{2}\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^{2}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^{3}\left(1-\frac{1}{ax}\right)^{9/2}}+\cdots$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^{2}\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^{2}\cdots}{7a\cdots}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^{2}\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^{2}\cdots}{7a\cdots}$$

$$=\frac{3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{a\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{3\left(28a-\frac{17}{x}\right)\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{9/2}}{35a^{2}\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{9\left(a-\frac{1}{x}\right)^{2}\cdots}{7a\cdots}$$

Mathematica [**A**]

time = 0.06, size = 109, normalized size = 0.41

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,(10-26ax-12a^2x^2+164a^3x^3+35a^4x^4)-105a^3x^3\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{35a^4\sqrt{1-\frac{1}{ax}}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(10 - 26*a*x - 12*a^2*x^2 + 164*a
^3*x^3 + 35*a^4*x^4) - 105*a^3*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(35*a^4*Sqr
t[1 - 1/(a*x)]*x^3)
```

Maple [A]
time = 0.06, size = 178, normalized size = 0.66

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(70a^{\frac{9}{2}}\sqrt{x\,(ax+1)}\,x^4+328a^{\frac{7}{2}}x^3\sqrt{x\,(ax+1)}-105\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^4x^4 \cdots\right)}{70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3a^{\frac{9}{2}}\sqrt{x\,(ax+1)}}$
risch	$\frac{(35a^5x^5+199a^4x^4+152a^3x^3-38a^2x^2-16ax+10)c^4\sqrt{\frac{c(ax-1)}{ax}}}{35x^3a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{3\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^4\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\cdots}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/70/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^4*(70*
a^(9/2)*(x*(a*x+1))^(1/2)*x^4+328*a^(7/2)*x^3*(x*(a*x+1))^(1/2)-105*ln(1/2*
(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4-24*a^(5/2)*x^2*(x*(a
*x+1))^(1/2)-52*a^(3/2)*x*(x*(a*x+1))^(1/2)+20*(x*(a*x+1))^(1/2)*a^(1/2))/x
^3/a^(9/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 437, normalized size = 1.63

$$\left[\frac{105\,\left(a^{4} c^{4} x^{4} - a^{3} c^{4} x^{3}\right)\sqrt{c}\,\log\left(-\frac{8\,a^{3} c x^{3} - 7\,a c x - 4\,\left(2\,a^{3} x^{3} + 3\,a^{2} x^{2} + a x\right)\sqrt{c}\,\sqrt{\frac{a x - 1}{a x + 1}}\,\sqrt{\frac{a c x - c}{a x}} - c}{a x - 1}\right) + 4\,\left(35\,a^{5} c^{4} x^{5} + 199\,a^{4} c^{4} x^{4} + 152\,a^{3} c^{4} x^{3} - 38\,a^{2} c^{4} x^{2} - 16\,a c^{4} x + 10\,c^{4}\right)\sqrt{\frac{a x - 1}{a x + 1}}\,\sqrt{\frac{a c x - c}{a x}}}{140\,\left(a^{5} x^{4} - a^{4} x^{3}\right)}, \frac{105\,\left(a^{4} c^{4} x^{4} - a^{3} c^{4} x^{3}\right)\sqrt{-c}\,\arctan\left(\frac{2\,\left(a^{2} x^{2} + a x\right)\sqrt{-c}\,\sqrt{\frac{a x - 1}{a x + 1}}\,\sqrt{\frac{a c x - c}{a x}}}{2\,a^{2} c x^{2} - a c x - c}\right) + 2\,\left(35\,a^{5} c^{4} x^{5} + 199\,a^{4} c^{4} x^{4} + 152\,a^{3} c^{4} x^{3} - 38\,a^{2} c^{4} x^{2} - 16\,a c^{4} x + 10\,c^{4}\right)\sqrt{\frac{a x - 1}{a x + 1}}\,\sqrt{\frac{a c x - c}{a x}}}{70\,\left(a^{5} x^{4} - a^{4} x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
 - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(35*a^5*c^4*x^5 + 199*a^4*c^4*x^4 + 1
52*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10*c^4)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/70*(105*(a^4*c^4*x^4
- a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(35*a^5*c^4
*x^5 + 199*a^4*c^4*x^4 + 152*a^3*c^4*x^3 - 38*a^2*c^4*x^2 - 16*a*c^4*x + 10
*c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3
)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^{9/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.456 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=237

$$\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{(c-$$

```
[Out] 1/3*(1+1/a/x)^(3/2)*(c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)-2/5*(1+1/a/x)^(5/2)*(
c-c/a/x)^(7/2)/a/(1-1/a/x)^(7/2)+(1+1/a/x)^(5/2)*(c-c/a/x)^(7/2)*x/(1-1/a/x
)^(7/2)-(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+(c-c/a/x
)^(7/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.292$, Rules used = {6317, 6314, 91, 81, 52, 65, 214}

$$-\frac{2\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(\frac{1}{ax}+1\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{x\left(\frac{1}{ax}+1\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(
a*x))^(3/2)*(c - c/(a*x))^(7/2))/(3*a*(1 - 1/(a*x))^(7/2)) - (2*(1 + 1/(a*x
))^(5/2)*(c - c/(a*x))^(7/2))/(5*a*(1 - 1/(a*x))^(7/2)) + ((1 + 1/(a*x))^(5
/2)*(c - c/(a*x))^(7/2)*x)/(1 - 1/(a*x))^(7/2) - ((c - c/(a*x))^(7/2)*ArcTa
nh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}\,dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(-\frac{1}{2a}+\frac{x}{a^2}\right)\left(1+\frac{x}{a}\right)^{3/2}}{x}\,dx,x,\frac{1}{x}\right.}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}x}{\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}}{2a\,(1}$$

$$=\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/2}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7}}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{a\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^{7/2}}{3a\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{7/}}{5a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Mathematica [A]
time = 0.06, size = 101, normalized size = 0.43

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,(-6+8ax+44a^2x^2+15a^3x^3)-15a^2x^2\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{15a^3\sqrt{1-\frac{1}{ax}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-6 + 8*a*x + 44*a^2*x^2 + 15*a^3
*x^3) - 15*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(15*a^3*Sqrt[1 - 1/(a*x)]*x
^2)
```

Maple [A]
time = 0.05, size = 161, normalized size = 0.68

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c^3 \left(30 a^{\frac{7}{2}} x^3 \sqrt{x(ax+1)}+88 a^{\frac{5}{2}} x^2 \sqrt{x(ax+1)}-15 \ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right) a^3 x^3+16\right.}{30\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1) x^2 a^{\frac{7}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(15a^4x^4+59a^3x^3+52a^2x^2+2ax-6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax(ax+1)}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(30*
a^(7/2)*x^3*(x*(a*x+1))^(1/2)+88*a^(5/2)*x^2*(x*(a*x+1))^(1/2)-15*ln(1/2*(2
*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+16*a^(3/2)*x*(x*(a*x+1
))^(1/2)-12*(x*(a*x+1))^(1/2)*a^(1/2))/x^2/a^(7/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 415, normalized size = 1.75

$$\left[\frac{15\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(15\,a^4c^3x^4+59\,a^3c^3x^3+52\,a^2c^3x^2+2\,ac^3x-6\,c^3)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{60\,(a^4x^3-a^3x^2)},\ \frac{15\,(a^3c^3x^3-a^2c^3x^2)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(15\,a^4c^3x^4+59\,a^3c^3x^3+52\,a^2c^3x^2+2\,ac^3x-6\,c^3)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{30\,(a^4x^3-a^3x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 + 52*a
^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(15*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 59*a^3*c^3*x^3 +
52*a^2*c^3*x^2 + 2*a*c^3*x - 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.457 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=156

$$-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -1/3*c^4*(1-1/a^2/x^2)^(3/2)/a/(c-c/a/x)^(3/2)+c^5*(1-1/a^2/x^2)^(5/2)*x/(c
-c/a/x)^(5/2)+c^(5/2)*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/
a-c^3*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6312, 893, 879, 889, 214}

$$\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^5x\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] -1/3*(c^4*(1 - 1/(a^2*x^2))^(3/2))/(a*(c - c/(a*x))^(3/2)) - (c^3*Sqrt[1 -
1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^5*(1 - 1/(a^2*x^2))^(5/2)*x)/(c -
c/(a*x))^(5/2) + (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c -
c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
```

```
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\left(c^3\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\text{Subst}\left(\int\frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x\left(c-\frac{cx}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^3\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^2\text{Subst}\left(\int\frac{\sqrt{c-}}{x\sqrt{1-}}\right.}{2a}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{c^4\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+}\right.}{}$$

$$=-\frac{c^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{c^3\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}{\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}}{}\right.}{a}$$

Mathematica [A]
time = 0.04, size = 89, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\left(2+2ax+3a^2x^2\right)+3ax\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 + 2*a*x + 3*a^2*x^2) + 3*a*x*A
rcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A]
time = 0.05, size = 144, normalized size = 0.92

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}+3\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2+4a^{\frac{3}{2}}x\sqrt{x(ax+1)}+4\sqrt{\cdots}\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\,a^{\frac{5}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(3a^3x^3+5a^2x^2+4ax+2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^2\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax(ax+1)}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)/x*c^2/a^(
5/2)*(6*a^(5/2)*x^2*(x*(a*x+1))^(1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)
+2*a*x+1)/a^(1/2))*a^2*x^2+4*a^(3/2)*x*(x*(a*x+1))^(1/2)+4*(x*(a*x+1))^(1/2
)*a^(1/2))/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 381, normalized size = 2.44

$$\left[\frac{3\left(a^2c^2x^2-ac^2x\right)\sqrt{c}\,\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(3a^3c^2x^3+5a^2c^2x^2+4ac^2x+2c^2\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{12\left(a^3x^2-a^2x\right)},\ -\frac{3\left(a^2c^2x^2-ac^2x\right)\sqrt{-c}\,\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)-2\left(3a^3c^2x^3+5a^2c^2x^2+4ac^2x+2c^2\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{6\left(a^3x^2-a^2x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x +
 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x
), -1/6*(3*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(3*a^3*c^2*x^3 + 5*a^2*c^2*x^2 + 4*a*c^2*x + 2*c^2)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.458 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=118

$$-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}+\frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] c^3*(1-1/a^2/x^2)^(3/2)*x/(c-c/a/x)^(3/2)+3*c^(3/2)*arctanh(c^(1/2)*(1-1/a^
2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a-3*c^2*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6312, 877, 879, 889, 214}

$$\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}+\frac{c^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-3*c^2*Sqrt[1 - 1/(a^2*x^2)])/(a*Sqrt[c - c/(a*x)]) + (c^3*(1 - 1/(a^2*x^2
))^(3/2)*x)/(c - c/(a*x))^(3/2) + (3*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a
^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
```

```
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{x^2\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(3c^2)\,\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(3c)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{(3c^3)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \sqrt{\cdots}\right)}{a^3}$$

$$= -\frac{3c^2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{ax}}} + \frac{c^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 66, normalized size = 0.56

$$-\frac{2\left(1+\frac{1}{ax}\right)^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1+\frac{1}{ax}\right)}{5a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*(1 + 1/(a*x))^(5/2)*(c - c/(a*x))^(3/2)*Hypergeometric2F1[2, 5/2, 7/2,
1 + 1/(a*x)])/(5*a*(1 - 1/(a*x))^(3/2))
```

Maple [A]
time = 0.05, size = 118, normalized size = 1.00

method	result	size
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, c\left(2a^{\frac{3}{2}}x\sqrt{x\left(ax+1\right)}+3\ln\left(\frac{2\sqrt{x\left(ax+1\right)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{x\left(ax+1\right)}\,\sqrt{a}\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{3}{2}}\sqrt{x\left(ax+1\right)}}$	118
risch	$\frac{(a^{2}x^{2}-ax-2)c\sqrt{\frac{c(ax-1)}{ax}}}{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}+\frac{3\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\left(ax+1\right)}}{2\sqrt{a^{2}c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c/a^(3/2)
*(2*a^(3/2)*x*(x*(a*x+1))^(1/2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*a*x-4*(x*(a*x+1))^(1/2)*a^(1/2))/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.39, size = 315, normalized size = 2.67

$$\left[\frac{3\,(acx-c)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}cx^{2}-acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)},\ -\frac{3\,(acx-c)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)-2\,(a^{2}cx^{2}-acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), -1/2*(3*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
```

```
^2*c*x^2 - a*c*x - c)) - 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.459 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1/a
/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6317, 6314, 100, 162, 65, 214, 212}

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}} -$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} -$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}} + \frac{5\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}}{a}$$

Mathematica [A]

time = 0.04, size = 93, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{ax}}\left(a\sqrt{1+\frac{1}{ax}}\,x+5\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 5*ArcTanh[Sqrt[1 + 1/(a*x)]] -
4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.15, size = 160, normalized size = 1.05

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\cdots}{\cdots}\right.\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(\cdots\right.}}{x-\frac{1}{a}}\right.}{a\sqrt{c}}\cdots\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(
a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)+5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a
*x+1)/a^(1/2))*a*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1)
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/(x*(a*x+1))^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.44, size = 512, normalized size = 3.37

$$\left[\frac{4\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+5(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4(a^2x-a)}, \frac{4\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)-5(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*
x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*
x - a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.460 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=215

$$-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\tan\ldots}{a\sqrt{c-\ldots}}$$

```
[Out] 7*arctanh((1+1/a/x)^(1/2))*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-5*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(1-1/a/x)^(1/2)/a/(c-c/a/x)^(1/2)-3*(1-1/a
/x)^(1/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)+a*x*(1-1/a/x)^(1/2)*(1+1/
a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6317, 6314, 100, 156, 162, 65, 214, 212}

$$\frac{ax\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{c-\frac{c}{ax}}}-\frac{5\sqrt{2}\sqrt{1-\frac{1}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (-3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/((a - x^(-1))*Sqrt[c - c/(a*x)]) +
 (a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*Sqrt[c - c/(a*x)])
 + (7*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[c - c/(a*x)]) -
 (5*Sqrt[2]*Sqrt[1 - 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[c
 - c/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= \frac{a\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{-\frac{7}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(a\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{\frac{7}{a}\cdots}{x\left(1-\frac{x}{a}\right)\cdots}\right.}{2\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(5\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{\cdots}{\left(1-\frac{x}{a}\right)\cdots}\right.}{a^2\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} - \frac{\left(7\sqrt{1-\frac{1}{ax}}\right)\text{Subst}\left(\int \frac{1}{-a+ax^2}\cdots\right.}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{3\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{a\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}} + \frac{7\sqrt{1-\frac{1}{ax}}\,\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\cdots\right.}{a\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A]
time = 0.06, size = 115, normalized size = 0.53

$$\frac{\sqrt{1-\frac{1}{ax}}\left(a\sqrt{1+\frac{1}{ax}}\,x(-3+ax)+7(-1+ax)\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-5\sqrt{2}\,(-1+ax)\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}\,(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x*(-3 + a*x) + 7*(-1 + a*x)*ArcTanh
[Sqrt[1 + 1/(a*x)]] - 5*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2
]]))/(a*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A]
time = 0.15, size = 259, normalized size = 1.20

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-5a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x-6\sqrt{x(ax+1)}\,a\right.}{2(}$
risch	$\frac{ax-1}{a\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{7\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}-\frac{5\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}}}{x-\frac{1}{a}}\right)}{2a^{2}\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^
(1/2)*a^(5/2)*(1/a)^(1/2)*x-5*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*
(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x-6*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2
)+7*a^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x
-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+5*2^
(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/
2))/a^(3/2)/c/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.44, size = 581, normalized size = 2.70

$$\left[\frac{7(a^2x^2-2ax+1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(a^3x^3-2a^2x^2-3ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}+\frac{5\sqrt{2}(a^2cx^2-2acx+c)\log\left(-\frac{17a^3x^3-3a^2x^2-13ax-\frac{4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}-1}{a^3x^3-3a^2x^2+3ax-1}\right)}{\sqrt{c}}}{4(a^3cx^2-2a^2cx+ac)}, \frac{5\sqrt{2}(a^2cx^2-2acx+c)\sqrt{-\frac{1}{c}}\arctan\left(\frac{2\sqrt{2}(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{-\frac{1}{c}}\sqrt{\frac{acx-c}{ax}}}{3a^2x^2-2ax-1}\right)-7(a^2x^2-2ax+1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2(a^3x^3-2a^2x^2-3ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(a^3cx^2-2a^2cx+ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] [1/4*(7*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^
3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)
/(a*x)) - c)/(a*x - 1)) + 4*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)) + 5*sqrt(2)*(a^2*c*x^2 - 2*a*c*x + c)*log(
-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)
*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/sqrt(c) - 1)/(a^3*x^3 -
3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^3*c*x^2 - 2*a^2*c*x + a*c), 1/2*(5*sqrt
(2)*(a^2*c*x^2 - 2*a*c*x + c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*s
qrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x^2 - 2*
a*x - 1)) - 7*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(
-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x
- c)) + 2*(a^3*x^3 - 2*a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]Warni
ng, choos
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.461 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\cdots\right.}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] 9*(1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-51/8*(1-1/a/x)
^(3/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)-2*a*(
1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)-15/4*(1-1/a/x)^(3/
2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(3/2)+a^2*(1-1/a/x)^(3/2)*x*(1+1/a/x)^
(1/2)/(a-1/x)^2/(c-c/a/x)^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6317, 6314, 100, 156, 162, 65, 214, 212}

$$\frac{a^2x\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{9\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{51\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{4\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (-2*a*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/((a - x^(-1))^2*(c - c/(a*x))^
(3/2)) - (15*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))*(c - c/
(a*x))^(3/2)) + (a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))
^2*(c - c/(a*x))^(3/2)) + (9*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]]
)/(a*(c - c/(a*x))^(3/2)) - (51*(1 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]/Sqrt[2]])/(4*Sqrt[2]*a*(c - c/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{3/2}\right)\operatorname{Subst}\left(\int \frac{}{x(1}\right.}{4\left(c-\frac{c}{ax}\right.}$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \cdots$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \cdots$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \cdots$$

$$= -\frac{2a\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{15\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{9}{\cdots}$$

Mathematica [A]

time = 0.08, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}\,x(15-23ax+4a^2x^2)+72(-1+ax)^2\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-51\sqrt{2}\,(-1+ax)^2\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{8ac\sqrt{c-\frac{c}{ax}}\,(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(15 - 23*a*x + 4*a^2*x^2) + 72*
(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 51*Sqrt[2]*(-1 + a*x)^2*ArcTanh[S
qrt[1 + 1/(a*x)]/Sqrt[2]]))/(8*a*c*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A]
time = 0.16, size = 373, normalized size = 1.36

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{9\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^2\sqrt{a^2c}}-\frac{51\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\,(x-}}{x-\frac{1}{a}}\right.}{16a^3\sqrt{c}}\right.}{}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{x\,(ax+1)}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-51a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x^2-92\sqrt{x\,(ax}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*(x
*(a*x+1))^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2-51*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1
/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-92*(x*(a*x+1))^(1/2)*a^
(5/2)*(1/a)^(1/2)*x+72*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^3*(1/a)^(1/2)*x^2+102*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x
+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x+60*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-1
44*a^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x+
72*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-51*2
^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1
/2))/a^(3/2)/c^2/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.42, size = 668, normalized size = 2.43

$$\left[\frac{51\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+72\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\left(4a^4x^4-19a^3x^3-8a^2x^2+15ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{32\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)}, \frac{51\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)-72\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+4\left(4a^4x^4-19a^3x^3-8a^2x^2+15ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\left(a^4c^2x^3-3a^3c^2x^2+3a^2c^2x-ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
[Out] [1/32*(51*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*c*
x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt
(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2
*x^2 + 3*a*x - 1)) + 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(8*a
^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^4*x^4 - 19*a^3
*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2), 1/16*(51*sqrt(2)*(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*
a*c*x - c)) - 72*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 4*(4*a^4*x^4 - 19*a^3*x^3 - 8*a^2*x^2 + 15*a*x)*sqrt
((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2
 + 3*a^2*c^2*x - a*c^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]Warni
ng, choos
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.462 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] 11*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-249/32*(1-1/a
/x)^(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-5/
3*a^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)-29/12*a*(1-
1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(5/2)-73/16*(1-1/a/x)^(5/2
)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a^3*(1-1/a/x)^(5/2)*x*(1+1/a/x)^(
1/2)/(a-1/x)^3/(c-c/a/x)^(5/2)
```

Rubi [A]

time = 0.14, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6317, 6314, 100, 156, 162, 65, 214, 212}

$$\frac{a^3x\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{11\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{249\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{16\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (-5*a^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(3*(a - x^(-1))^3*(c - c/(a*
x))^(5/2)) - (29*a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(12*(a - x^(-1))^
2*(c - c/(a*x))^(5/2)) - (73*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(16*(a
- x^(-1))*(c - c/(a*x))^(5/2)) + (a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]
*x)/((a - x^(-1))^3*(c - c/(a*x))^(5/2)) + (11*(1 - 1/(a*x))^(5/2)*ArcTanh[
Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - (249*(1 - 1/(a*x))^(5/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)^4}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{-\frac{11}{2a}-\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^4\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int\right.}{6\left(c-\right.}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} -$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} -$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} -$$

$$= -\frac{5a^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{29a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{12\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{73\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} -$$

Mathematica [A]
time = 0.10, size = 143, normalized size = 0.43

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}\,x(-219+554ax-415a^2x^2+48a^3x^3)+1056(-1+ax)^3\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-747\sqrt{2}\,(-1+ax)^3\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{96ac^2\sqrt{c-\frac{c}{ax}}\,(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a*x))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-219 + 554*a*x - 415*a^2*x^2 +
 48*a^3*x^3) + 1056*(-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]] - 747*Sqrt[2]*(
-1 + a*x)^3*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(96*a*c^2*Sqrt[c - c/(a*x)
]*(-1 + a*x)^3)
```

Maple [A]
time = 0.16, size = 480, normalized size = 1.43

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{11\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{2\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\frac{1}{a}\right)ac+2c}}{3a^7c\left(x-\frac{1}{a}\right)^3}-\frac{11}{}\right.}{}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(192\sqrt{x\,(ax+1)}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3-747a^{\frac{7}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x^3-1660\sqrt{x\,(a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(19
2*(x*(a*x+1))^(1/2)*a^(9/2)*(1/a)^(1/2)*x^3-747*a^(7/2)*2^(1/2)*ln((2*2^(1/
2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^3-1660*(x*(a*x+1))^(
1/2)*a^(7/2)*(1/a)^(1/2)*x^2+1056*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x
+1)/a^(1/2))*a^4*(1/a)^(1/2)*x^3+2241*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(
1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+2216*(x*(a*x+1))^(1/2)*a^(5/
2)*(1/a)^(1/2)*x-3168*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*a^3*(1/a)^(1/2)*x^2-2241*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x
```

```
+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x-876*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)+
3168*a^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*
x-1056*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+
747*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))
*a^(1/2))/a^(3/2)/c^3/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.46, size = 736, normalized size = 2.20

$$\left[\frac{747\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+1056\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\left(48a^5x^5-367a^4x^4+139a^3x^3+335a^2x^2-219ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{384\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}, \frac{747\sqrt{2}\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)-1056\left(a^4x^4-4a^3x^3+6a^2x^2-4ax+1\right)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+4\left(48a^5x^5-367a^4x^4+139a^3x^3+335a^2x^2-219ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{192\left(a^5c^3x^4-4a^4c^3x^3+6a^3c^3x^2-4a^2c^3x+ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
[Out] [1/384*(747*sqrt(2)*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(c)*l
og(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x
^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a
^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 -
4*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
 1)) + 8*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^3*x^3 + 335*a^2*x^2 - 219*a*x)*s
qrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3), 1/192*(747*sqrt(2)*(a^4*x^4 - 4
*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)
*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
2*a*c*x - c)) - 1056*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*sqrt(-c)
*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 4*(48*a^5*x^5 - 367*a^4*x^4 + 139*a^
3*x^3 + 335*a^2*x^2 - 219*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(
a*x)))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.463 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=221

$$-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\cdots 9$$

```
[Out] -9*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)-1/5*(80*a-7/x
)*(c-c/a/x)^(7/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+3/5*(a-1/x)^2*(c-c/a/
x)^(7/2)*(1+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)+(a-1/x)^3*(c-c/a/x)^(7/2)*x*(1
+1/a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6317, 6314, 100, 158, 152, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^3\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{\left(80a-\frac{7}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{9\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/5*((80*a - 7/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(a^2*(1 - 1/(a*x)
)^(7/2)) + (3*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(5*a^3*
(1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2
)*x)/(a^3*(1 - 1/(a*x))^(7/2)) - (9*(c - c/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\sqrt{1+\frac{x}{a}}}d\right.}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}x}{a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{(2}{}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{(}{}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{(}{}$$

$$=-\frac{\left(80a-\frac{7}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{3\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{(}{}$$

Mathematica [A]

time = 0.07, size = 101, normalized size = 0.46

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,(-2+16ax-92a^2x^2+5a^3x^3)-45a^2x^2\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{5a^3\sqrt{1-\frac{1}{ax}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + 16*a*x - 92*a^2*x^2 + 5*a^3
*x^3) - 45*a^2*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(5*a^3*Sqrt[1 - 1/(a*x)]*x^
2)
```

Maple [A]
time = 0.10, size = 161, normalized size = 0.73

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(10a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}-184a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}-45\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+}{2\sqrt{a}}\right.\right.}{10x^2a^{\frac{7}{2}}(ax-1)\sqrt{x(ax+1)}}$
risch	$\frac{(5a^4x^4-87a^3x^3-76a^2x^2+14ax-2)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{5x^2a^3(ax-1)}-\frac{9\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{2\sqrt{a^2c}\,(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(10*a^(7/2)*
x^3*(x*(a*x+1))^(1/2)-184*a^(5/2)*x^2*(x*(a*x+1))^(1/2)-45*ln(1/2*(2*(x*(a*
x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+32*a^(3/2)*x*(x*(a*x+1))^(1/2
)-4*(x*(a*x+1))^(1/2)*a^(1/2))/x^2/a^(7/2)/(a*x-1)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.36, size = 415, normalized size = 1.88

$$\left[\frac{45\,(a^{3}c^{3}x^{3} - a^{2}c^{3}x^{2})\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3} - 7\,acx - 4\,(2\,a^{3}x^{3} + 3\,a^{2}x^{2} + ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}} - c}{ax-1}\right) + 4\,(5\,a^{4}c^{3}x^{4} - 87\,a^{3}c^{3}x^{3} - 76\,a^{2}c^{3}x^{2} + 14\,ac^{3}x - 2\,c^{3})\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{20\,(a^{4}x^{3} - a^{3}x^{2})}, \frac{45\,(a^{3}c^{3}x^{3} - a^{2}c^{3}x^{2})\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2} + ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2} - acx - c}\right) + 2\,(5\,a^{4}c^{3}x^{4} - 87\,a^{3}c^{3}x^{3} - 76\,a^{2}c^{3}x^{2} + 14\,ac^{3}x - 2\,c^{3})\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{10\,(a^{4}x^{3} - a^{3}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x -
 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 76*a^
2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*x^3 - a^3*x^2), 1/10*(45*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(-c)*
arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(5*a^4*c^3*x^4 - 87*a^3*c^3*x^3 - 7
6*a^2*c^3*x^2 + 14*a*c^3*x - 2*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a\,x}\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.464 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

```
[Out] -7*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)-1/3*(16*a+1/x
)*(c-c/a/x)^(5/2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)+(a-1/x)^2*(c-c/a/x)^(
5/2)*x*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

Rubi [A]
time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 100, 152, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{\left(16a+\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{7\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/3*((16*a + x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(a^2*(1 - 1/(a
*x))^(5/2)) + ((a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)*x)/(a^2
*(1 - 1/(a*x))^(5/2)) - (7*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/
(a*(1 - 1/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx=\frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}\,dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(\frac{7}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\sqrt{1+\frac{x}{a}}}\,dx\right.}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}+$$

$$=-\frac{\left(16a+\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}x}{a^2\left(1-\frac{1}{ax}\right)^{5/2}}-$$

Mathematica [A]
time = 0.04, size = 89, normalized size = 0.55

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\left(2-22ax+3a^2x^2\right)-21ax\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{3a^2\sqrt{1-\frac{1}{ax}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^ArcCoth[a*x],x]
[Out] (c^2*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(2 - 22*a*x + 3*a^2*x^2) - 21*a*x
*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(3*a^2*Sqrt[1 - 1/(a*x)]*x)
```

Maple [A]
time = 0.10, size = 144, normalized size = 0.89

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^{2}\left(6a^{\frac{5}{2}}x^{2}\sqrt{x\,(ax+1)}-44a^{\frac{3}{2}}x\sqrt{x\,(ax+1)}-21\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right.}{6x\,a^{\frac{5}{2}}(ax-1)\sqrt{x\,(ax+1)}}$
risch	$\frac{(3a^{3}x^{3}-19a^{2}x^{2}-20ax+2)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^{2}(ax-1)}-\frac{7\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)c^{2}\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{ca}}{2\sqrt{a^{2}c}\,(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*a^(5/2)*x^
2*(x*(a*x+1))^(1/2)-44*a^(3/2)*x*(x*(a*x+1))^(1/2)-21*ln(1/2*(2*(x*(a*x+1))
^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2+4*(x*(a*x+1))^(1/2)*a^(1/2))/x/a^(
5/2)/(a*x-1)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.37, size = 381, normalized size = 2.37

$$\left[\frac{21\,(a^{2}c^{2}x^{2}-ac^{2}x)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^{3}c^{2}x^{3}-19\,a^{2}c^{2}x^{2}-20\,ac^{2}x+2\,c^{2})\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{12\,(a^{3}x^{2}-a^{2}x)},\ \frac{21\,(a^{2}c^{2}x^{2}-ac^{2}x)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(3\,a^{3}c^{2}x^{3}-19\,a^{2}c^{2}x^{2}-20\,ac^{2}x+2\,c^{2})\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{6\,(a^{3}x^{2}-a^{2}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*
x + 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^
2*x), 1/6*(21*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
```

```
x - c)) + 2*(3*a^3*c^2*x^3 - 19*a^2*c^2*x^2 - 20*a*c^2*x + 2*c^2)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-\frac{c}{a\,x}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.465 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -5*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)-2*(c-c/a/x)^(
3/2)*(1+1/a/x)^(1/2)/a/(1-1/a/x)^(3/2)+(c-c/a/x)^(3/2)*x*(1+1/a/x)^(1/2)/(1
-1/a/x)^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6317, 6314, 91, 81, 65, 214}

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{2\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))/(a*(1 - 1/(a*x))^(3/2)) + (Sqrt[
1 + 1/(a*x)]*(c - c/(a*x))^(3/2)*x)/(1 - 1/(a*x))^(3/2) - (5*(c - c/(a*x))^
(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx = \frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{-\frac{5}{2a}+\frac{x}{a^2}}{x\sqrt{1+\frac{x}{a}}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subs}}{2a\left(1\right.}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\left(5\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{Subs}}{(}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}}{a\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{3/2}x}{\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{5\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}}{a\left(1-\frac{1}{ax}\right.}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.50

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}(-2+ax)-5\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^ArcCoth[a*x],x]
[Out] (c*Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*(-2 + a*x) - 5*ArcTanh[Sqrt[1 + 1/(
a*x)]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.10, size = 118, normalized size = 0.84

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c\left(2a^{\frac{3}{2}}x\sqrt{x\,(ax+1)}-5\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-4\sqrt{x\,(ax+1)}\,\sqrt{a}\right.}{2a^{\frac{3}{2}}(ax-1)\sqrt{x\,(ax+1)}}$
risch	$\frac{(a^2x^2-ax-2)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}-\frac{5\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1}}{2\sqrt{a^2c}\,(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^(3/2)*x*(x
*(a*x+1))^(1/2)-5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x
-4*(x*(a*x+1))^(1/2)*a^(1/2))/a^(3/2)/(a*x-1)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.38, size = 315, normalized size = 2.25

$$\left[\frac{5\,(acx-c)\sqrt{c}\,\log\left(-\frac{8a^3cx^3-7acx-4\,(2a^3x^3+3a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2cx^2-acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\ \frac{5\,(acx-c)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2\,(a^2cx^2-acx-2\,c)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1))*sqrt(
(a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(5*(a*c*x - c)*sqrt(-c)*arctan(2*(a^2*
x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^
2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 - a*c*x - 2*c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-\frac{c}{a\,x}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.466 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=79

$$\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/
a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6312, 893, 889, 214}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{3\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.82

$$\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x-\frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

Maple [A]
time = 0.09, size = 101, normalized size = 1.28

method	result	si
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x\,(ax+1)}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{x\,(ax+1)}\,\sqrt{a}}$	10
risch	$\frac{x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1)}}{2\sqrt{a^2c}\,(ax-1)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^
(1/2)*a^(1/2)-3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/(x*(a*x+1))^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.
time = 0.38, size = 297, normalized size = 3.76

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},\ \frac{3\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.467 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] -arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+x*(1-1/a^2/
x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6312, 887, 889, 214}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(
a^2*x^2)])/Sqrt[c - c/(a*x)]]/(a*Sqrt[c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
```

```
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.85

$$\frac{\sqrt{1-\frac{1}{ax}}\left(a\sqrt{1+\frac{1}{ax}}\,x-\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]),x]
[Out] (Sqrt[1 - 1/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x - ArcTanh[Sqrt[1 + 1/(a*x)]]))/(a
*Sqrt[c - c/(a*x)])
```

Maple [A]
time = 0.10, size = 102, normalized size = 1.31

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-2\sqrt{x\,(ax+1)}\,\sqrt{a}+\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{a}\,c(ax-1)\sqrt{x\,(ax+1)}}$	102
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}-\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{cax\,(ax+1)}}{2a\sqrt{a^{2}c}\,\sqrt{\frac{c(ax-1)}{ax}}\,x}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(1/2)/c*(-2*
(x*(a*x+1))^(1/2)*a^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1
/2)))/(a*x-1)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(66) = 132$.
time = 0.46, size = 299, normalized size = 3.83

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}cx-ac)},\ \frac{(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(a^{2}x^{2}+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}cx-ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*c*x - a*c), 1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
```

```
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*c*x - a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(1/2), x)
```

3.468 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}} dx$

Optimal. Leaf size=151

$$\frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] (1-1/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(3/2)-(1-1/a/x)^(3/2)*
arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(3/2)*2^(1/2)+(1-1/a/x)^(3
/2)*x*(1+1/a/x)^(1/2)/(c-c/a/x)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6317, 6314, 105, 21, 85, 65, 214, 212}

$$\frac{x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] ((1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x)/(c - c/(a*x))^(3/2) + ((1 - 1/(a*
x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(3/2)) - (Sqrt[2]*(1
 - 1/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*(c - c/(a*x))^(3/2
))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{-\frac{1}{2a}-\frac{x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}}{}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(1-\frac{1}{ax}\right)^{3/2}\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\left(2\left(1-\frac{1}{ax}\right.\right.}{}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\sqrt{2}\left(1-\frac{1}{ax}\right)^{3/2}\tanh^{-1}}{a\left(c-\frac{c}{ax}\right)^{3}}$$

<u>**Mathematica**</u> **[A]**

time = 0.04, size = 91, normalized size = 0.60

$$\frac{\left(1-\frac{1}{ax}\right)^{3/2}\left(a\sqrt{1+\frac{1}{ax}}\,x+\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(3/2)),x]
```

```
[Out] ((1 - 1/(a*x))^(3/2)*(a*Sqrt[1 + 1/(a*x)]*x + ArcTanh[Sqrt[1 + 1/(a*x)]] -
Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*(c - c/(a*x))^(3/2))
```

Maple [A]
time = 0.18, size = 162, normalized size = 1.07

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-\sqrt{2}\,\ln\left(\frac{2\sqrt{\cdots}}{\cdots}\right.\right.}{2a^{\frac{3}{2}}c^{2}(ax-1)\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a^{2}\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(\cdots\right.}}{x-\frac{1}{a}}\right)}{2a^{3}\sqrt{c}}\cdots\right.}{cx\sqrt{\frac{c(ax-1)}{ax}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)/c^2*(2*
(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2
*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1)
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/(a*x-1)/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

Fricas [A]
time = 0.40, size = 522, normalized size = 3.46

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}+\frac{\sqrt{2}\,(acx-c)\log\left(-\frac{17\,a^3x^3-3\,a^2x^2-13\,ax-\frac{4\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}-1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)}{\sqrt{c}}}{4\,(a^2c^2x-ac^2)},\ \frac{\sqrt{2}\,(acx-c)\sqrt{-\frac{1}{c}}\,\arctan\left(\frac{2\sqrt{2}\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-\frac{1}{c}}\,\sqrt{\frac{acx-c}{ax}}}{3\,a^2x^2-2\,ax-1}\right)-(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^2c^2x-ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)) + sqrt(2)*(a*c*x - c)*log(-(17*a^3*x^3 - 3*a^2*x^2 - 13*a*x - 4*sqrt(2
)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x))/sqrt(c) - 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1))/sqrt(c))/(a^2*c^2*x
- a*c^2), 1/2*(sqrt(2)*(a*c*x - c)*sqrt(-1/c)*arctan(2*sqrt(2)*(a^2*x^2 + a
*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(3*a^2*x^2
 - 2*a*x - 1)) - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2
*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*c^
2*x - a*c^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(3/2), x)
```

3.469 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\tanh}{2\sqrt{2}\,a\,(}$$

```
[Out] 3*(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-9/4*(1-1/a/x)^
(5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)-3/2*(1
-1/a/x)^(5/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)+a*(1-1/a/x)^(5/2)*x*(
1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(5/2)
```

Rubi [A]

time = 0.11, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6317, 6314, 105, 21, 101, 162, 65, 214, 212}

$$\frac{ax\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{3\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{9\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{2\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (-3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)])/(2*(a - x^(-1))*(c - c/(a*x))^(5
/2)) + (a*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-1))*(c - c/(a*
x))^(5/2)) + (3*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(
a*x))^(5/2)) - (9*(1 - 1/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(
2*Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
```

```
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\operatorname{Subst}\left(\int \frac{-\frac{3}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^2}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\quad}{x(1-\cdots}\right.}{2a\left(c-\frac{c}{ax}\right.}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(9\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{\quad}{(1-\cdots}\right.}{4a^2\left(c-\frac{c}{ax}\right.}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(3\left(1-\frac{1}{ax}\right)^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{-a+\cdots}\right.}{\left(c-\frac{c}{ax}\right)^{5\cdots}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{2\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{3\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\cdots}\right.}{a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [A]
time = 0.07, size = 123, normalized size = 0.56

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}\,x(-3+2ax)+12(-1+ax)\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-9\sqrt{2}\,(-1+ax)\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{4ac^2\sqrt{c-\frac{c}{ax}}\,(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(-3 + 2*a*x) + 12*(-1 + a*x)*Ar
cTanh[Sqrt[1 + 1/(a*x)]] - 9*Sqrt[2]*(-1 + a*x)*ArcTanh[Sqrt[1 + 1/(a*x)]/S
qrt[2]]))/(4*a*c^2*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A]
time = 0.18, size = 264, normalized size = 1.21

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(8\sqrt{x(ax+1)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-9a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+1)}\,a+3ax+1}{ax-1}\right)x-12\right.}{}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{9\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(\right.}}{x-\frac{1}{a}}\right)}{8a^4\sqrt{c}}\right.}{c^2x\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(8*(x*(a*x+1))^
(1/2)*a^(5/2)*(1/a)^(1/2)*x-9*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*
(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x-12*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/
2)+12*a^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*x-12*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+9
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^
(1/2))/a^(3/2)/c^3/(a*x-1)^2/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(5/2), x)
```

Fricas [A]
time = 0.42, size = 596, normalized size = 2.72

$$\left[\frac{9\sqrt{2}\left(a^2x^2-2ax+1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+12\left(a^2x^2-2ax+1\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\left(2a^3x^3-a^2x^2-3ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16\left(a^3c^3x^2-2a^2c^3x+ac^3\right)}, \frac{9\sqrt{2}\left(a^2x^2-2ax+1\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)-12\left(a^2x^2-2ax+1\right)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+4\left(2a^3x^3-a^2x^2-3ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8\left(a^3c^3x^2-2a^2c^3x+ac^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c
*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
- 1)) + 12*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3), 1
/8*(9*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2
- 2*a*c*x - c)) - 12*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 -
 a*c*x - c)) + 4*(2*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x)))/(a^3*c^3*x^2 - 2*a^2*c^3*x + a*c^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a\,x}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(5/2), x)
```

3.470 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\Big(}{a\left(c-\frac{c}{ax}\right)^{7/2}}$$

```
[Out] 5*(1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-115/32*(1-1/a/
x)^(7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)-5/4
*a*(1-1/a/x)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)-35/16*(1-1/a/x
)^(7/2)*(1+1/a/x)^(1/2)/(a-1/x)/(c-c/a/x)^(7/2)+a^2*(1-1/a/x)^(7/2)*x*(1+1/
a/x)^(1/2)/(a-1/x)^2/(c-c/a/x)^(7/2)
```

Rubi [A]

time = 0.12, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6317, 6314, 105, 21, 101, 156, 162, 65, 214, 212}

$$\frac{a^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{35\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{5a\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{5\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{115\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{16\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (-5*a*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(4*(a - x^(-1))^2*(c - c/(a*x)
)^(7/2)) - (35*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)])/(16*(a - x^(-1))*(c -
 c/(a*x))^(7/2)) + (a^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x)/((a - x^(-
1))^2*(c - c/(a*x))^(7/2)) + (5*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x
)]])/(a*(c - c/(a*x))^(7/2)) - (115*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/
(a*x)]/Sqrt[2]])/(16*Sqrt[2]*a*(c - c/(a*x))^(7/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{-\frac{5}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^3}\,dx, x, \frac{1}{x}\right)}{2a\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(5\left(1-\frac{1}{ax}\right)^{7/2}\right)\text{Subst}\left(\int \frac{}{x(1\cdots}\right.}{4a\left(c-\frac{c}{ax}\cdots\right.}$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \cdots$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \cdots$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \cdots$$

$$= -\frac{5a\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{4\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{35\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}{16\left(a-\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x}{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}} + \cdots$$

Mathematica [A]
time = 0.09, size = 135, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2a\sqrt{1+\frac{1}{ax}}\,x(35-55ax+16a^2x^2)+160(-1+ax)^2\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-115\sqrt{2}\,(-1+ax)^2\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{32ac^3\sqrt{c-\frac{c}{ax}}\,(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*Sqrt[1 + 1/(a*x)]*x*(35 - 55*a*x + 16*a^2*x^2) + 16
0*(-1 + a*x)^2*ArcTanh[Sqrt[1 + 1/(a*x)]] - 115*Sqrt[2]*(-1 + a*x)^2*ArcTan
h[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(32*a*c^3*Sqrt[c - c/(a*x)]*(-1 + a*x)^2)
```

Maple [A]
time = 0.20, size = 371, normalized size = 1.34

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^4\sqrt{a^2c}}-\frac{115\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+}}{x-\frac{1}{a}}\right)}{64a^5\sqrt{c}}\right.}{}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(64\sqrt{x\,(ax+1)}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-115a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(64*(x*(a*x+1)
)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2-115*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/
2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2-220*(x*(a*x+1))^(1/2)*a^(5/2)*
(1/a)^(1/2)*x+160*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3
*(1/a)^(1/2)*x^2+230*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^
(1/2)*a+3*a*x+1)/(a*x-1))*x+140*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-320*a
^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*x+160*
ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)-115*2^(
```

```
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2
))/a^(3/2)/c^4/(a*x-1)^3/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

Fricas [A]
time = 0.43, size = 668, normalized size = 2.41

$$\left[\frac{115\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\left(3a^3x^3+4a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+160\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\left(16a^4x^4-39a^3x^3-20a^2x^2+35ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{128\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}, \frac{115\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)-160\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+4\left(16a^4x^4-39a^3x^3-20a^2x^2+35ax\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{64\left(a^4c^4x^3-3a^3c^4x^2+3a^2c^4x-ac^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/128*(115*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(17*a^3*
c*x^3 - 3*a^2*c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sq
rt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a
^2*x^2 + 3*a*x - 1)) + 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(
8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 8*(16*a^4*x^4 - 39
*a^3*x^3 - 20*a^2*x^2 + 35*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^4*c^4*x^3 - 3*a^3*c^4*x^2 + 3*a^2*c^4*x - a*c^4), 1/64*(115*sqrt
(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 160*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(2
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) + 4*(16*a^4*x^4 - 39*a^3*x^3 - 20*a^2*x^2 + 35*
a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*c^4*x^3 - 3*a^
3*c^4*x^2 + 3*a^2*c^4*x - a*c^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a*x))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a\,x}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a*x))^(7/2), x)
```

3.471 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=163

$$-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}\,c^{7/2}$$

```
[Out] -5/3*c^2*(c-c/a/x)^(3/2)/a+3/5*c*(c-c/a/x)^(5/2)/a+(c-c/a/x)^(7/2)*x-11*c^(
7/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+32*c^(7/2)*arctanh(1/2*(c-c/a/x)^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-21*c^3*(c-c/a/x)^(1/2)/a
```

Rubi [A]

time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6302, 6268, 25, 528, 382, 100, 159, 162, 65, 214}

$$-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+x\left(c-\frac{c}{ax}\right)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-21*c^3*Sqrt[c - c/(a*x)])/a - (5*c^2*(c - c/(a*x))^(3/2))/(3*a) + (3*c*(c
 - c/(a*x))^(5/2))/(5*a) + (c - c/(a*x))^(7/2)*x - (11*c^(7/2)*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx$$

$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}(1-ax)}{1+ax}dx$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}x}{1+ax}dx}{c}$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{9/2}}{a+\frac{1}{x}}dx}{c}$$

$$=-\frac{a\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{9/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{5/2}\left(\frac{11c^2}{2}+\frac{3c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{2\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{55c^3}{4}-\frac{25c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{5c}$$

$$=-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\frac{4\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\cdots}{\cdots}\right)}{\cdots}$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\cdots$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x+\cdots$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-(\cdots$$

$$=-\frac{21c^3\sqrt{c-\frac{c}{ax}}}{a}-\frac{5c^2\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\frac{3c\left(c-\frac{c}{ax}\right)^{5/2}}{5a}+\left(c-\frac{c}{ax}\right)^{7/2}x-\cdots$$

Mathematica [A]
time = 0.17, size = 125, normalized size = 0.77

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(-6+52ax-376a^2x^2+15a^3x^3\right)}{15a^3x^2}-\frac{11c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{32\sqrt{2}\,c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(-6 + 52*a*x - 376*a^2*x^2 + 15*a^3*x^3))/(15*a^3*x^
2) - (11*c^(7/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (32*Sqrt[2]*c^(7/2
)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(136) = 272.
time = 0.20, size = 281, normalized size = 1.72

method	result
risch	$\frac{(15a^4x^4-391a^3x^3+428a^2x^2-58ax+6)c^3\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\left(-\frac{11a^3\ln\left(\frac{-\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2cx^2-acx}\right)}{2\sqrt{a^2c}}-\frac{16a^2\sqrt{2}\ln\left(\frac{4c-3(x+\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(480\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}\,a^{\frac{7}{2}}x^4-1110\sqrt{\frac{1}{a}}\,\sqrt{ax^2-x}\,a^{\frac{7}{2}}x^4-480\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax-1)\ldots}}{ax+1}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(c*(a*x-1)/a/x)^(1/2)/x^3*c^3/a^(7/2)*(480*((a*x-1)*x)^(1/2)*(1/a)^(1/
2)*a^(7/2)*x^4-1110*(1/a)^(1/2)*(a*x^2-x)^(1/2)*a^(7/2)*x^4-480*2^(1/2)*ln(
(2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*x^4-72
0*(1/a)^(1/2)*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^3*x^4
+660*(1/a)^(1/2)*(a*x^2-x)^(3/2)*a^(5/2)*x^2+555*(1/a)^(1/2)*ln(1/2*(2*(a*x
^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^3*x^4-92*a^(3/2)*(a*x^2-x)^(3/2)*(1
/a)^(1/2)*x+12*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/(1/a)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(7/2)/(a*x + 1), x)
```

Fricas [A]
time = 0.34, size = 323, normalized size = 1.98

$$\left[\frac{480\,\sqrt{2}\,a^{2}c^{\frac{7}{2}}x^{2}\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+165\,a^{2}c^{\frac{7}{2}}x^{2}\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(15\,a^{3}c^{3}x^{3}-376\,a^{2}c^{3}x^{2}+52\,ac^{3}x-6\,c^{3})\sqrt{\frac{acx-c}{ax}}}{30\,a^{3}x^{2}},\ -\frac{480\,\sqrt{2}\,a^{2}\sqrt{-c}\,c^{3}x^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-165\,a^{2}\sqrt{-c}\,c^{3}x^{2}\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)-(15\,a^{3}c^{3}x^{3}-376\,a^{2}c^{3}x^{2}+52\,ac^{3}x-6\,c^{3})\sqrt{\frac{acx-c}{ax}}}{15\,a^{3}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/30*(480*sqrt(2)*a^2*c^(7/2)*x^2*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
 - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 165*a^2*c^(7/2)*x^2*log(-2*a*c*x +
2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(15*a^3*c^3*x^3 - 376*a^2*c^
3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2), -1/15*(480*
sqrt(2)*a^2*sqrt(-c)*c^3*x^2*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(
a*x))/c) - 165*a^2*sqrt(-c)*c^3*x^2*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))
/c) - (15*a^3*c^3*x^3 - 376*a^2*c^3*x^2 + 52*a*c^3*x - 6*c^3)*sqrt((a*c*x -
 c)/(a*x)))/(a^3*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(7/2)*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{7/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a*x))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

3.472 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=138

$$-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] 1/3*c*(c-c/a/x)^(3/2)/a+(c-c/a/x)^(5/2)*x-9*c^(5/2)*arctanh((c-c/a/x)^(1/2)
/c^(1/2))/a+16*c^(5/2)*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)
/a-7*c^2*(c-c/a/x)^(1/2)/a
```

Rubi [A]
time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6302, 6268, 25, 528, 382, 100, 159, 162, 65, 214}

$$-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{16\sqrt{2}\,c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+x\left(c-\frac{c}{ax}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*c^2*Sqrt[c - c/(a*x)])/a + (c*(c - c/(a*x))^(3/2))/(3*a) + (c - c/(a*x)
)^(5/2)*x - (9*c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (16*Sqrt[2]*
c^(5/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx$$

$$=-\int\frac{\left(c-\frac{c}{ax}\right)^{5/2}(1-ax)}{1+ax}dx$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}x}{1+ax}dx}{c}$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{7/2}}{a+\frac{1}{x}}dx}{c}$$

$$=-\frac{a\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{7/2}}{x^2(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}\left(\frac{9c^2}{2}+\frac{c^2x}{2a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{2\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}\left(\frac{27c^3}{4}-\frac{21c^3x}{4a}\right)}{x(a+x)}dx,x,\frac{1}{x}\right)}{3c}$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{4\text{Subst}\left(\int\frac{\frac{27c^4}{8}-\frac{69c^4x}{8a}}{x(a+x)\sqrt{c-}}\right.}{3c}$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x+\frac{(9c^3)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c-}}\right.}{2a}$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-(9c^2)\,\text{Subst}\left(\int\frac{1}{a-\frac{ax^2}{c}}\right.$$

$$=-\frac{7c^2\sqrt{c-\frac{c}{ax}}}{a}+\frac{c\left(c-\frac{c}{ax}\right)^{3/2}}{3a}+\left(c-\frac{c}{ax}\right)^{5/2}x-\frac{9c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{a}}}{\sqrt{c}}\right.}{a}$$

Mathematica [A]

time = 0.07, size = 116, normalized size = 0.84

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\,(2-26ax+3a^2x^2)-27ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+48\sqrt{2}\,ac^{5/2}x\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(2 - 26*a*x + 3*a^2*x^2) - 27*a*c^(5/2)*x*ArcTanh[Sq
rt[c - c/(a*x)]/Sqrt[c]] + 48*Sqrt[2]*a*c^(5/2)*x*ArcTanh[Sqrt[c - c/(a*x)]
/(Sqrt[2]*Sqrt[c])])/(3*a^2*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(115) = 230$.

time = 0.21, size = 257, normalized size = 1.86

method	result
risch	$\frac{(3a^3x^3-29a^2x^2+28ax-2)c^2\sqrt{\frac{c(ax-1)}{ax}}}{3x\,a^2(ax-1)}+\frac{\left(-\frac{9a^2\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{8a\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right.\cdots}{\cdots}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(48\sqrt{(ax-1)\,x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x^3-90a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^3-72\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\cdots}\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(c*(a*x-1)/a/x)^(1/2)/x^2*c^2/a^(5/2)*(48*((a*x-1)*x)^(1/2)*a^(5/2)*(1/
a)^(1/2)*x^3-90*a^(5/2)*(1/a)^(1/2)*(a*x^2-x)^(1/2)*x^3-72*ln(1/2*(2*((a*x-
1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2*x^3-48*a^(3/2)*2^(1/2
)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3+48*a^
(3/2)*(a*x^2-x)^(3/2)*(1/a)^(1/2)*x+45*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^3-4*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/
((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(5/2)/(a*x + 1), x)
```

Fricas [A]
time = 0.35, size = 285, normalized size = 2.07

$$\left[\frac{48\,\sqrt{2}\,ac^{\frac{5}{2}}x\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+27\,ac^{\frac{5}{2}}x\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(3\,a^2c^2x^2-26\,ac^2x+2\,c^2)\sqrt{\frac{acx-c}{ax}}}{6\,a^2x}, -\frac{48\,\sqrt{2}\,a\sqrt{-c}\,c^2x\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-27\,a\sqrt{-c}\,c^2x\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)-(3\,a^2c^2x^2-26\,ac^2x+2\,c^2)\sqrt{\frac{acx-c}{ax}}}{3\,a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(48*sqrt(2)*a*c^(5/2)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(
a*x)) + 3*a*c*x - c)/(a*x + 1)) + 27*a*c^(5/2)*x*log(-2*a*c*x + 2*a*sqrt(c)
*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*(3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*sq
rt((a*c*x - c)/(a*x)))/(a^2*x), -1/3*(48*sqrt(2)*a*sqrt(-c)*c^2*x*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 27*a*sqrt(-c)*c^2*x*arctan(
sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (3*a^2*c^2*x^2 - 26*a*c^2*x + 2*c^2)*
sqrt((a*c*x - c)/(a*x)))/(a^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(5/2)*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.473 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=113

$$-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] (c-c/a/x)^(3/2)*x-7*c^(3/2)*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a+8*c^(3/2)*ar
ctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)/a-c*(c-c/a/x)^(1/2)/a
```

Rubi [A]
time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6302, 6268, 25, 528, 382, 100, 159, 162, 65, 214}

$$-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+x\left(c-\frac{c}{ax}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((c*Sqrt[c - c/(a*x)])/a) + (c - c/(a*x))^(3/2)*x - (7*c^(3/2)*ArcTanh[Sqr
t[c - c/(a*x)]/Sqrt[c]])/a + (8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$$

$$=-\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}(1-ax)}{1+ax}\,dx$$

$$=\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}x}{1+ax}\,dx}{c}$$

$$=\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{5/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$=-\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}\left(\frac{7c^2}{2}-\frac{c^2x}{2a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{2\text{Subst}\left(\int \frac{\frac{7c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x+\frac{(7c^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}-$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-(7c)\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right.$$

$$=-\frac{c\sqrt{c-\frac{c}{ax}}}{a}+\left(c-\frac{c}{ax}\right)^{3/2}x-\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{8\sqrt{2}\,c^{3/}}{}$$

Mathematica [A]

time = 0.06, size = 95, normalized size = 0.84

$$\frac{c\sqrt{c-\frac{c}{ax}}\,(-2+ax)-7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+8\sqrt{2}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a*x)]*(-2 + a*x) - 7*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[
c]] + 8*Sqrt[2]*c^(3/2)*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(94) = 188$.
time = 0.19, size = 229, normalized size = 2.03

method	result
risch	$\frac{(a^2x^2-3ax+2)c\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\left(-\frac{7a\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{4\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\,(x}}{x+}\right.}{\sqrt{c}}\right.}{a(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,c\left(8\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x^2-10a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^2-12\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\right.}{2x\,a^{\frac{3}{2}}\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)/x*c/a^(3/2)*(8*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1
/2)*x^2-10*a^(3/2)*(1/a)^(1/2)*(a*x^2-x)^(1/2)*x^2-12*ln(1/2*(2*((a*x-1)*x)
^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a*x^2-8*a^(1/2)*2^(1/2)*ln((2*
2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^2+4*(a*x^2-x)^(
3/2)*a^(1/2)*(1/a)^(1/2)+5*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*
a*x-1)/a^(1/2))*a*x^2)/((a*x-1)*x)^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a*x))^(3/2)/(a*x + 1), x)
```

Fricas [A]
time = 0.34, size = 235, normalized size = 2.08

$$\left[\frac{8\,\sqrt{2}\,c^{\frac{3}{2}}\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+7\,c^{\frac{3}{2}}\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\,(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{2\,a},-\frac{8\,\sqrt{2}\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-7\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)-(acx-2\,c)\sqrt{\frac{acx-c}{ax}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(8*sqrt(2)*c^(3/2)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 + 3*a*c*x - c)/(a*x + 1)) + 7*c^(3/2)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a
*c*x - c)/(a*x)) + c) + 2*(a*c*x - 2*c)*sqrt((a*c*x - c)/(a*x)))/a, -(8*sqr
t(2)*sqrt(-c)*c*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 7*
sqrt(-c)*c*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - (a*c*x - 2*c)*sqrt(
(a*c*x - c)/(a*x)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**(3/2)*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{3/2}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.474 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=92

$$\sqrt{c-\frac{c}{ax}}\,x-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

```
[Out] -5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {6302, 6268, 25, 528, 382, 100, 162, 65, 214}

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{(5c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c \ldots}} \ldots\right)}{a}$$

$$= \sqrt{c-\frac{c}{ax}}\,x - 5\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) + 8\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}} \ldots\right)$$

$$= \sqrt{c-\frac{c}{ax}}\,x - \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}} \ldots\right)}{a}$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 1.00

$$\sqrt{c-\frac{c}{ax}}\,x - \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(75) = 150$.
time = 0.19, size = 190, normalized size = 2.07

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}} + \frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}} - \frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3(}{x+\frac{1}{a}}\right.}{a\sqrt{c}}\right.}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-4\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+6\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}+4\right.}{2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-4*((a*x
-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*
x-1)/a^(1/2))*a*(1/a)^(1/2)+4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)
^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)
```

Fricas [A]
time = 0.38, size = 219, normalized size = 2.38

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\,\sqrt{c}\,\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+5\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.475 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

```
[Out] -3*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(1/2)+2*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)/a/c^(1/2)+x*(c-c/a/x)^(1/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6302, 6268, 25, 528, 382, 101, 162, 65, 214}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx \\
&= -\int \frac{1-ax}{\sqrt{c-\frac{c}{ax}}\,(1+ax)}\,dx \\
&= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}\,x}{1+ax}\,dx}{c} \\
&= \frac{a\int \frac{\sqrt{c-\frac{c}{ax}}}{a+\frac{1}{x}}\,dx}{c} \\
&= -\frac{a\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c} \\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}+\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{c} \\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c}+\frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{2a}-\frac{2\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right.}{a} \\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{c}+\frac{4\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{a}}\right.}{c} \\
&= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a\sqrt{c}}
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 95, normalized size = 1.00

$$\frac{\sqrt{c-\frac{c}{ax}}\,x}{c}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a\sqrt{c}}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c - (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*Sqrt[c]
) + (2*Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*Sqrt[c])
```

Maple [A]
time = 0.18, size = 136, normalized size = 1.43

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-3\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-2\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax\cdots}{ax+\cdots}\right.\right.}{2\sqrt{(ax-1)\,x}\,c\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{-\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)}{2a\sqrt{a^{2}c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-3\,(\cdots}}{x+\frac{1}{a}}\right)}{a^{2}\sqrt{c}}\cdots\right.}{\sqrt{\frac{c(ax-1)}{ax}}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-3*ln(1
/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2*2^(1/2)*l
n((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((a
*x-1)*x)^(1/2)/c/a^(3/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a*x))), x)
```

Fricas [A]
time = 0.37, size = 234, normalized size = 2.46

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+2\,\sqrt{2}\,\sqrt{c}\,\log\left(-\frac{\frac{2\,\sqrt{2}\,ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right)+3\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac},\ -\frac{2\,\sqrt{2}\,c\sqrt{-\frac{1}{c}}\,\arctan\left(\frac{\sqrt{2}\,ax\sqrt{-\frac{1}{c}}\,\sqrt{\frac{acx-c}{ax}}}{ax-1}\right)-ax\sqrt{\frac{acx-c}{ax}}-3\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 2*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x
*sqrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + 3*sqrt(c)*log(-2
*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c), -(2*sqrt(2)*c*s
qrt(-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1))
- a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)
/(a*x))/c))/(a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(1/2)*(a*x + 1)), x)
```

3.476 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{ac^{3/2}}$$

```
[Out] -arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(3/2)+arctanh(1/2*(c-c/a/x)^(1/2)*2^(
1/2)/c^(1/2))*2^(1/2)/a/c^(3/2)+x*(c-c/a/x)^(1/2)/c^2
```

Rubi [A]
time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {6302, 6268, 25, 528, 382, 105, 21, 85, 65, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{ac^{3/2}}+\frac{x\sqrt{c-\frac{c}{ax}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
```

```
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\sqrt{c-\frac{c}{ax}}\,(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\sqrt{c-\frac{c}{ax}}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{\frac{c}{2}-\frac{cx}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2} + \frac{2\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^2}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Mathematica [A]
time = 0.04, size = 94, normalized size = 1.00

$$\frac{\sqrt{c-\frac{c}{ax}}\,x}{c^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x)/c^2 - ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]/(a*c^(3/2))
+ (Sqrt[2]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(a*c^(3/2))
```

Maple [A]
time = 0.23, size = 136, normalized size = 1.45

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{(ax}{ax+1}\right.\right.}{2a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\,c^2\sqrt{\frac{1}{a}}}$
risch	$\frac{ax-1}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^2\sqrt{a^2c}}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3}}{x+\frac{1}{a}}\right.}{2a^3\sqrt{c}}\right.}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2
)-ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-2^(1/
2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))
/((a*x-1)*x)^(1/2)/c^2/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(3/2)), x)
```

Fricas [A]
time = 0.38, size = 231, normalized size = 2.46

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}} + \sqrt{2}\,\sqrt{c}\,\log\left(-\frac{\frac{2\,\sqrt{2}\,ax\sqrt{\frac{acx-c}{ax}}}{\sqrt{c}}+3\,ax-1}{ax+1}\right) + \sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,ac^2}, -\frac{\sqrt{2}\,c\sqrt{-\frac{1}{c}}\,\arctan\left(\frac{\sqrt{2}\,ax\sqrt{-\frac{1}{c}}\,\sqrt{\frac{acx-c}{ax}}}{ax-1}\right) - ax\sqrt{\frac{acx-c}{ax}} - \sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*x*s
qrt((a*c*x - c)/(a*x))/sqrt(c) + 3*a*x - 1)/(a*x + 1)) + sqrt(c)*log(-2*a*c
*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/(a*c^2), -(sqrt(2)*c*sqrt(
-1/c)*arctan(sqrt(2)*a*x*sqrt(-1/c)*sqrt((a*c*x - c)/(a*x))/(a*x - 1)) - a*
x*sqrt((a*c*x - c)/(a*x)) - sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x)
)/c))/(a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a*x))^(3/2)*(a*x + 1)), x)
```

3.477 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

```
[Out] arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(5/2)+1/2*arctanh(1/2*(c-c/a/x)^(1/2)*
2^(1/2)/c^(1/2))/a/c^(5/2)*2^(1/2)-2/a/c^2/(c-c/a/x)^(1/2)+x/c^2/(c-c/a/x)^
(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6302, 6268, 25, 528, 382, 105, 157, 162, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}+\frac{x}{c^2\sqrt{c-\frac{c}{ax}}}-\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] -2/(a*c^2*Sqrt[c - c/(a*x)]) + x/(c^2*Sqrt[c - c/(a*x)]) + ArcTanh[Sqrt[c -
 c/(a*x)]/Sqrt[c]]/(a*c^(5/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c]
)]/(Sqrt[2]*a*c^(5/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{3/2}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{c}{2}-\frac{3cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\frac{c^2}{2}+\frac{c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2} - \frac{\text{Subst}\left(\int \frac{\cdots}{(a+x)\sqrt{\cdots}} \cdots\right)}{2\cdots}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{c^3} + \frac{\text{Subst}\left(\int \frac{1}{2a-\cdots} \cdots\right)}{\cdots}$$

$$= -\frac{2}{ac^2\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\sqrt{c-\frac{c}{ax}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{5/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\,ac^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 70, normalized size = 0.60

$$\frac{ax - {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a-\frac{1}{x}}{2a}\right) - {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{1}{ax}\right)}{ac^2\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (a*x - Hypergeometric2F1[-1/2, 1, 1/2, (a - x^(-1))/(2*a)] - Hypergeometric
2F1[-1/2, 1, 1/2, 1 - 1/(a*x)])/(a*c^2*Sqrt[c - c/(a*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(98) = 196.
time = 0.24, size = 368, normalized size = 3.17

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^3\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{a^5c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac\cdots}{\cdots}\right.}{\cdots}\right.}{c^2x\sqrt{\frac{c(ax-1)}{ax}}\cdots}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-8\sqrt{(ax-1)\,x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2+4((ax-1)x)^{\frac{3}{2}}a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-2\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a^3x^{2}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-8*((a*x-1)*x)^(1/2)*a^(7/2)*(1/a)^(1
/2)*x^2+4*((a*x-1)*x)^(3/2)*a^(5/2)*(1/a)^(1/2)-2*ln(1/2*(2*((a*x-1)*x)^(1/
2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^3*x^2+a^(5/2)*2^(1/2)*ln((2*2^(1
/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^2+16*((a*x-1)*x)^(1
/2)*a^(5/2)*(1/a)^(1/2)*x+4*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^
(1/2))*(1/a)^(1/2)*a^2*x-2*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-
1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x-8*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-2
*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)+2^(1/2
```

```
)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/
((a*x-1)*x)^(1/2)/c^3/(a*x-1)^2/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(5/2)), x)
```

Fricas [A]
time = 0.36, size = 287, normalized size = 2.47

$$\left[\frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+2\,(ax-1)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+4\,(a^2x^2-2\,ax)\sqrt{\frac{acx-c}{ax}}}{4\,(a^2c^3x-ac^3)},\ -\frac{\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+2\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\,(a^2x^2-2\,ax)\sqrt{\frac{acx-c}{ax}}}{2\,(a^2c^3x-ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="fricas")
[Out] [1/4*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 2*(a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a
*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(a^2*x^2 - 2*a*x)*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c^3*x - a*c^3), -1/2*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*(a*x - 1)*sqrt(-c)*arctan
(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(a^2*x^2 - 2*a*x)*sqrt((a*c*x - c)
/(a*x)))/(a^2*c^3*x - a*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(5/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{\left(c-\frac{c}{a\,x}\right)^{5/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a*x))^(5/2)*(a*x + 1)), x)
```

3.478 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}$$

```
[Out] -4/3/a/c^2/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(3/2)+3*arctanh((c-c/a/x)^(1/2)/
c^(1/2))/a/c^(7/2)+1/4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))/a/c^(7/
2)*2^(1/2)-7/2/a/c^3/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6302, 6268, 25, 528, 382, 105, 157, 162, 65, 214}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\,ac^{7/2}}-\frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] -4/(3*a*c^2*(c - c/(a*x))^(3/2)) - 7/(2*a*c^3*Sqrt[c - c/(a*x)]) + x/(c^2*(
c - c/(a*x))^(3/2)) + (3*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(a*c^(7/2)) +
ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]/(2*Sqrt[2]*a*c^(7/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{5/2}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{5/2}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3c}{2}-\frac{5cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{9c^2}{2}+\frac{6c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^4}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{9c^3}{2}-\frac{21c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x\right.}{3c^6}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x,\right.}{4ac^3}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{a}}\right.}{2c^4}$$

$$= -\frac{4}{3ac^2\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{7}{2ac^3\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{7/2}} + \frac{\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 79, normalized size = 0.54

$$\frac{x\left(3ax - {}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{a-\frac{1}{x}}{2a}\right) - 3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{1}{ax}\right)\right)}{3c^3\sqrt{c-\frac{c}{ax}}\,(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (x*(3*a*x - Hypergeometric2F1[-3/2, 1, -1/2, (a - x^(-1))/(2*a)] - 3*Hyperg
eometric2F1[-3/2, 1, -1/2, 1 - 1/(a*x)]))/(3*c^3*Sqrt[c - c/(a*x)]*(-1 + a*
x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(120) = 240$.
time = 0.24, size = 497, normalized size = 3.38

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}} + \frac{\left(\frac{3\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^4\sqrt{a^2c}} - \frac{17\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{6a^6c\left(x-\frac{1}{a}\right)} - \frac{\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\right.}{}\right.}{}\right.}{}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-84\sqrt{(ax-1)\,x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3+60((ax-1)x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x-36\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-84*((a*x-1)*x)^(1/2)*a^(9/2)*(1/a)^
(1/2)*x^3+60*((a*x-1)*x)^(3/2)*a^(7/2)*(1/a)^(1/2)*x-36*ln(1/2*(2*((a*x-1)*
x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^4*x^3+3*a^(7/2)*2^(1/2)*ln
((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3+252*((a*x
-1)*x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2-52*((a*x-1)*x)^(3/2)*a^(5/2)*(1/a)^(1/
2)+108*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^
3*x^2-9*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x
```

```
+1)/(a*x+1))*x^2-252*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x-108*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2*x+9*a^(3/2)*2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x+84*(
(a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)+36*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)
+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-3*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-
1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^4/(a*x-1)^3/(1
/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(7/2)), x)
```

Fricas [A]
time = 0.36, size = 359, normalized size = 2.44

$$\left[\frac{3\,\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{c}\,\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+36\,(a^2x^2-2\,ax+1)\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+4\,(6\,a^3x^3-29\,a^2x^2+21\,ax)\sqrt{\frac{acx-c}{ax}}}{24\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}, -\frac{3\,\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+36\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\,(6\,a^3x^3-29\,a^2x^2+21\,ax)\sqrt{\frac{acx-c}{ax}}}{12\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="fricas")
[Out] [1/24*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*
sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 36*(a^2*x^2 - 2*a*x + 1
)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 4*(6*
a^3*x^3 - 29*a^2*x^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4), -1/12*(3*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(1/
2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 36*(a^2*x^2 - 2*a*x + 1)*sq
rt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(6*a^3*x^3 - 29*a^2*x
^2 + 21*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(7/2),x)
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a\,x}\right)}^{7/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a*x))^(7/2)*(a*x + 1)), x)
```

3.479 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}$$

```
[Out] -6/5/a/c^2/(c-c/a/x)^(5/2)-11/6/a/c^3/(c-c/a/x)^(3/2)+x/c^2/(c-c/a/x)^(5/2)
+5*arctanh((c-c/a/x)^(1/2)/c^(1/2))/a/c^(9/2)+1/8*arctanh(1/2*(c-c/a/x)^(1/
2)*2^(1/2)/c^(1/2))/a/c^(9/2)*2^(1/2)-21/4/a/c^4/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6302, 6268, 25, 528, 382, 105, 157, 162, 65, 214}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{ac^{9/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}\,ac^{9/2}}-\frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}}-\frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] -6/(5*a*c^2*(c - c/(a*x))^(5/2)) - 11/(6*a*c^3*(c - c/(a*x))^(3/2)) - 21/(4
*a*c^4*Sqrt[c - c/(a*x)]) + x/(c^2*(c - c/(a*x))^(5/2)) + (5*ArcTanh[Sqrt[c
 - c/(a*x)]/Sqrt[c]])/(a*c^(9/2)) + ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt
[c])]/(4*Sqrt[2]*a*c^(9/2))
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{9/2}}\,dx$$

$$= -\int \frac{1-ax}{\left(c-\frac{c}{ax}\right)^{9/2}(1+ax)}\,dx$$

$$= \frac{a\int \frac{x}{\left(c-\frac{c}{ax}\right)^{7/2}(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{1}{\left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{7/2}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{5c}{2}-\frac{7cx}{2a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int \frac{\frac{25c^2}{2}+\frac{15c^2x}{a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{5c^4}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{75c^3}{2}-\frac{165c^3x}{4a}}{x(a+x)\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x\right.}{15c^6}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{6}{5ac^2\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{11}{6ac^3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{21}{4ac^4\sqrt{c-\frac{c}{ax}}} + \frac{x}{c^2\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{5\tanh^{-1}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 82, normalized size = 0.48

$$\frac{ax^2\left(5ax - {}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{a-\frac{1}{x}}{2a}\right) - 5\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};1-\frac{1}{ax}\right)\right)}{5c^4\sqrt{c-\frac{c}{ax}}\,(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a*x))^(9/2)),x]
```

```
[Out] (a*x^2*(5*a*x - Hypergeometric2F1[-5/2, 1, -3/2, (a - x^(-1))/(2*a)] - 5*Hy
pergeometric2F1[-5/2, 1, -3/2, 1 - 1/(a*x)]))/(5*c^4*Sqrt[c - c/(a*x)]*(-1
+ a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(141) = 282.
time = 0.25, size = 626, normalized size = 3.64

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{c(ax-1)}{ax}}} + \frac{\left(\frac{5\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2a^5\sqrt{a^2c}} - \frac{317\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+\left(x-\frac{1}{a}\right)ac}}{60a^7c\left(x-\frac{1}{a}\right)} - \frac{\sqrt{2}\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right.}{}\right.}{}\right.}{}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-1260\sqrt{(ax-1)\,x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^4+1020((ax-1)x)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^2-600\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-1260*((a*x-1)*x)^(1/2)*a^(11/2)*(1
/a)^(1/2)*x^4+1020*((a*x-1)*x)^(3/2)*a^(9/2)*(1/a)^(1/2)*x^2-600*ln(1/2*(2*
((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^5*x^4+15*a^(9/2)*
2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^4
+5040*((a*x-1)*x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^3-1792*((a*x-1)*x)^(3/2)*a^(7
/2)*(1/a)^(1/2)*x+2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*a^4*x^3-60*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)
```

```
*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3-7560*((a*x-1)*x)^(1/2)*a^(7/2)*(1/a)^(1/2
)*x^2+820*((a*x-1)*x)^(3/2)*a^(5/2)*(1/a)^(1/2)-3600*ln(1/2*(2*((a*x-1)*x)^
(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^3*x^2+90*a^(5/2)*2^(1/2)*ln((
2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^2+5040*((a*x-
1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x+2400*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2*x-60*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^
(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x-1260*((a*x-1)*x)^(1/2)*a^(3/2
)*(1/a)^(1/2)-600*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(
1/a)^(1/2)+15*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1
)/(a*x+1))*a^(1/2))/((a*x-1)*x)^(1/2)/c^5/(a*x-1)^4/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="maxima")
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a*x))^(9/2)), x)
```

Fricas [A]
time = 0.41, size = 431, normalized size = 2.51

$$\left[\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+600\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{c}\log\left(-2acx-2a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+4\left(60a^4x^4-497a^3x^3+740a^2x^2-315ax\right)\sqrt{\frac{acx-c}{ax}}}{240\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)},\ -\frac{15\sqrt{2}\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2c}\right)+600\left(a^3x^3-3a^2x^2+3ax-1\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)-2\left(60a^4x^4-497a^3x^3+740a^2x^2-315ax\right)\sqrt{\frac{acx-c}{ax}}}{120\left(a^4c^5x^3-3a^3c^5x^2+3a^2c^5x-ac^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="fricas")
[Out] [1/240*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-(2*sqrt(2
)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 600*(a^3*
x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c
*x - c)/(a*x)) + c) + 4*(60*a^4*x^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*
sqrt((a*c*x - c)/(a*x)))/(a^4*c^5*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5
), -1/120*(15*sqrt(2)*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 600*(a^3*x^3 - 3*a^2*x^2 + 3
*a*x - 1)*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*(60*a^4*x
^4 - 497*a^3*x^3 + 740*a^2*x^2 - 315*a*x)*sqrt((a*c*x - c)/(a*x)))/(a^4*c^5
*x^3 - 3*a^3*c^5*x^2 + 3*a^2*c^5*x - a*c^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{9}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)**(9/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x)))**(9/2)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a/x)^(9/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a\,x}\right)}^{9/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a*x))^(9/2)*(a*x + 1)), x)
```

3.480 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx$

Optimal. Leaf size=335

$$\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{65(a}{}$$

```
[Out] -15*(c-c/a/x)^(9/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(9/2)+10*(a-1/x)^4
*(c-c/a/x)^(9/2)/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+(a-1/x)^5*(c-c/a/x)^(9
/2)*x/a^5/(1-1/a/x)^(9/2)/(1+1/a/x)^(1/2)+5/7*(304*a-65/x)*(c-c/a/x)^(9/2)*
(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(9/2)+135/7*(a-1/x)^2*(c-c/a/x)^(9/2)*(1+1/a/
x)^(1/2)/a^3/(1-1/a/x)^(9/2)+65/7*(a-1/x)^3*(c-c/a/x)^(9/2)*(1+1/a/x)^(1/2)
/a^4/(1-1/a/x)^(9/2)
```

Rubi [A]
time = 0.13, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6317, 6314, 100, 155, 158, 152, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{\frac{1}{ax}+1}}+\frac{65\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}-\frac{15\left(c-\frac{c}{ax}\right)^{9/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^4*(c - c/(a*x))^(9/2))/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1
/(a*x)]) + (5*(304*a - 65/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(9/2))/(7*a^2*
(1 - 1/(a*x))^(9/2)) + (135*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(9/2))/(7*a^3*(1 - 1/(a*x))^(9/2)) + (65*(a - x^(-1))^3*Sqrt[1 + 1/(a*x)]*(
c - c/(a*x))^(9/2))/(7*a^4*(1 - 1/(a*x))^(9/2)) + ((a - x^(-1))^5*(c - c/(a
*x))^(9/2)*x)/(a^5*(1 - 1/(a*x))^(9/2)*Sqrt[1 + 1/(a*x)]) - (15*(c - c/(a*x
))^(9/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
```

```
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{9/2}\,dx=\frac{\left(c-\frac{c}{ax}\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{9/2}\,dx}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^6}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{9/2}\text{Subst}\left(\int\frac{\left(\frac{15}{2a}+\frac{5x}{2a^2}\right)\left(1-\frac{x}{a}\right)^4}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{9/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^5\left(c-\frac{c}{ax}\right)^{9/2}x}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{9/2}\right)\text{S}}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{65\left(a-\frac{1}{x}\right)^3\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^4\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{\left(a-\frac{1}{x}\right)^5}{a^5\left(1-\frac{1}{ax}\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{135\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^3\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{65\left(a-\frac{1}{x}\right.}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\right.}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\right.}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{9/2}}{a^5\left(1-\frac{1}{ax}\right)^{9/2}\sqrt{1+\frac{1}{ax}}}+\frac{5\left(304a-\frac{65}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{9/2}}{7a^2\left(1-\frac{1}{ax}\right)^{9/2}}+\frac{135\left(a-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.08, size = 140, normalized size = 0.42

$$\frac{c^4\sqrt{c-\frac{c}{ax}}\left(-2+20ax-110a^2x^2+720a^3x^3+1685a^4x^4+7a^5x^5-35a^4\sqrt{1+\frac{1}{ax}}\,x^4\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)+70a^4x^4\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)\right)}{7a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*Sqrt[c - c/(a*x)]*(-2 + 20*a*x - 110*a^2*x^2 + 720*a^3*x^3 + 1685*a^4*
x^4 + 7*a^5*x^5 - 35*a^4*Sqrt[1 + 1/(a*x)]*x^4*ArcTanh[Sqrt[1 + 1/(a*x)]] +
 70*a^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(7*a^5*Sqrt[1 -
1/(a^2*x^2)]*x^4)
```

Maple [A]

time = 0.12, size = 229, normalized size = 0.68

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^4\left(14a^{\frac{11}{2}}\sqrt{x(ax+1)}\,x^5+3510a^{\frac{9}{2}}\sqrt{x(ax+1)}\,x^4+1440a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}-105\right.}{}$
risch	$\frac{(7a^5x^5+859a^4x^4+720a^3x^3-110a^2x^2+20ax-2)c^4\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{7x^3a^4(ax-1)}+\frac{\left(-\frac{15a^4\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^4*(1
4*a^(11/2)*(x*(a*x+1))^(1/2)*x^5+3510*a^(9/2)*(x*(a*x+1))^(1/2)*x^4+1440*a^
(7/2)*x^3*(x*(a*x+1))^(1/2)-105*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1
)/a^(1/2))*a^5*x^5-105*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^4*x^4-220*a^(5/2)*x^2*(x*(a*x+1))^(1/2)+40*a^(3/2)*x*(x*(a*x+1))^(1/2)-
4*(x*(a*x+1))^(1/2)*a^(1/2))/x^3/a^(9/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.41, size = 437, normalized size = 1.30

$$\left[\frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(7a^5c^4x^5+1755a^4c^4x^4+720a^3c^4x^3-110a^2c^4x^2+20ac^4x-2c^4\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{28\left(a^5x^4-a^4x^3\right)},\ \frac{105\left(a^4c^4x^4-a^3c^4x^3\right)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2\left(7a^5c^4x^5+1755a^4c^4x^4+720a^3c^4x^3-110a^2c^4x^2+20ac^4x-2c^4\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{14\left(a^5x^4-a^4x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/28*(105*(a^4*c^4*x^4 - a^3*c^4*x^3)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(7*a^5*c^4*x^5 + 1755*a^4*c^4*x^4 + 72
0*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*c^4)*sqrt((a*x - 1)/(a*x +
 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3), 1/14*(105*(a^4*c^4*x^4 -
 a^3*c^4*x^3)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(7*a^5*c^4*x
^5 + 1755*a^4*c^4*x^4 + 720*a^3*c^4*x^3 - 110*a^2*c^4*x^2 + 20*a*c^4*x - 2*
c^4)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x^4 - a^4*x^3)
]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c-\frac{c}{a\,x}\right)^{9/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.481 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}\,dx$

Optimal. Leaf size=277

$$\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\right.}{a^4\left(1-\right.}$$

```
[Out] -13*(c-c/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(7/2)+10*(a-1/x)^3
*(c-c/a/x)^(7/2)/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+(a-1/x)^4*(c-c/a/x)^(7
/2)*x/a^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)+1/15*(1360*a-311/x)*(c-c/a/x)^(7/
2)*(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(7/2)+47/5*(a-1/x)^2*(c-c/a/x)^(7/2)*(1+1/
a/x)^(1/2)/a^3/(1-1/a/x)^(7/2)
```

Rubi [A]
time = 0.12, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6317, 6314, 100, 155, 158, 152, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{\frac{1}{ax}+1}\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}-\frac{13\left(c-\frac{c}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^3*(c - c/(a*x))^(7/2))/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1
/(a*x)]) + ((1360*a - 311/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2))/(15*a^2
*(1 - 1/(a*x))^(7/2)) + (47*(a - x^(-1))^2*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^
(7/2))/(5*a^3*(1 - 1/(a*x))^(7/2)) + ((a - x^(-1))^4*(c - c/(a*x))^(7/2)*x)
/(a^4*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - (13*(c - c/(a*x))^(7/2)*ArcT
anh[Sqrt[1 + 1/(a*x)]])/(a*(1 - 1/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{7/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{7/2}dx}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^5}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{7/2}\text{Subst}\left(\int\frac{\left(\frac{13}{2a}+\frac{3x}{2a^2}\right)\left(1-\frac{x}{a}\right)^3}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{7/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^4\left(c-\frac{c}{ax}\right)^{7/2}x}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{7/2}\right)}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{47\left(a-\frac{1}{x}\right)^2\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{5a^3\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{\left(a-\frac{1}{x}\right.}{a^4\left(1-\right.}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47(a}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47(a}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{7/2}}{a^4\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(1360a-\frac{311}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}{15a^2\left(1-\frac{1}{ax}\right)^{7/2}}+\frac{47(a}{}$$

<u>Mathematica</u> **[C]** Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.07, size = 132, normalized size = 0.48

$$\frac{c^3\sqrt{c-\frac{c}{ax}}\left(6-62ax+548a^2x^2+1441a^3x^3+15a^4x^4-45a^3\sqrt{1+\frac{1}{ax}}\,x^3\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)+150a^3x^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)\right)}{15a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a*x)]*(6 - 62*a*x + 548*a^2*x^2 + 1441*a^3*x^3 + 15*a^4*x^
4 - 45*a^3*Sqrt[1 + 1/(a*x)]*x^3*ArcTanh[Sqrt[1 + 1/(a*x)]] + 150*a^3*x^3*H
ypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(15*a^4*Sqrt[1 - 1/(a^2*x^2)]
*x^3)
```

Maple [A]
time = 0.11, size = 212, normalized size = 0.77

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^3\left(30a^{\frac{9}{2}}\sqrt{x(ax+1)}\,x^4+3182a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}-195\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax}{2\sqrt{a}}\right.\right.}{30(ax-1)}$
risch	$\frac{(15a^4x^4+631a^3x^3+548a^2x^2-62ax+6)c^3\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{15x^2a^3(ax-1)}+\frac{\left(-\frac{13a^3\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2\sqrt{a^2c}}+\frac{64a\sqrt{a^2c\,(}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^3*(3
0*a^(9/2)*(x*(a*x+1))^(1/2)*x^4+3182*a^(7/2)*x^3*(x*(a*x+1))^(1/2)-195*ln(1
/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^4*x^4+1096*a^(5/2)*x^2*
(x*(a*x+1))^(1/2)-195*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))
*a^3*x^3-124*a^(3/2)*x*(x*(a*x+1))^(1/2)+12*(x*(a*x+1))^(1/2)*a^(1/2))/x^2/
a^(7/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 415, normalized size = 1.50

$$\left[\frac{195\,(a^3c^3x^3-a^2c^3x^2)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(15\,a^4c^3x^4+1591\,a^3c^3x^3+548\,a^2c^3x^2-62\,ac^3x+6\,c^3)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{60\,(a^4x^3-a^3x^2)},\ \frac{195\,(a^3c^3x^3-a^2c^3x^2)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(15\,a^4c^3x^4+1591\,a^3c^3x^3+548\,a^2c^3x^2-62\,ac^3x+6\,c^3)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{30\,(a^4x^3-a^3x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/60*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x
- 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a
*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(15*a^4*c^3*x^4 + 1591*a^3*c^3*x^3 + 5
48*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)))/(a^4*x^3 - a^3*x^2), 1/30*(195*(a^3*c^3*x^3 - a^2*c^3*x^2)*sqr
t(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(15*a^4*c^3*x^4 + 1591*a^3*c^
3*x^3 + 548*a^2*c^3*x^2 - 62*a*c^3*x + 6*c^3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x)))/(a^4*x^3 - a^3*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a\,x}\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.482 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}\,dx$

Optimal. Leaf size=219

$$\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\text{ta}}{a\,(1}$$

```
[Out] -11*(c-c/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(5/2)+10*(a-1/x)^2
*(c-c/a/x)^(5/2)/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+(a-1/x)^3*(c-c/a/x)^(5
/2)*x/a^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)+1/3*(112*a-29/x)*(c-c/a/x)^(5/2)*
(1+1/a/x)^(1/2)/a^2/(1-1/a/x)^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6317, 6314, 100, 155, 152, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{11\left(c-\frac{c}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (10*(a - x^(-1))^2*(c - c/(a*x))^(5/2))/(a^3*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1
/(a*x)]) + ((112*a - 29/x)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2))/(3*a^2*(1
 - 1/(a*x))^(5/2)) + ((a - x^(-1))^3*(c - c/(a*x))^(5/2)*x)/(a^3*(1 - 1/(a*
x))^(5/2)*Sqrt[1 + 1/(a*x)]) - (11*(c - c/(a*x))^(5/2)*ArcTanh[Sqrt[1 + 1/(
a*x)]])/(a*(1 - 1/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{5/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{5/2}dx}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^4}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{5/2}\text{Subst}\left(\int\frac{\left(\frac{11}{2a}+\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{5/2}}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^3\left(c-\frac{c}{ax}\right)^{5/2}x}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{\left(2a\left(c-\frac{c}{ax}\right)^{5/2}\right)}{}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{(a-}{a^3(1-}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{(a-}{a^3(1-}$$

$$=\frac{10\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{5/2}}{a^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(112a-\frac{29}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}}{3a^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{(a-}{a^3(1-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.07, size = 124, normalized size = 0.57

$$\frac{c^2\sqrt{c-\frac{c}{ax}}\left(-2+32ax+103a^2x^2+3a^3x^3-3a^2\sqrt{1+\frac{1}{ax}}\,x^2\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)+30a^2x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)\right)}{3a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a*x)]*(-2 + 32*a*x + 103*a^2*x^2 + 3*a^3*x^3 - 3*a^2*Sqrt[
1 + 1/(a*x)]*x^2*ArcTanh[Sqrt[1 + 1/(a*x)]] + 30*a^2*x^2*Hypergeometric2F1[
-1/2, 1, 1/2, 1 + 1/(a*x)]))/(3*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A]
time = 0.11, size = 195, normalized size = 0.89

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,c^2\left(6a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}+266a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}-33\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right.\right.}{6(ax-1)^2xa^{\frac{5}{2}}\sqrt{x(ax}$
risch	$\frac{(3a^3x^3+37a^2x^2+32ax-2)c^2\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{3xa^2(ax-1)}+\frac{\left(-\frac{11a^2\ln\left(\frac{\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{2\sqrt{a^2c}}+\frac{32\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-}}{c\left(x+\frac{1}{a}\right)}\right.}{a^2(ax-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c^2*(6*
a^(7/2)*x^3*(x*(a*x+1))^(1/2)+266*a^(5/2)*x^2*(x*(a*x+1))^(1/2)-33*ln(1/2*(
2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^3*x^3+64*a^(3/2)*x*(x*(a*x+
1))^(1/2)-33*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-
4*(x*(a*x+1))^(1/2)*a^(1/2))/x/a^(5/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.40, size = 381, normalized size = 1.74

$$\left[\frac{33\,(a^2c^2x^2-ac^2x)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(3\,a^3c^2x^3+133\,a^2c^2x^2+32\,ac^2x-2\,c^2)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{12\,(a^3x^2-a^2x)},\ \frac{33\,(a^2c^2x^2-ac^2x)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(3\,a^3c^2x^3+133\,a^2c^2x^2+32\,ac^2x-2\,c^2)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{6\,(a^3x^2-a^2x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(
2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
- c)/(a*x)) - c)/(a*x - 1)) + 4*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2
*x - 2*c^2)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a
^2*x), 1/6*(33*(a^2*c^2*x^2 - a*c^2*x)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sq
rt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c
*x - c)) + 2*(3*a^3*c^2*x^3 + 133*a^2*c^2*x^2 + 32*a*c^2*x - 2*c^2)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x^2 - a^2*x)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7318 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a\,x}\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.483 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=158

$$\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -9*(c-c/a/x)^(3/2)*arctanh((1+1/a/x)^(1/2))/a/(1-1/a/x)^(3/2)+(21*a+1/x)*(c
-c/a/x)^(3/2)/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)+(a-1/x)^2*(c-c/a/x)^(3/2)
*x/a^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6317, 6314, 100, 151, 65, 214}

$$\frac{x\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((21*a + x^(-1))*(c - c/(a*x))^(3/2))/(a^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(
a*x)]) + ((a - x^(-1))^2*(c - c/(a*x))^(3/2)*x)/(a^2*(1 - 1/(a*x))^(3/2)*Sq
rt[1 + 1/(a*x)]) - (9*(c - c/(a*x))^(3/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(1
 - 1/(a*x))^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx=\frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=-\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^3}{x^2\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(c-\frac{c}{ax}\right)^{3/2}\text{Subst}\left(\int\frac{\left(\frac{9}{2a}-\frac{x}{2a^2}\right)\left(1-\frac{x}{a}\right)}{x\left(1+\frac{x}{a}\right)^{3/2}}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}$$

$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{S}}{2}$$

$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(9\left(c-\frac{c}{ax}\right)^{3/2}\right)\text{S}}{}$$

$$=\frac{\left(21a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{3/2}}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\left(a-\frac{1}{x}\right)^2\left(c-\frac{c}{ax}\right)^{3/2}x}{a^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{9\left(c-\frac{c}{ax}\right)^{3/2}\tanh}{a\left(1-\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 71, normalized size = 0.45

$$\frac{c\sqrt{c-\frac{c}{ax}}\left(2+10ax+a^2x^2+9ax\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)\right)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a*x))^(3/2)/E^(3*ArcCoth[a*x]),x]
[Out] (c*Sqrt[c - c/(a*x)]*(2 + 10*a*x + a^2*x^2 + 9*a*x*Hypergeometric2F1[-1/2,
1, 1/2, 1 + 1/(a*x)]))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 169, normalized size = 1.07

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\, c\left(2a^{\frac{5}{2}}x^{2}\sqrt{x\left(ax+1\right)}+38a^{\frac{3}{2}}x\sqrt{x\left(ax+1\right)}-9\ln\left(\frac{2\sqrt{x\left(ax+1\right)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^{2}x}{2(ax-1)^{2}a^{\frac{3}{2}}\sqrt{x\left(ax+1\right)}}$
risch	$\frac{(a^{2}x^{2}+3ax+2)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{a(ax-1)}+\frac{\left(-\frac{9a\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{16\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right.}{a(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*c*(2*a^
(5/2)*x^2*(x*(a*x+1))^(1/2)+38*a^(3/2)*x*(x*(a*x+1))^(1/2)-9*ln(1/2*(2*(x*(
a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-9*ln(1/2*(2*(x*(a*x+1))^(1/
2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+4*(x*(a*x+1))^(1/2)*a^(1/2))/a^(3/2)/(x*(a
*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 315, normalized size = 1.99

$$\left[\frac{9\left(acx-c\right)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\left(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax\right)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\left(a^{2}cx^{2}+19\,acx+2\,c\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\left(a^{2}x-a\right)},\ \frac{9\left(acx-c\right)\sqrt{-c}\,\arctan\left(\frac{2\left(a^{2}x^{2}+ax\right)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\left(a^{2}cx^{2}+19\,acx+2\,c\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\left(a^{2}x-a\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(9*(a*c*x - c)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a*x + 1))*sq
```

```
rt((a*c*x - c)/(a*x)))/(a^2*x - a), 1/2*(9*(a*c*x - c)*sqrt(-c)*arctan(2*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2
*a^2*c*x^2 - a*c*x - c)) + 2*(a^2*c*x^2 + 19*a*c*x + 2*c)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*x - a)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-\frac{c}{a\,x}\right)^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a*x))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.484 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=140

$$\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^(
1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 91, 79, 65, 214}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int -\right.}{2a\sqrt{1-}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int\right.}{\sqrt{1-}}$$

$$=\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+}\right.}{a\sqrt{1-\frac{1}{ax}}}$$

<u>Mathematica</u> [A]
time = 0.03, size = 67, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])

Maple [A]
time = 0.09, size = 146, normalized size = 1.04

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2a^{\frac{3}{2}}x\sqrt{x\,(ax+1)}-7\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{x\,(ax+1)}\,\sqrt{a}\right.}{2(ax-1)^{2}\sqrt{a}\,\sqrt{x\,(ax+1)}}$
risch	$\frac{x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{8\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{2}c\left(x+\frac{1}{a}\right)}\right)\sqrt{}}{ax-1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^
(3/2)*x*(x*(a*x+1))^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*a*x+18*(x*(a*x+1))^(1/2)*a^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2)))/a^(1/2)/(x*(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [A]
time = 0.38, size = 299, normalized size = 2.14

$$\left[\frac{7\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)},\ \frac{7\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(a^{2}x^{2}+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)

```
/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.485 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=118

$$\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

```
[Out] -5*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(1/2)+5*(c-c/a/
x)^(1/2)/a/c/(1-1/a^2/x^2)^(1/2)+x*(c-c/a/x)^(1/2)/c/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6312, 893, 883, 889, 214}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

```
[Out] (5*Sqrt[c - c/(a*x)])/(a*c*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a*x)]*x)/(
c*Sqrt[1 - 1/(a^2*x^2)]) - (5*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[
c - c/(a*x)]])/(a*Sqrt[c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 883

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*(p + 1)*(e*f + d*g))), x] + Dist[e^2*g*((m - n - 2)/(c*(p +
1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
```

```
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{5\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(5c)\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{5\sqrt{c-\frac{c}{ax}}}{ac\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a\sqrt{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 69, normalized size = 0.58

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax+5\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)\right)}{a\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]),x]
```

[Out] (Sqrt[1 - 1/(a*x)]*(a*x + 5*Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/
(a*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])

Maple [A]
time = 0.10, size = 149, normalized size = 1.26

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2a^{\frac{3}{2}}x\sqrt{x\left(ax+1\right)}-5\ln\left(\frac{2\sqrt{x\left(ax+1\right)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+10\sqrt{x\left(ax+1\right)}\,\sqrt{a}\right.}{2(ax-1)^{2}\sqrt{a}\, c\sqrt{x\left(ax+1\right)}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{5\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2a\sqrt{a^{2}c}}+\frac{4\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{3}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{cax\,(}}{\sqrt{\frac{c(ax-1)}{ax}}\, x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^
(3/2)*x*(x*(a*x+1))^(1/2)-5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*a*x+10*(x*(a*x+1))^(1/2)*a^(1/2)-5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2)))/a^(1/2)/c/(x*(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="maxima")

[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)

Fricas [A]
time = 0.38, size = 303, normalized size = 2.57

$$\left[\frac{5\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+5\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}cx-ac)},\ \frac{5\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(a^{2}x^{2}+5\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}cx-ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="fricas")

```
[Out] [1/4*(5*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c*x - a*c), 1/2*(5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 2*(a^2*x^2 + 5*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)))/(a^2*c*x - a*c)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a*x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(1/2), x)
```

3.486 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}}-\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))/a/c^(3/2)+3*x*(1-1/
a^2/x^2)^(1/2)/c/(c-c/a/x)^(1/2)-2*x*(c-c/a/x)^(1/2)/c^2/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A]
time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6312, 883, 887, 889, 214}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}-\frac{2x\sqrt{c-\frac{c}{ax}}}{c^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

```
[Out] (3*Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a*x)]) - (2*Sqrt[c - c/(a*x)]*x)
/(c^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/S
qrt[c - c/(a*x)]])/(a*c^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 883

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*(p + 1)*(e*f + d*g))), x] + Dist[e^2*g*((m - n - 2)/(c*(p +
```

```
1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^(p + 1), x]
, x] /; FreeQ[{a, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[p, -1] && RationalQ[n]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} + \frac{3\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$= \frac{3\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{ax}}} - \frac{2\sqrt{c-\frac{c}{ax}}\,x}{c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{ac^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 64, normalized size = 0.55

$$\frac{2\left(1-\frac{1}{ax}\right)^{3/2}\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1+\frac{1}{ax}\right)}{a\sqrt{1+\frac{1}{ax}}\,\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(3/2)),x]
```

[Out] (2*(1 - 1/(a*x))^(3/2)*Hypergeometric2F1[-1/2, 2, 1/2, 1 + 1/(a*x)])/(a*Sqr t[1 + 1/(a*x)]*(c - c/(a*x))^(3/2))

Maple [A]
time = 0.10, size = 149, normalized size = 1.27

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2a^{\frac{3}{2}}x\sqrt{x(ax+1)}-3\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+6\sqrt{x(ax+1)}\sqrt{a}\right.}{2(ax-1)^2\sqrt{a}\, c^2\sqrt{x(ax+1)}}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{ac\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(-\frac{3\ln\left(\frac{\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{2a^2\sqrt{a^2c}}+\frac{2\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{a^4c\left(x+\frac{1}{a}\right)}\right)a\sqrt{\frac{ax-1}{ax+1}}\sqrt{ca}}{cx\sqrt{\frac{c(ax-1)}{ax}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^ (3/2)*x*(x*(a*x+1))^(1/2)-3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^ (1/2))*a*x+6*(x*(a*x+1))^(1/2)*a^(1/2)-3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(1/2)/c^2/(x*(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="maxima")

[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)

Fricas [A]
time = 0.36, size = 311, normalized size = 2.66

$$\left[\frac{3(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx-4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(a^2x^2+3ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4(a^2c^2x-ac^2)}, \frac{3(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2(a^2x^2+3ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2(a^2c^2x-ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="fricas")

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*c^2*x - a*c^2), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2
+ a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*
x^2 - a*c*x - c)) + 2*(a^2*x^2 + 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x)))/(a^2*c^2*x - a*c^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(3/2), x)
```

3.487 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\,\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\,\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

```
[Out] -(1-1/a/x)^(5/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(5/2)-1/2*(1-1/a/x)^(
5/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(5/2)*2^(1/2)+2*(1-1/
a/x)^(5/2)/a/(c-c/a/x)^(5/2)/(1+1/a/x)^(1/2)+(1-1/a/x)^(5/2)*x/(c-c/a/x)^(5
/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6317, 6314, 105, 157, 162, 65, 214, 212}

$$\frac{x\left(1-\frac{1}{ax}\right)^{5/2}}{\sqrt{\frac{1}{ax}+1}\,\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{\frac{1}{ax}+1}\,\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (2*(1 - 1/(a*x))^(5/2))/(a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) + ((1 - 1
/(a*x))^(5/2)*x)/(Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(5/2)) - ((1 - 1/(a*x))^(
5/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(Sqrt[2]*a*(c - c/(a*x))^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{5/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{5/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{5/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{\frac{1}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(a\left(1-\frac{1}{ax}\right)^{5/2}\right)\text{Subst}\left(\int \frac{\frac{1}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{\cdots}}\cdots\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\cdots}}\cdots\right)}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}\text{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,x\cdots\right)}{\left(c-\frac{c}{ax}\right)^{5/2}}$$

$$= \frac{2\left(1-\frac{1}{ax}\right)^{5/2}}{a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{\left(1-\frac{1}{ax}\right)^{5/2}x}{\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{\left(1-\frac{1}{ax}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\cdots}{a\left(c-\frac{c}{ax}\right)^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 90, normalized size = 0.45

$$\frac{\sqrt{1-\frac{1}{ax}}\left(ax+{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+\frac{1}{x}}{2a}\right)+{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)\right)}{ac^2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(a*x + Hypergeometric2F1[-1/2, 1, 1/2, (a + x^(-1))/(2*a
)] + Hypergeometric2F1[-1/2, 1, 1/2, 1 + 1/(a*x)]))/(a*c^2*Sqrt[1 + 1/(a*x)
]*Sqrt[c - c/(a*x)])
```

Maple [A]
time = 0.19, size = 264, normalized size = 1.33

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\Bigg(-\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^3\sqrt{a^2c}}-\frac{\sqrt{2}\,\ln\Bigg(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\,(}{x-\frac{1}{a}}}{4a^4\sqrt{c}}}{c^2x\sqrt{\frac{c(ax-}{ax}}}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\Bigg(4\sqrt{x\,(ax+1)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x-a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x+8\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*(x
*(a*x+1))^(1/2)*a^(5/2)*(1/a)^(1/2)*x-a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(
1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x+8*(x*(a*x+1))^(1/2)*a^(3/2)*(1
/a)^(1/2)-2*a^2*(1/a)^(1/2)*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*x-2*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1
/2)-2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))
*a^(1/2))/a^(3/2)/c^3/(1/a)^(1/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

Fricas [A]
time = 0.47, size = 524, normalized size = 2.63

$$\left[\frac{\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+2\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\,(a^2x^2+2\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{8\,(a^2c^3x-ac^3)},\ \frac{\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\,a^2cx^2-2\,acx-c}\right)+2\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+4\,(a^2x^2+2\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^2c^3x-ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c*x
 - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1)
)*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 2*(a*x
- 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) +
 8*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*c^3*x - a*c^3), 1/4*(sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2
 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c
*x^2 - 2*a*c*x - c)) + 2*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) + 4*(a^2*x^2 + 2*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^2*c^3*x - a*c^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(5/2), x)
```

3.488 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx$

Optimal. Leaf size=267

$$\frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{ta}}{a\left(\right.}$$

```
[Out] (1-1/a/x)^(7/2)*arctanh((1+1/a/x)^(1/2))/a/(c-c/a/x)^(7/2)-11/8*(1-1/a/x)^(
7/2)*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))/a/(c-c/a/x)^(7/2)*2^(1/2)+7/4*(1-
1/a/x)^(7/2)/a/(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)-3/2*(1-1/a/x)^(7/2)/(a-1/x)/
(c-c/a/x)^(7/2)/(1+1/a/x)^(1/2)+a*(1-1/a/x)^(7/2)*x/(a-1/x)/(c-c/a/x)^(7/2)
/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6317, 6314, 105, 156, 157, 162, 65, 214, 212}

$$\frac{ax\left(1-\frac{1}{ax}\right)^{7/2}}{\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{\frac{1}{ax}+1}\left(c-\frac{c}{ax}\right)^{7/2}}+\frac{\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\left(c-\frac{c}{ax}\right)^{7/2}}-\frac{11\left(1-\frac{1}{ax}\right)^{7/2}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{4\sqrt{2}\,a\left(c-\frac{c}{ax}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (7*(1 - 1/(a*x))^(7/2))/(4*a*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) - (3*(1
 - 1/(a*x))^(7/2))/(2*(a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7/2)) +
 (a*(1 - 1/(a*x))^(7/2)*x)/((a - x^(-1))*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^(7
/2)) + ((1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*(c - c/(a*x))^(7
/2)) - (11*(1 - 1/(a*x))^(7/2)*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(4*Sqrt[
2]*a*(c - c/(a*x))^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{7/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{7/2}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{7/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{\left(1-\frac{1}{ax}\right)^{7/2}\text{Subst}\left(\int \frac{-\frac{1}{2a}-\frac{5x}{2a^2}}{x\left(1-\frac{x}{a}\right)^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{7/2}}$$

$$= -\frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{\left(a\left(1-\frac{1}{ax}\right)^{7/2}\right)}{}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\right.}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\right.}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\right.}$$

$$= \frac{7\left(1-\frac{1}{ax}\right)^{7/2}}{4a\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} - \frac{3\left(1-\frac{1}{ax}\right)^{7/2}}{2\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^{7/2}} + \frac{a\left(1-\frac{1}{ax}\right)^{7/2}x}{\left(a-\frac{1}{x}\right)\sqrt{1+\frac{1}{ax}}\left(c-\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.05, size = 121, normalized size = 0.45

$$\frac{\sqrt{1-\frac{1}{ax}}\left(2ax(-3+2ax)+11(-1+ax)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{a+\frac{1}{x}}{2a}\right)+(4-4ax)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1+\frac{1}{ax}\right)\right)}{4ac^3\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a*x))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a*x)]*(2*a*x*(-3 + 2*a*x) + 11*(-1 + a*x)*Hypergeometric2F1[-1
/2, 1, 1/2, (a + x^(-1))/(2*a)] + (4 - 4*a*x)*Hypergeometric2F1[-1/2, 1, 1/
2, 1 + 1/(a*x)]))/(4*a*c^3*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*(-1 + a*x))
```

Maple [A]
time = 0.19, size = 290, normalized size = 1.09

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16\sqrt{x\,(ax+1)}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2-11a^{\frac{5}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x\right.}{}$
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3\sqrt{\frac{c(ax-1)}{ax}}}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{2a^4\sqrt{a^2c}}-\frac{11\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3}}{x-\frac{1}{a}}\right.}{16a^5\sqrt{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*
(x*(a*x+1))^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2-11*a^(5/2)*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+4*(x*(a*x+1))^(1/2)*a
^(5/2)*(1/a)^(1/2)*x+8*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)
)*a^3*(1/a)^(1/2)*x^2-28*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-8*ln(1/2*(2*
(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2)+11*2^(1/2)*ln((2*
2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/a^(3/2)/
c^4/(1/a)^(1/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Fricas [A]
time = 0.40, size = 594, normalized size = 2.22

$$\left[\frac{11\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+8\,(a^2x^2-2\,ax+1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+8\,(4\,a^3x^3+a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{32\,(a^3c^4x^2-2\,a^2c^4x+ac^4)},\ \frac{11\,\sqrt{2}\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\arctan\left(\frac{2\,\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\,a^2cx^2-2\,acx-c}\right)-8\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+4\,(4\,a^3x^3+a^2x^2-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{16\,(a^3c^4x^2-2\,a^2c^4x+ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/32*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 8*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2
*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x -
 c)/(a*x)) - c)/(a*x - 1)) + 8*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4), 1
/16*(11*sqrt(2)*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) - 8*(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*
x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2
- a*c*x - c)) + 4*(4*a^3*x^3 + a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a/x)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a/x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a\,x}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a*x))^(7/2), x)
```

3.489 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{c-\frac{c}{ax}}\,x^{1+m}\,{}_2F_1\left(-\frac{1}{2},-1-m;-m;-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] x^(1+m)*hypergeom([-1/2, -1-m],[-m],-1/a/x)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x)
^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6317, 6316, 66}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(-\frac{1}{2},-m-1;-m;-\frac{1}{ax}\right)}{(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) && !IntegerQ[m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^m\,dx}{\sqrt{1-\frac{1}{ax}}}$$
$$=-\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int x^{-2-m}\sqrt{1+\frac{x}{a}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$=\frac{\sqrt{c-\frac{c}{ax}}\,x^{1+m}\,{}_2F_1\left(-\frac{1}{2},-1-m;-m;-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.00

$$\frac{\sqrt{c-\frac{c}{ax}}\,x^{1+m}\,{}_2F_1\left(-\frac{1}{2},-1-m;-m;-\frac{1}{ax}\right)}{(1+m)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^m,x]
[Out] (Sqrt[c - c/(a*x)]*x^(1 + m)*Hypergeometric2F1[-1/2, -1 - m, -m, -(1/(a*x))
])/((1 + m)*Sqrt[1 - 1/(a*x)])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{c-\frac{c}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((a*x + 1)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a
*x - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m \sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x^m*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.490 $\int e^{\coth^{-1}(ax)} \sqrt{c-\frac{c}{ax}}\, x^2\, dx$

Optimal. Leaf size=164

$$-\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] 1/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3-1/8*c*
x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)+1/12*c*x^2*(1-1/a^2/x^2)^(1/2)/a/
(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6313, 877, 887, 889, 214}

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}-\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] -1/8*(c*Sqrt[1 - 1/(a^2*x^2)]*x)/(a^2*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a
^2*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(3*S
qrt[c - c/(a*x)]) + (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c
 - c/(a*x)]])/(8*a^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
```

```
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^4\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \cdots\right.}{16a^3}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}} - \frac{c^2\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{c}\,\tanh^{-1}\left(\cdots\right.}{8a^3}$$

<u>Mathematica</u> [A]

time = 0.27, size = 147, normalized size = 0.90

$$\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(-3+2ax+8a^2x^2)}{-1+ax}-3\sqrt{c}\,\log(1-ax)+3\sqrt{c}\,\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3 + 2*a*x + 8*a^2*x^2
))/(-1 + a*x) - 3*Sqrt[c]*Log[1 - a*x] + 3*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1
 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a^3)
```

Maple [A]
time = 0.05, size = 121, normalized size = 0.74

method	result
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}+4a^{\frac{3}{2}}x\sqrt{x(ax+1)}-6\sqrt{x(ax+1)}\sqrt{a}+3\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}}{2\sqrt{a}}\right.\right.}{48\sqrt{\frac{ax-1}{ax+1}}\,a^{\frac{5}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(8a^2x^2+2ax-3)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{cax(ax+1)}}{16a^2\sqrt{a^2c}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(5/2)*(16*a^(5/2)*x^
2*(x*(a*x+1))^(1/2)+4*a^(3/2)*x*(x*(a*x+1))^(1/2)-6*(x*(a*x+1))^(1/2)*a^(1/
2)+3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.42, size = 337, normalized size = 2.05

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{96\,(a^4x-a^3)},\ -\frac{3\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(8\,a^4x^4+10\,a^3x^3-a^2x^2-3\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{48\,(a^4x-a^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/96*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a
^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c
)/(a*x - 1)) + 4*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x^2 - 3*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), -1/48*(3*(a*x - 1)*sqrt(
-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(8*a^4*x^4 + 10*a^3*x^3 - a^2*x
^2 - 3*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.491 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=124

$$\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] -1/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2+1/4*c
*x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c-c
/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6313, 877, 887, 889, 214}

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}+\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^
2)]*x^2)/(2*Sqrt[c - c/(a*x)]) - (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*
x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
```

```
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=-\left(c\text{Subst}\left(\int\frac{\sqrt{1-\frac{x^2}{a^2}}}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{4a}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{c^2\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^4}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Mathematica [A]
time = 0.16, size = 148, normalized size = 1.19

$$\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(1+2ax)+\sqrt{c}\,(-1+ax)\log(1-ax)+\sqrt{c}\,(1-ax)\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{8a^2(-1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)]*x,x]
[Out] (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(1 + 2*a*x) + Sqrt[c]*(-
1 + a*x)*Log[1 - a*x] + Sqrt[c]*(1 - a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2
```

```
*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2*(-1 + a*x)
)
```

Maple [A]
time = 0.04, size = 102, normalized size = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\, x \left(-4 a^{\frac{3}{2}} x \sqrt{x (ax+1)}-2 \sqrt{x (ax+1)}\, \sqrt{a}+\ln \left(\frac{2 \sqrt{x (ax+1)}\, \sqrt{a}+2 a x+1}{2 \sqrt{a}}\right)\right)}{8 \sqrt{\frac{ax-1}{ax+1}}\, a^{\frac{3}{2}} \sqrt{x (ax+1)}}$	102
risch	$\frac{(2ax+1) x \sqrt{\frac{c(ax-1)}{ax}}}{4a \sqrt{\frac{ax-1}{ax+1}}}-\frac{\ln \left(\frac{\frac{1}{2} a c+c\, a^{2} x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c\, x^{2}+a c x}\right) \sqrt{\frac{c(ax-1)}{ax}}\, \sqrt{c a x (ax+1)}}{8 a \sqrt{a^{2} c}\, \sqrt{\frac{ax-1}{ax+1}}\, (ax+1)}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(-4*a^(3/2)*x*
(x*(a*x+1))^(1/2)-2*(x*(a*x+1))^(1/2)*a^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a
^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.41, size = 317, normalized size = 2.56

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{16\,(a^3x-a^2)},\ \frac{(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{8\,(a^3x-a^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^2
*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/
(a*x - 1)) + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt
((a*c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x
^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2
*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a/x)**(1/2),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.492 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=78

$$\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6312, 877, 889, 214}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] + (Sqrt[c]*ArcTanh[(Sqrt[c]*S
qrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 877

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(f + g*x)^(n + 1)*((a + c*x^2)^p/(g*(
n + 1))), x] + Dist[c*(m/(e*g*(n + 1))), Int[(d + e*x)^(m + 1)*(f + g*x)^(n
 + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*
f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && G
tQ[p, 0] && LtQ[n, -1] &&  !(IntegerQ[n + p] && LeQ[n + p + 2, 0])
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} - \frac{c^2\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}\\
&= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}} + \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}
\end{aligned}$$

Mathematica [A]

time = 0.03, size = 66, normalized size = 0.85

$$\frac{\sqrt{c-\frac{c}{ax}}\left(1+ax+\sqrt{1+\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(1 + a*x + Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 87, normalized size = 1.12

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x\,(ax+1)}\,\sqrt{a}+\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{x\,(ax+1)}\,\sqrt{a}}$	87
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1)}}{2\sqrt{a^2c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^(1/2)*a^(1/2)+ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(x*(a*x+1))^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

time = 0.39, size = 295, normalized size = 3.78

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)},-\frac{(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x)))/(a^2*x - a), -1/2*((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-
c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x -
 c)) - 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))
/(a^2*x - a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.493 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=76

$$-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)-2*c*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6313, 879, 889, 214}

$$2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)-\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
 + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2}\\
&= -\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 132, normalized size = 1.74

$$\frac{-2a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,x+\sqrt{c}\,(1-ax)\log(1-ax)+\sqrt{c}\,(-1+ax)\log\left(2a^2\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{-1+ax}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
 a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
 c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Maple [A]
time = 0.04, size = 88, normalized size = 1.16

method	result	size
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax-2\sqrt{x(ax+1)}\sqrt{a}\right)}{\sqrt{\frac{ax-1}{ax+1}}\sqrt{x(ax+1)}\sqrt{a}}$	88
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{a\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\sqrt{cax(ax+1)}}{\sqrt{a^{2}c}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*(x*(a*x+1))^(1/2
)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-2*(x*(a*x+1))^(1/2)*a^(1/2))/(x*(a*x+1))^(1
/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(64) = 128.
time = 0.43, size = 275, normalized size = 3.62

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)},-\frac{(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.494 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

[Out] -2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)

Rubi [A]

time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6313, 663}

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]

[Out] (-2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2))

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)\\ &= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 1.22

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,(1+ax)}{-3+3ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + a*x))/(-3 + 3*a*x)
```

Maple [A]
time = 0.03, size = 41, normalized size = 1.11

method	result	size
gosper	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
default	$-\frac{2(ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}}$	41
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(a^2x^2+2ax+1)}{3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/x*(a*x+1)/((a*x-1)/(a*x+1))^(1/2)*(c*(a*x-1)/a/x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 58, normalized size = 1.57

$$-\frac{2\left(a^{2}x^{2}+2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\left(ax^{2}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] -2/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
)/(a*x^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^{2}\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 1.37, size = 47, normalized size = 1.27

$$-\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^{2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{3\,x\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] -(2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*((a*x - 1)/(a*x + 1))^(1/2))/(3*x*(a*x
- 1))
```

3.495 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=77

$$-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -2/15*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)+2/5*a^2*c*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6313, 809, 663}

$$\frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}-\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (-2*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2))/(15*(c - c/(a*x))^(3/2)) + (2*a^2*c*(1
 - 1/(a^2*x^2))^(3/2))/(5*Sqrt[c - c/(a*x)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
```

```
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\left(c\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}} - \frac{1}{5}(ac)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{15\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{2a^2c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{5\sqrt{c-\frac{c}{ax}}}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.75

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,(-3-ax+2a^2x^2)}{15x(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^3,x]
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-3 - a*x + 2*a^2*x^2))/(15*x*
(-1 + a*x))
```

Maple [A]
time = 0.03, size = 47, normalized size = 0.61

method	result	size
gosper	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}}$	47

default	$\frac{2(ax+1)(2ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2\sqrt{\frac{ax-1}{ax+1}}}$	47
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(2a^3x^3+a^2x^2-4ax-3)}{15\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(a*x+1)*(2*a*x-3)*(c*(a*x-1)/a/x)^(1/2)/x^2/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.88

$$\frac{2\left(2\,a^3x^3+a^2x^2-4\,ax-3\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{15\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] 2/15*(2*a^3*x^3 + a^2*x^2 - 4*a*x - 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*
x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 1.39, size = 53, normalized size = 0.69

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,{\left(a\,x+1\right)}^2\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{15\,x^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*(a*x + 1)^2*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))
/(15*x^2*(a*x - 1))
```

3.496 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=117

$$\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

```
[Out] 8/105*a^3*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/7*a*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)/x^2-8/35*a^3*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6313, 885, 809, 663}

$$-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7x^2\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (8*a^3*c^2*(1 - 1/(a^2*x^2))^(3/2))/(105*(c - c/(a*x))^(3/2)) - (8*a^3*c*(1
 - 1/(a^2*x^2))^(3/2))/(35*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^2))^
(3/2))/(7*(c - c/(a*x))^(3/2)*x^2)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 885

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*((a + c*x^2)^(
```

```
p + 1)/(c*(m - n - 1))), x] - Dist[n*((e*f + d*g)/(e*(m - n - 1))), Int[(d
+ e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
&& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Int
egerQ[n])
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\left(c\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2} + \frac{1}{7}(4ac)\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2} + \frac{1}{35}\left(4a^2c\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x\right.$$

$$= \frac{8a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{8a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{35\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

Mathematica [A]
time = 0.06, size = 66, normalized size = 0.56

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(15+3ax-4a^2x^2+8a^3x^3\right)}{105x^2(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(15 + 3*a*x - 4*a^2*x^2 + 8*a
^3*x^3))/(105*x^2*(-1 + a*x))
```

Maple [A]
time = 0.03, size = 55, normalized size = 0.47

method	result	size
gosper	$-\frac{2(ax+1)(8a^2x^2-12ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{\frac{ax-1}{ax+1}}}$	55
default	$-\frac{2(ax+1)(8a^2x^2-12ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3\sqrt{\frac{ax-1}{ax+1}}}$	55
risch	$-\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(8a^4x^4+4a^3x^3-a^2x^2+18ax+15)}{105\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(a*x+1)*(8*a^2*x^2-12*a*x+15)*(c*(a*x-1)/a/x)^(1/2)/x^3/((a*x-1)/(a*
x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.34, size = 77, normalized size = 0.66

$$-\frac{2\,(8\,a^4x^4+4\,a^3x^3-a^2x^2+18\,ax+15)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{105\,(ax^4-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] -2/105*(8*a^4*x^4 + 4*a^3*x^3 - a^2*x^2 + 18*a*x + 15)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [**B**]
time = 1.41, size = 100, normalized size = 0.85

$$-\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(8\,a^3\,x^3+12\,a^2\,x^2+11\,a\,x+29\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3}-\frac{88\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*(11*a*x + 12*a^2*x^2 + 8*a^3*x^3 + 29)*((c
*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (88*((a*x - 1)/(a*x + 1))^(1/2)*((c*(
a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.497 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=159

$$-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3}+\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2}$$

```
[Out] -16/315*a^4*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-2/9*a*c^2*(1-1/a^2/x^2)
^(3/2)/(c-c/a/x)^(3/2)/x^3+4/21*a^2*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)
/x^2+16/105*a^4*c*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6313, 885, 809, 663}

$$\frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21x^2\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9x^3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (-16*a^4*c^2*(1 - 1/(a^2*x^2))^(3/2))/(315*(c - c/(a*x))^(3/2)) + (16*a^4*c
*(1 - 1/(a^2*x^2))^(3/2))/(105*Sqrt[c - c/(a*x)]) - (2*a*c^2*(1 - 1/(a^2*x^
2))^(3/2))/(9*(c - c/(a*x))^(3/2)*x^3) + (4*a^2*c^2*(1 - 1/(a^2*x^2))^(3/2)
)/(21*(c - c/(a*x))^(3/2)*x^2)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 885

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*((a + c*x^2)^(
p + 1)/(c*(m - n - 1))), x] - Dist[n*((e*f + d*g)/(e*(m - n - 1))), Int[(d
+ e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
&& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Int
egerQ[n])
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\left(c\text{Subst}\left(\int \frac{x^3\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{1}{3}(2ac)\text{Subst}\left(\int \frac{x^2\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{21}\left(8a^2c\right)\text{Subst}\left(\int \frac{x\sqrt{1-\frac{x^2}{a^2}}}{\sqrt{c-\frac{cx}{a}}}\,dx\right.$$

$$= \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{21\left(c-\frac{c}{ax}\right)^{3/2}x^2} - \frac{1}{105}\left(8a^3c\right)\text{Sub}$$

$$= -\frac{16a^4c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{315\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{16a^4c\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{105\sqrt{c-\frac{c}{ax}}} - \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{9\left(c-\frac{c}{ax}\right)^{3/2}x^3} + \frac{4a^2c^2\left(1-\right.}{21\left(c-\frac{c}{ax}\right.}$$

Mathematica [A]

time = 0.07, size = 74, normalized size = 0.47

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,(-35-5ax+6a^2x^2-8a^3x^3+16a^4x^4)}{315x^3(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-35 - 5*a*x + 6*a^2*x^2 - 8*a
^3*x^3 + 16*a^4*x^4))/(315*x^3*(-1 + a*x))
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.40

method	result	size
gosper	$\frac{2(ax+1)(16a^3x^3-24a^2x^2+30ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
default	$\frac{2(ax+1)(16a^3x^3-24a^2x^2+30ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}}$	63
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(16a^5x^5+8a^4x^4-2a^3x^3+a^2x^2-40ax-35)}{315\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)x^4}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(a*x+1)*(16*a^3*x^3-24*a^2*x^2+30*a*x-35)*(c*(a*x-1)/a/x)^(1/2)/x^4/(
(a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.38, size = 84, normalized size = 0.53

$$\frac{2\left(16\,a^{5}x^{5}+8\,a^{4}x^{4}-2\,a^{3}x^{3}+a^{2}x^{2}-40\,ax-35\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{315\left(ax^{5}-x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] 2/315*(16*a^5*x^5 + 8*a^4*x^4 - 2*a^3*x^3 + a^2*x^2 - 40*a*x - 35)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 1.44, size = 108, normalized size = 0.68

$$\frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}\,\left(16\,a^4\,x^4+24\,a^3\,x^3+22\,a^2\,x^2+23\,a\,x-17\right)}{315\,x^4}-\frac{104\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{315\,x^4\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(23*a*x + 22*a^2
*x^2 + 24*a^3*x^3 + 16*a^4*x^4 - 17))/(315*x^4) - (104*((a*x - 1)/(a*x + 1)
)^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(315*x^4*(a*x - 1))
```

3.498 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=130

$$\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{75\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}$$

```
[Out] 75/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4+75/64*x*(c-c/a/x)^(1/2)/
a^3+25/32*x^2*(c-c/a/x)^(1/2)/a^2+5/8*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/a/
x)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 79, 44, 65, 214}

$$\frac{75\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}+\frac{75x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{25x^2\sqrt{c-\frac{c}{ax}}}{32a^2}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}+\frac{5x^3\sqrt{c-\frac{c}{ax}}}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (75*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (25*Sqrt[c - c/(a*x)]*x^2)/(32*a^2) + (
5*Sqrt[c - c/(a*x)]*x^3)/(8*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (75*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$$

$$=-\int\frac{\sqrt{c-\frac{c}{ax}}\,x^3(1+ax)}{1-ax}\,dx$$

$$=\frac{c\int\frac{x^2(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=\frac{c\int\frac{\left(a+\frac{1}{x}\right)x^3}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=-\frac{c\text{Subst}\left(\int\frac{a+x}{x^5\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(15c)\text{Subst}\left(\int\frac{1}{x^4\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a}$$

$$=\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(25c)\text{Subst}\left(\int\frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{16a^2}$$

$$=\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(75c)\text{Subst}\left(\int\frac{1}{x^2\sqrt{c-\cdots}}\right.}{64a^3}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{(75c)\cdots}{\cdots}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{75\text{Sub}\cdots}{\cdots}$$

$$=\frac{75\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{25\sqrt{c-\frac{c}{ax}}\,x^2}{32a^2}+\frac{5\sqrt{c-\frac{c}{ax}}\,x^3}{8a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{75\sqrt{c}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 50, normalized size = 0.38

$$\frac{\sqrt{c-\frac{c}{ax}}\left(a^4x^4+15\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\frac{1}{ax}\right)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a^4*x^4 + 15*Hypergeometric2F1[1/2, 4, 3/2, 1 - 1/(a*x)
]))/(4*a^4)
```

Maple [A]
time = 0.14, size = 172, normalized size = 1.32

method	result
risch	$\frac{(16a^3x^3+40a^2x^2+50ax+75)x\sqrt{\frac{c(ax-1)}{ax}}}{64a^3}+\frac{75\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{128a^3\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(32x(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}+112(a\,x^2-x)^{\frac{3}{2}}a^{\frac{5}{2}}+212\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-106\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+256a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\right.}{128\sqrt{(ax-1)\,x}\,a^{\frac{9}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(c*(a*x-1)/a/x)^(1/2)*x*(32*x*(a*x^2-x)^(3/2)*a^(7/2)+112*(a*x^2-x)^(
3/2)*a^(5/2)+212*(a*x^2-x)^(1/2)*a^(5/2)*x-106*(a*x^2-x)^(1/2)*a^(3/2)+256*
a^(3/2)*((a*x-1)*x)^(1/2)+128*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1
)/a^(1/2))-53*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-
1)*x)^(1/2)/a^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^3/(a*x - 1), x)
```

Fricas [A]
time = 0.36, size = 179, normalized size = 1.38

$$\left[\frac{2\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax\right)\sqrt{\frac{acx-c}{ax}}+75\,\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{128\,a^4},\ \frac{\left(16\,a^4x^4+40\,a^3x^3+50\,a^2x^2+75\,ax\right)\sqrt{\frac{acx-c}{ax}}-75\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{64\,a^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(2*(16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x - c)/
(a*x)) + 75*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) +
c))/a^4, 1/64*((16*a^4*x^4 + 40*a^3*x^3 + 50*a^2*x^2 + 75*a*x)*sqrt((a*c*x
- c)/(a*x)) - 75*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.44, size = 142, normalized size = 1.09

$$\frac{1}{64}\,\sqrt{a^2cx^2-acx}\left(2\left(4\,x\left(\frac{2\,x|a|}{a^2\mathrm{sgn}\left(x\right)}+\frac{5\,|a|}{a^3\mathrm{sgn}\left(x\right)}\right)+\frac{25\,|a|}{a^4\mathrm{sgn}\left(x\right)}\right)x+\frac{75\,|a|}{a^5\mathrm{sgn}\left(x\right)}\right)+\frac{75\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{128\,a^4}-\frac{75\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{128\,a^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(a^2*c*x^2 - a*c*x)*(2*(4*x*(2*x*abs(a)/(a^2*sgn(x)) + 5*abs(a)/(a
^3*sgn(x))) + 25*abs(a)/(a^4*sgn(x)))*x + 75*abs(a)/(a^5*sgn(x))) + 75/128*
sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^4 - 75/128*sqrt(c)*log(abs(-2*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^4*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^3*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.499 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=105

$$\frac{11\sqrt{c-\frac{c}{ax}}\,x}{8a^2}+\frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{11\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}$$

```
[Out] 11/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+11/8*x*(c-c/a/x)^(1/2)/a^
2+11/12*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 79, 44, 65, 214}

$$\frac{11\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{11x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}+\frac{11x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (11*Sqrt[c - c/(a*x)]*x)/(8*a^2) + (11*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 + (11*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{x(1+ax)}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= \frac{c\int \frac{\left(a+\frac{1}{x}\right)x^2}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^4\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 - \frac{(11c)\text{Subst}\left(\int \frac{1}{x^3\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{6a}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 - \frac{(11c)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}\,x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 - \frac{(11c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-}}\right.}{16a^3}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}\,x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 + \frac{11\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,\right.}{8a^2}$$

$$= \frac{11\sqrt{c-\frac{c}{ax}}\,x}{8a^2} + \frac{11\sqrt{c-\frac{c}{ax}}\,x^2}{12a} + \frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3 + \frac{11\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.}{8a^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 50, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(a^3x^3+11\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{1}{ax}\right)\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a^3*x^3 + 11*Hypergeometric2F1[1/2, 3, 3/2, 1 - 1/(a*x)
]))/(3*a^3)
```

Maple [A]
time = 0.14, size = 155, normalized size = 1.48

method	result
risch	$\frac{(8a^2x^2+22ax+33)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2}+\frac{11\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{16a^2\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16(a\,x^2-x)^{\frac{3}{2}}a^{\frac{5}{2}}+60\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x-30\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+96a^{\frac{3}{2}}\sqrt{(ax-1)\,x}+48a\ln\left(\frac{2\sqrt{(ax-1)}}{2\sqrt{}}\right.\right.}{48\sqrt{(ax-1)\,x}\,a^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(5/2)+60*(a*x^2-x)^(1/2)
*a^(5/2)*x-30*(a*x^2-x)^(1/2)*a^(3/2)+96*a^(3/2)*((a*x-1)*x)^(1/2)+48*a*ln(
1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-15*ln(1/2*(2*(a*x^2-x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/((a*x-1)*x)^(1/2)/a^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x^2/(a*x - 1), x)
```

Fricas [A]
time = 0.37, size = 163, normalized size = 1.55

$$\left[\frac{2\,(8\,a^3x^3+22\,a^2x^2+33\,ax)\sqrt{\frac{acx-c}{ax}}+33\,\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{48\,a^3},\ \frac{(8\,a^3x^3+22\,a^2x^2+33\,ax)\sqrt{\frac{acx-c}{ax}}-33\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{24\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) + 33*sqr
t(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^3, 1/24*(
(8*a^3*x^3 + 22*a^2*x^2 + 33*a*x)*sqrt((a*c*x - c)/(a*x)) - 33*sqrt(-c)*arc
tan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.44, size = 127, normalized size = 1.21

$$\frac{1}{24}\sqrt{a^2cx^2-acx}\left(2\,x\left(\frac{4\,x|a|}{a^2\mathrm{sgn}\,(x)}+\frac{11\,|a|}{a^3\mathrm{sgn}\,(x)}\right)+\frac{33\,|a|}{a^4\mathrm{sgn}\,(x)}\right)+\frac{11\,\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{16\,a^3}-\frac{11\,\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{16\,a^3\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(a^2*c*x^2 - a*c*x)*(2*x*(4*x*abs(a)/(a^2*sgn(x)) + 11*abs(a)/(a^3
*sgn(x))) + 33*abs(a)/(a^4*sgn(x))) + 11/16*sqrt(c)*log(abs(a)*abs(c))*sgn(
x)/a^3 - 11/16*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))
*sqrt(c)*abs(a) + a*c))/(a^3*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.500 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=80

$$\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

```
[Out] 7/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2+7/4*x*(c-c/a/x)^(1/2)/a+1/
2*x^2*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {6302, 6268, 25, 445, 457, 79, 44, 65, 214}

$$\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}+\frac{7x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (7*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (7*Sqrt[c]*ArcT
anh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$$

$$=-\int \frac{\sqrt{c-\frac{c}{ax}}\,x(1+ax)}{1-ax}\,dx$$

$$=\frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=\frac{c\int \frac{\left(a+\frac{1}{x}\right)x}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$=-\frac{c\text{Subst}\left(\int \frac{a+x}{x^3\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{(7c)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{4a}$$

$$=\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2-\frac{(7c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{8a^2}$$

$$=\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{7\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,x,\sqrt{c-\frac{c}{ax}}\right)}{4a}$$

$$=\frac{7\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}$$

Mathematica [A]

time = 0.05, size = 77, normalized size = 0.96

$$\frac{\sqrt{c-\frac{c}{ax}}\left(a\sqrt{1-\frac{1}{ax}}\,x(7+2ax)+7\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\right)\right)}{4a^2\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 - 1/(a*x)]*x*(7 + 2*a*x) + 7*ArcTanh[Sqrt[1 -
1/(a*x)]]))/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(64) = 128$.
time = 0.13, size = 139, normalized size = 1.74

method	result
risch	$\frac{(2ax+7)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{7\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{8a\sqrt{a^2c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x+16a^{\frac{3}{2}}\sqrt{(ax-1)\,x}-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}+8a\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-\text{l}\right.}{8\sqrt{(ax-1)\,x}\,a^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*(a*x-1)/a/x)^(1/2)*x*(4*(a*x^2-x)^(1/2)*a^(5/2)*x+16*a^(3/2)*((a*x-1
)*x)^(1/2)-2*(a*x^2-x)^(1/2)*a^(3/2)+8*a*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2
)+2*a*x-1)/a^(1/2))-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a)/
((a*x-1)*x)^(1/2)/a^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))*x/(a*x - 1), x)
```

Fricas [A]
time = 0.45, size = 147, normalized size = 1.84

$$\left[\frac{2\left(2a^2x^2+7ax\right)\sqrt{\frac{acx-c}{ax}}+7\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8a^2}, \frac{\left(2a^2x^2+7ax\right)\sqrt{\frac{acx-c}{ax}}-7\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{4a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(2*a^2*x^2 + 7*a*x)*sqrt((a*c*x - c)/(a*x)) + 7*sqrt(c)*log(-2*a*c*
x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2, 1/4*((2*a^2*x^2 + 7*a*
x)*sqrt((a*c*x - c)/(a*x)) - 7*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a
*x))/c))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.42, size = 112, normalized size = 1.40

$$\frac{1}{4}\sqrt{a^2cx^2-acx}\left(\frac{2x|a|}{a^2\mathrm{sgn}(x)}+\frac{7|a|}{a^3\mathrm{sgn}(x)}\right)+\frac{7\sqrt{c}\log\left(|a||c|\right)\mathrm{sgn}(x)}{8a^2}-\frac{7\sqrt{c}\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{8a^2\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a^2*c*x^2 - a*c*x)*(2*x*abs(a)/(a^2*sgn(x)) + 7*abs(a)/(a^3*sgn(x)
)) + 7/8*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a^2 - 7/8*sqrt(c)*log(abs(-2*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a^2*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.501 $\int e^{2 \coth^{-1}(ax)} \sqrt{c - \frac{c}{ax}}\, dx$

Optimal. Leaf size=50

$$\sqrt{c - \frac{c}{ax}}\, x + \frac{3\sqrt{c}\, \tanh^{-1}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

```
[Out] 3*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6268, 25, 528, 382, 79, 65, 214}

$$x\sqrt{c - \frac{c}{ax}} + \frac{3\sqrt{c}\, \tanh^{-1}\left(\frac{\sqrt{c - \frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{1-ax}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x^2\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \sqrt{c-\frac{c}{ax}}\,x - \frac{(3c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= \sqrt{c-\frac{c}{ax}}\,x + 3\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.00

$$\sqrt{c-\frac{c}{ax}}\,x + \frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] Sqrt[c - c/(a*x)]*x + (3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(42) = 84$.
time = 0.13, size = 120, normalized size = 2.40

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{3\ln\left(\frac{-\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{2\sqrt{a^{2}c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{a\,x^{2}-x}\,\sqrt{a}-4\sqrt{(ax-1)\,x}\,\sqrt{a}-\ln\left(\frac{2\sqrt{a\,x^{2}-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)-2\ln\left(\frac{2\sqrt{(ax-1)\,}}{2\sqrt{}}\right.\right.}{2\sqrt{(ax-1)\,x}\,\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*(a*x-1)/a/x)^(1/2)*x*(2*(a*x^2-x)^(1/2)*a^(1/2)-4*((a*x-1)*x)^(1/2)
*a^(1/2)-ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))-2*ln(1/2*(2*((
a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)))/((a*x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/(a*x - 1), x)
```

Fricas [A]
time = 0.38, size = 124, normalized size = 2.48

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+3\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-3\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 3*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x - c)/(a*x)) - 3*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2),x)

[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.
time = 0.44, size = 96, normalized size = 1.92

$$\frac{3\sqrt{c}\,\log\left(|a||c|\right)\mathrm{sgn}(x)}{2\,a}-\frac{3\sqrt{c}\,\log\left(\left|-2\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)\sqrt{c}\,|a|+ac\right|\right)}{2\,a\mathrm{sgn}\left(x\right)}+\frac{\sqrt{a^2cx^2-acx}\,|a|}{a^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2),x, algorithm="giac")

[Out] 3/2*sqrt(c)*log(abs(a)*abs(c))*sgn(x)/a - 3/2*sqrt(c)*log(abs(-2*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*sqrt(c)*abs(a) + a*c))/(a*sgn(x)) + sqrt(a^2*c*x^2 - a*c*x)*abs(a)/(a^2*sgn(x))

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1),x)

[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(a*x - 1), x)

3.502 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=47

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

```
[Out] 2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)+2*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6268, 25, 528, 457, 81, 65, 214}

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^2}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{a+x}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= 2\sqrt{c-\frac{c}{ax}} - c\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= 2\sqrt{c-\frac{c}{ax}} + (2a)\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(39) = 78$.
time = 0.13, size = 99, normalized size = 2.11

method	result	size
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{a\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{c\,(ax-1)\,ax}}{\sqrt{a^2c}\,(ax-1)}$	98
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-2a^{\frac{3}{2}}\sqrt{(ax-1)\,x}\,x^2+2(a\,x^2-x)^{\frac{3}{2}}\sqrt{a}-\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\,x^2\right)}{x\sqrt{(ax-1)\,x}\,\sqrt{a}}$	99

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] -(c*(a*x-1)/a/x)^(1/2)/x*(-2*a^(3/2)*((a*x-1)*x)^(1/2)*x^2+2*(a*x^2-x)^(3/2)*a^(1/2)-ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*x^2)/((a*x-1)*x)^(1/2)/a^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")

[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x), x)

Fricas [A]
time = 0.37, size = 111, normalized size = 2.36

$$\left[\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\sqrt{\frac{acx-c}{ax}},-2\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+2\sqrt{\frac{acx-c}{ax}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas")

[Out] [sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c) + 2*sqrt((a*c*x - c)/(a*x)), -2*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + 2*sqrt((a*c*x - c)/(a*x))]

Sympy [**A**]
time = 6.09, size = 39, normalized size = 0.83

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] -2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) + 2*sqrt(c - c/(a*x))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x+1\right)}{x\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.503 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=42

$$4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

```
[Out] -2/3*a*(c-c/a/x)^(3/2)/c+4*a*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6268, 25, 528, 455, 45}

$$4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] 4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx \\
&= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^2(1-ax)}\,dx \\
&= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^3}\,dx}{a} \\
&= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^2}\,dx}{a} \\
&= -\frac{c\text{Subst}\left(\int \frac{a+x}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{c\text{Subst}\left(\int \left(\frac{2a}{\sqrt{c-\frac{cx}{a}}}-\frac{a\sqrt{c-\frac{cx}{a}}}{c}\right)dx, x, \frac{1}{x}\right)}{a} \\
&= 4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.67

$$
\frac{2\sqrt{c-\frac{c}{ax}}\,(1+5ax)}{3x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 5*a*x))/(3*x)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.12, size = 173, normalized size = 4.12

method	result
gosper	$\frac{2(5ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x}$
trager	$\frac{2(5ax+1)\sqrt{-\frac{-acx+c}{ax}}}{3x}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2-4ax-1)}{3(ax-1)x}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-6\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}x^3-6a^{\frac{5}{2}}\sqrt{(ax-1)\,x}\,x^3+12a^{\frac{3}{2}}\,(a\,x^2-x)^{\frac{3}{2}}x+3\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^2x\right.}{3x^2\sqrt{(ax-1)\,x}\,\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*(a*x-1)/a/x)^(1/2)/x^2*(-6*(a*x^2-x)^(1/2)*a^(5/2)*x^3-6*a^(5/2)*((
a*x-1)*x)^(1/2)*x^3+12*a^(3/2)*(a*x^2-x)^(3/2)*x+3*ln(1/2*(2*(a*x^2-x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*a^2*x^3-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2
*a*x-1)/a^(1/2))*a^2*x^3+2*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)*x)^(1/2)/a^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^2), x)
```

Fricas [A]
time = 0.32, size = 28, normalized size = 0.67

$$\frac{2\,(5\,ax+1)\sqrt{\frac{acx-c}{ax}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.26, size = 24, normalized size = 0.57

$$\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\left(5\,a\,x+1\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*(5*a*x + 1))/(3*x)
```

3.504 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=69

$$4a^2\sqrt{c-\frac{c}{ax}}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}$$

```
[Out] -2*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2+4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6268, 25, 528, 457, 78}

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c}+4a^2\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] - (2*a^2*(c - c/(a*x))^(3/2))/c + (2*a^2*(c - c/(a*
x))^(5/2))/(5*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^3(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^4}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^3}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{x(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c\text{Subst}\left(\int \left(\frac{2a^2}{\sqrt{c-\frac{cx}{a}}} - \frac{3a^2\sqrt{c-\frac{cx}{a}}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{c^2}\right)dx, x, \frac{1}{x}\right)}{a}$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} - \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.52

$$\frac{2\sqrt{c-\frac{c}{ax}}\,(1+3ax+6a^2x^2)}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
[Out] (2*Sqrt[c - c/(a*x)]*(1 + 3*a*x + 6*a^2*x^2))/(5*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.12, size = 192, normalized size = 2.78

method	result
gosper	$\frac{2(6a^2x^2+3ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{5x^2}$
trager	$\frac{2(6a^2x^2+3ax+1)\sqrt{-\frac{-acx+c}{ax}}}{5x^2}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(6a^3x^3-3a^2x^2-2ax-1)}{5(ax-1)x^2}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(10\sqrt{(ax-1)\,x}\,a^{\frac{7}{2}}x^4+10a^{\frac{7}{2}}\sqrt{a\,x^2-x}\,x^4-20a^{\frac{5}{2}}\left(a\,x^2-x\right)^{\frac{3}{2}}x^2-5\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^3}{5x^3\sqrt{(ax-1)\,x}\,\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(c*(a*x-1)/a/x)^(1/2)/x^3*(10*((a*x-1)*x)^(1/2)*a^(7/2)*x^4+10*a^(7/2)*
(a*x^2-x)^(1/2)*x^4-20*a^(5/2)*(a*x^2-x)^(3/2)*x^2-5*ln(1/2*(2*(a*x^2-x)^(1
/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^3*x^4+5*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)
+2*a*x-1)/a^(1/2))*a^3*x^4-8*a^(3/2)*(a*x^2-x)^(3/2)*x-2*(a*x^2-x)^(3/2)*a^
(1/2))/((a*x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^3), x)
```

Fricas [A]
time = 0.34, size = 36, normalized size = 0.52

$$\frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{\frac{acx-c}{ax}}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt((a*c*x - c)/(a*x))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.29, size = 32, normalized size = 0.46

$$\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\left(6\,a^2\,x^2+3\,a\,x+1\right)}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*(3*a*x + 6*a^2*x^2 + 1))/(5*x^2)
```

3.505 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=96

$$4a^3\sqrt{c-\frac{c}{ax}}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

```
[Out] -10/3*a^3*(c-c/a/x)^(3/2)/c+8/5*a^3*(c-c/a/x)^(5/2)/c^2-2/7*a^3*(c-c/a/x)^(
7/2)/c^3+4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6268, 25, 528, 457, 78}

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^3\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] 4*a^3*Sqrt[c - c/(a*x)] - (10*a^3*(c - c/(a*x))^(3/2))/(3*c) + (8*a^3*(c -
c/(a*x))^(5/2))/(5*c^2) - (2*a^3*(c - c/(a*x))^(7/2))/(7*c^3)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^4(1-ax)}\,dx$$
$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^5}\,dx}{a}$$
$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^4}\,dx}{a}$$
$$= -\frac{c\text{Subst}\left(\int \frac{x^2(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{c\text{Subst}\left(\int \left(\frac{2a^3}{\sqrt{c-\frac{cx}{a}}} - \frac{5a^3\sqrt{c-\frac{cx}{a}}}{c} + \frac{4a^3\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{5/2}}{c^3}\right) dx, x$$
$$= 4a^3\sqrt{c-\frac{c}{ax}} - \frac{10a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{8a^3\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.46

$$\frac{2\sqrt{c-\frac{c}{ax}}\,(15+39ax+52a^2x^2+104a^3x^3)}{105x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
[Out] (2*Sqrt[c - c/(a*x)]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.13, size = 211, normalized size = 2.20

method	result
gosper	$\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{\frac{c(ax-1)}{ax}}}{105x^3}$
trager	$\frac{2(104a^3x^3+52a^2x^2+39ax+15)\sqrt{-\frac{-acx+c}{ax}}}{105x^3}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(104a^4x^4-52a^3x^3-13a^2x^2-24ax-15)}{105(ax-1)x^3}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(210\sqrt{(ax-1)x}\,a^{\frac{9}{2}}x^5+210\sqrt{a\,x^2-x}\,a^{\frac{9}{2}}x^5-420(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}x^3-105\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right.\right.}{105x^4\sqrt{(ax-1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(c*(a*x-1)/a/x)^(1/2)/x^4*(210*((a*x-1)*x)^(1/2)*a^(9/2)*x^5+210*(a*x
^2-x)^(1/2)*a^(9/2)*x^5-420*(a*x^2-x)^(3/2)*a^(7/2)*x^3-105*ln(1/2*(2*(a*x^
2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^4*x^5+105*ln(1/2*(2*((a*x-1)*x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*a^4*x^5-212*a^(5/2)*(a*x^2-x)^(3/2)*x^2-108*a^(
3/2)*(a*x^2-x)^(3/2)*x-30*(a*x^2-x)^(3/2)*a^(1/2))/((a*x-1)*x)^(1/2)/a^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^4), x)
```

Fricas [A]
time = 0.37, size = 44, normalized size = 0.46

$$\frac{2\,(104\,a^3x^3+52\,a^2x^2+39\,ax+15)\sqrt{\frac{acx-c}{ax}}}{105\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt((a*c*x - c)/(a*x))/x^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.28, size = 77, normalized size = 0.80

$$\frac{208\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{105}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{7\,x^3}+\frac{26\,a\,\sqrt{c-\frac{c}{a\,x}}}{35\,x^2}+\frac{104\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

```
[Out] (208*a^3*(c - c/(a*x))^(1/2))/105 + (2*(c - c/(a*x))^(1/2))/(7*x^3) + (26*a
*(c - c/(a*x))^(1/2))/(35*x^2) + (104*a^2*(c - c/(a*x))^(1/2))/(105*x)
```

3.506 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=121

$$4a^4\sqrt{c-\frac{c}{ax}}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}$$

```
[Out] -14/3*a^4*(c-c/a/x)^(3/2)/c+18/5*a^4*(c-c/a/x)^(5/2)/c^2-10/7*a^4*(c-c/a/x)
^(7/2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4+4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6268, 25, 528, 457, 78}

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] - (14*a^4*(c - c/(a*x))^(3/2))/(3*c) + (18*a^4*(c -
 c/(a*x))^(5/2))/(5*c^2) - (10*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c
 - c/(a*x))^(9/2))/(9*c^4)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1+ax)}{x^5(1-ax)}\,dx$$

$$= \frac{c\int \frac{1+ax}{\sqrt{c-\frac{c}{ax}}\,x^6}\,dx}{a}$$

$$= \frac{c\int \frac{a+\frac{1}{x}}{\sqrt{c-\frac{c}{ax}}\,x^5}\,dx}{a}$$

$$= -\frac{c\text{Subst}\left(\int \frac{x^3(a+x)}{\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{c\text{Subst}\left(\int \left(\frac{2a^4}{\sqrt{c-\frac{cx}{a}}} - \frac{7a^4\sqrt{c-\frac{cx}{a}}}{c} + \frac{9a^4\left(c-\frac{cx}{a}\right)^{3/2}}{c^2} - \frac{5a^4\left(c-\frac{cx}{a}\right)^{5/2}}{c^3} + \frac{a^4\left(c-\right.}{c^4}\right.\right.}{a}$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} - \frac{14a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{18a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{10a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4(}{}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.43

$$\frac{2\sqrt{c-\frac{c}{ax}}\,(35+85ax+102a^2x^2+136a^3x^3+272a^4x^4)}{315x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*a^4*x^4
))/(315*x^4)
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.13, size = 230, normalized size = 1.90

method	result
gosper	$\frac{2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4}$
trager	$\frac{2(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35)\sqrt{-\frac{-acx+c}{ax}}}{315x^4}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}\,(272a^5x^5-136a^4x^4-34a^3x^3-17a^2x^2-50ax-35)}{315(ax-1)x^4}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(-630a^{\frac{11}{2}}\sqrt{a\,x^2-x}\,x^6-630a^{\frac{11}{2}}\sqrt{(ax-1)\,x}\,x^6+315\ln\left(\frac{2\sqrt{a\,x^2-x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a^5x^6+1260a\ldots\right.}{31\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*(a*x-1)/a/x)^(1/2)/x^5*(-630*a^(11/2)*(a*x^2-x)^(1/2)*x^6-630*a^(
11/2)*((a*x-1)*x)^(1/2)*x^6+315*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/
a^(1/2))*a^5*x^6+1260*a^(9/2)*(a*x^2-x)^(3/2)*x^4-315*ln(1/2*(2*((a*x-1)*x)
^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^5*x^6+716*(a*x^2-x)^(3/2)*a^(7/2)*x^3+44
4*a^(5/2)*(a*x^2-x)^(3/2)*x^2+240*a^(3/2)*(a*x^2-x)^(3/2)*x+70*(a*x^2-x)^(3
/2)*a^(1/2))/((a*x-1)*x)^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a*x))/((a*x - 1)*x^5), x)
```

Fricas [A]
time = 0.37, size = 52, normalized size = 0.43

$$\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{\frac{acx-c}{ax}}}{315\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt((a*c*x -
 c)/(a*x))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax+1)}{x^5\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.37, size = 98, normalized size = 0.81

$$\frac{544\,a^4\,\sqrt{c-\frac{c}{a\,x}}}{315}+\frac{2\,\sqrt{c-\frac{c}{a\,x}}}{9\,x^4}+\frac{34\,a\,\sqrt{c-\frac{c}{a\,x}}}{63\,x^3}+\frac{68\,a^2\,\sqrt{c-\frac{c}{a\,x}}}{105\,x^2}+\frac{272\,a^3\,\sqrt{c-\frac{c}{a\,x}}}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] (544*a^4*(c - c/(a*x))^(1/2))/315 + (2*(c - c/(a*x))^(1/2))/(9*x^4) + (34*a
*(c - c/(a*x))^(1/2))/(63*x^3) + (68*a^2*(c - c/(a*x))^(1/2))/(105*x^2) + (
272*a^3*(c - c/(a*x))^(1/2))/(315*x)
```

3.507 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=313

$$\frac{149\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 363/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-4*arcta
nh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)
+149/64*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+107/96*x^2*(1
+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+17/24*x^3*(1+1/a/x)^(1/2)
*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/4*x^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/
(1-1/a/x)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6317, 6315, 100, 156, 162, 65, 214, 212}

$$\frac{363\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^4\sqrt{1-\frac{1}{ax}}}+\frac{149x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^4\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-\frac{1}{ax}}}+\frac{17x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{24a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] (149*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(64*a^3*Sqrt[1 - 1/(a*x)]) + (1
07*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]) + (1
7*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]) + (Sqrt
[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]) + (363*Sqrt[c -
c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2
]*Sqrt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^4*Sqrt[1 - 1/(a*
x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^5\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^4}{4\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{17}{2a}-\frac{15x}{2a^2}}{x^4\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{17\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^4}{4\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\right.}{12\sqrt{}}$$

$$=\frac{107\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^3}{24a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,\sqrt{c-}}{4\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{149\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\,\sqrt{c}}{24a\sqrt{1-}}$$

$$=\frac{149\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\,\sqrt{c}}{24a\sqrt{1-}}$$

$$=\frac{149\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x}{64a^3\sqrt{1-\frac{1}{ax}}}+\frac{107\sqrt{1+\frac{1}{ax}}\,\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}}+\frac{17\sqrt{1+\frac{1}{ax}}\,\sqrt{c}}{24a\sqrt{1-}}$$

Mathematica [A]

time = 0.53, size = 252, normalized size = 0.81

$$\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(447+214ax+136a^2x^2+48a^3x^3)}{-1+ax}-1089\sqrt{c}\log(1-ax)+768\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right)+1089\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)-768\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}\,a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-2ax+3a^2x^2)\right)}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^3,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(447 + 214*a*x + 136*a^
2*x^2 + 48*a^3*x^3))/(-1 + a*x) - 1089*Sqrt[c]*Log[1 - a*x] + 768*Sqrt[2]*S
qrt[c]*Log[(-1 + a*x)^2] + 1089*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x
^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 768*Sqrt[2]*Sqrt[c
]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c
*(-1 - 2*a*x + 3*a^2*x^2)])/(384*a^4)
```

Maple [A]

time = 0.14, size = 224, normalized size = 0.72

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(96\sqrt{x\left(ax+1\right)}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^3+272\sqrt{x\left(ax+1\right)}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^2+428\sqrt{x\left(ax+1\right)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x\cdots\right.}{384\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\cdots}$
risch	$\frac{\left(48a^3x^3+136a^2x^2+214ax+447\right)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{363\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{128a^3\sqrt{a^2c}}-\frac{2\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(96*(
x*(a*x+1))^(1/2)*a^(9/2)*(1/a)^(1/2)*x^3+272*(x*(a*x+1))^(1/2)*a^(7/2)*(1/a
)^(1/2)*x^2+428*(x*(a*x+1))^(1/2)*a^(5/2)*(1/a)^(1/2)*x+894*(x*(a*x+1))^(1/
2)*a^(3/2)*(1/a)^(1/2)-768*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1
/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+1089*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*
a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(9/2)/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.45, size = 568, normalized size = 1.81

$$\left[\frac{768\,\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+1089\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(48\,a^5x^5+184\,a^4x^4+350\,a^3x^3+661\,a^2x^2+447\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{768\,(a^5x-a^4)},\ \frac{768\,\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\,a^2cx^2-2\,acx-c}\right)-1089\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(48\,a^5x^5+184\,a^4x^4+350\,a^3x^3+661\,a^2x^2+447\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{384\,(a^5x-a^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/768*(768*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13
*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*
x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 1
089*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^
2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*
x - 1)) + 4*(48*a^5*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x
)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4), 1/384*(
768*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)
) - 1089*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1
)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(48*a^5
*x^5 + 184*a^4*x^4 + 350*a^3*x^3 + 661*a^2*x^2 + 447*a*x)*sqrt((a*x - 1)/(a
*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a/x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^3*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.508 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=261

$$\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}}+\frac{45\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\sqrt{1+\right.}{8a^3\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 45/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)-4*arctanh
(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+1
9/8*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+13/12*x^2*(1+1/a/
x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/3*x^3*(1+1/a/x)^(1/2)*(c-c/a/x
)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6317, 6315, 100, 156, 162, 65, 214, 212}

$$\frac{45\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^3\sqrt{1-\frac{1}{ax}}}+\frac{19x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^3\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{13x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] (19*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(8*a^2*Sqrt[1 - 1/(a*x)]) + (13*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]) + (45*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^3*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^4\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{-\frac{13}{2a}-\frac{11x}{2a^2}}{x^3\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\Big(\int}{6}$$

$$=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{3\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{19\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}+\frac{13\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]

time = 0.39, size = 244, normalized size = 0.93

$$\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(57+26ax+8a^2x^2)}{-1+ax}-135\sqrt{c}\log(1-ax)+96\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right)+135\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right)-96\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-2ax+3a^2x^2)\right)}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x^2,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(57 + 26*a*x + 8*a^2*x^
2))/(-1 + a*x) - 135*Sqrt[c]*Log[1 - a*x] + 96*Sqrt[2]*Sqrt[c]*Log[(-1 + a*
x)^2] + 135*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x
)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 96*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*S
qrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*
x^2)])/(48*a^3)
```

Maple [A]

time = 0.14, size = 202, normalized size = 0.77

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\, x\left(16\sqrt{x(ax+1)}\, a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\, x^2+52\sqrt{x(ax+1)}\, a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\, x+114\sqrt{x(ax+1)}\, a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-96\sqrt{\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{7}{2}}\sqrt{x(ax}$
risch	$\frac{(8a^2x^2+26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{45\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{16a^2\sqrt{a^2c}}-\frac{2\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\,(x}{x-}\right.}{a^3\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}(ax+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*(x
*(a*x+1))^(1/2)*a^(7/2)*(1/a)^(1/2)*x^2+52*(x*(a*x+1))^(1/2)*a^(5/2)*(1/a)^
(1/2)*x+114*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)-96*2^(1/2)*ln((2*2^(1/2)*
(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2)+135*ln(1/2*(2*(x*
(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^(1/2))/a^(7/2)/(x*(a*x+1))
^(1/2)/(1/a)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.46, size = 552, normalized size = 2.11

$$\left[\frac{96\,\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+135\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4+34\,a^3x^3+83\,a^2x^2+57\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{96\,(a^4x-a^3)},\ \frac{96\,\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\,a^2cx^2-2\,acx-c}\right)-135\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(8\,a^4x^4+34\,a^3x^3+83\,a^2x^2+57\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{48\,(a^4x-a^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="frica
s")
```

```
[Out] [1/96*(96*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a
*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 135
*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
 a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
 1)) + 4*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(96*sqrt(2)*(a*x - 1)*s
qrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))
*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 135*(a*x - 1)*sqrt(
-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x
 - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 + 34*a^3*x^3 + 83*a^
2*x^2 + 57*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x -
 a^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a/x)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^2*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.509 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=209

$$\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}+\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\,\mathrm{ta}}{a^2\sqrt{1}}$$

```
[Out] 23/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-4*arctanh
(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)+9
/4*x*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+1/2*x^2*(1+1/a/x)^(1
/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6317, 6315, 100, 156, 162, 65, 214, 212}

$$\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}}+\frac{9x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] (9*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]) + (Sqrt[1
 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]) + (23*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sq
rt[c - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a^2*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{9}{2a}-\frac{7x}{2a^2}}{x^2\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{}{x(1-}\right.}{2\sqrt{1-}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\right.}{a^3\sqrt{}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}-\frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\right.}{a^2\sqrt{1}}$$

$$=\frac{9\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}}+\frac{23\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\right.}{4a^2\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]

time = 0.31, size = 236, normalized size = 1.13

$$\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(9+2ax)}{-1+ax}-23\sqrt{c}\,\log(1-ax)+16\sqrt{2}\,\sqrt{c}\,\log\left((-1+ax)^2\right)+23\sqrt{c}\,\log\left(2a^2\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)-16\sqrt{2}\,\sqrt{c}\,\log\left(2\sqrt{2}\,a^2\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-2ax+3a^2x^2)\right)}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)]*x,x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(9 + 2*a*x))/(-1 + a*x)
 - 23*Sqrt[c]*Log[1 - a*x] + 16*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + 23*Sqrt
[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 -
 a*x + 2*a^2*x^2)] - 16*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 -
1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)])/(8*a^2)
```

Maple [A]
time = 0.14, size = 180, normalized size = 0.86

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{x(ax+1)}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\,x+18\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-16\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x(ax+\cdots}}{ax-1}\cdots\right.\right.}{8\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)a^{\frac{5}{2}}\sqrt{x(ax+1)}\,\sqrt{\frac{1}{a}}\cdots}$
risch	$\frac{(2ax+9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{23\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{8a\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)\cdots}}{x-\frac{1}{a}}\cdots\right.}{a^2\sqrt{c}\cdots}\cdots\right.}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*(x*(
a*x+1))^(1/2)*a^(5/2)*(1/a)^(1/2)*x+18*(x*(a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2
)-16*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1)
)*a^(1/2)+23*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*(1/a)^
(1/2))/a^(5/2)/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.44, size = 536, normalized size = 2.56

$$\left[\frac{16\sqrt{2}(ax-1)\sqrt{c}\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+23(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4(2a^3x^3+3a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4(2a^3x^3+11a^2x^2+9ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{16(a^3x-a^2)}, \frac{16\sqrt{2}(ax-1)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)-23(ax-1)\sqrt{-c}\arctan\left(\frac{2(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2(2a^3x^3+11a^2x^2+9ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{8(a^3x-a^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(16*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a
*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x
+ 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 23*
(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 +
a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x -
1)) + 4*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*
c*x - c)/(a*x)))/(a^3*x - a^2), 1/8*(16*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2
*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) - 23*(a*x - 1)*sqrt(-c)*arctan(2*(a^2
*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a
^2*c*x^2 - a*c*x - c)) + 2*(2*a^3*x^3 + 11*a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/
(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x*(c - c/(a*x))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.510 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 5*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)-4*arctanh(1/2*
(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+x*(1+1/a
/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6317, 6314, 100, 162, 65, 214, 212}

$$\frac{x\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x)/Sqrt[1 - 1/(a*x)] + (5*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*Sqrt[c
 - c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^2\left(1-\frac{x}{a}\right)}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{-\frac{5}{2a}-\frac{3x}{2a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(4\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\int\frac{1}{\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2\sqrt{1-\frac{1}{ax}}}-$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}-\frac{\left(5\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}\left(\int\frac{1}{-a+ax^2}\,dx,x,\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}-$$

$$=\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}}+\frac{5\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}}{a}$$

Mathematica [**A**]

time = 0.05, size = 93, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{ax}}\left(a\sqrt{1+\frac{1}{ax}}\,x+5\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)-4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(a*Sqrt[1 + 1/(a*x)]*x + 5*ArcTanh[Sqrt[1 + 1/(a*x)]] -
4*Sqrt[2]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]]))/(a*Sqrt[1 - 1/(a*x)])
```

Maple [A]
time = 0.13, size = 160, normalized size = 1.05

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}+5\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\cdots}{\cdots}\right.\right.}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{x(ax+1)}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$
risch	$\frac{x\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{5\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\,(\cdots}}{x-\frac{1}{a}}\right.}{a\sqrt{c}}\right.\cdots}{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(
a*x+1))^(1/2)*a^(3/2)*(1/a)^(1/2)+5*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a
*x+1)/a^(1/2))*a*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1)
)^(1/2)*a+3*a*x+1)/(a*x-1))*a^(1/2))/(x*(a*x+1))^(1/2)/a^(3/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.44, size = 512, normalized size = 3.37

$$\left[\frac{4\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+5\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)}, \frac{4\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\,a^2cx^2-2\,acx-c}\right)-5\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 5*(a*
x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x
)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1))
 + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
2*x - a), 1/2*(4*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x
)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 -
 2*a*c*x - c)) - 5*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) +
 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^2*
x - a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.511 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=146

$$\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{ax}}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-4*arctanh(1/2*(1
+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*(1+1/a/x)^
(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6317, 6315, 86, 162, 65, 214, 212}

$$\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]
```

```
[Out] (2*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(
a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)] - (4*Sqrt[2]*Sqrt[c - c
/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
```

```
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x\left(1-\frac{x}{a}\right)}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{\left(a\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(4\sqrt{c-\right.}{}}{}$$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(8\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - $$

$$= \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}} - \frac{4\sqrt{2}\sqrt{c-\frac{c}{ax}}}{\sqrt{}}$$

Mathematica [A]
time = 0.28, size = 218, normalized size = 1.49

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x}{-1+ax} - \sqrt{c}\log(1-ax) + 2\sqrt{2}\sqrt{c}\log\left((-1+ax)^2\right) + \sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2 + c(-1-ax+2a^2x^2)\right) - 2\sqrt{2}\sqrt{c}\log\left(2\sqrt{2}a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2 + c(-1-2ax+3a^2x^2)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x,x]

[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x)/(-1 + a*x) - Sqrt[c]*Log[1 - a*x] + 2*Sqrt[2]*Sqrt[c]*Log[(-1 + a*x)^2] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)] - 2*Sqrt[2]*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]

Maple [A]
time = 0.14, size = 159, normalized size = 1.09

method	result
default	$\frac{(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,ax-2\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\sqrt{\frac{1}{a}}\sqrt{x(ax+1)}\,a+3ax}{ax-1}\right.\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{x(ax+1)}\sqrt{a}\sqrt{\frac{1}{a}}}$
risch	$\frac{2\sqrt{\frac{c(ax-1)}{ax}}}{\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{\sqrt{a^2c}}-\frac{2\sqrt{2}\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\sqrt{c}\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3(}}{x-\frac{1}{a}}\right.}{\sqrt{c}}\right.}{\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*(1/a)^(1/2)*a*x-2*a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x+2*(x*(a*x+1))^(1/2)*a^(1/2)*(1/a)^(1/2))/(x*(a*x+1))^(1/2)/a^(1/2)/(1/a)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a*x))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)

Fricas [A]
time = 0.45, size = 490, normalized size = 3.36

$$\left[\frac{2\sqrt{2}\,(ax-1)\sqrt{c}\,\log\left(-\frac{17\,a^3cx^3-3\,a^2cx^2-13\,acx-4\,\sqrt{2}\,(3\,a^3x^3+4\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)},\ \frac{2\sqrt{2}\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\,a^2cx^2-2\,acx-c}\right)-(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] [1/2*(2*sqrt(2)*(a*x - 1)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 - 13*a*c
*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + (a*x
- 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*
sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) +
 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1),
(2*sqrt(2)*(a*x - 1)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c))
 - (a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x
 + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(a*x + 1)*sqr
t((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - c/(a*x))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.512 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=125

$$\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2/3*a*c^2*(1-1/a^2/x^2)^(3/2)/(c-c/a/x)^(3/2)-4*a*arctanh(1/2*c^(1/2)*(1-1/
a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a*c*(1-1/a^2/x^2)
^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6313, 679, 675, 214}

$$\frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*a*c^2*(1 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a*c*Sqrt[1 -
 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 675

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
```

```
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2c^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - (4c)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx}{a}}\sqrt{1-\frac{x^2}{a^2}}}\right.$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \frac{\left(8c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{2c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-}}{\sqrt{c}}\right.}{a}$$

$$= \frac{2ac^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4ac\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right.$$

Mathematica [A]
time = 0.19, size = 155, normalized size = 1.24

$$\frac{2a\left(\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,(1+7ax)+3\sqrt{2}\sqrt{c}\,(-1+ax)\log\left((-1+ax)^2\right)-3\sqrt{2}\sqrt{c}\,(-1+ax)\log\left(2\sqrt{2}\,a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c\left(-1-2ax+3a^2x^2\right)\right)\right)}{-3+3ax}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^2,x]
```

```
[Out] (2*a*(Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 + 7*a*x) + 3*Sqrt[2]*Sqrt[
c]*(-1 + a*x)*Log[(-1 + a*x)^2] - 3*Sqrt[2]*Sqrt[c]*(-1 + a*x)*Log[2*Sqrt[2
]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x +
 3*a^2*x^2)]))/(-3 + 3*a*x)
```

Maple [A]
time = 0.14, size = 140, normalized size = 1.12

method	result
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-3a\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\left(ax+1\right)}\,a+3ax+1}{ax-1}\right)x^{2}+7x\sqrt{x\left(ax+1\right)}\,a\sqrt{\frac{1}{a}}+\sqrt{x\left(ax+\right.}\right.}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x\sqrt{x\left(ax+1\right)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2\left(7a^{2}x^{2}+8ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}}{3x\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac+2c}}{x-\frac{1}{a}}\right)\sqrt{c}}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-3*a*2^(
1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^2+7*
x*(x*(a*x+1))^(1/2)*a*(1/a)^(1/2)+(x*(a*x+1))^(1/2)*(1/a)^(1/2))/x/(x*(a*x+
1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.38, size = 353, normalized size = 2.82

$$\left[\frac{3\sqrt{2}\left(a^{2}x^{2}-ax\right)\sqrt{c}\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\sqrt{2}\left(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+2\left(7\,a^{2}x^{2}+8\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\left(ax^{2}-x\right)},\ \frac{2\left(3\sqrt{2}\left(a^{2}x^{2}-ax\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\left(a^{2}x^{2}+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3\,a^{2}cx^{2}-2\,acx-c}\right)+\left(7\,a^{2}x^{2}+8\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}\right)}{3\left(ax^{2}-x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] [1/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*x^2 -
13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(
a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) +
 2*(7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a*x^2 - x), 2/3*(3*sqrt(2)*(a^2*x^2 - a*x)*sqrt(-c)*arctan(2*sqrt(2)*(a
^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3
*a^2*c*x^2 - 2*a*c*x - c)) + (7*a^2*x^2 + 8*a*x + 1)*sqrt((a*x - 1)/(a*x +
1))*sqrt((a*c*x - c)/(a*x)))/(a*x^2 - x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.513 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=170

$$\frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\,\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2/5*a^2*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^2*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)-4*a^2*arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/
(c-c/a/x)^(1/2))*2^(1/2)*c^(1/2)+4*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2
)
```

Rubi [A]
time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6313, 809, 679, 675, 214}

$$\frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\,\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a^2*c^3*(1 - 1/(a^2*x^2))^(5/2))/(5*(c - c/(a*x))^(5/2)) + (2*a^2*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) + (4*a^2*c*Sqrt[1 - 1/(a^2*x
^2)])/Sqrt[c - c/(a*x)] - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)])/(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 675

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\left(c^3\text{Subst}\left(\int \frac{x\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} - \left(ac^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \left(2ac^2\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x^2}{a^2}}}{\left(c-\frac{cx}{a}\right)^{3/2}}\,dx, x$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - (4ac)\text{Subst}\Bigg(\int$$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + \left(8c^2\right)\text{Subst}\Bigg($$

$$= \frac{2a^2c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{5\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^2c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - 4\sqrt{2}\,a^2\sqrt{c}\,\text{ta}$$

Mathematica [A]
time = 0.18, size = 162, normalized size = 0.95

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(3+11ax+38a^2x^2\right)}{15x(-1+ax)} + 2\sqrt{2}\,a^2\sqrt{c}\,\log\left((-1+ax)^2\right) - 2\sqrt{2}\,a^2\sqrt{c}\,\log\left(2\sqrt{2}\,a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2 + c\left(-1-2ax+3a^2x^2\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^3,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 11*a*x + 38*a^2*x^2))/(15
*x*(-1 + a*x)) + 2*Sqrt[2]*a^2*Sqrt[c]*Log[(-1 + a*x)^2] - 2*Sqrt[2]*a^2*Sq
rt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2
 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A]
time = 0.14, size = 165, normalized size = 0.97

method	result
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-15a^{2}\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x^{3}+38x^{2}\sqrt{x\,(ax+1)}\,a^{2}\sqrt{\frac{1}{a}}+11x\sqrt{}\right.}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^{2}\sqrt{x\,(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2(38a^{3}x^{3}+49a^{2}x^{2}+14ax+3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^{2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^{2}\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^{2}c\left(x-\frac{1}{a}\right)^{2}+3\left(x-\frac{1}{a}\right)ac}}{x-\frac{1}{a}}\right.}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-15*a^2
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^
3+38*x^2*(x*(a*x+1))^(1/2)*a^2*(1/a)^(1/2)+11*x*(x*(a*x+1))^(1/2)*a*(1/a)^(
1/2)+3*(x*(a*x+1))^(1/2)*(1/a)^(1/2))/x^2/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]

time = 0.39, size = 381, normalized size = 2.24

$$\left[\frac{15\sqrt{2}\,(a^{3}x^{3}-a^{2}x^{2})\sqrt{c}\,\log\left(-\frac{17\,a^{3}cx^{3}-3\,a^{2}cx^{2}-13\,acx-4\,\sqrt{2}\,(3\,a^{3}x^{3}+4\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+2\,(38\,a^{3}x^{3}+49\,a^{2}x^{2}+14\,ax+3)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{15\,(ax^{3}-x^{2})},\ \frac{2\left(15\,\sqrt{2}\,(a^{3}x^{3}-a^{2}x^{2})\sqrt{-c}\,\arctan\left(\frac{2\,\sqrt{2}\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\,a^{2}cx^{2}-2\,acx-c}\right)+(38\,a^{3}x^{3}+49\,a^{2}x^{2}+14\,ax+3)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}\right)}{15\,(ax^{3}-x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] [1/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
```

```
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(38*a^3*x^3 + 49*a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a*x^3 - x^2), 2/15*(15*sqrt(2)*(a^3*x^3 - a^2*x^2)
*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (38*a^3*x^3 + 49*
a^2*x^2 + 14*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a
*x^3 - x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.514 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=209

$$\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{2}}\right.$$

```
[Out] 4/7*a^3*c^3*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(5/2)+2/3*a^3*c^2*(1-1/a^2/x^2)^(
3/2)/(c-c/a/x)^(3/2)-2/7*a^3*c^2*(1-1/a^2/x^2)^(5/2)/(c-c/a/x)^(3/2)-4*a^3*
arctanh(1/2*c^(1/2)*(1-1/a^2/x^2)^(1/2)*2^(1/2)/(c-c/a/x)^(1/2))*2^(1/2)*c^
(1/2)+4*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6313, 1653, 809, 679, 675, 214}

$$\frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}}-\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}}+\frac{4a^3c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{2}\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (4*a^3*c^3*(1 - 1/(a^2*x^2))^(5/2))/(7*(c - c/(a*x))^(5/2)) + (2*a^3*c^2*(1
 - 1/(a^2*x^2))^(3/2))/(3*(c - c/(a*x))^(3/2)) - (2*a^3*c^2*(1 - 1/(a^2*x^2
))^(5/2))/(7*(c - c/(a*x))^(3/2)) + (4*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c
- c/(a*x)] - 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
(Sqrt[2]*Sqrt[c - c/(a*x)])]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 675

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> Dis
t[2*e, Subst[Int[1/(2*c*d + e^2*x^2), x], x, Sqrt[a + c*x^2]/Sqrt[d + e*x]]
, x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1653

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\left(c^3\text{Subst}\left(\int \frac{x^2\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{1}{7}(2a^4c)\,\text{Subst}\left(\int \frac{\left(\frac{3c^2}{2a^2}-\frac{5c^2x}{a^3}\right)\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right.$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - (a^2c^3)\,\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{a^2}\right)^{3/2}}{\left(c-\frac{cx}{a}\right)^{5/2}}\,dx,\right.$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} - (2a^2c^2)\,\text{Sub}$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-}}{\sqrt{c-\frac{c}{a}}}$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-}}{\sqrt{c-\frac{c}{a}}}$$

$$= \frac{4a^3c^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{5/2}} + \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{3\left(c-\frac{c}{ax}\right)^{3/2}} - \frac{2a^3c^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{7\left(c-\frac{c}{ax}\right)^{3/2}} + \frac{4a^3c\sqrt{1-}}{\sqrt{c-\frac{c}{a}}}$$

<u>Mathematica</u> [A]
time = 0.19, size = 170, normalized size = 0.81

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,(3+9ax+16a^2x^2+52a^3x^3)}{21x^2(-1+ax)} + 2\sqrt{2}\,a^3\sqrt{c}\,\log\left((-1+ax)^2\right) - 2\sqrt{2}\,a^3\sqrt{c}\,\log\left(2\sqrt{2}\,a^2\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2 + c(-1-2ax+3a^2x^2)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^4,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 + 9*a*x + 16*a^2*x^2 + 52*a
^3*x^3))/(21*x^2*(-1 + a*x)) + 2*Sqrt[2]*a^3*Sqrt[c]*Log[(-1 + a*x)^2] - 2*
```

```
Sqrt[2]*a^3*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c
- c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A]
time = 0.14, size = 187, normalized size = 0.89

method	result
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-21a^3\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x^4+52x^3\sqrt{x\,(ax+1)}\,a^3\sqrt{\frac{1}{a}}+16x^2\sqrt{x}\right.}{21\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^3\sqrt{x\,(ax+1)}\,\sqrt{\frac{1}{a}}}$
risch	$\frac{2(52a^4x^4+68a^3x^3+25a^2x^2+12ax+3)\sqrt{\frac{c(ax-1)}{ax}}}{21x^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^3\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+3\left(x-\right.}}{x-\frac{1}{a}}\right.}{\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-21*a^3
*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*x^
4+52*x^3*(x*(a*x+1))^(1/2)*a^3*(1/a)^(1/2)+16*x^2*(x*(a*x+1))^(1/2)*a^2*(1/
a)^(1/2)+9*x*(x*(a*x+1))^(1/2)*a*(1/a)^(1/2)+3*(x*(a*x+1))^(1/2)*(1/a)^(1/2
))/x^3/(x*(a*x+1))^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.42, size = 397, normalized size = 1.90

$$\left[\frac{21\sqrt{2}\,(a^4x^4-a^3x^3)\sqrt{c}\,\log\left(-\frac{17a^3cx^3-3a^2cx^2-13acx-4\sqrt{2}\,(3a^3x^3+4a^2x^2+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^3x^3-3a^2x^2+3ax-1}\right)+2\,(52a^4x^4+68a^3x^3+25a^2x^2+12ax+3)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{21\,(ax^4-x^3)},\ \frac{2\left(21\sqrt{2}\,(a^4x^4-a^3x^3)\sqrt{-c}\,\arctan\left(\frac{2\sqrt{2}\,(a^2x^2+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^2cx^2-2acx-c}\right)+(52a^4x^4+68a^3x^3+25a^2x^2+12ax+3)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}\right)}{21\,(ax^4-x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] [1/21*(21*sqrt(2)*(a^4*x^4 - a^3*x^3)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*c*
x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)) + 2*(52*a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3), 2/21*(21*sqrt(2)*(a^4*x
^4 - a^3*x^3)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^2 - 2*a*c*x - c)) + (52*
a^4*x^4 + 68*a^3*x^3 + 25*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*s
qrt((a*c*x - c)/(a*x)))/(a*x^4 - x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a*x))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.515 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=303

$$\frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2/5*a^4*(1+1/a/x)^(
5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-2/7*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)+2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-4
*a^4*arctanh(1/2*(1+1/a/x)^(1/2)*2^(1/2))*2^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)
^(1/2)+4*a^4*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6317, 6315, 90, 52, 65, 212}

$$\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}-\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}+\frac{4a^4\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{4\sqrt{2}a^4\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{ax}+1}}{\sqrt{2}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (4*a^4*Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*a^4*(1 +
 1/(a*x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a
*x))^(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) - (2*a^4*(1 + 1/(a*x))^
(7/2)*Sqrt[c - c/(a*x)])/(7*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(9/2)
*Sqrt[c - c/(a*x)])/(9*Sqrt[1 - 1/(a*x)]) - (4*Sqrt[2]*a^4*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]/Sqrt[2]])/Sqrt[1 - 1/(a*x)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int \frac{x^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int \left(-a^3\left(1+\frac{x}{a}\right)^{3/2}+\frac{a^3\left(1+\frac{x}{a}\right)^{3/2}}{1-\frac{x}{a}}+a^3\left(1+\frac{x}{a}\right)^{5/2}-a^3\left(1+\frac{x}{a}\right.\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{7\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{9/2}\sqrt{c-}}{9\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}} - \frac{2a^4\left(1+\frac{1}{ax}\right)^{7/2}\sqrt{c-}}{7\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-}}{5\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-}}{5\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{4a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}} + \frac{2a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-}}{5\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.20, size = 178, normalized size = 0.59

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,(35+95ax+138a^2x^2+236a^3x^3+788a^4x^4)}{315x^3(-1+ax)}+2\sqrt{2}\,a^4\sqrt{c}\,\log\left((-1+ax)^2\right)-2\sqrt{2}\,a^4\sqrt{c}\,\log\left(2\sqrt{2}\,a^2\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-2ax+3a^2x^2)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a*x)])/x^5,x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(35 + 95*a*x + 138*a^2*x^2 + 2
36*a^3*x^3 + 788*a^4*x^4))/(315*x^3*(-1 + a*x)) + 2*Sqrt[2]*a^4*Sqrt[c]*Log
[(-1 + a*x)^2] - 2*Sqrt[2]*a^4*Sqrt[c]*Log[2*Sqrt[2]*a^2*Sqrt[c]*Sqrt[1 - 1
/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - 2*a*x + 3*a^2*x^2)]
```

Maple [A]
time = 0.15, size = 209, normalized size = 0.69

method	result
risch	$\frac{2(788a^5x^5+1024a^4x^4+374a^3x^3+233a^2x^2+130ax+35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}-\frac{2a^4\sqrt{2}\,\ln\left(\frac{4c+3\left(x-\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\,(x-}{x-\frac{1}{a}}\right.}{\sqrt{c}}$
default	$\frac{2(ax-1)\sqrt{\frac{c(ax-1)}{ax}}\left(-315a^4\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{1}{a}}\,\sqrt{x\,(ax+1)}\,a+3ax+1}{ax-1}\right)x^5+788x^4\sqrt{x\,(ax+1)}\,a^4\sqrt{\frac{1}{a}}+236x\right.}{315\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)x^4\,\surd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(-315*a
^4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*(x*(a*x+1))^(1/2)*a+3*a*x+1)/(a*x-1))*
x^5+788*x^4*(x*(a*x+1))^(1/2)*a^4*(1/a)^(1/2)+236*x^3*(x*(a*x+1))^(1/2)*a^3
*(1/a)^(1/2)+138*x^2*(x*(a*x+1))^(1/2)*a^2*(1/a)^(1/2)+95*x*(x*(a*x+1))^(1/
2)*a*(1/a)^(1/2)+35*(x*(a*x+1))^(1/2)*(1/a)^(1/2))/x^4/(x*(a*x+1))^(1/2)/(1
/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a*x))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.39, size = 413, normalized size = 1.36

$$\left[\frac{315\sqrt{2}\left(a^{5}x^{5}-a^{4}x^{4}\right)\sqrt{c}\log\left(-\frac{17a^{3}cx^{3}-3a^{2}cx^{2}-13acx-4\sqrt{2}\left(3a^{3}x^{3}+4a^{2}x^{2}+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{a^{3}x^{3}-3a^{2}x^{2}+3ax-1}\right)+2\left(788a^{5}x^{5}+1024a^{4}x^{4}+374a^{3}x^{3}+233a^{2}x^{2}+130ax+35\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{315\left(ax^{5}-x^{4}\right)}, \frac{2\left(315\sqrt{2}\left(a^{5}x^{5}-a^{4}x^{4}\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\left(a^{2}x^{2}+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{3a^{2}cx^{2}-2acx-c}\right)+\left(788a^{5}x^{5}+1024a^{4}x^{4}+374a^{3}x^{3}+233a^{2}x^{2}+130ax+35\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}\right)}{315\left(ax^{5}-x^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="frica
s")
```

```
[Out] [1/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(c)*log(-(17*a^3*c*x^3 - 3*a^2*
c*x^2 - 13*a*c*x - 4*sqrt(2)*(3*a^3*x^3 + 4*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*
x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x
 - 1)) + 2*(788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2 + 130*a*
x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5 - x^4), 2
/315*(315*sqrt(2)*(a^5*x^5 - a^4*x^4)*sqrt(-c)*arctan(2*sqrt(2)*(a^2*x^2 +
a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(3*a^2*c*x^
2 - 2*a*c*x - c)) + (788*a^5*x^5 + 1024*a^4*x^4 + 374*a^3*x^3 + 233*a^2*x^2
 + 130*a*x + 35)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x^5
- x^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a/x)**(1/2)/x**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a/x)^(1/2)/x^5,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - c/(a*x))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.516 $\int e^{-\coth^{-1}(ax)} \sqrt{c-\frac{c}{ax}}\, x^m \, dx$

Optimal. Leaf size=126

$$\frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}}-\frac{(3+4m)\sqrt{c-\frac{c}{ax}}\,x^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -1/2*(3+4*m)*x^m*hypergeom([1/2, -m],[1-m],-1/a/x)*(c-c/a/x)^(1/2)/a/m/(1+m
)/(1-1/a/x)^(1/2)+x^(1+m)*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1+m)/(1-1/a/x)^(
1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6317, 6316, 80, 66}

$$\frac{\sqrt{\frac{1}{ax}+1}\,x^{m+1}\sqrt{c-\frac{c}{ax}}}{(m+1)\sqrt{1-\frac{1}{ax}}}-\frac{(4m+3)x^m\sqrt{c-\frac{c}{ax}}\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)}{2am(m+1)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a*x)])
 - ((3 + 4*m)*Sqrt[c - c/(a*x)]*x^m*Hypergeometric2F1[1/2, -m, 1 - m, -(1/(
a*x))])/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 6316

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_), x_Sy
mbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2
)/(x^(m + 2)*(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}
, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c,
0]) &&  !IntegerQ[m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^m\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^m\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int \frac{x^{-2-m}\left(1-\frac{x}{a}\right)}{\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}} + \frac{\left((3+4m)\sqrt{c-\frac{c}{ax}}\left(\frac{1}{x}\right)^m x^m\right)\text{Subst}\left(\int -\right.}{2a(1+m)\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{ax}}} - \frac{(3+4m)\sqrt{c-\frac{c}{ax}}\,x^m\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\right.}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]

time = 0.05, size = 93, normalized size = 0.74

$$\frac{\sqrt{c-\frac{c}{ax}}\,x^m\left(2am\sqrt{1+\frac{1}{ax}}\,x-(3+4m)\,{}_2F_1\left(\frac{1}{2},-m;1-m;-\frac{1}{ax}\right)\right)}{2am(1+m)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*x^m*(2*a*m*Sqrt[1 + 1/(a*x)]*x - (3 + 4*m)*Hypergeometri
c2F1[1/2, -m, 1 - m, -(1/(a*x))]))/(2*a*m*(1 + m)*Sqrt[1 - 1/(a*x)])
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^m\sqrt{c-\frac{c}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^m*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.517 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=164

$$\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

```
[Out] -11/8*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^3+11/8
*c*x*(1-1/a^2/x^2)^(1/2)/a^2/(c-c/a/x)^(1/2)-11/12*c*x^2*(1-1/a^2/x^2)^(1/2
)/a/(c-c/a/x)^(1/2)+1/3*c*x^3*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6313, 893, 887, 889, 214}

$$-\frac{11cx^2\sqrt{1-\frac{1}{a^2x^2}}}{12a\sqrt{c-\frac{c}{ax}}}+\frac{11cx\sqrt{1-\frac{1}{a^2x^2}}}{8a^2\sqrt{c-\frac{c}{ax}}}+\frac{cx^3\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] (11*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(8*a^2*Sqrt[c - c/(a*x)]) - (11*c*Sqrt[1 - 1
/(a^2*x^2)]*x^2)/(12*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2*x^2)]*x^3)/(
3*Sqrt[c - c/(a*x)]) - (11*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)])/
Sqrt[c - c/(a*x)]])/(8*a^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
```

```
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=-\frac{\mathrm{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{11\mathrm{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{6a}$$

$$=-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\mathrm{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right.}{8a^2}$$

$$=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{11\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}+\frac{(11c^2)\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$=\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x}{8a^2\sqrt{c-\frac{c}{ax}}}-\frac{11c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{12a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^3}{3\sqrt{c-\frac{c}{ax}}}-\frac{11\sqrt{c}\,\tanh^{-1}\cdots}{\cdots}$$

Mathematica [A]

time = 0.27, size = 147, normalized size = 0.90

$$\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(33-22ax+8a^2x^2)}{-1+ax}+33\sqrt{c}\,\log(1-ax)-33\sqrt{c}\,\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(33 - 22*a*x + 8*a^2*x^
2))/(-1 + a*x) + 33*Sqrt[c]*Log[1 - a*x] - 33*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqr
t[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(48*a
^3)
```

Maple [A]
time = 0.09, size = 133, normalized size = 0.81

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16a^{\frac{5}{2}}x^2\sqrt{x\,(ax+1)}-44a^{\frac{3}{2}}x\sqrt{x\,(ax+1)}+66\sqrt{x\,(ax+1)}\,\sqrt{a}-33\ln\left(\frac{2\sqrt{\ }}{\ }\right.\right.}{48a^{\frac{5}{2}}\,(ax-1)\sqrt{x\,(ax+1)}}$
risch	$\frac{(8a^2x^2-22ax+33)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{24a^2(ax-1)}-\frac{11\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{ca}}{16a^2\sqrt{a^2c}\,(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*a^(5/2)*x^
2*(x*(a*x+1))^(1/2)-44*a^(3/2)*x*(x*(a*x+1))^(1/2)+66*(x*(a*x+1))^(1/2)*a^(
1/2)-33*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(5/2)/(a*x
-1)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.36, size = 337, normalized size = 2.05

$$\left[\frac{33\,(ax-1)\sqrt{c}\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^{4}x^{4}-14\,a^{3}x^{3}+11\,a^{2}x^{2}+33\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{96\,(a^{4}x-a^{3})},\ \frac{33\,(ax-1)\sqrt{-c}\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(8\,a^{4}x^{4}-14\,a^{3}x^{3}+11\,a^{2}x^{2}+33\,ax)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{48\,(a^{4}x-a^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(33*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(8*a^4*x^4 - 14*a^3*x^3 + 11*a^2*x^2 + 33*a*x)*sqrt((a*x
- 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(33*(a*x - 1)*
sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((
a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 14*a^3*x^3 +
11*a^2*x^2 + 33*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^
4*x - a^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.518 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=124

$$-\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}}+\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

```
[Out] 7/4*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a^2-7/4*c*
x*(1-1/a^2/x^2)^(1/2)/a/(c-c/a/x)^(1/2)+1/2*c*x^2*(1-1/a^2/x^2)^(1/2)/(c-c/
a/x)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6313, 893, 887, 889, 214}

$$\frac{cx^2\sqrt{1-\frac{1}{a^2x^2}}}{2\sqrt{c-\frac{c}{ax}}}-\frac{7cx\sqrt{1-\frac{1}{a^2x^2}}}{4a\sqrt{c-\frac{c}{ax}}}+\frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] (-7*c*Sqrt[1 - 1/(a^2*x^2)]*x)/(4*a*Sqrt[c - c/(a*x)]) + (c*Sqrt[1 - 1/(a^2
*x^2)]*x^2)/(2*Sqrt[c - c/(a*x)]) + (7*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[1 - 1/
(a^2*x^2)])/Sqrt[c - c/(a*x)]])/(4*a^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 887

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e^2)*(d + e*x)^(m - 1)*(f + g*x)^(n + 1)*((a +
c*x^2)^(p + 1)/((n + 1)*(c*e*f + c*d*g))), x] - Dist[e*((m - n - 2)/((n + 1
)*(e*f + d*g))), Int[(d + e*x)^m*(f + g*x)^(n + 1)*(a + c*x^2)^p, x], x] /;
```

```
 FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e
^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} + \frac{7\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{4a}$$

$$= -\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} - \frac{7\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{8a^2}$$

$$= -\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} - \frac{(7c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\cdots}}{\sqrt{c\cdots}}\right.}{4a^4}$$

$$= -\frac{7c\sqrt{1-\frac{1}{a^2x^2}}\,x}{4a\sqrt{c-\frac{c}{ax}}} + \frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x^2}{2\sqrt{c-\frac{c}{ax}}} + \frac{7\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{4a^2}$$

Mathematica [A]
time = 0.23, size = 139, normalized size = 1.12

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(-7+2ax)}{-4+4ax} - \frac{7\sqrt{c}\,\log(1-ax)}{8a^2} + \frac{7\sqrt{c}\,\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^ArcCoth[a*x],x]
```

[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-7 + 2*a*x))/(-4 + 4*a*x) - (7*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (7*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(8*a^2)

Maple [A]
time = 0.09, size = 116, normalized size = 0.94

method	result
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{3}{2}}x\sqrt{x\,(ax+1)}-14\sqrt{x\,(ax+1)}\,\sqrt{a}+7\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right.}{8a^{\frac{3}{2}}(ax-1)\sqrt{x\,(ax+1)}}$
risch	$\frac{(2ax-7)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{7\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1}}{8a\sqrt{a^{2}c}\,(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x/a^(3/2)*(4*a^(3/2)*x*(x*(a*x+1))^(1/2)-14*(x*(a*x+1))^(1/2)*a^(1/2)+7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x-1)/(x*(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)

Fricas [A]
time = 0.38, size = 321, normalized size = 2.59

$$\left[\frac{7\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^{3}x^{3}-5\,a^{2}x^{2}-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{16\,(a^{3}x-a^{2})},-\frac{7\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)-2\,(2\,a^{3}x^{3}-5\,a^{2}x^{2}-7\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{8\,(a^{3}x-a^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")

[Out] [1/16*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(a*x - 1)) + 4*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x - 1)/(a*x + 1))*

```
sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(7*(a*x - 1)*sqrt(-c)*arctan(2
*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 5*a^2*x^2 - 7*a*x)*sqrt((a*x -
 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.519 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=79

$$\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

```
[Out] -3*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)/a+c*x*(1-1/
a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6312, 893, 889, 214}

$$\frac{cx\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (c*Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a*x)] - (3*Sqrt[c]*ArcTanh[(Sqrt[c]
*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]])/a
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 893

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(e*f - d*g)*(d + e*x)^(m - 2)*(f + g*x)^(n +
1)*((a + c*x^2)^(p + 1)/(c*g*(n + 1)*(e*f + d*g))), x] - Dist[e*((e*f*(p +
1) - d*g*(2*n + p + 3))/(g*(n + 1)*(e*f + d*g))), Int[(d + e*x)^(m - 1)*(f
+ g*x)^(n + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x]
&& NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p
 - 1, 0] && LtQ[n, -1] && IntegerQ[2*p]
```

Rule 6312

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^2), x], x, 1/x
], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] &
& (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1]) && IntegerQ[2*p]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{3\text{Subst}\left(\int\frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2a}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}+\frac{(3c^2)\,\text{Subst}\left(\int\frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx,x,\frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^3}$$

$$=\frac{c\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{ax}}}-\frac{3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.82

$$\frac{\sqrt{c-\frac{c}{ax}}\left(\sqrt{1+\frac{1}{ax}}\,x-\frac{3\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a*x)]*(Sqrt[1 + 1/(a*x)]*x - (3*ArcTanh[Sqrt[1 + 1/(a*x)]])/a)
)/Sqrt[1 - 1/(a*x)]
```

Maple [A]
time = 0.09, size = 101, normalized size = 1.28

method	result	si
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(2\sqrt{x(ax+1)}\,\sqrt{a}-3\ln\left(\frac{2\sqrt{x(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)\right)}{2(ax-1)\sqrt{x(ax+1)}\,\sqrt{a}}$	10
risch	$\frac{x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}-\frac{3\ln\left(\frac{\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax(ax+1)}}{2\sqrt{a^2c}\,(ax-1)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*(x*(a*x+1))^
(1/2)*a^(1/2)-3*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/(a*x
-1)/(x*(a*x+1))^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.
time = 0.36, size = 297, normalized size = 3.76

$$\left[\frac{3\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^2x-a)}, \frac{3\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(a^2x^2+ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^2x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)
/(a*x - 1)) + 4*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a), 1/2*(3*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqr
t(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*
x - c)) + 2*(a^2*x^2 + a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x
)))/(a^2*x - a)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.520 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=76

$$\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

```
[Out] 2*arctanh(c^(1/2)*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2))*c^(1/2)+2*c*(1-1/a^2
/x^2)^(1/2)/(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6313, 895, 889, 214}

$$\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}+2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*c*Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*
Sqrt[1 - 1/(a^2*x^2)])/Sqrt[c - c/(a*x)]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 895

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
```

```
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{x\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} - \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{a^2}+\frac{c^2x^2}{a^2}}\,dx, x, \frac{\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)}{a^2}\\
&= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}} + 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{ax}}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 132, normalized size = 1.74

$$
\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,x+\sqrt{c}\,(1-ax)\log(1-ax)+\sqrt{c}\,(-1+ax)\log\left(2a^2\sqrt{c}\,\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{-1+ax}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x + Sqrt[c]*(1 - a*x)*Log[1 -
a*x] + Sqrt[c]*(-1 + a*x)*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c -
c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(-1 + a*x)
```

Maple [A]
time = 0.09, size = 100, normalized size = 1.32

method	result	si
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+2\sqrt{x\,(ax+1)}\,\sqrt{a}\right)}{(ax-1)\sqrt{x\,(ax+1)}\,\sqrt{a}}$	10
risch	$\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{a\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{cax\,(ax+1)}}{\sqrt{a^{2}c}\,(ax-1)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(ln(1/2*(2*(x*(a*x+1)
)^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*(x*(a*x+1))^(1/2)*a^(1/2))/(a*x-1)/
(x*(a*x+1))^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(64) = 128.
time = 0.41, size = 275, normalized size = 3.62

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(ax-1)},\,-\frac{(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)-2\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) + 4*(a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/
(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x
 - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(a*
x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.521 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}$$

```
[Out] -8/3*a*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-2/3*a*(1-1/a^2/x^2)^(1/2)*(c-c
/a/x)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6313, 671, 663}

$$-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}-\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (-8*a*c*Sqrt[1 - 1/(a^2*x^2)])/(3*Sqrt[c - c/(a*x)]) - (2*a*Sqrt[1 - 1/(a^2
*x^2)]*Sqrt[c - c/(a*x)])/3
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
```

```
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}-\frac{4}{3}\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{8ac\sqrt{1-\frac{1}{a^2x^2}}}{3\sqrt{c-\frac{c}{ax}}}-\frac{2}{3}a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.66

$$
-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,(-1+5ax)}{-3+3ax}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^2),x]
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 5*a*x))/(-3 + 3*a*x)
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.77

method	result	size
gosper	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)x}$	54
default	$-\frac{2(ax+1)(5ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{3(ax-1)x}$	54

risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(5a^2x^2+4ax-1)}{3(ax-1)x}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(a*x+1)*(5*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A]
time = 0.39, size = 59, normalized size = 0.84

$$-\frac{2\left(5\,a^2x^2+4\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] -2/3*(5*a^2*x^2 + 4*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*
x))/(a*x^2 - x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(-1+\frac{1}{ax}\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x)))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Mupad [B]
time = 1.33, size = 54, normalized size = 0.77

$$-\frac{2\,\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(5\,a^2\,x^2+4\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
[Out] -(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(4*a*x + 5*a^2*x^2 - 1)
)/(3*x*(a*x - 1))
```

3.522 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}+\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

```
[Out] 2/5*a^2*(c-c/a/x)^(3/2)*(1-1/a^2/x^2)^(1/2)/c+8/5*a^2*c*(1-1/a^2/x^2)^(1/2)
/(c-c/a/x)^(1/2)+2/5*a^2*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6313, 809, 671, 663}

$$\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,\left(c-\frac{c}{ax}\right)^{3/2}}{5c}+\frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}+\frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (8*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(5*Sqrt[c - c/(a*x)]) + (2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*Sqrt[c - c/(a*x)])/5 + (2*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))
^(3/2))/(5*c)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{(3a)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{5c}$$

$$= \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} + \frac{1}{5}(4a)\text{Subst}\left(\int \frac{}{}\right.$$

$$= \frac{8a^2c\sqrt{1-\frac{1}{a^2x^2}}}{5\sqrt{c-\frac{c}{ax}}} + \frac{2}{5}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.51

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,(1-3ax+6a^2x^2)}{5x(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^3),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(1 - 3*a*x + 6*a^2*x^2))/(5*x*
(-1 + a*x))
```

Maple [A]
time = 0.08, size = 62, normalized size = 0.55

method	result	size
gosper	$\frac{2(ax+1)\left(6a^2x^2-3ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5(ax-1)x^2}$	62
default	$\frac{2(ax+1)\left(6a^2x^2-3ax+1\right)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{5(ax-1)x^2}$	62
risch	$\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,\left(6a^3x^3+3a^2x^2-2ax+1\right)}{5(ax-1)x^2}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(a*x+1)*(6*a^2*x^2-3*a*x+1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/
2)/(a*x-1)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

Fricas [A]
time = 0.34, size = 69, normalized size = 0.61

$$\frac{2\left(6\,a^3x^3+3\,a^2x^2-2\,ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{5\left(ax^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] 2/5*(6*a^3*x^3 + 3*a^2*x^2 - 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c
*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^3, x)
```

Mupad [B]
time = 1.35, size = 62, normalized size = 0.55

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(6\,a^3\,x^3+3\,a^2\,x^2-2\,a\,x+1\right)}{5\,x^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^3,x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(3*a^2*x^2 - 2*a*x + 6*a
^3*x^3 + 1))/(5*x^2*(a*x - 1))
```

3.523 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3}-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}\,x^2}$$

```
[Out] -104/105*a^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+2/7*c*(1-1/a^2/x^2)^(1/2
)/x^3/(c-c/a/x)^(1/2)-26/35*a*c*(1-1/a^2/x^2)^(1/2)/x^2/(c-c/a/x)^(1/2)-104
/105*a^3*(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6313, 895, 885, 809, 663}

$$-\frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35x^2\sqrt{c-\frac{c}{ax}}}+\frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7x^3\sqrt{c-\frac{c}{ax}}}-\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}-\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] (-104*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) - (104*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (2*c*Sqrt[1 - 1/(a^2*x^2)])/(7*Sq
rt[c - c/(a*x)]*x^3) - (26*a*c*Sqrt[1 - 1/(a^2*x^2)])/(35*Sqrt[c - c/(a*x)]
*x^2)
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 885

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-e)*(d + e*x)^(m - 1)*(f + g*x)^n*((a + c*x^2)^(
p + 1)/(c*(m - n - 1))), x] - Dist[n*((e*f + d*g)/(e*(m - n - 1))), Int[(d
+ e*x)^m*(f + g*x)^(n - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g
, m, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
&& EqQ[m + p, 0] && GtQ[n, 0] && NeQ[m - n - 1, 0] && (IntegerQ[2*p] || Int
egerQ[n])
```

Rule 895

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[e^2*(d + e*x)^(m - 2)*(f + g*x)^(n + 1)*((a + c*x
^2)^(p + 1)/(c*g*(n + p + 2))), x] - Dist[(e*f*(p + 1) - d*g*(2*n + p + 3))
/(g*(n + p + 2)), Int[(d + e*x)^(m - 1)*(f + g*x)^n*(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, f, g, m, n, p}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p - 1, 0] &&  !LtQ[n, -1] && Integer
Q[2*p]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - \frac{13}{7}\text{Subst}\left(\int \frac{x^2\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}\,x^2} + \frac{1}{35}(52a)\text{Subst}\left(\int \frac{x\sqrt{c-\frac{cx}{a}}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\right.$$

$$= -\frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - \frac{26ac\sqrt{1-\frac{1}{a^2x^2}}}{35\sqrt{c-\frac{c}{ax}}\,x^2} - \frac{1}{105}$$

$$= -\frac{104a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} - \frac{104}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}} + \frac{2c\sqrt{1-\frac{1}{a^2x^2}}}{7\sqrt{c-\frac{c}{ax}}\,x^3} - $$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.44

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{ax}}\,(-15+39ax-52a^2x^2+104a^3x^3)}{105x^2(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^ArcCoth[a*x]*x^4),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 39*a*x - 52*a^2*x^2 +
104*a^3*x^3))/(105*x^2*(-1 + a*x))
```

Maple [A]
time = 0.08, size = 70, normalized size = 0.47

method	result	size

gosper	$-\frac{2(ax+1)(104a^3x^3-52a^2x^2+39ax-15)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^3}$	70
default	$-\frac{2(ax+1)(104a^3x^3-52a^2x^2+39ax-15)\sqrt{\frac{c(ax-1)}{ax}}\,\sqrt{\frac{ax-1}{ax+1}}}{105(ax-1)x^3}$	70
risch	$-\frac{2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}\,(104a^4x^4+52a^3x^3-13a^2x^2+24ax-15)}{105(ax-1)x^3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(a*x+1)*(104*a^3*x^3-52*a^2*x^2+39*a*x-15)*(c*(a*x-1)/a/x)^(1/2)*((a
*x-1)/(a*x+1))^(1/2)/(a*x-1)/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

Fricas [A]
time = 0.35, size = 77, normalized size = 0.52

$$-\frac{2\left(104\,a^4x^4+52\,a^3x^3-13\,a^2x^2+24\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] -2/105*(104*a^4*x^4 + 52*a^3*x^3 - 13*a^2*x^2 + 24*a*x - 15)*sqrt((a*x - 1)
/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a*x))*sqrt((a*x - 1)/(a*x + 1))/x^4, x)
```

Mupad [B]
time = 1.41, size = 100, normalized size = 0.67

$$-\frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(104\,a^3\,x^3+156\,a^2\,x^2+143\,a\,x+167\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3}-\frac{304\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^4,x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*(143*a*x + 156*a^2*x^2 + 104*a^3*x^3 + 167
)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) - (304*((a*x - 1)/(a*x + 1))^(1/2)
*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.524 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=172

$$-\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\cdots}{\cdots}$$

```
[Out] 363/64*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^4-4*arctanh(1/2*(c-c/a/x)
^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^4-149/64*x*(c-c/a/x)^(1/2)/a^3+10
7/96*x^2*(c-c/a/x)^(1/2)/a^2-17/24*x^3*(c-c/a/x)^(1/2)/a+1/4*x^4*(c-c/a/x)^
(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {6302, 6268, 25, 528, 457, 100, 156, 162, 65, 214}

$$\frac{363\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{64a^4}-\frac{4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^4}-\frac{149x\sqrt{c-\frac{c}{ax}}}{64a^3}+\frac{107x^2\sqrt{c-\frac{c}{ax}}}{96a^2}+\frac{1}{4}x^4\sqrt{c-\frac{c}{ax}}-\frac{17x^3\sqrt{c-\frac{c}{ax}}}{24a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-149*Sqrt[c - c/(a*x)]*x)/(64*a^3) + (107*Sqrt[c - c/(a*x)]*x^2)/(96*a^2)
- (17*Sqrt[c - c/(a*x)]*x^3)/(24*a) + (Sqrt[c - c/(a*x)]*x^4)/4 + (363*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(64*a^4) - (4*Sqrt[2]*Sqrt[c]*ArcTa
nh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^4
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
```

```
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx=-\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$$

$$=-\int\frac{\sqrt{c-\frac{c}{ax}}\,x^3(1-ax)}{1+ax}\,dx$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^4}{1+ax}\,dx}{c}$$

$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{a+\frac{1}{x}}\,dx}{c}$$

$$=-\frac{a\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^5(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{\text{Subst}\left(\int\frac{\frac{17c^2}{2}-\frac{15c^2x}{2a}}{x^4(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{4c}$$

$$=-\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4-\frac{\text{Subst}\left(\int\frac{\frac{107c^3}{4}-\frac{85c^3x}{4a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{12ac^2}$$

$$=\frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4+\frac{\text{Subst}\left(\int\frac{\frac{447c^4}{8}-}{x^2(a+x)\sqrt{}}\right)}{24a}$$

$$=-\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4$$

$$=-\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4$$

$$=-\frac{149\sqrt{c-\frac{c}{ax}}\,x}{64a^3}+\frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2}-\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{24a}+\frac{1}{4}\sqrt{c-\frac{c}{ax}}\,x^4$$

$$\frac{149\sqrt{c-\frac{c}{ax}}\,x}{}\quad\frac{107\sqrt{c-\frac{c}{ax}}\,x^2}{}\quad\frac{17\sqrt{c-\frac{c}{ax}}\,x^3}{}\quad 1\quad\sqrt{c}$$

Mathematica [A]
time = 0.10, size = 116, normalized size = 0.67

$$\frac{a\sqrt{c-\frac{c}{ax}}\,x(-447+214ax-136a^2x^2+48a^3x^3)+1089\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-768\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-447 + 214*a*x - 136*a^2*x^2 + 48*a^3*x^3) + 1089*S
qrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 768*Sqrt[2]*Sqrt[c]*ArcTanh[Sqr
t[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/(192*a^4)
```

Maple [A]
time = 0.19, size = 259, normalized size = 1.51

method	result
risch	$\frac{(48a^3x^3-136a^2x^2+214ax-447)x\sqrt{\frac{c(ax-1)}{ax}}}{192a^3}+\frac{\left(\frac{363\ln\left(\frac{-\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2cx^2-acx}\right)}{128a^3\sqrt{a^2c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\cdots}{\cdots}\right)}{\cdots}\right.}{\cdots}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-96x(ax^2-x)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}+176(ax^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}-252\sqrt{ax^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+768\sqrt{(ax-1)x}\,a^{\frac{5}{2}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(c*(a*x-1)/a/x)^(1/2)*x*(-96*x*(a*x^2-x)^(3/2)*a^(9/2)*(1/a)^(1/2)+1
76*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)-252*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1
/2)*x+768*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)+126*(a*x^2-x)^(1/2)*a^(5/2)
*(1/a)^(1/2)-768*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2
)*a-3*a*x+1)/(a*x+1))-1152*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)
/a^(1/2))*(1/a)^(1/2)+63*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2)
)*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(11/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")

[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^3/(a*x + 1), x)

Fricas [A]
time = 0.35, size = 271, normalized size = 1.58

$$\left[\frac{768\,\sqrt{2}\,\sqrt{c}\,\log\left(\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+2\,(48\,a^4x^4-136\,a^3x^3+214\,a^2x^2-447\,ax)\sqrt{\frac{acx-c}{ax}}+1089\,\sqrt{c}\,\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{384\,a^4},\ \frac{768\,\sqrt{2}\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(48\,a^4x^4-136\,a^3x^3+214\,a^2x^2-447\,ax)\sqrt{\frac{acx-c}{ax}}-1089\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{192\,a^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/384*(768*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + 2*(48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) + 1089*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^4, 1/192*(768*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (48*a^4*x^4 - 136*a^3*x^3 + 214*a^2*x^2 - 447*a*x)*sqrt((a*c*x - c)/(a*x)) - 1089*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^4]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)

[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^3*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.525 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=147

$$\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{45\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}$$

```
[Out] -45/8*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^3+4*arctanh(1/2*(c-c/a/x)^
(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^3+19/8*x*(c-c/a/x)^(1/2)/a^2-13/12
*x^2*(c-c/a/x)^(1/2)/a+1/3*x^3*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6302, 6268, 25, 528, 457, 100, 156, 162, 65, 214}

$$-\frac{45\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{8a^3}+\frac{4\sqrt{2}\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)}{a^3}+\frac{19x\sqrt{c-\frac{c}{ax}}}{8a^2}+\frac{1}{3}x^3\sqrt{c-\frac{c}{ax}}-\frac{13x^2\sqrt{c-\frac{c}{ax}}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (19*Sqrt[c - c/(a*x)]*x)/(8*a^2) - (13*Sqrt[c - c/(a*x)]*x^2)/(12*a) + (Sqr
t[c - c/(a*x)]*x^3)/3 - (45*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(8*
a^3) + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^3
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] && !IntegerQ[p] && IntegerQ[n/2] && !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=-\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$$

$$=-\int \frac{\sqrt{c-\frac{c}{ax}}\,x^2(1-ax)}{1+ax}\,dx$$

$$=\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^3}{1+ax}\,dx}{c}$$

$$=\frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{a+\frac{1}{x}}\,dx}{c}$$

$$=-\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^4(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$

$$=\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{\text{Subst}\left(\int \frac{\frac{13c^2}{2}-\frac{11c^2x}{2a}}{x^3(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{3c}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{\text{Subst}\left(\int \frac{\frac{57c^3}{4}-\frac{39c^3x}{4a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx,x,\frac{1}{x}\right)}{6ac^2}$$

$$=\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{\text{Subst}\left(\int \frac{\frac{135c^4}{8}-\frac{57c^4}{8a}}{x(a+x)\sqrt{c-}}\right.}{6a^2c^3}$$

$$=\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3+\frac{(45c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-}}\right.}{16a^3}$$

$$=\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{45\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx,\right.}{8a^2}$$

$$=\frac{19\sqrt{c-\frac{c}{ax}}\,x}{8a^2}-\frac{13\sqrt{c-\frac{c}{ax}}\,x^2}{12a}+\frac{1}{3}\sqrt{c-\frac{c}{ax}}\,x^3-\frac{45\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.}{8a^3}$$

Mathematica [A]

time = 0.07, size = 108, normalized size = 0.73

$$\frac{a\sqrt{c-\frac{c}{ax}}\,x(57-26ax+8a^2x^2)-135\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)+96\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(57 - 26*a*x + 8*a^2*x^2) - 135*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/Sqrt[c]] + 96*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt
[2]*Sqrt[c])])/(24*a^3)
```

Maple [A]

time = 0.18, size = 237, normalized size = 1.61

method	result
risch	$\frac{(8a^2x^2-26ax+57)x\sqrt{\frac{c(ax-1)}{ax}}}{24a^2}+\frac{\left(-\frac{45\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{16a^2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a \ldots}}{\ldots}\right)}{a^3 \ldots} \ldots\right)}{ax-1 \ldots}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(16(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}-36\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+96\sqrt{(ax-1)\,x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+18\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\ldots} \ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*(a*x-1)/a/x)^(1/2)*x*(16*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1/2)-36*(a*
x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x+96*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)
+18*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)-96*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1
/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))-144*a^2*ln(1/2*(2*((a*x-1)*
x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)+9*ln(1/2*(2*(a*x^2-x)^(1/2)*
a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/((a*x-1)*x)^(1/2)/a^(9/2)/(1/a)^
(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x^2/(a*x + 1), x)
```

Fricas [A]
time = 0.34, size = 259, normalized size = 1.76

$$\left[\frac{96\sqrt{2}\sqrt{c}\log\left(-\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+3acx-c}{ax+1}\right)+2\left(8a^3x^3-26a^2x^2+57ax\right)\sqrt{\frac{acx-c}{ax}}+135\sqrt{c}\log\left(-2acx+2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{48a^3}, -\frac{96\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2c}\right)-\left(8a^3x^3-26a^2x^2+57ax\right)\sqrt{\frac{acx-c}{ax}}-135\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{24a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(96*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x
)) + 3*a*c*x - c)/(a*x + 1)) + 2*(8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*
c*x - c)/(a*x)) + 135*sqrt(c)*log(-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)
/(a*x)) + c))/a^3, -1/24*(96*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*s
qrt((a*c*x - c)/(a*x))/c) - (8*a^3*x^3 - 26*a^2*x^2 + 57*a*x)*sqrt((a*c*x -
 c)/(a*x)) - 135*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^2*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.526 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=122

$$-\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a}+\frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2+\frac{23\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a^2}$$

```
[Out] 23/4*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a^2-4*arctanh(1/2*(c-c/a/x)^(
1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a^2-9/4*x*(c-c/a/x)^(1/2)/a+1/2*x^2*(
c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6302, 6268, 25, 528, 457, 100, 156, 162, 65, 214}

$$\frac{23\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2}-\frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a^2}+\frac{1}{2}x^2\sqrt{c-\frac{c}{ax}}-\frac{9x\sqrt{c-\frac{c}{ax}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-9*Sqrt[c - c/(a*x)]*x)/(4*a) + (Sqrt[c - c/(a*x)]*x^2)/2 + (23*Sqrt[c]*Ar
cTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/(4*a^2) - (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt
[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a^2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$$
$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,x(1-ax)}{1+ax}\,dx$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x^2}{1+ax}\,dx}{c}$$
$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{a+\frac{1}{x}}\,dx}{c}$$
$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^3(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= \frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2 + \frac{\text{Subst}\left(\int \frac{\frac{9c^2}{2}-\frac{7c^2x}{2a}}{x^2(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2c}$$
$$= -\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2 - \frac{\text{Subst}\left(\int \frac{\frac{23c^3}{4}-\frac{9c^3x}{4a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2ac^2}$$
$$= -\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2 - \frac{(23c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{8a^2} +$$
$$= -\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2 + \frac{23\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)}{4a} -$$
$$= -\frac{9\sqrt{c-\frac{c}{ax}}\,x}{4a} + \frac{1}{2}\sqrt{c-\frac{c}{ax}}\,x^2 + \frac{23\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{4a^2} - \frac{4\sqrt{2}\,\sqrt{}}{}$$

Mathematica [A]

time = 0.06, size = 100, normalized size = 0.82

$$\frac{a\sqrt{c-\frac{c}{ax}}\,x(-9+2ax)+23\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-16\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (a*Sqrt[c - c/(a*x)]*x*(-9 + 2*a*x) + 23*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/
Sqrt[c]] - 16*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])
/(4*a^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(97) = 194$.
time = 0.19, size = 215, normalized size = 1.76

method	result
risch	$\frac{(2ax-9)x\sqrt{\frac{c(ax-1)}{ax}}}{4a}+\frac{\left(\frac{23\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{8a\sqrt{a^2c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2 \cdots}}{x+\frac{1}{a}}\right)}{a^2\sqrt{c}} \cdots\right)}{ax-1 \cdots}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(-4\sqrt{a\,x^2-x}\,a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x+16\sqrt{(ax-1)\,x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}+2\sqrt{a\,x^2-x}\,a^{\frac{5}{2}}\sqrt{\frac{1}{a}}-16a^{\frac{3}{2}}\sqrt{2}\,\ln\left(\frac{2\sqrt{2} \cdots}{\cdots}\right) \cdots\right)}{8\sqrt{(ax \cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*(a*x-1)/a/x)^(1/2)*x*(-4*(a*x^2-x)^(1/2)*a^(7/2)*(1/a)^(1/2)*x+16*(
(a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)+2*(a*x^2-x)^(1/2)*a^(5/2)*(1/a)^(1/2)-
16*a^(3/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(
a*x+1))-24*a^2*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^
(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2)/
((a*x-1)*x)^(1/2)/a^(7/2)/(1/a)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))*x/(a*x + 1), x)
```

Fricas [A]
time = 0.35, size = 239, normalized size = 1.96

$$\left[\frac{16\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}-3acx+c}{ax+1}\right)+2\left(2a^2x^2-9ax\right)\sqrt{\frac{acx-c}{ax}}+23\sqrt{c}\log\left(-2acx-2a\sqrt{c}x\sqrt{\frac{acx-c}{ax}}+c\right)}{8a^2}, \frac{16\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2c}\right)+\left(2a^2x^2-9ax\right)\sqrt{\frac{acx-c}{ax}}-23\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)}{4a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/8*(16*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x))
 - 3*a*c*x + c)/(a*x + 1)) + 2*(2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x))
+ 23*sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a^2
, 1/4*(16*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (2*a^2*x^2 - 9*a*x)*sqrt((a*c*x - c)/(a*x)) - 23*sqrt(-c)*arctan(s
qrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x*(c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.527 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=92

$$\sqrt{c-\frac{c}{ax}}\,x-\frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

```
[Out] -5*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)/a+4*arctanh(1/2*(c-c/a/x)^(1/2)
*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)/a+x*(c-c/a/x)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6268, 25, 528, 382, 100, 162, 65, 214}

$$x\sqrt{c-\frac{c}{ax}}-\frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a}+\frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4
*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{1+ax}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}x}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{a+\frac{1}{x}}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x^2(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{\text{Subst}\left(\int \frac{\frac{5c^2}{2}-\frac{3c^2x}{2a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \sqrt{c-\frac{c}{ax}}\,x + \frac{(5c)\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{2a} - \frac{(4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c\ldots}}\right.}{a}$$

$$= \sqrt{c-\frac{c}{ax}}\,x - 5\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) + 8\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\ldots\right.$$

$$= \sqrt{c-\frac{c}{ax}}\,x - \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\ldots\right.}{a}$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 1.00

$$\sqrt{c-\frac{c}{ax}}\,x - \frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)}{a} + \frac{4\sqrt{2}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c - c/(a*x)]/E^(2*ArcCoth[a*x]),x]

[Out] Sqrt[c - c/(a*x)]*x - (5*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]])/a + (4*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])])/a

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(75) = 150$.
time = 0.18, size = 189, normalized size = 2.05

method	result
risch	$x\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(-\frac{5\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{2\sqrt{a^2c}}-\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\,(}}{x+\frac{1}{a}}\right)}{a\sqrt{c}}\right.}{ax-1}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-2\sqrt{a\,x^2-x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}-6\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)a\sqrt{\frac{1}{a}}-4\sqrt{2}\right.}{2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/2*(c*(a*x-1)/a/x)^(1/2)*x*(4*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)-2*(a*x^2-x)^(1/2)*a^(3/2)*(1/a)^(1/2)-6*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2)-4*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(1/2)+ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(3/2)/(1/a)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")

[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/(a*x + 1), x)

Fricas [A]
time = 0.40, size = 219, normalized size = 2.38

$$\left[\frac{2\,ax\sqrt{\frac{acx-c}{ax}}+4\,\sqrt{2}\,\sqrt{c}\,\log\left(-\frac{2\,\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)+5\,\sqrt{c}\,\log\left(-2\,acx+2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)}{2\,a},\ \frac{ax\sqrt{\frac{acx-c}{ax}}-4\,\sqrt{2}\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+5\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{c}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*c*x - c)/(a*x)) + 4*sqrt(2)*sqrt(c)*log(-(2*sqrt(2)*a*s
qrt(c)*x*sqrt((a*c*x - c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) + 5*sqrt(c)*log(
-2*a*c*x + 2*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) + c))/a, (a*x*sqrt((a*c*x
- c)/(a*x)) - 4*sqrt(2)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x -
c)/(a*x))/c) + 5*sqrt(-c)*arctan(sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.528 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=86

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

```
[Out] 2*arctanh((c-c/a/x)^(1/2)/c^(1/2))*c^(1/2)-4*arctanh(1/2*(c-c/a/x)^(1/2)*2^
(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+2*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6268, 25, 445, 457, 86, 162, 65, 214}

$$2\sqrt{c-\frac{c}{ax}}+2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right)-4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{1+ax}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{x(a+x)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= 2\sqrt{c-\frac{c}{ax}} - \frac{a\text{Subst}\left(\int \frac{c^2-\frac{3c^2x}{a}}{x(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= 2\sqrt{c-\frac{c}{ax}} - c\text{Subst}\left(\int \frac{1}{x\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right) + (4c)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{\cdots}}\right.$$

$$= 2\sqrt{c-\frac{c}{ax}} + (2a)\text{Subst}\left(\int \frac{1}{a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right) - (8a)\text{Subst}\left(\int \frac{\cdots}{2a\cdots}\right.$$

$$= 2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right.$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 1.00

$$2\sqrt{c-\frac{c}{ax}} + 2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{c}}\right) - 4\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] 2*Sqrt[c - c/(a*x)] + 2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/Sqrt[c]] - 4*Sqrt
[2]*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(69) = 138$.
time = 0.18, size = 228, normalized size = 2.65

method	result
risch	$2\sqrt{\frac{c(ax-1)}{ax}}+\frac{\left(\frac{a\ln\left(\frac{-\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-acx}\right)}{\sqrt{a^2c}}+\frac{2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x\right.}}{x+\frac{1}{a}}\right.}{\sqrt{c}}\right.}{ax-1}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(2\sqrt{(ax-1)\,x}\,a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,x^2-4a^{\frac{3}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^2-3\ln\left(\frac{2\sqrt{(ax-1)\,x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\,a\right.}{x\sqrt{(a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(a*x-1)/a/x)^(1/2)/x*(2*((a*x-1)*x)^(1/2)*a^(3/2)*(1/a)^(1/2)*x^2-4*a^(
3/2)*(1/a)^(1/2)*(a*x^2-x)^(1/2)*x^2-3*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+
2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a*x^2-2*a^(1/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(
1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^2+2*(a*x^2-x)^(3/2)*a^(1/2)*(1
/a)^(1/2)+2*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))
*a*x^2)/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x), x)
```

Fricas [A]
time = 0.34, size = 203, normalized size = 2.36

$$\left[2\sqrt{2}\sqrt{c}\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\sqrt{c}\log\left(-2\,acx-2\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+c\right)+2\sqrt{\frac{acx-c}{ax}},\,4\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{c}\right)+2\sqrt{\frac{acx-c}{ax}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(2)*sqrt(c)*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a*x)) - 3*a
*c*x + c)/(a*x + 1)) + sqrt(c)*log(-2*a*c*x - 2*a*sqrt(c)*x*sqrt((a*c*x - c
)/(a*x)) + c) + 2*sqrt((a*c*x - c)/(a*x)), 4*sqrt(2)*sqrt(-c)*arctan(1/2*sq
rt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) - 2*sqrt(-c)*arctan(sqrt(-c)*sqrt
((a*c*x - c)/(a*x))/c) + 2*sqrt((a*c*x - c)/(a*x))]
```

Sympy [A]
time = 6.97, size = 80, normalized size = 0.93

$$-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{4\sqrt{2}\,c\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-\frac{c}{ax}}}{2\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c-\frac{c}{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] -2*c*atan(sqrt(c - c/(a*x))/sqrt(-c))/sqrt(-c) + 4*sqrt(2)*c*atan(sqrt(2)*s
qrt(c - c/(a*x))/(2*sqrt(-c)))/sqrt(-c) + 2*sqrt(c - c/(a*x))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.529 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=82

$$-4a\sqrt{c-\frac{c}{ax}}-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

```
[Out] -2/3*a*(c-c/a/x)^(3/2)/c+4*a*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^(1/2)-4*a*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6302, 6268, 25, 528, 455, 52, 65, 214}

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] -4*a*Sqrt[c - c/(a*x)] - (2*a*(c - c/(a*x))^(3/2))/(3*c) + 4*Sqrt[2]*a*Sqrt
[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^2(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^2}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - (2a)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx, x, \frac{1}{x}\right)$$

$$= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - (4ac)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{c-\frac{cx}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \left(8a^2\right)\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-\frac{c}{ax}}\right)$$

$$= -4a\sqrt{c-\frac{c}{ax}} - \frac{2a\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 0.84

$$\frac{2\sqrt{c-\frac{c}{ax}}\,(1-7ax)}{3x} + 4\sqrt{2}\,a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

[Out] (2*Sqrt[c - c/(a*x)]*(1 - 7*a*x))/(3*x) + 4*Sqrt[2]*a*Sqrt[c]*ArcTanh[Sqrt[
c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(67) = 134$.
time = 0.20, size = 254, normalized size = 3.10

method	result
risch	$-\frac{2(7a^2x^2-8ax+1)\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)} - \frac{2a\sqrt{2}\ \ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\ \sqrt{c}\ \sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+2c}}{x+\frac{1}{a}}\right)}{\sqrt{c}\ (ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(6\sqrt{(ax-1)x}\ a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\ x^3-18a^{\frac{5}{2}}\sqrt{\frac{1}{a}}\ \sqrt{a\,x^2-x}\ x^3-9\ln\left(\frac{2\sqrt{(ax-1)x}\ \sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\frac{1}{a}}\ \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)

[Out] 1/3*(c*(a*x-1)/a/x)^(1/2)/x^2*(6*((a*x-1)*x)^(1/2)*a^(5/2)*(1/a)^(1/2)*x^3-
18*a^(5/2)*(1/a)^(1/2)*(a*x^2-x)^(1/2)*x^3-9*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^
(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^2*x^3-6*a^(3/2)*2^(1/2)*ln((2*2^(1/2)
(1/a)^(1/2)((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^3+12*a^(3/2)*(1/a)^(1/
2)*(a*x^2-x)^(3/2)*x+9*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2)+2*a*x-
1)/a^(1/2))*a^2*x^3-2*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2
)/a^(1/2)/(1/a)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")

[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^2), x)

Fricas [A]
time = 0.37, size = 161, normalized size = 1.96

$$\left[\frac{2\left(3\sqrt{2}\,a\sqrt{c}\,x\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x},\ -\frac{2\left(6\sqrt{2}\,a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{2}\,\sqrt{-c}\,\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(7\,ax-1)\sqrt{\frac{acx-c}{ax}}\right)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a*sqrt(c)*x*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x - c)/(a
*x)) + 3*a*c*x - c)/(a*x + 1)) - (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x, -2
/3*(6*sqrt(2)*a*sqrt(-c)*x*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*
x))/c) + (7*a*x - 1)*sqrt((a*c*x - c)/(a*x)))/x]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.530 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=113

$$4a^2\sqrt{c-\frac{c}{ax}}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

```
[Out] 2/3*a^2*(c-c/a/x)^(3/2)/c+2/5*a^2*(c-c/a/x)^(5/2)/c^2-4*a^2*arctanh(1/2*(c-
c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^2*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 81, 52, 65, 214}

$$\frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^2\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] 4*a^2*Sqrt[c - c/(a*x)] + (2*a^2*(c - c/(a*x))^(3/2))/(3*c) + (2*a^2*(c - c
/(a*x))^(5/2))/(5*c^2) - 4*Sqrt[2]*a^2*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(S
qrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^3(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^2(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^3}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \frac{a^2\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(2a^2\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx,x,\frac{1}{x}\right)$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} + \left(4a^2c\right)\text{Subst}\left(\int \frac{\quad}{(a+\quad}\right.$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \left(8a^3\right)\text{Subst}\left(\int \frac{1}{2a-\quad}\right.$$

$$= 4a^2\sqrt{c-\frac{c}{ax}} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^2\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - 4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\quad\right.$$

Mathematica [A]

time = 0.11, size = 79, normalized size = 0.70

$$\frac{2\sqrt{c-\frac{c}{ax}}\,(3-11ax+38a^2x^2)}{15x^2}-4\sqrt{2}\,a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 11*a*x + 38*a^2*x^2))/(15*x^2) - 4*Sqrt[2]*a^2*Sq
rt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(94) = 188.
time = 0.20, size = 278, normalized size = 2.46

method	result
risch	$\frac{2(38a^3x^3-49a^2x^2+14ax-3)\sqrt{\frac{c(ax-1)}{ax}}}{15x^2(ax-1)}+\frac{2a^2\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\frac{1}{a}\right)ac+}}{x+\frac{1}{a}}\right.}{\sqrt{c}\,(ax-1)}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(30\sqrt{(ax-1)x}\,\sqrt{\frac{1}{a}}\,a^{\frac{7}{2}}x^4-90a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,\sqrt{a\,x^2-x}\,x^4-45\ln\left(\frac{2\sqrt{(ax-1)x}\,\sqrt{a}+2ax-1}{2\sqrt{a}}\right)\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(c*(a*x-1)/a/x)^(1/2)/x^3*(30*((a*x-1)*x)^(1/2)*(1/a)^(1/2)*a^(7/2)*x
^4-90*a^(7/2)*(1/a)^(1/2)*(a*x^2-x)^(1/2)*x^4-45*ln(1/2*(2*((a*x-1)*x)^(1/2
)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a)^(1/2)*a^3*x^4-30*2^(1/2)*ln((2*2^(1/2)*(1
/a)^(1/2)*((a*x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*a^(5/2)*x^4+60*a^(5/2)*(1/a
)^(1/2)*(a*x^2-x)^(3/2)*x^2+45*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2
)+2*a*x-1)/a^(1/2))*a^3*x^4-16*a^(3/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x+6*(a*x
^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^3), x)
```

Fricas [A]
time = 0.40, size = 181, normalized size = 1.60

$$\left[\frac{2\left(15\sqrt{2}\,a^2\sqrt{c}\,x^2\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2}, \frac{2\left(30\sqrt{2}\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+\left(38\,a^2x^2-11\,ax+3\right)\sqrt{\frac{acx-c}{ax}}\right)}{15\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [2/15*(15*sqrt(2)*a^2*sqrt(c)*x^2*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x
 - c)/(a*x)))/x^2, 2/15*(30*sqrt(2)*a^2*sqrt(-c)*x^2*arctan(1/2*sqrt(2)*sqr
t(-c)*sqrt((a*c*x - c)/(a*x))/c) + (38*a^2*x^2 - 11*a*x + 3)*sqrt((a*c*x -
c)/(a*x)))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(94) = 188.
time = 0.75, size = 278, normalized size = 2.46

$$\frac{4\sqrt{2}\,a^3c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}\left(x\right)}+\frac{2\left(60\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^4a^5c-45\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^3a^4c^{\frac{3}{2}}|a|+35\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^2a^5c^2-15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a^4c^{\frac{5}{2}}|a|+3\,a^5c^3\right)}{15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^5a^2|a|\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^3*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/15*(60*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4*a^5*c - 45*(sqrt(a^2*c)*x - sqrt(
```

```
a^2*c*x^2 - a*c*x))^3*a^4*c^(3/2)*abs(a) + 35*(sqrt(a^2*c)*x - sqrt(a^2*c*x
^2 - a*c*x))^2*a^5*c^2 - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^4*c
^(5/2)*abs(a) + 3*a^5*c^3)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^5*a^2
*abs(a)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.531 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=113

$$-4a^3\sqrt{c-\frac{c}{ax}}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

```
[Out] -2/3*a^3*(c-c/a/x)^(3/2)/c-2/7*a^3*(c-c/a/x)^(7/2)/c^3+4*a^3*arctanh(1/2*(c
-c/a/x)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^(1/2)-4*a^3*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6268, 25, 528, 457, 90, 52, 65, 214}

$$-\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}-\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c}-4a^3\sqrt{c-\frac{c}{ax}}+4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] -4*a^3*Sqrt[c - c/(a*x)] - (2*a^3*(c - c/(a*x))^(3/2))/(3*c) - (2*a^3*(c -
c/(a*x))^(7/2))/(7*c^3) + 4*Sqrt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^4(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^3(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^4}\,dx}{c}$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{a\text{Subst}\left(\int \left(\frac{a^2\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}-\frac{a\left(c-\frac{cx}{a}\right)^{5/2}}{c}\right)\,dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \frac{a^3\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= -\frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \left(2a^3\right)\text{Subst}\left(\int \frac{\sqrt{c-\frac{cx}{a}}}{a+x}\,dx,x,\frac{1}{x}\right)$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} - \left(4a^3c\right)\text{Subst}\left(\int \frac{\cdots}{(a+\cdots}\right.$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \left(8a^4\right)\text{Subst}\left(\int \frac{1\cdots}{2a-\cdots}\right.$$

$$= -4a^3\sqrt{c-\frac{c}{ax}} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{3/2}}{3c} - \frac{2a^3\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + 4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\cdots\right.$$

Mathematica [A]

time = 0.15, size = 87, normalized size = 0.77

$$\frac{2\sqrt{c-\frac{c}{ax}}\,(3-9ax+16a^2x^2-52a^3x^3)}{21x^3}+4\sqrt{2}\,a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(3 - 9*a*x + 16*a^2*x^2 - 52*a^3*x^3))/(21*x^3) + 4*Sq
rt[2]*a^3*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqrt[c])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(94) = 188$.
time = 0.19, size = 302, normalized size = 2.67

method	result
risch	$-\frac{2(52a^4x^4-68a^3x^3+25a^2x^2-12ax+3)\sqrt{\frac{c(ax-1)}{ax}}}{21x^3(ax-1)}-\frac{2a^3\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-3\left(x+\right.}}{x+\frac{1}{a}}\right.}{\sqrt{c}\,(ax-1)}$
default	$\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(42\sqrt{(ax-1)x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^5-126\sqrt{a\,x^2-x}\,a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^5+84(a\,x^2-x)^{\frac{3}{2}}a^{\frac{7}{2}}\sqrt{\frac{1}{a}}\,x^3-63\ln\left(\frac{2\sqrt{(ax-}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*(c*(a*x-1)/a/x)^(1/2)/x^4*(42*((a*x-1)*x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^
5-126*(a*x^2-x)^(1/2)*a^(9/2)*(1/a)^(1/2)*x^5+84*(a*x^2-x)^(3/2)*a^(7/2)*(1
/a)^(1/2)*x^3-63*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*(1/a
)^(1/2)*a^4*x^5-42*a^(7/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*x-1)*x)^(1
/2)*a-3*a*x+1)/(a*x+1))*x^5+63*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(1/2)*a^(1/2
)+2*a*x-1)/a^(1/2))*a^4*x^5-20*a^(5/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x^2+12*a
^(3/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x-6*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))
/((a*x-1)*x)^(1/2)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^4), x)
```

Fricas [A]
time = 0.35, size = 201, normalized size = 1.78

$$\left[\frac{2\left(21\sqrt{2}\,a^3\sqrt{c}\,x^3\log\left(-\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}+3\,acx-c}{ax+1}\right)-(52\,a^3x^3-16\,a^2x^2+9\,ax-3)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3}, -\frac{2\left(42\sqrt{2}\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2\,c}\right)+(52\,a^3x^3-16\,a^2x^2+9\,ax-3)\sqrt{\frac{acx-c}{ax}}\right)}{21\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [2/21*(21*sqrt(2)*a^3*sqrt(c)*x^3*log(-(2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x -
 c)/(a*x)) + 3*a*c*x - c)/(a*x + 1)) - (52*a^3*x^3 - 16*a^2*x^2 + 9*a*x - 3
)*sqrt((a*c*x - c)/(a*x)))/x^3, -2/21*(42*sqrt(2)*a^3*sqrt(-c)*x^3*arctan(1
/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (52*a^3*x^3 - 16*a^2*x^2 +
 9*a*x - 3)*sqrt((a*c*x - c)/(a*x)))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(94) = 188.
time = 0.80, size = 356, normalized size = 3.15

$$-\frac{4\sqrt{2}\,a^4c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2\,a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}\left(x\right)}-\frac{2\left(84\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^6a^7c-84\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^5a^6c^{\frac{3}{2}}|a|+112\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^4a^7c^2-105\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^3a^6c^{\frac{5}{2}}|a|+63\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^2a^7c^3-21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a^6c^{\frac{7}{2}}|a|+3\,a^7c^4\right)}{21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^7a^3|a|\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] -4*sqrt(2)*a^4*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) - 2/21*(84*(
```

```
sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^6*a^7*c - 84*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - a*c*x))^5*a^6*c^(3/2)*abs(a) + 112*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - a*c*x))^4*a^7*c^2 - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*
a^6*c^(5/2)*abs(a) + 63*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^7*c^3
 - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^6*c^(7/2)*abs(a) + 3*a^7*
c^4)/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^7*a^3*abs(a)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^4\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.532 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=163

$$4a^4\sqrt{c-\frac{c}{ax}}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\right.$$

```
[Out] 2/3*a^4*(c-c/a/x)^(3/2)/c+2/5*a^4*(c-c/a/x)^(5/2)/c^2-2/7*a^4*(c-c/a/x)^(7/
2)/c^3+2/9*a^4*(c-c/a/x)^(9/2)/c^4-4*a^4*arctanh(1/2*(c-c/a/x)^(1/2)*2^(1/2
)/c^(1/2))*2^(1/2)*c^(1/2)+4*a^4*(c-c/a/x)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6268, 25, 528, 457, 90, 52, 65, 214}

$$\frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4}-\frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2}+\frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c}+4a^4\sqrt{c-\frac{c}{ax}}-4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] 4*a^4*Sqrt[c - c/(a*x)] + (2*a^4*(c - c/(a*x))^(3/2))/(3*c) + (2*a^4*(c - c
/(a*x))^(5/2))/(5*c^2) - (2*a^4*(c - c/(a*x))^(7/2))/(7*c^3) + (2*a^4*(c -
c/(a*x))^(9/2))/(9*c^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(
Sqrt[2]*Sqrt[c])]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$$

$$= -\int \frac{\sqrt{c-\frac{c}{ax}}\,(1-ax)}{x^5(1+ax)}\,dx$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{x^4(1+ax)}\,dx}{c}$$

$$= \frac{a\int \frac{\left(c-\frac{c}{ax}\right)^{3/2}}{\left(a+\frac{1}{x}\right)x^5}\,dx}{c}$$

$$= -\frac{a\mathrm{Subst}\left(\int \frac{x^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{a\mathrm{Subst}\left(\int \left(a^2\left(c-\frac{cx}{a}\right)^{3/2} - \frac{a^3\left(c-\frac{cx}{a}\right)^{3/2}}{a+x} - \frac{a^2\left(c-\frac{cx}{a}\right)^{5/2}}{c} + \frac{a^2\left(c-\frac{cx}{a}\right)^{7/2}}{c^2}\right)\,dx, x,}{c}$$

$$= \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + \frac{a^4\mathrm{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{a+x}\right.}{c}$$

$$= \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{9/2}}{9c^4} + (2a$$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + 2a^4($$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + 2a^4($$

$$= 4a^4\sqrt{c-\frac{c}{ax}} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{3/2}}{3c} + \frac{2a^4\left(c-\frac{c}{ax}\right)^{5/2}}{5c^2} - \frac{2a^4\left(c-\frac{c}{ax}\right)^{7/2}}{7c^3} + 2a^4($$

Mathematica [A]

time = 0.18, size = 95, normalized size = 0.58

$$\frac{2\sqrt{c-\frac{c}{ax}}\,(35-95ax+138a^2x^2-236a^3x^3+788a^4x^4)}{315x^4}-4\sqrt{2}\,a^4\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-\frac{c}{ax}}}{\sqrt{2}\,\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (2*Sqrt[c - c/(a*x)]*(35 - 95*a*x + 138*a^2*x^2 - 236*a^3*x^3 + 788*a^4*x^4
))/(315*x^4) - 4*Sqrt[2]*a^4*Sqrt[c]*ArcTanh[Sqrt[c - c/(a*x)]/(Sqrt[2]*Sqr
t[c])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(136) = 272$.
time = 0.20, size = 326, normalized size = 2.00

method	result
risch	$\frac{2(788a^5x^5-1024a^4x^4+374a^3x^3-233a^2x^2+130ax-35)\sqrt{\frac{c(ax-1)}{ax}}}{315x^4(ax-1)}+\frac{2a^4\sqrt{2}\,\ln\left(\frac{4c-3\left(x+\frac{1}{a}\right)ac+2\sqrt{2}\,\sqrt{c}\,\sqrt{a^2c\left(x+\frac{1}{a}\right.}}{x+\frac{1}{a}}\right.}{}$
default	$-\frac{\sqrt{\frac{c(ax-1)}{ax}}\left(630\sqrt{(ax-1)x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^6-1890\sqrt{a\,x^2-x}\,a^{\frac{11}{2}}\sqrt{\frac{1}{a}}\,x^6+1260(a\,x^2-x)^{\frac{3}{2}}a^{\frac{9}{2}}\sqrt{\frac{1}{a}}\,x^4-945\ln\left(\frac{2\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*(a*x-1)/a/x)^(1/2)/x^5*(630*((a*x-1)*x)^(1/2)*a^(11/2)*(1/a)^(1/2
)*x^6-1890*(a*x^2-x)^(1/2)*a^(11/2)*(1/a)^(1/2)*x^6+1260*(a*x^2-x)^(3/2)*a^
(9/2)*(1/a)^(1/2)*x^4-945*ln(1/2*(2*((a*x-1)*x)^(1/2)*a^(1/2)+2*a*x-1)/a^(1
/2))*(1/a)^(1/2)*a^5*x^6-630*a^(9/2)*2^(1/2)*ln((2*2^(1/2)*(1/a)^(1/2)*((a*
x-1)*x)^(1/2)*a-3*a*x+1)/(a*x+1))*x^6+945*(1/a)^(1/2)*ln(1/2*(2*(a*x^2-x)^(
1/2)*a^(1/2)+2*a*x-1)/a^(1/2))*a^5*x^6-316*(a*x^2-x)^(3/2)*a^(7/2)*(1/a)^(1
/2)*x^3+156*a^(5/2)*(1/a)^(1/2)*(a*x^2-x)^(3/2)*x^2-120*a^(3/2)*(1/a)^(1/2)
*(a*x^2-x)^(3/2)*x+70*(a*x^2-x)^(3/2)*a^(1/2)*(1/a)^(1/2))/((a*x-1)*x)^(1/2
)/a^(1/2)/(1/a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a*x))/((a*x + 1)*x^5), x)
```

Fricas [A]
time = 0.35, size = 213, normalized size = 1.31

$$\left[\frac{2\left(315\sqrt{2}\,a^4\sqrt{c}\,x^4\log\left(\frac{2\sqrt{2}\,a\sqrt{c}\,x\sqrt{\frac{acx-c}{ax}}-3\,acx+c}{ax+1}\right)+\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-c}{ax}}\right)}{315\,x^4},\ \frac{2\left(630\sqrt{2}\,a^4\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{2}\sqrt{-c}\sqrt{\frac{acx-c}{ax}}}{2c}\right)+\left(788\,a^4x^4-236\,a^3x^3+138\,a^2x^2-95\,ax+35\right)\sqrt{\frac{acx-c}{ax}}\right)}{315\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [2/315*(315*sqrt(2)*a^4*sqrt(c)*x^4*log((2*sqrt(2)*a*sqrt(c)*x*sqrt((a*c*x
- c)/(a*x)) - 3*a*c*x + c)/(a*x + 1)) + (788*a^4*x^4 - 236*a^3*x^3 + 138*a^
2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/x^4, 2/315*(630*sqrt(2)*a^4*s
qrt(-c)*x^4*arctan(1/2*sqrt(2)*sqrt(-c)*sqrt((a*c*x - c)/(a*x))/c) + (788*a
^4*x^4 - 236*a^3*x^3 + 138*a^2*x^2 - 95*a*x + 35)*sqrt((a*c*x - c)/(a*x)))/
x^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(136) = 272.
time = 0.95, size = 434, normalized size = 2.66

$$\frac{4\sqrt{2}\,a^5c\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a+\sqrt{c}\,|a|\right)}{2a\sqrt{-c}}\right)}{\sqrt{-c}\,|a|\mathrm{sgn}\left(x\right)}+\frac{2\left(1260\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^8a^9c-1260\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^7a^8c^{\frac{3}{2}}|a|+2100\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^6a^9c^2-3150\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^5a^8c^{\frac{5}{2}}|a|+3528\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^4a^9c^3-2625\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^3a^8c^{\frac{7}{2}}|a|+1215\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^2a^9c^4-315\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)a^8c^{\frac{9}{2}}|a|+35\,a^9c^5\right)}{315\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-acx}\right)^9a^4|a|\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] 4*sqrt(2)*a^5*c*arctan(1/2*sqrt(2)*((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x
))*a + sqrt(c)*abs(a))/(a*sqrt(-c)))/(sqrt(-c)*abs(a)*sgn(x)) + 2/315*(1260
```

```
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^8*a^9*c - 1260*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - a*c*x))^7*a^8*c^(3/2)*abs(a) + 2100*(sqrt(a^2*c)*x - sqrt(
a^2*c*x^2 - a*c*x))^6*a^9*c^2 - 3150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*
x))^5*a^8*c^(5/2)*abs(a) + 3528*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^4
*a^9*c^3 - 2625*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^3*a^8*c^(7/2)*abs
(a) + 1215*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^2*a^9*c^4 - 315*(sqrt(
a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))*a^8*c^(9/2)*abs(a) + 35*a^9*c^5)/((sqrt
(a^2*c)*x - sqrt(a^2*c*x^2 - a*c*x))^9*a^4*abs(a)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,\left(a\,x-1\right)}{x^5\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
[Out] int(((c - c/(a*x))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.533 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx$

Optimal. Leaf size=303

$$-\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}-\frac{1115\sqrt{c-\frac{c}{ax}}\,x}{192a^3\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}+\frac{223\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}\,x^3}{24a\sqrt{1-\frac{1}{ax}}\,\sqrt{1+}}$$

```
[Out] 1115/64*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)-1115/6
4*(c-c/a/x)^(1/2)/a^4/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-1115/192*x*(c-c/a/x)^
(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+223/96*x^2*(c-c/a/x)^(1/2)/a^2/(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-25/24*x^3*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)/(
1+1/a/x)^(1/2)+1/4*x^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6317, 6315, 91, 79, 44, 53, 65, 214}

$$-\frac{1115\sqrt{c-\frac{c}{ax}}}{64a^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{1115\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{64a^4\sqrt{1-\frac{1}{ax}}}-\frac{1115x\sqrt{c-\frac{c}{ax}}}{192a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{223x^2\sqrt{c-\frac{c}{ax}}}{96a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{x^4\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{25x^3\sqrt{c-\frac{c}{ax}}}{24a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-1115*Sqrt[c - c/(a*x)])/(64*a^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (1
115*Sqrt[c - c/(a*x)]*x)/(192*a^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (2
23*Sqrt[c - c/(a*x)]*x^2)/(96*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2
5*Sqrt[c - c/(a*x)]*x^3)/(24*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt
[c - c/(a*x)]*x^4)/(4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (1115*Sqrt[c -
 c/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(64*a^4*Sqrt[1 - 1/(a*x)])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
```

```
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^3\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^3\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^5\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{25}{2a}+\frac{4x}{a^2}}{x^4\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{25\sqrt{c-\frac{c}{ax}}\,x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^4}{4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(223\sqrt{c-\frac{c}{ax}}\right)\text{S}}{48a}$$

$$=-\frac{223\sqrt{c-\frac{c}{ax}}\,x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{25\sqrt{c-\frac{c}{ax}}\,x^3}{24a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}}{4\sqrt{1-\frac{1}{ax}}\sqrt{1}}$$

$$=-\frac{223\sqrt{c-\frac{c}{ax}}\,x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x^2}{96a^2\sqrt{1-\frac{1}{ax}}}-\frac{25\sqrt{c-}}{24a\sqrt{1-}}$$

$$=-\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{64a^3\sqrt{1-\frac{1}{ax}}}-\frac{223\sqrt{c-\frac{c}{ax}}\,x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1115\sqrt{1+}}{96a^2\sqrt{}}$$

$$=-\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{64a^3\sqrt{1-\frac{1}{ax}}}-\frac{223\sqrt{c-\frac{c}{ax}}\,x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1115\sqrt{1+}}{96a^2\sqrt{}}$$

$$=-\frac{1115\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{64a^3\sqrt{1-\frac{1}{ax}}}-\frac{223\sqrt{c-\frac{c}{ax}}\,x^2}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{1115\sqrt{1+}}{96a^2\sqrt{}}$$

Mathematica [A]
time = 0.55, size = 167, normalized size = 0.55

$$\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2(-3345-1115ax+446a^2x^2-200a^3x^3+48a^4x^4)}{-1+a^2x^2}-3345\sqrt{c}\log(1-ax)+3345\sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}x^2+c(-1-ax+2a^2x^2)\right)}{384a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-3345 - 1115*a*x + 446
*a^2*x^2 - 200*a^3*x^3 + 48*a^4*x^4))/(-1 + a^2*x^2) - 3345*Sqrt[c]*Log[1 -
 a*x] + 3345*Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*
x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)])/(384*a^4)
```

Maple [A]
time = 0.10, size = 197, normalized size = 0.65

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(96a^{\frac{9}{2}}\sqrt{x\,(ax+1)}\,x^4-400a^{\frac{7}{2}}x^3\sqrt{x\,(ax+1)}+892a^{\frac{5}{2}}x^2\sqrt{x\,(ax+1)}-2230a^{\frac{3}{2}}x\right.}{384(ax-}$
risch	$\frac{(48a^3x^3-248a^2x^2+694ax-1809)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{192a^3(ax-1)}+\frac{\left(\frac{1115\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{128a^3\sqrt{a^2c}}-\frac{8\sqrt{a^2c\,(x}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(96
*a^(9/2)*(x*(a*x+1))^(1/2)*x^4-400*a^(7/2)*x^3*(x*(a*x+1))^(1/2)+892*a^(5/2
)*x^2*(x*(a*x+1))^(1/2)-2230*a^(3/2)*x*(x*(a*x+1))^(1/2)+3345*ln(1/2*(2*(x*
(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-6690*(x*(a*x+1))^(1/2)*a^(1/2)
+3345*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(7/2)/(x*(a*
x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 353, normalized size = 1.17

$$\left[\frac{3345\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx+4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-1115\,a^2x^2-3345\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{768\,(a^5x-a^4)}, -\frac{3345\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)-2\,(48\,a^5x^5-200\,a^4x^4+446\,a^3x^3-1115\,a^2x^2-3345\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{384\,(a^5x-a^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3345*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 +
 3*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
 - c)/(a*x - 1)) + 4*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2
 - 3345*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^
4), -1/384*(3345*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2
*(48*a^5*x^5 - 200*a^4*x^4 + 446*a^3*x^3 - 1115*a^2*x^2 - 3345*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^5*x - a^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^3*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.534 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx$

Optimal. Leaf size=251

$$\frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{c-\frac{c}{ax}}\,x}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{11}{}$$

```
[Out] -119/8*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)+119/8*(
c-c/a/x)^(1/2)/a^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+119/24*x*(c-c/a/x)^(1/2)
/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-19/12*x^2*(c-c/a/x)^(1/2)/a/(1-1/a/x)^
(1/2)/(1+1/a/x)^(1/2)+1/3*x^3*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/
2)
```

Rubi [A]

time = 0.20, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6317, 6315, 91, 79, 44, 53, 65, 214}

$$\frac{119\sqrt{c-\frac{c}{ax}}}{8a^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{119\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{8a^3\sqrt{1-\frac{1}{ax}}}+\frac{119x\sqrt{c-\frac{c}{ax}}}{24a^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{x^3\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{19x^2\sqrt{c-\frac{c}{ax}}}{12a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (119*Sqrt[c - c/(a*x)])/(8*a^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (119*
Sqrt[c - c/(a*x)]*x)/(24*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (19*Sqr
t[c - c/(a*x)]*x^2)/(12*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c -
c/(a*x)]*x^3)/(3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (119*Sqrt[c - c/(a*
x)]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(8*a^3*Sqrt[1 - 1/(a*x)])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
```

```
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x^2\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x^2\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^4\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{-\frac{19}{2a}+\frac{3x}{a^2}}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^3}{3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(119\sqrt{c-\frac{c}{ax}}\right)}{24}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{19\sqrt{c-\frac{c}{ax}}\,x^2}{12a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}\sqrt{}}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-}}{12a\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-}}{12a\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{119\sqrt{c-\frac{c}{ax}}\,x}{12a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{119\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}\,x}{8a^2\sqrt{1-\frac{1}{ax}}}-\frac{19\sqrt{c-}}{12a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.30, size = 159, normalized size = 0.63

$$\frac{\frac{2a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(357+119ax-38a^2x^2+8a^3x^3)}{-1+a^2x^2}+357\sqrt{c}\,\log(1-ax)-357\sqrt{c}\,\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{48a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((2*a^2*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(357 + 119*a*x - 38*a^2
*x^2 + 8*a^3*x^3))/(-1 + a^2*x^2) + 357*Sqrt[c]*Log[1 - a*x] - 357*Sqrt[c]*
Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x
 + 2*a^2*x^2)])/(48*a^3)
```

Maple [A]
time = 0.10, size = 180, normalized size = 0.72

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\,(16a^{\frac{7}{2}}x^3\sqrt{x(ax+1)}-76a^{\frac{5}{2}}x^2\sqrt{x(ax+1)}+238a^{\frac{3}{2}}x\sqrt{x(ax+1)}-357\ln(\frac{2\sqrt{\cdots}}{\cdots}\cdots}{48(ax-1)^2a^{\frac{5}{2}}\sqrt{x(ax+\cdots}}$
risch	$\frac{(8a^2x^2-46ax+165)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{24a^2(ax-1)}+\frac{(-\frac{119\ln\left(\frac{\frac{1}{2}ac+c\,a^2x}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2+acx}\right)}{16a^2\sqrt{a^2c}}+\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-(\cdots}}{a^4c\left(x+\frac{1}{a}\right)}\cdots}{ax-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(16*
a^(7/2)*x^3*(x*(a*x+1))^(1/2)-76*a^(5/2)*x^2*(x*(a*x+1))^(1/2)+238*a^(3/2)*
x*(x*(a*x+1))^(1/2)-357*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2
))*a*x+714*(x*(a*x+1))^(1/2)*a^(1/2)-357*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2
)+2*a*x+1)/a^(1/2)))/a^(5/2)/(x*(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]

time = 0.38, size = 337, normalized size = 1.34

$$\left[\frac{357\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^3cx^3-7\,acx-4\,(2\,a^3x^3+3\,a^2x^2+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2+357\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{96\,(a^4x-a^3)},\ \frac{357\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^2x^2+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^2cx^2-acx-c}\right)+2\,(8\,a^4x^4-38\,a^3x^3+119\,a^2x^2+357\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{48\,(a^4x-a^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(357*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3
*a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
 c)/(a*x - 1)) + 4*(8*a^4*x^4 - 38*a^3*x^3 + 119*a^2*x^2 + 357*a*x)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^4*x - a^3), 1/48*(357*(a*x -
 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sq
rt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(8*a^4*x^4 - 38*a^3*x^
3 + 119*a^2*x^2 + 357*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)
))/(a^4*x - a^3)]
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.535 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx$

Optimal. Leaf size=199

$$-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}}+\frac{47\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\sqrt{1+\right.}{4a^2\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 47/4*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)-47/4*(c-c
/a/x)^(1/2)/a^2/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-13/4*x*(c-c/a/x)^(1/2)/a/(1
-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+1/2*x^2*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/
a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6317, 6315, 91, 79, 53, 65, 214}

$$-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}+\frac{47\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{4a^2\sqrt{1-\frac{1}{ax}}}+\frac{x^2\sqrt{c-\frac{c}{ax}}}{2\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}-\frac{13x\sqrt{c-\frac{c}{ax}}}{4a\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-47*Sqrt[c - c/(a*x)])/(4*a^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (13*S
qrt[c - c/(a*x)]*x)/(4*a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x^2)/(2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (47*Sqrt[c - c/(a*x)
]*ArcTanh[Sqrt[1 + 1/(a*x)]])/(4*a^2*Sqrt[1 - 1/(a*x)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,x\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,x\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^2}{x^3\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$=\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int\frac{-\frac{13}{2a}+\frac{2x}{a^2}}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{ax}}}$$

$$=-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{\left(47\sqrt{c-\frac{c}{ax}}\right)\text{Sul}}{8a^2\,}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

$$=-\frac{47\sqrt{c-\frac{c}{ax}}}{4a^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{13\sqrt{c-\frac{c}{ax}}\,x}{4a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x^2}{2\sqrt{1-\frac{1}{ax}}\sqrt{1+}}$$

Mathematica [A]
time = 0.26, size = 151, normalized size = 0.76

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2(-47-13ax+2a^2x^2)}{-4+4a^2x^2}-\frac{47\sqrt{c}\,\log(1-ax)}{8a^2}+\frac{47\sqrt{c}\,\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2+c(-1-ax+2a^2x^2)\right)}{8a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a*x)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2*(-47 - 13*a*x + 2*a^2*x^2))/(-
4 + 4*a^2*x^2) - (47*Sqrt[c]*Log[1 - a*x])/(8*a^2) + (47*Sqrt[c]*Log[2*a^2*
Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x
^2)])/(8*a^2)
```

Maple [A]
time = 0.10, size = 163, normalized size = 0.82

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\,x\left(4a^{\frac{5}{2}}x^{2}\sqrt{x(ax+1)}-26a^{\frac{3}{2}}x\sqrt{x(ax+1)}+47\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a\right.}{8(ax-1)^{2}a^{\frac{3}{2}}\sqrt{x(ax+1)}}$
risch	$\frac{(2ax-15)x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{4a(ax-1)}+\frac{\left(\frac{47\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{8a\sqrt{a^{2}c}}-\frac{8\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{3}c\left(x+\frac{1}{a}\right)}\right.}{ax-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(4*a^
(5/2)*x^2*(x*(a*x+1))^(1/2)-26*a^(3/2)*x*(x*(a*x+1))^(1/2)+47*ln(1/2*(2*(x*
(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x-94*(x*(a*x+1))^(1/2)*a^(1/2)+4
7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2)))/a^(3/2)/(x*(a*x+1)
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a*x))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.40, size = 321, normalized size = 1.61

$$\left[\frac{47\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx+4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(2\,a^{3}x^{3}-13\,a^{2}x^{2}-47\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{16\,(a^{3}x-a^{2})},\ -\frac{47\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)-2\,(2\,a^{3}x^{3}-13\,a^{2}x^{2}-47\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{8\,(a^{3}x-a^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(47*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*
a^2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) -
c)/(a*x - 1)) + 4*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt((a*x - 1)/(a*x + 1
))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2), -1/8*(47*(a*x - 1)*sqrt(-c)*arct
an(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(2*a^2*c*x^2 - a*c*x - c)) - 2*(2*a^3*x^3 - 13*a^2*x^2 - 47*a*x)*sqrt(
(a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a^3*x - a^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.536 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=140

$$\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -7*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/a/(1-1/a/x)^(1/2)+9*(c-c/a/x)^(
1/2)/a/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+x*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1
+1/a/x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6317, 6314, 91, 79, 65, 214}

$$\frac{x\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{a\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*Sqrt[c - c/(a*x)])/(a*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (Sqrt[c - c
/(a*x)]*x)/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (7*Sqrt[c - c/(a*x)]*Arc
Tanh[Sqrt[1 + 1/(a*x)]])/(a*Sqrt[1 - 1/(a*x)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
```

```
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x^2\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{-\frac{7}{2a}+\frac{x}{a^2}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int -\ldots\right.}{2a\sqrt{1-\ldots}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\left(7\sqrt{c-\frac{c}{ax}}\right)\text{Subst}\left(\int \ldots\right.}{\sqrt{1-\ldots}}$$

$$= \frac{9\sqrt{c-\frac{c}{ax}}}{a\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{c-\frac{c}{ax}}\,x}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{7\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\ldots}\right.}{a\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{ax}}\left(9+ax-7\sqrt{1+\frac{1}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/E^(3*ArcCoth[a*x]),x]
```

[Out] (Sqrt[c - c/(a*x)]*(9 + a*x - 7*Sqrt[1 + 1/(a*x)]*ArcTanh[Sqrt[1 + 1/(a*x)]
]))/(a*Sqrt[1 - 1/(a^2*x^2)])

Maple [A]
time = 0.09, size = 146, normalized size = 1.04

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\, x\left(2a^{\frac{3}{2}}x\sqrt{x\,(ax+1)}-7\ln\left(\frac{2\sqrt{x\,(ax+1)}\,\sqrt{a}+2ax+1}{2\sqrt{a}}\right)ax+18\sqrt{x\,(ax+1)}\,\sqrt{\vphantom{a}}\right.}{2(ax-1)^{2}\sqrt{a}\,\sqrt{x\,(ax+1)}}$
risch	$\frac{x(ax+1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(-\frac{7\ln\left(\frac{\frac{1}{2}ac+c\,a^{2}x}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}+acx}\right)}{2\sqrt{a^{2}c}}+\frac{8\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-\left(x+\frac{1}{a}\right)ac}}{a^{2}c\left(x+\frac{1}{a}\right)}\right)\sqrt{\vphantom{a}}}{ax-1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*x*(2*a^
(3/2)*x*(x*(a*x+1))^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^
(1/2))*a*x+18*(x*(a*x+1))^(1/2)*a^(1/2)-7*ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/
2)+2*a*x+1)/a^(1/2)))/a^(1/2)/(x*(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [A]
time = 0.37, size = 299, normalized size = 2.14

$$\left[\frac{7\,(ax-1)\sqrt{c}\,\log\left(-\frac{8\,a^{3}cx^{3}-7\,acx-4\,(2\,a^{3}x^{3}+3\,a^{2}x^{2}+ax)\sqrt{c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)+4\,(a^{2}x^{2}+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{4\,(a^{2}x-a)},\ \frac{7\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{2\,(a^{2}x^{2}+ax)\sqrt{-c}\,\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,a^{2}cx^{2}-acx-c}\right)+2\,(a^{2}x^{2}+9\,ax)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{2\,(a^{2}x-a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] [1/4*(7*(a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x - 4*(2*a^3*x^3 + 3*a^
2*x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)

```
/(a*x - 1)) + 4*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c
)/(a*x)))/(a^2*x - a), 1/2*(7*(a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*s
qrt(-c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*
c*x - c)) + 2*(a^2*x^2 + 9*a*x)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/
(a*x)))/(a^2*x - a)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.537 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx$

Optimal. Leaf size=134

$$-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{1+\frac{1}{ax}}\right)}{\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 2*arctanh((1+1/a/x)^(1/2))*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-8*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-2*(1+1/a/x)^(1/2)*(c-c/a/x)^(1/2)/(1-1/a/
x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {6317, 6315, 89, 65, 214}

$$-\frac{2\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{c-\frac{c}{ax}}\tanh^{-1}\left(\sqrt{\frac{1}{ax}+1}\right)}{\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (-8*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (2*Sqrt[1 +
1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (2*Sqrt[c - c/(a*x)]*ArcTan
h[Sqrt[1 + 1/(a*x)]])/Sqrt[1 - 1/(a*x)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^2}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int \left(-\frac{4}{a\left(1+\frac{x}{a}\right)^{3/2}}+\frac{1}{a\sqrt{1+\frac{x}{a}}}+\frac{1}{x\sqrt{1+\frac{x}{a}}}\right)dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int \frac{}{x\sqrt{}}\right.}{\sqrt{1-}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} - \frac{\left(2a\sqrt{c-\frac{c}{ax}}\right)\mathrm{Subst}}{\sqrt{}}$$

$$= -\frac{8\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{2\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{2\sqrt{c-\frac{c}{ax}}\,\tanh^{-1}\left(\sqrt{\right.}}{\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.27, size = 131, normalized size = 0.98

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x(1+5ax)}{-1+a^2x^2} - \sqrt{c}\log(1-ax) + \sqrt{c}\log\left(2a^2\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,x^2 + c\left(-1-ax+2a^2x^2\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x),x]

[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x*(1 + 5*a*x))/(-1 + a^2*x^2) - Sqrt[c]*Log[1 - a*x] + Sqrt[c]*Log[2*a^2*Sqrt[c]*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*x^2 + c*(-1 - a*x + 2*a^2*x^2)]

Maple [A]
time = 0.10, size = 151, normalized size = 1.13

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(ax-1)}{ax}}\left(10a^{\frac{3}{2}}x\sqrt{x(ax+1)}-\ln\left(\frac{2\sqrt{x(ax+1)}\sqrt{a}+2ax+1}{2\sqrt{a}}\right)a^2x^2-\ln\left(\frac{2\sqrt{x(ax+1)}\cdots}{2\sqrt{a}\cdots}\right.\cdots\right.}{(ax-1)^2\sqrt{a}\sqrt{x(ax+1)}}$
risch	$-\frac{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}+\frac{\left(\frac{a\ln\left(\frac{\frac{1}{2}ac+ca^2x}{\sqrt{a^2c}}+\sqrt{a^2cx^2+acx}\right)}{\sqrt{a^2c}}-\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-\left(x+\frac{1}{a}\right)ac}}{ac\left(x+\frac{1}{a}\right)}\right)\sqrt{\frac{a\cdots}{a\cdots}}\cdots}{ax-1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)

[Out] -((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2*(a*x+1)*(c*(a*x-1)/a/x)^(1/2)*(10*a^(3/2)*x*(x*(a*x+1))^(1/2)-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a^2*x^2-ln(1/2*(2*(x*(a*x+1))^(1/2)*a^(1/2)+2*a*x+1)/a^(1/2))*a*x+2*(x*(a*x+1))^(1/2)*a^(1/2))/a^(1/2)/(x*(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)

Fricas [A]
time = 0.37, size = 277, normalized size = 2.07

$$\left[\frac{(ax-1)\sqrt{c}\log\left(-\frac{8a^3cx^3-7acx+4\left(2a^3x^3+3a^2x^2+ax\right)\sqrt{c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}-c}{ax-1}\right)-4\left(5ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2\left(ax-1\right)}, -\frac{(ax-1)\sqrt{-c}\arctan\left(\frac{2\left(a^2x^2+ax\right)\sqrt{-c}\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{2a^2cx^2-acx-c}\right)+2\left(5ax+1\right)\sqrt{\frac{ax-1}{ax+1}}\sqrt{\frac{acx-c}{ax}}}{ax-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fricas")

```
[Out] [1/2*((a*x - 1)*sqrt(c)*log(-(8*a^3*c*x^3 - 7*a*c*x + 4*(2*a^3*x^3 + 3*a^2*
x^2 + a*x)*sqrt(c)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)) - c)/(
a*x - 1)) - 4*(5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))
)/(a*x - 1), -((a*x - 1)*sqrt(-c)*arctan(2*(a^2*x^2 + a*x)*sqrt(-c)*sqrt((a
*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(2*a^2*c*x^2 - a*c*x - c)) + 2*(
5*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x)))/(a*x - 1)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a\,x}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

```
[Out] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.538 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx$

Optimal. Leaf size=109

$$\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -16/3*a*(c-c/a/x)^(3/2)/c/(1-1/a^2/x^2)^(1/2)-2/3*a*(c-c/a/x)^(5/2)/c^2/(1-
1/a^2/x^2)^(1/2)+64/3*a*(c-c/a/x)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6313, 671, 663}

$$-\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}}+\frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (64*a*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a^2*x^2)]) - (16*a*(c - c/(a*x))^(3
/2))/(3*c*Sqrt[1 - 1/(a^2*x^2)]) - (2*a*(c - c/(a*x))^(5/2))/(3*c^2*Sqrt[1
- 1/(a^2*x^2)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && IntegerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{8\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c^2}$$

$$= -\frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}} - \frac{32\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{64a\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{16a\left(c-\frac{c}{ax}\right)^{3/2}}{3c\sqrt{1-\frac{1}{a^2x^2}}} - \frac{2a\left(c-\frac{c}{ax}\right)^{5/2}}{3c^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 0.53

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-1+10ax+23a^2x^2\right)}{-3+3a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^2),x]
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-1 + 10*a*x + 23*a^2*x^2))/(-3 + 3*a^2*x^2)
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.57

method	result	size
gosper	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
default	$\frac{2(ax+1)(23a^2x^2+10ax-1)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3x(ax-1)^2}$	62
risch	$\frac{2(11a^2x^2+10ax-1)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{3x(ax-1)}+\frac{8a^2x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(a*x+1)*(23*a^2*x^2+10*a*x-1)*(c*(a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(
3/2)/x/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A]
time = 0.34, size = 59, normalized size = 0.54

$$\frac{2\left(23\,a^2x^2+10\,ax-1\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{3\left(ax^2-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] 2/3*(23*a^2*x^2 + 10*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a
*x))/(a*x^2 - x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.36, size = 54, normalized size = 0.50

$$\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(23\,a^2\,x^2+10\,a\,x-1\right)}{3\,x\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] (2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(10*a*x + 23*a^2*x^2 - 1
))/(3*x*(a*x - 1))
```

3.539 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx$

Optimal. Leaf size=150

$$-\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -a^2*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)-7/5*a^2*(c-c/a/x)^(3/2)*(1-1/a
^2/x^2)^(1/2)/c-224/15*a^2*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)-56/15*a^2*
(1-1/a^2/x^2)^(1/2)*(c-c/a/x)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6313, 803, 671, 663}

$$-\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c}-\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}-\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (-224*a^2*c*Sqrt[1 - 1/(a^2*x^2)])/(15*Sqrt[c - c/(a*x)]) - (56*a^2*Sqrt[1
- 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/15 - (7*a^2*Sqrt[1 - 1/(a^2*x^2)]*(c - c/
(a*x))^(3/2))/(5*c) - (a^2*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d*g + e*f)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(
p + 1))), x] - Dist[e*((m*(d*g + e*f) + 2*e*f*(p + 1))/(2*c*d*(p + 1))), In
t[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && EqQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c^3}$$

$$= -\frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(7a)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{2c^2}$$

$$= -\frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} - \frac{(28a)\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}}\right.}{5c}$$

$$= -\frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{5c} - \frac{a^2\left(c-\frac{c}{ax}\right)^{7}}{c^3\sqrt{1-\frac{1}{a^2x}}}$$

$$= -\frac{224a^2c\sqrt{1-\frac{1}{a^2x^2}}}{15\sqrt{c-\frac{c}{ax}}} - \frac{56}{15}a^2\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} - \frac{7a^2\sqrt{1-\frac{1}{a^2x^2}}\left(c-\right.}{5c}$$

Mathematica [A]
time = 0.07, size = 70, normalized size = 0.47

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\,(3-16ax+79a^2x^2+158a^3x^3)}{15x\,(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^3),x]
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(3 - 16*a*x + 79*a^2*x^2 + 15
8*a^3*x^3))/(15*x*(-1 + a^2*x^2))
```

Maple [A]
time = 0.10, size = 70, normalized size = 0.47

method	result	size
gosper	$-\frac{2(ax+1)(158a^3x^3+79a^2x^2-16ax+3)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^2(ax-1)^2}$	70
default	$-\frac{2(ax+1)(158a^3x^3+79a^2x^2-16ax+3)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{15x^2(ax-1)^2}$	70
risch	$-\frac{2(98a^3x^3+79a^2x^2-16ax+3)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{15x^2(ax-1)}-\frac{8a^3x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -2/15*(a*x+1)*(158*a^3*x^3+79*a^2*x^2-16*a*x+3)*(c*(a*x-1)/a/x)^(1/2)*((a*x
-1)/(a*x+1))^(3/2)/x^2/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="maxima"
)
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A]
time = 0.36, size = 69, normalized size = 0.46

$$-\frac{2\,(158\,a^3x^3+79\,a^2x^2-16\,ax+3)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{15\,(ax^3-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] -2/15*(158*a^3*x^3 + 79*a^2*x^2 - 16*a*x + 3)*sqrt((a*x - 1)/(a*x + 1))*sqr
t((a*c*x - c)/(a*x))/(a*x^3 - x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.44, size = 62, normalized size = 0.41

$$-\frac{2\sqrt{c-\frac{c}{a\,x}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(158\,a^3\,x^3+79\,a^2\,x^2-16\,a\,x+3\right)}{15\,x^2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

```
[Out] -(2*(c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)*(79*a^2*x^2 - 16*a*x +
158*a^3*x^3 + 3))/(15*x^2*(a*x - 1))
```

3.540 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx$

Optimal. Leaf size=188

$$\frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)}{7c^2}$$

```
[Out] a^3*(c-c/a/x)^(7/2)/c^3/(1-1/a^2/x^2)^(1/2)+59/35*a^3*(c-c/a/x)^(3/2)*(1-1/
a^2/x^2)^(1/2)/c+2/7*a^3*(c-c/a/x)^(5/2)*(1-1/a^2/x^2)^(1/2)/c^2+1888/105*a
^3*c*(1-1/a^2/x^2)^(1/2)/(c-c/a/x)^(1/2)+472/105*a^3*(1-1/a^2/x^2)^(1/2)*(c
-c/a/x)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6313, 1649, 809, 671, 663}

$$\frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2}+\frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c}+\frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}+\frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (1888*a^3*c*Sqrt[1 - 1/(a^2*x^2)])/(105*Sqrt[c - c/(a*x)]) + (472*a^3*Sqrt[
1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)])/105 + (59*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c
- c/(a*x))^(3/2))/(35*c) + (2*a^3*Sqrt[1 - 1/(a^2*x^2)]*(c - c/(a*x))^(5/2)
)/(7*c^2) + (a^3*(c - c/(a*x))^(7/2))/(c^3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] /; FreeQ[{a, c, d
, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + p, 0]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*(Simplify[m + p]/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !Integ
erQ[p] && IGtQ[Simplify[m + p], 0]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1649

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a*e + c*d*x, x], f = PolynomialRemainder
[Pq, a*e + c*d*x, x]}, Simp[(-d)*f*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*e*
(p + 1))), x] + Dist[d/(2*a*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p
+ 1)*ExpandToSum[2*a*e*(p + 1)*Q + f*(m + 2*p + 2), x], x], x]] /; FreeQ[{a
, c, d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
 && GtQ[m, 0]
```

Rule 6313

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^n, Subst[Int[(c + d*x)^(p - n)*((1 - x^2/a^2)^(n/2)/x^(m
+ 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, p}, x] && EqQ[c + a*d, 0] && Inte
gerQ[(n - 1)/2] && IntegerQ[m] && (IntegerQ[p] || EqQ[p, n/2] || EqQ[p, n/2
 + 1] || LtQ[-5, m, -1]) && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2\left(c-\frac{cx}{a}\right)^{7/2}}{\left(1-\frac{x^2}{a^2}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{7a^2}{2}-ax\right)\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{(59a^2)\,\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{5/2}}{\sqrt{1-\frac{x^2}{a^2}}}\,d\right.}{14c^2}$$

$$= \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{5/2}}{7c^2} + \frac{a^3\left(c-\frac{c}{ax}\right)^{7/2}}{c^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\left(c-\frac{c}{ax}\right)^{3/2}}{35c} + \frac{2a^3\sqrt{1-\frac{1}{a^2}}}{7}$$

$$= \frac{1888a^3c\sqrt{1-\frac{1}{a^2x^2}}}{105\sqrt{c-\frac{c}{ax}}} + \frac{472}{105}a^3\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}} + \frac{59a^3\sqrt{1-\frac{1}{a^2x^2}}\,(c-}{35c}$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 0.41

$$\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(-15+66ax-167a^2x^2+668a^3x^3+1336a^4x^4\right)}{105x^2\left(-1+a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(-15 + 66*a*x - 167*a^2*x^2 +
668*a^3*x^3 + 1336*a^4*x^4))/(105*x^2*(-1 + a^2*x^2))
```

Maple [A]
time = 0.11, size = 78, normalized size = 0.41

method	result	size
gosper	$\frac{2(ax+1)\left(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
default	$\frac{2(ax+1)\left(1336a^4x^4+668a^3x^3-167a^2x^2+66ax-15\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{105x^3(ax-1)^2}$	78
risch	$\frac{2\left(916a^4x^4+668a^3x^3-167a^2x^2+66ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{105x^3(ax-1)}+\frac{8a^4x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(a*x+1)*(1336*a^4*x^4+668*a^3*x^3-167*a^2*x^2+66*a*x-15)*(c*(a*x-1)/a
/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A]
time = 0.34, size = 77, normalized size = 0.41

$$\frac{2\left(1336\,a^4x^4+668\,a^3x^3-167\,a^2x^2+66\,ax-15\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{105\left(ax^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] 2/105*(1336*a^4*x^4 + 668*a^3*x^3 - 167*a^2*x^2 + 66*a*x - 15)*sqrt((a*x -
1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^4 - x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.41, size = 100, normalized size = 0.53

$$\frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(1336\,a^3\,x^3+2004\,a^2\,x^2+1837\,a\,x+1903\right)\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3}+\frac{3776\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{105\,x^3\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2)*(1837*a*x + 2004*a^2*x^2 + 1336*a^3*x^3 + 19
03)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3) + (3776*((a*x - 1)/(a*x + 1))^(1
/2)*((c*(a*x - 1))/(a*x))^(1/2))/(105*x^3*(a*x - 1))
```

3.541 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx$

Optimal. Leaf size=289

$$-\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}-\frac{32a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(1+\frac{1}{ax}\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 50/3*a^4*(1+1/a/x)^(3/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-38/5*a^4*(1+1/a/x)
^(5/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)+2*a^4*(1+1/a/x)^(7/2)*(c-c/a/x)^(1/2
)/(1-1/a/x)^(1/2)-2/9*a^4*(1+1/a/x)^(9/2)*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)-8
*a^4*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)-32*a^4*(1+1/a/x)^(1/2)
*(c-c/a/x)^(1/2)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6317, 6315, 90}

$$-\frac{2a^4\left(\frac{1}{ax}+1\right)^{9/2}\sqrt{c-\frac{c}{ax}}}{9\sqrt{1-\frac{1}{ax}}}+\frac{2a^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{38a^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{c-\frac{c}{ax}}}{5\sqrt{1-\frac{1}{ax}}}+\frac{50a^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{c-\frac{c}{ax}}}{3\sqrt{1-\frac{1}{ax}}}-\frac{32a^4\sqrt{\frac{1}{ax}+1}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}}-\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-8*a^4*Sqrt[c - c/(a*x)])/(Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) - (32*a^4*
Sqrt[1 + 1/(a*x)]*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] + (50*a^4*(1 + 1/(a*
x))^(3/2)*Sqrt[c - c/(a*x)])/(3*Sqrt[1 - 1/(a*x)]) - (38*a^4*(1 + 1/(a*x))^
(5/2)*Sqrt[c - c/(a*x)])/(5*Sqrt[1 - 1/(a*x)]) + (2*a^4*(1 + 1/(a*x))^(7/2)
*Sqrt[c - c/(a*x)])/Sqrt[1 - 1/(a*x)] - (2*a^4*(1 + 1/(a*x))^(9/2)*Sqrt[c -
 c/(a*x)])/(9*Sqrt[1 - 1/(a*x)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6315

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.)*(x_)^(m_.), x_S
ymbol] :> Dist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^(m + 2)*
(1 - x/a)^(n/2))), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2
```

```
- a^2*d^2, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegerQ[
m]
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{ax}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}}{x^5}\,dx}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \frac{x^3\left(1-\frac{x}{a}\right)^2}{\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{\sqrt{c-\frac{c}{ax}}\,\text{Subst}\left(\int \left(-\frac{4a^3}{\left(1+\frac{x}{a}\right)^{3/2}}+\frac{16a^3}{\sqrt{1+\frac{x}{a}}}-25a^3\sqrt{1+\frac{x}{a}}+19a^3\left(1+\right.\right.\right.}{\sqrt{1-\frac{1}{ax}}}$$

$$= -\frac{8a^4\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} - \frac{32a^4\sqrt{1+\frac{1}{ax}}\sqrt{c-\frac{c}{ax}}}{\sqrt{1-\frac{1}{ax}}} + \frac{50a^4\left(1+\frac{1}{ax}\right)^{3/2}\sqrt{c-}}{3\sqrt{1-\frac{1}{ax}}}$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 0.30

$$-\frac{2a\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{ax}}\left(5-20ax+41a^2x^2-82a^3x^3+328a^4x^4+656a^5x^5\right)}{45x^3\left(-1+a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a*x)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (-2*a*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a*x)]*(5 - 20*a*x + 41*a^2*x^2 - 82
*a^3*x^3 + 328*a^4*x^4 + 656*a^5*x^5))/(45*x^3*(-1 + a^2*x^2))
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.30

method	result	size
gosper	$-\frac{2(ax+1)\left(656a^5x^5+328a^4x^4-82a^3x^3+41a^2x^2-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45(ax-1)^2x^4}$	86
default	$-\frac{2(ax+1)\left(656a^5x^5+328a^4x^4-82a^3x^3+41a^2x^2-20ax+5\right)\sqrt{\frac{c(ax-1)}{ax}}\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{45(ax-1)^2x^4}$	86
risch	$-\frac{2\left(476a^5x^5+328a^4x^4-82a^3x^3+41a^2x^2-20ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{45x^4(ax-1)}-\frac{8a^5x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(ax-1)}{ax}}}{ax-1}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(a*x+1)*(656*a^5*x^5+328*a^4*x^4-82*a^3*x^3+41*a^2*x^2-20*a*x+5)*(c*(
a*x-1)/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a*x))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Fricas [A]
time = 0.35, size = 85, normalized size = 0.29

$$-\frac{2\left(656\,a^5x^5+328\,a^4x^4-82\,a^3x^3+41\,a^2x^2-20\,ax+5\right)\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{acx-c}{ax}}}{45\left(ax^5-x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] -2/45*(656*a^5*x^5 + 328*a^4*x^4 - 82*a^3*x^3 + 41*a^2*x^2 - 20*a*x + 5)*sq
rt((a*x - 1)/(a*x + 1))*sqrt((a*c*x - c)/(a*x))/(a*x^5 - x^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 1.41, size = 108, normalized size = 0.37

$$-\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}\,\left(656\,a^4\,x^4+984\,a^3\,x^3+902\,a^2\,x^2+943\,a\,x+923\right)}{45\,x^4}-\frac{1856\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,\sqrt{\frac{c\,\left(a\,x-1\right)}{a\,x}}}{45\,x^4\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] - (2*((a*x - 1)/(a*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2)*(943*a*x + 902
*a^2*x^2 + 984*a^3*x^3 + 656*a^4*x^4 + 923))/(45*x^4) - (1856*((a*x - 1)/(a
*x + 1))^(1/2)*((c*(a*x - 1))/(a*x))^(1/2))/(45*x^4*(a*x - 1))
```

3.542 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)\,dx$

Optimal. Leaf size=185

$$c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{n/2}x-\frac{2c(1-n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1,\frac{n}{2};\frac{2+n}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an}-\frac{2^{n/2}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

```
[Out] c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(1/2*n)*x-2*c*(1-n)*(1+1/a/x)^(1/2*n)*hyper
geom([1, 1/2*n],[1+1/2*n],(a+1/x)/(a-1/x))/a/n/((1-1/a/x)^(1/2*n))-2^(1/2*n
)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([1-1/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/
a)/a/(2-n)
```

Rubi [A]
time = 0.08, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6314, 130, 71, 98, 133}

$$-\frac{c2^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}{}_2F_1\left(1-\frac{n}{2},1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}-\frac{2c(1-n)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{an}+cx\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] c*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^(n/2)*x - (2*c*(1 - n)*(1 + 1/(a*x)
)^(n/2)*Hypergeometric2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a
*n*(1 - 1/(a*x))^(n/2)) - (2^(n/2)*c*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric
2F1[1 - n/2, 1 - n/2, 2 - n/2, (a - x^(-1))/(2*a)])/(a*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 130

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_Symbol] :> Dist[b*(d/f^2), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)
, x], x] + Dist[(b*e - a*f)*((d*e - c*f)/f^2), Int[(a + b*x)^(m - 1)*((c +
d*x)^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)dx &= -\left(c\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\
&= -\frac{2^{2-\frac{n}{2}}c\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{1}{2}(-2+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 155, normalized size = 0.84

$$
\frac{ce^{n\coth^{-1}(ax)}\left(-e^{2\coth^{-1}(ax)}n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)+e^{2\coth^{-1}(ax)}(-1+n)n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(anx+{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)+(-1+n)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{an(2+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x)),x]
```

```
[Out] (c*E^(n*ArcCoth[a*x])*(-(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2,
 2 + n/2, -E^(2*ArcCoth[a*x])]) + E^(2*ArcCoth[a*x])*(-1 + n)*n*Hypergeomet
```

```
ric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*n*x + Hypergeo
metric2F1[1, n/2, 1 + n/2, -E^(2*ArcCoth[a*x])] + (-1 + n)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*n*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="fricas")
[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int ae^{n\,\mathrm{acoth}\,(ax)}\,dx+\int\left(-\frac{e^{n\,\mathrm{acoth}\,(ax)}}{x}\right)\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x),x)
[Out] c*(Integral(a*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x, x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x),x, algorithm="giac")
[Out] integrate((c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x)),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x)), x)
```

3.543 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=113

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c}-\frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

```
[Out] (1+1/a/x)^(1+1/2*n)*x/c/((1-1/a/x)^(1/2*n))-2*(1+n)*(1+1/a/x)^(1/2*n)*hyper
geom([1, -1/2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c/n/((1-1/a/x)^(1/2*n))
```

Rubi [A]

time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6314, 98, 133}

$$\frac{x\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}}{c}-\frac{2(n+1)\left(1-\frac{1}{ax}\right)^{-n/2}\left(\frac{1}{ax}+1\right)^{n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] ((1 + 1/(a*x))^((2 + n)/2)*x)/(c*(1 - 1/(a*x))^(n/2)) - (2*(1 + n)*(1 + 1/(
a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/(a + x^(-1))
])/(a*c*n*(1 - 1/(a*x))^(n/2))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{ax}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} - \frac{(1+n)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx,x,\frac{1}{x}\right)}{ac}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c} - \frac{2(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{acn}$$

Mathematica [A]
time = 0.25, size = 97, normalized size = 0.86

$$\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n(1+n)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+(1+n)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{acn(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*(1 + n)*Hypergeometric2F1[1, 1 +
n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + (1 + n)*Hypergeom
etric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="fricas")
```

```
[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*x - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a \int \frac{xe^{n \operatorname{acoth}(ax)}}{ax-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x),x)
```

```
[Out] a*Integral(x*exp(n*acoth(a*x))/(a*x - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n \operatorname{acoth}(a\,x)}}{c-\frac{c}{a\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x)),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x)), x)
```

3.544 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}+\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2}-\frac{2(2+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}{}_2F_1\left(1\right.}{ac^2n}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(1+1/2*n)/a/c^2/(2+n)+(1-1/a/x)^(-1-1
/2*n)*(1+1/a/x)^(1+1/2*n)*x/c^2-2*(2+n)*(1+1/a/x)^(1/2*n)*hypergeom([1, -1/
2*n],[1-1/2*n],(a-1/x)/(a+1/x))/a/c^2/n/((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6314, 105, 160, 12, 133}

$$-\frac{2(n+2)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{ac^2n}-\frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)}+\frac{x\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2))/(a*c^2*(2 +
n))) + ((1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*x)/c^2 - (2*(2 +
 n)*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (a - x^(-1))/
(a + x^(-1))])/(a*c^2*n*(1 - 1/(a*x))^(n/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{c^2}\\
&= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2}+\frac{\text{Subst}\left(\int \frac{\left(-\frac{2+n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x}\,dx,x,\frac{1}{x}\right)}{c^2}\\
&= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}+\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2}-\frac{a\text{Subst}\left(\int \frac{(2+n)^2}{\cdots}\right.}{\cdots}\\
&= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}+\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2}-\frac{(2+n)\text{Subst}\left(\int\right.}{\cdots}\\
&= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}}{ac^2(2+n)}+\frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}x}{c^2}-\frac{2(2+n)\left(1-\frac{1}{ax}\right.}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 113, normalized size = 0.68

$$\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\left(n(1+ax)(-3+2ax+n(-1+ax))-2(2+n)^2(-1+ax)\,{}_2F_1\left(1,-\frac{n}{2};1-\frac{n}{2};\frac{-1+ax}{1+ax}\right)\right)}{ac^2n(2+n)(-1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^2,x]
```

```
[Out] ((1 + 1/(a*x))^(n/2)*(n*(1 + a*x)*(-3 + 2*a*x + n*(-1 + a*x)) - 2*(2 + n)^2
*(-1 + a*x)*Hypergeometric2F1[1, -1/2*n, 1 - n/2, (-1 + a*x)/(1 + a*x)]))/(
a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)*(-1 + a*x))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*x^2 - 2*a*c^2*x + c
^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2 e^{n \operatorname{acoth}(ax)}}{a^2x^2-2ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**2,x)
```

```
[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 2*a*x + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{\left(c-\frac{c}{a\,x}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^2,x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^2, x)
```

3.545 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}F_1\left(\frac{2+n}{2};\frac{1}{2}(-3+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(1-\frac{1}{ax}\right)^{3/2}}$$

```
[Out] -2^(5/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^(3/2)*AppellF1(1+1/2*n,-3/2+1/
2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(1-1/a/x)^(3/2)
```

Rubi [A]

time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6317, 6314, 141}

$$-\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(1-\frac{1}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(5/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^(3/2)*AppellF1[(2
+ n)/2, (-3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(1 - 1/(a*x))^(3/2)))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^{3/2}dx &= \frac{\left(c-\frac{c}{ax}\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^{3/2}dx}{\left(1-\frac{1}{ax}\right)^{3/2}}\\ &= -\frac{\left(c-\frac{c}{ax}\right)^{3/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{ax}\right)^{3/2}}\\ &= -\frac{2^{\frac{5}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^{3/2}F_1\left(\frac{2+n}{2};\frac{1}{2}(-3+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(1-\frac{1}{ax}\right)^{3/2}}\end{aligned}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^(3/2),x]
[Out] $Aborted
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="maxima")

[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="fricas")

[Out] integral((a*c*x - c)*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*x), x)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(3/2),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(3/2),x, algorithm="giac")

[Out] integrate((c - c/(a*x))^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-\frac{c}{a\,x}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2),x)

[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^(3/2), x)

3.546 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{ax}}\,F_1\left(\frac{2+n}{2};\frac{1}{2}(-1+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{1-\frac{1}{ax}}}$$

```
[Out] -2^(3/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,-1/2+1/2*n,2,2+1/2*n,1/
2*(a+1/x)/a,1+1/a/x)*(c-c/a/x)^(1/2)/a/(2+n)/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6317, 6314, 141}

$$-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n-1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{1-\frac{1}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
```

```
[Out] -((2^(3/2 - n/2)*(1 + 1/(a*x))^((2 + n)/2)*Sqrt[c - c/(a*x)]*AppellF1[(2 +
n)/2, (-1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n
)*Sqrt[1 - 1/(a*x)]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{ax}}\,dx=\frac{\sqrt{c-\frac{c}{ax}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{ax}}\,dx}{\sqrt{1-\frac{1}{ax}}}$$
$$=-\frac{\sqrt{c-\frac{c}{ax}}\,\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{ax}}}$$
$$=-\frac{2^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\sqrt{c-\frac{c}{ax}}\,F_1\left(\frac{2+n}{2};\frac{1}{2}(-1+n),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{1-\frac{1}{ax}}}$$

Mathematica [**F**]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a*x)],x]
[Out] $Aborted
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\sqrt{c-\frac{c}{ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-1+\frac{1}{ax}\right)}\, e^{n\operatorname{acoth}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**(1/2),x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a*x))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\operatorname{acoth}(a\,x)}\,\sqrt{c-\frac{c}{a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^(1/2), x)
```

3.547 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{1+n}{2},2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\sqrt{c-\frac{c}{ax}}}$$

```
[Out] -2^(1/2-1/2*n)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,1/2+1/2*n,2,2+1/2*n,1/2
*(a+1/x)/a,1+1/a/x)*(1-1/a/x)^(1/2)/a/(2+n)/(c-c/a/x)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6317, 6314, 141}

$$-\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+1}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\sqrt{c-\frac{c}{ax}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
```

```
[Out] -((2^(1/2 - n/2)*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2 +
n)/2, (1 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 + n)
*Sqrt[c - c/(a*x)]))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx = \frac{\sqrt{1-\frac{1}{ax}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{ax}}}\,dx}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{\sqrt{1-\frac{1}{ax}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{ax}}}$$

$$= -\frac{2^{\frac{1}{2}-\frac{n}{2}}\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}} F_1\left(\frac{2+n}{2}; \frac{1+n}{2}, 2; \frac{4+n}{2}; \frac{a+\frac{1}{x}}{2a}, 1+\frac{1}{ax}\right)}{a(2+n)\sqrt{c-\frac{c}{ax}}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a*x)],x]
[Out] $Aborted
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\sqrt{c-\frac{c}{ax}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="fricas")
[Out] integral(a*x*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a*c*x -
 c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^(1/2), x)
```

3.548 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{2^{-\frac{1}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{3+n}{2},2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(c-\frac{c}{ax}\right)^{3/2}}$$

```
[Out] -2^(-1/2-1/2*n)*(1-1/a/x)^(3/2)*(1+1/a/x)^(1+1/2*n)*AppellF1(1+1/2*n,3/2+1/
2*n,2,2+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/(c-c/a/x)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6317, 6314, 141}

$$-\frac{2^{-\frac{n}{2}-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}F_1\left(\frac{n+2}{2};\frac{n+3}{2},2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)\left(c-\frac{c}{ax}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
```

```
[Out] -((2^(-1/2 - n/2)*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^((2 + n)/2)*AppellF1[(2
 + n)/2, (3 + n)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
 n)*(c - c/(a*x))^(3/2)))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{ax}\right)^{3/2}}\,dx = \frac{\left(1-\frac{1}{ax}\right)^{3/2}\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{ax}\right)^{3/2}}\,dx}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{\left(1-\frac{1}{ax}\right)^{3/2}\operatorname{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx, x, \frac{1}{x}\right)}{\left(c-\frac{c}{ax}\right)^{3/2}}$$

$$= -\frac{2^{-\frac{1}{2}-\frac{n}{2}}\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}F_1\left(\frac{2+n}{2};\frac{3+n}{2},2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)\left(c-\frac{c}{ax}\right)^{3/2}}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a*x))^(3/2),x]
[Out] $Aborted
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{ax}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a*c*x - c)/(a*x))/(a^2
*c^2*x^2 - 2*a*c^2*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}{(ax)}}}{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a/x)**(3/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(-c*(-1 + 1/(a*x)))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a/x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a*x))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{\left(c-\frac{c}{a\,x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - c/(a*x))^(3/2), x)
```

3.549 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=110

$$-\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^pF_1\left(\frac{2+n}{2};\frac{1}{2}(n-2p),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n)}$$

```
[Out] -2^(1-1/2*n+p)*(1+1/a/x)^(1+1/2*n)*(c-c/a/x)^p*AppellF1(1+1/2*n,1/2*n-p,2,2
+1/2*n,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n)/((1-1/a/x)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6317, 6314, 141}

$$-\frac{2^{-\frac{n}{2}+p+1}\left(\frac{1}{ax}+1\right)^{\frac{n+2}{2}}\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^pF_1\left(\frac{n+2}{2};\frac{1}{2}(n-2p),2;\frac{n+4}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -((2^(1 - n/2 + p)*(1 + 1/(a*x))^((2 + n)/2)*(c - c/(a*x))^p*AppellF1[(2 +
n)/2, (n - 2*p)/2, 2, (4 + n)/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(2 +
n)*(1 - 1/(a*x))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
```

```
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$

$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}dx, x, \frac{1}{x}\right)\right.$$

$$= -\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{\frac{2+n}{2}}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{2+n}{2};\frac{1}{2}(n-2p),2;\frac{4+n}{2};\frac{a+\frac{1}{x}}{2a},1+\right.}{a(2+n)}$$

Mathematica [F]
time = 1.01, size = 0, normalized size = 0.00

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a*x))^p, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a/x)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.550 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=67

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p{}_2F_1\left(2,1+p;2+p;1+\frac{1}{ax}\right)}{a(1+p)}$$

```
[Out] -(1+1/a/x)^(1+p)*(c-c/a/x)^p*hypergeom([2, 1+p],[2+p],1+1/a/x)/a/(1+p)/((1-
1/a/x)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {6317, 6314, 67}

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{p+1}\left(c-\frac{c}{ax}\right)^p{}_2F_1\left(2,p+1;p+2;1+\frac{1}{ax}\right)}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^p}{x^2}dx,x,\frac{1}{x}\right)\right)$$
$$= -\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(2,1+p;2+p;1+\frac{1}{ax}\right)}{a(1+p)}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.00

$$-\frac{\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+p}\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(2,1+p;2+p;1+\frac{1}{ax}\right)}{a(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] -(((1 + 1/(a*x))^(1 + p)*(c - c/(a*x))^p*Hypergeometric2F1[2, 1 + p, 2 + p,
 1 + 1/(a*x)])/(a*(1 + p)*(1 - 1/(a*x))^p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int e^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
[Out] int(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^p*((a*c*x - c)/(a*x))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} e^{2p\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(c-c/a/x)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a/x)^p,x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{2\,p\,\mathrm{acoth}(a\,x)}\,{\left(c-\frac{c}{a\,x}\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.551 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=93

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1-p}\left(c-\frac{c}{ax}\right)^p F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(1-p)}$$

```
[Out] -4^p*(1+1/a/x)^(1-p)*(c-c/a/x)^p*AppellF1(1-p,-2*p,2,2-p,1/2*(a+1/x)/a,1+1/
a/x)/a/(1-p)/((1-1/a/x)^p)
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {6317, 6314, 141}

$$-\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(\frac{1}{ax}+1\right)^{1-p}F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] -((4^p*(1 + 1/(a*x))^(1 - p)*(c - c/(a*x))^p*AppellF1[1 - p, -2*p, 2, 2 - p
, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - p)*(1 - 1/(a*x))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2p}\left(1+\frac{x}{a}\right)^{-p}}{x^2}dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{4^p\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1-p}\left(c-\frac{c}{ax}\right)^p F_1\left(1-p;-2p,2;2-p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(1-p)}\end{aligned}$$

Mathematica [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]),x]
[Out] Integrate[(c - c/(a*x))^p/E^(2*p*ArcCoth[a*x]), x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{ax}\right)^p e^{-2p\,\mathrm{arccoth}(ax)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
[Out] int((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a*c*x - c)/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\right)^{p} e^{-2p \operatorname{acoth}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p/exp(2*p*acoth(a*x)),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{-2\,p\,\operatorname{acoth}(a\,x)}\,{\left(c-\frac{c}{a\,x}\right)}^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p,x)
[Out] int(exp(-2*p*acoth(a*x))*(c - c/(a*x))^p, x)
```

3.552 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=57

$$\left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}$$

```
[Out] (c-c/a/x)^p*x+(2-p)*(c-c/a/x)^p*hypergeom([1, p],[1+p],1-1/a/x)/a/p
```

Rubi [A]
time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6302, 6268, 25, 528, 382, 79, 67}

$$\frac{(2-p)\left(c-\frac{c}{ax}\right)^p {}_2F_1\left(1,p;p+1;1-\frac{1}{ax}\right)}{ap}+x\left(c-\frac{c}{ax}\right)^p$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
```

```
[Out] (c - c/(a*x))^p*x + ((2 - p)*(c - c/(a*x))^p*Hypergeometric2F1[1, p, 1 + p,
 1 - 1/(a*x)])/(a*p)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx\\
&= -\int \frac{\left(c-\frac{c}{ax}\right)^p(1+ax)}{1-ax}\,dx\\
&= \frac{c\int \frac{\left(c-\frac{c}{ax}\right)^{-1+p}(1+ax)}{x}\,dx}{a}\\
&= \frac{c\int \left(a+\frac{1}{x}\right)\left(c-\frac{c}{ax}\right)^{-1+p}\,dx}{a}\\
&= -\frac{c\text{Subst}\left(\int \frac{(a+x)\left(c-\frac{cx}{a}\right)^{-1+p}}{x^2}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^p x-\frac{(c(2-p))\text{Subst}\left(\int \frac{\left(c-\frac{cx}{a}\right)^{-1+p}}{x}\,dx,x,\frac{1}{x}\right)}{a}\\
&= \left(c-\frac{c}{ax}\right)^p x+\frac{(2-p)\left(c-\frac{c}{ax}\right)^p\,{}_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)}{ap}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.81

$$\frac{\left(c-\frac{c}{ax}\right)^p\left(apx-(-2+p)\,_2F_1\left(1,p;1+p;1-\frac{1}{ax}\right)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a*x))^p,x]
[Out] ((c - c/(a*x))^p*(a*p*x - (-2 + p)*Hypergeometric2F1[1, p, 1 + p, 1 - 1/(a*
x)]))/(a*p)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int\frac{(ax+1)\left(c-\frac{c}{ax}\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)
[Out] int(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="fricas")
[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p/(a*x - 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.10, size = 272, normalized size = 4.77

$$a\left(\begin{cases}\frac{0^px}{a}+\frac{0^p\log(ax-1)}{a^2}-\frac{a^{-p}c^ppx^2x^{-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^px}{a}+\frac{0^p\log(-ax+1)}{a^2}-\frac{a^{-p}c^ppx^2x^{-p}e^{i\pi p}\Gamma(p)\Gamma(2-p)\,_2F_1\left(\begin{matrix}1-p,2-p\\3-p\end{matrix}\middle|ax\right)}{\Gamma(3-p)\Gamma(p+1)} & \text{otherwise}\end{cases}\right)+\begin{cases}\frac{0^p\log(ax-1)}{a}-\frac{a^{-p}c^ppxx^{-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{for } |ax|>1\\ \frac{0^p\log(-ax+1)}{a}-\frac{a^{-p}c^ppxx^{-p}e^{i\pi p}\Gamma(p)\Gamma(1-p)\,_2F_1\left(\begin{matrix}1-p,1-p\\2-p\end{matrix}\middle|ax\right)}{\Gamma(2-p)\Gamma(p+1)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)**p,x)

[Out] a*Piecewise((0**p*x/a + 0**p*log(a*x - 1)/a**2 - c**p*p*x**2*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(a**p*x**p*gamma(3 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*x/a + 0**p*log(-a*x + 1)/a**2 - c**p*p*x**2*exp(I*pi*p)*gamma(p)*gamma(2 - p)*hyper((1 - p, 2 - p), (3 - p,), a*x)/(a**p*x**p*gamma(3 - p)*gamma(p + 1)), True)) + Piecewise((0**p*log(a*x - 1)/a - c**p*p*x*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(a**p*x**p*gamma(2 - p)*gamma(p + 1)), Abs(a*x) > 1), (0**p*log(-a*x + 1)/a - c**p*p*x*exp(I*pi*p)*gamma(p)*gamma(1 - p)*hyper((1 - p, 1 - p), (2 - p,), a*x)/(a**p*x**p*gamma(2 - p)*gamma(p + 1)), True))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a/x)^p,x, algorithm="giac")

[Out] integrate((a*x + 1)*(c - c/(a*x))^p/(a*x - 1), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c-\frac{c}{a\,x}\right)}^p\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1),x)

[Out] int(((c - c/(a*x))^p*(a*x + 1))/(a*x - 1), x)

3.553 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=90

$$-\frac{2^{\frac{1}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{3a}$$

```
[Out] -1/3*2^(1/2+p)*(1+1/a/x)^(3/2)*(c-c/a/x)^p*AppellF1(3/2,1/2-p,2,5/2,1/2*(a+
1/x)/a,1+1/a/x)/a/((1-1/a/x)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6317, 6314, 141}

$$-\frac{2^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{-p} F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
```

```
[Out] -1/3*(2^(1/2 + p)*(1 + 1/(a*x))^(3/2)*(c - c/(a*x))^p*AppellF1[3/2, 1/2 - p
, 2, 5/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx = \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\sqrt{1+\frac{x}{a}}}{x^2}dx, x, \frac{1}{x}\right)\right)$$
$$= -\frac{2^{\frac{1}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\left(1+\frac{1}{ax}\right)^{3/2}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{3}{2};\frac{1}{2}-p,2;\frac{5}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{3a}$$

Mathematica [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p,x]
[Out] Integrate[E^ArcCoth[a*x]*(c - c/(a*x))^p, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(c-\frac{c}{ax}\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a/x)**p,x)
```

```
[Out] Integral((-c*(-1 + 1/(a*x)))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a/x)^p,x, algorithm="giac")
```

```
[Out] integrate((c - c/(a*x))^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{p}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a*x))^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.554 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\sqrt{1+\frac{1}{ax}}\,\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{1}{2};-\frac{1}{2}-p,2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a}$$

```
[Out] -2^(3/2+p)*(c-c/a/x)^p*AppellF1(1/2,-1/2-p,2,3/2,1/2*(a+1/x)/a,1+1/a/x)*(1+
1/a/x)^(1/2)/a/((1-1/a/x)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6317, 6314, 141}

$$-\frac{2^{p+\frac{3}{2}}\sqrt{\frac{1}{ax}+1}\,\left(1-\frac{1}{ax}\right)^{-p}F_1\left(\frac{1}{2};-p-\frac{1}{2},2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)\left(c-\frac{c}{ax}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
```

```
[Out] -((2^(3/2 + p)*Sqrt[1 + 1/(a*x)]*(c - c/(a*x))^p*AppellF1[1/2, -1/2 - p, 2,
 3/2, (a + x^(-1))/(2*a), 1 + 1/(a*x)])/(a*(1 - 1/(a*x))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6314

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_))^(p_.), x_Symbol] :> D
ist[-c^p, Subst[Int[(1 + d*(x/c))^p*((1 + x/a)^(n/2)/(x^2*(1 - x/a)^(n/2)))
, x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&
 !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6317

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
 :> Dist[(c + d/x)^p/(1 + d/(c*x))^p, Int[u*(1 + d/(c*x))^p*E^(n*ArcCoth[a*
x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !Inte
```

```
gerQ[n/2] &&  !(IntegerQ[p] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx &= \left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{ax}\right)^p dx\\ &= -\left(\left(\left(1-\frac{1}{ax}\right)^{-p}\left(c-\frac{c}{ax}\right)^p\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{2^{\frac{3}{2}+p}\left(1-\frac{1}{ax}\right)^{-p}\sqrt{1+\frac{1}{ax}}\left(c-\frac{c}{ax}\right)^p F_1\left(\frac{1}{2};-\frac{1}{2}-p,2;\frac{3}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a}\end{aligned}$$

Mathematica [**F**]
time = 1.18, size = 0, normalized size = 0.00

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x],x]
[Out] Integrate[(c - c/(a*x))^p/E^ArcCoth[a*x], x]
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{ax}\right)^p \sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
[Out] int((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral(((a*c*x - c)/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(-1 + 1/(a*x)))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a*x))^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(c-\frac{c}{a\,x}\right)}^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a*x))^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.555 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$

Optimal. Leaf size=114

$$\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+p;3+p;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}-\frac{\left(c-\frac{c}{ax}\right)^{2+p}\,{}_2F_1\left(1,2+p;3+p;1-\frac{1}{ax}\right)}{ac^2}$$

```
[Out] (c-c/a/x)^(2+p)*x/c^2+1/2*(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1/2*(a-1
/x)/a)/a/c^2/(2+p)-(c-c/a/x)^(2+p)*hypergeom([1, 2+p],[3+p],1-1/a/x)/a/c^2
```

Rubi [A]
time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6302, 6268, 25, 528, 382, 105, 162, 67, 70}

$$\frac{\left(c-\frac{c}{ax}\right)^{p+2}\,{}_2F_1\left(1,p+2;p+3;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(p+2)}-\frac{\left(c-\frac{c}{ax}\right)^{p+2}\,{}_2F_1\left(1,p+2;p+3;1-\frac{1}{ax}\right)}{ac^2}+\frac{x\left(c-\frac{c}{ax}\right)^{p+2}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a*x))^(2 + p)*x)/c^2 + ((c - c/(a*x))^(2 + p)*Hypergeometric2F1[1,
 2 + p, 3 + p, (a - x^(-1))/(2*a)])/(2*a*c^2*(2 + p)) - ((c - c/(a*x))^(2 +
 p)*Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x)])/(a*c^2)
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] && !(IntegerQ[m] && NegQ[n])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 6268

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_))^(p_), x_Symbol]
:> Int[u*(c + d/x)^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x] /; FreeQ[{a, c,
d, p}, x] && EqQ[c^2 - a^2*d^2, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !G
tQ[c, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{ax}\right)^p\,dx$$
$$=-\int\frac{\left(c-\frac{c}{ax}\right)^p(1-ax)}{1+ax}\,dx$$
$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{1+p}x}{1+ax}\,dx}{c}$$
$$=\frac{a\int\frac{\left(c-\frac{c}{ax}\right)^{1+p}}{a+\frac{1}{x}}\,dx}{c}$$
$$=-\frac{a\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x^2(a+x)}\,dx,x,\frac{1}{x}\right)}{c}$$
$$=\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{1+p}\left(c(2+p)+\frac{c(1+p)x}{a}\right)}{x(a+x)}\,dx,x,\frac{1}{x}\right)}{c^2}$$
$$=\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}-\frac{\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{1+p}}{a+x}\,dx,x,\frac{1}{x}\right)}{ac}+\frac{(2+p)\text{Subst}\left(\int\frac{\left(c-\frac{cx}{a}\right)^{1+p}}{x}\,dx\right.}{ac}$$
$$=\frac{\left(c-\frac{c}{ax}\right)^{2+p}x}{c^2}+\frac{\left(c-\frac{c}{ax}\right)^{2+p}{}_2F_1\left(1,2+p;3+p;\frac{a-\frac{1}{x}}{2a}\right)}{2ac^2(2+p)}-\frac{\left(c-\frac{c}{ax}\right)^{2+p}{}_2F_1(1}{}$$

Mathematica [A]
time = 0.03, size = 87, normalized size = 0.76

$$\frac{\left(c-\frac{c}{ax}\right)^p(-1+ax)^2\left({}_2F_1\left(1,2+p;3+p;\frac{a-\frac{1}{x}}{2a}\right)+2(2+p)\left(ax-{}_2F_1\left(1,2+p;3+p;1-\frac{1}{ax}\right)\right)\right)}{2a^3(2+p)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a*x))^p/E^(2*ArcCoth[a*x]),x]
[Out] ((c - c/(a*x))^p*(-1 + a*x)^2*(Hypergeometric2F1[1, 2 + p, 3 + p, (a - x^(-
1))/(2*a)] + 2*(2 + p)*(a*x - Hypergeometric2F1[1, 2 + p, 3 + p, 1 - 1/(a*x
)])))/(2*a^3*(2 + p)*x^2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\left(c-\frac{c}{ax}\right)^p(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
[Out] int((c-c/a/x)^p*(a*x-1)/(a*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] integral((a*x - 1)*((a*c*x - c)/(a*x))^p/(a*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(-1+\frac{1}{ax}\right)\right)^{p}(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)**p*(a*x-1)/(a*x+1),x)
[Out] Integral((-c*(-1 + 1/(a*x)))**p*(a*x - 1)/(a*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a/x)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(c - c/(a*x))^p/(a*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a\,x}\right)^{p}\,(a\,x-1)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a*x))^p*(a*x - 1))/(a*x + 1), x)
```

3.556 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal. Leaf size=393

$$\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{64}a^3$$

```
[Out] 5/72*a^6*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-7/72*a^7*c^4*(1-1/a/x)^(5
/2)*(1+1/a/x)^(11/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(11/2)*x^9+3
5/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+35/384*a*c^4*(1+1/a/x)
^(3/2)*x^2*(1-1/a/x)^(1/2)+7/192*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2
)+1/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/144*a^4*c^4*(1+1/a/x)^
(9/2)*x^5*(1-1/a/x)^(1/2)-5/144*a^5*c^4*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2
)+35/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{7}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{5}{72}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{5}{144}a^5c^4x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{1}{144}a^4c^4x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{64}a^3c^4x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{7}{192}a^2c^4x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{35}{384}ac^4x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{35}{128}c^4x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{35c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (35*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(5/2)*x^3)/192 + (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/6
4 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/144 - (5*a^5*c^4*Sq
rt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/144 + (5*a^6*c^4*(1 - 1/(a*x))^(3
/2)*(1 + 1/(a*x))^(11/2)*x^7)/72 - (7*a^7*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a
*x))^(11/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(11/2)*x^9
)/9 + (35*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = \left(a^8c^4\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4 x^8\,dx$$

$$= -\left(\left(a^8c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^{10}}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}x^9 + \frac{1}{9}\left(7a^7c^4\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\right.}{x^9}\right.$$

$$= -\frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x^8 + \frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{11/2}$$

$$= \frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7 - \frac{7}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}$$

$$= -\frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6 + \frac{5}{72}a^6c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}$$

$$= \frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 - \frac{5}{144}a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6 +$$

$$= \frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 + \frac{1}{144}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 -$$

$$= \frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 +$$

$$= \frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 +$$

$$= \frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{7}{192}a^2c^4$$

$$= \frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{7}{192}a^2c^4$$

Mathematica [A]
time = 0.09, size = 111, normalized size = 0.28

$$\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(128-837ax-512a^2x^2+978a^3x^3+768a^4x^4-600a^5x^5-512a^6x^6+144a^7x^7+128a^8x^8)+315\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 - 837*a*x - 512*a^2*x^2 + 978*a^3*x^3
+ 768*a^4*x^4 - 600*a^5*x^5 - 512*a^6*x^6 + 144*a^7*x^7 + 128*a^8*x^8) + 31
5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

Maple [A]
time = 0.11, size = 279, normalized size = 0.71

method	result
risch	$\frac{(128a^8x^8+144a^7x^7-512a^6x^6-600a^5x^5+768a^4x^4+978a^3x^3-512a^2x^2-837ax+128)(ax-1)c^4}{1152a\sqrt{\frac{ax-1}{ax+1}}}+\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c}{128\sqrt{a^2}\sqrt{\frac{ax}{ax}}}$
default	$\frac{(ax-1)c^4\left(128(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+144(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5-384\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4-456(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+384(\right.}{115}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1152*(a*x-1)*c^4/a*(128*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^6*x^6+144*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2)*a^5*x^5-384*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-45
6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+384*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a
^2*x^2+522*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+256*(a^2*x^2-1)^(3/2)*(a^2)^(1
/2)-315*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-384*((a*x+1)*(a*x-1))^(3/2)*(a^2)
^(1/2)+315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1
)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 415, normalized size = 1.06

$$\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10458\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-23202\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+32768\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+23202\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-10458\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-315\,c^4\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{9(ax-1)a^2}{ax+1}-\frac{36(ax-1)^2a^2}{(ax+1)^2}+\frac{84(ax-1)^3a^2}{(ax+1)^3}-\frac{126(ax-1)^4a^2}{(ax+1)^4}+\frac{126(ax-1)^5a^2}{(ax+1)^5}-\frac{84(ax-1)^6a^2}{(ax+1)^6}+\frac{36(ax-1)^7a^2}{(ax+1)^7}-\frac{9(ax-1)^8a^2}{(ax+1)^8}+\frac{(ax-1)^9a^2}{(ax+1)^9}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 27
30*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/2
) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 32768*c^4*((a*x - 1)/(a*x + 1)
)^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*x
 + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x - 1
)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2 +
 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a*
x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7*
a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1)
^9 - a^2))*a
```

Fricas [A]
time = 0.36, size = 169, normalized size = 0.43

$$\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(128\,a^9c^4x^9+272\,a^8c^4x^8-368\,a^7c^4x^7-1112\,a^6c^4x^6+168\,a^5c^4x^5+1746\,a^4c^4x^4+466\,a^3c^4x^3-1349\,a^2c^4x^2-709\,ac^4x+128\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (128*a^9*c^4*x^9 + 272*a^8*c^4*x^8 - 368*a^7*c^4*x^7
 - 1112*a^6*c^4*x^6 + 168*a^5*c^4*x^5 + 1746*a^4*c^4*x^4 + 466*a^3*c^4*x^3
- 1349*a^2*c^4*x^2 - 709*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^8x^8}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(6*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x**6
/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8/sqrt(a*x/(a*x +
 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.43, size = 218, normalized size = 0.55

$$-\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{128\,|a|\mathrm{sgn}\left(ax+1\right)}-\frac{1}{1152}\sqrt{a^2x^2-1}\left(\left(\left(\frac{837\,c^4}{\mathrm{sgn}\left(ax+1\right)}+2\left(\frac{256\,ac^4}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{489\,a^2c^4}{\mathrm{sgn}\left(ax+1\right)}+4\left(\frac{96\,a^3c^4}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{75\,a^4c^4}{\mathrm{sgn}\left(ax+1\right)}+2\left(\frac{32\,a^5c^4}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{8\,a^7c^4x}{\mathrm{sgn}\left(ax+1\right)}+\frac{9\,a^6c^4}{\mathrm{sgn}\left(ax+1\right)}\right)x\right)x\right)x\right)x\right)x\right)x\right)x-\frac{128\,c^4}{a\mathrm{sgn}\left(ax+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) -
 1/1152*sqrt(a^2*x^2 - 1)*((837*c^4/sgn(a*x + 1) + 2*(256*a*c^4/sgn(a*x + 1
) - (489*a^2*c^4/sgn(a*x + 1) + 4*(96*a^3*c^4/sgn(a*x + 1) - (75*a^4*c^4/sg
n(a*x + 1) + 2*(32*a^5*c^4/sgn(a*x + 1) - (8*a^7*c^4*x/sgn(a*x + 1) + 9*a^6
*c^4/sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x - 128*c^4/(a*sgn(a*x + 1)))
```

Mupad [B]
time = 1.37, size = 362, normalized size = 0.92

$$\frac{\frac{455\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}-\frac{35\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}+\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{32}+\frac{512\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{9}-\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{32}+\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{32}-\frac{455\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/2}}{96}+\frac{35\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/2}}{64}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}+\frac{9\,a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}-\frac{a\,{\left(a\,x-1\right)}^9}{{\left(a\,x+1\right)}^9}}+\frac{35\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((455*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (35*c^4*((a*x - 1)/(a*x + 1))^(
1/2))/64 - (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (1289*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 - (1289*c^4*
((a*x - 1)/(a*x + 1))^(11/2))/32 + (581*c^4*((a*x - 1)/(a*x + 1))^(13/2))/3
2 - (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (35*c^4*((a*x - 1)/(a*x + 1
))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a*x + 1
)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^4 - (1
26*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*a*(a*x
 - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9)/(a*x
 + 1)^9) + (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.557 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal. Leaf size=313

$$\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{56}a^3c^3$$

```
[Out] 5/42*a^5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(9/2)*x^7+5/16*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+5
/48*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/24*a^2*c^3*(1+1/a/x)^(5/2)*
x^3*(1-1/a/x)^(1/2)+1/56*a^3*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-1/14*a
^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)+5/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/
a/x)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{5}{42}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{14}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{56}a^3c^3x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{24}a^2c^3x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{5}{48}ac^3x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{5}{16}c^3x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (5*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/48 + (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/56 - (a^
4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/14 + (5*a^5*c^3*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/42 - (a^6*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1
/(a*x))^(9/2)*x^7)/7 + (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])
/(16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\left(\left(a^6c^3\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7 - \frac{1}{7}\left(5a^5c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\right.}{x^7}\right.$$

$$= \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7$$

$$= -\frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}a^5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6$$

$$= \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5 + \frac{5}{42}$$

$$= \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{14}$$

$$= \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{56}a$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{}$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{}$$

$$= \frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 + \frac{1}{24}a^2c^3\sqrt{}$$

Mathematica [A]
time = 0.10, size = 95, normalized size = 0.30

$$\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(48-231ax-144a^2x^2+182a^3x^3+144a^4x^4-56a^5x^5-48a^6x^6)+105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 - 231*a*x - 144*a^2*x^2 + 182*a^3*x^3 +
 144*a^4*x^4 - 56*a^5*x^5 - 48*a^6*x^6) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

Maple [A]
time = 0.10, size = 231, normalized size = 0.74

method	result
risch	$-\frac{(48a^6x^6+56a^5x^5-144a^4x^4-182a^3x^3+144a^2x^2+231ax-48)(ax-1)c^3}{336a\sqrt{\frac{ax-1}{ax+1}}}+\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax+1)(ax}}{16\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$-\frac{(ax-1)c^3\left(48\sqrt{a^2}(a^2x^2-1)^{\frac{3}{2}}a^4x^4+56(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^3x^3-96(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^2x^2-126\sqrt{a^2}(a^2x^2-1)^{\frac{3}{2}}ax-64(a^2x\right.}{336a\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/336*(a*x-1)*c^3/a*(48*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4+56*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*a^3*x^3-96*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-126*(
a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+105*(a^2)
^(1/2)*(a^2*x^2-1)^(1/2)*a*x+112*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-105*ln
((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1
/2)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 337, normalized size = 1.08

$$\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-3072\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-1981\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+\frac{35(ax-1)^3a^2}{(ax+1)^3}-\frac{35(ax-1)^4a^2}{(ax+1)^4}+\frac{21(ax-1)^5a^2}{(ax+1)^5}-\frac{7(ax-1)^6a^2}{(ax+1)^6}+\frac{(ax-1)^7a^2}{(ax+1)^7}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 700
*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) -
3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2)
 + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1)))
/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1)
^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(a
*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 - a
^2))*a
```

Fricas [A]
time = 0.34, size = 148, normalized size = 0.47

$$\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(48\,a^7c^3x^7+104\,a^6c^3x^6-88\,a^5c^3x^5-326\,a^4c^3x^4-38\,a^3c^3x^3+375\,a^2c^3x^2+183\,ac^3x-48\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (48*a^7*c^3*x^7 + 104*a^6*c^3*x^6 - 88*a^5*c^3*x^5 -
326*a^4*c^3*x^4 - 38*a^3*c^3*x^3 + 375*a^2*c^3*x^2 + 183*a*c^3*x - 48*c^3)*
sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{3a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^6x^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(-3*a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6/s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)), x))
```

Giac [A]
time = 0.45, size = 178, normalized size = 0.57

$$-\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{16\,|a|\mathrm{sgn}\left(ax+1\right)}-\frac{1}{336}\sqrt{a^2x^2-1}\left(\left(2\left(\frac{72\,ac^3}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{91\,a^2c^3}{\mathrm{sgn}\left(ax+1\right)}+4\left(\frac{18\,a^3c^3}{\mathrm{sgn}\left(ax+1\right)}-\left(\frac{6\,a^5c^3x}{\mathrm{sgn}\left(ax+1\right)}+\frac{7\,a^4c^3}{\mathrm{sgn}\left(ax+1\right)}\right)x\right)x\right)x\right)x+\frac{231\,c^3}{\mathrm{sgn}\left(ax+1\right)}\right)x-\frac{48\,c^3}{a\mathrm{sgn}\left(ax+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) - 1
/336*sqrt(a^2*x^2 - 1)*((2*(72*a*c^3/sgn(a*x + 1) - (91*a^2*c^3/sgn(a*x + 1
) + 4*(18*a^3*c^3/sgn(a*x + 1) - (6*a^5*c^3*x/sgn(a*x + 1) + 7*a^4*c^3/sgn(
a*x + 1))*x)*x)*x)*x + 231*c^3/sgn(a*x + 1))*x - 48*c^3/(a*sgn(a*x + 1)))
```

Mupad [B]
time = 1.31, size = 289, normalized size = 0.92

$$\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{6}+\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{24}+\frac{128\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{7}-\frac{283\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{24}+\frac{25\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{6}-\frac{5\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((5*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/8 - (25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 + (283*c^3*((a*x -
1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(a*x + 1))^(7/2))/7 - (283*c^
3*((a*x - 1)/(a*x + 1))^(9/2))/24 + (25*c^3*((a*x - 1)/(a*x + 1))^(11/2))/6
 - (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*x + 1) +
 (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 + (35*a*(a
*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a*x - 1)^6)
/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

3.558 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal. Leaf size=233

$$\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1$$

```
[Out] 1/5*a^4*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+3/8*c^2*arctanh((1-1/a/x)^(
1/2)*(1+1/a/x)^(1/2))/a+1/8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/20*
a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-3/20*a^3*c^2*(1+1/a/x)^(7/2)*x^
4*(1-1/a/x)^(1/2)+3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{20}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{3}{8}c^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 + (a*c^2*Sqrt[1 - 1/(a*x)]*
(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)
*x^3)/20 - (3*a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (a^4*
c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/5 + (3*c^2*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5+\frac{1}{5}\left(3a^3c^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)}{x^5}\right.$$

$$=-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-$$

$$=\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a$$

$$=\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{3}{20}a^3$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-}$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-}$$

$$=\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{20}a^2c^2\sqrt{1-}$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.34

$$\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\left(8-25ax-16a^2x^2+10a^3x^3+8a^4x^4\right)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 25*a*x - 16*a^2*x^2 + 10*a^3*x^3 + 8*a
^4*x^4) + 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [A]
time = 0.10, size = 183, normalized size = 0.79

method	result
risch	$\frac{(8a^4x^4+10a^3x^3-16a^2x^2-25ax+8)(ax-1)c^2}{40a\sqrt{\frac{ax-1}{ax+1}}}+\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax+1)(ax-1)}}{8\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$
default	$\frac{(ax-1)c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+30\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-45\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-40((ax+1)(ax\right.}{120a\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)(ax-1)}\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(a*x-1)*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+30*(a^2)^(1/2
)*(a^2*x^2-1)^(3/2)*a*x+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-45*(a^2)^(1/2)*(a^
2*x^2-1)^(1/2)*a*x-40*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+45*ln((a^2*x+(a^2
*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)
*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 259, normalized size = 1.11

$$\frac{1}{40}a\left(\frac{15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+128c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-15c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{5(ax-1)a^2}{ax+1}-\frac{10(ax-1)^2a^2}{(ax+1)^2}+\frac{10(ax-1)^3a^2}{(ax+1)^3}-\frac{5(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5a^2}{(ax+1)^5}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a*
x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^2
*((a*x - 1)/(a*x + 1))^(7/2) + 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^2
*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*x
 + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^2
))
```

Fricas [A]
time = 0.35, size = 125, normalized size = 0.54

$$\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(8\,a^5c^2x^5+18\,a^4c^2x^4-6\,a^3c^2x^3-41\,a^2c^2x^2-17\,ac^2x+8\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) + (8*a^5*c^2*x^5 + 18*a^4*c^2*x^4 - 6*a^3*c^2*x^3 - 41*a^2
*c^2*x^2 - 17*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{a^4x^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/sqrt(a*x/(a*
x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.44, size = 137, normalized size = 0.59

$$\frac{1}{40}\sqrt{a^2x^2-1}\left(\left(2\left(\left(\frac{4\,a^3c^2x}{\operatorname{sgn}(ax+1)}+\frac{5\,a^2c^2}{\operatorname{sgn}(ax+1)}\right)x-\frac{8\,ac^2}{\operatorname{sgn}(ax+1)}\right)x-\frac{25\,c^2}{\operatorname{sgn}(ax+1)}\right)x+\frac{8\,c^2}{a\operatorname{sgn}(ax+1)}\right)-\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\operatorname{sgn}(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/40*sqrt(a^2*x^2 - 1)*((2*((4*a^3*c^2*x/sgn(a*x + 1) + 5*a^2*c^2/sgn(a*x +
 1))*x - 8*a*c^2/sgn(a*x + 1))*x - 25*c^2/sgn(a*x + 1))*x + 8*c^2/(a*sgn(a*
x + 1))) - 3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x
+ 1))
```

Mupad [B]
time = 1.26, size = 214, normalized size = 0.92

$$\frac{\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}-\frac{3\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{32\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{5}-\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{2}+\frac{3\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}+\frac{3\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((7*c^2*((a*x - 1)/(a*x + 1))^(3/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(1/2)
)/4 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 - (7*c^2*((a*x - 1)/(a*x + 1))
^(7/2))/2 + (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (3*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.559 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=145

$$\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{c\tanh^{-1}\left(\sqrt{1\right.}}{}$$

```
[Out] 1/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/6*a*c*(1+1/a/x)^(3/2)*x^
2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/2*c*x*(1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
```

```
[Out] (c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (a*c*Sqrt[1 - 1/(a*x)]*(1 + 1
/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^3)/3
+ (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\left(\left(a^2c\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{3}(ac)\text{Subst}\left(\int \frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{2}c\text{Subst}$$

$$= \frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}$$

$$= \frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}$$

$$= \frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.42

$$\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(2-3ax-2a^2x^2)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2),x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 2*a^2*x^2) + 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A]
time = 0.05, size = 119, normalized size = 0.82

method	result	size
risch	$-\frac{(2a^2x^2+3ax-2)(ax-1)c}{6a\sqrt{\frac{ax-1}{ax+1}}}+\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	108
default	$-\frac{(ax-1)c\left(3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)*c*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+2*((a*x+1)*(a*x-1))^(3/
2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/(
(a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A]
time = 0.25, size = 171, normalized size = 1.18

$$-\frac{1}{6}\,a\left(\frac{2\left(3\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-8\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) - 8*c*((a*x - 1)/(a*x + 1))^(3/2
) - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)
^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.35, size = 91, normalized size = 0.63

$$\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(2\,a^3cx^3+5\,a^2cx^2+acx-2\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x +
 1)) - 1) - (2*a^3*c*x^3 + 5*a^2*c*x^2 + a*c*x - 2*c)*sqrt((a*x - 1)/(a*x +
 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{a^2x^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int \left(-\frac{1}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/
sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.41, size = 90, normalized size = 0.62

$$-\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,acx}{\operatorname{sgn}(ax+1)}+\frac{3\,c}{\operatorname{sgn}(ax+1)}\right)x-\frac{2\,c}{a\operatorname{sgn}(ax+1)}\right)-\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a^2*x^2 - 1)*((2*a*c*x/sgn(a*x + 1) + 3*c/sgn(a*x + 1))*x - 2*c/(
a*sgn(a*x + 1))) - 1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sg
n(a*x + 1))
```

Mupad [B]
time = 0.07, size = 131, normalized size = 0.90

$$\frac{c\operatorname{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{c\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{8\,c\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-c\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{a\,(a\,x-1)^3}{(a\,x+1)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (c*((a*x - 1)/(a*x + 1))^(1/2) +
 (8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(5/2))/(a -
(3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)
/(a*x + 1)^3)
```

3.560 $\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

```
[Out] 1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {6318}

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcCoth[a*x]/(a*c)
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{\coth^{-1}(ax)}}{ac}$$

Mathematica [A]
time = 0.03, size = 13, normalized size = 1.00

$$\frac{e^{\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] E^ArcCoth[a*x]/(a*c)
```

Maple [A]
time = 0.19, size = 23, normalized size = 1.77

method	result	size
gosper	$\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\, ac}$	23
default	$\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\, ac}$	23
trager	$\frac{(ax+1)\sqrt{-\frac{-ax+1}{ax+1}}}{ac(ax-1)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/((a*x-1)/(a*x+1))^(1/2)/a/c
```

Maxima [A]
time = 0.26, size = 22, normalized size = 1.69

$$\frac{1}{ac\sqrt{\frac{ax-1}{ax+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] 1/(a*c*sqrt((a*x - 1)/(a*x + 1)))
```

Fricas [A]
time = 0.33, size = 34, normalized size = 2.62

$$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] (a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x - a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(1/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [**B**]
time = 1.22, size = 22, normalized size = 1.69

$$\frac{1}{a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] 1/(a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.561 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx$

Optimal. Leaf size=51

$$\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2(1-a^2x^2)}$$

```
[Out] 2/3/((a*x-1)/(a*x+1))^(1/2)/a/c^2-1/3/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/
c^2/(-a^2*x^2+1)
```

Rubi [A]

time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6320, 6318}

$$\frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{(1-2ax)e^{\coth^{-1}(ax)}}{3ac^2(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^ArcCoth[a*x])/(3*a*c^2) - (E^ArcCoth[a*x]*(1 - 2*a*x))/(3*a*c^2*(1 - a
^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2} dx &= -\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2(1-a^2x^2)}+\frac{2\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2} dx}{3c}\\ &= \frac{2e^{\coth^{-1}(ax)}}{3ac^2}-\frac{e^{\coth^{-1}(ax)}(1-2ax)}{3ac^2(1-a^2x^2)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.98

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1-2ax+2a^2x^2)}{3c^2(-1+ax)^2(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 - 2*a*x + 2*a^2*x^2))/(3*c^2*(-1 + a*x)^2*(1 +
 a*x))
```

Maple [A]
time = 0.18, size = 52, normalized size = 1.02

method	result	size
trager	$\frac{(2a^2x^2-2ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^2(ax-1)^2}$	47
gosper	$\frac{2a^2x^2-2ax-1}{3(a^2x^2-1)c^2\sqrt{\frac{ax-1}{ax+1}}\,a}$	49
default	$\frac{2a^2x^2-2ax-1}{3\sqrt{\frac{ax-1}{ax+1}}\,c^2(ax+1)(ax-1)a}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*a^2*x^2-2*a*x-1)/((a*x-1)/(a*x+1))^(1/2)/c^2/(a*x+1)/(a*x-1)/a
```

Maxima [A]
time = 0.26, size = 65, normalized size = 1.27

$$\frac{1}{12}\,a\left(\frac{3\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{\frac{6\,(ax-1)}{ax+1}-1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*(3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2) + (6*(a*x - 1)/(a*x + 1) - 1)
/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A]
time = 0.33, size = 58, normalized size = 1.14

$$\frac{(2\,a^2x^2-2\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/3*(2*a^2*x^2 - 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - 2*a^2*
c^2*x + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
[Out] Integral(1/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x**2*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 - c)^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 0.07, size = 50, normalized size = 0.98

$$\frac{-2\,a^2\,x^2+2\,a\,x+1}{(3\,a\,c^2-3\,a^3\,c^2\,x^2)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (2*a*x - 2*a^2*x^2 + 1)/((3*a*c^2 - 3*a^3*c^2*x^2)*((a*x - 1)/(a*x + 1))^(1
/2))
```

3.562 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=85

$$\frac{8e^{\coth^{-1}(ax)}}{15ac^3}-\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3\left(1-a^2x^2\right)}$$

```
[Out] 8/15/((a*x-1)/(a*x+1))^(1/2)/a/c^3-1/15/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)/
a/c^3/(-a^2*x^2+1)^2-4/15/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^3/(-a^2*x^
2+1)
```

Rubi [A]

time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6320, 6318}

$$-\frac{(1-4ax)e^{\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}-\frac{4(1-2ax)e^{\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)}+\frac{8e^{\coth^{-1}(ax)}}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] (8*E^ArcCoth[a*x])/(15*a*c^3) - (E^ArcCoth[a*x]*(1 - 4*a*x))/(15*a*c^3*(1 -
 a^2*x^2)^2) - (4*E^ArcCoth[a*x]*(1 - 2*a*x))/(15*a*c^3*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3(1-a^2x^2)^2} + \frac{4\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{5c}$$
$$= -\frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3(1-a^2x^2)^2} - \frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3(1-a^2x^2)} + \frac{8\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{15c^2}$$
$$= \frac{8e^{\coth^{-1}(ax)}}{15ac^3} - \frac{e^{\coth^{-1}(ax)}(1-4ax)}{15ac^3(1-a^2x^2)^2} - \frac{4e^{\coth^{-1}(ax)}(1-2ax)}{15ac^3(1-a^2x^2)}$$

Mathematica [A]
time = 0.06, size = 66, normalized size = 0.78

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(3+12ax-12a^2x^2-8a^3x^3+8a^4x^4)}{15c^3(-1+ax)^3(1+ax)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^3,x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(3 + 12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4))
/(15*c^3*(-1 + a*x)^3*(1 + a*x)^2)
```

Maple [A]
time = 0.19, size = 68, normalized size = 0.80

method	result	size
gosper	$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15(a^2x^2-1)^2c^3\sqrt{\frac{ax-1}{ax+1}}\,a}$	65
default	$\frac{8a^4x^4-8a^3x^3-12a^2x^2+12ax+3}{15\sqrt{\frac{ax-1}{ax+1}}\,c^3(ax+1)^2a(ax-1)^2}$	68
trager	$\frac{(8a^4x^4-8a^3x^3-12a^2x^2+12ax+3)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^3(ax+1)(ax-1)^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
[Out] 1/15*(8*a^4*x^4-8*a^3*x^3-12*a^2*x^2+12*a*x+3)/((a*x-1)/(a*x+1))^(1/2)/c^3/
(a*x+1)^2/a/(a*x-1)^2
```

Maxima [A]
time = 0.26, size = 99, normalized size = 1.16

$$-\frac{1}{240}\,a\left(\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-12\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{\frac{20\,(ax-1)}{ax+1}-\frac{90\,(ax-1)^2}{(ax+1)^2}-3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/240*a*(5*(((a*x - 1)/(a*x + 1))^(3/2) - 12*sqrt((a*x - 1)/(a*x + 1)))/(a
^2*c^3) + (20*(a*x - 1)/(a*x + 1) - 90*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a^2*c^
3*((a*x - 1)/(a*x + 1))^(5/2)))
```

Fricas [A]
time = 0.33, size = 86, normalized size = 1.01

$$\frac{(8\,a^4x^4-8\,a^3x^3-12\,a^2x^2+12\,ax+3)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*a^4*x^4 - 8*a^3*x^3 - 12*a^2*x^2 + 12*a*x + 3)*sqrt((a*x - 1)/(a*x
+ 1))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(1/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*c*x^2 - c)^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 0.07, size = 60, normalized size = 0.71

$$\frac{8\,a^4\,x^4-8\,a^3\,x^3-12\,a^2\,x^2+12\,a\,x+3}{15\,a\,c^3\,{\left(a\,x+1\right)}^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (12*a*x - 12*a^2*x^2 - 8*a^3*x^3 + 8*a^4*x^4 + 3)/(15*a*c^3*(a*x + 1)^4*((a
*x - 1)/(a*x + 1))^(5/2))
```

3.563 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=119

$$\frac{16e^{\coth^{-1}(ax)}}{35ac^4}-\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4\left(1-a^2x^2\right)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4\left(1-a^2x^2\right)^2}-\frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4\left(1-a^2x^2\right)}$$

```
[Out] 16/35/((a*x-1)/(a*x+1))^(1/2)/a/c^4-1/35/((a*x-1)/(a*x+1))^(1/2)*(-6*a*x+1)
/a/c^4/(-a^2*x^2+1)^3-2/35/((a*x-1)/(a*x+1))^(1/2)*(-4*a*x+1)/a/c^4/(-a^2*x
^2+1)^2-8/35/((a*x-1)/(a*x+1))^(1/2)*(-2*a*x+1)/a/c^4/(-a^2*x^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6320, 6318}

$$-\frac{(1-6ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}-\frac{8(1-2ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)}-\frac{2(1-4ax)e^{\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^2}+\frac{16e^{\coth^{-1}(ax)}}{35ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (16*E^ArcCoth[a*x])/(35*a*c^4) - (E^ArcCoth[a*x]*(1 - 6*a*x))/(35*a*c^4*(1
- a^2*x^2)^3) - (2*E^ArcCoth[a*x]*(1 - 4*a*x))/(35*a*c^4*(1 - a^2*x^2)^2) -
 (8*E^ArcCoth[a*x]*(1 - 2*a*x))/(35*a*c^4*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx &= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4(1-a^2x^2)^3}+\frac{6\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{7c} \\
&= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4(1-a^2x^2)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4(1-a^2x^2)^2}+\frac{24\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{35c^2} \\
&= -\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4(1-a^2x^2)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4(1-a^2x^2)^2}-\frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4(1-a^2x^2)}+\frac{16\int \frac{e^{\coth^{-1}(ax)}}{c-a^2cx^2}}{35c^3} \\
&= \frac{16e^{\coth^{-1}(ax)}}{35ac^4}-\frac{e^{\coth^{-1}(ax)}(1-6ax)}{35ac^4(1-a^2x^2)^3}-\frac{2e^{\coth^{-1}(ax)}(1-4ax)}{35ac^4(1-a^2x^2)^2}-\frac{8e^{\coth^{-1}(ax)}(1-2ax)}{35ac^4(1-a^2x^2)}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 0.69

$$
\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5-30ax+30a^2x^2+40a^3x^3-40a^4x^4-16a^5x^5+16a^6x^6)}{35c^4(-1+ax)^4(1+ax)^3}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^4,x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-5 - 30*a*x + 30*a^2*x^2 + 40*a^3*x^3 - 40*a^4*x^
4 - 16*a^5*x^5 + 16*a^6*x^6))/(35*c^4*(-1 + a*x)^4*(1 + a*x)^3)
```

Maple [A]
time = 0.19, size = 84, normalized size = 0.71

method	result	size
gosper	$\frac{16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5}{35(a^2x^2-1)^3c^4\sqrt{\frac{ax-1}{ax+1}}\,a}$	81
default	$\frac{16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5}{35\sqrt{\frac{ax-1}{ax+1}}\,c^4(ax+1)^3(ax-1)^3a}$	84
trager	$\frac{(16a^6x^6-16a^5x^5-40a^4x^4+40a^3x^3+30a^2x^2-30ax-5)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^4(ax+1)^2(ax-1)^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(16*a^6*x^6-16*a^5*x^5-40*a^4*x^4+40*a^3*x^3+30*a^2*x^2-30*a*x-5)/((a*
x-1)/(a*x+1))^(1/2)/c^4/(a*x+1)^3/(a*x-1)^3/a
```

Maxima [A]
time = 0.25, size = 132, normalized size = 1.11

$$\frac{1}{2240}\,a\left(\frac{7\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+75\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{42\,(ax-1)}{ax+1}-\frac{175\,(ax-1)^2}{(ax+1)^2}+\frac{700\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/2240*a*(7*(((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) +
 75*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + (42*(a*x - 1)/(a*x + 1) - 175*(a
*x - 1)^2/(a*x + 1)^2 + 700*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^4*((a*x - 1
)/(a*x + 1))^(7/2)))
```

Fricas [A]
time = 0.38, size = 134, normalized size = 1.13

$$\frac{(16\,a^6x^6-16\,a^5x^5-40\,a^4x^4+40\,a^3x^3+30\,a^2x^2-30\,ax-5)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(16*a^6*x^6 - 16*a^5*x^5 - 40*a^4*x^4 + 40*a^3*x^3 + 30*a^2*x^2 - 30*a
*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^
4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(1/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x**6*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))), x)/c**4
```

<u>Giac</u> [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 - c)^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

<u>Mupad</u> [B]
time = 0.06, size = 142, normalized size = 1.19

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{32\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{320\,a\,c^4}-\frac{\frac{5\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{20\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{6\,\left(a\,x-1\right)}{5\,\left(a\,x+1\right)}+\frac{1}{7}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((a*x - 1)/(a*x + 1))^(3/2)/(
32*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - ((5*(a*x - 1)^2)/(a*x
 + 1)^2 - (20*(a*x - 1)^3)/(a*x + 1)^3 - (6*(a*x - 1))/(5*(a*x + 1)) + 1/7)
/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2))
```

3.564 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^5\,dx$

Optimal. Leaf size=84

$$-\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

```
[Out] -16/7*c^5*(a*x+1)^7/a+4*c^5*(a*x+1)^8/a-8/3*c^5*(a*x+1)^9/a+4/5*c^5*(a*x+1)
^10/a-1/11*c^5*(a*x+1)^11/a
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{4c^5(ax+1)^{10}}{5a}-\frac{8c^5(ax+1)^9}{3a}+\frac{4c^5(ax+1)^8}{a}-\frac{16c^5(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (-16*c^5*(1 + a*x)^7)/(7*a) + (4*c^5*(1 + a*x)^8)/a - (8*c^5*(1 + a*x)^9)/(
3*a) + (4*c^5*(1 + a*x)^10)/(5*a) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx = -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^5\,dx$$
$$= -\left(c^5\int(1-ax)^4(1+ax)^6\,dx\right)$$
$$= -\left(c^5\int\left(16(1+ax)^6-32(1+ax)^7+24(1+ax)^8-8(1+ax)^9+(1+ax\right.\right.$$
$$= -\frac{16c^5(1+ax)^7}{7a}+\frac{4c^5(1+ax)^8}{a}-\frac{8c^5(1+ax)^9}{3a}+\frac{4c^5(1+ax)^{10}}{5a}-\frac{c^5(1+}{11}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.56

$$-\frac{c^5(1+ax)^7\left(281-812ax+938a^2x^2-504a^3x^3+105a^4x^4\right)}{1155a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
[Out] -1/1155*(c^5*(1 + a*x)^7*(281 - 812*a*x + 938*a^2*x^2 - 504*a^3*x^3 + 105*a
^4*x^4))/a
```

Maple [A]
time = 0.22, size = 85, normalized size = 1.01

method	result
gosper	$-\frac{c^5x\left(105a^{10}x^{10}+231a^9x^9-385a^8x^8-1155a^7x^7+330a^6x^6+2310a^5x^5+462a^4x^4-2310a^3x^3-1155a^2x^2+1155ax+1155\right)}{1155}$
default	$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{1}{5}a^9x^{10}+\frac{1}{3}a^8x^9+a^7x^8-\frac{2}{7}a^6x^7-2a^5x^6-\frac{2}{5}a^4x^5+2a^3x^4+a^2x^3-a\,x^2-x\right)$
norman	$a^7c^5x^8+c^5a^2x^3-c^5x-a\,c^5x^2-\frac{2}{5}a^4c^5x^5-2a^5c^5x^6-\frac{2}{7}a^6c^5x^7+\frac{1}{3}a^8c^5x^9-\frac{1}{5}a^9c^5x^{10}-\frac{1}{11}a^{10}c^5$
risch	$a^7c^5x^8+c^5a^2x^3-c^5x-a\,c^5x^2-\frac{2}{5}a^4c^5x^5-2a^5c^5x^6-\frac{2}{7}a^6c^5x^7+\frac{1}{3}a^8c^5x^9-\frac{1}{5}a^9c^5x^{10}-\frac{1}{11}a^{10}c^5$
meijerg	$\frac{c^5\left(-\frac{xa\left(2520a^{10}x^{10}+2772a^9x^9+3080a^8x^8+3465a^7x^7+3960a^6x^6+4620a^5x^5+5544a^4x^4+6930a^3x^3+9240a^2x^2+13860ax+27720\right)}{27720}-\ln(-ax+1)\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
[Out] c^5*(-1/11*a^10*x^11-1/5*a^9*x^10+1/3*a^8*x^9+a^7*x^8-2/7*a^6*x^7-2*a^5*x^6
-2/5*a^4*x^5+2*a^3*x^4+a^2*x^3-a*x^2-x)
```

Maxima [A]
time = 0.26, size = 113, normalized size = 1.35

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

Fricas [A]
time = 0.34, size = 113, normalized size = 1.35

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

Sympy [A]
time = 0.04, size = 119, normalized size = 1.42

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{a^9c^5x^{10}}{5}+\frac{a^8c^5x^9}{3}+a^7c^5x^8-\frac{2a^6c^5x^7}{7}-2a^5c^5x^6-\frac{2a^4c^5x^5}{5}+2a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - a**9*c**5*x**10/5 + a**8*c**5*x**9/3 + a**7*c**5*x**
8 - 2*a**6*c**5*x**7/7 - 2*a**5*c**5*x**6 - 2*a**4*c**5*x**5/5 + 2*a**3*c**
5*x**4 + a**2*c**5*x**3 - a*c**5*x**2 - c**5*x
```

Giac [A]
time = 0.42, size = 113, normalized size = 1.35

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{1}{5}\,a^9c^5x^{10}+\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8-\frac{2}{7}\,a^6c^5x^7-2\,a^5c^5x^6-\frac{2}{5}\,a^4c^5x^5+2\,a^3c^5x^4+a^2c^5x^3-ac^5x^2-c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/11*a^10*c^5*x^11 - 1/5*a^9*c^5*x^10 + 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 - 2/
7*a^6*c^5*x^7 - 2*a^5*c^5*x^6 - 2/5*a^4*c^5*x^5 + 2*a^3*c^5*x^4 + a^2*c^5*x
^3 - a*c^5*x^2 - c^5*x
```

Mupad [B]
time = 1.25, size = 113, normalized size = 1.35

$$-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{a^9\,c^5\,x^{10}}{5}+\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8-\frac{2\,a^6\,c^5\,x^7}{7}-2\,a^5\,c^5\,x^6-\frac{2\,a^4\,c^5\,x^5}{5}+2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3-a\,c^5\,x^2-c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] a^2*c^5*x^3 - a*c^5*x^2 - c^5*x + 2*a^3*c^5*x^4 - (2*a^4*c^5*x^5)/5 - 2*a^5
*c^5*x^6 - (2*a^6*c^5*x^7)/7 + a^7*c^5*x^8 + (a^8*c^5*x^9)/3 - (a^9*c^5*x^1
0)/5 - (a^10*c^5*x^11)/11
```

3.565 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal. Leaf size=69

$$-\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}$$

```
[Out] -4/3*c^4*(a*x+1)^6/a+12/7*c^4*(a*x+1)^7/a-3/4*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+
1)^9/a
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 45}

$$\frac{c^4(ax+1)^9}{9a}-\frac{3c^4(ax+1)^8}{4a}+\frac{12c^4(ax+1)^7}{7a}-\frac{4c^4(ax+1)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (-4*c^4*(1 + a*x)^6)/(3*a) + (12*c^4*(1 + a*x)^7)/(7*a) - (3*c^4*(1 + a*x)^
8)/(4*a) + (c^4*(1 + a*x)^9)/(9*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$= -\left(c^4\int (1-ax)^3(1+ax)^5\,dx\right)$$
$$= -\left(c^4\int \left(8(1+ax)^5-12(1+ax)^6+6(1+ax)^7-(1+ax)^8\right)\,dx\right)$$
$$= -\frac{4c^4(1+ax)^6}{3a}+\frac{12c^4(1+ax)^7}{7a}-\frac{3c^4(1+ax)^8}{4a}+\frac{c^4(1+ax)^9}{9a}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.57

$$\frac{c^4(1+ax)^6\left(-65+138ax-105a^2x^2+28a^3x^3\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(1 + a*x)^6*(-65 + 138*a*x - 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A]
time = 0.20, size = 63, normalized size = 0.91

method	result
gosper	$\frac{c^4x\left(28a^8x^8+63a^7x^7-72a^6x^6-252a^5x^5+378a^3x^3+168a^2x^2-252ax-252\right)}{252}$
default	$c^4\left(\frac{1}{9}a^8x^9+\frac{1}{4}a^7x^8-\frac{2}{7}a^6x^7-a^5x^6+\frac{3}{2}a^3x^4+\frac{2}{3}a^2x^3-a\,x^2-x\right)$
norman	$-c^4x+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9-c^4a\,x^2$
risch	$-c^4x+\frac{2}{3}a^2c^4x^3+\frac{3}{2}a^3c^4x^4-a^5c^4x^6-\frac{2}{7}a^6c^4x^7+\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9-c^4a\,x^2$
meijerg	$-\frac{c^4\left(-\frac{ax\left(280a^8x^8+315a^7x^7+360a^6x^6+420a^5x^5+504a^4x^4+630a^3x^3+840a^2x^2+1260ax+2520\right)}{2520}-\ln(-ax+1)\right)}{a}+\frac{4c^4\left(-\frac{ax\left(120a^6x^6+14\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(1/9*a^8*x^9+1/4*a^7*x^8-2/7*a^6*x^7-a^5*x^6+3/2*a^3*x^4+2/3*a^2*x^3-a*
x^2-x)
```

Maxima [A]
time = 0.26, size = 82, normalized size = 1.19

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Fricas [A]
time = 0.32, size = 82, normalized size = 1.19

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Sympy [A]
time = 0.03, size = 87, normalized size = 1.26

$$\frac{a^8c^4x^9}{9}+\frac{a^7c^4x^8}{4}-\frac{2a^6c^4x^7}{7}-a^5c^4x^6+\frac{3a^3c^4x^4}{2}+\frac{2a^2c^4x^3}{3}-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 - a**5*c**4*x**6 +
 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 - a*c**4*x**2 - c**4*x
```

Giac [A]
time = 0.40, size = 82, normalized size = 1.19

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7-a^5c^4x^6+\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3-ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 - a^5*c^4*x^6 + 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 - a*c^4*x^2 - c^4*x
```

Mupad [B]
time = 0.04, size = 82, normalized size = 1.19

$$\frac{a^8\,c^4\,x^9}{9}+\frac{a^7\,c^4\,x^8}{4}-\frac{2\,a^6\,c^4\,x^7}{7}-a^5\,c^4\,x^6+\frac{3\,a^3\,c^4\,x^4}{2}+\frac{2\,a^2\,c^4\,x^3}{3}-a\,c^4\,x^2-c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (2*a^2*c^4*x^3)/3 - a*c^4*x^2 - c^4*x + (3*a^3*c^4*x^4)/2 - a^5*c^4*x^6 - (
2*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

3.566 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal. Leaf size=52

$$-\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

```
[Out] -4/5*c^3*(a*x+1)^5/a+2/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6302, 6275, 45}

$$-\frac{c^3(ax+1)^7}{7a}+\frac{2c^3(ax+1)^6}{3a}-\frac{4c^3(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-4*c^3*(1 + a*x)^5)/(5*a) + (2*c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/
(7*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\ &= -\left(c^3\int (1-ax)^2(1+ax)^4\,dx\right)\\ &= -\left(c^3\int \left(4(1+ax)^4-4(1+ax)^5+(1+ax)^6\right)\,dx\right)\\ &= -\frac{4c^3(1+ax)^5}{5a}+\frac{2c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.60

$$-\frac{c^3(1+ax)^5\left(29-40ax+15a^2x^2\right)}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/105*(c^3*(1 + a*x)^5*(29 - 40*a*x + 15*a^2*x^2))/a
```

Maple [A]
time = 0.19, size = 54, normalized size = 1.04

method	result
gosper	$-\frac{c^3x\left(15a^6x^6+35a^5x^5-21a^4x^4-105a^3x^3-35a^2x^2+105ax+105\right)}{105}$
default	$c^3\left(-\frac{1}{7}a^6x^7-\frac{1}{3}a^5x^6+\frac{1}{5}a^4x^5+a^3x^4+\frac{1}{3}a^2x^3-a\,x^2-x\right)$
norman	$a^3c^3x^4-c^3x+\frac{1}{3}a^2c^3x^3+\frac{1}{5}a^4c^3x^5-\frac{1}{3}a^5c^3x^6-\frac{1}{7}a^6c^3x^7-c^3a\,x^2$
risch	$a^3c^3x^4-c^3x+\frac{1}{3}a^2c^3x^3+\frac{1}{5}a^4c^3x^5-\frac{1}{3}a^5c^3x^6-\frac{1}{7}a^6c^3x^7-c^3a\,x^2$
meijerg	$\frac{c^3\left(-\frac{ax\left(120a^6x^6+140a^5x^5+168a^4x^4+210a^3x^3+280a^2x^2+420ax+840\right)}{840}-\ln(-ax+1)\right)}{a}-\frac{3c^3\left(-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\mathrm{l}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/7*a^6*x^7-1/3*a^5*x^6+1/5*a^4*x^5+a^3*x^4+1/3*a^2*x^3-a*x^2-x)
```

Maxima [A]
time = 0.26, size = 70, normalized size = 1.35

$$-\frac{1}{7}\,a^6c^3x^7-\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5+a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3-ac^3x^2-c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Fricas [A]

time = 0.35, size = 70, normalized size = 1.35

$$-\frac{1}{7}\,a^{6}c^{3}x^{7}-\frac{1}{3}\,a^{5}c^{3}x^{6}+\frac{1}{5}\,a^{4}c^{3}x^{5}+a^{3}c^{3}x^{4}+\frac{1}{3}\,a^{2}c^{3}x^{3}-ac^{3}x^{2}-c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Sympy [A]

time = 0.03, size = 70, normalized size = 1.35

$$-\frac{a^{6}c^{3}x^{7}}{7}-\frac{a^{5}c^{3}x^{6}}{3}+\frac{a^{4}c^{3}x^{5}}{5}+a^{3}c^{3}x^{4}+\frac{a^{2}c^{3}x^{3}}{3}-ac^{3}x^{2}-c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - a**5*c**3*x**6/3 + a**4*c**3*x**5/5 + a**3*c**3*x**4 +
a**2*c**3*x**3/3 - a*c**3*x**2 - c**3*x
```

Giac [A]

time = 0.41, size = 70, normalized size = 1.35

$$-\frac{1}{7}\,a^{6}c^{3}x^{7}-\frac{1}{3}\,a^{5}c^{3}x^{6}+\frac{1}{5}\,a^{4}c^{3}x^{5}+a^{3}c^{3}x^{4}+\frac{1}{3}\,a^{2}c^{3}x^{3}-ac^{3}x^{2}-c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 - 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 + a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 - a*c^3*x^2 - c^3*x
```

Mupad [B]

time = 0.04, size = 70, normalized size = 1.35

$$-\frac{a^6\,c^3\,x^7}{7}-\frac{a^5\,c^3\,x^6}{3}+\frac{a^4\,c^3\,x^5}{5}+a^3\,c^3\,x^4+\frac{a^2\,c^3\,x^3}{3}-a\,c^3\,x^2-c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (a^2*c^3*x^3)/3 - a*c^3*x^2 - c^3*x + a^3*c^3*x^4 + (a^4*c^3*x^5)/5 - (a^5*
c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

3.567 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal. Leaf size=35

$$-\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}$$

```
[Out] -1/2*c^2*(a*x+1)^4/a+1/5*c^2*(a*x+1)^5/a
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 45}

$$\frac{c^2(ax+1)^5}{5a}-\frac{c^2(ax+1)^4}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/2*(c^2*(1 + a*x)^4)/a + (c^2*(1 + a*x)^5)/(5*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx\\
&= -\left(c^2\int (1-ax)(1+ax)^3\,dx\right)\\
&= -\left(c^2\int \left(2(1+ax)^3-(1+ax)^4\right)\,dx\right)\\
&= -\frac{c^2(1+ax)^4}{2a}+\frac{c^2(1+ax)^5}{5a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.66

$$\frac{c^2(1+ax)^4(-3+2ax)}{10a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
[Out] (c^2*(1 + a*x)^4*(-3 + 2*a*x))/(10*a)
```

Maple [A]
time = 0.18, size = 31, normalized size = 0.89

method	result
gosper	$\frac{c^2x(2a^4x^4+5a^3x^3-10ax-10)}{10}$
default	$c^2\left(\frac{1}{5}a^4x^5+\frac{1}{2}a^3x^4-a\,x^2-x\right)$
norman	$-c^2x-a\,c^2x^2+\frac{1}{2}a^3c^2x^4+\frac{1}{5}a^4c^2x^5$
risch	$-c^2x-a\,c^2x^2+\frac{1}{2}a^3c^2x^4+\frac{1}{5}a^4c^2x^5$
meijerg	$-\frac{c^2\left(-\frac{ax\left(12a^4x^4+15a^3x^3+20a^2x^2+30ax+60\right)}{60}-\ln(-ax+1)\right)}{a}+\frac{2c^2\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a}-\frac{c^2(-ax-\ln(-ax+1)}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] c^2*(1/5*a^4*x^5+1/2*a^3*x^4-a*x^2-x)
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.09

$$\frac{1}{5}\,a^4c^2x^5+\frac{1}{2}\,a^3c^2x^4-ac^2x^2-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Fricas [A]
time = 0.31, size = 38, normalized size = 1.09

$$\frac{1}{5}a^4c^2x^5 + \frac{1}{2}a^3c^2x^4 - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Sympy [A]
time = 0.02, size = 36, normalized size = 1.03

$$\frac{a^4c^2x^5}{5} + \frac{a^3c^2x^4}{2} - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**2,x)
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4/2 - a*c**2*x**2 - c**2*x
```

Giac [A]
time = 0.40, size = 38, normalized size = 1.09

$$\frac{1}{5}a^4c^2x^5 + \frac{1}{2}a^3c^2x^4 - ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
[Out] 1/5*a^4*c^2*x^5 + 1/2*a^3*c^2*x^4 - a*c^2*x^2 - c^2*x
```

Mupad [B]
time = 0.05, size = 38, normalized size = 1.09

$$\frac{a^4\,c^2\,x^5}{5} + \frac{a^3\,c^2\,x^4}{2} - a\,c^2\,x^2 - c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1))/(a*x - 1),x)
[Out] (a^3*c^2*x^4)/2 - a*c^2*x^2 - c^2*x + (a^4*c^2*x^5)/5
```

3.568 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=15

$$-\frac{c(1+ax)^3}{3a}$$

[Out] -1/3*c*(a*x+1)^3/a

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6302, 6275, 32}

$$-\frac{c(ax+1)^3}{3a}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]

[Out] -1/3*(c*(1 + a*x)^3)/a

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx\\ &= -\left(c\int (1+ax)^2\,dx\right)\\ &= -\frac{c(1+ax)^3}{3a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.53

$$-cx - acx^2 - \frac{1}{3}a^2cx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2),x]
[Out] -(c*x) - a*c*x^2 - (a^2*c*x^3)/3
```

Maple [A]
time = 0.10, size = 14, normalized size = 0.93

method	result	size
default	$-\frac{c(ax+1)^3}{3a}$	14
gosper	$-\frac{cx(a^2x^2+3ax+3)}{3}$	18
norman	$-cx - ac\,x^2 - \frac{1}{3}a^2c\,x^3$	22
risch	$-\frac{a^2c\,x^3}{3} - ac\,x^2 - cx - \frac{c}{3a}$	28
meijerg	$\frac{c\left(-\frac{ax\left(4a^2x^2+6ax+12\right)}{12}-\ln(-ax+1)\right)}{a} - \frac{c(-ax-\ln(-ax+1))}{a} - \frac{c\left(\frac{ax(3ax+6)}{6}+\ln(-ax+1)\right)}{a} + \frac{c\ln(-ax+1)}{a}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/3*c*(a*x+1)^3/a
```

Maxima [A]
time = 0.26, size = 21, normalized size = 1.40

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Fricas [A]
time = 0.35, size = 21, normalized size = 1.40

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Sympy [A]
time = 0.01, size = 20, normalized size = 1.33

$$-\frac{a^2cx^3}{3} - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c),x)
[Out] -a**2*c*x**3/3 - a*c*x**2 - c*x
```

Giac [A]
time = 0.41, size = 21, normalized size = 1.40

$$-\frac{1}{3}a^2cx^3 - acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -1/3*a^2*c*x^3 - a*c*x^2 - c*x
```

Mupad [B]
time = 0.03, size = 17, normalized size = 1.13

$$-\frac{c\,x\,\left(a^2\,x^2+3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1))/(a*x - 1),x)
[Out] -(c*x*(3*a*x + a^2*x^2 + 3))/3
```

3.569 $\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{ac(1-ax)}$$

```
[Out] -1/a/c/(-a*x+1)
```

Rubi [A]
time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 32}

$$-\frac{1}{ac(1-ax)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] -(1/(a*c*(1 - a*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^2}\,dx}{c}\\
&= -\frac{1}{ac(1-ax)}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.
time = 0.01, size = 18, normalized size = 1.12

$$\frac{e^{2\coth^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(2*ArcCoth[a*x])/(2*a*c)
```

Maple [A]
time = 0.14, size = 15, normalized size = 0.94

method	result	size
norman	$\frac{x}{c(ax-1)}$	13
gosper	$\frac{1}{ac(ax-1)}$	15
default	$\frac{1}{ac(ax-1)}$	15
risch	$\frac{1}{ac(ax-1)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/c/(a*x-1)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.81

$$\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/(a^2*c*x - a*c)
```

Fricas [A]
time = 0.33, size = 13, normalized size = 0.81

$$\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] 1/(a^2*c*x - a*c)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.62

$$\frac{1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c),x)
[Out] 1/(a**2*c*x - a*c)
```

Giac [A]
time = 0.40, size = 14, normalized size = 0.88

$$\frac{1}{(ax - 1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
[Out] 1/((a*x - 1)*a*c)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.88

$$-\frac{1}{a\,\left(c - a\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)*(a*x - 1)),x)
[Out] -1/(a*(c - a*c*x))
```

3.570 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] -1/4/a/c^2/(-a*x+1)^2-1/4/a/c^2/(-a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6275, 46, 213}

$$-\frac{1}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/4*1/(a*c^2*(1 - a*x)^2) - 1/(4*a*c^2*(1 - a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^2}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^3(1+ax)}\,dx}{c^2}\\
&= -\frac{\int \left(-\frac{1}{2(-1+ax)^3}+\frac{1}{4(-1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^2}\\
&= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}\\
&= -\frac{1}{4ac^2(1-ax)^2}-\frac{1}{4ac^2(1-ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.69

$$\frac{-2+ax-(-1+ax)^2\tanh^{-1}(ax)}{4ac^2(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
[Out] (-2 + a*x - (-1 + a*x)^2*ArcTanh[a*x])/(4*a*c^2*(-1 + a*x)^2)
```

Maple [A]
time = 0.14, size = 52, normalized size = 1.02

method	result	size
risch	$\frac{\frac{x}{4}-\frac{1}{2a}}{(ax-1)^2c^2}+\frac{\ln(-ax+1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	51
default	$\frac{-\frac{\ln(ax+1)}{8a}-\frac{1}{4a(ax-1)^2}+\frac{1}{4a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^2}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^2}{2c}-\frac{a^2x^3}{4c}}{(ax+1)c(ax-1)^2}+\frac{\ln(ax-1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/c^2*(-1/8/a*ln(a*x+1)-1/4/a/(a*x-1)^2+1/4/a/(a*x-1)+1/8/a*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 63, normalized size = 1.24

$$\frac{ax-2}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A]
time = 0.35, size = 76, normalized size = 1.49

$$\frac{2\,ax-(a^2x^2-2\,ax+1)\log\left(ax+1\right)+(a^2x^2-2\,ax+1)\log\left(ax-1\right)-4}{8\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 4)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [A]
time = 0.15, size = 54, normalized size = 1.06

$$\frac{ax-2}{4a^3c^2x^2-8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (a*x - 2)/(4*a**3*c**2*x**2 - 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

Giac [A]
time = 0.43, size = 51, normalized size = 1.00

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax-2}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x -
 2)/((a*x - 1)^2*a*c^2)
```

Mupad [B]
time = 1.26, size = 46, normalized size = 0.90

$$\frac{\frac{x}{4}-\frac{1}{2\,a}}{a^2\,c^2\,x^2-2\,a\,c^2\,x+c^2}-\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^2*(a*x - 1)),x)
[Out] (x/4 - 1/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

3.571 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] -1/12/a/c^3/(-a*x+1)^3-1/8/a/c^3/(-a*x+1)^2-3/16/a/c^3/(-a*x+1)+1/16/a/c^3/
(a*x+1)-1/4*arctanh(a*x)/a/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6275, 46, 213}

$$-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(ax+1)}-\frac{1}{8ac^3(1-ax)^2}-\frac{1}{12ac^3(1-ax)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/12*1/(a*c^3*(1 - a*x)^3) - 1/(8*a*c^3*(1 - a*x)^2) - 3/(16*a*c^3*(1 - a*
x)) + 1/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx \\
&= -\frac{\int \frac{1}{(1-ax)^4(1+ax)^2}\,dx}{c^3} \\
&= -\frac{\int \left(\frac{1}{4(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^3} \\
&= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3} \\
&= -\frac{1}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^2}-\frac{3}{16ac^3(1-ax)}+\frac{1}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.73

$$\frac{4+ax-6a^2x^2+3a^3x^3-3(-1+ax)^3(1+ax)\tanh^{-1}(ax)}{12ac^3(-1+ax)^3(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
[Out] (4 + a*x - 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)^3*(1 + a*x)*ArcTanh[a*x])/(
12*a*c^3*(-1 + a*x)^3*(1 + a*x))
```

Maple [A]
time = 0.15, size = 76, normalized size = 0.88

method	result	size
default	$\frac{\frac{1}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{12a(ax-1)^3}-\frac{1}{8a(ax-1)^2}+\frac{3}{16a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^3}$	76
risch	$\frac{\frac{a^2x^3}{4}-\frac{a\,x^2}{2}+\frac{x}{12}+\frac{1}{3a}}{(ax-1)^2(a^2x^2-1)c^3}-\frac{\ln(ax+1)}{8a\,c^3}+\frac{\ln(-ax+1)}{8a\,c^3}$	76
norman	$\frac{\frac{3x}{4c}+\frac{a\,x^2}{4c}-\frac{11a^2x^3}{12c}-\frac{a^3x^4}{12c}+\frac{a^4x^5}{3c}}{(ax-1)^3(ax+1)^2c^2}+\frac{\ln(ax-1)}{8c^3a}-\frac{\ln(ax+1)}{8a\,c^3}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/16/a/(a*x+1)-1/8/a*ln(a*x+1)+1/12/a/(a*x-1)^3-1/8/a/(a*x-1)^2+3/16
/a/(a*x-1)+1/8/a*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.06

$$\frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}-\frac{\log(ax+1)}{8\,ac^3}+\frac{\log(ax-1)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 - 6*a^2*x^2 + a*x + 4)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Fricas [A]
time = 0.34, size = 121, normalized size = 1.41

$$\frac{6\,a^3x^3-12\,a^2x^2+2\,ax-3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax+1)+3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax-1)+8}{24\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 - 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x - 1) + 8)/(a^5*c
^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A]
time = 0.22, size = 85, normalized size = 0.99

$$-\frac{-3a^3x^3+6a^2x^2-ax-4}{12a^5c^3x^4-24a^4c^3x^3+24a^2c^3x-12ac^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(-3*a**3*x**3 + 6*a**2*x**2 - a*x - 4)/(12*a**5*c**3*x**4 - 24*a**4*c**3*x
**3 + 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c
**3)
```

Giac [A]
time = 0.42, size = 74, normalized size = 0.86

$$-\frac{\log(|ax+1|)}{8\,ac^3}+\frac{\log(|ax-1|)}{8\,ac^3}+\frac{3\,a^3x^3-6\,a^2x^2+ax+4}{12\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 - 6*a^2*x^2 + a*x + 4)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

Mupad [B]
time = 0.09, size = 73, normalized size = 0.85

$$-\frac{\frac{x}{12}-\frac{a\,x^2}{2}+\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4+2\,a^3\,c^3\,x^3-2\,a\,c^3\,x+c^3}-\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^3*(a*x - 1)),x)
```

```
[Out] - (x/12 - (a*x^2)/2 + 1/(3*a) + (a^2*x^3)/4)/(c^3 + 2*a^3*c^3*x^3 - a^4*c^3
*x^4 - 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

3.572 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=121

$$-\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}+\frac{5}{64ac^4(1+ax)}-\frac{15}{}$$

```
[Out] -1/32/a/c^4/(-a*x+1)^4-1/16/a/c^4/(-a*x+1)^3-3/32/a/c^4/(-a*x+1)^2-5/32/a/c
^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2+5/64/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^
4
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6275, 46, 213}

$$-\frac{5}{32ac^4(1-ax)}+\frac{5}{64ac^4(ax+1)}-\frac{3}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}-\frac{1}{16ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}-\frac{15\tanh^{-1}(ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/32*1/(a*c^4*(1 - a*x)^4) - 1/(16*a*c^4*(1 - a*x)^3) - 3/(32*a*c^4*(1 - a
*x)^2) - 5/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) + 5/(64*a*c^4*(1
 + a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$

$$= -\frac{\int \frac{1}{(1-ax)^5(1+ax)^3}\,dx}{c^4}$$

$$= -\frac{\int \left(-\frac{1}{8(-1+ax)^5}+\frac{3}{16(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{5}{32(-1+ax)^2}+\frac{1}{32(1+ax)^3}+\frac{5}{64(1+ax)^2}-\frac{15}{64(-1+a\ldots}\right.}{c^4}$$

$$= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+a\ldots}$$

$$= -\frac{1}{32ac^4(1-ax)^4}-\frac{1}{16ac^4(1-ax)^3}-\frac{3}{32ac^4(1-ax)^2}-\frac{5}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+a\ldots}$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.68

$$-\frac{16+17ax-50a^2x^2+10a^3x^3+30a^4x^4-15a^5x^5+15(-1+ax)^4(1+ax)^2\tanh^{-1}(ax)}{64ac^4(-1+ax)^4(1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/64*(16 + 17*a*x - 50*a^2*x^2 + 10*a^3*x^3 + 30*a^4*x^4 - 15*a^5*x^5 + 15
*(-1 + a*x)^4*(1 + a*x)^2*ArcTanh[a*x])/(a*c^4*(-1 + a*x)^4*(1 + a*x)^2)
```

Maple [A]
time = 0.16, size = 100, normalized size = 0.83

method	result	size
risch	$\frac{\frac{15a^4x^5}{64}-\frac{15a^3x^4}{32}-\frac{5a^2x^3}{32}+\frac{25a\,x^2}{32}-\frac{17x}{64}-\frac{1}{4a}}{(ax-1)^2(a^2x^2-1)^2c^4}-\frac{15\ln(ax+1)}{128a\,c^4}+\frac{15\ln(-ax+1)}{128a\,c^4}$	92
default	$\frac{\frac{1}{64a(ax+1)^2}+\frac{5}{64a(ax+1)}-\frac{15\ln(ax+1)}{128a}-\frac{1}{32a(ax-1)^4}+\frac{1}{16a(ax-1)^3}-\frac{3}{32a(ax-1)^2}+\frac{5}{32a(ax-1)}+\frac{15\ln(ax-1)}{128a}}{c^4}$	100
norman	$\frac{-\frac{49x}{64c}-\frac{15a\,x^2}{64c}+\frac{11a^2x^3}{8c}+\frac{a^3x^4}{8c}-\frac{63a^4x^5}{64c}-\frac{a^5x^6}{64c}+\frac{a^6x^7}{4c}}{(ax-1)^4(ax+1)^3c^3}+\frac{15\ln(ax-1)}{128c^4a}-\frac{15\ln(ax+1)}{128a\,c^4}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/64/a/(a*x+1)^2+5/64/a/(a*x+1)-15/128/a*ln(a*x+1)-1/32/a/(a*x-1)^4+
1/16/a/(a*x-1)^3-3/32/a/(a*x-1)^2+5/32/a/(a*x-1)+15/128/a*ln(a*x-1))
```

Maxima [A]
time = 0.28, size = 140, normalized size = 1.16

$$\frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}-\frac{15\,\log{(ax+1)}}{128\,ac^4}+\frac{15\,\log{(ax-1)}}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/(a^7
*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(103) = 206$.
time = 0.34, size = 217, normalized size = 1.79

$$\frac{30\,a^5x^5-60\,a^4x^4-20\,a^3x^3+100\,a^2x^2-34\,ax-15\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\log{(ax+1)}+15\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\log{(ax-1)}-32}{128\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 - 60*a^4*x^4 - 20*a^3*x^3 + 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1)
 + 15*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log
(a*x - 1) - 32)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3
- a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A]
time = 0.33, size = 141, normalized size = 1.17

$$\frac{15a^5x^5-30a^4x^4-10a^3x^3+50a^2x^2-17ax-16}{64a^7c^4x^6-128a^6c^4x^5-64a^5c^4x^4+256a^4c^4x^3-64a^3c^4x^2-128a^2c^4x+64ac^4}+\frac{\frac{15\log{(x-\frac{1}{a})}}{128}-\frac{15\log{(x+\frac{1}{a})}}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 - 30*a**4*x**4 - 10*a**3*x**3 + 50*a**2*x**2 - 17*a*x - 16)/(
64*a**7*c**4*x**6 - 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 + 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 - 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Giac [A]
time = 0.41, size = 91, normalized size = 0.75

$$-\frac{15 \log(|ax+1|)}{128\,ac^4}+\frac{15 \log(|ax-1|)}{128\,ac^4}+\frac{15\,a^5x^5-30\,a^4x^4-10\,a^3x^3+50\,a^2x^2-17\,ax-16}{64\,(ax+1)^2(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 - 30*a^4*x^4 - 10*a^3*x^3 + 50*a^2*x^2 - 17*a*x - 16)/((a*x +
1)^2*(a*x - 1)^4*a*c^4)
```

Mupad [B]
time = 1.30, size = 121, normalized size = 1.00

$$\frac{\frac{17\,x}{64}-\frac{25\,a\,x^2}{32}+\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}+\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5+a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3+a^2\,c^4\,x^2+2\,a\,c^4\,x-c^4}-\frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^4*(a*x - 1)),x)
```

```
[Out] ((17*x)/64 - (25*a*x^2)/32 + 1/(4*a) + (5*a^2*x^3)/32 + (15*a^3*x^4)/32 - (
15*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^4*x^4 + 2*a^5*c^
4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

3.573 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal. Leaf size=393

$$-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{44}$$

```
[Out] -5/72*a^7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(13/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(5
/2)*(1+1/a/x)^(13/2)*x^9-55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
)/a-55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-11/192*a^2*c^4*(1+1/a/
x)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/448*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(
1/2)-11/1008*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-5/1008*a^5*c^4*(1+
1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)+5/168*a^6*c^4*(1+1/a/x)^(13/2)*x^7*(1-1/a
/x)^(1/2)-55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{13/2}+\frac{5}{168}a^6c^4x^7\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{13/2}-\frac{5a^5c^4x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{1008}-\frac{11a^4c^4x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{1008}-\frac{11}{448}a^3c^4x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{192}a^2c^4x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{55}{384}ac^4x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{55}{128}c^4x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{55c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (-55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (55*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/
(a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x
^4)/448 - (11*a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/1008 - (5*
a^5*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/1008 + (5*a^6*c^4*Sqrt[
1 - 1/(a*x)]*(1 + 1/(a*x))^(13/2)*x^7)/168 - (5*a^7*c^4*(1 - 1/(a*x))^(3/2)
*(1 + 1/(a*x))^(13/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^
(13/2)*x^9)/9 - (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*
a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13/2}x^9+\frac{1}{9}\left(5a^7c^4\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\right.}{x^9}\right.$$

$$=-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{13}$$

$$=\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-\frac{5}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{13/2}x$$

$$=-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}+\frac{5}{168}a^6c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{13/2}x^7-$$

$$=-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-\frac{5a^5c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6}{1008}+\frac{5}{168}$$

$$=-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{11a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5}{1008}-$$

$$=-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{448}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$=-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{192}a$$

$$=-\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{11}{}$$

Mathematica [A]
time = 0.09, size = 111, normalized size = 0.28

$$\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-3712+4599ax+10240a^2x^2+3066a^3x^3-8448a^4x^4-7224a^5x^5+1024a^6x^6+3024a^7x^7+896a^8x^8)-3465\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 + 4599*a*x + 10240*a^2*x^2 + 3066*a^
3*x^3 - 8448*a^4*x^4 - 7224*a^5*x^5 + 1024*a^6*x^6 + 3024*a^7*x^7 + 896*a^8
*x^8) - 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

Maple [A]
time = 0.11, size = 288, normalized size = 0.73

method	result
risch	$\frac{(896a^8x^8+3024a^7x^7+1024a^6x^6-7224a^5x^5-8448a^4x^4+3066a^3x^3+10240a^2x^2+4599ax-3712)(ax-1)c^4}{8064a\sqrt{\frac{ax-1}{ax+1}}}-\frac{55\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x\dots}\right.}{128\sqrt{\dots}}$
default	$\frac{(ax-1)^2c^4\left(896(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+3024(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+1920\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4-4200(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-\dots\right.}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8064*(a*x-1)^2*c^4/a*(896*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^6*x^6+3024*(a^2
*x^2-1)^(3/2)*(a^2)^(1/2)*a^5*x^5+1920*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^
4-4200*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-6528*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*a^2*x^2-1134*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+8064*((a*x+1)*(a*x-1))^
(3/2)*(a^2)^(1/2)-4352*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+3465*(a^2)^(1/2)*(a^2*
x^2-1)^(1/2)*a*x-3465*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*a)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 415, normalized size = 1.06

$$-\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+115038\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-255222\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+360448\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-334602\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-115038\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3465\,c^4\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{9(ax-1)a^2}{ax+1}-\frac{36(ax-1)^2a^2}{(ax+1)^2}+\frac{84(ax-1)^3a^2}{(ax+1)^3}-\frac{126(ax-1)^4a^2}{(ax+1)^4}+\frac{126(ax-1)^5a^2}{(ax+1)^5}-\frac{84(ax-1)^6a^2}{(ax+1)^6}+\frac{36(ax-1)^7a^2}{(ax+1)^7}-\frac{9(ax-1)^8a^2}{(ax+1)^8}+\frac{(ax-1)^9a^2}{(ax+1)^9}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqr
t((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2)
- 30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))
^(13/2) - 255222*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 360448*c^4*((a*x - 1)/(
a*x + 1))^(9/2) - 334602*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
 - 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*s
qrt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(
a*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)
^4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*
(a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a
^2/(a*x + 1)^9 - a^2))*a
```

Fricas [A]
time = 0.34, size = 170, normalized size = 0.43

$$-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(896\,a^9c^4x^9+3920\,a^8c^4x^8+4048\,a^7c^4x^7-6200\,a^6c^4x^6-15672\,a^5c^4x^5-5382\,a^4c^4x^4+13306\,a^3c^4x^3+14839\,a^2c^4x^2+887\,ac^4x-3712\,c^4\right)\sqrt{\frac{ax-1}{ax+1}}}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) - 1) - (896*a^9*c^4*x^9 + 3920*a^8*c^4*x^8 + 4048*a^7*c^
4*x^7 - 6200*a^6*c^4*x^6 - 15672*a^5*c^4*x^5 - 5382*a^4*c^4*x^4 + 13306*a^3
*c^4*x^3 + 14839*a^2*c^4*x^2 + 887*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-\frac{4a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^6x^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^8x^8}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4*x**4
/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6*x**6/(a*x*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)
+ Integral(a**8*x**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x))
```

Giac [A]
time = 0.41, size = 216, normalized size = 0.55

$$\frac{55\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{128\,|a|\mathrm{sgn}\left(ax+1\right)}+\frac{1}{8064}\,\sqrt{a^2x^2-1}\left(\left(\frac{4599\,c^4}{\mathrm{sgn}\left(ax+1\right)}+2\left(\frac{5120\,ac^4}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{1533\,a^2c^4}{\mathrm{sgn}\left(ax+1\right)}-4\left(\frac{1056\,a^3c^4}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{903\,a^4c^4}{\mathrm{sgn}\left(ax+1\right)}-2\left(\frac{64\,a^5c^4}{\mathrm{sgn}\left(ax+1\right)}+7\left(\frac{8\,a^7c^4x}{\mathrm{sgn}\left(ax+1\right)}+\frac{27\,a^6c^4}{\mathrm{sgn}\left(ax+1\right)}\right)x\right)x\right)x\right)x\right)x\right)x-\frac{3712\,c^4}{a\mathrm{sgn}\left(ax+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) +
1/8064*sqrt(a^2*x^2 - 1)*((4599*c^4/sgn(a*x + 1) + 2*(5120*a*c^4/sgn(a*x +
1) + (1533*a^2*c^4/sgn(a*x + 1) - 4*(1056*a^3*c^4/sgn(a*x + 1) + (903*a^4*c
^4/sgn(a*x + 1) - 2*(64*a^5*c^4/sgn(a*x + 1) + 7*(8*a^7*c^4*x/sgn(a*x + 1)
+ 27*a^6*c^4/sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x - 3712*c^4/(a*sgn(a*x + 1)))
```

Mupad [B]
time = 0.21, size = 362, normalized size = 0.92

$$\frac{\frac{55\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{715\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}+\frac{913\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}+\frac{18589\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{224}-\frac{5632\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{63}+\frac{14179\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{224}-\frac{913\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{32}+\frac{715\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/2}}{96}-\frac{55\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/2}}{64}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}+\frac{9\,a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}-\frac{a\,{\left(a\,x-1\right)}^9}{{\left(a\,x+1\right)}^9}}-\frac{55\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((55*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (715*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/96 + (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (18589*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (14179
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/224 - (913*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/32 + (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (55*c^4*((a*x - 1)/(a
*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^
4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*
a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9
)/(a*x + 1)^9) - (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.574 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal. Leaf size=313

$$-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3$$

```
[Out] -1/7*a^6*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)*x^7-9/16*c^3*arctanh((1-1/a/x
)^(1/2)*(1+1/a/x)^(1/2))/a-3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-3
/40*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-9/280*a^3*c^3*(1+1/a/x)^(7/
2)*x^4*(1-1/a/x)^(1/2)-1/70*a^4*c^3*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)+1/1
4*a^5*c^3*(1+1/a/x)^(11/2)*x^6*(1-1/a/x)^(1/2)-9/16*c^3*x*(1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{1}{14}a^5c^3x^6\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}-\frac{1}{70}a^4c^3x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{9}{280}a^3c^3x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{3}{40}a^2c^3x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{16}ac^3x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{9}{16}c^3x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{9c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (3*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x)
)^(5/2)*x^3)/40 - (9*a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/280
 - (a^4*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/70 + (a^5*c^3*Sqrt[1
 - 1/(a*x)]*(1 + 1/(a*x))^(11/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(3/2)*(1
+ 1/(a*x))^(11/2)*x^7)/7 - (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x
)]])/(16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\left(\left(a^6c^3\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$=\left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7-\frac{1}{7}\left(3a^5c^3\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\,(1-}{x^7}\right.$$

$$=\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}x^7$$

$$=-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{14}a^5c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}x^6-$$

$$=-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-\frac{1}{70}a^4c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-$$

$$=-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{9}{280}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4-$$

$$=-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3$$

$$=-\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{40}a^2c^3$$

Mathematica [A]
time = 0.07, size = 95, normalized size = 0.30

$$-\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(368-245ax-656a^2x^2-350a^3x^3+208a^4x^4+280a^5x^5+80a^6x^6)+315\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 - 245*a*x - 656*a^2*x^2 - 350*a
^3*x^3 + 208*a^4*x^4 + 280*a^5*x^5 + 80*a^6*x^6) + 315*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x]))/a
```

Maple [A]
time = 0.10, size = 240, normalized size = 0.77

method	result
risch	$-\frac{(80a^6x^6+280a^5x^5+208a^4x^4-350a^3x^3-656a^2x^2-245ax+368)(ax-1)c^3}{560a\sqrt{\frac{ax-1}{ax+1}}}-\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{(ax+1)(a}}{16\sqrt{a^2}\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{(ax-1)^2c^3\left(80\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4+280(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3+288(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-70\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+192(a\right.}{560a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/560*(a*x-1)^2*c^3/a*(80*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4+280*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+288*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-7
0*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+192*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-315*(
a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-560*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+31
5*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1)
)^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 337, normalized size = 1.08

$$-\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+5943\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-9216\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+8393\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-315\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+\frac{35(ax-1)^3a^2}{(ax+1)^3}-\frac{35(ax-1)^4a^2}{(ax+1)^4}+\frac{21(ax-1)^5a^2}{(ax+1)^5}-\frac{7(ax-1)^6a^2}{(ax+1)^6}+\frac{(ax-1)^7a^2}{(ax+1)^7}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 21
00*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 5943*c^3*((a*x - 1)/(a*x + 1))^(9/2)
- 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 8393*c^3*((a*x - 1)/(a*x + 1))^(5/
2) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
 1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2
/(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7
- a^2))*a
```

Fricas [A]
time = 0.36, size = 147, normalized size = 0.47

$$-\frac{315\,c^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(80\,a^{7}c^{3}x^{7}+360\,a^{6}c^{3}x^{6}+488\,a^{5}c^{3}x^{5}-142\,a^{4}c^{3}x^{4}-1006\,a^{3}c^{3}x^{3}-901\,a^{2}c^{3}x^{2}+123\,ac^{3}x+368\,c^{3}\right)\sqrt{\frac{ax-1}{ax+1}}}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (80*a^7*c^3*x^7 + 360*a^6*c^3*x^6 + 488*a^5*c^3*x^5
- 142*a^4*c^3*x^4 - 1006*a^3*c^3*x^3 - 901*a^2*c^3*x^2 + 123*a*c^3*x + 368*
c^3)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^{3}\left(\int\frac{3a^{2}x^{2}}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^{4}x^{4}}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^{6}x^{6}}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4*x**
4/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1)), x) + Integral(a**6*x**6/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
Integral(-1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*
x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [A]
time = 0.42, size = 177, normalized size = 0.57

$$\frac{9\,c^{3}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)}{16\,|a|\mathrm{sgn}\left(ax+1\right)}+\frac{1}{560}\,\sqrt{a^{2}x^{2}-1}\left(\left(2\left(\frac{328\,ac^{3}}{\mathrm{sgn}\left(ax+1\right)}+\left(\frac{175\,a^{2}c^{3}}{\mathrm{sgn}\left(ax+1\right)}-4\left(\frac{26\,a^{3}c^{3}}{\mathrm{sgn}\left(ax+1\right)}+5\left(\frac{2\,a^{5}c^{3}x}{\mathrm{sgn}\left(ax+1\right)}+\frac{7\,a^{4}c^{3}}{\mathrm{sgn}\left(ax+1\right)}\right)x\right)x\right)x\right)x+\frac{245\,c^{3}}{\mathrm{sgn}\left(ax+1\right)}\right)x-\frac{368\,c^{3}}{a\mathrm{sgn}\left(ax+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + 1/
560*sqrt(a^2*x^2 - 1)*((2*(328*a*c^3/sgn(a*x + 1) + (175*a^2*c^3/sgn(a*x +
1) - 4*(26*a^3*c^3/sgn(a*x + 1) + 5*(2*a^5*c^3*x/sgn(a*x + 1) + 7*a^4*c^3/s
gn(a*x + 1))*x)*x)*x)*x + 245*c^3/sgn(a*x + 1))*x - 368*c^3/(a*sgn(a*x + 1)
))
```

Mupad [B]
time = 0.12, size = 289, normalized size = 0.92

$$-\frac{\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}-\frac{9\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{1199\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}-\frac{1152\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{35}+\frac{849\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{40}-\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{2}+\frac{9\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}-\frac{9\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - ((15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(1
/2))/8 + (1199*c^3*((a*x - 1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)/(
a*x + 1))^(7/2))/35 + (849*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 - (15*c^3*((
a*x - 1)/(a*x + 1))^(11/2))/2 + (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a
- (7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x -
 1)^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a
*x + 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7) -
(9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

3.575 $\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2$$

```
[Out] -7/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/24*a*c^2*(1+1/a/x)^(3
/2)*x^2*(1-1/a/x)^(1/2)-7/60*a^2*c^2*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-1/
20*a^3*c^2*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)+1/5*a^4*c^2*(1+1/a/x)^(9/2)*
x^5*(1-1/a/x)^(1/2)-7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{5}a^4c^2x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{20}a^3c^2x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{7}{60}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{7}{24}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{8}c^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{7c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (-7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/24 - (7*a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/60 - (a^3*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/20 + (
a^4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/5 - (7*c^2*ArcTanh[Sqrt[
1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\frac{1}{5}\left(a^3c^2\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{7/2}}{x^5\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right.$$

$$=-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{5}a^4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5+\ldots$$

$$=-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{20}a^3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\ldots$$

$$=-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\ldots$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\ldots$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\ldots$$

$$=-\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{24}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{60}a^2c^2\ldots$$

Mathematica [A]

time = 0.06, size = 79, normalized size = 0.34

$$\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-136+15ax+112a^2x^2+90a^3x^3+24a^4x^4)-105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 + 15*a*x + 112*a^2*x^2 + 90*a^3*x^3 +
 24*a^4*x^4) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A]
time = 0.10, size = 192, normalized size = 0.82

method	result
risch	$\frac{(24a^4x^4+90a^3x^3+112a^2x^2+15ax-136)(ax-1)c^2}{120a\sqrt{\frac{ax-1}{ax+1}}}-\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{(ax+1)(ax-1)}}{8\sqrt{a^2}\,\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)^2c^2\Bigg(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+90\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+105\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+120((ax+1)}{120a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(a*x-1)^2*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+90*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*a*x+16*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)+105*(a^2)^(1/2)*
(a^2*x^2-1)^(1/2)*a*x+120*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-105*ln((a^2*x
+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x-1)/(a*x+1))^(3/2)/(a*
x+1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 259, normalized size = 1.11

$$-\frac{1}{120}\,a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-490\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+896\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-790\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{5(ax-1)a^2}{ax+1}-\frac{10(ax-1)^2a^2}{(ax+1)^2}+\frac{10(ax-1)^3a^2}{(ax+1)^3}-\frac{5(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5a^2}{(ax+1)^5}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 4
```

```
90*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) -
790*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5
*(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*
a^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)
^5 - a^2))
```

Fricas [A]
time = 0.36, size = 126, normalized size = 0.54

$$-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(24\,a^5c^2x^5+114\,a^4c^2x^4+202\,a^3c^2x^3+127\,a^2c^2x^2-121\,ac^2x-136\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) - (24*a^5*c^2*x^5 + 114*a^4*c^2*x^4 + 202*a^3*c^2*x^3
+ 127*a^2*c^2*x^2 - 121*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4x^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1
) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4*x**4/(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a
*x + 1))/(a*x + 1)), x) + Integral(1/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))
```

Giac [A]
time = 0.44, size = 138, normalized size = 0.59

$$\frac{1}{120}\sqrt{a^2x^2-1}\left(\left(2\left(3\left(\frac{4\,a^3c^2x}{\mathrm{sgn}\,(ax+1)}+\frac{15\,a^2c^2}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{56\,ac^2}{\mathrm{sgn}\,(ax+1)}\right)x+\frac{15\,c^2}{\mathrm{sgn}\,(ax+1)}\right)x-\frac{136\,c^2}{a\mathrm{sgn}\,(ax+1)}\right)+\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{8\,|a|\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/120*sqrt(a^2*x^2 - 1)*((2*(3*(4*a^3*c^2*x/sgn(a*x + 1) + 15*a^2*c^2/sgn(a
*x + 1))*x + 56*a*c^2/sgn(a*x + 1))*x + 15*c^2/sgn(a*x + 1))*x - 136*c^2/(a
```

```
*sgn(a*x + 1))) + 7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*s
gn(a*x + 1))
```

Mupad [B]
time = 1.25, size = 214, normalized size = 0.92

$$\frac{\frac{7c^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{79\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{224\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{49\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}-\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}-\frac{7\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((7*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 + (79*c^2*((a*x - 1)/(a*x + 1))^(3/2
))/6 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (49*c^2*((a*x - 1)/(a*x +
 1))^(7/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (7*c^2*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.576 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=145

$$-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{5c\tanh^{-1}\left(\cdots\right.}{\cdots}$$

```
[Out] -5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/6*a*c*(1+1/a/x)^(3/2)*x
^2*(1-1/a/x)^(1/2)-1/3*a^2*c*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-5/2*c*x*(1
-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{3}a^2cx^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{5}{6}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 - (5*a*c*Sqrt[1 - 1/(a*x)]*(
1 + 1/(a*x))^(3/2)*x^2)/6 - (a^2*c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)*x^
3)/3 - (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\left(\left(a^2c\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{3}(5ac)\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{2}(5c)$$

$$=-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-}$$

$$=-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-}$$

$$=-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{6}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\sqrt{1-}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.42

$$-\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(22+9ax+2a^2x^2)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -1/6*(c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(22 + 9*a*x + 2*a^2*x^2) + 15*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/a
```

Maple [A]
time = 0.09, size = 183, normalized size = 1.26

method	result
risch	$-\frac{(2a^2x^2+9ax+22)(ax-1)c}{6a\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c\left(9\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a+24\sqrt{a^2}\,\sqrt{(ax+}\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)^2*c*(9*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+2*((a*x+1)*(a*x-1))^(
3/2)*(a^2)^(1/2)-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a+
24*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+24*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(1/2))/(a^2)^(1/2)))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x
-1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 171, normalized size = 1.18

$$\frac{1}{6}\,a\left(\frac{2\left(15\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+33\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(15*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 33*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.35, size = 91, normalized size = 0.63

$$-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(2\,a^3cx^3+11\,a^2cx^2+31\,acx+22\,c\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*
x + 1)) - 1) + (2*a^3*c*x^3 + 11*a^2*c*x^2 + 31*a*c*x + 22*c)*sqrt((a*x - 1
)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{a^2x^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - s
qrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*
x + 1)), x))
```

Giac [A]
time = 0.41, size = 90, normalized size = 0.62

$$-\frac{1}{6}\sqrt{a^2x^2-1}\left(\left(\frac{2\,acx}{\operatorname{sgn}(ax+1)}+\frac{9\,c}{\operatorname{sgn}(ax+1)}\right)x+\frac{22\,c}{a\operatorname{sgn}(ax+1)}\right)+\frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{2\,|a|\operatorname{sgn}(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a^2*x^2 - 1)*((2*a*c*x/sgn(a*x + 1) + 9*c/sgn(a*x + 1))*x + 22*c/
(a*sgn(a*x + 1))) + 5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*s
gn(a*x + 1))
```

Mupad [B]
time = 0.07, size = 133, normalized size = 0.92

$$-\frac{11\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+5\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,(a\,x-1)}{a\,x+1}+\frac{3\,a\,{(a\,x-1)}^2}{{(a\,x+1)}^2}-\frac{a\,{(a\,x-1)}^3}{{(a\,x+1)}^3}}-\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] - (11*c*((a*x - 1)/(a*x + 1))^(1/2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3
+ 5*c*((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a
*x - 1)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (5*c*atanh(((a*x -
1)/(a*x + 1))^(1/2)))/a
```

3.577 $\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

```
[Out] 1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6318}

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(3*ArcCoth[a*x])/(3*a*c)
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.00

$$\frac{e^{3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] E^(3*ArcCoth[a*x])/(3*a*c)
```

Maple [A]
time = 0.13, size = 24, normalized size = 1.33

method	result	size
gosper	$\frac{1}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} ac}$	24
default	$\frac{1}{3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} ac}$	24
trager	$\frac{(ax+1)^{2} \sqrt{-\frac{-ax+1}{ax+1}}}{3ac(ax-1)^{2}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((a*x-1)/(a*x+1))^(3/2)/a/c
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.28

$$\frac{1}{3\,ac\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/3/(a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(23) = 46$.
time = 0.35, size = 51, normalized size = 2.83

$$\frac{(a^2x^2 + 2\,ax + 1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3cx^2 - 2\,a^2cx + ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*x^2 + 2*a*x + 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c*x^2 - 2*a^2*c*x
+ a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(1/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**2*x
**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.
time = 0.42, size = 49, normalized size = 2.72

$$\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x - 1)^
3*a*c)
```

Mupad [B]
time = 0.03, size = 23, normalized size = 1.28

$$\frac{1}{3\,a\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] 1/(3*a*c*((a*x - 1)/(a*x + 1))^(3/2))
```

3.578 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}+\frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}$$

```
[Out] -2/15/((a*x-1)/(a*x+1))^(3/2)/a/c^2+1/5/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/
a/c^2/(-a^2*x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6320, 6318}

$$\frac{(3-2ax)e^{3\coth^{-1}(ax)}}{5ac^2(1-a^2x^2)}-\frac{2e^{3\coth^{-1}(ax)}}{15ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-2*E^(3*ArcCoth[a*x]))/(15*a*c^2) + (E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(5*a*
c^2*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)} - \frac{2\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c}$$
$$= -\frac{2e^{3\coth^{-1}(ax)}}{15ac^2} + \frac{e^{3\coth^{-1}(ax)}(3-2ax)}{5ac^2(1-a^2x^2)}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.78

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(7-6ax+2a^2x^2)}{15c^2(-1+ax)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(7 - 6*a*x + 2*a^2*x^2))/(c^2*(-1 + a*x)^3)
```

Maple [A]
time = 0.19, size = 52, normalized size = 0.95

method	result	size
gosper	$-\frac{2a^2x^2-6ax+7}{15(a^2x^2-1)c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	49
default	$-\frac{2a^2x^2-6ax+7}{15\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)c^2(ax-1)a}$	52
trager	$-\frac{(ax+1)(2a^2x^2-6ax+7)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^2(ax-1)^3}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/15*(2*a^2*x^2-6*a*x+7)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/c^2/(a*x-1)/a
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.00

$$\frac{\frac{10\,(ax-1)}{ax+1} - \frac{15\,(ax-1)^2}{(ax+1)^2} - 3}{60\,ac^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(10*(a*x - 1)/(a*x + 1) - 15*(a*x - 1)^2/(a*x + 1)^2 - 3)/(a*c^2*((a*x
 - 1)/(a*x + 1))^(5/2))
```

Fricas [A]
time = 0.38, size = 77, normalized size = 1.40

$$-\frac{(2\,a^3x^3-4\,a^2x^2+ax+7)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^4*c^2*
x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(1/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**4*x*
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

Giac [A]
time = 0.44, size = 65, normalized size = 1.18

$$-\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2-5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x-1\right)^5ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 - 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x - 1)^5*a*c^2)
```

Mupad [B]
time = 0.04, size = 55, normalized size = 1.00

$$-\frac{\frac{(a\,x-1)^2}{(a\,x+1)^2}-\frac{2\,(a\,x-1)}{3\,(a\,x+1)}+\frac{1}{5}}{4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] -((a*x - 1)^2/(a*x + 1)^2 - (2*(a*x - 1))/(3*(a*x + 1)) + 1/5)/(4*a*c^2*((a
*x - 1)/(a*x + 1))^(5/2))
```

3.579 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=91

$$-\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}-\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3\left(1-a^2x^2\right)}$$

```
[Out] -8/35/((a*x-1)/(a*x+1))^(3/2)/a/c^3-1/7/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)/
a/c^3/(-a^2*x^2+1)^2+12/35/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^3/(-a^2*x
^2+1)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6320, 6318}

$$-\frac{(3-4ax)e^{3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{12(3-2ax)e^{3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}-\frac{8e^{3\coth^{-1}(ax)}}{35ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (-8*E^(3*ArcCoth[a*x]))/(35*a*c^3) - (E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(7*a*
c^3*(1 - a^2*x^2)^2) + (12*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(35*a*c^3*(1 - a
^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3(1-a^2x^2)^2} + \frac{12\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{7c}$$
$$= -\frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3(1-a^2x^2)^2} + \frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3(1-a^2x^2)} - \frac{24\int \frac{e^{3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}$$
$$= -\frac{8e^{3\coth^{-1}(ax)}}{35ac^3} - \frac{e^{3\coth^{-1}(ax)}(3-4ax)}{7ac^3(1-a^2x^2)^2} + \frac{12e^{3\coth^{-1}(ax)}(3-2ax)}{35ac^3(1-a^2x^2)}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.73

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-13+4ax+20a^2x^2-24a^3x^3+8a^4x^4)}{35c^3(-1+ax)^4(1+ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-13 + 4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a
^4*x^4))/(c^3*(-1 + a*x)^4*(1 + a*x))
```

Maple [A]
time = 0.19, size = 68, normalized size = 0.75

method	result	size
trager	$-\frac{(8a^4x^4-24a^3x^3+20a^2x^2+4ax-13)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^3(ax-1)^4}$	63
gosper	$-\frac{8a^4x^4-24a^3x^3+20a^2x^2+4ax-13}{35(a^2x^2-1)^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	65
default	$-\frac{8a^4x^4-24a^3x^3+20a^2x^2+4ax-13}{35\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3a(ax-1)^2}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(8*a^4*x^4-24*a^3*x^3+20*a^2*x^2+4*a*x-13)/((a*x-1)/(a*x+1))^(3/2)/(a
*x+1)^2/c^3/a/(a*x-1)^2
```

Maxima [A]
time = 0.27, size = 97, normalized size = 1.07

$$-\frac{1}{560}\,a\left(\frac{35\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{\frac{28\,(ax-1)}{ax+1}-\frac{70\,(ax-1)^2}{(ax+1)^2}+\frac{140\,(ax-1)^3}{(ax+1)^3}-5}{a^2c^3\,\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*(35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3) + (28*(a*x - 1)/(a*x + 1)
- 70*(a*x - 1)^2/(a*x + 1)^2 + 140*(a*x - 1)^3/(a*x + 1)^3 - 5)/(a^2*c^3*((
a*x - 1)/(a*x + 1))^(7/2)))
```

Fricas [A]
time = 0.34, size = 96, normalized size = 1.05

$$-\frac{\left(8\,a^4x^4-24\,a^3x^3+20\,a^2x^2+4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\,\left(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(8*a^4*x^4 - 24*a^3*x^3 + 20*a^2*x^2 + 4*a*x - 13)*sqrt((a*x - 1)/(a*
x + 1))/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(1/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**6*x
**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x +
1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3
*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-1/((a^2*c*x^2 - c)^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B]
time = 1.26, size = 60, normalized size = 0.66

$$-\frac{8\,a^4\,x^4-24\,a^3\,x^3+20\,a^2\,x^2+4\,a\,x-13}{35\,a\,c^3\,{\left(a\,x+1\right)}^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] -(4*a*x + 20*a^2*x^2 - 24*a^3*x^3 + 8*a^4*x^4 - 13)/(35*a*c^3*(a*x + 1)^4*(
(a*x - 1)/(a*x + 1))^(7/2))
```

3.580 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^4}\,dx$

Optimal. Leaf size=127

$$-\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}-\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4\left(1-a^2x^2\right)^3}-\frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4\left(1-a^2x^2\right)}$$

```
[Out] -16/63/((a*x-1)/(a*x+1))^(3/2)/a/c^4-1/9/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+1)
/a/c^4/(-a^2*x^2+1)^3-10/63/((a*x-1)/(a*x+1))^(3/2)*(-4*a*x+3)/a/c^4/(-a^2*
x^2+1)^2+8/21/((a*x-1)/(a*x+1))^(3/2)*(-2*a*x+3)/a/c^4/(-a^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6320, 6318}

$$-\frac{10(3-4ax)e^{3\coth^{-1}(ax)}}{63ac^4\left(1-a^2x^2\right)^2}+\frac{8(3-2ax)e^{3\coth^{-1}(ax)}}{21ac^4\left(1-a^2x^2\right)}-\frac{(1-2ax)e^{3\coth^{-1}(ax)}}{9ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{3\coth^{-1}(ax)}}{63ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (-16*E^(3*ArcCoth[a*x]))/(63*a*c^4) - (E^(3*ArcCoth[a*x])*(1 - 2*a*x))/(9*a
*c^4*(1 - a^2*x^2)^3) - (10*E^(3*ArcCoth[a*x])*(3 - 4*a*x))/(63*a*c^4*(1 -
a^2*x^2)^2) + (8*E^(3*ArcCoth[a*x])*(3 - 2*a*x))/(21*a*c^4*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{9c}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{40\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{21c^2}$$

$$= -\frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{8e^{3\coth^{-1}(ax)}(3-2ax)}{21ac^4(1-a^2x^2)} - \frac{16\int \frac{e^{3\coth^{-1}}}{c-a^2}}{21c}$$

$$= -\frac{16e^{3\coth^{-1}(ax)}}{63ac^4} - \frac{e^{3\coth^{-1}(ax)}(1-2ax)}{9ac^4(1-a^2x^2)^3} - \frac{10e^{3\coth^{-1}(ax)}(3-4ax)}{63ac^4(1-a^2x^2)^2} + \frac{8e^{3\coth^{-1}(ax)}(3-2}{21ac^4(1-a^2x^2)}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 0.65

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(19+6ax-66a^2x^2+56a^3x^3+24a^4x^4-48a^5x^5+16a^6x^6)}{63c^4(-1+ax)^5(1+ax)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -1/63*(Sqrt[1 - 1/(a^2*x^2)]*x*(19 + 6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a
^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6))/(c^4*(-1 + a*x)^5*(1 + a*x)^2)
```

Maple [A]
time = 0.19, size = 84, normalized size = 0.66

method	result	size
gosper	$-\frac{16a^6x^6-48a^5x^5+24a^4x^4+56a^3x^3-66a^2x^2+6ax+19}{63(a^2x^2-1)^3c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	81
default	$-\frac{16a^6x^6-48a^5x^5+24a^4x^4+56a^3x^3-66a^2x^2+6ax+19}{63\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^3c^4(ax-1)^3a}$	84
trager	$-\frac{(16a^6x^6-48a^5x^5+24a^4x^4+56a^3x^3-66a^2x^2+6ax+19)\sqrt{-\frac{-ax+1}{ax+1}}}{63a\,c^4(ax+1)(ax-1)^5}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/63*(16*a^6*x^6-48*a^5*x^5+24*a^4*x^4+56*a^3*x^3-66*a^2*x^2+6*a*x+19)/((a
*x-1)/(a*x+1))^(3/2)/(a*x+1)^3/c^4/(a*x-1)^3/a
```

Maxima [A]
time = 0.26, size = 131, normalized size = 1.03

$$\frac{1}{4032}\,a\left(\frac{21\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-18\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{\frac{54\,(ax-1)}{ax+1}-\frac{189\,(ax-1)^2}{(ax+1)^2}+\frac{420\,(ax-1)^3}{(ax+1)^3}-\frac{945\,(ax-1)^4}{(ax+1)^4}-7}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*(21*(((a*x - 1)/(a*x + 1))^(3/2) - 18*sqrt((a*x - 1)/(a*x + 1)))/(
a^2*c^4) + (54*(a*x - 1)/(a*x + 1) - 189*(a*x - 1)^2/(a*x + 1)^2 + 420*(a*x
 - 1)^3/(a*x + 1)^3 - 945*(a*x - 1)^4/(a*x + 1)^4 - 7)/(a^2*c^4*((a*x - 1)/
(a*x + 1))^(9/2)))
```

Fricas [A]
time = 0.38, size = 124, normalized size = 0.98

$$-\frac{\left(16\,a^6x^6-48\,a^5x^5+24\,a^4x^4+56\,a^3x^3-66\,a^2x^2+6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/63*(16*a^6*x^6 - 48*a^5*x^5 + 24*a^4*x^4 + 56*a^3*x^3 - 66*a^2*x^2 + 6*a
*x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4
*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\frac{a^9x^9\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{6a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{4a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(1/(a**9*x**9*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a**8*x*
*8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**7*x**7*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + 4*a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1
))/(a*x + 1) + 6*a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 6*
a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 4*a**3*x**3*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
```

```
a*x + 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
[Out] integrate(1/((a^2*c*x^2 - c)^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B]
time = 1.26, size = 76, normalized size = 0.60

$$-\frac{16\,a^6\,x^6-48\,a^5\,x^5+24\,a^4\,x^4+56\,a^3\,x^3-66\,a^2\,x^2+6\,a\,x+19}{63\,a\,c^4\,{\left(a\,x+1\right)}^6\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] -(6*a*x - 66*a^2*x^2 + 56*a^3*x^3 + 24*a^4*x^4 - 48*a^5*x^5 + 16*a^6*x^6 +
19)/(63*a*c^4*(a*x + 1)^6*((a*x - 1)/(a*x + 1))^(9/2))
```

3.581 $\int e^{4 \coth^{-1}(ax)}\left(c-a^2 c x^2\right)^5 \, dx$

Optimal. Leaf size=66

$$\frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

```
[Out] c^5*(a*x+1)^8/a-4/3*c^5*(a*x+1)^9/a+3/5*c^5*(a*x+1)^10/a-1/11*c^5*(a*x+1)^1
1/a
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$-\frac{c^5(ax+1)^{11}}{11a}+\frac{3c^5(ax+1)^{10}}{5a}-\frac{4c^5(ax+1)^9}{3a}+\frac{c^5(ax+1)^8}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] (c^5*(1 + a*x)^8)/a - (4*c^5*(1 + a*x)^9)/(3*a) + (3*c^5*(1 + a*x)^10)/(5*a
) - (c^5*(1 + a*x)^11)/(11*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^5\,dx = \int e^{4\tanh^{-1}(ax)}(c-a^2cx^2)^5\,dx$$
$$= c^5\int (1-ax)^3(1+ax)^7\,dx$$
$$= c^5\int \left(8(1+ax)^7-12(1+ax)^8+6(1+ax)^9-(1+ax)^{10}\right)\,dx$$
$$= \frac{c^5(1+ax)^8}{a}-\frac{4c^5(1+ax)^9}{3a}+\frac{3c^5(1+ax)^{10}}{5a}-\frac{c^5(1+ax)^{11}}{11a}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.59

$$-\frac{c^5(1+ax)^8\left(-29+67ax-54a^2x^2+15a^3x^3\right)}{165a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^5,x]
```

```
[Out] -1/165*(c^5*(1 + a*x)^8*(-29 + 67*a*x - 54*a^2*x^2 + 15*a^3*x^3))/a
```

Maple [A]
time = 0.28, size = 75, normalized size = 1.14

method	result
default	$c^5\left(-\frac{1}{11}a^{10}x^{11}-\frac{2}{5}a^9x^{10}-\frac{1}{3}a^8x^9+a^7x^8+2a^6x^7-\frac{14}{5}a^4x^5-2a^3x^4+a^2x^3+2a\,x^2+x\right)$
gosper	$-\frac{c^5x\left(15a^{10}x^{10}+66a^9x^9+55a^8x^8-165a^7x^7-330a^6x^6+462a^4x^4+330a^3x^3-165a^2x^2-330ax-165\right)}{165}$
risch	$-\frac{1}{11}a^{10}c^5x^{11}-\frac{2}{5}a^9c^5x^{10}-\frac{1}{3}a^8c^5x^9+a^7c^5x^8+2a^6c^5x^7-\frac{14}{5}a^4c^5x^5-2c^5a^3x^4+c^5a^2x^3+2a\,c^5x^2$ -
norman	$\frac{-c^5x+a^7c^5x^8+c^5a^2x^3-a\,c^5x^2+\frac{4}{5}a^4c^5x^5-\frac{14}{5}a^5c^5x^6-2a^6c^5x^7+\frac{4}{3}a^8c^5x^9+\frac{1}{15}a^9c^5x^{10}-\frac{17}{55}a^{10}c^5x^{11}-\frac{1}{11}a^{11}c^5x^{12}+3c^5a^3x^4}{ax-1}$
meijerg	$\frac{c^5\left(-\frac{xa\left(-2730x^{11}a^{11}-3276a^{10}x^{10}-4004a^9x^9-5005a^8x^8-6435a^7x^7-8580a^6x^6-12012a^5x^5-18018a^4x^4-30030a^3x^3-60060a^2x^2-180180ax+36\right.}{30030(-ax+1)}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(-1/11*a^10*x^11-2/5*a^9*x^10-1/3*a^8*x^9+a^7*x^8+2*a^6*x^7-14/5*a^4*x^
5-2*a^3*x^4+a^2*x^3+2*a*x^2+x)
```

Maple [A]

Maxima [A]
time = 0.26, size = 101, normalized size = 1.53

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="maxima")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Fricas [A]
time = 0.35, size = 101, normalized size = 1.53

$$-\frac{1}{11}\,a^{10}c^5x^{11}-\frac{2}{5}\,a^9c^5x^{10}-\frac{1}{3}\,a^8c^5x^9+a^7c^5x^8+2\,a^6c^5x^7-\frac{14}{5}\,a^4c^5x^5-2\,a^3c^5x^4+a^2c^5x^3+2\,ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] -1/11*a^10*c^5*x^11 - 2/5*a^9*c^5*x^10 - 1/3*a^8*c^5*x^9 + a^7*c^5*x^8 + 2*
a^6*c^5*x^7 - 14/5*a^4*c^5*x^5 - 2*a^3*c^5*x^4 + a^2*c^5*x^3 + 2*a*c^5*x^2
+ c^5*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(54) = 108$.
time = 0.04, size = 109, normalized size = 1.65

$$-\frac{a^{10}c^5x^{11}}{11}-\frac{2a^9c^5x^{10}}{5}-\frac{a^8c^5x^9}{3}+a^7c^5x^8+2a^6c^5x^7-\frac{14a^4c^5x^5}{5}-2a^3c^5x^4+a^2c^5x^3+2ac^5x^2+c^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**5,x)
```

```
[Out] -a**10*c**5*x**11/11 - 2*a**9*c**5*x**10/5 - a**8*c**5*x**9/3 + a**7*c**5*x
**8 + 2*a**6*c**5*x**7 - 14*a**4*c**5*x**5/5 - 2*a**3*c**5*x**4 + a**2*c**5
*x**3 + 2*a*c**5*x**2 + c**5*x
```

Giac [A]
time = 0.41, size = 102, normalized size = 1.55

$$-\frac{\left(15\,c^5+\frac{231\,c^5}{ax-1}+\frac{1540\,c^5}{(ax-1)^2}+\frac{5775\,c^5}{(ax-1)^3}+\frac{13200\,c^5}{(ax-1)^4}+\frac{18480\,c^5}{(ax-1)^5}+\frac{14784\,c^5}{(ax-1)^6}+\frac{5280\,c^5}{(ax-1)^7}\right)(ax-1)^{11}}{165\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/165*(15*c^5 + 231*c^5/(a*x - 1) + 1540*c^5/(a*x - 1)^2 + 5775*c^5/(a*x -
 1)^3 + 13200*c^5/(a*x - 1)^4 + 18480*c^5/(a*x - 1)^5 + 14784*c^5/(a*x - 1)
^6 + 5280*c^5/(a*x - 1)^7)*(a*x - 1)^11/a
```

Mupad [B]
time = 1.24, size = 101, normalized size = 1.53

$$-\frac{a^{10}\,c^5\,x^{11}}{11}-\frac{2\,a^9\,c^5\,x^{10}}{5}-\frac{a^8\,c^5\,x^9}{3}+a^7\,c^5\,x^8+2\,a^6\,c^5\,x^7-\frac{14\,a^4\,c^5\,x^5}{5}-2\,a^3\,c^5\,x^4+a^2\,c^5\,x^3+2\,a\,c^5\,x^2+c^5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^5*x + 2*a*c^5*x^2 + a^2*c^5*x^3 - 2*a^3*c^5*x^4 - (14*a^4*c^5*x^5)/5 + 2*
a^6*c^5*x^7 + a^7*c^5*x^8 - (a^8*c^5*x^9)/3 - (2*a^9*c^5*x^10)/5 - (a^10*c^
5*x^11)/11
```

3.582 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal. Leaf size=52

$$\frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}$$

```
[Out] 4/7*c^4*(a*x+1)^7/a-1/2*c^4*(a*x+1)^8/a+1/9*c^4*(a*x+1)^9/a
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 45}

$$\frac{c^4(ax+1)^9}{9a}-\frac{c^4(ax+1)^8}{2a}+\frac{4c^4(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (4*c^4*(1 + a*x)^7)/(7*a) - (c^4*(1 + a*x)^8)/(2*a) + (c^4*(1 + a*x)^9)/(9*
a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$=c^4\int (1-ax)^2(1+ax)^6\,dx$$
$$=c^4\int \left(4(1+ax)^6-4(1+ax)^7+(1+ax)^8\right)\,dx$$
$$=\frac{4c^4(1+ax)^7}{7a}-\frac{c^4(1+ax)^8}{2a}+\frac{c^4(1+ax)^9}{9a}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.60

$$\frac{c^4(1+ax)^7\left(23-35ax+14a^2x^2\right)}{126a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^4,x]
```

```
[Out] (c^4*(1 + a*x)^7*(23 - 35*a*x + 14*a^2*x^2))/(126*a)
```

Maple [A]
time = 0.24, size = 69, normalized size = 1.33

method	result
gosper	$\frac{c^4x\left(14a^8x^8+63a^7x^7+72a^6x^6-84a^5x^5-252a^4x^4-126a^3x^3+168a^2x^2+252ax+126\right)}{126}$
default	$c^4\left(\frac{1}{9}a^8x^9+\frac{1}{2}a^7x^8+\frac{4}{7}a^6x^7-\frac{2}{3}a^5x^6-2a^4x^5-a^3x^4+\frac{4}{3}a^2x^3+2a\,x^2+x\right)$
risch	$\frac{1}{9}a^8c^4x^9+\frac{1}{2}a^7c^4x^8+\frac{4}{7}a^6c^4x^7-\frac{2}{3}a^5c^4x^6-2a^4c^4x^5-a^3c^4x^4+\frac{4}{3}a^2c^4x^3+2c^4a\,x^2+c^4x$
norman	$\frac{-c^4x+a^4c^4x^5+\frac{2}{3}a^2c^4x^3+\frac{7}{3}a^3c^4x^4-\frac{4}{3}a^5c^4x^6-\frac{26}{21}a^6c^4x^7+\frac{1}{14}a^7c^4x^8+\frac{7}{18}a^8c^4x^9+\frac{1}{9}a^9c^4x^{10}-c^4a\,x^2}{ax-1}$
meijerg	$-\frac{c^4\left(-\frac{xa\left(-308a^9x^9-385a^8x^8-495a^7x^7-660a^6x^6-924a^5x^5-1386a^4x^4-2310a^3x^3-4620a^2x^2-13860ax+27720\right)}{2772(-ax+1)}-10\ln(-ax+1)\right)}{a}+\frac{3c^4\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(1/9*a^8*x^9+1/2*a^7*x^8+4/7*a^6*x^7-2/3*a^5*x^6-2*a^4*x^5-a^3*x^4+4/3*
a^2*x^3+2*a*x^2+x)
```

Maxima [A]
time = 0.27, size = 92, normalized size = 1.77

$$\frac{1}{9}\,a^8c^4x^9+\frac{1}{2}\,a^7c^4x^8+\frac{4}{7}\,a^6c^4x^7-\frac{2}{3}\,a^5c^4x^6-2\,a^4c^4x^5-a^3c^4x^4+\frac{4}{3}\,a^2c^4x^3+2\,ac^4x^2+c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Fricas [A]
time = 0.33, size = 92, normalized size = 1.77

$$\frac{1}{9}a^8c^4x^9 + \frac{1}{2}a^7c^4x^8 + \frac{4}{7}a^6c^4x^7 - \frac{2}{3}a^5c^4x^6 - 2a^4c^4x^5 - a^3c^4x^4 + \frac{4}{3}a^2c^4x^3 + 2ac^4x^2 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 + 1/2*a^7*c^4*x^8 + 4/7*a^6*c^4*x^7 - 2/3*a^5*c^4*x^6 - 2*a
^4*c^4*x^5 - a^3*c^4*x^4 + 4/3*a^2*c^4*x^3 + 2*a*c^4*x^2 + c^4*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.
time = 0.03, size = 100, normalized size = 1.92

$$\frac{a^8c^4x^9}{9} + \frac{a^7c^4x^8}{2} + \frac{4a^6c^4x^7}{7} - \frac{2a^5c^4x^6}{3} - 2a^4c^4x^5 - a^3c^4x^4 + \frac{4a^2c^4x^3}{3} + 2ac^4x^2 + c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**4,x)
```

```
[Out] a**8*c**4*x**9/9 + a**7*c**4*x**8/2 + 4*a**6*c**4*x**7/7 - 2*a**5*c**4*x**6
/3 - 2*a**4*c**4*x**5 - a**3*c**4*x**4 + 4*a**2*c**4*x**3/3 + 2*a*c**4*x**2
 + c**4*x
```

Giac [A]
time = 0.41, size = 90, normalized size = 1.73

$$\frac{\left(14\,c^4 + \frac{189\,c^4}{ax-1} + \frac{1080\,c^4}{(ax-1)^2} + \frac{3360\,c^4}{(ax-1)^3} + \frac{6048\,c^4}{(ax-1)^4} + \frac{6048\,c^4}{(ax-1)^5} + \frac{2688\,c^4}{(ax-1)^6}\right)(ax-1)^9}{126\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/126*(14*c^4 + 189*c^4/(a*x - 1) + 1080*c^4/(a*x - 1)^2 + 3360*c^4/(a*x -
1)^3 + 6048*c^4/(a*x - 1)^4 + 6048*c^4/(a*x - 1)^5 + 2688*c^4/(a*x - 1)^6)*
(a*x - 1)^9/a
```

Mupad [B]
time = 0.05, size = 92, normalized size = 1.77

$$\frac{a^8\,c^4\,x^9}{9} + \frac{a^7\,c^4\,x^8}{2} + \frac{4\,a^6\,c^4\,x^7}{7} - \frac{2\,a^5\,c^4\,x^6}{3} - 2\,a^4\,c^4\,x^5 - a^3\,c^4\,x^4 + \frac{4\,a^2\,c^4\,x^3}{3} + 2\,a\,c^4\,x^2 + c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c - a^2*c*x^2)^4*(a*x + 1)^2)/(a*x - 1)^2,x)

[Out] c^4*x + 2*a*c^4*x^2 + (4*a^2*c^4*x^3)/3 - a^3*c^4*x^4 - 2*a^4*c^4*x^5 - (2*a^5*c^4*x^6)/3 + (4*a^6*c^4*x^7)/7 + (a^7*c^4*x^8)/2 + (a^8*c^4*x^9)/9

3.583 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal. Leaf size=35

$$\frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

```
[Out] 1/3*c^3*(a*x+1)^6/a-1/7*c^3*(a*x+1)^7/a
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6302, 6275, 45}

$$\frac{c^3(ax+1)^6}{3a}-\frac{c^3(ax+1)^7}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (c^3*(1 + a*x)^6)/(3*a) - (c^3*(1 + a*x)^7)/(7*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$$
$$=c^3\int(1-ax)(1+ax)^5\,dx$$
$$=c^3\int\left(2(1+ax)^5-(1+ax)^6\right)\,dx$$
$$=\frac{c^3(1+ax)^6}{3a}-\frac{c^3(1+ax)^7}{7a}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.66

$$-\frac{c^3(1+ax)^6(-4+3ax)}{21a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/21*(c^3*(1 + a*x)^6*(-4 + 3*a*x))/a
```

Maple [A]
time = 0.23, size = 45, normalized size = 1.29

method	result
gosper	$-\frac{c^3x\left(3a^6x^6+14a^5x^5+21a^4x^4-35a^2x^2-42ax-21\right)}{21}$
default	$c^3\left(-\frac{1}{7}a^6x^7-\frac{2}{3}a^5x^6-a^4x^5+\frac{5}{3}a^2x^3+2a\,x^2+x\right)$
risch	$-\frac{1}{7}a^6c^3x^7-\frac{2}{3}a^5c^3x^6-a^4c^3x^5+\frac{5}{3}a^2c^3x^3+2c^3a\,x^2+c^3x$
norman	$\frac{-c^3x+a^4c^3x^5+\frac{1}{3}a^2c^3x^3+\frac{5}{3}a^3c^3x^4-\frac{1}{3}a^5c^3x^6-\frac{11}{21}a^6c^3x^7-\frac{1}{7}a^7c^3x^8-c^3a\,x^2}{ax-1}$
meijerg	$\frac{c^3\left(-\frac{ax\left(-45a^7x^7-60a^6x^6-84a^5x^5-126a^4x^4-210a^3x^3-420a^2x^2-1260ax+2520\right)}{315(-ax+1)}-8\ln(-ax+1)\right)}{a}-\frac{2c^3\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3\right.}{70(-ax+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/7*a^6*x^7-2/3*a^5*x^6-a^4*x^5+5/3*a^2*x^3+2*a*x^2+x)
```

Maxima [A]
time = 0.26, size = 59, normalized size = 1.69

$$-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

Fricas [A]
time = 0.36, size = 59, normalized size = 1.69

$$-\frac{1}{7}\,a^6c^3x^7-\frac{2}{3}\,a^5c^3x^6-a^4c^3x^5+\frac{5}{3}\,a^2c^3x^3+2\,ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 - 2/3*a^5*c^3*x^6 - a^4*c^3*x^5 + 5/3*a^2*c^3*x^3 + 2*a*c^
3*x^2 + c^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.
time = 0.03, size = 63, normalized size = 1.80

$$-\frac{a^6c^3x^7}{7}-\frac{2a^5c^3x^6}{3}-a^4c^3x^5+\frac{5a^2c^3x^3}{3}+2ac^3x^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -a**6*c**3*x**7/7 - 2*a**5*c**3*x**6/3 - a**4*c**3*x**5 + 5*a**2*c**3*x**3/
3 + 2*a*c**3*x**2 + c**3*x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(31) = 62$.
time = 0.41, size = 78, normalized size = 2.23

$$-\frac{\left(3\,c^3+\frac{35\,c^3}{ax-1}+\frac{168\,c^3}{(ax-1)^2}+\frac{420\,c^3}{(ax-1)^3}+\frac{560\,c^3}{(ax-1)^4}+\frac{336\,c^3}{(ax-1)^5}\right)(ax-1)^7}{21\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/21*(3*c^3 + 35*c^3/(a*x - 1) + 168*c^3/(a*x - 1)^2 + 420*c^3/(a*x - 1)^3
 + 560*c^3/(a*x - 1)^4 + 336*c^3/(a*x - 1)^5)*(a*x - 1)^7/a
```

Mupad [B]
time = 0.03, size = 59, normalized size = 1.69

$$-\frac{a^6\,c^3\,x^7}{7}-\frac{2\,a^5\,c^3\,x^6}{3}-a^4\,c^3\,x^5+\frac{5\,a^2\,c^3\,x^3}{3}+2\,a\,c^3\,x^2+c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^3*x + 2*a*c^3*x^2 + (5*a^2*c^3*x^3)/3 - a^4*c^3*x^5 - (2*a^5*c^3*x^6)/3 -
 (a^6*c^3*x^7)/7
```

3.584 $\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(1+ax)^5}{5a}$$

[Out] 1/5*c^2*(a*x+1)^5/a

Rubi [A]

time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 32}

$$\frac{c^2(ax+1)^5}{5a}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]

[Out] (c^2*(1 + a*x)^5)/(5*a)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx\\
&= c^2\int(1+ax)^4\,dx\\
&= \frac{c^2(1+ax)^5}{5a}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(17) = 34.
time = 0.01, size = 49, normalized size = 2.88

$$c^2x + 2ac^2x^2 + 2a^2c^2x^3 + a^3c^2x^4 + \frac{1}{5}a^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
[Out] c^2*x + 2*a*c^2*x^2 + 2*a^2*c^2*x^3 + a^3*c^2*x^4 + (a^4*c^2*x^5)/5
```

Maple [A]
time = 0.19, size = 16, normalized size = 0.94

method	result
default	$\frac{c^2(ax+1)^5}{5a}$
gosper	$\frac{c^2x(a^4x^4+5a^3x^3+10a^2x^2+10ax+5)}{5}$
risch	$\frac{a^4c^2x^5}{5} + a^3c^2x^4 + 2a^2c^2x^3 + 2a\,c^2x^2 + c^2x + \frac{c^2}{5a}$
norman	$\frac{-c^2x+a^3c^2x^4-a\,c^2x^2+\frac{4}{5}a^4c^2x^5+\frac{1}{5}a^5c^2x^6}{ax-1}$
meijerg	$-\frac{c^2\left(-\frac{ax\left(-14a^5x^5-21a^4x^4-35a^3x^3-70a^2x^2-210ax+420\right)}{70(-ax+1)}-6\ln(-ax+1)\right)}{a} + \frac{c^2\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/5*c^2*(a*x+1)^5/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.
time = 0.26, size = 47, normalized size = 2.76

$$\frac{1}{5}\,a^4c^2x^5 + a^3c^2x^4 + 2\,a^2c^2x^3 + 2\,ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.
time = 0.35, size = 47, normalized size = 2.76

$$\frac{1}{5}\,a^4c^2x^5 + a^3c^2x^4 + 2\,a^2c^2x^3 + 2\,ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*a^4*c^2*x^5 + a^3*c^2*x^4 + 2*a^2*c^2*x^3 + 2*a*c^2*x^2 + c^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(12) = 24$.
time = 0.03, size = 48, normalized size = 2.82

$$\frac{a^4c^2x^5}{5} + a^3c^2x^4 + 2a^2c^2x^3 + 2ac^2x^2 + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**2,x)
```

```
[Out] a**4*c**2*x**5/5 + a**3*c**2*x**4 + 2*a**2*c**2*x**3 + 2*a*c**2*x**2 + c**2
*x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(15) = 30$.
time = 0.40, size = 64, normalized size = 3.76

$$\frac{\left(c^2 + \frac{10\,c^2}{ax-1} + \frac{40\,c^2}{(ax-1)^2} + \frac{80\,c^2}{(ax-1)^3} + \frac{80\,c^2}{(ax-1)^4}\right)(ax-1)^5}{5\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*(c^2 + 10*c^2/(a*x - 1) + 40*c^2/(a*x - 1)^2 + 80*c^2/(a*x - 1)^3 + 80*
c^2/(a*x - 1)^4)*(a*x - 1)^5/a
```

Mupad [B]
time = 0.03, size = 47, normalized size = 2.76

$$\frac{a^4\,c^2\,x^5}{5} + a^3\,c^2\,x^4 + 2\,a^2\,c^2\,x^3 + 2\,a\,c^2\,x^2 + c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] c^2*x + 2*a*c^2*x^2 + 2*a^2*c^2*x^3 + a^3*c^2*x^4 + (a^4*c^2*x^5)/5
```

3.585 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=46

$$-4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}$$

```
[Out] -4*c*x-c*(a*x+1)^2/a-1/3*c*(a*x+1)^3/a-8*c*ln(-a*x+1)/a
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {6302, 6275, 45}

$$-\frac{c(ax+1)^3}{3a}-\frac{c(ax+1)^2}{a}-\frac{8c\log(1-ax)}{a}-4cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] -4*c*x - (c*(1 + a*x)^2)/a - (c*(1 + a*x)^3)/(3*a) - (8*c*Log[1 - a*x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=\int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx$$
$$=c\int\frac{(1+ax)^3}{1-ax}\,dx$$
$$=c\int\left(-4+\frac{8}{1-ax}-2(1+ax)-(1+ax)^2\right)dx$$
$$=-4cx-\frac{c(1+ax)^2}{a}-\frac{c(1+ax)^3}{3a}-\frac{8c\log(1-ax)}{a}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.78

$$-7cx-2acx^2-\frac{1}{3}a^2cx^3-\frac{8c\log(1-ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2),x]
[Out] -7*c*x - 2*a*c*x^2 - (a^2*c*x^3)/3 - (8*c*Log[1 - a*x])/a
```

Maple [A]
time = 0.15, size = 32, normalized size = 0.70

method	result
default	$c\left(-\frac{a^2x^3}{3}-2a\,x^2-7x-\frac{8\ln(ax-1)}{a}\right)$
risch	$-\frac{a^2c\,x^3}{3}-2ac\,x^2-7cx-\frac{8c\ln(ax-1)}{a}$
norman	$\frac{7cx-5ac\,x^2-\frac{5}{3}a^2c\,x^3-\frac{1}{3}a^3c\,x^4}{ax-1}-\frac{8c\ln(ax-1)}{a}$
meijerg	$\frac{c\left(-\frac{ax\left(-5a^3x^3-10a^2x^2-30ax+60\right)}{15(-ax+1)}-4\ln(-ax+1)\right)}{a}-\frac{2c\left(\frac{ax\left(-2a^2x^2-6ax+12\right)}{-4ax+4}+3\ln(-ax+1)\right)}{a}+\frac{2c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] c*(-1/3*a^2*x^3-2*a*x^2-7*x-8/a*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.72

$$-\frac{1}{3}\,a^2cx^3-2\,acx^2-7\,cx-\frac{8\,c\log(ax-1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 - 2*a*c*x^2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Fricas [A]
time = 0.38, size = 37, normalized size = 0.80

$$-\frac{a^3cx^3 + 6\,a^2cx^2 + 21\,acx + 24\,c\log\left(ax - 1\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*(a^3*c*x^3 + 6*a^2*c*x^2 + 21*a*c*x + 24*c*log(a*x - 1))/a
```

Sympy [A]
time = 0.06, size = 36, normalized size = 0.78

$$-\frac{a^2cx^3}{3} - 2acx^2 - 7cx - \frac{8c\log\left(ax - 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c),x)
[Out] -a**2*c*x**3/3 - 2*a*c*x**2 - 7*c*x - 8*c*log(a*x - 1)/a
```

Giac [A]
time = 0.42, size = 60, normalized size = 1.30

$$-\frac{(ax-1)^3\left(c + \frac{9\,c}{ax-1} + \frac{36\,c}{(ax-1)^2}\right)}{3\,a} + \frac{8\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c),x, algorithm="giac")
[Out] -1/3*(a*x - 1)^3*(c + 9*c/(a*x - 1) + 36*c/(a*x - 1)^2)/a + 8*c*log(abs(a*x
 - 1)/((a*x - 1)^2*abs(a)))/a
```

Mupad [B]
time = 0.05, size = 33, normalized size = 0.72

$$-7\,c\,x - \frac{a^2\,c\,x^3}{3} - \frac{8\,c\,\ln\left(a\,x - 1\right)}{a} - 2\,a\,c\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] - 7*c*x - (a^2*c*x^3)/3 - (8*c*log(a*x - 1))/a - 2*a*c*x^2
```

3.586 $\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=13

$$\frac{x}{c(1-ax)^2}$$

```
[Out] x/c/(-a*x+1)^2
```

Rubi [A]
time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 34}

$$\frac{x}{c(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] x/(c*(1 - a*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{c-a^2cx^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx\\
&= \frac{\int \frac{1+ax}{(1-ax)^3}\,dx}{c}\\
&= \frac{x}{c(1-ax)^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.92

$$\frac{(1+ax)^2}{4ac(1-ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2),x]
```

```
[Out] (1 + a*x)^2/(4*a*c*(1 - a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.
time = 0.18, size = 28, normalized size = 2.15

method	result	size
gosper	$\frac{x}{(ax-1)^2 c}$	13
norman	$\frac{x}{(ax-1)^2 c}$	13
risch	$\frac{x}{(ax-1)^2 c}$	13
default	$\frac{\frac{1}{a(ax-1)}+\frac{1}{a(ax-1)^2}}{c}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/a/(a*x-1)+1/a/(a*x-1)^2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 1.46

$$\frac{x}{a^2cx^2 - 2\,acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Fricas [A]
time = 0.33, size = 19, normalized size = 1.46

$$\frac{x}{a^2cx^2 - 2\,acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] x/(a^2*c*x^2 - 2*a*c*x + c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.
time = 0.09, size = 17, normalized size = 1.31

$$\frac{x}{a^2cx^2 - 2acx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c),x)
```

```
[Out] x/(a**2*c*x**2 - 2*a*c*x + c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 0.39, size = 27, normalized size = 2.08

$$\frac{\frac{1}{(ax-1)a} + \frac{1}{(ax-1)^2a}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] (1/((a*x - 1)*a) + 1/((a*x - 1)^2*a))/c
```

Mupad [B]
time = 0.05, size = 12, normalized size = 0.92

$$\frac{x}{c\,(a\,x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)*(a*x - 1)^2),x)
```

```
[Out] x/(c*(a*x - 1)^2)
```

3.587 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3ac^2(1-ax)^3}$$

[Out] 1/3/a/c^2/(-a*x+1)^3

Rubi [A]
time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 32}

$$\frac{1}{3ac^2(1-ax)^3}$$

Antiderivative was successfully verified.

[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]

[Out] 1/(3*a*c^2*(1 - a*x)^3)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx\\ &= \frac{\int \frac{1}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{1}{3ac^2(1-ax)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.94

$$-\frac{1}{3ac^2(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/3*1/(a*c^2*(-1 + a*x)^3)
```

Maple [A]
time = 0.18, size = 16, normalized size = 0.89

method	result	size
gosper	$-\frac{1}{3(ax-1)^3c^2a}$	16
default	$-\frac{1}{3(ax-1)^3c^2a}$	16
risch	$-\frac{1}{3(ax-1)^3c^2a}$	16
norman	$\frac{-\frac{x}{c}+\frac{2a^2x^3}{3c}-\frac{a^3x^4}{3c}}{(ax+1)(ax-1)^3c}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a*x-1)^3/c^2/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.
time = 0.27, size = 41, normalized size = 2.28

$$-\frac{1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.
time = 0.34, size = 41, normalized size = 2.28

$$-\frac{1}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.
time = 0.11, size = 42, normalized size = 2.33

$$-\frac{1}{3a^4c^2x^3 - 9a^3c^2x^2 + 9a^2c^2x - 3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] -1/(3*a**4*c**2*x**3 - 9*a**3*c**2*x**2 + 9*a**2*c**2*x - 3*a*c**2)
```

Giac [A]
time = 0.41, size = 15, normalized size = 0.83

$$-\frac{1}{3\left(ax-1\right)^3ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/3/((a*x - 1)^3*a*c^2)
```

Mupad [B]
time = 0.06, size = 40, normalized size = 2.22

$$\frac{1}{-3\,a^4\,c^2\,x^3+9\,a^3\,c^2\,x^2-9\,a^2\,c^2\,x+3\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^2*(a*x - 1)^2),x)
```

```
[Out] 1/(3*a*c^2 - 9*a^2*c^2*x + 9*a^3*c^2*x^2 - 3*a^4*c^2*x^3)
```

3.588 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=87

$$\frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

```
[Out] 1/8/a/c^3/(-a*x+1)^4+1/12/a/c^3/(-a*x+1)^3+1/16/a/c^3/(-a*x+1)^2+1/16/a/c^3
/(-a*x+1)+1/16*arctanh(a*x)/a/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\frac{1}{16ac^3(1-ax)}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{8ac^3(1-ax)^4}+\frac{\tanh^{-1}(ax)}{16ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(8*a*c^3*(1 - a*x)^4) + 1/(12*a*c^3*(1 - a*x)^3) + 1/(16*a*c^3*(1 - a*x)^
2) + 1/(16*a*c^3*(1 - a*x)) + ArcTanh[a*x]/(16*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx \\
&= \frac{\int \frac{1}{(1-ax)^5(1+ax)}\,dx}{c^3} \\
&= \frac{\int \left(-\frac{1}{2(-1+ax)^5}+\frac{1}{4(-1+ax)^4}-\frac{1}{8(-1+ax)^3}+\frac{1}{16(-1+ax)^2}-\frac{1}{16(-1+a^2x^2)}\right)dx}{c^3} \\
&= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}-\frac{\int \frac{1}{-1+a^2x^2}\,dx}{16c^3} \\
&= \frac{1}{8ac^3(1-ax)^4}+\frac{1}{12ac^3(1-ax)^3}+\frac{1}{16ac^3(1-ax)^2}+\frac{1}{16ac^3(1-ax)}+\frac{\tanh^{-1}(ax)}{16ac^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.60

$$\frac{16-19ax+12a^2x^2-3a^3x^3+3(-1+ax)^4\tanh^{-1}(ax)}{48ac^3(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
[Out] (16 - 19*a*x + 12*a^2*x^2 - 3*a^3*x^3 + 3*(-1 + a*x)^4*ArcTanh[a*x])/(48*a*
c^3*(-1 + a*x)^4)
```

Maple [A]
time = 0.18, size = 76, normalized size = 0.87

method	result	size
risch	$\frac{-\frac{a^2x^3}{16}+\frac{a\,x^2}{4}-\frac{19x}{48}+\frac{1}{3a}}{(ax-1)^4c^3}+\frac{\ln(-ax-1)}{32c^3a}-\frac{\ln(ax-1)}{32c^3a}$	65
default	$\frac{\frac{\ln(ax+1)}{32a}+\frac{1}{8a(ax-1)^4}-\frac{1}{12a(ax-1)^3}+\frac{1}{16a(ax-1)^2}-\frac{1}{16a(ax-1)}-\frac{\ln(ax-1)}{32a}}{c^3}$	76
norman	$\frac{\frac{15x}{16c}+\frac{a\,x^2}{8c}-\frac{31a^2x^3}{24c}+\frac{11a^3x^4}{24c}+\frac{29a^4x^5}{48c}-\frac{a^5x^6}{3c}}{(ax-1)^4(ax+1)^2c^2}-\frac{\ln(ax-1)}{32c^3a}+\frac{\ln(ax+1)}{32a\,c^3}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/32/a*ln(a*x+1)+1/8/a/(a*x-1)^4-1/12/a/(a*x-1)^3+1/16/a/(a*x-1)^2-1
/16/a/(a*x-1)-1/32/a*ln(a*x-1))
```

Maxima [**A**]
time = 0.29, size = 102, normalized size = 1.17

$$-\frac{3\,a^{3}x^{3}-12\,a^{2}x^{2}+19\,ax-16}{48\,(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3})}+\frac{\log{(ax+1)}}{32\,ac^{3}}-\frac{\log{(ax-1)}}{32\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^3*x^3 - 12*a^2*x^2 + 19*a*x - 16)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 +
 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + 1/32*log(a*x + 1)/(a*c^3) - 1/32*lo
g(a*x - 1)/(a*c^3)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(73) = 146.
time = 0.37, size = 147, normalized size = 1.69

$$-\frac{6\,a^{3}x^{3}-24\,a^{2}x^{2}+38\,ax-3\,(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1)\log{(ax+1)}+3\,(a^{4}x^{4}-4\,a^{3}x^{3}+6\,a^{2}x^{2}-4\,ax+1)\log{(ax-1)}-32}{96\,(a^{5}c^{3}x^{4}-4\,a^{4}c^{3}x^{3}+6\,a^{3}c^{3}x^{2}-4\,a^{2}c^{3}x+ac^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(6*a^3*x^3 - 24*a^2*x^2 + 38*a*x - 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2
 - 4*a*x + 1)*log(a*x + 1) + 3*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1
)*log(a*x - 1) - 32)/(a^5*c^3*x^4 - 4*a^4*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c
^3*x + a*c^3)
```

Sympy [**A**]
time = 0.24, size = 99, normalized size = 1.14

$$-\frac{3a^{3}x^{3}-12a^{2}x^{2}+19ax-16}{48a^{5}c^{3}x^{4}-192a^{4}c^{3}x^{3}+288a^{3}c^{3}x^{2}-192a^{2}c^{3}x+48ac^{3}}-\frac{\frac{\log{\left(x-\frac{1}{a}\right)}}{32}-\frac{\log{\left(x+\frac{1}{a}\right)}}{32}}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(3*a**3*x**3 - 12*a**2*x**2 + 19*a*x - 16)/(48*a**5*c**3*x**4 - 192*a**4*c
**3*x**3 + 288*a**3*c**3*x**2 - 192*a**2*c**3*x + 48*a*c**3) - (log(x - 1/a
)/32 - log(x + 1/a)/32)/(a*c**3)
```

Giac [**A**]
time = 0.41, size = 91, normalized size = 1.05

$$\frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^{3}}-\frac{\frac{3\,a^{3}c^{9}}{ax-1}-\frac{3\,a^{3}c^{9}}{(ax-1)^{2}}+\frac{4\,a^{3}c^{9}}{(ax-1)^{3}}-\frac{6\,a^{3}c^{9}}{(ax-1)^{4}}}{48\,a^{4}c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(3*a^3*c^9/(a*x - 1) - 3*a^3
*c^9/(a*x - 1)^2 + 4*a^3*c^9/(a*x - 1)^3 - 6*a^3*c^9/(a*x - 1)^4)/(a^4*c^12
)
```

Mupad [B]
time = 0.09, size = 83, normalized size = 0.95

$$\frac{\operatorname{atanh}(a\,x)}{16\,a\,c^3}-\frac{\frac{19\,x}{48}-\frac{a\,x^2}{4}-\frac{1}{3\,a}+\frac{a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^3*(a*x - 1)^2),x)
```

```
[Out] atanh(a*x)/(16*a*c^3) - ((19*x)/48 - (a*x^2)/4 - 1/(3*a) + (a^2*x^3)/16)/(c
^3 + 6*a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x)
```

3.589 $\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=122

$$\frac{1}{20ac^4(1-ax)^5}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^2}+\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{3\tan}{3}$$

```
[Out] 1/20/a/c^4/(-a*x+1)^5+1/16/a/c^4/(-a*x+1)^4+1/16/a/c^4/(-a*x+1)^3+1/16/a/c^
4/(-a*x+1)^2+5/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+3/32*arctanh(a*x)/a/c^4
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6275, 46, 213}

$$\frac{5}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}+\frac{1}{16ac^4(1-ax)^2}+\frac{1}{16ac^4(1-ax)^3}+\frac{1}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{3\tanh^{-1}(ax)}{32ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] 1/(20*a*c^4*(1 - a*x)^5) + 1/(16*a*c^4*(1 - a*x)^4) + 1/(16*a*c^4*(1 - a*x)
^3) + 1/(16*a*c^4*(1 - a*x)^2) + 5/(64*a*c^4*(1 - a*x)) - 1/(64*a*c^4*(1 +
a*x)) + (3*ArcTanh[a*x])/(32*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$$
$$= \frac{\int \frac{1}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$
$$= \frac{\int \left(\frac{1}{4(-1+ax)^6} - \frac{1}{4(-1+ax)^5} + \frac{3}{16(-1+ax)^4} - \frac{1}{8(-1+ax)^3} + \frac{5}{64(-1+ax)^2} + \frac{1}{64(1+ax)^2} - \frac{3}{32(-1+a^2x^2)}\right)}{c^4}$$
$$= \frac{1}{20ac^4(1-ax)^5} + \frac{1}{16ac^4(1-ax)^4} + \frac{1}{16ac^4(1-ax)^3} + \frac{1}{16ac^4(1-ax)^2} + \frac{5}{64ac^4(1-ax}$$
$$= \frac{1}{20ac^4(1-ax)^5} + \frac{1}{16ac^4(1-ax)^4} + \frac{1}{16ac^4(1-ax)^3} + \frac{1}{16ac^4(1-ax)^2} + \frac{5}{64ac^4(1-ax}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.66

$$\frac{-48+47ax+20a^2x^2-80a^3x^3+60a^4x^4-15a^5x^5+15(-1+ax)^5(1+ax)\tanh^{-1}(ax)}{160ac^4(-1+ax)^5(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
[Out] (-48 + 47*a*x + 20*a^2*x^2 - 80*a^3*x^3 + 60*a^4*x^4 - 15*a^5*x^5 + 15*(-1
+ a*x)^5*(1 + a*x)*ArcTanh[a*x])/(160*a*c^4*(-1 + a*x)^5*(1 + a*x))
```

Maple [A]
time = 0.20, size = 100, normalized size = 0.82

method	result	size
risch	$\frac{-\frac{3a^4x^5}{32}+\frac{3a^3x^4}{8}-\frac{a^2x^3}{2}+\frac{a\,x^2}{8}+\frac{47x}{160}-\frac{3}{10a}}{(ax-1)^4(a^2x^2-1)c^4}-\frac{3\ln(ax-1)}{64c^4a}+\frac{3\ln(-ax-1)}{64c^4a}$	92
default	$\frac{-\frac{1}{64a(ax+1)}+\frac{3\ln(ax+1)}{64a}-\frac{1}{20a(ax-1)^5}+\frac{1}{16a(ax-1)^4}-\frac{1}{16a(ax-1)^3}+\frac{1}{16a(ax-1)^2}-\frac{5}{64a(ax-1)}-\frac{3\ln(ax-1)}{64a}}{c^4}$	100
norman	$\frac{-\frac{a^3x^4}{2c}-\frac{29x}{32c}-\frac{3a\,x^2}{16c}+\frac{59a^2x^3}{32c}-\frac{263a^4x^5}{160c}+\frac{63a^5x^6}{80c}+\frac{81a^6x^7}{160c}-\frac{3a^7x^8}{10c}}{(ax-1)^5(ax+1)^3c^3}-\frac{3\ln(ax-1)}{64c^4a}+\frac{3\ln(ax+1)}{64a\,c^4}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/64/a/(a*x+1)+3/64/a*ln(a*x+1)-1/20/a/(a*x-1)^5+1/16/a/(a*x-1)^4-1
/16/a/(a*x-1)^3+1/16/a/(a*x-1)^2-5/64/a/(a*x-1)-3/64/a*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 130, normalized size = 1.07

$$-\frac{15\,a^5x^5-60\,a^4x^4+80\,a^3x^3-20\,a^2x^2-47\,ax+48}{160\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)}+\frac{3\,\log\left(ax+1\right)}{64\,ac^4}-\frac{3\,\log\left(ax-1\right)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] -1/160*(15*a^5*x^5 - 60*a^4*x^4 + 80*a^3*x^3 - 20*a^2*x^2 - 47*a*x + 48)/(a
^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x -
a*c^4) + 3/64*log(a*x + 1)/(a*c^4) - 3/64*log(a*x - 1)/(a*c^4)
```

Fricas [A]
time = 0.38, size = 191, normalized size = 1.57

$$-\frac{30\,a^5x^5-120\,a^4x^4+160\,a^3x^3-40\,a^2x^2-94\,ax-15\,(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1)\log\left(ax+1\right)+15\,(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1)\log\left(ax-1\right)+96}{320\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/320*(30*a^5*x^5 - 120*a^4*x^4 + 160*a^3*x^3 - 40*a^2*x^2 - 94*a*x - 15*(
a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 15*
(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) + 96
)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*
x - a*c^4)
```

Sympy [A]
time = 0.33, size = 129, normalized size = 1.06

$$\frac{-15a^5x^5+60a^4x^4-80a^3x^3+20a^2x^2+47ax-48}{160a^7c^4x^6-640a^6c^4x^5+800a^5c^4x^4-800a^3c^4x^2+640a^2c^4x-160ac^4}+\frac{-\frac{3\log\left(x-\frac{1}{a}\right)}{64}+\frac{3\log\left(x+\frac{1}{a}\right)}{64}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (-15*a**5*x**5 + 60*a**4*x**4 - 80*a**3*x**3 + 20*a**2*x**2 + 47*a*x - 48)/
(160*a**7*c**4*x**6 - 640*a**6*c**4*x**5 + 800*a**5*c**4*x**4 - 800*a**3*c*
*4*x**2 + 640*a**2*c**4*x - 160*a*c**4) + (-3*log(x - 1/a)/64 + 3*log(x + 1
/a)/64)/(a*c**4)
```

Giac [A]
time = 0.41, size = 127, normalized size = 1.04

$$\frac{3\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4}+\frac{1}{128\,ac^4\left(\frac{2}{ax-1}+1\right)}-\frac{\frac{25\,a^9c^{16}}{ax-1}-\frac{20\,a^9c^{16}}{(ax-1)^2}+\frac{20\,a^9c^{16}}{(ax-1)^3}-\frac{20\,a^9c^{16}}{(ax-1)^4}+\frac{16\,a^9c^{16}}{(ax-1)^5}}{320\,a^{10}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 3/64*log(abs(-2/(a*x - 1) - 1))/(a*c^4) + 1/128/(a*c^4*(2/(a*x - 1) + 1)) -
 1/320*(25*a^9*c^16/(a*x - 1) - 20*a^9*c^16/(a*x - 1)^2 + 20*a^9*c^16/(a*x
- 1)^3 - 20*a^9*c^16/(a*x - 1)^4 + 16*a^9*c^16/(a*x - 1)^5)/(a^10*c^20)
```

Mupad [B]
time = 1.30, size = 111, normalized size = 0.91

$$\frac{3\,\mathrm{atanh}(a\,x)}{32\,a\,c^4}-\frac{\frac{47\,x}{160}+\frac{a\,x^2}{8}-\frac{3}{10\,a}-\frac{a^2\,x^3}{2}+\frac{3\,a^3\,x^4}{8}-\frac{3\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - a^2*c*x^2)^4*(a*x - 1)^2),x)
```

```
[Out] (3*atanh(a*x))/(32*a*c^4) - ((47*x)/160 + (a*x^2)/8 - 3/(10*a) - (a^2*x^3)/
2 + (3*a^3*x^4)/8 - (3*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 +
4*a^5*c^4*x^5 - a^6*c^4*x^6 - 4*a*c^4*x)
```

3.590 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$

Optimal. Leaf size=393

$$-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}$$

```
[Out] -5/48*a^5*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x^6+1/8*a^6*c^4*(1-1/a/x)^(5/
2)*(1+1/a/x)^(9/2)*x^7-1/8*a^7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x^8+1/9*
a^8*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(9/2)*x^9-35/128*c^4*arctanh((1-1/a/x)^(1
/2)*(1+1/a/x)^(1/2))/a-35/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-7/1
92*a^2*c^4*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)-1/64*a^3*c^4*(1+1/a/x)^(7/2)
*x^4*(1-1/a/x)^(1/2)+1/16*a^4*c^4*(1+1/a/x)^(9/2)*x^5*(1-1/a/x)^(1/2)-35/12
8*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{8}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{8}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{5}{48}a^5c^4x^6\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{1}{16}a^4c^4x^5\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}-\frac{1}{64}a^3c^4x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{7}{192}a^2c^4x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{35}{384}ac^4x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{35}{128}c^4x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{35c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

```
[Out] (-35*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 - (35*a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 - (7*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/
64 + (a^4*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2)*x^5)/16 - (5*a^5*c^4*(1
 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)*x^6)/48 + (a^6*c^4*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(9/2)*x^7)/8 - (a^7*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
9/2)*x^8)/8 + (a^8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(9/2)*x^9)/9 - (35
*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}x^9+\left(a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)}{x^9}\right.$$

$$=-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{9/2}$$

$$=\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}x^7-\frac{1}{8}a^7c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x^8$$

$$=-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6+\frac{1}{8}a^6c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}$$

$$=\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-\frac{5}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x^6$$

$$=-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{1}{16}a^4c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}x^5-$$

$$=-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4$$

$$=-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}$$

$$=-\frac{35}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{35}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{7}{192}$$

Mathematica [A]
time = 0.09, size = 111, normalized size = 0.28

$$\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(128+837ax-512a^2x^2-978a^3x^3+768a^4x^4+600a^5x^5-512a^6x^6-144a^7x^7+128a^8x^8)-315\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{1152a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(128 + 837*a*x - 512*a^2*x^2 - 978*a^3*x^3
+ 768*a^4*x^4 + 600*a^5*x^5 - 512*a^6*x^6 - 144*a^7*x^7 + 128*a^8*x^8) - 31
5*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1152*a)
```

Maple [A]
time = 0.10, size = 279, normalized size = 0.71

method	result
risch	$\frac{(128a^8x^8-144a^7x^7-512a^6x^6+600a^5x^5+768a^4x^4-978a^3x^3-512a^2x^2+837ax+128)(ax+1)c^4\sqrt{\frac{ax-1}{ax+1}}}{1152a}-\frac{35\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x\right.}}{}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-128(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+144(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+384\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4-456(a^2x^2-1)^{\frac{3}{2}}\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1152*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4/a*(-128*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2)*a^6*x^6+144*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^5*x^5+384*(a^2)^(1/2)*(a
^2*x^2-1)^(3/2)*a^4*x^4-456*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-384*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+522*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+384*
((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-256*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-315*(
a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2
))/(a^2)^(1/2))*a)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 415, normalized size = 1.06

$$-\frac{1}{1152}\left(\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10458\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-23202\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-32768\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+23202\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-10458\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2730\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-315\,c^4\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{9(ax-1)a^2}{ax+1}-\frac{36(ax-1)^2a^2}{(ax+1)^2}+\frac{84(ax-1)^3a^2}{(ax+1)^3}-\frac{126(ax-1)^4a^2}{(ax+1)^4}+\frac{126(ax-1)^5a^2}{(ax+1)^5}-\frac{84(ax-1)^6a^2}{(ax+1)^6}+\frac{36(ax-1)^7a^2}{(ax+1)^7}-\frac{9(ax-1)^8a^2}{(ax+1)^8}+\frac{(ax-1)^9a^2}{(ax+1)^9}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^4*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^4*((a*x - 1)/(a*x + 1))^(17/2) - 2
730*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10458*c^4*((a*x - 1)/(a*x + 1))^(13/
2) - 23202*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 32768*c^4*((a*x - 1)/(a*x + 1
))^(9/2) + 23202*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 10458*c^4*((a*x - 1)/(a*
x + 1))^(5/2) + 2730*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^4*sqrt((a*x -
1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a*x + 1)^2
+ 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^4 + 126*(a
*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(a*x - 1)^7
*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^2/(a*x + 1
)^9 - a^2))*a
```

Fricas [A]
time = 0.37, size = 170, normalized size = 0.43

$$-\frac{315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(128\,a^9c^4x^9-16\,a^8c^4x^8-656\,a^7c^4x^7+88\,a^6c^4x^6+1368\,a^5c^4x^5-210\,a^4c^4x^4-1490\,a^3c^4x^3+325\,a^2c^4x^2+965\,ac^4x+128\,c^4)\sqrt{\frac{ax-1}{ax+1}}}{1152\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1152*(315*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^4*log(sqrt((a*x
 - 1)/(a*x + 1)) - 1) - (128*a^9*c^4*x^9 - 16*a^8*c^4*x^8 - 656*a^7*c^4*x^7
 + 88*a^6*c^4*x^6 + 1368*a^5*c^4*x^5 - 210*a^4*c^4*x^4 - 1490*a^3*c^4*x^3 +
 325*a^2*c^4*x^2 + 965*a*c^4*x + 128*c^4)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^4\left(\int\left(-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int 6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(-4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-4*a**6*x**6
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**8*x**8*sqrt(a*x/(a*x +
 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.43, size = 196, normalized size = 0.50

$$\frac{35\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{128\,|a|}+\frac{1}{1152}\sqrt{a^2x^2-1}\left(\frac{128\,c^4\mathrm{sgn}(ax+1)}{a}+(837\,c^4\mathrm{sgn}(ax+1)-2\,(256\,ac^4\mathrm{sgn}(ax+1)+(489\,a^2c^4\mathrm{sgn}(ax+1)-4\,(96\,a^3c^4\mathrm{sgn}(ax+1)+(75\,a^4c^4\mathrm{sgn}(ax+1)-2\,(32\,a^5c^4\mathrm{sgn}(ax+1)-(8\,a^7c^4x\mathrm{sgn}(ax+1)-9\,a^6c^4\mathrm{sgn}(ax+1))x)x)x)x)x)x)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

[Out] 35/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/
1152*sqrt(a^2*x^2 - 1)*(128*c^4*sgn(a*x + 1)/a + (837*c^4*sgn(a*x + 1) - 2*
(256*a*c^4*sgn(a*x + 1) + (489*a^2*c^4*sgn(a*x + 1) - 4*(96*a^3*c^4*sgn(a*x
 + 1) + (75*a^4*c^4*sgn(a*x + 1) - 2*(32*a^5*c^4*sgn(a*x + 1) - (8*a^7*c^4*
x*sgn(a*x + 1) - 9*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)

Mupad [B]
time = 1.34, size = 362, normalized size = 0.92

$$\frac{\frac{35\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{455\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}+\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}-\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{32}+\frac{512\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{9}+\frac{1289\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{32}-\frac{581\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{32}+\frac{455\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/2}}{96}-\frac{35\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/2}}{64}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}+\frac{9\,a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}-\frac{a\,{\left(a\,x-1\right)}^9}{{\left(a\,x+1\right)}^9}}-\frac{35\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] ((35*c^4*((a*x - 1)/(a*x + 1))^(1/2))/64 - (455*c^4*((a*x - 1)/(a*x + 1))^(
3/2))/96 + (581*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 - (1289*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/32 + (512*c^4*((a*x - 1)/(a*x + 1))^(9/2))/9 + (1289*c^4*
((a*x - 1)/(a*x + 1))^(11/2))/32 - (581*c^4*((a*x - 1)/(a*x + 1))^(13/2))/3
2 + (455*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 - (35*c^4*((a*x - 1)/(a*x + 1
))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a*x + 1
)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^4 - (1
26*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*a*(a*x
 - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9)/(a*x
 + 1)^9) - (35*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)

3.591 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^3\,dx$

Optimal. Leaf size=313

$$-\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{1}{8}a^3c^3$$

```
[Out] -1/6*a^4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5+1/6*a^5*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(7/2)*x^6-1/7*a^6*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^7-5/16*
c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-5/48*a*c^3*(1+1/a/x)^(3/2)*x
^2*(1-1/a/x)^(1/2)-1/24*a^2*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+1/8*a^3
*c^3*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/2)-5/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{6}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{6}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{1}{8}a^3c^3x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{1}{24}a^2c^3x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{5}{48}ac^3x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{5}{16}c^3x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{5c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

```
[Out] (-5*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 - (5*a*c^3*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2)*x^2)/48 - (a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2)*x^3)/24 + (a^3*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^4)/8 - (a^
4*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/6 + (a^5*c^3*(1 - 1/(a*x
))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/6 - (a^6*c^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(
a*x))^(7/2)*x^7)/7 - (5*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(
16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx = -\left(\left(a^6c^3\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3 x^6\,dx\right)$$

$$= \left(a^6c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^8}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7 - \left(a^5c^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\right.}{x^7}\right.$$

$$= \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6 - \frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7$$

$$= -\frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 + \frac{1}{6}a^5c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}$$

$$= \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 - \frac{1}{6}a^4c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5 +$$

$$= -\frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 + \frac{1}{8}a^3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4 -$$

$$= -\frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{24}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3 +$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x - \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{24}a^2c$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x - \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{24}a^2c$$

$$= -\frac{5}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x - \frac{5}{48}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2 - \frac{1}{24}a^2c$$

Mathematica [A]
time = 0.10, size = 95, normalized size = 0.30

$$\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(48+231ax-144a^2x^2-182a^3x^3+144a^4x^4+56a^5x^5-48a^6x^6)-105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{336a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 231*a*x - 144*a^2*x^2 - 182*a^3*x^3 +
 144*a^4*x^4 + 56*a^5*x^5 - 48*a^6*x^6) - 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)
])*x]))/(336*a)
```

Maple [A]
time = 0.10, size = 231, normalized size = 0.74

method	result
risch	$-\frac{(48a^6x^6-56a^5x^5-144a^4x^4+182a^3x^3+144a^2x^2-231ax-48)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{336a}-\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{\frac{ax-1}{ax+1}}}{16\sqrt{a^2}\,(ax-1}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(48\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4-56(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-96(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+126\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}\right.}{336a\sqrt{(ax+1)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/336*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3/a*(48*(a^2)^(1/2)*(a^2*x^2-1)^(3
/2)*a^4*x^4-56*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-96*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*a^2*x^2+126*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+112*((a*x+1)*(a*x-
1))^(3/2)*(a^2)^(1/2)-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-105*(a^2)^(1/2)*(a^2
*x^2-1)^(1/2)*a*x+105*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))
*a)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 337, normalized size = 1.08

$$-\frac{1}{336}\left(\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1981\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+3072\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-1981\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+700\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+\frac{35(ax-1)^3a^2}{(ax+1)^3}-\frac{35(ax-1)^4a^2}{(ax+1)^4}+\frac{21(ax-1)^5a^2}{(ax+1)^5}-\frac{7(ax-1)^6a^2}{(ax+1)^6}+\frac{(ax-1)^7a^2}{(ax+1)^7}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^3*log(sqrt((
a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 70
```

```
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1981*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
 3072*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 1981*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 700*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x - 1
)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/(
a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
a^2))*a
```

Fricas [A]
time = 0.35, size = 147, normalized size = 0.47

$$-\frac{105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(48\,a^7c^3x^7-8\,a^6c^3x^6-200\,a^5c^3x^5+38\,a^4c^3x^4+326\,a^3c^3x^3-87\,a^2c^3x^2-279\,ac^3x-48\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{336\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/336*(105*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) - 1) + (48*a^7*c^3*x^7 - 8*a^6*c^3*x^6 - 200*a^5*c^3*x^5 +
38*a^4*c^3*x^4 + 326*a^3*c^3*x^3 - 87*a^2*c^3*x^2 - 279*a*c^3*x - 48*c^3)*s
qrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(-3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(a**6*x**6*s
qrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x/(a*x + 1) - 1/(a*
x + 1)), x))
```

Giac [A]
time = 0.41, size = 161, normalized size = 0.51

$$\frac{5\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{16\,|a|}+\frac{1}{336}\sqrt{a^2x^2-1}\left(\frac{48\,c^3\mathrm{sgn}(ax+1)}{a}+\left(231\,c^3\mathrm{sgn}(ax+1)-2\left(72\,ac^3\mathrm{sgn}(ax+1)+\left(91\,a^2c^3\mathrm{sgn}(ax+1)-4\left(18\,a^3c^3\mathrm{sgn}(ax+1)-\left(6\,a^5c^3x\mathrm{sgn}(ax+1)-7\,a^4c^3\mathrm{sgn}(ax+1)\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 5/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/33
6*sqrt(a^2*x^2 - 1)*(48*c^3*sgn(a*x + 1)/a + (231*c^3*sgn(a*x + 1) - 2*(72*
```

```
a*c^3*sgn(a*x + 1) + (91*a^2*c^3*sgn(a*x + 1) - 4*(18*a^3*c^3*sgn(a*x + 1)
- (6*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

Mupad [B]
time = 0.11, size = 289, normalized size = 0.92

$$-\frac{\frac{25\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{5\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{283\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{24}+\frac{128\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{7}+\frac{283\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{24}-\frac{25\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{6}+\frac{5\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}-\frac{5\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - ((25*c^3*((a*x - 1)/(a*x + 1))^(3/2))/6 - (5*c^3*((a*x - 1)/(a*x + 1))^(1
/2))/8 - (283*c^3*((a*x - 1)/(a*x + 1))^(5/2))/24 + (128*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/7 + (283*c^3*((a*x - 1)/(a*x + 1))^(9/2))/24 - (25*c^3*((a*x
 - 1)/(a*x + 1))^(11/2))/6 + (5*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (
7*a*(a*x - 1))/(a*x + 1) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)
^3)/(a*x + 1)^3 + (35*a*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x
+ 1)^5 + (7*a*(a*x - 1)^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7) - (5*
c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a)
```

3.592 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a^3c^2\left(1-\right.$$

```
[Out] -1/4*a^3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(5/2
)*(1+1/a/x)^(5/2)*x^5-3/8*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-1/
8*a*c^2*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+1/4*a^2*c^2*(1+1/a/x)^(5/2)*x^3
*(1-1/a/x)^(1/2)-3/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{4}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{1}{4}a^2c^2x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{1}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{3}{8}c^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

```
[Out] (-3*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (a*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2)*x^2)/8 + (a^2*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
)*x^3)/4 - (a^3*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/4 + (a^4*c
^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/5 - (3*c^2*ArcTanh[Sqrt[1 -
 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\left(a^3c^2\right)\mathrm{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)}{x^5}\right.$$

$$=-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}$$

$$=\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a^3c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4+$$

$$=-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{1}{4}a$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{}$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{}$$

$$=-\frac{3}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{1}{4}a^2c^2\sqrt{}$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.34

$$\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(8+25ax-16a^2x^2-10a^3x^3+8a^4x^4)-15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 + 25*a*x - 16*a^2*x^2 - 10*a^3*x^3 + 8*a
^4*x^4) - 15*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(40*a)
```

Maple [A]
time = 0.10, size = 183, normalized size = 0.79

method	result
risch	$\frac{(8a^4x^4-10a^3x^3-16a^2x^2+25ax+8)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{40a} - \frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1}}{8\sqrt{a^2}\,(ax-1)}$
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-30\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+45\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-40(\right.}{120a\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*a^2*x^2-30*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+16*(a^2*x^2-1)^(3/2)*(a^2)^
(1/2)+45*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-40*((a*x+1)*(a*x-1))^(3/2)*(a^2)
^(1/2)-45*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/((a*x+1)
*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 259, normalized size = 1.11

$$-\frac{1}{40}\,a\left(\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-128\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-15\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{5\,(ax-1)a^2}{ax+1}-\frac{10\,(ax-1)^2a^2}{(ax+1)^2}+\frac{10\,(ax-1)^3a^2}{(ax+1)^3}-\frac{5\,(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5a^2}{(ax+1)^5}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/40*a*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c^2*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(15*c^2*((a*x - 1)/(a*x + 1))^(9/2) - 70*c^
2*((a*x - 1)/(a*x + 1))^(7/2) - 128*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 70*c^
2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*(a*x -
 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a^2/(a*
x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^5 - a^
2))
```

Fricas [A]
time = 0.38, size = 126, normalized size = 0.54

$$-\frac{15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(8\,a^5c^2x^5-2\,a^4c^2x^4-26\,a^3c^2x^3+9\,a^2c^2x^2+33\,ac^2x+8\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(15*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c^2*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (8*a^5*c^2*x^5 - 2*a^4*c^2*x^4 - 26*a^3*c^2*x^3 + 9*a^2
*c^2*x^2 + 33*a*c^2*x + 8*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx+\int a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integra
l(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.42, size = 126, normalized size = 0.54

$$\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{8\,|a|}+\frac{1}{40}\sqrt{a^2x^2-1}\left(\left(25\,c^2\mathrm{sgn}(ax+1)-2\left(8\,ac^2\mathrm{sgn}(ax+1)-\left(4\,a^3c^2x\mathrm{sgn}(ax+1)-5\,a^2c^2\mathrm{sgn}(ax+1)\right)x\right)x\right)x+\frac{8\,c^2\mathrm{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + 1/40*
sqrt(a^2*x^2 - 1)*((25*c^2*sgn(a*x + 1) - 2*(8*a*c^2*sgn(a*x + 1) - (4*a^3*
c^2*x*sgn(a*x + 1) - 5*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 8*c^2*sgn(a*x + 1)/a
)
```

Mupad [B]
time = 0.08, size = 214, normalized size = 0.92

$$\frac{\frac{3\,c^2\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2}+\frac{32\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{5}+\frac{7\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{2}-\frac{3\,c^2\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,(a\,x-1)}{a\,x+1}+\frac{10\,a\,(a\,x-1)^2}{(a\,x+1)^2}-\frac{10\,a\,(a\,x-1)^3}{(a\,x+1)^3}+\frac{5\,a\,(a\,x-1)^4}{(a\,x+1)^4}-\frac{a\,(a\,x-1)^5}{(a\,x+1)^5}}-\frac{3\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((3*c^2*((a*x - 1)/(a*x + 1))^(1/2))/4 - (7*c^2*((a*x - 1)/(a*x + 1))^(3/2)
)/2 + (32*c^2*((a*x - 1)/(a*x + 1))^(5/2))/5 + (7*c^2*((a*x - 1)/(a*x + 1))
^(7/2))/2 - (3*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))/(a*
x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^3 +
(5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) - (3*c^2*atanh
(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.593 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=145

$$-\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{c\tanh^{-1}}{}$$

```
[Out] -1/3*a^2*c*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-1/2*c*arctanh((1-1/a/x)^(1/2
)*(1+1/a/x)^(1/2))/a+1/2*a*c*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-1/2*c*x*(1
-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{1}{2}acx^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{1}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x) + (a*c*Sqrt[1 - 1/(a*x)]*(1
+ 1/(a*x))^(3/2)*x^2)/2 - (a^2*c*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)*x^
3)/3 - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\left(\left(a^2c\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-(ac)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}{x^3}\,d\right.$$

$$= \frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3+\frac{1}{2}c\text{S}$$

$$= -\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\right.$$

$$= -\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\right.$$

$$= -\frac{1}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{1}{2}ac\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{1}{3}a^2c\left(1-\right.$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.42

$$\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(2+3ax-2a^2x^2)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^ArcCoth[a*x],x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 2*a^2*x^2) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A]
time = 0.05, size = 119, normalized size = 0.82

method	result	size
risch	$-\frac{(2a^2x^2-3ax-2)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{6a}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax-1)}$	108
default	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-3\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a\right)}{6\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-
2*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-3*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(
1/2))/(a^2)^(1/2))*a)/((a*x+1)*(a*x-1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 171, normalized size = 1.18

$$\frac{1}{6}a\left(\frac{2\left(3c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+8c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3(ax-1)a^2}{ax+1}-\frac{3(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}-\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(3*c*((a*x - 1)/(a*x + 1))^(5/2) + 8*c*((a*x - 1)/(a*x + 1))^(3/2)
 - 3*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x - 1)^
2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) - 3*c*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.36, size = 91, normalized size = 0.63

$$-\frac{3c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(2a^3cx^3-a^2cx^2-5acx-2c)\sqrt{\frac{ax-1}{ax+1}}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*c*log(sqrt((a*x - 1)/(a*x
+ 1)) - 1) + (2*a^3*c*x^3 - a^2*c*x^2 - 5*a*c*x - 2*c)*sqrt((a*x - 1)/(a*x
+ 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] -c*(Integral(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sq
rt(a*x/(a*x + 1) - 1/(a*x + 1)), x))
```

Giac [A]
time = 0.42, size = 82, normalized size = 0.57

$$\frac{c\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}-\frac{1}{6}\sqrt{a^{2}x^{2}-1}\left((2\,acx\operatorname{sgn}(ax+1)-3\,c\operatorname{sgn}(ax+1))x-\frac{2\,c\operatorname{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sqr
t(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 3*c*sgn(a*x + 1))*x - 2*c*sgn(a*x +
 1)/a)
```

Mupad [B]
time = 1.24, size = 132, normalized size = 0.91

$$-\frac{\frac{8\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}-c\,\sqrt{\frac{a\,x-1}{a\,x+1}}+c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}-\frac{c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] - ((8*c*((a*x - 1)/(a*x + 1))^(3/2))/3 - c*((a*x - 1)/(a*x + 1))^(1/2) + c*
((a*x - 1)/(a*x + 1))^(5/2))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1
)^2)/(a*x + 1)^2 - (a*(a*x - 1)^3)/(a*x + 1)^3) - (c*atanh(((a*x - 1)/(a*x
+ 1))^(1/2)))/a
```

3.594 $\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=16

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

```
[Out] -1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6318}

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]
[Out] -(1/(a*c*E^ArcCoth[a*x]))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx=-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Mathematica [A]
time = 0.04, size = 16, normalized size = 1.00

$$-\frac{e^{-\coth^{-1}(ax)}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)),x]
[Out] -(1/(a*c*E^ArcCoth[a*x]))
```

Maple [A]
time = 0.18, size = 24, normalized size = 1.50

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$	24
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$	24
trager	$-\frac{\sqrt{-\frac{-ax+1}{ax+1}}}{ac}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/c*((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.44

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Fricas [A]
time = 0.34, size = 23, normalized size = 1.44

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] -sqrt((a*x - 1)/(a*x + 1))/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.03, size = 23, normalized size = 1.44

$$-\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2),x)
```

```
[Out] -((a*x - 1)/(a*x + 1))^(1/2)/(a*c)
```

3.595 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}+\frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2\,(1-a^2x^2)}$$

```
[Out] -2/3/a/c^2*((a*x-1)/(a*x+1))^(1/2)+1/3*(2*a*x+1)/a/c^2*((a*x-1)/(a*x+1))^(1
/2)/(-a^2*x^2+1)
```

Rubi [A]

time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6320, 6318}

$$\frac{(2ax+1)e^{-\coth^{-1}(ax)}}{3ac^2\,(1-a^2x^2)}-\frac{2e^{-\coth^{-1}(ax)}}{3ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
```

```
[Out] -2/(3*a*c^2*E^ArcCoth[a*x]) + (1 + 2*a*x)/(3*a*c^2*E^ArcCoth[a*x]*(1 - a^2*
x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2(1-a^2x^2)} + \frac{2\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{3c}$$
$$= -\frac{2e^{-\coth^{-1}(ax)}}{3ac^2} + \frac{e^{-\coth^{-1}(ax)}(1+2ax)}{3ac^2(1-a^2x^2)}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.87

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+2ax+2a^2x^2)}{3(-1+ax)(c+acx)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^2),x]
[Out] -1/3*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + 2*a*x + 2*a^2*x^2))/((-1 + a*x)*(c + a*
c*x)^2)
```

Maple [A]
time = 0.20, size = 52, normalized size = 0.95

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2+2ax-1)}{3(a^2x^2-1)a\,c^2}$	49
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(2a^2x^2+2ax-1)}{3c^2(ax+1)(ax-1)a}$	52
trager	$-\frac{(2a^2x^2+2ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3a\,c^2(ax-1)(ax+1)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -1/3*((a*x-1)/(a*x+1))^(1/2)*(2*a^2*x^2+2*a*x-1)/c^2/(a*x+1)/(a*x-1)/a
```

Maxima [A]
time = 0.27, size = 67, normalized size = 1.22

$$\frac{1}{12}\,a\left(\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-6\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}-\frac{3}{a^2c^2\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*((((a*x - 1)/(a*x + 1))^(3/2) - 6*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^
2) - 3/(a^2*c^2*sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A]
time = 0.36, size = 50, normalized size = 0.91

$$-\frac{(2\,a^2x^2+2\,ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a^2*x^2 + 2*a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 - a*c^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x**2 + 1), x
)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^2, x)
```

Mupad [B]
time = 0.05, size = 55, normalized size = 1.00

$$-\frac{\frac{6\,(a\,x-1)}{a\,x+1}-\frac{(a\,x-1)^2}{(a\,x+1)^2}+3}{12\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^2,x)
[Out] -((6*(a*x - 1))/(a*x + 1) - (a*x - 1)^2/(a*x + 1)^2 + 3)/(12*a*c^2*((a*x -
1)/(a*x + 1))^(1/2))
```

3.596 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3} \, dx$

Optimal. Leaf size=91

$$-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}+\frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3\left(1-a^2x^2\right)^2}+\frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3\left(1-a^2x^2\right)}$$

```
[Out] -8/15/a/c^3*((a*x-1)/(a*x+1))^(1/2)+1/15*(4*a*x+1)/a/c^3*((a*x-1)/(a*x+1))^
(1/2)/(-a^2*x^2+1)^2+4/15*(2*a*x+1)/a/c^3*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^2
+1)
```

Rubi [A]

time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\frac{4(2ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)}+\frac{(4ax+1)e^{-\coth^{-1}(ax)}}{15ac^3\left(1-a^2x^2\right)^2}-\frac{8e^{-\coth^{-1}(ax)}}{15ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -8/(15*a*c^3*E^ArcCoth[a*x]) + (1 + 4*a*x)/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^
2*x^2)^2) + (4*(1 + 2*a*x))/(15*a*c^3*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2} + \frac{4\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{5c}$$
$$= \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2} + \frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3(1-a^2x^2)} + \frac{8\int \frac{e^{-\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{15c^2}$$
$$= -\frac{8e^{-\coth^{-1}(ax)}}{15ac^3} + \frac{e^{-\coth^{-1}(ax)}(1+4ax)}{15ac^3(1-a^2x^2)^2} + \frac{4e^{-\coth^{-1}(ax)}(1+2ax)}{15ac^3(1-a^2x^2)}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 0.70

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(3-12ax-12a^2x^2+8a^3x^3+8a^4x^4)}{15(-1+ax)^2(c+acx)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/15*(Sqrt[1 - 1/(a^2*x^2)]*x*(3 - 12*a*x - 12*a^2*x^2 + 8*a^3*x^3 + 8*a^4
*x^4))/((-1 + a*x)^2*(c + a*c*x)^3)
```

Maple [A]
time = 0.21, size = 68, normalized size = 0.75

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)}{15(a^2x^2-1)^2c^3a}$	65
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)}{15c^3(ax+1)^2a(ax-1)^2}$	68
trager	$-\frac{(8a^4x^4+8a^3x^3-12a^2x^2-12ax+3)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^3(ax-1)^2(ax+1)^2}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*((a*x-1)/(a*x+1))^(1/2)*(8*a^4*x^4+8*a^3*x^3-12*a^2*x^2-12*a*x+3)/c^3
/(a*x+1)^2/a/(a*x-1)^2
```

Maxima [A]
time = 0.27, size = 102, normalized size = 1.12

$$-\frac{1}{240}\,a\left(\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-20\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+90\sqrt{\frac{ax-1}{ax+1}}}{a^{2}c^{3}}+\frac{5\left(\frac{12\,(ax-1)}{ax+1}-1\right)}{a^{2}c^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/240*a*((3*((a*x - 1)/(a*x + 1))^(5/2) - 20*((a*x - 1)/(a*x + 1))^(3/2) +
 90*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 5*(12*(a*x - 1)/(a*x + 1) - 1)/(
a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A]
time = 0.35, size = 76, normalized size = 0.84

$$-\frac{\left(8\,a^{4}x^{4}+8\,a^{3}x^{3}-12\,a^{2}x^{2}-12\,ax+3\right)\sqrt{\frac{ax-1}{ax+1}}}{15\left(a^{5}c^{3}x^{4}-2\,a^{3}c^{3}x^{2}+ac^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(8*a^4*x^4 + 8*a^3*x^3 - 12*a^2*x^2 - 12*a*x + 3)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^{6}x^{6}-3a^{4}x^{4}+3a^{2}x^{2}-1}\,dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x**4 + 3*a*
*2*x**2 - 1), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^3, x)
```

Mupad [B]
time = 1.24, size = 109, normalized size = 1.20

$$\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{12\,a\,c^3}-\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3}-\frac{\frac{4\,(a\,x-1)}{a\,x+1}-\frac{1}{3}}{16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^3,x)
[Out] ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^3) - (3*((a*x - 1)/(a*x + 1))^(1/2))/(8
*a*c^3) - ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) - ((4*(a*x - 1))/(a*x + 1)
 - 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2))
```

3.597 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=127

$$-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}+\frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4\left(1-a^2x^2\right)^3}+\frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4\left(1-a^2x^2\right)^2}+\frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4\left(1-a^2x^2\right)}$$

```
[Out] -16/35/a/c^4*((a*x-1)/(a*x+1))^(1/2)+1/35*(6*a*x+1)/a/c^4*((a*x-1)/(a*x+1))
^(1/2)/(-a^2*x^2+1)^3+2/35*(4*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^
2+1)^2+8/35*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(1/2)/(-a^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\frac{8(2ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)}+\frac{2(4ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^2}+\frac{(6ax+1)e^{-\coth^{-1}(ax)}}{35ac^4\left(1-a^2x^2\right)^3}-\frac{16e^{-\coth^{-1}(ax)}}{35ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -16/(35*a*c^4*E^ArcCoth[a*x]) + (1 + 6*a*x)/(35*a*c^4*E^ArcCoth[a*x]*(1 - a
^2*x^2)^3) + (2*(1 + 4*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2)^2) + (8
*(1 + 2*a*x))/(35*a*c^4*E^ArcCoth[a*x]*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{6\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{7c}$$

$$= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{24\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{35c^2}$$

$$= \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{8e^{-\coth^{-1}(ax)}(1+2ax)}{35ac^4(1-a^2x^2)} + \frac{16\int \frac{e^{-\coth^{-1}}}{c-a^2c}}{35c^3}$$

$$= -\frac{16e^{-\coth^{-1}(ax)}}{35ac^4} + \frac{e^{-\coth^{-1}(ax)}(1+6ax)}{35ac^4(1-a^2x^2)^3} + \frac{2e^{-\coth^{-1}(ax)}(1+4ax)}{35ac^4(1-a^2x^2)^2} + \frac{8e^{-\coth^{-1}(ax)}(1+2}{35ac^4(1-a^2x^2}$$

Mathematica [A]
time = 0.09, size = 80, normalized size = 0.63

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-5+30ax+30a^2x^2-40a^3x^3-40a^4x^4+16a^5x^5+16a^6x^6)}{35(-1+ax)^3(c+acx)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^4),x]
```

```
[Out] -1/35*(Sqrt[1 - 1/(a^2*x^2)]*x*(-5 + 30*a*x + 30*a^2*x^2 - 40*a^3*x^3 - 40*
a^4*x^4 + 16*a^5*x^5 + 16*a^6*x^6))/((-1 + a*x)^3*(c + a*c*x)^4)
```

Maple [A]
time = 0.22, size = 84, normalized size = 0.66

method	result	size
gosper	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5)}{35(a^2x^2-1)^3c^4a}$	81
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5)}{35c^4(ax+1)^3(ax-1)^3a}$	84
trager	$-\frac{(16a^6x^6+16a^5x^5-40a^4x^4-40a^3x^3+30a^2x^2+30ax-5)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^4(ax-1)^3(ax+1)^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*((a*x-1)/(a*x+1))^(1/2)*(16*a^6*x^6+16*a^5*x^5-40*a^4*x^4-40*a^3*x^3+
30*a^2*x^2+30*a*x-5)/c^4/(a*x+1)^3/(a*x-1)^3/a
```

Maxima [A]
time = 0.26, size = 135, normalized size = 1.06

$$\frac{1}{2240}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+175\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-700\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{7\left(\frac{10\,(ax-1)}{ax+1}-\frac{75\,(ax-1)^2}{(ax+1)^2}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/2240*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 42*((a*x - 1)/(a*x + 1))^(5/2) +
 175*((a*x - 1)/(a*x + 1))^(3/2) - 700*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^4)
 + 7*(10*(a*x - 1)/(a*x + 1) - 75*(a*x - 1)^2/(a*x + 1)^2 - 1)/(a^2*c^4*((a
*x - 1)/(a*x + 1))^(5/2)))
```

Fricas [A]
time = 0.35, size = 104, normalized size = 0.82

$$-\frac{\left(16\,a^6x^6+16\,a^5x^5-40\,a^4x^4-40\,a^3x^3+30\,a^2x^2+30\,ax-5\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] -1/35*(16*a^6*x^6 + 16*a^5*x^5 - 40*a^4*x^4 - 40*a^3*x^3 + 30*a^2*x^2 + 30*
a*x - 5)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4
*x^2 - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x**6 + 6*a**
4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x^2 - c)^4, x)
```

Mupad [B]
time = 0.04, size = 148, normalized size = 1.17

$$\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{64\,a\,c^4}-\frac{5\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{160\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{448\,a\,c^4}-\frac{\frac{15\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{2\,\left(a\,x-1\right)}{a\,x+1}+\frac{1}{5}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^4,x)
```

```
[Out] (5*((a*x - 1)/(a*x + 1))^(3/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(1/2)
)/(16*a*c^4) - (3*((a*x - 1)/(a*x + 1))^(5/2))/(160*a*c^4) + ((a*x - 1)/(a*
x + 1))^(7/2)/(448*a*c^4) - ((15*(a*x - 1)^2)/(a*x + 1)^2 - (2*(a*x - 1))/(
a*x + 1) + 1/5)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2))
```

3.598 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal. Leaf size=73

$$\frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}$$

```
[Out] 4/3*c^4*(-a*x+1)^6/a-12/7*c^4*(-a*x+1)^7/a+3/4*c^4*(-a*x+1)^8/a-1/9*c^4*(-a
*x+1)^9/a
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6302, 6275, 45}

$$-\frac{c^4(1-ax)^9}{9a}+\frac{3c^4(1-ax)^8}{4a}-\frac{12c^4(1-ax)^7}{7a}+\frac{4c^4(1-ax)^6}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^4*(1 - a*x)^6)/(3*a) - (12*c^4*(1 - a*x)^7)/(7*a) + (3*c^4*(1 - a*x)^8
)/(4*a) - (c^4*(1 - a*x)^9)/(9*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx$$
$$= -\left(c^4\int(1-ax)^5(1+ax)^3\,dx\right)$$
$$= -\left(c^4\int\left(8(1-ax)^5-12(1-ax)^6+6(1-ax)^7-(1-ax)^8\right)\,dx\right)$$
$$= \frac{4c^4(1-ax)^6}{3a}-\frac{12c^4(1-ax)^7}{7a}+\frac{3c^4(1-ax)^8}{4a}-\frac{c^4(1-ax)^9}{9a}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.53

$$\frac{c^4(-1+ax)^6\left(65+138ax+105a^2x^2+28a^3x^3\right)}{252a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(2*ArcCoth[a*x]),x]
[Out] (c^4*(-1 + a*x)^6*(65 + 138*a*x + 105*a^2*x^2 + 28*a^3*x^3))/(252*a)
```

Maple [A]
time = 0.24, size = 61, normalized size = 0.84

method	result
gosper	$\frac{c^4x\left(28a^8x^8-63a^7x^7-72a^6x^6+252a^5x^5-378a^3x^3+168a^2x^2+252ax-252\right)}{252}$
default	$c^4\left(\frac{1}{9}a^8x^9-\frac{1}{4}a^7x^8-\frac{2}{7}a^6x^7+a^5x^6-\frac{3}{2}a^3x^4+\frac{2}{3}a^2x^3+a\,x^2-x\right)$
norman	$a^5c^4x^6+c^4a\,x^2-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
risch	$a^5c^4x^6+c^4a\,x^2-c^4x+\frac{2}{3}a^2c^4x^3-\frac{3}{2}a^3c^4x^4-\frac{2}{7}a^6c^4x^7-\frac{1}{4}a^7c^4x^8+\frac{1}{9}a^8c^4x^9$
meijerg	$\frac{c^4\left(\frac{ax\left(280a^8x^8-315a^7x^7+360a^6x^6-420a^5x^5+504a^4x^4-630a^3x^3+840a^2x^2-1260ax+2520\right)}{2520}-\ln(ax+1)\right)}{a}-\frac{c^4\left(-\frac{ax\left(-315a^7x^7+360a^6x^6-4\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^4*(1/9*a^8*x^9-1/4*a^7*x^8-2/7*a^6*x^7+a^5*x^6-3/2*a^3*x^4+2/3*a^2*x^3+a*
x^2-x)
```

Maxima [A]
time = 0.26, size = 80, normalized size = 1.10

$$\frac{1}{9}\,a^8c^4x^9-\frac{1}{4}\,a^7c^4x^8-\frac{2}{7}\,a^6c^4x^7+a^5c^4x^6-\frac{3}{2}\,a^3c^4x^4+\frac{2}{3}\,a^2c^4x^3+ac^4x^2-c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Fricas [A]
time = 0.32, size = 80, normalized size = 1.10

$$\frac{1}{9}a^8c^4x^9 - \frac{1}{4}a^7c^4x^8 - \frac{2}{7}a^6c^4x^7 + a^5c^4x^6 - \frac{3}{2}a^3c^4x^4 + \frac{2}{3}a^2c^4x^3 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Sympy [A]
time = 0.04, size = 87, normalized size = 1.19

$$\frac{a^8c^4x^9}{9} - \frac{a^7c^4x^8}{4} - \frac{2a^6c^4x^7}{7} + a^5c^4x^6 - \frac{3a^3c^4x^4}{2} + \frac{2a^2c^4x^3}{3} + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] a**8*c**4*x**9/9 - a**7*c**4*x**8/4 - 2*a**6*c**4*x**7/7 + a**5*c**4*x**6 -
 3*a**3*c**4*x**4/2 + 2*a**2*c**4*x**3/3 + a*c**4*x**2 - c**4*x
```

Giac [A]
time = 0.41, size = 80, normalized size = 1.10

$$\frac{1}{9}a^8c^4x^9 - \frac{1}{4}a^7c^4x^8 - \frac{2}{7}a^6c^4x^7 + a^5c^4x^6 - \frac{3}{2}a^3c^4x^4 + \frac{2}{3}a^2c^4x^3 + ac^4x^2 - c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/9*a^8*c^4*x^9 - 1/4*a^7*c^4*x^8 - 2/7*a^6*c^4*x^7 + a^5*c^4*x^6 - 3/2*a^3
*c^4*x^4 + 2/3*a^2*c^4*x^3 + a*c^4*x^2 - c^4*x
```

Mupad [B]
time = 0.05, size = 80, normalized size = 1.10

$$\frac{a^8\,c^4\,x^9}{9} - \frac{a^7\,c^4\,x^8}{4} - \frac{2\,a^6\,c^4\,x^7}{7} + a^5\,c^4\,x^6 - \frac{3\,a^3\,c^4\,x^4}{2} + \frac{2\,a^2\,c^4\,x^3}{3} + a\,c^4\,x^2 - c^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] a*c^4*x^2 - c^4*x + (2*a^2*c^4*x^3)/3 - (3*a^3*c^4*x^4)/2 + a^5*c^4*x^6 - (
2*a^6*c^4*x^7)/7 - (a^7*c^4*x^8)/4 + (a^8*c^4*x^9)/9
```

3.599 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=55

$$\frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}$$

```
[Out] 4/5*c^3*(-a*x+1)^5/a-2/3*c^3*(-a*x+1)^6/a+1/7*c^3*(-a*x+1)^7/a
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6302, 6275, 45}

$$\frac{c^3(1-ax)^7}{7a}-\frac{2c^3(1-ax)^6}{3a}+\frac{4c^3(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (4*c^3*(1 - a*x)^5)/(5*a) - (2*c^3*(1 - a*x)^6)/(3*a) + (c^3*(1 - a*x)^7)/(
7*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx\\
&= -\left(c^3\int (1-ax)^4(1+ax)^2\,dx\right)\\
&= -\left(c^3\int \left(4(1-ax)^4-4(1-ax)^5+(1-ax)^6\right)\,dx\right)\\
&= \frac{4c^3(1-ax)^5}{5a}-\frac{2c^3(1-ax)^6}{3a}+\frac{c^3(1-ax)^7}{7a}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.56

$$
-\frac{c^3(-1+ax)^5\left(29+40ax+15a^2x^2\right)}{105a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(2*ArcCoth[a*x]),x]
[Out] -1/105*(c^3*(-1 + a*x)^5*(29 + 40*a*x + 15*a^2*x^2))/a
```

Maple [A]
time = 0.19, size = 54, normalized size = 0.98

method	result
gosper	$-\frac{c^3x\left(15a^6x^6-35a^5x^5-21a^4x^4+105a^3x^3-35a^2x^2-105ax+105\right)}{105}$
default	$c^3\left(-\frac{1}{7}a^6x^7+\frac{1}{3}a^5x^6+\frac{1}{5}a^4x^5-a^3x^4+\frac{1}{3}a^2x^3+a\,x^2-x\right)$
norman	$c^3a\,x^2-c^3x+\frac{1}{3}a^2c^3x^3-a^3c^3x^4+\frac{1}{5}a^4c^3x^5+\frac{1}{3}a^5c^3x^6-\frac{1}{7}a^6c^3x^7$
risch	$c^3a\,x^2-c^3x+\frac{1}{3}a^2c^3x^3-a^3c^3x^4+\frac{1}{5}a^4c^3x^5+\frac{1}{3}a^5c^3x^6-\frac{1}{7}a^6c^3x^7$
meijerg	$-\frac{c^3\left(\frac{ax\left(120a^6x^6-140a^5x^5+168a^4x^4-210a^3x^3+280a^2x^2-420ax+840\right)}{840}-\ln(ax+1)\right)}{a}+\frac{c^3\left(-\frac{ax\left(-70a^5x^5+84a^4x^4-105a^3x^3+140a^2x^2-\right.}{420}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^3*(-1/7*a^6*x^7+1/3*a^5*x^6+1/5*a^4*x^5-a^3*x^4+1/3*a^2*x^3+a*x^2-x)
```

Maxima [A]
time = 0.26, size = 70, normalized size = 1.27

$$
-\frac{1}{7}\,a^6c^3x^7+\frac{1}{3}\,a^5c^3x^6+\frac{1}{5}\,a^4c^3x^5-a^3c^3x^4+\frac{1}{3}\,a^2c^3x^3+ac^3x^2-c^3x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Fricas [A]
time = 0.33, size = 70, normalized size = 1.27

$$-\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Sympy [A]
time = 0.03, size = 70, normalized size = 1.27

$$-\frac{a^6c^3x^7}{7} + \frac{a^5c^3x^6}{3} + \frac{a^4c^3x^5}{5} - a^3c^3x^4 + \frac{a^2c^3x^3}{3} + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**6*c**3*x**7/7 + a**5*c**3*x**6/3 + a**4*c**3*x**5/5 - a**3*c**3*x**4 +
a**2*c**3*x**3/3 + a*c**3*x**2 - c**3*x
```

Giac [A]
time = 0.39, size = 70, normalized size = 1.27

$$-\frac{1}{7}a^6c^3x^7 + \frac{1}{3}a^5c^3x^6 + \frac{1}{5}a^4c^3x^5 - a^3c^3x^4 + \frac{1}{3}a^2c^3x^3 + ac^3x^2 - c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/7*a^6*c^3*x^7 + 1/3*a^5*c^3*x^6 + 1/5*a^4*c^3*x^5 - a^3*c^3*x^4 + 1/3*a^
2*c^3*x^3 + a*c^3*x^2 - c^3*x
```

Mupad [B]
time = 0.04, size = 70, normalized size = 1.27

$$-\frac{a^6\,c^3\,x^7}{7} + \frac{a^5\,c^3\,x^6}{3} + \frac{a^4\,c^3\,x^5}{5} - a^3\,c^3\,x^4 + \frac{a^2\,c^3\,x^3}{3} + a\,c^3\,x^2 - c^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] a*c^3*x^2 - c^3*x + (a^2*c^3*x^3)/3 - a^3*c^3*x^4 + (a^4*c^3*x^5)/5 + (a^5*
c^3*x^6)/3 - (a^6*c^3*x^7)/7
```

3.600 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal. Leaf size=37

$$\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

[Out] 1/2*c^2*(-a*x+1)^4/a-1/5*c^2*(-a*x+1)^5/a

Rubi [A]
time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6302, 6275, 45}

$$\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

Antiderivative was successfully verified.

[In] Int[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]

[Out] (c^2*(1 - a*x)^4)/(2*a) - (c^2*(1 - a*x)^5)/(5*a)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$$
$$=-\left(c^2\int(1-ax)^3(1+ax)\,dx\right)$$
$$=-\left(c^2\int\left(2(1-ax)^3-(1-ax)^4\right)\,dx\right)$$
$$=\frac{c^2(1-ax)^4}{2a}-\frac{c^2(1-ax)^5}{5a}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.81

$$\frac{1}{10}c^2x\left(-10+10ax-5a^3x^3+2a^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*x*(-10 + 10*a*x - 5*a^3*x^3 + 2*a^4*x^4))/10
```

Maple [A]
time = 0.18, size = 30, normalized size = 0.81

method	result
gosper	$\frac{c^2x\left(2a^4x^4-5a^3x^3+10ax-10\right)}{10}$
default	$c^2\left(\frac{1}{5}a^4x^5-\frac{1}{2}a^3x^4+a\,x^2-x\right)$
norman	$a\,c^2x^2-c^2x-\frac{1}{2}a^3c^2x^4+\frac{1}{5}a^4c^2x^5$
risch	$a\,c^2x^2-c^2x-\frac{1}{2}a^3c^2x^4+\frac{1}{5}a^4c^2x^5$
meijerg	$\frac{c^2\left(\frac{ax\left(12a^4x^4-15a^3x^3+20a^2x^2-30ax+60\right)}{60}-\ln(ax+1)\right)}{a}-\frac{c^2\left(-\frac{ax\left(-15a^3x^3+20a^2x^2-30ax+60\right)}{60}+\ln(ax+1)\right)}{a}-\frac{2c^2\left(\frac{ax\left(4a^2x^2-6ax\right.}{12}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/5*a^4*x^5-1/2*a^3*x^4+a*x^2-x)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 1.00

$$\frac{1}{5}\,a^4c^2x^5-\frac{1}{2}\,a^3c^2x^4+ac^2x^2-c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Fricas [A]
time = 0.33, size = 37, normalized size = 1.00

$$\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Sympy [A]
time = 0.03, size = 36, normalized size = 0.97

$$\frac{a^4c^2x^5}{5} - \frac{a^3c^2x^4}{2} + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*(a*x-1)/(a*x+1),x)
[Out] a**4*c**2*x**5/5 - a**3*c**2*x**4/2 + a*c**2*x**2 - c**2*x
```

Giac [A]
time = 0.40, size = 37, normalized size = 1.00

$$\frac{1}{5}a^4c^2x^5 - \frac{1}{2}a^3c^2x^4 + ac^2x^2 - c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/5*a^4*c^2*x^5 - 1/2*a^3*c^2*x^4 + a*c^2*x^2 - c^2*x
```

Mupad [B]
time = 0.05, size = 37, normalized size = 1.00

$$\frac{a^4\,c^2\,x^5}{5} - \frac{a^3\,c^2\,x^4}{2} + a\,c^2\,x^2 - c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^2*(a*x - 1))/(a*x + 1),x)
[Out] a*c^2*x^2 - c^2*x - (a^3*c^2*x^4)/2 + (a^4*c^2*x^5)/5
```

3.601 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=16

$$\frac{c(1-ax)^3}{3a}$$

```
[Out] 1/3*c*(-a*x+1)^3/a
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6302, 6275, 32}

$$\frac{c(1-ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*(1 - a*x)^3)/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)\,dx\\ &= -\left(c\int (1-ax)^2\,dx\right)\\ &= \frac{c(1-ax)^3}{3a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.38

$$-cx + acx^2 - \frac{1}{3}a^2cx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)/E^(2*ArcCoth[a*x]),x]
[Out] -(c*x) + a*c*x^2 - (a^2*c*x^3)/3
```

Maple [A]
time = 0.10, size = 14, normalized size = 0.88

method	result	size
default	$-\frac{c(ax-1)^3}{3a}$	14
gosper	$-\frac{cx(a^2x^2-3ax+3)}{3}$	18
norman	$ac\,x^2 - cx - \frac{1}{3}a^2c\,x^3$	21
risch	$-\frac{a^2c\,x^3}{3} + ac\,x^2 - cx + \frac{c}{3a}$	27
meijerg	$-\frac{c\left(\frac{ax\left(4a^2x^2-6ax+12\right)}{12}-\ln(ax+1)\right)}{a} + \frac{c\left(-\frac{ax(-3ax+6)}{6}+\ln(ax+1)\right)}{a} + \frac{c(ax-\ln(ax+1))}{a} - \frac{c\ln(ax+1)}{a}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] -1/3*c*(a*x-1)^3/a
```

Maxima [A]
time = 0.27, size = 20, normalized size = 1.25

$$-\frac{1}{3}a^2cx^3 + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Fricas [A]
time = 0.34, size = 20, normalized size = 1.25

$$-\frac{1}{3}a^2cx^3 + acx^2 - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Sympy [**A**]
time = 0.01, size = 19, normalized size = 1.19

$$-\frac{a^2cx^3}{3}+acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**2*c*x**3/3 + a*c*x**2 - c*x
```

Giac [**A**]
time = 0.40, size = 20, normalized size = 1.25

$$-\frac{1}{3}\,a^2cx^3+acx^2-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/3*a^2*c*x^3 + a*c*x^2 - c*x
```

Mupad [**B**]
time = 0.04, size = 17, normalized size = 1.06

$$-\frac{c\,x\,\left(a^2\,x^2-3\,a\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] -(c*x*(a^2*x^2 - 3*a*x + 3))/3
```

3.602 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=14

$$\frac{1}{ac(1+ax)}$$

[Out] 1/a/c/(a*x+1)

Rubi [A]
time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6302, 6275, 32}

$$\frac{1}{ac(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]

[Out] 1/(a*c*(1 + a*x))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{c-a^2cx^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-a^2cx^2}\,dx\\ &= -\frac{\int \frac{1}{(1+ax)^2}\,dx}{c}\\ &= \frac{1}{ac(1+ax)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.
time = 0.01, size = 18, normalized size = 1.29

$$-\frac{e^{-2\coth^{-1}(ax)}}{2ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/2*1/(a*c*E^(2*ArcCoth[a*x]))
```

Maple [A]
time = 0.14, size = 15, normalized size = 1.07

method	result	size
norman	$-\frac{x}{c(ax+1)}$	14
gosper	$\frac{1}{ac(ax+1)}$	15
default	$\frac{1}{ac(ax+1)}$	15
risch	$\frac{1}{ac(ax+1)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/c/(a*x+1)
```

Maxima [A]
time = 0.25, size = 12, normalized size = 0.86

$$\frac{1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/(a^2*c*x + a*c)
```

Fricas [A]
time = 0.34, size = 12, normalized size = 0.86

$$\frac{1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/(a^2*c*x + a*c)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.71

$$\frac{1}{a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c),x)
```

```
[Out] 1/(a**2*c*x + a*c)
```

Giac [A]
time = 0.41, size = 14, normalized size = 1.00

$$\frac{1}{(ax+1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 1/((a*x + 1)*a*c)
```

Mupad [B]
time = 0.05, size = 12, normalized size = 0.86

$$\frac{1}{a\ (c+a\,c\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)*(a*x + 1)),x)
```

```
[Out] 1/(a*(c + a*c*x))
```

3.603 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

```
[Out] 1/4/a/c^2/(a*x+1)^2+1/4/a/c^2/(a*x+1)-1/4*arctanh(a*x)/a/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6275, 46, 213}

$$\frac{1}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}-\frac{\tanh^{-1}(ax)}{4ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 1/(4*a*c^2*(1 + a*x)^2) + 1/(4*a*c^2*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)(1+ax)^3}\,dx}{c^2}\\
&= -\frac{\int \left(\frac{1}{2(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^2}\\
&= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^2}\\
&= \frac{1}{4ac^2(1+ax)^2}+\frac{1}{4ac^2(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.67

$$\frac{2+ax-(1+ax)^2\tanh^{-1}(ax)}{4a(c+acx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
[Out] (2 + a*x - (1 + a*x)^2*ArcTanh[a*x])/(4*a*(c + a*c*x)^2)
```

Maple [A]
time = 0.14, size = 52, normalized size = 1.06

method	result	size
risch	$\frac{\frac{x}{4}+\frac{1}{2a}}{(ax+1)^2c^2}+\frac{\ln(-ax+1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	51
default	$\frac{\frac{1}{4a(ax+1)^2}+\frac{1}{4a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{\ln(ax-1)}{8a}}{c^2}$	52
norman	$\frac{-\frac{3}{4ac}+\frac{a\,x^2}{2c}+\frac{a^2x^3}{4c}}{(ax-1)c(ax+1)^2}+\frac{\ln(ax-1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/c^2*(1/4/a/(a*x+1)^2+1/4/a/(a*x+1)-1/8/a*ln(a*x+1)+1/8/a*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 63, normalized size = 1.29

$$\frac{ax+2}{4\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}-\frac{\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) - 1/8*log(a*x + 1)/(a*c^2
) + 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A]
time = 0.34, size = 76, normalized size = 1.55

$$\frac{2\,ax-(a^2x^2+2\,ax+1)\log\left(ax+1\right)+(a^2x^2+2\,ax+1)\log\left(ax-1\right)+4}{8\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + (a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 4)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [A]
time = 0.14, size = 54, normalized size = 1.10

$$\frac{ax+2}{4a^3c^2x^2+8a^2c^2x+4ac^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (a*x + 2)/(4*a**3*c**2*x**2 + 8*a**2*c**2*x + 4*a*c**2) + (log(x - 1/a)/8 -
 log(x + 1/a)/8)/(a*c**2)
```

Giac [A]
time = 0.41, size = 51, normalized size = 1.04

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}+\frac{ax+2}{4\,(ax+1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) + 1/4*(a*x +
 2)/((a*x + 1)^2*a*c^2)
```

Mupad [B]
time = 0.07, size = 46, normalized size = 0.94

$$\frac{\frac{x}{4}+\frac{1}{2a}}{a^2c^2x^2+2ac^2x+c^2}-\frac{\operatorname{atanh}(ax)}{4ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^2*(a*x + 1)),x)
```

```
[Out] (x/4 + 1/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) - atanh(a*x)/(4*a*c^2)
```

3.604 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

```
[Out] -1/16/a/c^3/(-a*x+1)+1/12/a/c^3/(a*x+1)^3+1/8/a/c^3/(a*x+1)^2+3/16/a/c^3/(a
*x+1)-1/4*arctanh(a*x)/a/c^3
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6275, 46, 213}

$$-\frac{1}{16ac^3(1-ax)}+\frac{3}{16ac^3(ax+1)}+\frac{1}{8ac^3(ax+1)^2}+\frac{1}{12ac^3(ax+1)^3}-\frac{\tanh^{-1}(ax)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] -1/16*1/(a*c^3*(1 - a*x)) + 1/(12*a*c^3*(1 + a*x)^3) + 1/(8*a*c^3*(1 + a*x)
^2) + 3/(16*a*c^3*(1 + a*x)) - ArcTanh[a*x]/(4*a*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx\\
&= -\frac{\int \frac{1}{(1-ax)^2(1+ax)^4}\,dx}{c^3}\\
&= -\frac{\int \left(\frac{1}{16(-1+ax)^2}+\frac{1}{4(1+ax)^4}+\frac{1}{4(1+ax)^3}+\frac{3}{16(1+ax)^2}-\frac{1}{4(-1+a^2x^2)}\right)dx}{c^3}\\
&= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}+\frac{\int \frac{1}{-1+a^2x^2}\,dx}{4c^3}\\
&= -\frac{1}{16ac^3(1-ax)}+\frac{1}{12ac^3(1+ax)^3}+\frac{1}{8ac^3(1+ax)^2}+\frac{3}{16ac^3(1+ax)}-\frac{\tanh^{-1}(ax)}{4ac^3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.73

$$\frac{-4+ax+6a^2x^2+3a^3x^3-3(-1+ax)(1+ax)^3\tanh^{-1}(ax)}{12a(-1+ax)(c+acx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
[Out] (-4 + a*x + 6*a^2*x^2 + 3*a^3*x^3 - 3*(-1 + a*x)*(1 + a*x)^3*ArcTanh[a*x])/
(12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A]
time = 0.14, size = 76, normalized size = 0.90

method	result	size
default	$\frac{\frac{1}{12a(ax+1)^3}+\frac{1}{8a(ax+1)^2}+\frac{3}{16a(ax+1)}-\frac{\ln(ax+1)}{8a}+\frac{1}{16a(ax-1)}+\frac{\ln(ax-1)}{8a}}{c^3}$	76
risch	$\frac{\frac{a^2x^3}{4}+\frac{a\,x^2}{2}+\frac{x}{12}-\frac{1}{3a}}{(ax+1)^2(a^2x^2-1)c^3}-\frac{\ln(ax+1)}{8a\,c^3}+\frac{\ln(-ax+1)}{8a\,c^3}$	76
norman	$\frac{-\frac{3x}{4c}+\frac{a\,x^2}{4c}+\frac{11a^2x^3}{12c}-\frac{a^3x^4}{12c}-\frac{a^4x^5}{3c}}{(ax-1)^2(ax+1)^3c^2}+\frac{\ln(ax-1)}{8c^3a}-\frac{\ln(ax+1)}{8a\,c^3}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/12/a/(a*x+1)^3+1/8/a/(a*x+1)^2+3/16/a/(a*x+1)-1/8/a*ln(a*x+1)+1/16
/a/(a*x-1)+1/8/a*ln(a*x-1))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.08

$$\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}-\frac{\log\left(ax+1\right)}{8\,ac^3}+\frac{\log\left(ax-1\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^3*x^3 + 6*a^2*x^2 + a*x - 4)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2
*c^3*x - a*c^3) - 1/8*log(a*x + 1)/(a*c^3) + 1/8*log(a*x - 1)/(a*c^3)
```

Fricas [A]
time = 0.37, size = 121, normalized size = 1.44

$$\frac{6\,a^3x^3+12\,a^2x^2+2\,ax-3\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(ax+1\right)+3\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(ax-1\right)-8}{24\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 + 12*a^2*x^2 + 2*a*x - 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*
log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) - 8)/(a^5*c
^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [A]
time = 0.22, size = 85, normalized size = 1.01

$$-\frac{-3a^3x^3-6a^2x^2-ax+4}{12a^5c^3x^4+24a^4c^3x^3-24a^2c^3x-12ac^3}-\frac{-\frac{\log\left(x-\frac{1}{a}\right)}{8}+\frac{\log\left(x+\frac{1}{a}\right)}{8}}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(-3*a**3*x**3 - 6*a**2*x**2 - a*x + 4)/(12*a**5*c**3*x**4 + 24*a**4*c**3*x
**3 - 24*a**2*c**3*x - 12*a*c**3) - (-log(x - 1/a)/8 + log(x + 1/a)/8)/(a*c
**3)
```

Giac [A]
time = 0.41, size = 74, normalized size = 0.88

$$-\frac{\log\left(|ax+1|\right)}{8\,ac^3}+\frac{\log\left(|ax-1|\right)}{8\,ac^3}+\frac{3\,a^3x^3+6\,a^2x^2+ax-4}{12\,(ax+1)^3(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*log(abs(a*x + 1))/(a*c^3) + 1/8*log(abs(a*x - 1))/(a*c^3) + 1/12*(3*a^
3*x^3 + 6*a^2*x^2 + a*x - 4)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

Mupad [B]
time = 1.26, size = 73, normalized size = 0.87

$$-\frac{\frac{x}{12}+\frac{a\,x^2}{2}-\frac{1}{3\,a}+\frac{a^2\,x^3}{4}}{-a^4\,c^3\,x^4-2\,a^3\,c^3\,x^3+2\,a\,c^3\,x+c^3}-\frac{\mathrm{atanh}(a\,x)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^3*(a*x + 1)),x)
```

```
[Out] - (x/12 + (a*x^2)/2 - 1/(3*a) + (a^2*x^3)/4)/(c^3 - 2*a^3*c^3*x^3 - a^4*c^3
*x^4 + 2*a*c^3*x) - atanh(a*x)/(4*a*c^3)
```

3.605 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+ax)^2}+\frac{5}{32ac^4(1+ax)}-\frac{15\tanh^{-1}(ax)}{64ac^4}$$

```
[Out] -1/64/a/c^4/(-a*x+1)^2-5/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4+1/16/a/c^4/
(a*x+1)^3+3/32/a/c^4/(a*x+1)^2+5/32/a/c^4/(a*x+1)-15/64*arctanh(a*x)/a/c^4
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6275, 46, 213}

$$-\frac{5}{64ac^4(1-ax)}+\frac{5}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{3}{32ac^4(ax+1)^2}+\frac{1}{16ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}-\frac{15\tanh^{-1}(ax)}{64ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] -1/64*1/(a*c^4*(1 - a*x)^2) - 5/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1 + a*x
)^4) + 1/(16*a*c^4*(1 + a*x)^3) + 3/(32*a*c^4*(1 + a*x)^2) + 5/(32*a*c^4*(1
 + a*x)) - (15*ArcTanh[a*x])/(64*a*c^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6275

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
 Dist[c^p, Int[(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a,
 c, d, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] || GtQ[c, 0])
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx\\ &= -\frac{\int \frac{1}{(1-ax)^3(1+ax)^5}\,dx}{c^4}\\ &= -\frac{\int \left(-\frac{1}{32(-1+ax)^3}+\frac{5}{64(-1+ax)^2}+\frac{1}{8(1+ax)^5}+\frac{3}{16(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{5}{32(1+ax)^2}-\frac{15}{64(-1+a^2x}\right.}{c^4}\\ &= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+}\\ &= -\frac{1}{64ac^4(1-ax)^2}-\frac{5}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}+\frac{1}{16ac^4(1+ax)^3}+\frac{3}{32ac^4(1+}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.67

$$\frac{16-17ax-50a^2x^2-10a^3x^3+30a^4x^4+15a^5x^5-15(-1+ax)^2(1+ax)^4\tanh^{-1}(ax)}{64a(-1+ax)^2(c+acx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
[Out] (16 - 17*a*x - 50*a^2*x^2 - 10*a^3*x^3 + 30*a^4*x^4 + 15*a^5*x^5 - 15*(-1 +
 a*x)^2*(1 + a*x)^4*ArcTanh[a*x])/(64*a*(-1 + a*x)^2*(c + a*c*x)^4)
```

Maple [A]
time = 0.16, size = 100, normalized size = 0.84

method	result	size
risch	$\frac{\frac{15a^4x^5}{64}+\frac{15a^3x^4}{32}-\frac{5a^2x^3}{32}-\frac{25a\,x^2}{32}-\frac{17x}{64}+\frac{1}{4a}}{(ax+1)^2(a^2x^2-1)^2c^4}-\frac{15\ln(ax+1)}{128a\,c^4}+\frac{15\ln(-ax+1)}{128a\,c^4}$	92
default	$\frac{\frac{1}{32a(ax+1)^4}+\frac{1}{16a(ax+1)^3}+\frac{3}{32a(ax+1)^2}+\frac{5}{32a(ax+1)}-\frac{15\ln(ax+1)}{128a}-\frac{1}{64a(ax-1)^2}+\frac{5}{64a(ax-1)}+\frac{15\ln(ax-1)}{128a}}{c^4}$	100
norman	$\frac{\frac{49x}{64c}-\frac{15a\,x^2}{64c}-\frac{11a^2x^3}{8c}+\frac{a^3x^4}{8c}+\frac{63a^4x^5}{64c}-\frac{a^5x^6}{64c}-\frac{a^6x^7}{4c}}{(ax-1)^3(ax+1)^4c^3}+\frac{15\ln(ax-1)}{128c^4a}-\frac{15\ln(ax+1)}{128a\,c^4}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/32/a/(a*x+1)^4+1/16/a/(a*x+1)^3+3/32/a/(a*x+1)^2+5/32/a/(a*x+1)-15
/128/a*ln(a*x+1)-1/64/a/(a*x-1)^2+5/64/a/(a*x-1)+15/128/a*ln(a*x-1))
```

Maxima [A]
time = 0.28, size = 140, normalized size = 1.18

$$\frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}-\frac{15\,\log{(ax+1)}}{128\,ac^4}+\frac{15\,\log{(ax-1)}}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/(a^7
*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^
2*c^4*x + a*c^4) - 15/128*log(a*x + 1)/(a*c^4) + 15/128*log(a*x - 1)/(a*c^4
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.
time = 0.37, size = 217, normalized size = 1.82

$$\frac{30\,a^5x^5+60\,a^4x^4-20\,a^3x^3-100\,a^2x^2-34\,ax-15\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\log{(ax+1)}+15\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\log{(ax-1)}+32}{128\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(30*a^5*x^5 + 60*a^4*x^4 - 20*a^3*x^3 - 100*a^2*x^2 - 34*a*x - 15*(a^
6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
 + 15*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log
(a*x - 1) + 32)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3
- a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A]
time = 0.32, size = 141, normalized size = 1.18

$$\frac{15a^5x^5+30a^4x^4-10a^3x^3-50a^2x^2-17ax+16}{64a^7c^4x^6+128a^6c^4x^5-64a^5c^4x^4-256a^4c^4x^3-64a^3c^4x^2+128a^2c^4x+64ac^4}+\frac{\frac{15\log{(x-\frac{1}{a})}}{128}-\frac{15\log{(x+\frac{1}{a})}}{128}}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (15*a**5*x**5 + 30*a**4*x**4 - 10*a**3*x**3 - 50*a**2*x**2 - 17*a*x + 16)/(
64*a**7*c**4*x**6 + 128*a**6*c**4*x**5 - 64*a**5*c**4*x**4 - 256*a**4*c**4*
x**3 - 64*a**3*c**4*x**2 + 128*a**2*c**4*x + 64*a*c**4) + (15*log(x - 1/a)/
128 - 15*log(x + 1/a)/128)/(a*c**4)
```

Giac [A]
time = 0.40, size = 91, normalized size = 0.76

$$-\frac{15\,\log\left(|ax+1|\right)}{128\,ac^4}+\frac{15\,\log\left(|ax-1|\right)}{128\,ac^4}+\frac{15\,a^5x^5+30\,a^4x^4-10\,a^3x^3-50\,a^2x^2-17\,ax+16}{64\,(ax+1)^4(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -15/128*log(abs(a*x + 1))/(a*c^4) + 15/128*log(abs(a*x - 1))/(a*c^4) + 1/64
*(15*a^5*x^5 + 30*a^4*x^4 - 10*a^3*x^3 - 50*a^2*x^2 - 17*a*x + 16)/((a*x +
1)^4*(a*x - 1)^2*a*c^4)
```

Mupad [B]
time = 1.28, size = 121, normalized size = 1.02

$$-\frac{\frac{17\,x}{64}+\frac{25\,a\,x^2}{32}-\frac{1}{4\,a}+\frac{5\,a^2\,x^3}{32}-\frac{15\,a^3\,x^4}{32}-\frac{15\,a^4\,x^5}{64}}{a^6\,c^4\,x^6+2\,a^5\,c^4\,x^5-a^4\,c^4\,x^4-4\,a^3\,c^4\,x^3-a^2\,c^4\,x^2+2\,a\,c^4\,x+c^4}-\frac{15\,\mathrm{atanh}(a\,x)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^4*(a*x + 1)),x)
```

```
[Out] - ((17*x)/64 + (25*a*x^2)/32 - 1/(4*a) + (5*a^2*x^3)/32 - (15*a^3*x^4)/32 -
 (15*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^4*x^4 + 2*a^5*
c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) - (15*atanh(a*x))/(64*a*c^4)
```

3.606 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^4\,dx$

Optimal. Leaf size=393

$$\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c$$

```
[Out] 11/48*a^4*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)*x^5-11/48*a^5*c^4*(1-1/a/x)^(
5/2)*(1+1/a/x)^(7/2)*x^6+11/56*a^6*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)*x^7-
11/72*a^7*c^4*(1-1/a/x)^(9/2)*(1+1/a/x)^(7/2)*x^8+1/9*a^8*c^4*(1-1/a/x)^(11
/2)*(1+1/a/x)^(7/2)*x^9+55/128*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))
/a+55/384*a*c^4*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)+11/192*a^2*c^4*(1+1/a/x
)^(5/2)*x^3*(1-1/a/x)^(1/2)-11/64*a^3*c^4*(1+1/a/x)^(7/2)*x^4*(1-1/a/x)^(1/
2)+55/128*c^4*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{9}a^8c^4x^9\left(1-\frac{1}{ax}\right)^{11/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{72}a^7c^4x^8\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{56}a^6c^4x^7\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{48}a^5c^4x^6\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{48}a^4c^4x^5\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{11}{64}a^3c^4x^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{11}{192}a^2c^4x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{55}{384}ac^4x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{55}{128}c^4x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{55c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (55*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/128 + (55*a*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2)*x^2)/384 + (11*a^2*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(5/2)*x^3)/192 - (11*a^3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x^
4)/64 + (11*a^4*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)*x^5)/48 - (11*a
^5*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)*x^6)/48 + (11*a^6*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(7/2)*x^7)/56 - (11*a^7*c^4*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(7/2)*x^8)/72 + (a^8*c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^
(7/2)*x^9)/9 + (55*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(128*a
)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^4\,dx=\left(a^8c^4\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^4x^8\,dx$$

$$=-\left(\left(a^8c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^{10}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}x^9+\frac{1}{9}\left(11a^7c^4\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1\right.}{x^9}\right.$$

$$=-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x^8+\frac{1}{9}a^8c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{7/2}$$

$$=\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}x^7-\frac{11}{72}a^7c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}$$

$$=-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x^6+\frac{11}{56}a^6c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/}$$

$$=\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5-\frac{11}{48}a^5c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}$$

$$=-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+\frac{11}{48}a^4c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}x^5$$

$$=\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-\frac{11}{64}a^3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x^4+$$

$$=\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3-$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2$$

$$=\frac{55}{128}c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{55}{384}ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{11}{192}a^2$$

Mathematica [A]
time = 0.09, size = 111, normalized size = 0.28

$$\frac{c^4\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-3712-4599ax+10240a^2x^2-3066a^3x^3-8448a^4x^4+7224a^5x^5+1024a^6x^6-3024a^7x^7+896a^8x^8)+3465\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{8064a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-3712 - 4599*a*x + 10240*a^2*x^2 - 3066*a^
3*x^3 - 8448*a^4*x^4 + 7224*a^5*x^5 + 1024*a^6*x^6 - 3024*a^7*x^7 + 896*a^8
*x^8) + 3465*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(8064*a)
```

Maple [A]
time = 0.10, size = 288, normalized size = 0.73

method	result
risch	$\frac{(896a^8x^8-3024a^7x^7+1024a^6x^6+7224a^5x^5-8448a^4x^4-3066a^3x^3+10240a^2x^2-4599ax-3712)(ax+1)c^4\sqrt{\frac{ax-1}{ax+1}}}{8064a}+\frac{55\ln\left(\frac{a^2}{\sqrt{a}}\right.}{}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(896(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-3024(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^5x^5+1920\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4+4200(a^2x^2-1)^{\frac{3}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8064*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^4/a*(896*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2)*a^6*x^6-3024*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^5*x^5+1920*(a^2)^(1/2)*
(a^2*x^2-1)^(3/2)*a^4*x^4+4200*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-6528*(
a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+1134*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x
+8064*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-4352*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)-3465*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+3465*ln((a^2*x+(a^2*x^2-1)^(1/2)*(
a^2)^(1/2))/(a^2)^(1/2))*a)/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 415, normalized size = 1.06

$$\frac{1}{8064}\left(\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3465\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(3465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{17}{2}}-30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+115038\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}+334602\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-360448\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+255222\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-115038\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+30030\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-3465\,c^4\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{9(ax-1)a^2}{ax+1}-\frac{36(ax-1)^2a^2}{(ax+1)^2}+\frac{84(ax-1)^3a^2}{(ax+1)^3}-\frac{126(ax-1)^4a^2}{(ax+1)^4}+\frac{126(ax-1)^5a^2}{(ax+1)^5}-\frac{84(ax-1)^6a^2}{(ax+1)^6}+\frac{36(ax-1)^7a^2}{(ax+1)^7}-\frac{9(ax-1)^8a^2}{(ax+1)^8}+\frac{(ax-1)^9a^2}{(ax+1)^9}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 3465*c^4*log(sqrt
((a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(3465*c^4*((a*x - 1)/(a*x + 1))^(17/2) -
```

```
30030*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 115038*c^4*((a*x - 1)/(a*x + 1))^
(13/2) + 334602*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 360448*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 255222*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 115038*c^4*((a*x
- 1)/(a*x + 1))^(5/2) + 30030*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 3465*c^4*sq
rt((a*x - 1)/(a*x + 1)))/(9*(a*x - 1)*a^2/(a*x + 1) - 36*(a*x - 1)^2*a^2/(a
*x + 1)^2 + 84*(a*x - 1)^3*a^2/(a*x + 1)^3 - 126*(a*x - 1)^4*a^2/(a*x + 1)^
4 + 126*(a*x - 1)^5*a^2/(a*x + 1)^5 - 84*(a*x - 1)^6*a^2/(a*x + 1)^6 + 36*(
a*x - 1)^7*a^2/(a*x + 1)^7 - 9*(a*x - 1)^8*a^2/(a*x + 1)^8 + (a*x - 1)^9*a^
2/(a*x + 1)^9 - a^2))*a
```

Fricas [A]
time = 0.39, size = 169, normalized size = 0.43

$$\frac{3465\,c^{4}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3465\,c^{4}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(896\,a^{9}c^{4}x^{9}-2128\,a^{8}c^{4}x^{8}-2000\,a^{7}c^{4}x^{7}+8248\,a^{6}c^{4}x^{6}-1224\,a^{5}c^{4}x^{5}-11514\,a^{4}c^{4}x^{4}+7174\,a^{3}c^{4}x^{3}+5641\,a^{2}c^{4}x^{2}-8311\,ac^{4}x-3712\,c^{4}\right)\sqrt{\frac{ax-1}{ax+1}}}{8064\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8064*(3465*c^4*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3465*c^4*log(sqrt((a*
x - 1)/(a*x + 1)) - 1) + (896*a^9*c^4*x^9 - 2128*a^8*c^4*x^8 - 2000*a^7*c^4
*x^7 + 8248*a^6*c^4*x^6 - 1224*a^5*c^4*x^5 - 11514*a^4*c^4*x^4 + 7174*a^3*c
^4*x^3 + 5641*a^2*c^4*x^2 - 8311*a*c^4*x - 3712*c^4)*sqrt((a*x - 1)/(a*x +
1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^{4}\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{4a^{2}x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{4a^{3}x^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{6a^{4}x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{6a^{5}x^{5}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{4a^{6}x^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{4a^{7}x^{7}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^{8}x^{8}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^{9}x^{9}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(4*a**2*x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-6*a**4*x**4*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(6*a**5*x**5*sqrt(a*x/(
a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(4*a**6*x**6*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-4*a**7*x**7*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**8*x**8*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1), x) + Integral(a**9*x**9*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1), x))
```

Giac [A]
time = 0.43, size = 198, normalized size = 0.50

$$-\frac{55\,c^{4}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\mathrm{sgn}(ax+1)}{128\,|a|}-\frac{1}{8064}\sqrt{a^{2}x^{2}-1}\left(\frac{3712\,c^{4}\mathrm{sgn}(ax+1)}{a}+\left(4599\,c^{4}\mathrm{sgn}(ax+1)-2\left(5120\,ac^{4}\mathrm{sgn}(ax+1)-\left(1533\,a^{2}c^{4}\mathrm{sgn}(ax+1)+4\left(1056\,a^{3}c^{4}\mathrm{sgn}(ax+1)-\left(903\,a^{4}c^{4}\mathrm{sgn}(ax+1)+2\left(64\,a^{5}c^{4}\mathrm{sgn}(ax+1)+7\left(8\,a^{7}c^{4}x\mathrm{sgn}(ax+1)-27\,a^{6}c^{4}\mathrm{sgn}(ax+1)\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -55/128*c^4*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1
/8064*sqrt(a^2*x^2 - 1)*(3712*c^4*sgn(a*x + 1)/a + (4599*c^4*sgn(a*x + 1) -
 2*(5120*a*c^4*sgn(a*x + 1) - (1533*a^2*c^4*sgn(a*x + 1) + 4*(1056*a^3*c^4*
sgn(a*x + 1) - (903*a^4*c^4*sgn(a*x + 1) + 2*(64*a^5*c^4*sgn(a*x + 1) + 7*(
8*a^7*c^4*x*sgn(a*x + 1) - 27*a^6*c^4*sgn(a*x + 1))*x)*x)*x)*x)*x)*x)*x)
```

Mupad [B]
time = 0.17, size = 362, normalized size = 0.92

$$\frac{\frac{715\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96}-\frac{55\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{64}-\frac{913\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32}+\frac{14179\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{224}-\frac{5632\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{63}+\frac{18589\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{224}+\frac{913\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{32}-\frac{715\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/2}}{96}+\frac{55\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{17/2}}{64}}{a-\frac{9\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{36\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{84\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{126\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{126\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{84\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{36\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}+\frac{9\,a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}-\frac{a\,{\left(a\,x-1\right)}^9}{{\left(a\,x+1\right)}^9}}+\frac{55\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((715*c^4*((a*x - 1)/(a*x + 1))^(3/2))/96 - (55*c^4*((a*x - 1)/(a*x + 1))^(
1/2))/64 - (913*c^4*((a*x - 1)/(a*x + 1))^(5/2))/32 + (14179*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/224 - (5632*c^4*((a*x - 1)/(a*x + 1))^(9/2))/63 + (18589
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/224 + (913*c^4*((a*x - 1)/(a*x + 1))^(13
/2))/32 - (715*c^4*((a*x - 1)/(a*x + 1))^(15/2))/96 + (55*c^4*((a*x - 1)/(a
*x + 1))^(17/2))/64)/(a - (9*a*(a*x - 1))/(a*x + 1) + (36*a*(a*x - 1)^2)/(a
*x + 1)^2 - (84*a*(a*x - 1)^3)/(a*x + 1)^3 + (126*a*(a*x - 1)^4)/(a*x + 1)^
4 - (126*a*(a*x - 1)^5)/(a*x + 1)^5 + (84*a*(a*x - 1)^6)/(a*x + 1)^6 - (36*
a*(a*x - 1)^7)/(a*x + 1)^7 + (9*a*(a*x - 1)^8)/(a*x + 1)^8 - (a*(a*x - 1)^9
)/(a*x + 1)^9) + (55*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(64*a)
```

3.607 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=313

$$\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{3}{8}a^3c^3\Big(1-$$

```
[Out] 3/8*a^3*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x^4-3/10*a^4*c^3*(1-1/a/x)^(5/2
)*(1+1/a/x)^(5/2)*x^5+3/14*a^5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x^6-1/7*
a^6*c^3*(1-1/a/x)^(9/2)*(1+1/a/x)^(5/2)*x^7+9/16*c^3*arctanh((1-1/a/x)^(1/2
)*(1+1/a/x)^(1/2))/a+3/16*a*c^3*(1+1/a/x)^(3/2)*x^2*(1-1/a/x)^(1/2)-3/8*a^2
*c^3*(1+1/a/x)^(5/2)*x^3*(1-1/a/x)^(1/2)+9/16*c^3*x*(1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{7}a^6c^3x^7\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{14}a^5c^3x^6\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{10}a^4c^3x^5\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{8}a^3c^3x^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}-\frac{3}{8}a^2c^3x^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{3}{16}ac^3x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{9}{16}c^3x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{9c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (9*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/16 + (3*a*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2)*x^2)/16 - (3*a^2*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2)*x^3)/8 + (3*a^3*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x^4)/8 -
 (3*a^4*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2)*x^5)/10 + (3*a^5*c^3*(1
 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)*x^6)/14 - (a^6*c^3*(1 - 1/(a*x))^(9/2
)*(1 + 1/(a*x))^(5/2)*x^7)/7 + (9*c^3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/(16*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx=-\left(\left(a^6c^3\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)$$

$$=\left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^8}\,dx,x,\frac{1}{x}\right)$$

$$=-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x^7-\frac{1}{7}\left(9a^5c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\right.}{x^7}\right.$$

$$=\frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x^6-\frac{1}{7}a^6c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}x$$

$$=-\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x^5+\frac{3}{14}a^5c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/}$$

$$=\frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4-\frac{3}{10}a^4c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}x$$

$$=-\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{3}{8}a^3c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x^4-$$

$$=\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}x^3+\frac{3}{8}a^3$$

$$=\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1}$$

$$=\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1}$$

$$=\frac{9}{16}c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{3}{16}ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2-\frac{3}{8}a^2c^3\sqrt{1}$$

Mathematica [A]

time = 0.07, size = 95, normalized size = 0.30

$$-\frac{c^3\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(368+245ax-656a^2x^2+350a^3x^3+208a^4x^4-280a^5x^5+80a^6x^6)-315\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{560a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/560*(c^3*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(368 + 245*a*x - 656*a^2*x^2 + 350*a
^3*x^3 + 208*a^4*x^4 - 280*a^5*x^5 + 80*a^6*x^6) - 315*Log[(1 + Sqrt[1 - 1/
(a^2*x^2)])*x]))/a
```

Maple [A]
time = 0.10, size = 240, normalized size = 0.77

method	result
risch	$-\frac{(80a^6x^6-280a^5x^5+208a^4x^4+350a^3x^3-656a^2x^2+245ax+368)(ax+1)c^3\sqrt{\frac{ax-1}{ax+1}}}{560a}+\frac{9\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^3\sqrt{\frac{ax}{ax}}}{16\sqrt{a^2}\,(a}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(-80\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4+280(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^3x^3-288(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-70\sqrt{a^2}\,(a^2x^2-1)\right.}{560a(ax-1)\sqrt{(ax-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/560*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^3/a*(-80*(a^2)^(1/2)*(a^2*x^2-1)^
(3/2)*a^4*x^4+280*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-288*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*a^2*x^2-70*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+560*((a*x+1)*(a
*x-1))^(3/2)*(a^2)^(1/2)-192*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-315*(a^2)^(1/2)*
(a^2*x^2-1)^(1/2)*a*x+315*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1
/2))*a)/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 337, normalized size = 1.08

$$\frac{1}{560}\left(\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(315\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}-2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-8393\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+9216\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-5943\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+2100\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-315\,c^3\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{7(ax-1)a^2}{ax+1}-\frac{21(ax-1)^2a^2}{(ax+1)^2}+\frac{35(ax-1)^3a^2}{(ax+1)^3}-\frac{35(ax-1)^4a^2}{(ax+1)^4}+\frac{21(ax-1)^5a^2}{(ax+1)^5}-\frac{7(ax-1)^6a^2}{(ax+1)^6}+\frac{(ax-1)^7a^2}{(ax+1)^7}-a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 315*c^3*log(sqrt((a
*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(315*c^3*((a*x - 1)/(a*x + 1))^(13/2) - 210
```

```
0*c^3*((a*x - 1)/(a*x + 1))^(11/2) - 8393*c^3*((a*x - 1)/(a*x + 1))^(9/2) +
 9216*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 5943*c^3*((a*x - 1)/(a*x + 1))^(5/2
) + 2100*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 315*c^3*sqrt((a*x - 1)/(a*x + 1)
))/(7*(a*x - 1)*a^2/(a*x + 1) - 21*(a*x - 1)^2*a^2/(a*x + 1)^2 + 35*(a*x -
1)^3*a^2/(a*x + 1)^3 - 35*(a*x - 1)^4*a^2/(a*x + 1)^4 + 21*(a*x - 1)^5*a^2/
(a*x + 1)^5 - 7*(a*x - 1)^6*a^2/(a*x + 1)^6 + (a*x - 1)^7*a^2/(a*x + 1)^7 -
 a^2))*a
```

Fricas [A]
time = 0.34, size = 148, normalized size = 0.47

$$\frac{315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-315\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(80\,a^7c^3x^7-200\,a^6c^3x^6-72\,a^5c^3x^5+558\,a^4c^3x^4-306\,a^3c^3x^3-411\,a^2c^3x^2+613\,ac^3x+368\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(315*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 315*c^3*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) - (80*a^7*c^3*x^7 - 200*a^6*c^3*x^6 - 72*a^5*c^3*x^5 +
558*a^4*c^3*x^4 - 306*a^3*c^3*x^3 - 411*a^2*c^3*x^2 + 613*a*c^3*x + 368*c^3
)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
-a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**2*x**
2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(3*a**3*x**3*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(3*a**4*x**4*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-3*a**5*x**5*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**6*x**6*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x**7*sqrt(a*x/(a*x + 1) -
 1/(a*x + 1))/(a*x + 1), x))
```

Giac [A]
time = 0.40, size = 162, normalized size = 0.52

$$-\frac{9\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{16\,|a|}-\frac{1}{560}\,\sqrt{a^2x^2-1}\left(\frac{368\,c^3\mathrm{sgn}(ax+1)}{a}+\left(245\,c^3\mathrm{sgn}(ax+1)-2\left(328\,ac^3\mathrm{sgn}(ax+1)-\left(175\,a^2c^3\mathrm{sgn}(ax+1)+4\left(26\,a^3c^3\mathrm{sgn}(ax+1)+5\left(2\,a^5c^3x\mathrm{sgn}(ax+1)-7\,a^4c^3\mathrm{sgn}(ax+1)\right)x\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -9/16*c^3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/5
60*sqrt(a^2*x^2 - 1)*(368*c^3*sgn(a*x + 1)/a + (245*c^3*sgn(a*x + 1) - 2*(3
28*a*c^3*sgn(a*x + 1) - (175*a^2*c^3*sgn(a*x + 1) + 4*(26*a^3*c^3*sgn(a*x +
 1) + 5*(2*a^5*c^3*x*sgn(a*x + 1) - 7*a^4*c^3*sgn(a*x + 1))*x)*x)*x)*x)*x)
```

Mupad [B]
time = 0.13, size = 289, normalized size = 0.92

$$\frac{9\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{\frac{9\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}-\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{2}+\frac{849\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}-\frac{1152\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{35}+\frac{1199\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{40}+\frac{15\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{2}-\frac{9\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{8}}{a-\frac{7\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{21\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{35\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{35\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{21\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{7\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (9*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(8*a) - ((9*c^3*((a*x - 1)/(a*x
+ 1))^(1/2))/8 - (15*c^3*((a*x - 1)/(a*x + 1))^(3/2))/2 + (849*c^3*((a*x -
1)/(a*x + 1))^(5/2))/40 - (1152*c^3*((a*x - 1)/(a*x + 1))^(7/2))/35 + (1199
*c^3*((a*x - 1)/(a*x + 1))^(9/2))/40 + (15*c^3*((a*x - 1)/(a*x + 1))^(11/2)
)/2 - (9*c^3*((a*x - 1)/(a*x + 1))^(13/2))/8)/(a - (7*a*(a*x - 1))/(a*x + 1
) + (21*a*(a*x - 1)^2)/(a*x + 1)^2 - (35*a*(a*x - 1)^3)/(a*x + 1)^3 + (35*a
*(a*x - 1)^4)/(a*x + 1)^4 - (21*a*(a*x - 1)^5)/(a*x + 1)^5 + (7*a*(a*x - 1)
^6)/(a*x + 1)^6 - (a*(a*x - 1)^7)/(a*x + 1)^7)
```

3.608 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx$

Optimal. Leaf size=233

$$\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{7}{20}a^3c^2\left(\right.$$

```
[Out] 7/12*a^2*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)*x^3-7/20*a^3*c^2*(1-1/a/x)^(5/
2)*(1+1/a/x)^(3/2)*x^4+1/5*a^4*c^2*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)*x^5+7/8*
c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/8*a*c^2*(1+1/a/x)^(3/2)*x^
2*(1-1/a/x)^(1/2)+7/8*c^2*x*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6326, 6330, 96, 94, 214}

$$\frac{1}{5}a^4c^2x^5\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{20}a^3c^2x^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7}{12}a^2c^2x^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{7}{8}ac^2x^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}+\frac{7}{8}c^2x\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{7c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (7*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/8 - (7*a*c^2*Sqrt[1 - 1/(a*x)
]*(1 + 1/(a*x))^(3/2)*x^2)/8 + (7*a^2*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))
^(3/2)*x^3)/12 - (7*a^3*c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x^4)/20
 + (a^4*c^2*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)*x^5)/5 + (7*c^2*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(8*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx=\left(a^4c^2\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx$$

$$=-\left(\left(a^4c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^6}\,dx,x,\frac{1}{x}\right)\right)$$

$$=\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}x^5+\frac{1}{5}\left(7a^3c^2\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1-}}{x^5}\right.$$

$$=-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x^4+\frac{1}{5}a^4c^2\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}$$

$$=\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-\frac{7}{20}a^3c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}$$

$$=-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}x^3-$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\right.$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\right.$$

$$=\frac{7}{8}c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x-\frac{7}{8}ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x^2+\frac{7}{12}a^2c^2\left(1-\right.$$

Mathematica [A]

time = 0.06, size = 79, normalized size = 0.34

$$\frac{c^2\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-136-15ax+112a^2x^2-90a^3x^3+24a^4x^4)+105\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-136 - 15*a*x + 112*a^2*x^2 - 90*a^3*x^3 +
 24*a^4*x^4) + 105*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a)
```

Maple [A]
time = 0.10, size = 192, normalized size = 0.82

method	result
risch	$\frac{(24a^4x^4-90a^3x^3+112a^2x^2-15ax-136)(ax+1)c^2\sqrt{\frac{ax-1}{ax+1}}}{120a}+\frac{7\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax\cdots}}{8\sqrt{a^2}\,(ax-1)\cdots}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(24(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-90\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}ax+16(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-105\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\cdots\right.}{120a(ax-1)\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^2/a*(24*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*a^2*x^2-90*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+16*(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)-105*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+120*((a*x+1)*(a*x-1))^(3/2)*(
a^2)^(1/2)+105*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a)/(a*
x-1)/((a*x+1)*(a*x-1))^(1/2)/(a^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 259, normalized size = 1.11

$$\frac{1}{120}a\left(\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2\left(105\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+790\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-896\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+490\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^2\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{5(ax-1)a^2}{ax+1}-\frac{10(ax-1)^2a^2}{(ax+1)^2}+\frac{10(ax-1)^3a^2}{(ax+1)^3}-\frac{5(ax-1)^4a^2}{(ax+1)^4}+\frac{(ax-1)^5a^2}{(ax+1)^5}-a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/120*a*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 - 105*c^2*log(sqrt(
(a*x - 1)/(a*x + 1)) - 1)/a^2 - 2*(105*c^2*((a*x - 1)/(a*x + 1))^(9/2) + 79
0*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 896*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 4
```

```
90*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^2*sqrt((a*x - 1)/(a*x + 1)))/(5*
(a*x - 1)*a^2/(a*x + 1) - 10*(a*x - 1)^2*a^2/(a*x + 1)^2 + 10*(a*x - 1)^3*a
^2/(a*x + 1)^3 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 + (a*x - 1)^5*a^2/(a*x + 1)^
5 - a^2))
```

Fricas [A]
time = 0.35, size = 125, normalized size = 0.54

$$\frac{105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(24\,a^5c^2x^5-66\,a^4c^2x^4+22\,a^3c^2x^3+97\,a^2c^2x^2-151\,ac^2x-136\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{120\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(105*c^2*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) - 1) + (24*a^5*c^2*x^5 - 66*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 9
7*a^2*c^2*x^2 - 151*a*c^2*x - 136*c^2)*sqrt((a*x - 1)/(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(
a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(2*a**2*x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-2*a**3*x**3*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**4*x**4*sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x**5*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Giac [A]
time = 0.42, size = 126, normalized size = 0.54

$$-\frac{7\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{8\,|a|}-\frac{1}{120}\,\sqrt{a^2x^2-1}\left(\left(15\,c^2\mathrm{sgn}(ax+1)-2\left(56\,ac^2\mathrm{sgn}(ax+1)+3\left(4\,a^3c^2x\mathrm{sgn}(ax+1)-15\,a^2c^2\mathrm{sgn}(ax+1)\right)x\right)x\right)x+\frac{136\,c^2\mathrm{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -7/8*c^2*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/12
0*sqrt(a^2*x^2 - 1)*((15*c^2*sgn(a*x + 1) - 2*(56*a*c^2*sgn(a*x + 1) + 3*(4
*a^3*c^2*x*sgn(a*x + 1) - 15*a^2*c^2*sgn(a*x + 1))*x)*x)*x + 136*c^2*sgn(a*
x + 1)/a)
```

Mupad [B]
time = 1.27, size = 214, normalized size = 0.92

$$\frac{\frac{49\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6}-\frac{7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}-\frac{224\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{15}+\frac{79\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{6}+\frac{7\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{4}}{a-\frac{5\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{10\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{10\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}}+\frac{7\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((49*c^2*((a*x - 1)/(a*x + 1))^(3/2))/6 - (7*c^2*((a*x - 1)/(a*x + 1))^(1/2
))/4 - (224*c^2*((a*x - 1)/(a*x + 1))^(5/2))/15 + (79*c^2*((a*x - 1)/(a*x +
 1))^(7/2))/6 + (7*c^2*((a*x - 1)/(a*x + 1))^(9/2))/4)/(a - (5*a*(a*x - 1))
/(a*x + 1) + (10*a*(a*x - 1)^2)/(a*x + 1)^2 - (10*a*(a*x - 1)^3)/(a*x + 1)^
3 + (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (a*(a*x - 1)^5)/(a*x + 1)^5) + (7*c^2*a
tanh(((a*x - 1)/(a*x + 1))^(1/2)))/(4*a)
```

3.609 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=145

$$-\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x+\frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2-\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3+\frac{5c\tanh^{-1}\left(\sqrt{\ldots}\right.}{\ldots}$$

```
[Out] 5/2*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+5/6*a*c*(1-1/a/x)^(3/2)*x^
2*(1+1/a/x)^(1/2)-1/3*a^2*c*(1-1/a/x)^(5/2)*x^3*(1+1/a/x)^(1/2)-5/2*c*x*(1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6326, 6330, 96, 94, 214}

$$-\frac{1}{3}a^2cx^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}+\frac{5}{6}acx^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}-\frac{5}{2}cx\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}+\frac{5c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]*x)/2 + (5*a*c*(1 - 1/(a*x))^(3/2)
*Sqrt[1 + 1/(a*x)]*x^2)/6 - (a^2*c*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)]*x^
3)/3 + (5*c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(2*a)
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx = -\left(\left(a^2c\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$

$$= \left(a^2c\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^4\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3 - \frac{1}{3}(5ac)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}}{x^3\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}\,x^3 + \frac{1}{2}(5c)\text{Su}$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{a}\right.$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{a}\right.$$

$$= -\frac{5}{2}c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\,x + \frac{5}{6}ac\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x^2 - \frac{1}{3}a^2c\left(1-\frac{1}{a}\right.$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.42

$$\frac{c\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-22+9ax-2a^2x^2)+15\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/E^(3*ArcCoth[a*x]),x]
[Out] (c*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-22 + 9*a*x - 2*a^2*x^2) + 15*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/(6*a)
```

Maple [A]
time = 0.10, size = 183, normalized size = 1.26

method	result
risch	$-\frac{(2a^2x^2-9ax+22)(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{6a}+\frac{5\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)c\sqrt{\frac{ax-1}{ax+1}}\sqrt{(ax+1)(ax-1)}}{2\sqrt{a^2}\,(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c\left(9\sqrt{a^2}\sqrt{a^2x^2-1}\,ax-2((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}-9\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\sqrt{a^2}}{\sqrt{a^2}}\right)a-24\sqrt{a^2}\cdots\right.}{6(ax-1)\sqrt{(ax+1)(ax-1)}\,a\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c*(9*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*
x-2*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-9*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)
^(1/2))/(a^2)^(1/2))*a-24*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+24*a*ln((a^2*
x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/(a*x-1)/((a*x+1)*(a*x-
1))^(1/2)/a/(a^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 171, normalized size = 1.18

$$\frac{1}{6}a\left(\frac{2\left(33\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-40\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15\,c\sqrt{\frac{ax-1}{ax+1}}\right)}{\frac{3\,(ax-1)a^2}{ax+1}-\frac{3\,(ax-1)^2a^2}{(ax+1)^2}+\frac{(ax-1)^3a^2}{(ax+1)^3}-a^2}+\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(2*(33*c*((a*x - 1)/(a*x + 1))^(5/2) - 40*c*((a*x - 1)/(a*x + 1))^(3/
2) + 15*c*sqrt((a*x - 1)/(a*x + 1)))/(3*(a*x - 1)*a^2/(a*x + 1) - 3*(a*x -
1)^2*a^2/(a*x + 1)^2 + (a*x - 1)^3*a^2/(a*x + 1)^3 - a^2) + 15*c*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 15*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.36, size = 92, normalized size = 0.63

$$\frac{15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(2\,a^3cx^3-7\,a^2cx^2+13\,acx+22\,c)\sqrt{\frac{ax-1}{ax+1}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(15*c*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*c*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (2*a^3*c*x^3 - 7*a^2*c*x^2 + 13*a*c*x + 22*c)*sqrt((a*x - 1)/
(a*x + 1)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx+\int\left(-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\left(-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] -c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(-a**2*x**2*sqr
t(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x**3*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x))
```

Giac [A]
time = 0.41, size = 82, normalized size = 0.57

$$-\frac{5\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{2\,|a|}-\frac{1}{6}\sqrt{a^2x^2-1}\left((2\,acx\operatorname{sgn}(ax+1)-9\,c\operatorname{sgn}(ax+1))x+\frac{22\,c\operatorname{sgn}(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -5/2*c*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) - 1/6*sq
rt(a^2*x^2 - 1)*((2*a*c*x*sgn(a*x + 1) - 9*c*sgn(a*x + 1))*x + 22*c*sgn(a*x
 + 1)/a)
```

Mupad [B]
time = 0.07, size = 133, normalized size = 0.92

$$\frac{5\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{5\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}-\frac{40\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+11\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{a-\frac{3\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{3\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (5*c*((a*x - 1)/(a*x + 1))^(1/
2) - (40*c*((a*x - 1)/(a*x + 1))^(3/2))/3 + 11*c*((a*x - 1)/(a*x + 1))^(5/2
))/(a - (3*a*(a*x - 1))/(a*x + 1) + (3*a*(a*x - 1)^2)/(a*x + 1)^2 - (a*(a*x
 - 1)^3)/(a*x + 1)^3)
```

3.610 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

```
[Out] -1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6318}

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/3*1/(a*c*E^(3*ArcCoth[a*x]))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.00

$$-\frac{e^{-3\coth^{-1}(ax)}}{3ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)),x]
```

```
[Out] -1/3*1/(a*c*E^(3*ArcCoth[a*x]))
```

Maple [A]
time = 0.13, size = 24, normalized size = 1.33

method	result	size
gosper	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3ac}$	24
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3ac}$	24
trager	$-\frac{(ax-1)\sqrt{-\frac{-ax+1}{ax+1}}}{3ac(ax+1)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/3/a/c*((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.26, size = 23, normalized size = 1.28

$$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] -1/3*((a*x - 1)/(a*x + 1))^(3/2)/(a*c)
```

Fricas [A]
time = 0.34, size = 34, normalized size = 1.89

$$-\frac{(ax-1)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2cx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] -1/3*(a*x - 1)*sqrt((a*x - 1)/(a*x + 1))/(a^2*c*x + a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c),x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x**2 - a*x
- 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2
*x**2 - a*x - 1), x))/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(23) = 46.
time = 0.43, size = 49, normalized size = 2.72

$$\frac{2\left(3\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+1\right)}{3\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^3ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*(3*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 1)/(((a + sqrt(a^2 - 1/x^2))*x + 1)^
3*a*c)
```

Mupad [B]
time = 1.21, size = 23, normalized size = 1.28

$$-\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3\,a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2),x)
```

```
[Out] -((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c)
```

3.611 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^2} dx$

Optimal. Leaf size=55

$$\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2} - \frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2\left(1-a^2x^2\right)}$$

```
[Out] 2/15/a/c^2*((a*x-1)/(a*x+1))^(3/2)+1/5*(-2*a*x-3)/a/c^2*((a*x-1)/(a*x+1))^(
3/2)/(-a^2*x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6320, 6318}

$$\frac{2e^{-3\coth^{-1}(ax)}}{15ac^2} - \frac{(2ax+3)e^{-3\coth^{-1}(ax)}}{5ac^2\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
```

```
[Out] 2/(15*a*c^2*E^(3*ArcCoth[a*x])) - (3 + 2*a*x)/(5*a*c^2*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2(1-a^2x^2)} - \frac{2\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{5c}$$
$$= \frac{2e^{-3\coth^{-1}(ax)}}{15ac^2} - \frac{e^{-3\coth^{-1}(ax)}(3+2ax)}{5ac^2(1-a^2x^2)}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.78

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(7+6ax+2a^2x^2)}{15c^2(1+ax)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^2),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(7 + 6*a*x + 2*a^2*x^2))/(15*c^2*(1 + a*x)^3)
```

Maple [A]
time = 0.18, size = 52, normalized size = 0.95

method	result	size
trager	$\frac{(2a^2x^2+6ax+7)\sqrt{-\frac{-ax+1}{ax+1}}}{15a\,c^2(ax+1)^2}$	47
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^2x^2+6ax+7)}{15(a^2x^2-1)a\,c^2}$	49
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(2a^2x^2+6ax+7)}{15(ax-1)c^2a(ax+1)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/15*((a*x-1)/(a*x+1))^(3/2)*(2*a^2*x^2+6*a*x+7)/(a*x-1)/c^2/a/(a*x+1)
```

Maxima [A]
time = 0.26, size = 60, normalized size = 1.09

$$\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}} - 10\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} + 15\sqrt{\frac{ax-1}{ax+1}}}{60\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(3*((a*x - 1)/(a*x + 1))^(5/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 15*s
qrt((a*x - 1)/(a*x + 1)))/(a*c^2)
```

Fricas [A]
time = 0.35, size = 58, normalized size = 1.05

$$\frac{(2\,a^2x^2+6\,ax+7)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(2*a^2*x^2 + 6*a*x + 7)*sqrt((a*x - 1)/(a*x + 1))/(a^3*c^2*x^2 + 2*a^2
*c^2*x + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**
3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x)
)/c**2
```

Giac [A]
time = 0.44, size = 65, normalized size = 1.18

$$-\frac{4\left(10\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)^2x^2+5\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)}{15\left(\left(a+\sqrt{a^2-\frac{1}{x^2}}\right)x+1\right)^5ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -4/15*(10*(a + sqrt(a^2 - 1/x^2))^2*x^2 + 5*(a + sqrt(a^2 - 1/x^2))*x + 1)/
(((a + sqrt(a^2 - 1/x^2))*x + 1)^5*a*c^2)
```

Mupad [B]
time = 0.05, size = 60, normalized size = 1.09

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}-10\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}+3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{60\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^2,x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2) - 10*((a*x - 1)/(a*x + 1))^(3/2) + 3*((a*x
- 1)/(a*x + 1))^(5/2))/(60*a*c^2)
```

3.612 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=91

$$\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}+\frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3\left(1-a^2x^2\right)^2}-\frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3\left(1-a^2x^2\right)}$$

```
[Out] 8/35/a/c^3*((a*x-1)/(a*x+1))^(3/2)+1/7*(4*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(3
/2)/(-a^2*x^2+1)^2-12/35*(2*a*x+3)/a/c^3*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2+
1)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$-\frac{12(2ax+3)e^{-3\coth^{-1}(ax)}}{35ac^3\left(1-a^2x^2\right)}+\frac{(4ax+3)e^{-3\coth^{-1}(ax)}}{7ac^3\left(1-a^2x^2\right)^2}+\frac{8e^{-3\coth^{-1}(ax)}}{35ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] 8/(35*a*c^3*E^(3*ArcCoth[a*x])) + (3 + 4*a*x)/(7*a*c^3*E^(3*ArcCoth[a*x])*(
1 - a^2*x^2)^2) - (12*(3 + 2*a*x))/(35*a*c^3*E^(3*ArcCoth[a*x])*(1 - a^2*x^
2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3(1-a^2x^2)^2} + \frac{12\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{7c}$$
$$= \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3(1-a^2x^2)^2} - \frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3(1-a^2x^2)} - \frac{24\int \frac{e^{-3\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{35c^2}$$
$$= \frac{8e^{-3\coth^{-1}(ax)}}{35ac^3} + \frac{e^{-3\coth^{-1}(ax)}(3+4ax)}{7ac^3(1-a^2x^2)^2} - \frac{12e^{-3\coth^{-1}(ax)}(3+2ax)}{35ac^3(1-a^2x^2)}$$

Mathematica [A]
time = 0.06, size = 66, normalized size = 0.73

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-13-4ax+20a^2x^2+24a^3x^3+8a^4x^4)}{35c^3(-1+ax)(1+ax)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^3),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-13 - 4*a*x + 20*a^2*x^2 + 24*a^3*x^3 + 8*a^4*x^4
))/(35*c^3*(-1 + a*x)*(1 + a*x)^4)
```

Maple [A]
time = 0.19, size = 68, normalized size = 0.75

method	result	size
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13)}{35(a^2x^2-1)^2c^3a}$	65
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13)}{35(ax-1)^2c^3a(ax+1)^2}$	68
trager	$\frac{(8a^4x^4+24a^3x^3+20a^2x^2-4ax-13)\sqrt{-\frac{-ax+1}{ax+1}}}{35a\,c^3(ax-1)(ax+1)^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*((a*x-1)/(a*x+1))^(3/2)*(8*a^4*x^4+24*a^3*x^3+20*a^2*x^2-4*a*x-13)/(a*
x-1)^2/c^3/a/(a*x+1)^2
```

Maxima [A]
time = 0.27, size = 103, normalized size = 1.13

$$-\frac{1}{560}\,a\left(\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-28\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+70\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-140\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{35}{a^2c^3\sqrt{\frac{ax-1}{ax+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*((5*((a*x - 1)/(a*x + 1))^(7/2) - 28*((a*x - 1)/(a*x + 1))^(5/2) +
 70*((a*x - 1)/(a*x + 1))^(3/2) - 140*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c^3)
- 35/(a^2*c^3*sqrt((a*x - 1)/(a*x + 1))))
```

Fricas [A]
time = 0.36, size = 86, normalized size = 0.95

$$\frac{\left(8\,a^4x^4+24\,a^3x^3+20\,a^2x^2-4\,ax-13\right)\sqrt{\frac{ax-1}{ax+1}}}{35\left(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(8*a^4*x^4 + 24*a^3*x^3 + 20*a^2*x^2 - 4*a*x - 13)*sqrt((a*x - 1)/(a*x
 + 1))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a*
*5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x) + Integral
(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a**5*x**5
 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x))/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^3, x)
```

Mupad [B]
time = 1.21, size = 116, normalized size = 1.27

$$\frac{1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^3}-\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{20\,a\,c^3}-\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{112\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^3,x)
```

```
[Out] 1/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2)) + ((a*x - 1)/(a*x + 1))^(1/2)/(4*a
*c^3) - ((a*x - 1)/(a*x + 1))^(3/2)/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)
/(20*a*c^3) - ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3)
```

3.613 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=127

$$\frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}+\frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3}+\frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2}-\frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4(1-a^2x^2)}$$

```
[Out] 16/63/a/c^4*((a*x-1)/(a*x+1))^(3/2)+1/9*(2*a*x+1)/a/c^4*((a*x-1)/(a*x+1))^(
3/2)/(-a^2*x^2+1)^3+10/63*(4*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2
+1)^2-8/21*(2*a*x+3)/a/c^4*((a*x-1)/(a*x+1))^(3/2)/(-a^2*x^2+1)
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\frac{(2ax+1)e^{-3\coth^{-1}(ax)}}{9ac^4(1-a^2x^2)^3}-\frac{8(2ax+3)e^{-3\coth^{-1}(ax)}}{21ac^4(1-a^2x^2)}+\frac{10(4ax+3)e^{-3\coth^{-1}(ax)}}{63ac^4(1-a^2x^2)^2}+\frac{16e^{-3\coth^{-1}(ax)}}{63ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] 16/(63*a*c^4*E^(3*ArcCoth[a*x])) + (1 + 2*a*x)/(9*a*c^4*E^(3*ArcCoth[a*x])*
(1 - a^2*x^2)^3) + (10*(3 + 4*a*x))/(63*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x
^2)^2) - (8*(3 + 2*a*x))/(21*a*c^4*E^(3*ArcCoth[a*x])*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{9c}$$

$$= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} + \frac{40\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{21c^2}$$

$$= \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} - \frac{8e^{-3\coth^{-1}(ax)}(3+2ax)}{21ac^4(1-a^2x^2)} - \frac{16\int e^{-}}{}$$

$$= \frac{16e^{-3\coth^{-1}(ax)}}{63ac^4} + \frac{e^{-3\coth^{-1}(ax)}(1+2ax)}{9ac^4(1-a^2x^2)^3} + \frac{10e^{-3\coth^{-1}(ax)}(3+4ax)}{63ac^4(1-a^2x^2)^2} - \frac{8e^{-3\coth^{-1}(ax)}(3}{21ac^4(1-a}$$

Mathematica [A]
time = 0.09, size = 82, normalized size = 0.65

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(19-6ax-66a^2x^2-56a^3x^3+24a^4x^4+48a^5x^5+16a^6x^6)}{63c^4(-1+ax)^2(1+ax)^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^4),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(19 - 6*a*x - 66*a^2*x^2 - 56*a^3*x^3 + 24*a^4*x^4
 + 48*a^5*x^5 + 16*a^6*x^6))/(63*c^4*(-1 + a*x)^2*(1 + a*x)^5)
```

Maple [A]
time = 0.20, size = 84, normalized size = 0.66

method	result	size
gosper	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19)}{63(a^2x^2-1)^3c^4a}$	81
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19)}{63(ax-1)^3c^4(ax+1)^3a}$	84
trager	$\frac{(16a^6x^6+48a^5x^5+24a^4x^4-56a^3x^3-66a^2x^2-6ax+19)\sqrt{-\frac{-ax+1}{ax+1}}}{63a\,c^4(ax-1)^2(ax+1)^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/63*((a*x-1)/(a*x+1))^(3/2)*(16*a^6*x^6+48*a^5*x^5+24*a^4*x^4-56*a^3*x^3-6
6*a^2*x^2-6*a*x+19)/(a*x-1)^3/c^4/(a*x+1)^3/a
```

Maxima [A]
time = 0.27, size = 136, normalized size = 1.07

$$\frac{1}{4032}a\left(\frac{7\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-54\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+189\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-420\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+945\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}+\frac{21\left(\frac{18(ax-1)}{ax+1}-1\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/4032*a*((7*((a*x - 1)/(a*x + 1))^(9/2) - 54*((a*x - 1)/(a*x + 1))^(7/2) +
 189*((a*x - 1)/(a*x + 1))^(5/2) - 420*((a*x - 1)/(a*x + 1))^(3/2) + 945*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^4) + 21*(18*(a*x - 1)/(a*x + 1) - 1)/(a^2*c
^4*((a*x - 1)/(a*x + 1))^(3/2)))
```

Fricas [A]
time = 0.35, size = 134, normalized size = 1.06

$$\frac{\left(16\,a^6x^6+48\,a^5x^5+24\,a^4x^4-56\,a^3x^3-66\,a^2x^2-6\,ax+19\right)\sqrt{\frac{ax-1}{ax+1}}}{63\left(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/63*(16*a^6*x^6 + 48*a^5*x^5 + 24*a^4*x^4 - 56*a^3*x^3 - 66*a^2*x^2 - 6*a*
x + 19)*sqrt((a*x - 1)/(a*x + 1))/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^
4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^9x^9+a^8x^8-4a^7x^7-4a^6x^6+6a^5x^5+6a^4x^4-4a^3x^3-4a^2x^2+ax+1}\right)dx+\int\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^9x^9+a^8x^8-4a^7x^7-4a^6x^6+6a^5x^5+6a^4x^4-4a^3x^3-4a^2x^2+ax+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**4,x)
```

```
[Out] (Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**9*x**9 + a**8*x**8 - 4*a**
7*x**7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 - 4*a**3*x**3 - 4*a**2*x**
2 + a*x + 1), x) + Integral(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**9*x**
9 + a**8*x**8 - 4*a**7*x**7 - 4*a**6*x**6 + 6*a**5*x**5 + 6*a**4*x**4 - 4*a
**3*x**3 - 4*a**2*x**2 + a*x + 1), x))/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^4,x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(a^2*c*x^2 - c)^4, x)
```

Mupad [B]
time = 0.04, size = 155, normalized size = 1.22

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{48\,a\,c^4}+\frac{3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{64\,a\,c^4}-\frac{3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{224\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{576\,a\,c^4}+\frac{\frac{6\,(a\,x-1)}{a\,x+1}-\frac{1}{3}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^4,x)
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - (5*((a*x - 1)/(a*x + 1))^(3/2
))/(48*a*c^4) + (3*((a*x - 1)/(a*x + 1))^(5/2))/(64*a*c^4) - (3*((a*x - 1)/
(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*c^4) + (
(6*(a*x - 1))/(a*x + 1) - 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(3/2))
```

3.614 $\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$

Optimal. Leaf size=229

$$\frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] 8/3*(a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(a*x+1
)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(a*x+1)^8*(-a^2*c*x
^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(a*x+1)^9*(-a^2*c*x^2+c)^(9/2)
/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^10/(1-
1/a^2/x^2)^(9/2)/x^9
```

Rubi [A]

time = 0.14, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6328, 45}

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(ax+1)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (8*(1 + a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^4(1+ax)^5\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(1+ax)^5-32(1+ax)^6+24(1+ax)^7-8(1+ax)^8+\right.}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{8(1+ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-\right.}{a^{10}\left(1-\frac{1}{a^2x^2}\right.}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.34

$$\frac{c^4(1+ax)^6\sqrt{c-a^2cx^2}\left(193-528ax+588a^2x^2-308a^3x^3+63a^4x^4\right)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*(1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 - 528*a*x + 588*a^2*x^2 - 308*a^3
*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.13, size = 113, normalized size = 0.49

method	result	size
default	$\frac{\left(63a^9x^9+70a^8x^8-315a^7x^7-360a^6x^6+630a^5x^5+756a^4x^4-630a^3x^3-840a^2x^2+315ax+630\right)x\,c^4\sqrt{-c\left(a^2x^2-1\right)}}{630(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	113

gosper	$\frac{x(63a^9x^9+70a^8x^8-315a^7x^7-360a^6x^6+630a^5x^5+756a^4x^4-630a^3x^3-840a^2x^2+315ax+630)(-a^2c\,x^2+c)^{\frac{9}{2}}}{630(ax-1)^4(ax+1)^5\sqrt{\frac{ax-1}{ax+1}}}$	116

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/630*(63*a^9*x^9+70*a^8*x^8-315*a^7*x^7-360*a^6*x^6+630*a^5*x^5+756*a^4*x^4-630*a^3*x^3-840*a^2*x^2+315*a*x+630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")

[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)

Fricas [A]
time = 0.35, size = 117, normalized size = 0.51

$$\frac{(63\,a^9c^4x^{10} + 70\,a^8c^4x^9 - 315\,a^7c^4x^8 - 360\,a^6c^4x^7 + 630\,a^5c^4x^6 + 756\,a^4c^4x^5 - 630\,a^3c^4x^4 - 840\,a^2c^4x^3 + 315\,ac^4x^2 + 630\,c^4x)\sqrt{-a^2c}}{630\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")

[Out] 1/630*(63*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 - 360*a^6*c^4*x^7 + 630*a^5*c^4*x^6 + 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 - 840*a^2*c^4*x^3 + 315*a*c^4*x^2 + 630*c^4*x)*sqrt(-a^2*c)/a

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(9/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{9/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.615 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal. Leaf size=183

$$-\frac{8(1+ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] -8/5*(a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(a*x+1)^6
*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(a*x+1)^7*(-a^2*c*x^2
+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^2+c)^(7/2)/a^
8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A]
time = 0.13, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6328, 45}

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(ax+1)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-8*(1 + a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx&=\frac{\left(c-a^2cx^2\right)^{7/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\&=\frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^3(1+ax)^4\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\&=\frac{\left(c-a^2cx^2\right)^{7/2}\int\left(-8(1+ax)^4+12(1+ax)^5-6(1+ax)^6+(1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\&=-\frac{8(1+ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1+ax)^7\left(c-\right.}{7a^8\left(1-\frac{1}{a^2x^2}\right.}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.39

$$-\frac{c^3(1+ax)^5\sqrt{c-a^2cx^2}\left(-93+185ax-135a^2x^2+35a^3x^3\right)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2),x]
[Out] -1/280*(c^3*(1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(-93 + 185*a*x - 135*a^2*x^2 +
35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 97, normalized size = 0.53

method	result	size
default	$-\frac{\left(35a^7x^7+40a^6x^6-140a^5x^5-168a^4x^4+210a^3x^3+280a^2x^2-140ax-280\right)x\,c^3\sqrt{-c\left(a^2x^2-1\right)}}{280(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	97
gosper	$\frac{x\left(35a^7x^7+40a^6x^6-140a^5x^5-168a^4x^4+210a^3x^3+280a^2x^2-140ax-280\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{280(ax+1)^4(ax-1)^3\sqrt{\frac{ax-1}{ax+1}}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(35*a^7*x^7+40*a^6*x^6-140*a^5*x^5-168*a^4*x^4+210*a^3*x^3+280*a^2*x
^2-140*a*x-280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 95, normalized size = 0.52

$$-\frac{\left(35\,a^{7}c^{3}x^{8}+40\,a^{6}c^{3}x^{7}-140\,a^{5}c^{3}x^{6}-168\,a^{4}c^{3}x^{5}+210\,a^{3}c^{3}x^{4}+280\,a^{2}c^{3}x^{3}-140\,ac^{3}x^{2}-280\,c^{3}x\right)\sqrt{-a^{2}c}}{280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 + 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 - 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 + 280*a^2*c^3*x^3 - 140*a*c^3*x^2 - 280*c^3*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.616 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal. Leaf size=136

$$\frac{(1+ax)^4\,(c-a^2cx^2)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\,(c-a^2cx^2)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\,(c-a^2cx^2)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] (a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(a*x+1)^5*(-
a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)^6*(-a^2*c*x^2+c)
^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A]
time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6327, 6328, 45}

$$\frac{(ax+1)^6\,(c-a^2cx^2)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(ax+1)^5\,(c-a^2cx^2)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(ax+1)^4\,(c-a^2cx^2)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^2(1+ax)^3\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(4(1+ax)^3-4(1+ax)^4+(1+ax)^5\right)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{(1+ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2c\right.}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.46

$$\frac{c^2(1+ax)^4\left(11-14ax+5a^2x^2\right)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2),x]
[Out] (c^2*(1 + a*x)^4*(11 - 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.12, size = 81, normalized size = 0.60

method	result	size
default	$\frac{(5a^5x^5+6a^4x^4-15a^3x^3-20a^2x^2+15ax+30)x\,c^2\sqrt{-c\left(a^2x^2-1\right)}}{30(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	81
gosper	$\frac{x(5a^5x^5+6a^4x^4-15a^3x^3-20a^2x^2+15ax+30)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30(ax+1)^3(ax-1)^2\sqrt{\frac{ax-1}{ax+1}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

[Out] 1/30*(5*a^5*x^5+6*a^4*x^4-15*a^3*x^3-20*a^2*x^2+15*a*x+30)*x*c^2*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)

Fricas [A]
time = 0.36, size = 73, normalized size = 0.54

$$\frac{\left(5\,a^5c^2x^6+6\,a^4c^2x^5-15\,a^3c^2x^4-20\,a^2c^2x^3+15\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] 1/30*(5*a^5*c^2*x^6 + 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 + 15*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(5/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.617 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal. Leaf size=93

$$-\frac{2(1+ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -2/3*(a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(a*x+1)
^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A]
time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6327, 6328, 45}

$$\frac{(ax+1)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(ax+1)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*(1 + a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{3/2}\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)(1+ax)^2\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= \frac{\left(c-a^2cx^2\right)^{3/2}\int\left(-2(1+ax)^2+(1+ax)^3\right)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\ &= -\frac{2(1+ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.57

$$-\frac{c(1+ax)^3(-5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/12*(c*(1 + a*x)^3*(-5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Maple [A]
time = 0.11, size = 63, normalized size = 0.68

method	result	size
default	$-\frac{\left(3a^3x^3+4a^2x^2-6ax-12\right)xc\sqrt{-c\left(a^2x^2-1\right)}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	63
gosper	$\frac{x\left(3a^3x^3+4a^2x^2-6ax-12\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{12(ax-1)(ax+1)^2\sqrt{\frac{ax-1}{ax+1}}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(3*a^3*x^3+4*a^2*x^2-6*a*x-12)*x*c*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a
*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.34, size = 43, normalized size = 0.46

$$-\frac{(3\,a^3cx^4+4\,a^2cx^3-6\,acx^2-12\,cx)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/12*(3*a^3*c*x^4 + 4*a^2*c*x^3 - 6*a*c*x^2 - 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-c(ax-1)\,(ax+1))^{\frac{3}{2}}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.618 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6327, 6328}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.60

$$\frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.66

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c\,(a^2x^2-1)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x+2)*x*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^2c}\,(ax^2+2\,x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.619 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

[Out] x*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6328, 31}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]

[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{1}{-1+ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Maple [A]
time = 0.09, size = 51, normalized size = 1.34

method	result	size
default	$-\frac{\ln(ax-1)\sqrt{-c\left(a^2x^2-1\right)}}{ca(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(a*x-1)*(-c*(a^2*x^2-1))^(1/2)/c/a/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.58

$$-\frac{\sqrt{-a^2c}\,\log\left(ax-1\right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] -sqrt(-a^2*c)*log(a*x - 1)/(a^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.620 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6327, 6328, 46, 213}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}}-\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\,(c-a^2cx^2)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\,(c-a^2cx^2)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.62

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-1+(-1+ax)\tanh^{-1}(ax)\right)}{2c(-1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (-1 + a*x)*ArcTanh[a*x]))/(c*(-1 + a*x)
*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 84, normalized size = 0.92

method	result	size

default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(\ln \left(a x +1\right) a x -x \ln \left(a x -1\right) a -\ln \left(a x +1\right)+\ln \left(a x -1\right)-2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} a}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a*x-x*ln(a*x-
1)*a-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.40, size = 86, normalized size = 0.95

$$-\frac{(a^{2} x - a)\sqrt{-c}\, \log\left(\frac{a^{2} c x^{2} + 2\, \sqrt{-a^{2} c}\, \sqrt{-c}\, x + c}{a^{2} x^{2} - 1}\right) + 2\, \sqrt{-a^{2} c}}{4\, (a^{3} c^{2} x - a^{2} c^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}}\, \left(- c\, (a x - 1)\, (a x + 1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.621 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-}}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6327, 6328, 46, 213}

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
- (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\,(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}}+\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\,(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}}-
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 0.45

$$
-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\tanh^{-1}(ax)\right)}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 169, normalized size = 0.92

method	result
default	$\frac{\sqrt{-c\,(a^2x^2-1)}\,(3\ln(ax+1)a^3x^3-3x^3\ln(ax-1)a^3-3\ln(ax+1)a^2x^2+3x^2\ln(ax-1)a^2-6a^2x^2-3\ln(ax+1)ax+3x\ln(ax-1)a}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3(ax+1)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*a^
3*x^3-3*x^3*ln(a*x-1)*a^3-3*ln(a*x+1)*a^2*x^2+3*x^2*ln(a*x-1)*a^2-6*a^2*x^2
-3*ln(a*x+1)*a*x+3*x*ln(a*x-1)*a+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x^2-
1)/c^3/(a*x+1)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.37, size = 136, normalized size = 0.74

$$-\frac{3\,(a^4x^3-a^3x^2-a^2x+a)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\,(3\,a^2x^2-3\,ax-2)\sqrt{-a^2c}}{16\,(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.622 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right.}$$

```
[Out] 1/24*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(
1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^
2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7
/(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a
^2*c*x^2+c)^(7/2)+5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2
+c)^(7/2)
```

Rubi [A]
time = 0.16, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6327, 6328, 46, 213}

$$\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\tanh^{-1}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
+ (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2))
- (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (
5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^4(1+ax)^3}\,dx}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{8(-1+ax)^4}-\frac{3}{16(-1+ax)^3}+\frac{3}{16(-1+ax)^2}+\frac{1}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{1}{16}\right.}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)\left(c-a^2cx^2\right)^{7/2}}$$

Mathematica [A]
time = 0.06, size = 101, normalized size = 0.36

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-8-25ax+25a^2x^2+15a^3x^3-15a^4x^4+15(-1+ax)^3(1+ax)^2\tanh^{-1}(ax)\right)}{48c^3(-1+ax)^3(1+ax)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/48*(Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 25*a*x + 25*a^2*x^2 + 15*a^3*x^3 - 15*
a^4*x^4 + 15*(-1 + a*x)^3*(1 + a*x)^2*ArcTanh[a*x]))/(c^3*(-1 + a*x)^3*(1 +
 a*x)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 241, normalized size = 0.87

method	result
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(15 \ln(ax+1) a^{5} x^{5}-15 x^{5} \ln(ax-1) a^{5}-15 \ln(ax+1) a^{4} x^{4}+15 x^{4} \ln(ax-1) a^{4}-30 a^{4} x^{4}-30 \ln(ax+1) a^{3} x^{3}+30 x\right.}{96 \sqrt{\frac{ax-1}{ax+1}}\,(ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)
*a^5*x^5-15*x^5*ln(a*x-1)*a^5-15*ln(a*x+1)*a^4*x^4+15*x^4*ln(a*x-1)*a^4-30*
a^4*x^4-30*ln(a*x+1)*a^3*x^3+30*x^3*ln(a*x-1)*a^3+30*a^3*x^3+30*ln(a*x+1)*a
^2*x^2-30*x^2*ln(a*x-1)*a^2+50*a^2*x^2+15*ln(a*x+1)*a*x-15*x*ln(a*x-1)*a-50
*a*x-15*ln(a*x+1)+15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x+1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.36, size = 191, normalized size = 0.69

$$-\frac{15\left(a^{6} x^{5}-a^{5} x^{4}-2 a^{4} x^{3}+2 a^{3} x^{2}+a^{2} x-a\right) \sqrt{-c} \log \left(\frac{a^{2} c x^{2}+2 \sqrt{-a^{2} c} \sqrt{-c}\, x+c}{a^{2} x^{2}-1}\right)+2\left(15 a^{4} x^{4}-15 a^{3} x^{3}-25 a^{2} x^{2}+25 a x+8\right) \sqrt{-a^{2} c}}{96\left(a^{7} c^{4} x^{5}-a^{6} c^{4} x^{4}-2 a^{5} c^{4} x^{3}+2 a^{4} c^{4} x^{2}+a^{3} c^{4} x-a^{2} c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/96*(15*(a^6*x^5 - a^5*x^4 - 2*a^4*x^3 + 2*a^3*x^2 + a^2*x - a)*sqrt(-c)*
log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15*a^4*
x^4 - 15*a^3*x^3 - 25*a^2*x^2 + 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 - a^
6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 + a^3*c^4*x - a^2*c^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.623 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal. Leaf size=176

$$-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x(c-a^2cx^2)^{3/2}-\frac{77}{480}c^2x(c-a^2cx^2)^{5/2}-\frac{11}{80}cx(c-a^2cx^2)^{7/2}+\frac{11(c-a^2cx^2)^{9/}}{90a}$$

[Out] -77/384*c^3*x*(-a^2*c*x^2+c)^(3/2)-77/480*c^2*x*(-a^2*c*x^2+c)^(5/2)-11/80*c*x*(-a^2*c*x^2+c)^(7/2)+11/90*(-a^2*c*x^2+c)^(9/2)/a+1/10*(a*x+1)*(-a^2*c*x^2+c)^(9/2)/a-77/256*c^(9/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-77/256*c^4*x*(-a^2*c*x^2+c)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$-\frac{77c^{9/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{256a}-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x(c-a^2cx^2)^{3/2}-\frac{77}{480}c^2x(c-a^2cx^2)^{5/2}-\frac{11}{80}cx(c-a^2cx^2)^{7/2}+\frac{(ax+1)(c-a^2cx^2)^{9/2}}{10a}+\frac{11(c-a^2cx^2)^{9/2}}{90a}$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]

[Out] (-77*c^4*x*Sqrt[c - a^2*c*x^2])/256 - (77*c^3*x*(c - a^2*c*x^2)^(3/2))/384 - (77*c^2*x*(c - a^2*c*x^2)^(5/2))/480 - (11*c*x*(c - a^2*c*x^2)^(7/2))/80 + (11*(c - a^2*c*x^2)^(9/2))/(90*a) + ((1 + a*x)*(c - a^2*c*x^2)^(9/2))/(10*a) - (77*c^(9/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(256*a)

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx$$
$$=-\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{7/2}\,dx\right)$$
$$=\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{1}{10}(11c)\int(1+ax)\left(c-a^2cx^2\right)^{7/2}\,dx$$
$$=\frac{11\left(c-a^2cx^2\right)^{9/2}}{90a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{1}{10}(11c)\int\left(c-a^2cx^2\right)^{7/2}\,d$$
$$=-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{11\left(c-a^2cx^2\right)^{9/2}}{90a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{9/2}}{10a}-\frac{1}{80}$$
$$=-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+\frac{11\left(c-a^2cx^2\right)^{9/2}}{90a}+\frac{(1+}{}$$
$$=-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-\frac{11}{80}cx\left(c-a^2cx^2\right)^{7/2}+$$
$$=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-$$
$$=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-$$
$$=-\frac{77}{256}c^4x\sqrt{c-a^2cx^2}-\frac{77}{384}c^3x\left(c-a^2cx^2\right)^{3/2}-\frac{77}{480}c^2x\left(c-a^2cx^2\right)^{5/2}-$$

Mathematica [A]
time = 0.10, size = 167, normalized size = 0.95

$$\frac{c^4\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}\,(2560-10615ax-2185a^2x^2+16390a^3x^3+9210a^4x^4-15048a^5x^5-10552a^6x^6+7216a^7x^7+5584a^8x^8-1408a^9x^9-1152a^{10}x^{10})+6930\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{11520a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(2560 - 10615*a*x - 2185*a^2*x^2 +
16390*a^3*x^3 + 9210*a^4*x^4 - 15048*a^5*x^5 - 10552*a^6*x^6 + 7216*a^7*x^7
 + 5584*a^8*x^8 - 1408*a^9*x^9 - 1152*a^10*x^10) + 6930*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(11520*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(144) = 288$.
time = 0.21, size = 454, normalized size = 2.58

method	result
risch	$-\frac{\left(1152 a^{9} x^{9}+2560 a^{8} x^{8}-3024 a^{7} x^{7}-10240 a^{6} x^{6}+312 a^{5} x^{5}+15360 a^{4} x^{4}+6150 a^{3} x^{3}-10240 a^{2} x^{2}-8055 a x+2560\right)\left(a^{2} x^{2}-1\right) c^{5}}{11520 a \sqrt{-c\left(a^{2} x^{2}-1\right)}}-\frac{77 \text{ar}}{}$

default	$\frac{x\left(-a^2 c\,x^2+c\right)^{\frac{9}{2}}}{10}+\frac{9c\left(\frac{x\left(-a^2 c\,x^2+c\right)^{\frac{7}{2}}}{8}+\frac{7c\left(\frac{x\left(-a^2 c\,x^2+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(-a^2 c\,x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2 c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a\cdots}}{\sqrt{-a^2c\cdots}}\right)}{2\sqrt{a^2c\cdots}}\right)}{4}\right)}{6}\right)}{8}\right)}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x*(-a^2*c*x^2+c)^(9/2)+9/10*c*(1/8*x*(-a^2*c*x^2+c)^(7/2)+7/8*c*(1/6*x
*(-a^2*c*x^2+c)^(5/2)+5/6*c*(1/4*x*(-a^2*c*x^2+c)^(3/2)+3/4*c*(1/2*x*(-a^2*
c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1
/2))))))+2/a*(1/9*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(9/2)-a*c*(-1/16*(-2*a^2
*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(7/2)+7/8*c*(-1/12
*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(5/2)+5/6*
c*(-1/8*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(3/
2)+3/4*c*(-1/4*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a
*c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x
-1/a)*a*c)^(1/2)))))))
```

Maxima [A]
time = 0.51, size = 192, normalized size = 1.09

$$\frac{1}{10}\left(-a^{2}cx^{2}+c\right)^{\frac{9}{2}}x-\frac{11}{80}\left(-a^{2}cx^{2}+c\right)^{\frac{7}{2}}cx-\frac{77}{480}\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}c^{2}x-\frac{77}{384}\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}c^{3}x-\frac{35}{64}\sqrt{a^{2}cx^{2}-4\,acx+3\,c}\,c^{4}x+\frac{63}{256}\sqrt{-a^{2}cx^{2}+c}\,c^{4}x-\frac{35\,c^{6}\arcsin\left(ax-2\right)}{64\,a\left(-c\right)^{\frac{3}{2}}}+\frac{63\,c^{\frac{9}{2}}\arcsin\left(ax\right)}{256\,a}+\frac{2\left(-a^{2}cx^{2}+c\right)^{\frac{9}{2}}}{9\,a}+\frac{35\sqrt{a^{2}cx^{2}-4\,acx+3\,c}\,c^{4}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/10*(-a^2*c*x^2 + c)^(9/2)*x - 11/80*(-a^2*c*x^2 + c)^(7/2)*c*x - 77/480*(
-a^2*c*x^2 + c)^(5/2)*c^2*x - 77/384*(-a^2*c*x^2 + c)^(3/2)*c^3*x - 35/64*s
qrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4*x + 63/256*sqrt(-a^2*c*x^2 + c)*c^4*x -
35/64*c^6*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 63/256*c^(9/2)*arcsin(a*x)/a + 2
/9*(-a^2*c*x^2 + c)^(9/2)/a + 35/32*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^4/a
```

Fricas [A]
time = 0.41, size = 329, normalized size = 1.87

$$\left[\frac{3465\sqrt{-c}\,c^{4}\log\left(2\,a^{2}cx^{2}-2\sqrt{-a^{2}cx^{2}+c}\,a\sqrt{-c}\,x-c\right)+2\left(1152\,a^{9}c^{4}x^{9}+2560\,a^{8}c^{4}x^{8}-3024\,a^{7}c^{4}x^{7}-10240\,a^{6}c^{4}x^{6}+312\,a^{5}c^{4}x^{5}+15360\,a^{4}c^{4}x^{4}+6150\,a^{3}c^{4}x^{3}-10240\,a^{2}c^{4}x^{2}-8055\,ac^{4}x+2560\,c^{4}\right)\sqrt{-a^{2}cx^{2}+c}}{23040\,a},\ \frac{3465\,c^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-a^{2}cx^{2}+c}\,a\sqrt{c}\,x}{a^{2}cx^{2}-c}\right)+\left(1152\,a^{9}c^{4}x^{9}+2560\,a^{8}c^{4}x^{8}-3024\,a^{7}c^{4}x^{7}-10240\,a^{6}c^{4}x^{6}+312\,a^{5}c^{4}x^{5}+15360\,a^{4}c^{4}x^{4}+6150\,a^{3}c^{4}x^{3}-10240\,a^{2}c^{4}x^{2}-8055\,ac^{4}x+2560\,c^{4}\right)\sqrt{-a^{2}cx^{2}+c}}{11520\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(3465*sqrt(-c)*c^4*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt
(-c)*x - c) + 2*(1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 1
0240*a^6*c^4*x^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 -
 10240*a^2*c^4*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a, 1/11
520*(3465*c^(9/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c))
+ (1152*a^9*c^4*x^9 + 2560*a^8*c^4*x^8 - 3024*a^7*c^4*x^7 - 10240*a^6*c^4*x
^6 + 312*a^5*c^4*x^5 + 15360*a^4*c^4*x^4 + 6150*a^3*c^4*x^3 - 10240*a^2*c^4
*x^2 - 8055*a*c^4*x + 2560*c^4)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 150.00, size = 1340, normalized size = 7.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] a**8*c**4*Piecewise((I*a**2*sqrt(c)*x**11/(10*sqrt(a**2*x**2 - 1)) - 9*I*sq
rt(c)*x**9/(80*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**7/(480*a**2*sqrt(a**2*x*
*2 - 1)) - 7*I*sqrt(c)*x**5/(1920*a**4*sqrt(a**2*x**2 - 1)) - 7*I*sqrt(c)*x
**3/(768*a**6*sqrt(a**2*x**2 - 1)) + 7*I*sqrt(c)*x/(256*a**8*sqrt(a**2*x**2
 - 1)) - 7*I*sqrt(c)*acosh(a*x)/(256*a**9), Abs(a**2*x**2) > 1), (-a**2*sqr
t(c)*x**11/(10*sqrt(-a**2*x**2 + 1)) + 9*sqrt(c)*x**9/(80*sqrt(-a**2*x**2 +
 1)) + sqrt(c)*x**7/(480*a**2*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**5/(1920*
a**4*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**3/(768*a**6*sqrt(-a**2*x**2 + 1))
 - 7*sqrt(c)*x/(256*a**8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*asin(a*x)/(256*a
**9), True)) + 2*a**7*c**4*Piecewise((x**8*sqrt(-a**2*c*x**2 + c)/9 - x**6*
sqrt(-a**2*c*x**2 + c)/(63*a**2) - 2*x**4*sqrt(-a**2*c*x**2 + c)/(105*a**4)
 - 8*x**2*sqrt(-a**2*c*x**2 + c)/(315*a**6) - 16*sqrt(-a**2*c*x**2 + c)/(31
5*a**8), Ne(a, 0)), (sqrt(c)*x**8/8, True)) - 2*a**6*c**4*Piecewise((I*a**2
*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqrt(c)*x**7/(48*sqrt(a**2*x**2
 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**3/(
384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(128*a**6*sqrt(a**2*x**2 - 1)
) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*
x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/(48*sqrt(-a**2*x**2 + 1)) +
sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**3/(384*a**4*sqr
t(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c
)*asin(a*x)/(128*a**7), True)) - 6*a**5*c**4*Piecewise((x**6*sqrt(-a**2*c*x
**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a**2) - 4*x**2*sqrt(-a**2*c*x*
*2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(105*a**6), Ne(a, 0)), (sqrt(
c)*x**6/6, True)) + 6*a**3*c**4*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 -
x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4),
 Ne(a, 0)), (sqrt(c)*x**4/4, True)) + 2*a**2*c**4*Piecewise((I*a**2*sqrt(c)
*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) +
I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), A
bs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c
)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) +
 sqrt(c)*asin(a*x)/(8*a**3), True)) - 2*a*c**4*Piecewise((0, Eq(c, 0)), (sq
rt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True))
- c**4*Piecewise((I*sqrt(c)*x*sqrt(a**2*x**2 - 1)/2 - I*sqrt(c)*acosh(a*x)/
(2*a), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**3/(2*sqrt(-a**2*x**2 + 1)) +
sqrt(c)*x/(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A]

time = 0.43, size = 164, normalized size = 0.93

$$\frac{77\,c^5\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{256\,\sqrt{-c}\,|a|}+\frac{1}{11520}\,\sqrt{-a^2cx^2+c}\left(\frac{2560\,c^4}{a}-\left(8055\,c^4+2\left(5120\,ac^4-\left(3075\,a^2c^4+4\left(1920\,a^3c^4+\left(39\,a^4c^4-2\left(640\,a^5c^4+\left(189\,a^6c^4-8\left(9\,a^8c^4x+20\,a^7c^4\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 77/256*c^5*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/11520*sqrt(-a^2*c*x^2 + c)*(2560*c^4/a - (8055*c^4 + 2*(5120*a*c^4 -
 (3075*a^2*c^4 + 4*(1920*a^3*c^4 + (39*a^4*c^4 - 2*(640*a^5*c^4 + (189*a^6*
c^4 - 8*(9*a^8*c^4*x + 20*a^7*c^4)*x)*x)*x)*x)*x)*x)*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{9/2}\,\left(a\,x+1\right)}{a\,x-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(9/2)*(a*x + 1))/(a*x - 1), x)
```

3.624 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=153

$$-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}$$

```
[Out] -15/64*c^2*x*(-a^2*c*x^2+c)^(3/2)-3/16*c*x*(-a^2*c*x^2+c)^(5/2)+9/56*(-a^2*
c*x^2+c)^(7/2)/a+1/8*(a*x+1)*(-a^2*c*x^2+c)^(7/2)/a-45/128*c^(7/2)*arctan(a
*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-45/128*c^3*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$-\frac{45c^{7/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{128a}-\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{7/2}}{8a}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-45*c^3*x*Sqrt[c - a^2*c*x^2])/128 - (15*c^2*x*(c - a^2*c*x^2)^(3/2))/64 -
 (3*c*x*(c - a^2*c*x^2)^(5/2))/16 + (9*(c - a^2*c*x^2)^(7/2))/(56*a) + ((1
+ a*x)*(c - a^2*c*x^2)^(7/2))/(8*a) - (45*c^(7/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(128*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$$
$$= -\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{5/2}\,dx\right)$$
$$= \frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int(1+ax)\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= \frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{8}(9c)\int\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$= -\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{7/2}}{8a}-\frac{1}{16}($$
$$= -\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9\left(c-a^2cx^2\right)^{7/2}}{56a}+\frac{(1+a}{}$$
$$= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9(}{}$$
$$= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9(}{}$$
$$= -\frac{45}{128}c^3x\sqrt{c-a^2cx^2}-\frac{15}{64}c^2x\left(c-a^2cx^2\right)^{3/2}-\frac{3}{16}cx\left(c-a^2cx^2\right)^{5/2}+\frac{9(}{}$$

<u>**Mathematica**</u> **[A]**
time = 0.14, size = 151, normalized size = 0.99

$$\frac{c^3\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}\,(256-837ax-187a^2x^2+978a^3x^3+558a^4x^4-600a^5x^5-424a^6x^6+144a^7x^7+112a^8x^8)+630\sqrt{1-ax}\,\mathrm{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{896a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(256 - 837*a*x - 187*a^2*x^2 + 978*
a^3*x^3 + 558*a^4*x^4 - 600*a^5*x^5 - 424*a^6*x^6 + 144*a^7*x^7 + 112*a^8*x
^8) + 630*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(896*a*Sqrt[1 - a*x
]*Sqrt[1 - a^2*x^2])
```

<u>**Maple**</u> **[B]** Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(125) = 250$.
time = 0.20, size = 375, normalized size = 2.45

method	result

risch	$\frac{(112a^7x^7+256a^6x^6-168a^5x^5-768a^4x^4-210a^3x^3+768a^2x^2+581ax-256)(a^2x^2-1)c^4}{896a\sqrt{-c(a^2x^2-1)}}-\frac{45\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c^4}{128\sqrt{a^2c}}$
default	$\frac{x(-a^2cx^2+c)^{\frac{7}{2}}}{8}+\frac{7c\left(\frac{x(-a^2cx^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(-a^2cx^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2cx^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}\right)}{6}\right)}{8}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(-a^2*c*x^2+c)^(7/2)+7/8*c*(1/6*x*(-a^2*c*x^2+c)^(5/2)+5/6*c*(1/4*x*(
-a^2*c*x^2+c)^(3/2)+3/4*c*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*a
rctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))))+2/a*(1/7*(-a^2*c*(x-1/a)^2-2
*(x-1/a)*a*c)^(7/2)-a*c*(-1/12*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(x-1/
a)^2-2*(x-1/a)*a*c)^(5/2)+5/6*c*(-1/8*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*
c*(x-1/a)^2-2*(x-1/a)*a*c)^(3/2)+3/4*c*(-1/4*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c
*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2))))))
```

Maxima [A]

time = 0.48, size = 173, normalized size = 1.13

$$\frac{1}{8}\left(-a^{2} c x^{2}+c\right)^{\frac{7}{2}} x-\frac{3}{16}\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} c x-\frac{15}{64}\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} c^{2} x-\frac{5}{8} \sqrt{a^{2} c x^{2}-4 a c x+3 c}\, c^{3} x+\frac{35}{128} \sqrt{-a^{2} c x^{2}+c}\, c^{3} x-\frac{5 c^{5} \arcsin (a x-2)}{8 a(-c)^{\frac{3}{2}}}+\frac{35 c^{\frac{7}{2}} \arcsin (a x)}{128 a}+\frac{2\left(-a^{2} c x^{2}+c\right)^{\frac{7}{2}}}{7 a}+\frac{5 \sqrt{a^{2} c x^{2}-4 a c x+3 c}\, c^{3}}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(-a^2*c*x^2 + c)^(7/2)*x - 3/16*(-a^2*c*x^2 + c)^(5/2)*c*x - 15/64*(-a^
2*c*x^2 + c)^(3/2)*c^2*x - 5/8*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^3*x + 35/1
28*sqrt(-a^2*c*x^2 + c)*c^3*x - 5/8*c^5*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 35
/128*c^(7/2)*arcsin(a*x)/a + 2/7*(-a^2*c*x^2 + c)^(7/2)/a + 5/4*sqrt(a^2*c*
x^2 - 4*a*c*x + 3*c)*c^3/a
```

Fricas [A]
time = 0.38, size = 286, normalized size = 1.87

$$\left[\frac{315 \sqrt{-c}\, c^{3} \log \left(2 a^{2} c x^{2}-2 \sqrt{-a^{2} c x^{2}+c}\, a \sqrt{-c}\, x-c\right)-2\left(112 a^{7} c^{3} x^{7}+256 a^{6} c^{3} x^{6}-168 a^{5} c^{3} x^{5}-768 a^{4} c^{3} x^{4}-210 a^{3} c^{3} x^{3}+768 a^{2} c^{3} x^{2}+581 a c^{3} x-256 c^{3}\right) \sqrt{-a^{2} c x^{2}+c}}{1792 a}, \frac{315 c^{\frac{7}{2}} \arctan \left(\frac{\sqrt{-a^{2} c x^{2}+c}\, a \sqrt{c}\, x}{a^{2} c x^{2}-c}\right)-\left(112 a^{7} c^{3} x^{7}+256 a^{6} c^{3} x^{6}-168 a^{5} c^{3} x^{5}-768 a^{4} c^{3} x^{4}-210 a^{3} c^{3} x^{3}+768 a^{2} c^{3} x^{2}+581 a c^{3} x-256 c^{3}\right) \sqrt{-a^{2} c x^{2}+c}}{896 a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/1792*(315*sqrt(-c)*c^3*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 2*(112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^5 - 768*a^
4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256*c^3)*sqrt
(-a^2*c*x^2 + c))/a, 1/896*(315*c^(7/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(
c)*x/(a^2*c*x^2 - c)) - (112*a^7*c^3*x^7 + 256*a^6*c^3*x^6 - 168*a^5*c^3*x^
5 - 768*a^4*c^3*x^4 - 210*a^3*c^3*x^3 + 768*a^2*c^3*x^2 + 581*a*c^3*x - 256
*c^3)*sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 33.01, size = 1090, normalized size = 7.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] -a**6*c**3*Piecewise((I*a**2*sqrt(c)*x**9/(8*sqrt(a**2*x**2 - 1)) - 7*I*sqr
t(c)*x**7/(48*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**5/(192*a**2*sqrt(a**2*x**
2 - 1)) - 5*I*sqrt(c)*x**3/(384*a**4*sqrt(a**2*x**2 - 1)) + 5*I*sqrt(c)*x/(
128*a**6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*acosh(a*x)/(128*a**7), Abs(a**2
*x**2) > 1), (-a**2*sqrt(c)*x**9/(8*sqrt(-a**2*x**2 + 1)) + 7*sqrt(c)*x**7/
(48*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**5/(192*a**2*sqrt(-a**2*x**2 + 1)) +
5*sqrt(c)*x**3/(384*a**4*sqrt(-a**2*x**2 + 1)) - 5*sqrt(c)*x/(128*a**6*sqrt
(-a**2*x**2 + 1)) + 5*sqrt(c)*asin(a*x)/(128*a**7), True)) - 2*a**5*c**3*Pi
```

```
ecewise((x**6*sqrt(-a**2*c*x**2 + c)/7 - x**4*sqrt(-a**2*c*x**2 + c)/(35*a*
*2) - 4*x**2*sqrt(-a**2*c*x**2 + c)/(105*a**4) - 8*sqrt(-a**2*c*x**2 + c)/(
105*a**6), Ne(a, 0)), (sqrt(c)*x**6/6, True)) + a**4*c**3*Piecewise((I*a**2
*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt(c)*x**5/(24*sqrt(a**2*x**2
 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(16*a**
4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(16*a**5), Abs(a**2*x**2) > 1
), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1)) + 5*sqrt(c)*x**5/(24*sqrt(-
a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a**2*x**2 + 1)) - sqrt(c)*x/(
16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(16*a**5), True)) + 4*a**
3*c**3*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5 - x**2*sqrt(-a**2*c*x**2 +
c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4), Ne(a, 0)), (sqrt(c)*x**4
/4, True)) + a**2*c**3*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1
)) - 3*I*sqrt(c)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a*
*2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*
sqrt(c)*x**5/(4*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 +
 1)) - sqrt(c)*x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3)
, True)) - 2*a*c**3*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)),
 (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)) - c**3*Piecewise((I*sqrt(c)
*x*sqrt(a**2*x**2 - 1)/2 - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1),
 (-a**2*sqrt(c)*x**3/(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x/(2*sqrt(-a**2*x**
2 + 1)) + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A]
time = 0.42, size = 141, normalized size = 0.92

$$\frac{45\,c^4\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{128\,\sqrt{-c}\,|a|}+\frac{1}{896}\,\sqrt{-a^2cx^2+c}\left(\frac{256\,c^3}{a}-\left(581\,c^3+2\,\left(384\,ac^3-\left(105\,a^2c^3+4\,\left(96\,a^3c^3+\left(21\,a^4c^3-2\,\left(7\,a^6c^3x+16\,a^5c^3\right)x\right)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 45/128*c^4*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a
)) + 1/896*sqrt(-a^2*c*x^2 + c)*(256*c^3/a - (581*c^3 + 2*(384*a*c^3 - (105
*a^2*c^3 + 4*(96*a^3*c^3 + (21*a^4*c^3 - 2*(7*a^6*c^3*x + 16*a^5*c^3)*x)*x)
*x)*x)*x)*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,\left(a\,x+1\right)}{a\,x-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.625 $\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=130

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

```
[Out] -7/24*c*x*(-a^2*c*x^2+c)^(3/2)+7/30*(-a^2*c*x^2+c)^(5/2)/a+1/6*(a*x+1)*(-a^
2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-
7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.292$, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$-\frac{7c^{5/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{(ax+1)\left(c-a^2cx^2\right)^{5/2}}{6a}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 + (7*(
c - a^2*c*x^2)^(5/2))/(30*a) + ((1 + a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$=-\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\right)$$
$$=\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int(1+ax)\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}(7$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)}{}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)}{}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}+\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}+\frac{(1+ax)}{}$$

Mathematica [A]
time = 0.16, size = 135, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}\,(96-231ax-57a^2x^2+182a^3x^3+106a^4x^4-56a^5x^5-40a^6x^6)+210\sqrt{1-ax}\,\mathrm{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(96 - 231*a*x - 57*a^2*x^2 + 182*a^
3*x^3 + 106*a^4*x^4 - 56*a^5*x^5 - 40*a^6*x^6) + 210*Sqrt[1 - a*x]*ArcSin[S
qrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(106) = 212$.
time = 0.19, size = 296, normalized size = 2.28

method	result
risch	$-\frac{(40a^5x^5+96a^4x^4-10a^3x^3-192a^2x^2-135ax+96)(a^2x^2-1)c^3}{240a\sqrt{-c(a^2x^2-1)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2cx^2+c}}\right)c^3}{16\sqrt{a^2c}}$

default	$\frac{x\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^{2} c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2} c}\,x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{2\sqrt{a^{2} c}}\right)}{4}\right)}{6}+\frac{2\left(-a^{2} c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right.\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)+5/6*c*(1/4*x*(-a^2*c*x^2+c)^(3/2)+3/4*c*(1/2*x*(
-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))))+2/a*(1/5*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(5/2)-a*c*(-1/8*(-2*a
^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(3/2)+3/4*c*(-1/
4*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)+1/2
*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1
/2)))))
```

Maxima [A]

time = 0.48, size = 154, normalized size = 1.18

$$\frac{1}{6}\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}x-\frac{7}{24}\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}cx-\frac{3}{4}\sqrt{a^{2}cx^{2}-4\,acx+3\,c}\,c^{2}x+\frac{5}{16}\sqrt{-a^{2}cx^{2}+c}\,c^{2}x-\frac{3\,c^{4}\arcsin\left(ax-2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}}+\frac{5\,c^{\frac{5}{2}}\arcsin\left(ax\right)}{16\,a}+\frac{2\left(-a^{2}cx^{2}+c\right)^{\frac{5}{2}}}{5\,a}+\frac{3\sqrt{a^{2}cx^{2}-4\,acx+3\,c}\,c^{2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(a
^2*c*x^2 - 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x - 3/4*c^4
*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a + 2/5*(-a^2*c*
x^2 + c)^(5/2)/a + 3/2*sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c^2/a
```

Fricas [A]

time = 0.37, size = 241, normalized size = 1.85

$$\left[\frac{105\sqrt{-c}\,c^{2}\log\left(2\,a^{2}cx^{2}-2\sqrt{-a^{2}cx^{2}+c}\,a\sqrt{-c}\,x-c\right)+2\left(40\,a^{5}c^{2}x^{5}+96\,a^{4}c^{2}x^{4}-10\,a^{3}c^{2}x^{3}-192\,a^{2}c^{2}x^{2}-135\,ac^{2}x+96\,c^{2}\right)\sqrt{-a^{2}cx^{2}+c}}{480\,a},\ \frac{105\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a^{2}cx^{2}+c}\,a\sqrt{c}\,x}{a^{2}cx^{2}-c}\right)+\left(40\,a^{5}c^{2}x^{5}+96\,a^{4}c^{2}x^{4}-10\,a^{3}c^{2}x^{3}-192\,a^{2}c^{2}x^{2}-135\,ac^{2}x+96\,c^{2}\right)\sqrt{-a^{2}cx^{2}+c}}{240\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) + 2*(40*a^5*c^2*x^5 + 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^
2*x^2 - 135*a*c^2*x + 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*a
rctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 +
 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 192*a^2*c^2*x^2 - 135*a*c^2*x + 96*c^2)*
sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 9.15, size = 476, normalized size = 3.66

$$a^4c^2\left(\begin{cases} \frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}\,x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2|>1 \\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}\,x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\operatorname{asin}(ax)}{16a^5} & \text{otherwise} \end{cases}\right) + 2a^3c^2\left(\begin{cases} \frac{x^4\sqrt{-a^2cx^2+c}}{5} - \frac{x^2\sqrt{-a^2cx^2+c}}{15a^2} - \frac{2\sqrt{-a^2cx^2+c}}{15a^4} & \text{for } a\neq 0 \\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise} \end{cases}\right) - 2ac^2\left(\begin{cases} 0 & \text{for } c=0 \\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0 \\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise} \end{cases}\right) - c^2\left(\begin{cases} \frac{i\sqrt{c}\,x\sqrt{a^2x^2-1}}{2} - \frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{2a} & \text{for } |a^2x^2|>1 \\ -\frac{a^2\sqrt{c}\,x^3}{2\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,x}{2\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\operatorname{asin}(ax)}{2a} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt
(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
- 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(1
6*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
+ 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a*
*2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(16*a**5), True)) + 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
- x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4
), Ne(a, 0)), (sqrt(c)*x**4/4, True)) - 2*a*c**2*Piecewise((0, Eq(c, 0)), (
sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)
) - c**2*Piecewise((I*sqrt(c)*x*sqrt(a**2*x**2 - 1)/2 - I*sqrt(c)*acosh(a*x
)/(2*a), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**3/(2*sqrt(-a**2*x**2 + 1))
+ sqrt(c)*x/(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A]
time = 0.42, size = 116, normalized size = 0.89

$$\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{16\sqrt{-c}\,|a|} - \frac{1}{240}\sqrt{-a^2cx^2+c}\left(\left(135\,c^2+2\left(96\,ac^2+\left(5\,a^2c^2-4\left(5\,a^4c^2x+12\,a^3c^2\right)x\right)x\right)x\right)x-\frac{96\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 + 2*(96*a*c^2 + (5*a^2*c^2 - 4*(5*a
^4*c^2*x + 12*a^3*c^2)*x)*x)*x)*x - 96*c^2/a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.626 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal. Leaf size=107

$$-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5(c-a^2cx^2)^{3/2}}{12a}+\frac{(1+ax)\,(c-a^2cx^2)^{3/2}}{4a}-\frac{5c^{3/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

```
[Out] 5/12*(-a^2*c*x^2+c)^(3/2)/a+1/4*(a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/2)*
arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6276, 685, 655, 201, 223, 209}

$$-\frac{5c^{3/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{(ax+1)\,(c-a^2cx^2)^{3/2}}{4a}+\frac{5(c-a^2cx^2)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 + (5*(c - a^2*c*x^2)^(3/2))/(12*a) + ((1 + a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx\\
&= -\left(c\int (1+ax)^2\sqrt{c-a^2cx^2}\,dx\right)\\
&= \frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int (1+ax)\sqrt{c-a^2cx^2}\,dx\\
&= \frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int \sqrt{c-a^2cx^2}\,dx\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)\mathrm{S}\\
&= -\frac{5}{8}cx\sqrt{c-a^2cx^2}+\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{(1+ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\tan}{}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 117, normalized size = 1.09

$$\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}\,(16-25ax-7a^2x^2+10a^3x^3+6a^4x^4)+30\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(16 - 25*a*x - 7*a^2*x^2 + 10*a^3*x^3
 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqrt
[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(87) = 174.
time = 0.19, size = 217, normalized size = 2.03

method	result
risch	$\frac{(6a^3x^3+16a^2x^2+9ax-16)(a^2x^2-1)c^2}{24a\sqrt{-c(a^2x^2-1)}}-\frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c^2}{8\sqrt{a^2c}}$
default	$\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}+\frac{\frac{2\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3}-2ac\left(-\frac{(-2a\ldots}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)+3/4*c*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))+2/a*(1/3*(-a^2*c*(x-1/a
)^2-2*(x-1/a)*a*c)^(3/2)-a*c*(-1/4*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c*(-a^2*c*(
x-1/a)^2-2*(x-1/a)*a*c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-
a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2))))
```

Maxima [A]
time = 0.48, size = 131, normalized size = 1.22

$$\frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x-\sqrt{a^2cx^2-4\,acx+3\,c}\;cx+\frac{3}{8}\sqrt{-a^2cx^2+c}\;cx-\frac{c^3\arcsin\left(ax-2\right)}{a\left(-c\right)^{\frac{3}{2}}}+\frac{3\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{8\,a}+\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a}+\frac{2\sqrt{a^2cx^2-4\,acx+3\,c}\;c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 - 4*a*c*x + 3*c)*c*x + 3/8*sq
rt(-a^2*c*x^2 + c)*c*x - c^3*arcsin(a*x - 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)*a
rcsin(a*x)/a + 2/3*(-a^2*c*x^2 + c)^(3/2)/a + 2*sqrt(a^2*c*x^2 - 4*a*c*x +
3*c)*c/a
```

Fricas [A]
time = 0.38, size = 180, normalized size = 1.68

$$\left[\frac{15\sqrt{-c}\,c\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\,(6\,a^3cx^3+16\,a^2cx^2+9\,acx-16\,c)\sqrt{-a^2cx^2+c}}{48\,a},\ \frac{15\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)-(6\,a^3cx^3+16\,a^2cx^2+9\,acx-16\,c)\sqrt{-a^2cx^2+c}}{24\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - 2*(6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) - (6*a^3*c*x^3 + 16*a^2*c*x^2 + 9*a*c*x - 16*c)*sqrt(-a^2*c*x^2 + c))/a
]
```

Sympy [C] Result contains complex when optimal does not.
time = 4.91, size = 340, normalized size = 3.18

$$-a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)-2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{i\sqrt{c}\,x\sqrt{a^2x^2-1}}{2}-\frac{i\sqrt{c}\operatorname{acosh}(ax)}{2a} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^3}{2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,x}{2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] -a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c
)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
 I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4
*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*
x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) - 2*a*
c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
 c)**(3/2)/(3*a**2*c), True)) - c*Piecewise((I*sqrt(c)*x*sqrt(a**2*x**2 - 1
)/2 - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**3/
(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x/(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asi
n(a*x)/(2*a), True))
```

Giac [A]
time = 0.41, size = 85, normalized size = 0.79

$$-\frac{1}{24}\,\sqrt{-a^2cx^2+c}\,\left(\left(2\,\left(3\,a^2cx+8\,ac\right)x+9\,c\right)x-\frac{16\,c}{a}\right)+\frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x + 8*a*c)*x + 9*c)*x - 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.627 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)^
(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {6302, 6276, 685, 655, 223, 209}

$$-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
```

```
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int\frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int\frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\text{Subst}\left(\int\frac{1}{1+a^2cx^2}\,dx,x,-\right.\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.88

$$
\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])

Maple [A]
time = 0.19, size = 136, normalized size = 1.58

method	result
risch	$-\frac{(ax+4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))

Maxima [A]
time = 0.47, size = 47, normalized size = 0.55

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\,x-\frac{3\sqrt{c}\,\arcsin(ax)}{2a}+\frac{2\sqrt{-a^2cx^2+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2 + c)/a

Fricas [A]
time = 0.38, size = 134, normalized size = 1.56

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{-c}\,\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4a},\ \frac{\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.41, size = 62, normalized size = 0.72

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.628 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=59

$$-\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\frac{\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)-2*(a*x+1)/a/(-a^2*c*x^2+
c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6302, 6276, 667, 223, 209}

$$\frac{\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}-\frac{2(ax+1)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*(1 + a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*
c*x^2]]/(a*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{3/2}}\,dx\right)\\ &= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= -\frac{2(1+ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 1.39

$$-\frac{2\sqrt{1-a^2x^2}\left(\sqrt{1+ax}+\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\,\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*(Sqrt[1 + a*x] + Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/S
qrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.14, size = 79, normalized size = 1.34

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}}{a^2c\left(x-\frac{1}{a}\right)}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x-1/a
)*(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)
```

Maxima [A]
time = 0.47, size = 40, normalized size = 0.68

$$\frac{2\sqrt{-a^2cx^2+c}}{a^2cx-ac}+\frac{\arcsin(ax)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x - a*c) + arcsin(a*x)/(a*sqrt(c))
```

Fricas [A]
time = 0.37, size = 153, normalized size = 2.59

$$\left[-\frac{(ax-1)\sqrt{-c}\,\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-4\,\sqrt{-a^2cx^2+c}}{2\,(a^2cx-ac)},-\frac{(ax-1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)-2\,\sqrt{-a^2cx^2+c}}{a^2cx-ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*x - 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x - a*c), -((a*x - 1)*sqrt(c)*ar
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x - a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x+1}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - a^2*c*x^2)^(1/2)*(a*x - 1)), x)
```

3.629 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2(1+ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] -2/3*(a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6302, 6276, 667, 197}

$$-\frac{x}{3c\sqrt{c-a^2cx^2}}-\frac{2(ax+1)}{3a\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (-2*(1 + a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{5/2}}\,dx\right)\\
&= -\frac{2(1+ax)}{3a\,(c-a^2cx^2)^{3/2}}-\frac{1}{3}\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx\\
&= -\frac{2(1+ax)}{3a\,(c-a^2cx^2)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 64, normalized size = 1.25

$$
-\frac{(2-ax)\sqrt{1+ax}\,\sqrt{1-a^2x^2}}{3ac(1-ax)^{3/2}\sqrt{c-a^2cx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/3*((2 - a*x)*Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2])/(a*c*(1 - a*x)^(3/2)*Sqrt[
c - a^2*c*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(43) = 86.
time = 0.18, size = 127, normalized size = 2.49

method	result
gosper	$\frac{(ax+1)^2(ax-2)}{3a(-a^2c\,x^2+c)^{\frac{3}{2}}}$
trager	$\frac{(ax-2)\sqrt{-a^2c\,x^2+c}}{3c^2(ax-1)^2a}$
default	$\frac{x}{c\sqrt{-a^2c\,x^2+c}}+\frac{\frac{2}{3ac\left(x-\frac{1}{a}\right)\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}}+\frac{2\left(-2a^2c\left(x-\frac{1}{a}\right)-2ac\right)}{3a\,c^2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/c/(-a^2*c*x^2+c)^(1/2)+2/a*(1/3/a/c/(x-1/a)/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a
*c)^(1/2)+1/3/a/c^2*(-2*a^2*c*(x-1/a)-2*a*c)/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*
c)^(1/2))
```

Maxima [A]
time = 0.26, size = 61, normalized size = 1.20

$$-\frac{x}{3\sqrt{-a^2cx^2+c}\,c}+\frac{2}{3\left(\sqrt{-a^2cx^2+c}\,a^2cx-\sqrt{-a^2cx^2+c}\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x - sqrt(
-a^2*c*x^2 + c)*a*c)
```

Fricas [A]
time = 0.39, size = 47, normalized size = 0.92

$$\frac{\sqrt{-a^2cx^2+c}\,(ax-2)}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x - 2)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(43) = 86.
time = 0.43, size = 148, normalized size = 2.90

$$\frac{\left(ac-3\sqrt{-a^2c}\,\sqrt{c}\right)\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}-\sqrt{-a^2c}\,ac^2\right)}-\frac{2\left(2\,a^2c+3\,a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}+\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^3\mathrm{csgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(a*c - 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) - sqrt(-a^2*c)*a*c^2
) - 2/3*(2*a^2*c + 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqrt
(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) + sqrt(-a^2*c + c/x^2) - sqrt(
c)/x)^3*c*sgn(x))
```

Mupad [B]
time = 1.29, size = 33, normalized size = 0.65

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-2\right)}{3\,a\,c^2\,{\left(a\,x-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(3/2)*(a*x - 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x - 2))/(3*a*c^2*(a*x - 1)^2)
```

3.630 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2(1+ax)}{5a\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

```
[Out] -2/5*(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2/
(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6302, 6276, 667, 198, 197}

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2(ax+1)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*(1 + a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2))
- (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{7/2}}\,dx\right)\\
&= -\frac{2(1+ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{3}{5}\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx\\
&= -\frac{2(1+ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}-\frac{2\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{5c}\\
&= -\frac{2(1+ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.72

$$-\frac{2+ax-4a^2x^2+2a^3x^3}{5ac^2(-1+ax)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/5*(2 + a*x - 4*a^2*x^2 + 2*a^3*x^3)/(a*c^2*(-1 + a*x)^2*Sqrt[c - a^2*c*x
^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(62) = 124$.
time = 0.20, size = 206, normalized size = 2.78

method	result
gosper	$-\frac{(ax+1)^2(2a^3x^3-4a^2x^2+ax+2)}{5a(-a^2c\,x^2+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^3x^3-4a^2x^2+ax+2)\sqrt{-a^2c\,x^2+c}}{5c^3(ax-1)^3a(ax+1)}$
default	$\frac{x}{3c(-a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{-a^2c\,x^2+c}}+\frac{\frac{2}{5ac\left(x-\frac{1}{a}\right)\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\frac{8a\left(-\frac{-2a^2c\left(x-\frac{1}{a}\right)-2ac}{6a^2c^2\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}-\cdots\right.}{\cdots}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x/c/(-a^2*c*x^2+c)^(3/2)+2/3*x/c^2/(-a^2*c*x^2+c)^(1/2)+2/a*(1/5/a/c/(x
-1/a)/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(3/2)-4/5*a*(-1/6*(-2*a^2*c*(x-1/a)-
2*a*c)/a^2/c^2/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(3/2)-1/3/a^2/c^3*(-2*a^2*c
*(x-1/a)-2*a*c)/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.29, size = 80, normalized size = 1.08

$$\frac{2}{5\left((-a^2cx^2+c)^{\frac{3}{2}}a^2cx-(-a^2cx^2+c)^{\frac{3}{2}}ac\right)}-\frac{2\,x}{5\,\sqrt{-a^2cx^2+c}\,c^2}-\frac{x}{5\,(-a^2cx^2+c)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x - (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/(
sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

Fricas [A]
time = 0.40, size = 75, normalized size = 1.01

$$\frac{(2\,a^3x^3-4\,a^2x^2+ax+2)\sqrt{-a^2cx^2+c}}{5\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 - 4*a^2*x^2 + a*x + 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 - 2
*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x - 1)), x)
```

Mupad [B]
time = 1.38, size = 56, normalized size = 0.76

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(2\,a^3\,x^3-4\,a^2\,x^2+a\,x+2\right)}{5\,a\,c^3\,{\left(a\,x-1\right)}^3\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(5/2)*(a*x - 1)),x)
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x - 4*a^2*x^2 + 2*a^3*x^3 + 2))/(5*a*c^3*(a*x - 1
)^3*(a*x + 1))
```

3.631 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{2(1+ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

```
[Out] -2/7*(a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^2
/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6302, 6276, 667, 198, 197}

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{2(ax+1)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (-2*(1 + a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2))
- (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{9/2}}\,dx\right)\\
&= -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}}-\frac{5}{7}\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx\\
&= -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}}-\frac{x}{7c\,(c-a^2cx^2)^{5/2}}-\frac{4\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{7c}\\
&= -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}}-\frac{x}{7c\,(c-a^2cx^2)^{5/2}}-\frac{4x}{21c^2\,(c-a^2cx^2)^{3/2}}-\frac{8\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{21c^2}\\
&= -\frac{2(1+ax)}{7a\,(c-a^2cx^2)^{7/2}}-\frac{x}{7c\,(c-a^2cx^2)^{5/2}}-\frac{4x}{21c^2\,(c-a^2cx^2)^{3/2}}-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 96, normalized size = 0.99

$$-\frac{\sqrt{1-a^2x^2}\,(6+9ax-24a^2x^2+4a^3x^3+16a^4x^4-8a^5x^5)}{21ac^3(1-ax)^{7/2}(1+ax)^{3/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(6 + 9*a*x - 24*a^2*x^2 + 4*a^3*x^3 + 16*a^4*x^4 -
 8*a^5*x^5))/(a*c^3*(1 - a*x)^(7/2)*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(81) = 162.
time = 0.19, size = 292, normalized size = 3.01

method	result
gosper	$\frac{(ax+1)^2(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6)}{21a(-a^2cx^2+c)^{\frac{7}{2}}}$
trager	$\frac{(8a^5x^5-16a^4x^4-4a^3x^3+24a^2x^2-9ax-6)\sqrt{-a^2cx^2+c}}{21c^4(ax-1)^4(ax+1)^2a}$
default	$\frac{x}{5c(-a^2cx^2+c)^{\frac{5}{2}}}+\frac{\frac{4x}{15c(-a^2cx^2+c)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2cx^2+c}}}{c}+\frac{\frac{2}{7ac\left(x-\frac{1}{a}\right)\left(-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}-\frac{12a\left(-\frac{-2a^2c}{10a^2c^2\left(-a^2c(x\right.}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x/c/(-a^2*c*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(-a^2*c*x^2+c)^(3/2)+2/3*x/c^2/
(-a^2*c*x^2+c)^(1/2))+2/a*(1/7/a/c/(x-1/a)/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)
^(5/2)-6/7*a*(-1/10*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c^2/(-a^2*c*(x-1/a)^2-2*(x
-1/a)*a*c)^(5/2)+4/5/c*(-1/6*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c^2/(-a^2*c*(x-1/
a)^2-2*(x-1/a)*a*c)^(3/2)-1/3/a^2/c^3*(-2*a^2*c*(x-1/a)-2*a*c)/(-a^2*c*(x-1
/a)^2-2*(x-1/a)*a*c)^(1/2))))
```

Maxima [A]
time = 0.27, size = 99, normalized size = 1.02

$$\frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx-(-a^2cx^2+c)^{\frac{5}{2}}ac\right)}-\frac{8\,x}{21\,\sqrt{-a^2cx^2+c}\,c^3}-\frac{4\,x}{21\,(-a^2cx^2+c)^{\frac{3}{2}}c^2}-\frac{x}{7\,(-a^2cx^2+c)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x - (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x/
(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/((
-a^2*c*x^2 + c)^(5/2)*c)
```

Fricas [A]
time = 0.52, size = 124, normalized size = 1.28

$$\frac{(8\,a^5x^5-16\,a^4x^4-4\,a^3x^3+24\,a^2x^2-9\,ax-6)\sqrt{-a^2cx^2+c}}{21\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 - 16*a^4*x^4 - 4*a^3*x^3 + 24*a^2*x^2 - 9*a*x - 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a
^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x - 1)), x)
```

Mupad [B]
time = 1.45, size = 134, normalized size = 1.38

$$\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,{\left(a\,x-1\right)}^3}-\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,{\left(a\,x-1\right)}^4}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}+\frac{5}{28\,a\,c^4}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}+\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(7/2)*(a*x - 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x - 1)^3) - (c - a^2*c*x^2)^(1/2)/(28*a*
c^4*(a*x - 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) + 5/(28*a*c^4)))
/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*
(a*x + 1))
```

3.632 $\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2(1+ax)}{9a\left(c-a^2cx^2\right)^{9/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

```
[Out] -2/9*(a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^2
/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6302, 6276, 667, 198, 197}

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{15c^2\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{9c\left(c-a^2cx^2\right)^{7/2}}-\frac{2(ax+1)}{9a\left(c-a^2cx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-2*(1 + a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2))
- (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2
)) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{(c-a^2cx^2)^{11/2}}\,dx\right)\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}}-\frac{7}{9}\int \frac{1}{(c-a^2cx^2)^{9/2}}\,dx\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}}-\frac{x}{9c\,(c-a^2cx^2)^{7/2}}-\frac{2\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx}{3c}\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}}-\frac{x}{9c\,(c-a^2cx^2)^{7/2}}-\frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}}-\frac{8\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{15c^2}\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}}-\frac{x}{9c\,(c-a^2cx^2)^{7/2}}-\frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}}-\frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}}\\
&= -\frac{2(1+ax)}{9a\,(c-a^2cx^2)^{9/2}}-\frac{x}{9c\,(c-a^2cx^2)^{7/2}}-\frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}}-\frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 112, normalized size = 0.93

$$\frac{\sqrt{1-a^2x^2}\,(-10-25ax+60a^2x^2+10a^3x^3-80a^4x^4+24a^5x^5+32a^6x^6-16a^7x^7)}{45ac^4(1-ax)^{9/2}(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-10 - 25*a*x + 60*a^2*x^2 + 10*a^3*x^3 - 80*a^4*x^4 + 2
4*a^5*x^5 + 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(9/2)*(1 + a*x)^(
5/2)*Sqrt[c - a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(100) = 200$.
time = 0.20, size = 378, normalized size = 3.15

method	result
gosper	$-\frac{(ax+1)^{2}\left(16a^{7}x^{7}-32a^{6}x^{6}-24a^{5}x^{5}+80a^{4}x^{4}-10a^{3}x^{3}-60a^{2}x^{2}+25ax+10\right)}{45a\left(-a^{2}c\,x^{2}+c\right)^{\frac{9}{2}}}$
trager	$\frac{\left(16a^{7}x^{7}-32a^{6}x^{6}-24a^{5}x^{5}+80a^{4}x^{4}-10a^{3}x^{3}-60a^{2}x^{2}+25ax+10\right)\sqrt{-a^{2}c\,x^{2}+c}}{45c^{5}(ax-1)^{5}(ax+1)^{3}a}$
default	$\frac{x}{7c\left(-a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(-a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(-a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^{2}\sqrt{-a^{2}c\,x^{2}+c}}\right)}{7c}}{c}+\frac{2}{9ac\left(x-\frac{1}{a}\right)\left(-a^{2}c\left(x-\frac{1}{a}\right)^{2}-2\left(x-\frac{1}{a}\right)ac\right)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x/c/(-a^2*c*x^2+c)^(7/2)+6/7/c*(1/5*x/c/(-a^2*c*x^2+c)^(5/2)+4/5/c*(1/3
*x/c/(-a^2*c*x^2+c)^(3/2)+2/3*x/c^2/(-a^2*c*x^2+c)^(1/2)))+2/a*(1/9/a/c/(x-
1/a)/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(7/2)-8/9*a*(-1/14*(-2*a^2*c*(x-1/a)-
2*a*c)/a^2/c^2/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(7/2)+6/7/c*(-1/10*(-2*a^2*
c*(x-1/a)-2*a*c)/a^2/c^2/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(5/2)+4/5/c*(-1/6
*(-2*a^2*c*(x-1/a)-2*a*c)/a^2/c^2/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(3/2)-1/
3/a^2/c^3*(-2*a^2*c*(x-1/a)-2*a*c)/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))
))
```

Maxima [A]
time = 0.27, size = 118, normalized size = 0.98

$$\frac{2}{9\left((-a^{2}cx^{2}+c)^{\frac{7}{2}}a^{2}cx-(-a^{2}cx^{2}+c)^{\frac{7}{2}}ac\right)}-\frac{16\,x}{45\,\sqrt{-a^{2}cx^{2}+c}\,c^{4}}-\frac{8\,x}{45\,(-a^{2}cx^{2}+c)^{\frac{3}{2}}c^{3}}-\frac{2\,x}{15\,(-a^{2}cx^{2}+c)^{\frac{5}{2}}c^{2}}-\frac{x}{9\,(-a^{2}cx^{2}+c)^{\frac{7}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x - (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*x
/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*x/
((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

Fricas [A]
time = 0.66, size = 152, normalized size = 1.27

$$\frac{\left(16\,a^{7}x^{7}-32\,a^{6}x^{6}-24\,a^{5}x^{5}+80\,a^{4}x^{4}-10\,a^{3}x^{3}-60\,a^{2}x^{2}+25\,ax+10\right)\sqrt{-a^{2}cx^{2}+c}}{45\,\left(a^{9}c^{5}x^{8}-2\,a^{8}c^{5}x^{7}-2\,a^{7}c^{5}x^{6}+6\,a^{6}c^{5}x^{5}-6\,a^{4}c^{5}x^{3}+2\,a^{3}c^{5}x^{2}+2\,a^{2}c^{5}x-ac^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(16*a^7*x^7 - 32*a^6*x^6 - 24*a^5*x^5 + 80*a^4*x^4 - 10*a^3*x^3 - 60*a
^2*x^2 + 25*a*x + 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 - 2*a^8*c^5*x^7 - 2
*a^7*c^5*x^6 + 6*a^6*c^5*x^5 - 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 + 2*a^2*c^5*x
- a*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{9}{2}}\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x - 1)), x)
```

Mupad [B]
time = 1.51, size = 177, normalized size = 1.48

$$\frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,{\left(a\,x-1\right)}^5}-\frac{5\,\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,{\left(a\,x-1\right)}^4}+\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}+\frac{5}{24\,a\,c^5}\right)}{{\left(a\,x-1\right)}^3\,{\left(a\,x+1\right)}^3}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}-\frac{5}{144\,a\,c^5}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}+\frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - a^2*c*x^2)^(9/2)*(a*x - 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(72*a*c^5*(a*x - 1)^5) - (5*(c - a^2*c*x^2)^(1/2))/(1
44*a*c^5*(a*x - 1)^4) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) + 5/(24*a*
c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5) -
 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2))/(
45*c^5*(a*x - 1)*(a*x + 1))
```

3.633 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal. Leaf size=185

$$-\frac{8(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1+ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1+ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] -8/7*(a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(a*x+1
)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(a*x+1)^9*(-a^2*c
*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(a*x+1)^10*(-a^2*c*x^2+c)^(
9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A]
time = 0.14, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 45}

$$\frac{(ax+1)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(ax+1)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(ax+1)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(ax+1)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (-8*(1 + a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 + a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 + a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 + a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^3(1+ax)^6\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(-8(1+ax)^6+12(1+ax)^7-6(1+ax)^8+(1+ax)^9\right)\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

$$=-\frac{8(1+ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1+ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1+ax)^9\left(c-\right.}{3a^{10}\left(1-\frac{1}{a^2x^2}\right.}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.38

$$\frac{c^4(1+ax)^7\sqrt{c-a^2cx^2}\left(-44+98ax-77a^2x^2+21a^3x^3\right)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2),x]
[Out] (c^4*(1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(-44 + 98*a*x - 77*a^2*x^2 + 21*a^3*x^
3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 102, normalized size = 0.55

method	result	size
gosper	$\frac{x\left(21a^9x^9+70a^8x^8-240a^6x^6-210a^5x^5+252a^4x^4+420a^3x^3-315ax-210\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}}{210(ax-1)^3(ax+1)^6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$\frac{\left(21a^9x^9+70a^8x^8-240a^6x^6-210a^5x^5+252a^4x^4+420a^3x^3-315ax-210\right)x\,c^4\sqrt{-c\left(a^2x^2-1\right)}\,(ax-1)}{210(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210*(21*a^9*x^9+70*a^8*x^8-240*a^6*x^6-210*a^5*x^5+252*a^4*x^4+420*a^3*x^
3-315*a*x-210)*x*c^4*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/(a*x+1)^2/((a*x-1)/(a*x
+1))^(3/2)
```

Maxima [A]
time = 0.30, size = 204, normalized size = 1.10

$$\frac{\left(21\, a^{11} \sqrt{-c}\, c^{4} x^{11} + 49\, a^{10} \sqrt{-c}\, c^{4} x^{10} - 70\, a^{9} \sqrt{-c}\, c^{4} x^{9} - 240\, a^{8} \sqrt{-c}\, c^{4} x^{8} + 30\, a^{7} \sqrt{-c}\, c^{4} x^{7} + 462\, a^{6} \sqrt{-c}\, c^{4} x^{6} + 168\, a^{5} \sqrt{-c}\, c^{4} x^{5} - 420\, a^{4} \sqrt{-c}\, c^{4} x^{4} - 315\, a^{3} \sqrt{-c}\, c^{4} x^{3} + 105\, a^{2} \sqrt{-c}\, c^{4} x^{2} + 210\, \sqrt{-c}\, c^{4}\right) (a x + 1)^{2}}{210\, (a^{3} x^{2} + 2\, a^{2} x + a)(a x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 + 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 - 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 + 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 - 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 + 105*a^2*sqrt(-c)*c^4*x^2 + 210*sqrt(-c)*c^4)*(a*x
+ 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.51

$$\frac{\left(21\, a^{9} c^{4} x^{10} + 70\, a^{8} c^{4} x^{9} - 240\, a^{6} c^{4} x^{7} - 210\, a^{5} c^{4} x^{6} + 252\, a^{4} c^{4} x^{5} + 420\, a^{3} c^{4} x^{4} - 315\, a c^{4} x^{2} - 210\, c^{4} x\right) \sqrt{-a^{2} c}}{210\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="fric
as")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 + 70*a^8*c^4*x^9 - 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 + 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 - 210*c^4*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(9/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{9/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.634 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal. Leaf size=139

$$\frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] 2/3*(a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(a*x+1)^
7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(a*x+1)^8*(-a^2*c*x^
2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A]
time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6327, 6328, 45}

$$\frac{(ax+1)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(ax+1)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (2*(1 + a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 + a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 + a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{7/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^2(1+ax)^5\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(c-a^2cx^2\right)^{7/2}\int\left(4(1+ax)^5-4(1+ax)^6+(1+ax)^7\right)\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{2(1+ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1+ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1+ax)^8\left(c-a^2c\right.}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7\!/}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 0.45

$$-\frac{c^3(1+ax)^6\left(37-54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2),x]
[Out] -1/168*(c^3*(1 + a*x)^6*(37 - 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a^
2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 102, normalized size = 0.73

method	result	size
gosper	$\frac{x\left(21a^7x^7+72a^6x^6+28a^5x^5-168a^4x^4-210a^3x^3+56a^2x^2+252ax+168\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}}{168(ax+1)^5(ax-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	100
default	$-\frac{\left(21a^7x^7+72a^6x^6+28a^5x^5-168a^4x^4-210a^3x^3+56a^2x^2+252ax+168\right)x\,c^3\sqrt{-c\left(a^2x^2-1\right)}\,(ax-1)}{168(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/168*(21*a^7*x^7+72*a^6*x^6+28*a^5*x^5-168*a^4*x^4-210*a^3*x^3+56*a^2*x^2
+252*a*x+168)*x*c^3*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/(a*x+1)^2/((a*x-1)/(a*x+
1))^(3/2)
```

Maxima [A]
time = 0.29, size = 172, normalized size = 1.24

$$-\frac{\left(21\,a^9\sqrt{-c}\,c^3x^9+51\,a^8\sqrt{-c}\,c^3x^8-44\,a^7\sqrt{-c}\,c^3x^7-196\,a^6\sqrt{-c}\,c^3x^6-42\,a^5\sqrt{-c}\,c^3x^5+266\,a^4\sqrt{-c}\,c^3x^4+196\,a^3\sqrt{-c}\,c^3x^3-84\,a^2\sqrt{-c}\,c^3x^2-168\,\sqrt{-c}\,c^3\right)(ax+1)^2}{168\,(a^3x^2+2\,a^2x+a)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 + 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 - 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 + 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 - 84*a^2*sqrt(-c)*c^3*x^2 - 168*sq
rt(-c)*c^3)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.68

$$-\frac{\left(21\,a^7c^3x^8+72\,a^6c^3x^7+28\,a^5c^3x^6-168\,a^4c^3x^5-210\,a^3c^3x^4+56\,a^2c^3x^3+252\,ac^3x^2+168\,c^3x\right)\sqrt{-a^2c}}{168\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="fric
as")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 + 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 - 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 + 56*a^2*c^3*x^3 + 252*a*c^3*x^2 + 168*c^3*x)*sqrt(-a^2*c
)/a
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{7/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.635 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal. Leaf size=93

$$-\frac{2(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] -2/5*(a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(a*x+1)
^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A]
time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\frac{(ax+1)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (-2*(1 + a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 + a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)(1+ax)^4\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(-2(1+ax)^4+(1+ax)^5\right)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\ &= -\frac{2(1+ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1+ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.59

$$\frac{c^2(1+ax)^5(-7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2),x]
[Out] (c^2*(1 + a*x)^5*(-7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.92

method	result	size
gosper	$\frac{x\left(5a^5x^5+18a^4x^4+15a^3x^3-20a^2x^2-45ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{30(ax-1)(ax+1)^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	84
default	$\frac{\left(5a^5x^5+18a^4x^4+15a^3x^3-20a^2x^2-45ax-30\right)x\,c^2\sqrt{-c\left(a^2x^2-1\right)}\,(ax-1)}{30(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/30*(5*a^5*x^5+18*a^4*x^4+15*a^3*x^3-20*a^2*x^2-45*a*x-30)*x*c^2*(-c*(a^2*
x^2-1))^(1/2)*(a*x-1)/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [**A**]
time = 0.28, size = 140, normalized size = 1.51

$$\frac{\left(5\,a^{7}\sqrt{-c}\,c^{2}x^{7}+13\,a^{6}\sqrt{-c}\,c^{2}x^{6}-3\,a^{5}\sqrt{-c}\,c^{2}x^{5}-35\,a^{4}\sqrt{-c}\,c^{2}x^{4}-25\,a^{3}\sqrt{-c}\,c^{2}x^{3}+15\,a^{2}\sqrt{-c}\,c^{2}x^{2}+30\,\sqrt{-c}\,c^{2}\right)(ax+1)^{2}}{30\,(a^{3}x^{2}+2\,a^{2}x+a)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 + 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 - 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 + 15*a^2*sqrt(-c)*
c^2*x^2 + 30*sqrt(-c)*c^2)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*x - 1))
```

Fricas [**A**]
time = 0.35, size = 73, normalized size = 0.78

$$\frac{\left(5\,a^{5}c^{2}x^{6}+18\,a^{4}c^{2}x^{5}+15\,a^{3}c^{2}x^{4}-20\,a^{2}c^{2}x^{3}-45\,ac^{2}x^{2}-30\,c^{2}x\right)\sqrt{-a^{2}c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 + 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 - 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.636 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal. Leaf size=46

$$\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] 1/4*(a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A]
time = 0.12, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 32}

$$\frac{(ax+1)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 + a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int(1+ax)^3\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{(1+ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A]
time = 0.04, size = 69, normalized size = 1.50

$$-\frac{ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2\sqrt{c-a^2cx^2}\,\left(4+6ax+4a^2x^2+a^3x^3\right)}{-4+4a^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
[Out] -((a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2*Sqrt[c - a^2*c*x^2]*(4 + 6*a*x + 4*a^2*x^2
 + a^3*x^3))/(-4 + 4*a^2*x^2))
```

Maple [A]
time = 0.10, size = 48, normalized size = 1.04

method	result	size
default	$-\frac{(ax-1)(ax+1)^2\sqrt{-c\left(a^2x^2-1\right)}\,c}{4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a}$	48
gosper	$\frac{x\left(a^3x^3+4a^2x^2+6ax+4\right)\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{4(ax+1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*c/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(40) = 80.
time = 0.29, size = 97, normalized size = 2.11

$$-\frac{\left(a^5\sqrt{-c}\,cx^5+3\,a^4\sqrt{-c}\,cx^4+2\,a^3\sqrt{-c}\,cx^3-2\,a^2\sqrt{-c}\,cx^2-4\,\sqrt{-c}\,c\right)(ax+1)^2}{4\left(a^3x^2+2\,a^2x+a\right)(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 + 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 - 2*
a^2*sqrt(-c)*c*x^2 - 4*sqrt(-c)*c)*(a*x + 1)^2/((a^3*x^2 + 2*a^2*x + a)*(a*
x - 1))
```

Fricas [A]
time = 0.36, size = 42, normalized size = 0.91

$$-\frac{(a^3cx^4+4\,a^2cx^3+6\,acx^2+4\,cx)\sqrt{-a^2c}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*(a^3*c*x^4 + 4*a^2*c*x^3 + 6*a*c*x^2 + 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-c(ax-1)\,(ax+1))^{\frac{3}{2}}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.637 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=113

$$\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 45}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(6+ax)+8\log(1-ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/(a
^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 67, normalized size = 0.59

method	result	size
default	$\frac{\left(a^2x^2+6ax+8\ln(ax-1)\right)\sqrt{-c\left(a^2x^2-1\right)}\,(ax-1)}{2a(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 33, normalized size = 0.29

$$\frac{\left(a^2x^2+6\,ax+8\,\log\left(ax-1\right)\right)\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.638 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] 2*x*(1-1/a^2/x^2)^(1/2)/(-a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(-a*x+1)*(1-1/a^2
/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(1-ax)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 - a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 - a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{1+ax}{(-1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \left(\frac{2}{(-1+ax)^2}+\frac{1}{-1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1-ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1-ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.67

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+(-1+ax)\log(1-ax))}{(-1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + (-1 + a*x)*Log[1 - a*x]))/((-1 + a*x)*Sqrt[c
 - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.81

method	result	size
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(x\ln(ax-1)a-\ln(ax-1)-2\right)}{ac(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(x*ln(a*x-1)*a-ln(a*x-1)-2)/a/c/(a*x+1)^2/((a*x-1)/
(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.38, size = 39, normalized size = 0.49

$$-\frac{\sqrt{-a^2c}\left((ax-1)\log\left(ax-1\right)-2\right)}{a^3cx-a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] -sqrt(-a^2*c)*((a*x - 1)*log(a*x - 1) - 2)/(a^3*c*x - a^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(((a*x - 1)/(a*x + 1))**(3/2)*sqrt(-c*(a*x - 1)*(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.639 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 32}

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 - a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^3}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)^2\,(c-a^2cx^2)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.09

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)^3(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)^3*(1 + a
*x))
```

Maple [A]
time = 0.09, size = 56, normalized size = 1.19

method	result	size
gosper	$-\frac{ax-1}{2a\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(-a^2c\,x^2+c)^{\frac{3}{2}}}$	39
default	$-\frac{\sqrt{-c\,(a^2x^2-1)}}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)(a^2x^2-1)c^2a}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)/c^2
/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 39, normalized size = 0.83

$$-\frac{\sqrt{-a^2c}}{2\left(a^4c^2x^2-2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B]
time = 1.55, size = 90, normalized size = 1.91

$$\frac{\left(\frac{1}{2\,a^3\,c}+\frac{x}{2\,a^2\,c}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}+x^2\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x\,\sqrt{c-a^2\,c\,x^2}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((1/(2*a^3*c) + x/(2*a^2*c))*((a*x - 1)/(a*x + 1))^(1/2))/((c - a^2*c*x^2)^
(1/2)/a^2 + x^2*(c - a^2*c*x^2)^(1/2) - (2*x*(c - a^2*c*x^2)^(1/2))/a)
```

3.640 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=185

$$\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}(a$$

```
[Out] 1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-
1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^
(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arc
tanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.14, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6327, 6328, 46, 213}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(1-ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 - a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^4(1+ax)}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{2(-1+ax)^4}-\frac{1}{4(-1+ax)^3}+\frac{1}{8(-1+ax)^2}-\frac{1}{8(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)(c-a^2cx^2)^{5/2}}-\\
&= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1-ax)^3(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)(c-a^2cx^2)^{5/2}}+
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.38

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-10+9ax-3a^2x^2+3(-1+ax)^3\tanh^{-1}(ax)\right)}{24c^2(-1+ax)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-10 + 9*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^3*ArcTanh[
a*x]))/(24*c^2*(-1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 169, normalized size = 0.91

method	result
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(3 \ln \left(a x +1\right) a^{3} x^{3}-3 x^{3} \ln \left(a x -1\right) a^{3}-9 \ln \left(a x +1\right) a^{2} x^{2}+9 x^{2} \ln \left(a x -1\right) a^{2}-6 a^{2} x^{2}+9 \ln \left(a x +1\right) a x -9 x \ln \left(a x -1\right)\right.}{48 \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}} \left(a x -1\right) \left(a x +1\right) \left(a^{2} x^{2}-1\right) c^{3} a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/48/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*a^3*x^3-3*x^3*ln(a*x-1)*a^3-9*ln(a*x+1)*a^2*x^2+9*x^2*ln(a*x-1)*a^2-6*a^2*x^2+9*ln(a*x+1)*a*x-9*x*ln(a*x-1)*a+18*a*x-3*ln(a*x+1)+3*ln(a*x-1)-20)/(a^2*x^2-1)/c^3/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Fricas [A]
time = 0.35, size = 139, normalized size = 0.75

$$-\frac{3 \left(a^{4} x^{3}-3 a^{3} x^{2}+3 a^{2} x-a\right) \sqrt{-c}\, \log \left(\frac{a^{2} c x^{2}-2 \sqrt{-a^{2} c}\, \sqrt{-c}\, x+c}{a^{2} x^{2}-1}\right)-2 \left(3 a^{2} x^{2}-9 a x+10\right) \sqrt{-a^{2} c}}{48 \left(a^{5} c^{3} x^{3}-3 a^{4} c^{3} x^{2}+3 a^{3} c^{3} x-a^{2} c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] -1/48*(3*(a^4*x^3 - 3*a^3*x^2 + 3*a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 - 9*a*x + 10)*sqrt(-a^2*c))/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2*c^3)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.641 $\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=278

$$-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right.}$$

```
[Out] -1/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*
(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x
^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x
^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(
-a^2*c*x^2+c)^(7/2)-5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x
^2+c)^(7/2)
```

Rubi [A]
time = 0.16, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6327, 6328, 46, 213}

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(1-ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(1-ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\tanh^{-1}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/16*(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/((1 - a*x)^4*(c - a^2*c*x^2)^(7/2))
 - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 - a*x)^3*(c - a^2*c*x^2)^(7/2))
 - (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)^2*(c - a^2*c*x^2)^(7/2
)) - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2))
+ (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) -
(5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{(c-a^2cx^2)^{7/2}}\\
&= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^5(1+ax)^2}\,dx}{(c-a^2cx^2)^{7/2}}\\
&= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{4(-1+ax)^5}-\frac{1}{4(-1+ax)^4}+\frac{3}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{32(1+ax)^2}+\right.}{(c-a^2cx^2)^{7/2}}\\
&= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3(c-a^2cx^2)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2(c-a^2cx^2}\\
&= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^4(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1-ax)^3(c-a^2cx^2)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2(c-a^2cx^2}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 0.36

$$
\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(32-15ax-35a^2x^2+45a^3x^3-15a^4x^4+15(-1+ax)^4(1+ax)\tanh^{-1}(ax)\right)}{96c^3(-1+ax)^4(1+ax)\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(32 - 15*a*x - 35*a^2*x^2 + 45*a^3*x^3 - 15*a^4*x^4 + 15*(-1 + a*x)^4*(1 + a*x)*ArcTanh[a*x]))/(96*c^3*(-1 + a*x)^4*(1 + a*x)*Sqrt[c - a^2*c*x^2])

Maple [A]
time = 0.10, size = 241, normalized size = 0.87

method	result
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(15 \ln \left(a x+1\right) a^{5} x^{5}-15 x^{5} \ln \left(a x-1\right) a^{5}-45 \ln \left(a x+1\right) a^{4} x^{4}+45 x^{4} \ln \left(a x-1\right) a^{4}-30 a^{4} x^{4}+30 \ln \left(a x+1\right) a^{3} x^{3}-30 a\right.}{192\left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}} \left(a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/192/((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1)*a^5*x^5-15*x^5*ln(a*x-1)*a^5-45*ln(a*x+1)*a^4*x^4+45*x^4*ln(a*x-1)*a^4-30*a^4*x^4+30*ln(a*x+1)*a^3*x^3-30*x^3*ln(a*x-1)*a^3+90*a^3*x^3+30*ln(a*x+1)*a^2*x^2-30*x^2*ln(a*x-1)*a^2-70*a^2*x^2-45*ln(a*x+1)*a*x+45*x*ln(a*x-1)*a-30*a*x+15*ln(a*x+1)-15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")

[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

Fricas [A]
time = 0.37, size = 190, normalized size = 0.68

$$-\frac{15\,(a^6x^5-3\,a^5x^4+2\,a^4x^3+2\,a^3x^2-3\,a^2x+a)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\,(15\,a^4x^4-45\,a^3x^3+35\,a^2x^2+15\,ax-32)\sqrt{-a^2c}}{192\,(a^7c^4x^5-3\,a^6c^4x^4+2\,a^5c^4x^3+2\,a^4c^4x^2-3\,a^3c^4x+a^2c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")

[Out] -1/192*(15*(a^6*x^5 - 3*a^5*x^4 + 2*a^4*x^3 + 2*a^3*x^2 - 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^4*x^4 - 45*a^3*x^3 + 35*a^2*x^2 + 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.642 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal. Leaf size=234

$$\frac{8(1-ax)^6\,(c-a^2cx^2)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\,(c-a^2cx^2)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\,(c-a^2cx^2)^{9/2}}{a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{8(1-ax)^9\,(c-a^2cx^2)^{9/2}}{9a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] 8/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-32/7*(-a*x
+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3*(-a*x+1)^8*(-a^2*
c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-8/9*(-a*x+1)^9*(-a^2*c*x^2+c)^(
9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+c)^(9/2)/a^1
0/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A]

time = 0.14, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 45}

$$\frac{(1-ax)^{10}\,(c-a^2cx^2)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^9\,(c-a^2cx^2)^{9/2}}{9a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\,(c-a^2cx^2)^{9/2}}{a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{32(1-ax)^7\,(c-a^2cx^2)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{8(1-ax)^6\,(c-a^2cx^2)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
```

```
[Out] (8*(1 - a*x)^6*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
- (32*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (8*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(9*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^5(1+ax)^4\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(16(-1+ax)^5+32(-1+ax)^6+24(-1+ax)^7+8(-1\right.}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{8(1-ax)^6\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{32(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-\right.}{a^{10}\left(1-\frac{1}{a^2x\right.}}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.34

$$\frac{c^4(-1+ax)^6\sqrt{c-a^2cx^2}\left(193+528ax+588a^2x^2+308a^3x^3+63a^4x^4\right)}{630a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^ArcCoth[a*x],x]
[Out] (c^4*(-1 + a*x)^6*Sqrt[c - a^2*c*x^2]*(193 + 528*a*x + 588*a^2*x^2 + 308*a^
3*x^3 + 63*a^4*x^4))/(630*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 113, normalized size = 0.48

method	result
default	$\frac{\left(63a^9x^9-70a^8x^8-315a^7x^7+360a^6x^6+630a^5x^5-756a^4x^4-630a^3x^3+840a^2x^2+315ax-630\right)x\,c^4\sqrt{-c\left(a^2x^2-1\right)}\sqrt{\frac{ax-1}{ax+1}}}{630ax-630}$
gosper	$\frac{x\left(63a^9x^9-70a^8x^8-315a^7x^7+360a^6x^6+630a^5x^5-756a^4x^4-630a^3x^3+840a^2x^2+315ax-630\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}\sqrt{\frac{ax-1}{ax+1}}}{630(ax+1)^4(ax-1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/630*(63*a^9*x^9-70*a^8*x^8-315*a^7*x^7+360*a^6*x^6+630*a^5*x^5-756*a^4*x^
4-630*a^3*x^3+840*a^2*x^2+315*a*x-630)*x*c^4*(-c*(a^2*x^2-1))^(1/2)*((a*x-1
)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.33, size = 117, normalized size = 0.50

$$\frac{\left(63\,a^{9}c^{4}x^{10}-70\,a^{8}c^{4}x^{9}-315\,a^{7}c^{4}x^{8}+360\,a^{6}c^{4}x^{7}+630\,a^{5}c^{4}x^{6}-756\,a^{4}c^{4}x^{5}-630\,a^{3}c^{4}x^{4}+840\,a^{2}c^{4}x^{3}+315\,ac^{4}x^{2}-630\,c^{4}x\right)\sqrt{-a^{2}c}}{630\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/630*(63*a^9*c^4*x^10 - 70*a^8*c^4*x^9 - 315*a^7*c^4*x^8 + 360*a^6*c^4*x^7
 + 630*a^5*c^4*x^6 - 756*a^4*c^4*x^5 - 630*a^3*c^4*x^4 + 840*a^2*c^4*x^3 +
315*a*c^4*x^2 - 630*c^4*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-a^2\,c\,x^2\right)}^{9/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.643 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^{7/2}\,dx$

Optimal. Leaf size=187

$$-\frac{8(1-ax)^5(c-a^2cx^2)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^6(c-a^2cx^2)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1-ax)^7(c-a^2cx^2)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8(c-a^2cx^2)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] -8/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+2*(-a*x+1)
^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-6/7*(-a*x+1)^7*(-a^2*c*
x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c*x^2+c)^(7/2
)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A]
time = 0.14, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 45}

$$\frac{(1-ax)^8(c-a^2cx^2)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{6(1-ax)^7(c-a^2cx^2)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6(c-a^2cx^2)^{7/2}}{a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{8(1-ax)^5(c-a^2cx^2)^{7/2}}{5a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-8*(1 - a*x)^5*(c - a^2*c*x^2)^(7/2))/(5*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (6*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx = \frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^4(1+ax)^3\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

$$= \frac{\left(c-a^2cx^2\right)^{7/2}\int\left(8(-1+ax)^4+12(-1+ax)^5+6(-1+ax)^6+(-1+a\right.}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{8(1-ax)^5\left(c-a^2cx^2\right)^{7/2}}{5a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{6(1-ax)^7\,(c}{7a^8\,(1-\frac{}{a^2}}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.38

$$-\frac{c^3(-1+ax)^5\sqrt{c-a^2cx^2}\left(93+185ax+135a^2x^2+35a^3x^3\right)}{280a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^ArcCoth[a*x],x]
[Out] -1/280*(c^3*(-1 + a*x)^5*Sqrt[c - a^2*c*x^2]*(93 + 185*a*x + 135*a^2*x^2 +
35*a^3*x^3))/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 97, normalized size = 0.52

method	result	size
default	$-\frac{\left(35a^7x^7-40a^6x^6-140a^5x^5+168a^4x^4+210a^3x^3-280a^2x^2-140ax+280\right)x\,c^3\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{280(ax-1)}$	97
gosper	$\frac{x\left(35a^7x^7-40a^6x^6-140a^5x^5+168a^4x^4+210a^3x^3-280a^2x^2-140ax+280\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}\sqrt{\frac{ax-1}{ax+1}}}{280(ax-1)^4(ax+1)^3}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(35*a^7*x^7-40*a^6*x^6-140*a^5*x^5+168*a^4*x^4+210*a^3*x^3-280*a^2*x
^2-140*a*x+280)*x*c^3*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.33, size = 95, normalized size = 0.51

$$-\frac{(35\,a^7c^3x^8 - 40\,a^6c^3x^7 - 140\,a^5c^3x^6 + 168\,a^4c^3x^5 + 210\,a^3c^3x^4 - 280\,a^2c^3x^3 - 140\,ac^3x^2 + 280\,c^3x)\sqrt{-a^2c}}{280\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/280*(35*a^7*c^3*x^8 - 40*a^6*c^3*x^7 - 140*a^5*c^3*x^6 + 168*a^4*c^3*x^5
 + 210*a^3*c^3*x^4 - 280*a^2*c^3*x^3 - 140*a*c^3*x^2 + 280*c^3*x)*sqrt(-a^2
*c)/a
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.644 $\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=139

$$\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] (-a*x+1)^4*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5-4/5*(-a*x+1)^5*
(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+1)^6*(-a^2*c*x^2
+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6327, 6328, 45}

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(5/2))/(a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) - (4*
(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5) + ((
1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^3(1+ax)^2\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{\left(c-a^2cx^2\right)^{5/2}\int\left(4(-1+ax)^3+4(-1+ax)^4+(-1+ax)^5\right)dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\
&= \frac{(1-ax)^4\left(c-a^2cx^2\right)^{5/2}}{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}-\frac{4(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2\right.}{6a^6\left(1-\frac{1}{a^2x^2}\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.45

$$
\frac{c^2(-1+ax)^4\left(11+14ax+5a^2x^2\right)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^ArcCoth[a*x],x]
[Out] (c^2*(-1 + a*x)^4*(11 + 14*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sq
rt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 81, normalized size = 0.58

method	result	size
default	$\frac{\left(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30\right)x\,c^2\sqrt{-c\left(a^2x^2-1\right)}\sqrt{\frac{ax-1}{ax+1}}}{30ax-30}$	81
gosper	$\frac{x\left(5a^5x^5-6a^4x^4-15a^3x^3+20a^2x^2+15ax-30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\frac{ax-1}{ax+1}}}{30(ax-1)^3(ax+1)^2}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(5*a^5*x^5-6*a^4*x^4-15*a^3*x^3+20*a^2*x^2+15*a*x-30)*x*c^2*(-c*(a^2*x
^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.34, size = 73, normalized size = 0.53

$$\frac{(5\,a^5c^2x^6 - 6\,a^4c^2x^5 - 15\,a^3c^2x^4 + 20\,a^2c^2x^3 + 15\,ac^2x^2 - 30\,c^2x)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] 1/30*(5*a^5*c^2*x^6 - 6*a^4*c^2*x^5 - 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 + 15*
a*c^2*x^2 - 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.645 $\int e^{-\coth^{-1}(ax)}(c - a^2cx^2)^{3/2}\, dx$

Optimal. Leaf size=95

$$-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -2/3*(-a*x+1)^3*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3+1/4*(-a*x+
1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 45}

$$\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] (-2*(1 - a*x)^3*(c - a^2*c*x^2)^(3/2))/(3*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
+ ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)^2(1+ax)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int\left(2(-1+ax)^2+(-1+ax)^3\right)\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=-\frac{2(1-ax)^3\left(c-a^2cx^2\right)^{3/2}}{3a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}+\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.56

$$-\frac{c(-1+ax)^3(5+3ax)\sqrt{c-a^2cx^2}}{12a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^ArcCoth[a*x],x]
[Out] -1/12*(c*(-1 + a*x)^3*(5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x)
```

Maple [A]
time = 0.12, size = 63, normalized size = 0.66

method	result	size
default	$-\frac{(3a^3x^3-4a^2x^2-6ax+12)xc\sqrt{-c\left(a^2x^2-1\right)}\sqrt{\frac{ax-1}{ax+1}}}{12(ax-1)}$	63
gosper	$\frac{x\left(3a^3x^3-4a^2x^2-6ax+12\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\frac{ax-1}{ax+1}}}{12(ax+1)(ax-1)^2}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(3*a^3*x^3-4*a^2*x^2-6*a*x+12)*x*c*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a
*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.33, size = 43, normalized size = 0.45

$$-\frac{(3\,a^3cx^4-4\,a^2cx^3-6\,acx^2+12\,cx)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
[Out] -1/12*(3*a^3*c*x^4 - 4*a^2*c*x^3 - 6*a*c*x^2 + 12*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.646 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6327, 6328}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.59

$$\frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.09, size = 45, normalized size = 0.65

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c\,(a^2x^2-1)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x-2)*x*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.32, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^2c}\,(ax^2-2\,x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.647 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] x*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 31}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{1}{1+ax}\,dx}{\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Maple [A]
time = 0.09, size = 51, normalized size = 1.38

method	result	size
default	$-\frac{\ln(ax+1)\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{c(ax-1)a}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/c/(a*x-1)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.36, size = 22, normalized size = 0.59

$$-\frac{\sqrt{-a^2c}\,\log\left(ax+1\right)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c)*log(a*x + 1)/(a^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.648 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a^2*(1-1/a
^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6327, 6328, 46, 213}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 + a*x)*(c - a^2*c*x^2)^(3/2)) - (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)(1+ax)^2}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(-\frac{1}{2(1+ax)^2}+\frac{1}{2(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 0.60

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-1+(1+ax)\tanh^{-1}(ax)\right)}{2(c+acx)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (1 + a*x)*ArcTanh[a*x]))/(2*(c + a*c*x)*Sqrt
[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 84, normalized size = 0.93

method	result	size

default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\, \sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(\ln \left(ax+1\right) a x-x \ln \left(ax-1\right) a+\ln \left(ax+1\right)-\ln \left(ax-1\right)-2\right)}{4 \left(a^{2} x^{2}-1\right) c^{2} a}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a*x-x*ln(a*x
-1)*a+ln(a*x+1)-ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.92

$$-\frac{\left(a^{2} x+a\right) \sqrt{-c}\, \log\left(\frac{a^{2} c x^{2}-2 \sqrt{-a^{2} c}\, \sqrt{-c}\, x+c}{a^{2} x^{2}-1}\right)-2 \sqrt{-a^{2} c}}{4 \left(a^{3} c^{2} x+a^{2} c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/4*((a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) - 2*sqrt(-a^2*c))/(a^3*c^2*x + a^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ax-1}{ax+1}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

3.649 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}(}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] 1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/
a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1-1/a^2/x^2)^(5/
2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arctanh
(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6327, 6328, 46, 213}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a^
4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) - (a^4
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (3*a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^2(1+ax)^3}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{8(-1+ax)^2}+\frac{1}{4(1+ax)^3}+\frac{1}{4(1+ax)^2}-\frac{3}{8(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)(c-a^2cx^2)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2(c-a^2cx^2)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)(c-a^2cx^2)^{5/2}} - \frac{(3}{}$$

$$= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)(c-a^2cx^2)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2(c-a^2cx^2)^{5/2}} - \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1+ax)(c-a^2cx^2)^{5/2}} + \frac{3a}{}$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 0.44

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2-3ax-3a^2x^2+3(-1+ax)(1+ax)^2\tanh^{-1}(ax)\right)}{8(-1+ax)(c+acx)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)*(1 + a*x)^2*
ArcTanh[a*x]))/(8*(-1 + a*x)*(c + a*c*x)^2*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 169, normalized size = 0.92

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(a^2x^2-1)}\,(3\ln(ax+1)a^3x^3-3x^3\ln(ax-1)a^3+3\ln(ax+1)a^2x^2-3x^2\ln(ax-1)a^2-6a^2x^2-3\ln(ax+1)ax+}{16(ax+1)(a^2x^2-1)c^3a(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*a
^3*x^3-3*x^3*ln(a*x-1)*a^3+3*ln(a*x+1)*a^2*x^2-3*x^2*ln(a*x-1)*a^2-6*a^2*x^
2-3*ln(a*x+1)*a*x+3*x*ln(a*x-1)*a-6*a*x-3*ln(a*x+1)+3*ln(a*x-1)+4)/(a^2*x^2
-1)/c^3/a/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 0.36, size = 137, normalized size = 0.75

$$-\frac{3\,(a^4x^3+a^3x^2-a^2x-a)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)-2\,(3\,a^2x^2+3\,ax-2)\sqrt{-a^2c}}{16\,(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/16*(3*(a^4*x^3 + a^3*x^2 - a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(3*a^2*x^2 + 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

3.650 $\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=276

$$-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x}{32(1+ax)^2\left(c-a^2cx\right.}$$

```
[Out] -1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(
1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)+1/24*a^6*(1-1/a^2/x^2)
^(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)+3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/
(a*x+1)^2/(-a^2*c*x^2+c)^(7/2)+3/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a
^2*c*x^2+c)^(7/2)-5/16*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2
+c)^(7/2)
```

Rubi [A]
time = 0.16, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6327, 6328, 46, 213}

$$-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(1-ax)\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}+\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{24(ax+1)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\tanh^{-1}(ax)}{16\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/32*(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/((1 - a*x)^2*(c - a^2*c*x^2)^(7/2))
 - (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) +
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(24*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) +
(3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2))
+ (3*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)*(c - a^2*c*x^2)^(7/2))
- (5*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(16*(c - a^2*c*x^2)^(7/2
))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^3(1+ax)^4}\,dx}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{16(-1+ax)^3}-\frac{1}{8(-1+ax)^2}-\frac{1}{8(1+ax)^4}-\frac{3}{16(1+ax)^3}-\frac{3}{16(1+ax)^2}+\frac{}{16(\cdot}\right.}{(c-a^2cx^2)^{7/2}}$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)(c-a^2cx^2)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3(c-a^2cx^2)^{7/2}}$$

$$= -\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)^2(c-a^2cx^2)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{8(1-ax)(c-a^2cx^2)^{7/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{24(1+ax)^3(c-a^2cx^2)^{7/2}}$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 0.36

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-8+25ax+25a^2x^2-15a^3x^3-15a^4x^4+15(-1+ax)^2(1+ax)^3\tanh^{-1}(ax)\right)}{48(-1+ax)^2(c+acx)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(-8 + 25*a*x + 25*a^2*x^2 - 15*a^3*x^3 - 15*a^4*x^
4 + 15*(-1 + a*x)^2*(1 + a*x)^3*ArcTanh[a*x]))/(48*(-1 + a*x)^2*(c + a*c*x)
^3*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 241, normalized size = 0.87

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(15\ln(ax+1)a^{5}x^{5}-15x^{5}\ln(ax-1)a^{5}+15\ln(ax+1)a^{4}x^{4}-15x^{4}\ln(ax-1)a^{4}-30a^{4}x^{4}-30\ln(ax-\right.}{96(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*((a*x-1)/(a*x+1))^(1/2)/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(15*ln(a*x+1
)*a^5*x^5-15*x^5*ln(a*x-1)*a^5+15*ln(a*x+1)*a^4*x^4-15*x^4*ln(a*x-1)*a^4-30
*a^4*x^4-30*ln(a*x+1)*a^3*x^3+30*x^3*ln(a*x-1)*a^3-30*a^3*x^3-30*ln(a*x+1)*
a^2*x^2+30*x^2*ln(a*x-1)*a^2+50*a^2*x^2+15*ln(a*x+1)*a*x-15*x*ln(a*x-1)*a+5
0*a*x+15*ln(a*x+1)-15*ln(a*x-1)-16)/(a^2*x^2-1)/c^4/a/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A]
time = 0.36, size = 186, normalized size = 0.67

$$-\frac{15\left(a^{6}x^{5}+a^{5}x^{4}-2\,a^{4}x^{3}-2\,a^{3}x^{2}+a^{2}x+a\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{2}-2\,\sqrt{-a^{2}c}\,\sqrt{-c}\,x+c}{a^{2}x^{2}-1}\right)-2\left(15\,a^{4}x^{4}+15\,a^{3}x^{3}-25\,a^{2}x^{2}-25\,ax+8\right)\sqrt{-a^{2}c}}{96\left(a^{7}c^{4}x^{5}+a^{6}c^{4}x^{4}-2\,a^{5}c^{4}x^{3}-2\,a^{4}c^{4}x^{2}+a^{3}c^{4}x+a^{2}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/96*(15*(a^6*x^5 + a^5*x^4 - 2*a^4*x^3 - 2*a^3*x^2 + a^2*x + a)*sqrt(-c)*
log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(15*a^4*
x^4 + 15*a^3*x^3 - 25*a^2*x^2 - 25*a*x + 8)*sqrt(-a^2*c))/(a^7*c^4*x^5 + a^
6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(-a^2*c*x^2 + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-a^2\,c\,x^2\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - a^2*c*x^2)^(7/2), x)
```

3.651 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^{5/2}\,dx$

Optimal. Leaf size=131

$$-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7c^{5/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}$$

```
[Out] -7/24*c*x*(-a^2*c*x^2+c)^(3/2)-7/30*(-a^2*c*x^2+c)^(5/2)/a-1/6*(-a*x+1)*(-a
^2*c*x^2+c)^(5/2)/a-7/16*c^(5/2)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a
-7/16*c^2*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6277, 685, 655, 201, 223, 209}

$$-\frac{7c^{5/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{16a}-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*c^2*x*Sqrt[c - a^2*c*x^2])/16 - (7*c*x*(c - a^2*c*x^2)^(3/2))/24 - (7*(
c - a^2*c*x^2)^(5/2))/(30*a) - ((1 - a*x)*(c - a^2*c*x^2)^(5/2))/(6*a) - (7
*c^(5/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(16*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$$
$$=-\left(c\int(1-ax)^2\left(c-a^2cx^2\right)^{3/2}\,dx\right)$$
$$=-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int(1-ax)\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{6}(7c)\int\left(c-a^2cx^2\right)^{3/2}\,d$$
$$=-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{5/2}}{6a}-\frac{1}{8}($$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax}{}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax}{}$$
$$=-\frac{7}{16}c^2x\sqrt{c-a^2cx^2}-\frac{7}{24}cx\left(c-a^2cx^2\right)^{3/2}-\frac{7\left(c-a^2cx^2\right)^{5/2}}{30a}-\frac{(1-ax}{}$$

Mathematica [A]
time = 0.09, size = 136, normalized size = 1.04

$$\frac{c^2\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}\left(96+39ax-327a^2x^2+202a^3x^3+86a^4x^4-136a^5x^5+40a^6x^6\right)+210\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{240a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(96 + 39*a*x - 327*a^2*x^2 + 202*
a^3*x^3 + 86*a^4*x^4 - 136*a^5*x^5 + 40*a^6*x^6)) + 210*Sqrt[1 - a*x]*ArcSi
n[Sqrt[1 - a*x]/Sqrt[2]]))/(240*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(107) = 214.
time = 0.20, size = 275, normalized size = 2.10

method	result
risch	$-\frac{\left(40a^5x^5-96a^4x^4-10a^3x^3+192a^2x^2-135ax-96\right)\left(a^2x^2-1\right)c^3}{240a\sqrt{-c\left(a^2x^2-1\right)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c^3}{16\sqrt{a^2c}}$

default	$\frac{x\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^{2} c\,x^{2}+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^{2} c}\,x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{2\sqrt{a^{2} c}}\right)}{4}\right)}{6}-\frac{2\left(\frac{\left(-a^{2} c\left(x+\frac{1}{a}\right)^{2}+2\left(x \cdots\right.\right.}{5} \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(-a^2*c*x^2+c)^(5/2)+5/6*c*(1/4*x*(-a^2*c*x^2+c)^(3/2)+3/4*c*(1/2*x*(
-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+
c)^(1/2))))-2/a*(1/5*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(5/2)+a*c*(-1/8*(-2*a
^2*c*(x+1/a)+2*a*c)/a^2/c*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(3/2)+3/4*c*(-1/
4*(-2*a^2*c*(x+1/a)+2*a*c)/a^2/c*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+1/2
*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1
/2)))))
```

Maxima [A]
time = 0.47, size = 154, normalized size = 1.18

$$\frac{1}{6}\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} x-\frac{7}{24}\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} c x-\frac{3}{4}\sqrt{a^{2} c x^{2}+4\,a c x+3\,c}\,c^{2} x+\frac{5}{16}\sqrt{-a^{2} c x^{2}+c}\,c^{2} x+\frac{3\,c^{4}\arcsin\left(a x+2\right)}{4\,a\left(-c\right)^{\frac{3}{2}}}+\frac{5\,c^{\frac{5}{2}}\arcsin\left(a x\right)}{16\,a}-\frac{2\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}}}{5\,a}-\frac{3\sqrt{a^{2} c x^{2}+4\,a c x+3\,c}\,c^{2}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/6*(-a^2*c*x^2 + c)^(5/2)*x - 7/24*(-a^2*c*x^2 + c)^(3/2)*c*x - 3/4*sqrt(a
^2*c*x^2 + 4*a*c*x + 3*c)*c^2*x + 5/16*sqrt(-a^2*c*x^2 + c)*c^2*x + 3/4*c^4
*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 5/16*c^(5/2)*arcsin(a*x)/a - 2/5*(-a^2*c*
x^2 + c)^(5/2)/a - 3/2*sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c^2/a
```

Fricas [A]
time = 0.37, size = 241, normalized size = 1.84

$$\left[\frac{105\sqrt{-c}\,c^{2}\log\left(2\,a^{2} c x^{2}-2\sqrt{-a^{2} c x^{2}+c}\,a\sqrt{-c}\,x-c\right)+2\left(40\,a^{5} c^{2} x^{5}-96\,a^{4} c^{2} x^{4}-10\,a^{3} c^{2} x^{3}+192\,a^{2} c^{2} x^{2}-135\,a c^{2} x-96\,c^{2}\right)\sqrt{-a^{2} c x^{2}+c}}{480\,a},\ \frac{105\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a^{2} c x^{2}+c}\,a\sqrt{c}\,x}{a^{2} c x^{2}-c}\right)+\left(40\,a^{5} c^{2} x^{5}-96\,a^{4} c^{2} x^{4}-10\,a^{3} c^{2} x^{3}+192\,a^{2} c^{2} x^{2}-135\,a c^{2} x-96\,c^{2}\right)\sqrt{-a^{2} c x^{2}+c}}{240\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/480*(105*sqrt(-c)*c^2*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c
)*x - c) + 2*(40*a^5*c^2*x^5 - 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^
2*x^2 - 135*a*c^2*x - 96*c^2)*sqrt(-a^2*c*x^2 + c))/a, 1/240*(105*c^(5/2)*a
rctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + (40*a^5*c^2*x^5 -
 96*a^4*c^2*x^4 - 10*a^3*c^2*x^3 + 192*a^2*c^2*x^2 - 135*a*c^2*x - 96*c^2)*
sqrt(-a^2*c*x^2 + c))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 8.34, size = 476, normalized size = 3.63

$$a^4c^2\left(\begin{cases} \frac{ia^2\sqrt{c}\,x^7}{6\sqrt{a^2x^2-1}} - \frac{5i\sqrt{c}\,x^5}{24\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,x^3}{48a^2\sqrt{a^2x^2-1}} + \frac{i\sqrt{c}\,x}{16a^4\sqrt{a^2x^2-1}} - \frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{16a^5} & \text{for } |a^2x^2|>1 \\ -\frac{a^2\sqrt{c}\,x^7}{6\sqrt{-a^2x^2+1}} + \frac{5\sqrt{c}\,x^5}{24\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,x^3}{48a^2\sqrt{-a^2x^2+1}} - \frac{\sqrt{c}\,x}{16a^4\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\operatorname{asin}(ax)}{16a^5} & \text{otherwise} \end{cases}\right) - 2a^3c^2\left(\begin{cases} \frac{x^4\sqrt{-a^2cx^2+c}}{5} - \frac{x^2\sqrt{-a^2cx^2+c}}{15a^2} - \frac{2\sqrt{-a^2cx^2+c}}{15a^4} & \text{for } a\neq 0 \\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise} \end{cases}\right) + 2ac^2\left(\begin{cases} 0 & \text{for } c=0 \\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0 \\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise} \end{cases}\right) - c^2\left(\begin{cases} \frac{i\sqrt{c}\,x\sqrt{a^2x^2-1}}{2} - \frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{2a} & \text{for } |a^2x^2|>1 \\ -\frac{a^2\sqrt{c}\,x^3}{2\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,x}{2\sqrt{-a^2x^2+1}} + \frac{\sqrt{c}\,\operatorname{asin}(ax)}{2a} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] a**4*c**2*Piecewise((I*a**2*sqrt(c)*x**7/(6*sqrt(a**2*x**2 - 1)) - 5*I*sqrt
(c)*x**5/(24*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*x**3/(48*a**2*sqrt(a**2*x**2
- 1)) + I*sqrt(c)*x/(16*a**4*sqrt(a**2*x**2 - 1)) - I*sqrt(c)*acosh(a*x)/(1
6*a**5), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**7/(6*sqrt(-a**2*x**2 + 1))
+ 5*sqrt(c)*x**5/(24*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x**3/(48*a**2*sqrt(-a*
*2*x**2 + 1)) - sqrt(c)*x/(16*a**4*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x
)/(16*a**5), True)) - 2*a**3*c**2*Piecewise((x**4*sqrt(-a**2*c*x**2 + c)/5
- x**2*sqrt(-a**2*c*x**2 + c)/(15*a**2) - 2*sqrt(-a**2*c*x**2 + c)/(15*a**4
), Ne(a, 0)), (sqrt(c)*x**4/4, True)) + 2*a*c**2*Piecewise((0, Eq(c, 0)), (
sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 + c)**(3/2)/(3*a**2*c), True)
) - c**2*Piecewise((I*sqrt(c)*x*sqrt(a**2*x**2 - 1)/2 - I*sqrt(c)*acosh(a*x
)/(2*a), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**3/(2*sqrt(-a**2*x**2 + 1))
+ sqrt(c)*x/(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(2*a), True))
```

Giac [A]
time = 0.44, size = 117, normalized size = 0.89

$$\frac{7\,c^3\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\,\right|\right)}{16\,\sqrt{-c}\,|a|} - \frac{1}{240}\,\sqrt{-a^2cx^2+c}\,\left(\left(135\,c^2-2\,\left(96\,ac^2-\left(5\,a^2c^2-4\,\left(5\,a^4c^2x-12\,a^3c^2\right)x\right)x\right)x\right)x+\frac{96\,c^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 7/16*c^3*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
 - 1/240*sqrt(-a^2*c*x^2 + c)*((135*c^2 - 2*(96*a*c^2 - (5*a^2*c^2 - 4*(5*a
^4*c^2*x - 12*a^3*c^2)*x)*x)*x)*x + 96*c^2/a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a^2*c*x^2)^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.652 $\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$

Optimal. Leaf size=108

$$-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}$$

```
[Out] -5/12*(-a^2*c*x^2+c)^(3/2)/a-1/4*(-a*x+1)*(-a^2*c*x^2+c)^(3/2)/a-5/8*c^(3/2
)*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a-5/8*c*x*(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6277, 685, 655, 201, 223, 209}

$$-\frac{5c^{3/2}\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a}-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-5*c*x*Sqrt[c - a^2*c*x^2])/8 - (5*(c - a^2*c*x^2)^(3/2))/(12*a) - ((1 - a
*x)*(c - a^2*c*x^2)^(3/2))/(4*a) - (5*c^(3/2)*ArcTan[(a*Sqrt[c]*x)/Sqrt[c -
 a^2*c*x^2]])/(8*a)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx$$
$$=-\left(c\int(1-ax)^2\sqrt{c-a^2cx^2}\,dx\right)$$
$$=-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int(1-ax)\sqrt{c-a^2cx^2}\,dx$$
$$=-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{4}(5c)\int\sqrt{c-a^2cx^2}\,dx$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{1}{8}\left(5c^2\right)$$
$$=-\frac{5}{8}cx\sqrt{c-a^2cx^2}-\frac{5\left(c-a^2cx^2\right)^{3/2}}{12a}-\frac{(1-ax)\left(c-a^2cx^2\right)^{3/2}}{4a}-\frac{5c^{3/2}\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.06, size = 117, normalized size = 1.08

$$\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{1+ax}\left(-16+7ax+25a^2x^2-22a^3x^3+6a^4x^4\right)+30\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{24a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(Sqrt[1 + a*x]*(-16 + 7*a*x + 25*a^2*x^2 - 22*a^3*x^
3 + 6*a^4*x^4) + 30*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(24*a*Sqr
t[1 - a*x]*Sqrt[1 - a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(88) = 176$.
time = 0.20, size = 202, normalized size = 1.87

method	result
risch	$\frac{(6a^3x^3-16a^2x^2+9ax+16)(a^2x^2-1)c^2}{24a\sqrt{-c(a^2x^2-1)}}-\frac{5\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c^2}{8\sqrt{a^2c}}$
default	$\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{4}-\frac{2\left(\frac{\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}{3}+ac\left(-\frac{(-2\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)+3/4*c*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))-2/a*(1/3*(-a^2*c*(x+1/a
)^2+2*(x+1/a)*a*c)^(3/2)+a*c*(-1/4*(-2*a^2*c*(x+1/a)+2*a*c)/a^2/c*(-a^2*c*(
x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-
a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 130, normalized size = 1.20

$$\frac{1}{4}\left(-a^2cx^2+c\right)^{\frac{3}{2}}x-\sqrt{a^2cx^2+4\,acx+3\,c}\,cx+\frac{3}{8}\sqrt{-a^2cx^2+c}\,cx+\frac{c^3\arcsin\left(ax+2\right)}{a\left(-c\right)^{\frac{3}{2}}}+\frac{3\,c^{\frac{3}{2}}\arcsin\left(ax\right)}{8\,a}-\frac{2\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a}-\frac{2\sqrt{a^2cx^2+4\,acx+3\,c}\,c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 1/4*(-a^2*c*x^2 + c)^(3/2)*x - sqrt(a^2*c*x^2 + 4*a*c*x + 3*c)*c*x + 3/8*sq
rt(-a^2*c*x^2 + c)*c*x + c^3*arcsin(a*x + 2)/(a*(-c)^(3/2)) + 3/8*c^(3/2)*a
rcsin(a*x)/a - 2/3*(-a^2*c*x^2 + c)^(3/2)/a - 2*sqrt(a^2*c*x^2 + 4*a*c*x +
3*c)*c/a
```

Fricas [A]
time = 0.38, size = 180, normalized size = 1.67

$$\left[\frac{15\,\sqrt{-c}\,c\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\,(6\,a^3cx^3-16\,a^2cx^2+9\,acx+16\,c)\sqrt{-a^2cx^2+c}}{48\,a},\ \frac{15\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)-(6\,a^3cx^3-16\,a^2cx^2+9\,acx+16\,c)\sqrt{-a^2cx^2+c}}{24\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-c)*c*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c) - 2*(6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c)
)/a, 1/24*(15*c^(3/2)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 -
c)) - (6*a^3*c*x^3 - 16*a^2*c*x^2 + 9*a*c*x + 16*c)*sqrt(-a^2*c*x^2 + c))/a
]
```

Sympy [C] Result contains complex when optimal does not.
time = 4.93, size = 338, normalized size = 3.13

$$-a^2c\left(\begin{cases}\frac{ia^2\sqrt{c}\,x^5}{4\sqrt{a^2x^2-1}}-\frac{3i\sqrt{c}\,x^3}{8\sqrt{a^2x^2-1}}+\frac{i\sqrt{c}\,x}{8a^2\sqrt{a^2x^2-1}}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{8a^3} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^5}{4\sqrt{-a^2x^2+1}}+\frac{3\sqrt{c}\,x^3}{8\sqrt{-a^2x^2+1}}-\frac{\sqrt{c}\,x}{8a^2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{8a^3} & \text{otherwise}\end{cases}\right)+2ac\left(\begin{cases}0 & \text{for } c=0\\ \frac{\sqrt{c}\,x^2}{2} & \text{for } a^2=0\\ -\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3a^2c} & \text{otherwise}\end{cases}\right)-c\left(\begin{cases}\frac{i\sqrt{c}\,x\sqrt{a^2x^2-1}}{2}-\frac{i\sqrt{c}\,\operatorname{acosh}(ax)}{2a} & \text{for } |a^2x^2|>1\\ -\frac{a^2\sqrt{c}\,x^3}{2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,x}{2\sqrt{-a^2x^2+1}}+\frac{\sqrt{c}\,\operatorname{asin}(ax)}{2a} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] -a**2*c*Piecewise((I*a**2*sqrt(c)*x**5/(4*sqrt(a**2*x**2 - 1)) - 3*I*sqrt(c
)*x**3/(8*sqrt(a**2*x**2 - 1)) + I*sqrt(c)*x/(8*a**2*sqrt(a**2*x**2 - 1)) -
 I*sqrt(c)*acosh(a*x)/(8*a**3), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**5/(4
*sqrt(-a**2*x**2 + 1)) + 3*sqrt(c)*x**3/(8*sqrt(-a**2*x**2 + 1)) - sqrt(c)*
x/(8*a**2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asin(a*x)/(8*a**3), True)) + 2*a*
c*Piecewise((0, Eq(c, 0)), (sqrt(c)*x**2/2, Eq(a**2, 0)), (-(-a**2*c*x**2 +
 c)**(3/2)/(3*a**2*c), True)) - c*Piecewise((I*sqrt(c)*x*sqrt(a**2*x**2 - 1
)/2 - I*sqrt(c)*acosh(a*x)/(2*a), Abs(a**2*x**2) > 1), (-a**2*sqrt(c)*x**3/
(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*x/(2*sqrt(-a**2*x**2 + 1)) + sqrt(c)*asi
n(a*x)/(2*a), True))
```

Giac [A]
time = 0.42, size = 85, normalized size = 0.79

$$-\frac{1}{24}\,\sqrt{-a^2cx^2+c}\left(\left(2\left(3\,a^2cx-8\,ac\right)x+9\,c\right)x+\frac{16\,c}{a}\right)+\frac{5\,c^2\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8\,\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*a^2*c*x - 8*a*c)*x + 9*c)*x + 16*c/a) + 5
/8*c^2*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{3/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.653 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)^
(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6277, 685, 655, 223, 209}

$$-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
```

```
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,
\right.\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 100, normalized size = 1.15

$$\frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}\,(4-5ax+a^2x^2)+6\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])

Maple [A]
time = 0.19, size = 127, normalized size = 1.46

method	result
risch	$-\frac{(ax-4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{}{\sqrt{-a^2c}}\right.}{}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))

Maxima [A]
time = 0.46, size = 47, normalized size = 0.54

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\,x-\frac{3\sqrt{c}\,\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2cx^2+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")

[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2 + c)/a

Fricas [A]
time = 0.35, size = 134, normalized size = 1.54

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(ax-4)+3\sqrt{-c}\,\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}\,(ax-4)+3\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A]
time = 0.42, size = 62, normalized size = 0.71

$$\frac{1}{2}\,\sqrt{-a^2cx^2+c}\,\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\,\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.654 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=60

$$\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))/a/c^(1/2)+2*(-a*x+1)/a/(-a^2*c*x^2
+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6302, 6277, 667, 223, 209}

$$\frac{\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}+\frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*(1 - a*x))/(a*Sqrt[c - a^2*c*x^2]) + ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c
*x^2]]/(a*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx\\ &= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx\right)\\ &= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\ &= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{x}{\sqrt{c-a^2cx^2}}\right)\\ &= \frac{2(1-ax)}{a\sqrt{c-a^2cx^2}}+\frac{\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 100, normalized size = 1.67

$$-\frac{2\sqrt{1-a^2x^2}\left((-1+ax)\sqrt{1+ax}+\sqrt{1-ax}\,(1+ax)\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{a\sqrt{1-ax}\,(1+ax)\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*((-1 + a*x)*Sqrt[1 + a*x] + Sqrt[1 - a*x]*(1 + a*x)*A
rcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*(1 + a*x)*Sqrt[c - a^2*c*x^
2])
```

Maple [A]
time = 0.14, size = 73, normalized size = 1.22

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{a^{2}c}\, x}{\sqrt{-a^{2}c\,x^{2}+c}}\right)}{\sqrt{a^{2}c}}+\frac{2\sqrt{-a^{2}c\left(x+\frac{1}{a}\right)^{2}+2\left(x+\frac{1}{a}\right)ac}}{a^{2}c\left(x+\frac{1}{a}\right)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))+2/a^2/c/(x+1/a
)*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)
```

Maxima [A]
time = 0.47, size = 39, normalized size = 0.65

$$\frac{2\sqrt{-a^{2}cx^{2}+c}}{a^{2}cx+ac}+\frac{\arcsin(ax)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-a^2*c*x^2 + c)/(a^2*c*x + a*c) + arcsin(a*x)/(a*sqrt(c))
```

Fricas [A]
time = 0.34, size = 151, normalized size = 2.52

$$\left[-\frac{(ax+1)\sqrt{-c}\,\log\left(2\,a^{2}cx^{2}-2\,\sqrt{-a^{2}cx^{2}+c}\,a\sqrt{-c}\,x-c\right)-4\,\sqrt{-a^{2}cx^{2}+c}}{2\,(a^{2}cx+ac)},-\frac{(ax+1)\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^{2}cx^{2}+c}\,a\sqrt{c}\,x}{a^{2}cx^{2}-c}\right)-2\,\sqrt{-a^{2}cx^{2}+c}}{a^{2}cx+ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*x + 1)*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-
c)*x - c) - 4*sqrt(-a^2*c*x^2 + c))/(a^2*c*x + a*c), -((a*x + 1)*sqrt(c)*ar
ctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) - 2*sqrt(-a^2*c*x^2
+ c))/(a^2*c*x + a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax-1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral((a*x - 1)/(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a\,x-1}{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - a^2*c*x^2)^(1/2)*(a*x + 1)), x)
```

3.655 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

```
[Out] 2/3*(-a*x+1)/a/(-a^2*c*x^2+c)^(3/2)-1/3*x/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {6302, 6277, 667, 197}

$$\frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{3c\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (2*(1 - a*x))/(3*a*(c - a^2*c*x^2)^(3/2)) - x/(3*c*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{\left(c-a^2cx^2\right)^{5/2}}\,dx\right) \\
&= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}} - \frac{1}{3}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}}\,dx \\
&= \frac{2(1-ax)}{3a\left(c-a^2cx^2\right)^{3/2}} - \frac{x}{3c\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.21

$$
\frac{\sqrt{1-ax}\,(2+ax)\sqrt{1-a^2x^2}}{3ac(1+ax)^{3/2}\sqrt{c-a^2cx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[1 - a*x]*(2 + a*x)*Sqrt[1 - a^2*x^2])/(3*a*c*(1 + a*x)^(3/2)*Sqrt[c -
 a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(44) = 88.
time = 0.19, size = 115, normalized size = 2.21

method	result
gosper	$\frac{(ax-1)^2(ax+2)}{3a\left(-a^2cx^2+c\right)^{\frac{3}{2}}}$
trager	$\frac{(ax+2)\sqrt{-a^2cx^2+c}}{3c^2(ax+1)^2a}$
default	$\frac{x}{c\sqrt{-a^2cx^2+c}} - \frac{2\left(-\frac{1}{3ac\left(x+\frac{1}{a}\right)\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}} - \frac{-2a^2c\left(x+\frac{1}{a}\right)+2ac}{3ac^2\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right.}\cdots\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/c/(-a^2*c*x^2+c)^(1/2)-2/a*(-1/3/a/c/(x+1/a)/(-a^2*c*(x+1/a)^2+2*(x+1/a)*
a*c)^(1/2)-1/3/a/c^2*(-2*a^2*c*(x+1/a)+2*a*c)/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a
*c)^(1/2))
```

Maxima [A]
time = 0.26, size = 60, normalized size = 1.15

$$-\frac{x}{3\sqrt{-a^2cx^2+c}\,c}+\frac{2}{3\left(\sqrt{-a^2cx^2+c}\,a^2cx+\sqrt{-a^2cx^2+c}\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*x/(sqrt(-a^2*c*x^2 + c)*c) + 2/3/(sqrt(-a^2*c*x^2 + c)*a^2*c*x + sqrt(
-a^2*c*x^2 + c)*a*c)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 0.90

$$\frac{\sqrt{-a^2cx^2+c}\,(ax+2)}{3\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*(a*x + 2)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax-1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(3/2)*(a*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(43) = 86.
time = 0.43, size = 148, normalized size = 2.85

$$-\frac{\left(ac+3\sqrt{-a^2c}\,\sqrt{c}\right)\mathrm{sgn}(x)}{3\left(a^2c^{\frac{5}{2}}+\sqrt{-a^2c}\,ac^2\right)}+\frac{2\left(2\,a^2c-3\,a\sqrt{c}\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)+3\left(\sqrt{-a^2c+\frac{c}{x^2}}-\frac{\sqrt{c}}{x}\right)^2\right)}{3\left(a\sqrt{c}-\sqrt{-a^2c+\frac{c}{x^2}}+\frac{\sqrt{c}}{x}\right)^3c\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*c + 3*sqrt(-a^2*c)*sqrt(c))*sgn(x)/(a^2*c^(5/2) + sqrt(-a^2*c)*a*c^
2) + 2/3*(2*a^2*c - 3*a*sqrt(c)*(sqrt(-a^2*c + c/x^2) - sqrt(c)/x) + 3*(sqr
t(-a^2*c + c/x^2) - sqrt(c)/x)^2)/((a*sqrt(c) - sqrt(-a^2*c + c/x^2) + sqrt
(c)/x)^3*c*sgn(x))
```

Mupad [B]
time = 1.28, size = 33, normalized size = 0.63

$$\frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+2\right)}{3\,a\,c^2\,{\left(a\,x+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(3/2)*(a*x + 1)),x)
```

```
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x + 2))/(3*a*c^2*(a*x + 1)^2)
```

3.656 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}$$

```
[Out] 2/5*(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-1/5*x/c/(-a^2*c*x^2+c)^(3/2)-2/5*x/c^2/
(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {6302, 6277, 667, 198, 197}

$$-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}-\frac{x}{5c\left(c-a^2cx^2\right)^{3/2}}+\frac{2(1-ax)}{5a\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (2*(1 - a*x))/(5*a*(c - a^2*c*x^2)^(5/2)) - x/(5*c*(c - a^2*c*x^2)^(3/2)) -
 (2*x)/(5*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx\\ &= -\left(c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{7/2}}\,dx\right)\\ &= \frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{3}{5}\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx\\ &= \frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}-\frac{2\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{5c}\\ &= \frac{2(1-ax)}{5a\,(c-a^2cx^2)^{5/2}}-\frac{x}{5c\,(c-a^2cx^2)^{3/2}}-\frac{2x}{5c^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 1.05

$$-\frac{\sqrt{1-a^2x^2}\,(-2+ax+4a^2x^2+2a^3x^3)}{5ac^2\sqrt{1-ax}\,(1+ax)^{5/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/5*(Sqrt[1 - a^2*x^2]*(-2 + a*x + 4*a^2*x^2 + 2*a^3*x^3))/(a*c^2*Sqrt[1 -
 a*x]*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(63) = 126$.
time = 0.19, size = 188, normalized size = 2.51

method	result
gosper	$-\frac{(ax-1)^2(2a^3x^3+4a^2x^2+ax-2)}{5a(-a^2c\,x^2+c)^{\frac{5}{2}}}$
trager	$\frac{(2a^3x^3+4a^2x^2+ax-2)\sqrt{-a^2c\,x^2+c}}{5c^3(ax+1)^3a(ax-1)}$
default	$\frac{x}{3c(-a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{-a^2c\,x^2+c}}-\frac{2\left(-\frac{1}{5ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{3}{2}}}+\frac{4a\left(-\frac{-2a^2c\left(x+\frac{1}{a}\right)+2ac}{6a^2c^2\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)\right.}}{}\right)}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x/c/(-a^2*c*x^2+c)^(3/2)+2/3*x/c^2/(-a^2*c*x^2+c)^(1/2)-2/a*(-1/5/a/c/(
x+1/a)/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(3/2)+4/5*a*(-1/6*(-2*a^2*c*(x+1/a)
+2*a*c)/a^2/c^2/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(3/2)-1/3/a^2/c^3*(-2*a^2*
c*(x+1/a)+2*a*c)/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.26, size = 79, normalized size = 1.05

$$\frac{2}{5\left((-a^2cx^2+c)^{\frac{3}{2}}a^2cx+(-a^2cx^2+c)^{\frac{3}{2}}ac\right)}-\frac{2\,x}{5\,\sqrt{-a^2cx^2+c}\,c^2}-\frac{x}{5\,(-a^2cx^2+c)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5/((-a^2*c*x^2 + c)^(3/2)*a^2*c*x + (-a^2*c*x^2 + c)^(3/2)*a*c) - 2/5*x/(
sqrt(-a^2*c*x^2 + c)*c^2) - 1/5*x/((-a^2*c*x^2 + c)^(3/2)*c)
```

Fricas [A]
time = 0.39, size = 75, normalized size = 1.00

$$\frac{(2\,a^3x^3+4\,a^2x^2+ax-2)\sqrt{-a^2cx^2+c}}{5\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*a^3*x^3 + 4*a^2*x^2 + a*x - 2)*sqrt(-a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2
*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - 1}{\left(-c\left(ax - 1\right)\left(ax + 1\right)\right)^{\frac{5}{2}}\left(ax + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(5/2)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(5/2)*(a*x + 1)), x)
```

Mupad [B]
time = 1.40, size = 56, normalized size = 0.75

$$\frac{\sqrt{c - a^2\,c\,x^2}\,\left(2\,a^3\,x^3 + 4\,a^2\,x^2 + a\,x - 2\right)}{5\,a\,c^3\,\left(a\,x - 1\right)\,{\left(a\,x + 1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(5/2)*(a*x + 1)),x)
[Out] ((c - a^2*c*x^2)^(1/2)*(a*x + 4*a^2*x^2 + 2*a^3*x^3 - 2))/(5*a*c^3*(a*x - 1
)*(a*x + 1)^3)
```

3.657 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}$$

```
[Out] 2/7*(-a*x+1)/a/(-a^2*c*x^2+c)^(7/2)-1/7*x/c/(-a^2*c*x^2+c)^(5/2)-4/21*x/c^2
/(-a^2*c*x^2+c)^(3/2)-8/21*x/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6302, 6277, 667, 198, 197}

$$-\frac{8x}{21c^3\sqrt{c-a^2cx^2}}-\frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{x}{7c\left(c-a^2cx^2\right)^{5/2}}+\frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (2*(1 - a*x))/(7*a*(c - a^2*c*x^2)^(7/2)) - x/(7*c*(c - a^2*c*x^2)^(5/2)) -
 (4*x)/(21*c^2*(c - a^2*c*x^2)^(3/2)) - (8*x)/(21*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx \\
&= -\left(c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{9/2}}\,dx\right) \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{5}{7}\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{7c} \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8\int \frac{1}{(c-a^2cx^2)^{3/2}}\,dx}{21c^2} \\
&= \frac{2(1-ax)}{7a\left(c-a^2cx^2\right)^{7/2}} - \frac{x}{7c\left(c-a^2cx^2\right)^{5/2}} - \frac{4x}{21c^2\left(c-a^2cx^2\right)^{3/2}} - \frac{8x}{21c^3\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 96, normalized size = 0.98

$$-\frac{\sqrt{1-a^2x^2}\,(-6+9ax+24a^2x^2+4a^3x^3-16a^4x^4-8a^5x^5)}{21ac^3(1-ax)^{3/2}(1+ax)^{7/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
[Out] -1/21*(Sqrt[1 - a^2*x^2]*(-6 + 9*a*x + 24*a^2*x^2 + 4*a^3*x^3 - 16*a^4*x^4
- 8*a^5*x^5))/(a*c^3*(1 - a*x)^(3/2)*(1 + a*x)^(7/2)*Sqrt[c - a^2*c*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(82) = 164.
time = 0.20, size = 268, normalized size = 2.73

method	result
gosper	$\frac{(ax-1)^2(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)}{21a(-a^2cx^2+c)^{\frac{7}{2}}}$
trager	$\frac{(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)\sqrt{-a^2cx^2+c}}{21c^4(ax+1)^4(ax-1)^2a}$
default	$\frac{x}{5c(-a^2cx^2+c)^{\frac{5}{2}}}+\frac{\frac{4x}{15c(-a^2cx^2+c)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2cx^2+c}}}{c}-\frac{2\left(-\frac{1}{7ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac\right)^{\frac{5}{2}}}+\frac{6a\left(-\frac{-2}{10a^2c^2\left(-a^2\right.}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x/c/(-a^2*c*x^2+c)^(5/2)+4/5/c*(1/3*x/c/(-a^2*c*x^2+c)^(3/2)+2/3*x/c^2/
(-a^2*c*x^2+c)^(1/2))-2/a*(-1/7/a/c/(x+1/a)/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c
)^(5/2)+6/7*a*(-1/10*(-2*a^2*c*(x+1/a)+2*a*c)/a^2/c^2/(-a^2*c*(x+1/a)^2+2*(
x+1/a)*a*c)^(5/2)+4/5/c*(-1/6*(-2*a^2*c*(x+1/a)+2*a*c)/a^2/c^2/(-a^2*c*(x+1
/a)^2+2*(x+1/a)*a*c)^(3/2)-1/3/a^2/c^3*(-2*a^2*c*(x+1/a)+2*a*c)/(-a^2*c*(x+
1/a)^2+2*(x+1/a)*a*c)^(1/2))))
```

Maxima [A]
time = 0.26, size = 98, normalized size = 1.00

$$\frac{2}{7\left((-a^2cx^2+c)^{\frac{5}{2}}a^2cx+(-a^2cx^2+c)^{\frac{5}{2}}ac\right)}-\frac{8x}{21\sqrt{-a^2cx^2+c}\,c^3}-\frac{4x}{21(-a^2cx^2+c)^{\frac{3}{2}}c^2}-\frac{x}{7(-a^2cx^2+c)^{\frac{5}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7/((-a^2*c*x^2 + c)^(5/2)*a^2*c*x + (-a^2*c*x^2 + c)^(5/2)*a*c) - 8/21*x/
(sqrt(-a^2*c*x^2 + c)*c^3) - 4/21*x/((-a^2*c*x^2 + c)^(3/2)*c^2) - 1/7*x/((
-a^2*c*x^2 + c)^(5/2)*c)
```

Fricas [A]
time = 0.50, size = 124, normalized size = 1.27

$$\frac{(8a^5x^5+16a^4x^4-4a^3x^3-24a^2x^2-9ax+6)\sqrt{-a^2cx^2+c}}{21(a^7c^4x^6+2a^6c^4x^5-a^5c^4x^4-4a^4c^4x^3-a^3c^4x^2+2a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(8*a^5*x^5 + 16*a^4*x^4 - 4*a^3*x^3 - 24*a^2*x^2 - 9*a*x + 6)*sqrt(-a^
2*c*x^2 + c)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a
^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(7/2)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(7/2)*(a*x + 1)), x)
```

Mupad [B]
time = 1.45, size = 134, normalized size = 1.37

$$\frac{\sqrt{c-a^2\,c\,x^2}}{14\,a\,c^4\,{\left(a\,x+1\right)}^3}+\frac{\sqrt{c-a^2\,c\,x^2}}{28\,a\,c^4\,{\left(a\,x+1\right)}^4}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{11\,x}{42\,c^4}-\frac{5}{28\,a\,c^4}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}+\frac{8\,x\,\sqrt{c-a^2\,c\,x^2}}{21\,c^4\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(7/2)*(a*x + 1)),x)
```

```
[Out] (c - a^2*c*x^2)^(1/2)/(14*a*c^4*(a*x + 1)^3) + (c - a^2*c*x^2)^(1/2)/(28*a*
c^4*(a*x + 1)^4) - ((c - a^2*c*x^2)^(1/2)*((11*x)/(42*c^4) - 5/(28*a*c^4)))
/((a*x - 1)^2*(a*x + 1)^2) + (8*x*(c - a^2*c*x^2)^(1/2))/(21*c^4*(a*x - 1)*
(a*x + 1))
```

3.658 $\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}}-\frac{x}{9c\,(c-a^2cx^2)^{7/2}}-\frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}}-\frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}}-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}$$

```
[Out] 2/9*(-a*x+1)/a/(-a^2*c*x^2+c)^(9/2)-1/9*x/c/(-a^2*c*x^2+c)^(7/2)-2/15*x/c^2
/(-a^2*c*x^2+c)^(5/2)-8/45*x/c^3/(-a^2*c*x^2+c)^(3/2)-16/45*x/c^4/(-a^2*c*x
^2+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {6302, 6277, 667, 198, 197}

$$-\frac{16x}{45c^4\sqrt{c-a^2cx^2}}-\frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}}-\frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}}-\frac{x}{9c\,(c-a^2cx^2)^{7/2}}+\frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (2*(1 - a*x))/(9*a*(c - a^2*c*x^2)^(9/2)) - x/(9*c*(c - a^2*c*x^2)^(7/2)) -
 (2*x)/(15*c^2*(c - a^2*c*x^2)^(5/2)) - (8*x)/(45*c^3*(c - a^2*c*x^2)^(3/2)
) - (16*x)/(45*c^4*Sqrt[c - a^2*c*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 667

```
Int[((d_) + (e_.)*(x_))^2*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(
d + e*x)*((a + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[e^2*((p + 2)/(c*(p +
1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c
*d^2 + a*e^2, 0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx\\ &= -\left(c\int \frac{(1-ax)^2}{(c-a^2cx^2)^{11/2}}\,dx\right)\\ &= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{7}{9}\int \frac{1}{(c-a^2cx^2)^{9/2}}\,dx\\ &= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2\int \frac{1}{(c-a^2cx^2)^{7/2}}\,dx}{3c}\\ &= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}} - \frac{8\int \frac{1}{(c-a^2cx^2)^{5/2}}\,dx}{15c^2}\\ &= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}} - \frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}} -\\ &= \frac{2(1-ax)}{9a\,(c-a^2cx^2)^{9/2}} - \frac{x}{9c\,(c-a^2cx^2)^{7/2}} - \frac{2x}{15c^2\,(c-a^2cx^2)^{5/2}} - \frac{8x}{45c^3\,(c-a^2cx^2)^{3/2}} -\end{aligned}$$

Mathematica [A]
time = 0.05, size = 112, normalized size = 0.93

$$\frac{\sqrt{1-a^2x^2}\,(10-25ax-60a^2x^2+10a^3x^3+80a^4x^4+24a^5x^5-32a^6x^6-16a^7x^7)}{45ac^4(1-ax)^{5/2}(1+ax)^{9/2}\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^(9/2)),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(10 - 25*a*x - 60*a^2*x^2 + 10*a^3*x^3 + 80*a^4*x^4 + 24
*a^5*x^5 - 32*a^6*x^6 - 16*a^7*x^7))/(45*a*c^4*(1 - a*x)^(5/2)*(1 + a*x)^(9
/2)*Sqrt[c - a^2*c*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(101) = 202$.
time = 0.20, size = 348, normalized size = 2.88

method	result
gosper	$-\frac{(ax-1)^2(16a^7x^7+32a^6x^6-24a^5x^5-80a^4x^4-10a^3x^3+60a^2x^2+25ax-10)}{45a(-a^2cx^2+c)^{\frac{9}{2}}}$
trager	$\frac{(16a^7x^7+32a^6x^6-24a^5x^5-80a^4x^4-10a^3x^3+60a^2x^2+25ax-10)\sqrt{-a^2cx^2+c}}{45c^5(ax+1)^5(ax-1)^3a}$
default	$\frac{x}{7c(-a^2cx^2+c)^{\frac{7}{2}}}+\frac{\frac{6x}{35c(-a^2cx^2+c)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c(-a^2cx^2+c)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{-a^2cx^2+c}}\right)}{7c}}{c}-\frac{2\left(-\frac{1}{9ac\left(x+\frac{1}{a}\right)\left(-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x/c/(-a^2*c*x^2+c)^(7/2)+6/7/c*(1/5*x/c/(-a^2*c*x^2+c)^(5/2)+4/5/c*(1/3
*x/c/(-a^2*c*x^2+c)^(3/2)+2/3*x/c^2/(-a^2*c*x^2+c)^(1/2)))-2/a*(-1/9/a/c/(x
+1/a)/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(7/2)+8/9*a*(-1/14*(-2*a^2*c*(x+1/a)
+2*a*c)/a^2/c^2/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(7/2)+6/7/c*(-1/10*(-2*a^2
*c*(x+1/a)+2*a*c)/a^2/c^2/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(5/2)+4/5/c*(-1/
6*(-2*a^2*c*(x+1/a)+2*a*c)/a^2/c^2/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(3/2)-1
/3/a^2/c^3*(-2*a^2*c*(x+1/a)+2*a*c)/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))
)))
```

Maxima [A]
time = 0.27, size = 117, normalized size = 0.97

$$\frac{2}{9\left((-a^2cx^2+c)^{\frac{7}{2}}a^2cx+(-a^2cx^2+c)^{\frac{7}{2}}ac\right)}-\frac{16\,x}{45\,\sqrt{-a^2cx^2+c}\,c^4}-\frac{8\,x}{45\,(-a^2cx^2+c)^{\frac{3}{2}}c^3}-\frac{2\,x}{15\,(-a^2cx^2+c)^{\frac{5}{2}}c^2}-\frac{x}{9\,(-a^2cx^2+c)^{\frac{7}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/9/((-a^2*c*x^2 + c)^(7/2)*a^2*c*x + (-a^2*c*x^2 + c)^(7/2)*a*c) - 16/45*x
/(sqrt(-a^2*c*x^2 + c)*c^4) - 8/45*x/((-a^2*c*x^2 + c)^(3/2)*c^3) - 2/15*x/
((-a^2*c*x^2 + c)^(5/2)*c^2) - 1/9*x/((-a^2*c*x^2 + c)^(7/2)*c)
```

Fricas [A]
time = 0.64, size = 152, normalized size = 1.26

$$\frac{(16\,a^7x^7+32\,a^6x^6-24\,a^5x^5-80\,a^4x^4-10\,a^3x^3+60\,a^2x^2+25\,ax-10)\sqrt{-a^2cx^2+c}}{45\,(a^9c^5x^8+2\,a^8c^5x^7-2\,a^7c^5x^6-6\,a^6c^5x^5+6\,a^4c^5x^3+2\,a^3c^5x^2-2\,a^2c^5x-ac^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(16*a^7*x^7 + 32*a^6*x^6 - 24*a^5*x^5 - 80*a^4*x^4 - 10*a^3*x^3 + 60*a
^2*x^2 + 25*a*x - 10)*sqrt(-a^2*c*x^2 + c)/(a^9*c^5*x^8 + 2*a^8*c^5*x^7 - 2
*a^7*c^5*x^6 - 6*a^6*c^5*x^5 + 6*a^4*c^5*x^3 + 2*a^3*c^5*x^2 - 2*a^2*c^5*x
- a*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax-1}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{9}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a**2*c*x**2+c)**(9/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(a*x - 1)*(a*x + 1))**(9/2)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*x - 1)/((-a^2*c*x^2 + c)^(9/2)*(a*x + 1)), x)
```

Mupad [B]
time = 1.49, size = 177, normalized size = 1.46

$$\frac{5\sqrt{c-a^2\,c\,x^2}}{144\,a\,c^5\,{\left(a\,x+1\right)}^4}+\frac{\sqrt{c-a^2\,c\,x^2}}{72\,a\,c^5\,{\left(a\,x+1\right)}^5}+\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{31\,x}{120\,c^5}-\frac{5}{24\,a\,c^5}\right)}{{\left(a\,x-1\right)}^3\,{\left(a\,x+1\right)}^3}-\frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{8\,x}{45\,c^5}+\frac{5}{144\,a\,c^5}\right)}{{\left(a\,x-1\right)}^2\,{\left(a\,x+1\right)}^2}+\frac{16\,x\,\sqrt{c-a^2\,c\,x^2}}{45\,c^5\,\left(a\,x-1\right)\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - a^2*c*x^2)^(9/2)*(a*x + 1)),x)
```

```
[Out] (5*(c - a^2*c*x^2)^(1/2))/(144*a*c^5*(a*x + 1)^4) + (c - a^2*c*x^2)^(1/2)/(
72*a*c^5*(a*x + 1)^5) + ((c - a^2*c*x^2)^(1/2)*((31*x)/(120*c^5) - 5/(24*a*
c^5)))/((a*x - 1)^3*(a*x + 1)^3) - ((c - a^2*c*x^2)^(1/2)*((8*x)/(45*c^5) +
 5/(144*a*c^5)))/((a*x - 1)^2*(a*x + 1)^2) + (16*x*(c - a^2*c*x^2)^(1/2))/(
45*c^5*(a*x - 1)*(a*x + 1))
```

3.659 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{9/2}\,dx$

Optimal. Leaf size=189

$$-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$

```
[Out] -8/7*(-a*x+1)^7*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+3/2*(-a*x
+1)^8*(-a^2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9-2/3*(-a*x+1)^9*(-a^
2*c*x^2+c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9+1/10*(-a*x+1)^10*(-a^2*c*x^2+
c)^(9/2)/a^10/(1-1/a^2/x^2)^(9/2)/x^9
```

Rubi [A]

time = 0.14, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 45}

$$\frac{(1-ax)^{10}\left(c-a^2cx^2\right)^{9/2}}{10a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{2(1-ax)^9\left(c-a^2cx^2\right)^{9/2}}{3a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}x^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-8*(1 - a*x)^7*(c - a^2*c*x^2)^(9/2))/(7*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^9)
 + (3*(1 - a*x)^8*(c - a^2*c*x^2)^(9/2))/(2*a^10*(1 - 1/(a^2*x^2))^(9/2)*x^
9) - (2*(1 - a*x)^9*(c - a^2*c*x^2)^(9/2))/(3*a^10*(1 - 1/(a^2*x^2))^(9/2)*
x^9) + ((1 - a*x)^10*(c - a^2*c*x^2)^(9/2))/(10*a^10*(1 - 1/(a^2*x^2))^(9/2
)*x^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{9/2}\,dx=\frac{\left(c-a^2cx^2\right)^{9/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int(-1+ax)^6(1+ax)^3\,dx}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=\frac{\left(c-a^2cx^2\right)^{9/2}\int\left(8(-1+ax)^6+12(-1+ax)^7+6(-1+ax)^8+(-1+a\right.}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}$$
$$=-\frac{8(1-ax)^7\left(c-a^2cx^2\right)^{9/2}}{7a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}+\frac{3(1-ax)^8\left(c-a^2cx^2\right)^{9/2}}{2a^{10}\left(1-\frac{1}{a^2x^2}\right)^{9/2}x^9}-\frac{2(1-ax)^9\left(c\right.}{3a^{10}\left(1-\frac{1}{a^2\right.}}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.38

$$\frac{c^4(-1+ax)^7\sqrt{c-a^2cx^2}\left(44+98ax+77a^2x^2+21a^3x^3\right)}{210a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(9/2)/E^(3*ArcCoth[a*x]),x]
[Out] (c^4*(-1 + a*x)^7*Sqrt[c - a^2*c*x^2]*(44 + 98*a*x + 77*a^2*x^2 + 21*a^3*x^
3))/(210*a^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 102, normalized size = 0.54

method	result	size
gosper	$\frac{x\left(21a^9x^9-70a^8x^8+240a^6x^6-210a^5x^5-252a^4x^4+420a^3x^3-315ax+210\right)\left(-a^2c\,x^2+c\right)^{\frac{9}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210(ax+1)^3(ax-1)^6}$	100
default	$\frac{\left(21a^9x^9-70a^8x^8+240a^6x^6-210a^5x^5-252a^4x^4+420a^3x^3-315ax+210\right)x\,c^4\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{210(ax-1)^2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210*(21*a^9*x^9-70*a^8*x^8+240*a^6*x^6-210*a^5*x^5-252*a^4*x^4+420*a^3*x^
3-315*a*x+210)*x*c^4*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)
/(a*x-1)^2
```

Maxima [**A**]
time = 0.29, size = 204, normalized size = 1.08

$$\frac{(21\,a^{11}\sqrt{-c}\,c^4x^{11}-49\,a^{10}\sqrt{-c}\,c^4x^{10}-70\,a^9\sqrt{-c}\,c^4x^9+240\,a^8\sqrt{-c}\,c^4x^8+30\,a^7\sqrt{-c}\,c^4x^7-462\,a^6\sqrt{-c}\,c^4x^6+168\,a^5\sqrt{-c}\,c^4x^5+420\,a^4\sqrt{-c}\,c^4x^4-315\,a^3\sqrt{-c}\,c^4x^3-105\,a^2\sqrt{-c}\,c^4x^2-210\sqrt{-c}\,c^4)(ax-1)^2}{210\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/210*(21*a^11*sqrt(-c)*c^4*x^11 - 49*a^10*sqrt(-c)*c^4*x^10 - 70*a^9*sqrt(
-c)*c^4*x^9 + 240*a^8*sqrt(-c)*c^4*x^8 + 30*a^7*sqrt(-c)*c^4*x^7 - 462*a^6*
sqrt(-c)*c^4*x^6 + 168*a^5*sqrt(-c)*c^4*x^5 + 420*a^4*sqrt(-c)*c^4*x^4 - 31
5*a^3*sqrt(-c)*c^4*x^3 - 105*a^2*sqrt(-c)*c^4*x^2 - 210*sqrt(-c)*c^4)*(a*x
- 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Fricas [**A**]
time = 0.34, size = 95, normalized size = 0.50

$$\frac{(21\,a^9c^4x^{10}-70\,a^8c^4x^9+240\,a^6c^4x^7-210\,a^5c^4x^6-252\,a^4c^4x^5+420\,a^3c^4x^4-315\,ac^4x^2+210\,c^4x)\sqrt{-a^2c}}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/210*(21*a^9*c^4*x^10 - 70*a^8*c^4*x^9 + 240*a^6*c^4*x^7 - 210*a^5*c^4*x^6
 - 252*a^4*c^4*x^5 + 420*a^3*c^4*x^4 - 315*a*c^4*x^2 + 210*c^4*x)*sqrt(-a^2
*c)/a
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^{9/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.660 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=142

$$\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}$$

```
[Out] 2/3*(-a*x+1)^6*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7-4/7*(-a*x+1
)^7*(-a^2*c*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7+1/8*(-a*x+1)^8*(-a^2*c
*x^2+c)^(7/2)/a^8/(1-1/a^2/x^2)^(7/2)/x^7
```

Rubi [A]
time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6327, 6328, 45}

$$\frac{(1-ax)^8\left(c-a^2cx^2\right)^{7/2}}{8a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}+\frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (2*(1 - a*x)^6*(c - a^2*c*x^2)^(7/2))/(3*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7) -
 (4*(1 - a*x)^7*(c - a^2*c*x^2)^(7/2))/(7*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
+ ((1 - a*x)^8*(c - a^2*c*x^2)^(7/2))/(8*a^8*(1 - 1/(a^2*x^2))^(7/2)*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{7/2}\,dx &= \frac{\left(c-a^2cx^2\right)^{7/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(c-a^2cx^2\right)^{7/2}\int(-1+ax)^5(1+ax)^2\,dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{\left(c-a^2cx^2\right)^{7/2}\int\left(4(-1+ax)^5+4(-1+ax)^6+(-1+ax)^7\right)dx}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\\ &= \frac{2(1-ax)^6\left(c-a^2cx^2\right)^{7/2}}{3a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}-\frac{4(1-ax)^7\left(c-a^2cx^2\right)^{7/2}}{7a^8\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}+\frac{(1-ax)^8\left(c-a\right.}{8a^8\left(1-\frac{1}{a^2x^2}\right)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.44

$$-\frac{c^3(-1+ax)^6\left(37+54ax+21a^2x^2\right)\sqrt{c-a^2cx^2}}{168a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(7/2)/E^(3*ArcCoth[a*x]),x]
[Out] -1/168*(c^3*(-1 + a*x)^6*(37 + 54*a*x + 21*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(a
^2*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 102, normalized size = 0.72

method	result	size
gosper	$\frac{x\left(21a^7x^7-72a^6x^6+28a^5x^5+168a^4x^4-210a^3x^3-56a^2x^2+252ax-168\right)\left(-a^2c\,x^2+c\right)^{\frac{7}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168(ax+1)^2(ax-1)^5}$	100
default	$-\frac{\left(21a^7x^7-72a^6x^6+28a^5x^5+168a^4x^4-210a^3x^3-56a^2x^2+252ax-168\right)x\,c^3\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{168(ax-1)^2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/168*(21*a^7*x^7-72*a^6*x^6+28*a^5*x^5+168*a^4*x^4-210*a^3*x^3-56*a^2*x^2
+252*a*x-168)*x*c^3*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/
(a*x-1)^2
```

Maxima [A]
time = 0.28, size = 172, normalized size = 1.21

$$-\frac{\left(21\,a^{9}\sqrt{-c}\,c^{3}x^{9}-51\,a^{8}\sqrt{-c}\,c^{3}x^{8}-44\,a^{7}\sqrt{-c}\,c^{3}x^{7}+196\,a^{6}\sqrt{-c}\,c^{3}x^{6}-42\,a^{5}\sqrt{-c}\,c^{3}x^{5}-266\,a^{4}\sqrt{-c}\,c^{3}x^{4}+196\,a^{3}\sqrt{-c}\,c^{3}x^{3}+84\,a^{2}\sqrt{-c}\,c^{3}x^{2}+168\,\sqrt{-c}\,c^{3}\right)(ax-1)^{2}}{168\,(a^{3}x^{2}-2\,a^{2}x+a)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/168*(21*a^9*sqrt(-c)*c^3*x^9 - 51*a^8*sqrt(-c)*c^3*x^8 - 44*a^7*sqrt(-c)
*c^3*x^7 + 196*a^6*sqrt(-c)*c^3*x^6 - 42*a^5*sqrt(-c)*c^3*x^5 - 266*a^4*sqr
t(-c)*c^3*x^4 + 196*a^3*sqrt(-c)*c^3*x^3 + 84*a^2*sqrt(-c)*c^3*x^2 + 168*sq
rt(-c)*c^3)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Fricas [A]
time = 0.34, size = 95, normalized size = 0.67

$$-\frac{\left(21\,a^{7}c^{3}x^{8}-72\,a^{6}c^{3}x^{7}+28\,a^{5}c^{3}x^{6}+168\,a^{4}c^{3}x^{5}-210\,a^{3}c^{3}x^{4}-56\,a^{2}c^{3}x^{3}+252\,ac^{3}x^{2}-168\,c^{3}x\right)\sqrt{-a^{2}c}}{168\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/168*(21*a^7*c^3*x^8 - 72*a^6*c^3*x^7 + 28*a^5*c^3*x^6 + 168*a^4*c^3*x^5
- 210*a^3*c^3*x^4 - 56*a^2*c^3*x^3 + 252*a*c^3*x^2 - 168*c^3*x)*sqrt(-a^2*c
)/a
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.661 $\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx$

Optimal. Leaf size=95

$$-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}$$

```
[Out] -2/5*(-a*x+1)^5*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5+1/6*(-a*x+
1)^6*(-a^2*c*x^2+c)^(5/2)/a^6/(1-1/a^2/x^2)^(5/2)/x^5
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6327, 6328, 45}

$$\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-2*(1 - a*x)^5*(c - a^2*c*x^2)^(5/2))/(5*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
+ ((1 - a*x)^6*(c - a^2*c*x^2)^(5/2))/(6*a^6*(1 - 1/(a^2*x^2))^(5/2)*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{5/2}\,dx&=\frac{\left(c-a^2cx^2\right)^{5/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\&=\frac{\left(c-a^2cx^2\right)^{5/2}\int(-1+ax)^4(1+ax)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\&=\frac{\left(c-a^2cx^2\right)^{5/2}\int\left(2(-1+ax)^4+(-1+ax)^5\right)\,dx}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\\&=-\frac{2(1-ax)^5\left(c-a^2cx^2\right)^{5/2}}{5a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}+\frac{(1-ax)^6\left(c-a^2cx^2\right)^{5/2}}{6a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.58

$$\frac{c^2(-1+ax)^5(7+5ax)\sqrt{c-a^2cx^2}}{30a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(-1 + a*x)^5*(7 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(30*a^2*Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Maple [A]
time = 0.11, size = 86, normalized size = 0.91

method	result	size
gosper	$\frac{x\left(5a^5x^5-18a^4x^4+15a^3x^3+20a^2x^2-45ax+30\right)\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30(ax+1)(ax-1)^4}$	84
default	$\frac{\left(5a^5x^5-18a^4x^4+15a^3x^3+20a^2x^2-45ax+30\right)x\,c^2\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{30(ax-1)^2}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(5*a^5*x^5-18*a^4*x^4+15*a^3*x^3+20*a^2*x^2-45*a*x+30)*x*c^2*(-c*(a^2*
x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2
```

Maxima [A]
time = 0.28, size = 140, normalized size = 1.47

$$\frac{\left(5\,a^7\sqrt{-c}\,c^2x^7-13\,a^6\sqrt{-c}\,c^2x^6-3\,a^5\sqrt{-c}\,c^2x^5+35\,a^4\sqrt{-c}\,c^2x^4-25\,a^3\sqrt{-c}\,c^2x^3-15\,a^2\sqrt{-c}\,c^2x^2-30\,\sqrt{-c}\,c^2\right)(ax-1)^2}{30\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/30*(5*a^7*sqrt(-c)*c^2*x^7 - 13*a^6*sqrt(-c)*c^2*x^6 - 3*a^5*sqrt(-c)*c^2
*x^5 + 35*a^4*sqrt(-c)*c^2*x^4 - 25*a^3*sqrt(-c)*c^2*x^3 - 15*a^2*sqrt(-c)*
c^2*x^2 - 30*sqrt(-c)*c^2)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*x + 1))
```

Fricas [A]
time = 0.35, size = 73, normalized size = 0.77

$$\frac{\left(5\,a^5c^2x^6-18\,a^4c^2x^5+15\,a^3c^2x^4+20\,a^2c^2x^3-45\,ac^2x^2+30\,c^2x\right)\sqrt{-a^2c}}{30\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/30*(5*a^5*c^2*x^6 - 18*a^4*c^2*x^5 + 15*a^3*c^2*x^4 + 20*a^2*c^2*x^3 - 45
*a*c^2*x^2 + 30*c^2*x)*sqrt(-a^2*c)/a
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.662 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal. Leaf size=47

$$\frac{(1-ax)^4\,(c-a^2cx^2)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] 1/4*(-a*x+1)^4*(-a^2*c*x^2+c)^(3/2)/a^4/(1-1/a^2/x^2)^(3/2)/x^3
```

Rubi [A]
time = 0.12, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6328, 32}

$$\frac{(1-ax)^4\,(c-a^2cx^2)^{3/2}}{4a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((1 - a*x)^4*(c - a^2*c*x^2)^(3/2))/(4*a^4*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{\left(c-a^2cx^2\right)^{3/2}\int(-1+ax)^3\,dx}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$
$$=\frac{(1-ax)^4\left(c-a^2cx^2\right)^{3/2}}{4a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A]
time = 0.04, size = 69, normalized size = 1.47

$$-\frac{ac\sqrt{1-\frac{1}{a^2x^2}}\,x^2\sqrt{c-a^2cx^2}\,(-4+6ax-4a^2x^2+a^3x^3)}{-4+4a^2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -((a*c*Sqrt[1 - 1/(a^2*x^2)]*x^2*Sqrt[c - a^2*c*x^2]*(-4 + 6*a*x - 4*a^2*x^
2 + a^3*x^3))/(-4 + 4*a^2*x^2))
```

Maple [A]
time = 0.09, size = 48, normalized size = 1.02

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)^2(ax+1)\sqrt{-c\left(a^2x^2-1\right)}\,c}{4a}$	48
gosper	$\frac{x\left(a^3x^3-4a^2x^2+6ax-4\right)\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4(ax-1)^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(3/2)*(a*x-1)^2*(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*c/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(40) = 80.
time = 0.29, size = 97, normalized size = 2.06

$$-\frac{\left(a^5\sqrt{-c}\,cx^5-3\,a^4\sqrt{-c}\,cx^4+2\,a^3\sqrt{-c}\,cx^3+2\,a^2\sqrt{-c}\,cx^2+4\,\sqrt{-c}\,c\right)(ax-1)^2}{4\,(a^3x^2-2\,a^2x+a)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/4*(a^5*sqrt(-c)*c*x^5 - 3*a^4*sqrt(-c)*c*x^4 + 2*a^3*sqrt(-c)*c*x^3 + 2*
a^2*sqrt(-c)*c*x^2 + 4*sqrt(-c)*c)*(a*x - 1)^2/((a^3*x^2 - 2*a^2*x + a)*(a*
x + 1))
```

Fricas [A]
time = 0.34, size = 42, normalized size = 0.89

$$-\frac{(a^3cx^4 - 4\,a^2cx^3 + 6\,acx^2 - 4\,cx)\sqrt{-a^2c}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/4*(a^3*c*x^4 - 4*a^2*c*x^3 + 6*a*c*x^2 - 4*c*x)*sqrt(-a^2*c)/a
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c-a^2\,c\,x^2\right)}^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.663 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] -3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6327, 6328, 45}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int\frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int\left(-3+ax+\frac{4}{1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(-6+ax)+8\log(1+ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 67, normalized size = 0.60

method	result	size
default	$\frac{\left(a^2x^2-6ax+8\ln(ax+1)\right)\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a(ax-1)^2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/a/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.34, size = 33, normalized size = 0.29

$$\frac{\left(a^{2} x^{2}-6\,ax+8\,\log\left(ax+1\right)\right)\sqrt{-a^{2}c}}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.664 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1+ax)\sqrt{c-a^2cx^2}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

```
[Out] 2*x*(1-1/a^2/x^2)^(1/2)/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+x*ln(a*x+1)*(1-1/a^2/x
^2)^(1/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 45}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}}{(ax+1)\sqrt{c-a^2cx^2}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\,\log(ax+1)}{\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*x)/((1 + a*x)*Sqrt[c - a^2*c*x^2]) + (Sqrt[1 - 1/(
a^2*x^2)]*x*Log[1 + a*x])/Sqrt[c - a^2*c*x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{-1+ax}{(1+ax)^2}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \left(-\frac{2}{(1+ax)^2}+\frac{1}{1+ax}\right)dx}{\sqrt{c-a^2cx^2}}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\,x}{(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\log(1+ax)}{\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.68

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+(1+ax)\log(1+ax))}{(1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 + (1 + a*x)*Log[1 + a*x]))/((1 + a*x)*Sqrt[c -
a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.81

method	result	size
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(\ln(ax+1)ax+\ln(ax+1)+2)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{ac(ax-1)^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a*x+ln(a*x+1)+2)*((a*x-1)/(a*x+1))^(3/2)
/a/c/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [A]
time = 0.34, size = 38, normalized size = 0.49

$$-\frac{\sqrt{-a^2c}\,((ax+1)\log{(ax+1)}+2)}{a^3cx+a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas
")
```

```
[Out] -sqrt(-a^2*c)*((a*x + 1)*log(a*x + 1) + 2)/(a^3*c*x + a^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

3.665 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(a*x+1)^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]

time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 32}

$$-\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(ax+1)^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/2*(a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/((1 + a*x)^2*(c - a^2*c*x^2)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(1+ax)^3}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1+ax)^2\left(c-a^2cx^2\right)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 1.11

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}{2c^2(-1+ax)(1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2)),x]
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])/(c^2*(-1 + a*x)*(1 + a*x
)^3)
```

Maple [A]
time = 0.10, size = 56, normalized size = 1.22

method	result	size
gosper	$-\frac{(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a(-a^2c\,x^2+c)^{\frac{3}{2}}}$	39
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\left(a^2x^2-1\right)}}{2(ax-1)(a^2x^2-1)a\,c^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)/(a^2*x^2-1)/a/c
^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 0.33, size = 39, normalized size = 0.85

$$-\frac{\sqrt{-a^2c}}{2\left(a^4c^2x^2+2\,a^3c^2x+a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*sqrt(-a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(((a*x - 1)/(a*x + 1))**(3/2)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B]
time = 1.47, size = 58, normalized size = 1.26

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a^2\,c\,\left(x\,\sqrt{c-a^2\,c\,x^2}+\frac{\sqrt{c-a^2\,c\,x^2}}{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(2*a^2*c*(x*(c - a^2*c*x^2)^(1/2) + (c - a^2*c*
x^2)^(1/2)/a))
```

3.666 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}(a$$

```
[Out] 1/6*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)^3/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1
/a^2/x^2)^(5/2)*x^5/(a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(5
/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arctan
h(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]

time = 0.14, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6327, 6328, 46, 213}

$$\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{6(ax+1)^3\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(6*(1 + a*x)^3*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)^2*(c - a^2*c*x^2)^(5/2)) + (a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^4*
(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)(1+ax)^4}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{2(1+ax)^4}-\frac{1}{4(1+ax)^3}-\frac{1}{8(1+ax)^2}+\frac{1}{8(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}+\\
&= \frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{6(1+ax)^3(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}-\end{aligned}$$

Mathematica [A]
time = 0.05, size = 71, normalized size = 0.39

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-10-9ax-3a^2x^2+3(1+ax)^3\tanh^{-1}(ax)\right)}{24c^2(1+ax)^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/24*(Sqrt[1 - 1/(a^2*x^2)]*x*(-10 - 9*a*x - 3*a^2*x^2 + 3*(1 + a*x)^3*Arc
Tanh[a*x]))/(c^2*(1 + a*x)^3*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 169, normalized size = 0.93

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\sqrt{-c\left(a^{2}x^{2}-1\right)}\,(3\ln(ax+1)a^{3}x^{3}-3x^{3}\ln(ax-1)a^{3}+9\ln(ax+1)a^{2}x^{2}-9x^{2}\ln(ax-1)a^{2}-6a^{2}x^{2}+9\ln(ax+1)ax-9x\ln}{48(ax-1)(ax+1)(a^{2}x^{2}-1)c^{3}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)/(a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a
*x+1)*a^3*x^3-3*x^3*ln(a*x-1)*a^3+9*ln(a*x+1)*a^2*x^2-9*x^2*ln(a*x-1)*a^2-6
*a^2*x^2+9*ln(a*x+1)*a*x-9*x*ln(a*x-1)*a-18*a*x+3*ln(a*x+1)-3*ln(a*x-1)-20)
/(a^2*x^2-1)/c^3/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 0.38, size = 136, normalized size = 0.75

$$-\frac{3\left(a^{4}x^{3}+3\,a^{3}x^{2}+3\,a^{2}x+a\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{2}+2\,\sqrt{-a^{2}c}\,\sqrt{-c}\,x+c}{a^{2}x^{2}-1}\right)+2\left(3\,a^{2}x^{2}+9\,ax+10\right)\sqrt{-a^{2}c}}{48\left(a^{5}c^{3}x^{3}+3\,a^{4}c^{3}x^{2}+3\,a^{3}c^{3}x+a^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/48*(3*(a^4*x^3 + 3*a^3*x^2 + 3*a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sq
rt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 + 9*a*x + 10)*sqrt
(-a^2*c))/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(5/2), x)
```

3.667 $\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=275

$$\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1+ax)^2\left(c-a^2cx^2\right)^{7/2}}$$

```
[Out] 1/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(-a*x+1)/(-a^2*c*x^2+c)^(7/2)-1/16*a^6*(1-
1/a^2/x^2)^(7/2)*x^7/(a*x+1)^4/(-a^2*c*x^2+c)^(7/2)-1/12*a^6*(1-1/a^2/x^2)^
(7/2)*x^7/(a*x+1)^3/(-a^2*c*x^2+c)^(7/2)-3/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(
a*x+1)^2/(-a^2*c*x^2+c)^(7/2)-1/8*a^6*(1-1/a^2/x^2)^(7/2)*x^7/(a*x+1)/(-a^2
*c*x^2+c)^(7/2)+5/32*a^6*(1-1/a^2/x^2)^(7/2)*x^7*arctanh(a*x)/(-a^2*c*x^2+c
)^(7/2)
```

Rubi [A]

time = 0.15, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6327, 6328, 46, 213}

$$\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(1-ax)\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{8(ax+1)\left(c-a^2cx^2\right)^{7/2}}-\frac{3a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{32(ax+1)^2\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{12(ax+1)^3\left(c-a^2cx^2\right)^{7/2}}-\frac{a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}{16(ax+1)^4\left(c-a^2cx^2\right)^{7/2}}+\frac{5a^6x^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\tanh^{-1}(ax)}{32\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] (a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 - a*x)*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(16*(1 + a*x)^4*(c - a^2*c*x^2)^(7/2)) - (a
^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(12*(1 + a*x)^3*(c - a^2*c*x^2)^(7/2)) - (3
*a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(32*(1 + a*x)^2*(c - a^2*c*x^2)^(7/2)) -
(a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7)/(8*(1 + a*x)*(c - a^2*c*x^2)^(7/2)) + (5*
a^6*(1 - 1/(a^2*x^2))^(7/2)*x^7*ArcTanh[a*x])/(32*(c - a^2*c*x^2)^(7/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}\,dx}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{1}{(-1+ax)^2(1+ax)^5}\,dx}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{\left(a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7\right)\int \left(\frac{1}{32(-1+ax)^2}+\frac{1}{4(1+ax)^5}+\frac{1}{4(1+ax)^4}+\frac{3}{16(1+ax)^3}+\frac{1}{8(1+ax)^2}-\frac{}{32(-}\right.}{(c-a^2cx^2)^{7/2}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\,(c-a^2cx^2)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\,(c-a^2cx^2)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\,(c-a^2cx^2)^{7/}}$$

$$= \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{32(1-ax)\,(c-a^2cx^2)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{16(1+ax)^4\,(c-a^2cx^2)^{7/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{7/2}x^7}{12(1+ax)^3\,(c-a^2cx^2)^{7/}}$$

Mathematica [A]
time = 0.07, size = 99, normalized size = 0.36

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(32+15ax-35a^2x^2-45a^3x^3-15a^4x^4+15(-1+ax)(1+ax)^4\tanh^{-1}(ax)\right)}{96c^3(-1+ax)(1+ax)^4\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^(7/2)),x]
```

```
[Out] -1/96*(Sqrt[1 - 1/(a^2*x^2)]*x*(32 + 15*a*x - 35*a^2*x^2 - 45*a^3*x^3 - 15*
a^4*x^4 + 15*(-1 + a*x)*(1 + a*x)^4*ArcTanh[a*x]))/(c^3*(-1 + a*x)*(1 + a*x
)^4*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 241, normalized size = 0.88

method	result
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} \sqrt{-c\left(a^2x^2-1\right)}\,\left(15\ln(ax+1)a^5x^5-15x^5\ln(ax-1)a^5+45\ln(ax+1)a^4x^4-45x^4\ln(ax-1)a^4-30a^4x^4+30\ln(ax+1)a^3\right.}{192(ax-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/(a*x+1)^2*(-c*(a^2*x^2-1))^(1/2)*(1
5*ln(a*x+1)*a^5*x^5-15*x^5*ln(a*x-1)*a^5+45*ln(a*x+1)*a^4*x^4-45*x^4*ln(a*x
-1)*a^4-30*a^4*x^4+30*ln(a*x+1)*a^3*x^3-30*x^3*ln(a*x-1)*a^3-90*a^3*x^3-30*
ln(a*x+1)*a^2*x^2+30*x^2*ln(a*x-1)*a^2-70*a^2*x^2-45*ln(a*x+1)*a*x+45*x*ln(
a*x-1)*a+30*a*x-15*ln(a*x+1)+15*ln(a*x-1)+64)/(a^2*x^2-1)/c^4/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima
")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A]
time = 0.35, size = 193, normalized size = 0.70

$$-\frac{15\left(a^6x^5+3\,a^5x^4+2\,a^4x^3-2\,a^3x^2-3\,a^2x-a\right)\sqrt{-c}\,\log\left(\frac{a^2cx^2+2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\left(15\,a^4x^4+45\,a^3x^3+35\,a^2x^2-15\,ax-32\right)\sqrt{-a^2c}}{192\left(a^7c^4x^5+3\,a^6c^4x^4+2\,a^5c^4x^3-2\,a^4c^4x^2-3\,a^3c^4x-a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/192*(15*(a^6*x^5 + 3*a^5*x^4 + 2*a^4*x^3 - 2*a^3*x^2 - 3*a^2*x - a)*sqrt
(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(15
*a^4*x^4 + 45*a^3*x^3 + 35*a^2*x^2 - 15*a*x - 32)*sqrt(-a^2*c))/(a^7*c^4*x^
5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 - 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(-a**2*c*x**2+c)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(-a^2*c*x^2 + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2),x)
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - a^2*c*x^2)^(7/2), x)
```

3.668 $\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=76

$$\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^(
1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 45}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c
*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int x^2(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(x^2+ax^3\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.59

$$\frac{x^2(4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^2*Sqrt[c - a^2*c*x^2],x]
[Out] (x^2*(4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.63

method	result	size
gosper	$\frac{x^3(3ax+4)\sqrt{-a^2c\,x^2+c}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{(3ax+4)x^3\sqrt{-c\left(a^2x^2-1\right)}}{12(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/12*(3*a*x+4)*x^3*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 25, normalized size = 0.33

$$\frac{(3\,ax^4 + 4\,x^3)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/12*(3*a*x^4 + 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.669 $\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=74

$$\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/
2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6327, 6328, 45}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x
^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{\sqrt{c-a^2cx^2}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{\sqrt{c-a^2cx^2}\int\left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.58

$$\frac{x(3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x*Sqrt[c - a^2*c*x^2],x]
[Out] (x*(3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.65

method	result	size
gosper	$\frac{x^{2}(2ax+3)\sqrt{-a^{2}c\,x^{2}+c}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	47
default	$\frac{(2ax+3)x^{2}\sqrt{-c\left(a^{2}x^{2}-1\right)}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*(2*a*x+3)*x^2*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 25, normalized size = 0.34

$$\frac{\left(2\,ax^3+3\,x^2\right)\sqrt{-a^2c}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.670 $\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6327, 6328}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])/(2*
Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.60

$$\frac{(2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2],x]
[Out] ((2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.09, size = 45, normalized size = 0.66

method	result	size
gosper	$\frac{x(ax+2)\sqrt{-a^2c\,x^2+c}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44
default	$\frac{(ax+2)x\sqrt{-c\left(a^2x^2-1\right)}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x+2)*x*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^2c}\,(ax^2+2\,x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 + 2*x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c-a^2\,c\,x^2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.671 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 45}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.61

$$
\frac{\sqrt{c-a^2cx^2}\,(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x,x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 44, normalized size = 0.64

method	result	size
default	$\frac{(ax+\ln(x))\sqrt{-c\left(a^2x^2-1\right)}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

```
[Out] (a*x+ln(x))*(-c*(a^2*x^2-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 18, normalized size = 0.26

$$\frac{\sqrt{-a^2c}\,(ax+\log(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + log(x))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.672 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/
x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {6327, 6328, 45}

$$\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) + (Sqrt[c - a^2*c*x^2]
*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.59

$$\frac{\sqrt{c-a^2cx^2}\,(-1+ax\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - a^2*c*x^2])/x^2,x]
[Out] (Sqrt[c - a^2*c*x^2]*(-1 + a*x*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.66

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{-c\left(a^2x^2-1\right)}}{x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

```
[Out] (a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.36, size = 22, normalized size = 0.30

$$\frac{\sqrt{-a^2c}\,(ax\log(x)-1)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) - 1)/(a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x^2\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x**2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.673 $\int e^{2\coth^{-1}(ax)} x^3 \sqrt{c-a^2cx^2}\, dx$

Optimal. Leaf size=137

$$\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8+5ax)\sqrt{c-a^2cx^2}}{20a^4}-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] -3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*x
^2+c)^(1/2)/a^2+1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/2)
+3/20*(5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A]
time = 0.29, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6286, 1823, 847, 794, 223, 209}

$$\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{x^3\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}+\frac{3(5ax+8)\sqrt{c-a^2cx^2}}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) + (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 + 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) - (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = -\int e^{2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^3(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x^3(-9a^2c-10a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}$$
$$= \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x^2(30a^3c^2+36a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x(-72a^4c^3-90a^5c^3x}{\sqrt{c-a^2cx^2}}}{60a^6c^2}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8+5ax)\sqrt{c-}}{20a^4}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8+5ax)\sqrt{c-}}{20a^4}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} + \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8+5ax)\sqrt{c-}}{20a^4}$$

Mathematica [A]
time = 0.15, size = 96, normalized size = 0.70

$$\frac{\sqrt{c-a^2cx^2}\,(24+15ax+12a^2x^2+10a^3x^3+4a^4x^4)+15\sqrt{c}\,\mathrm{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(24 + 15*a*x + 12*a^2*x^2 + 10*a^3*x^3 + 4*a^4*x^4) +
15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*
a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(113) = 226.
time = 0.21, size = 270, normalized size = 1.97

method	result

risch	$-\frac{(4a^4x^4+10a^3x^3+12a^2x^2+15ax+24)(a^2x^2-1)c}{20a^4\sqrt{-c(a^2x^2-1)}}-\frac{3\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{4a^3\sqrt{a^2c}}$
default	$-\frac{x^2(-a^2c\,x^2+c)^{\frac{3}{2}}}{5a^2c}-\frac{4(-a^2c\,x^2+c)^{\frac{3}{2}}}{5c\,a^4}+\frac{-\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{2a^2c}+\frac{\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}}{2a^2}}{a}+\frac{x\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c-4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)+2/a*(-1/
4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+1/4/a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))+2/a^3*(1/2*x*(-a
^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2)))+2/a^4*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.47, size = 117, normalized size = 0.85

$$-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x^2}{5\,a^2c}+\frac{5\,\sqrt{-a^2cx^2+c}\,x}{4\,a^3}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{2\,a^3c}-\frac{3\,\sqrt{c}\,\arcsin(ax)}{4\,a^4}+\frac{2\,\sqrt{-a^2cx^2+c}}{a^4}-\frac{4\,(-a^2cx^2+c)^{\frac{3}{2}}}{5\,a^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) + 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 -
1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) - 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sqrt
(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

Fricas [A]
time = 0.34, size = 184, normalized size = 1.34

$$\left[\frac{2\,(4\,a^4x^4+10\,a^3x^3+12\,a^2x^2+15\,ax+24)\sqrt{-a^2cx^2+c}+15\,\sqrt{-c}\,\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{40\,a^4},\ \frac{(4\,a^4x^4+10\,a^3x^3+12\,a^2x^2+15\,ax+24)\sqrt{-a^2cx^2+c}+15\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{20\,a^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/40*(2*(4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-a^2*c*x^
2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
- c))/a^4, 1/20*((4*a^4*x^4 + 10*a^3*x^3 + 12*a^2*x^2 + 15*a*x + 24)*sqrt(-
```

```
a^2*c*x^2 + c) + 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*
x^2 - c)))/a^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.674 $\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}+\frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\,\mathrm{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] -7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3+2/3*x^2*(-a^2*c*x
^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)+1/24*(21*a*x+32)*(-a^2*c*x^2+c)^
(1/2)/a^3
```

Rubi [A]
time = 0.26, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6286, 1823, 847, 794, 223, 209}

$$\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{7\sqrt{c}\,\mathrm{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}+\frac{(21ax+32)\sqrt{c-a^2cx^2}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 + ((32 + 21
*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[
c - a^2*c*x^2]])/(8*a^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx \\
&= -\left(c\int \frac{x^2(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x^2(-7a^2c-8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2} \\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x(16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c} \\
&= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\int
\end{aligned}
$$

$$
= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c)\mathrm{Su}
$$

$$
= \frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{(32+21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{7\sqrt{c}\ \mathrm{t}
$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.79

$$
\frac{\sqrt{c-a^2cx^2}\,(32+21ax+16a^2x^2+6a^3x^3)+21\sqrt{c}\,\mathrm{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)}{24a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*Ar
cTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [A]
time = 0.21, size = 185, normalized size = 1.65

method	result
risch	$-\frac{(6a^3x^3+16a^2x^2+21ax+32)(a^2x^2-1)c}{24a^3\sqrt{-c(a^2x^2-1)}}-\frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{8a^2\sqrt{a^2c}}$

default	$-\frac{x(-a^2c\,x^2+c)^{\frac{3}{2}}}{4a^2c}+\frac{\frac{9x\sqrt{-a^2c\,x^2+c}}{8}+\frac{9c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{8\sqrt{a^2c}}}{a^2}-\frac{2(-a^2c\,x^2+c)^{\frac{3}{2}}}{3a^3c}+\frac{2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+9/4/a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c
/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))-2/3/a^3*(-a^2*
c*x^2+c)^(3/2)/c+2/a^3*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.48, size = 93, normalized size = 0.83

$$\frac{9\sqrt{-a^2cx^2+c}\,x}{8\,a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4\,a^2c}-\frac{7\sqrt{c}\,\arcsin(ax)}{8\,a^3}+\frac{2\sqrt{-a^2cx^2+c}}{a^3}-\frac{2\,(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/8
*sqrt(c)*arcsin(a*x)/a^3 + 2*sqrt(-a^2*c*x^2 + c)/a^3 - 2/3*(-a^2*c*x^2 + c
)^(3/2)/(a^3*c)
```

Fricas [A]
time = 0.35, size = 168, normalized size = 1.50

$$\left[\frac{2\,(6\,a^3x^3+16\,a^2x^2+21\,ax+32)\sqrt{-a^2cx^2+c}+21\,\sqrt{-c}\,\log\left(2\,a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{48\,a^3},\ \frac{(6\,a^3x^3+16\,a^2x^2+21\,ax+32)\sqrt{-a^2cx^2+c}+21\,\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{24\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*s
qrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/
24*((6*a^3*x^3 + 16*a^2*x^2 + 21*a*x + 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.40, size = 84, normalized size = 0.75

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x+\frac{8}{a}\right)x+\frac{21}{a^2}\right)x+\frac{32}{a^3}\right)+\frac{7c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x + 8/a)*x + 21/a^2)*x + 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.675 $\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=85

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] -arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+c
)^(1/2)+1/3*(3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A]
time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {6302, 6286, 1823, 794, 223, 209}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}+\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(3ax+5)\sqrt{c-a^2cx^2}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) - (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\int \frac{x\left(-5a^2c-6a^3cx\right)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{c\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{}{\sqrt{c-}}\right.}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5+3ax)\sqrt{c-a^2cx^2}}{3a^2}-\frac{\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.93

$$
\frac{\left(5+3ax+a^2x^2\right)\sqrt{c-a^2cx^2}+3\sqrt{c}\,\text{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\left(-1+a^2x^2\right)}\right)}{3a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((5 + 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] + 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(71) = 142$.
time = 0.20, size = 163, normalized size = 1.92

method	result
risch	$-\frac{\left(a^{2} x^{2}+3 a x+5\right) \left(a^{2} x^{2}-1\right) c}{3 a^{2} \sqrt{-c \left(a^{2} x^{2}-1\right)}}-\frac{\arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c\, x^{2}+c}}\right) c}{a \sqrt{a^{2} c}}$
default	$-\frac{\left(-a^{2} c\, x^{2}+c \right)^{\frac{3}{2}}}{3 a^{2} c}+\frac{x \sqrt{-a^{2} c\, x^{2}+c}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c\, x^{2}+c}}\right)}{\sqrt{a^{2} c}}}{a}+\frac{2 \sqrt{-a^{2} c \left(x-\frac{1}{a}\right)^{2}-2 \left(x-\frac{1}{a}\right) a c}-\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c+2/a*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*
c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))+2/a^2*((-a^2*c*(x-1/
a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*
(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.46, size = 70, normalized size = 0.82

$$\frac{\sqrt{-a^2cx^2+c}\,x}{a}-\frac{\sqrt{c}\,\arcsin\left(ax\right)}{a^2}+\frac{2\sqrt{-a^2cx^2+c}}{a^2}-\frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{3\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*x/a - sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c)
/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

Fricas [A]
time = 0.36, size = 150, normalized size = 1.76

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,\left(a^2x^2+3\,ax+5\right)+3\sqrt{-c}\,\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{6\,a^2},\ \frac{\sqrt{-a^2cx^2+c}\,\left(a^2x^2+3\,ax+5\right)+3\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{3\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c
*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2
+ c)*(a^2*x^2 + 3*a*x + 5) + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c
)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.40, size = 72, normalized size = 0.85

$$\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x+\frac{3}{a}\right)x+\frac{5}{a^2}\right)+\frac{c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{a\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x + 3/a)*x + 5/a^2) + c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.676 $\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=86

$$\frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a+3/2*(-a^2*c*x^2+c)^
(1/2)/a+1/2*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6302, 6276, 685, 655, 223, 209}

$$-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}+\frac{(ax+1)\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c-a^2cx^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(2*a) + ((1 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3*
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
```

```
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1+ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,-
\end{aligned}
$$

$$
= \frac{3\sqrt{c-a^2cx^2}}{2a}+\frac{(1+ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}
$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.88

$$
\frac{\sqrt{c-a^2cx^2}\left((4+ax)\sqrt{1-a^2x^2}+6\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

[Out] (Sqrt[c - a^2*c*x^2]*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/
Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])

Maple [A]
time = 0.20, size = 136, normalized size = 1.58

method	result
risch	$-\frac{(ax+4)(a^2x^2-1)c}{2a\sqrt{-c\,(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}+\frac{2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}-\frac{2ac\arctan\left(\frac{}{\sqrt{-a^2c}}\right)}{}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2
*c*x^2+c)^(1/2))+2/a*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a^2*c)^(1
/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))

Maxima [A]
time = 0.47, size = 47, normalized size = 0.55

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\,x-\frac{3\sqrt{c}\,\arcsin(ax)}{2\,a}+\frac{2\sqrt{-a^2cx^2+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")

[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a + 2*sqrt(-a^2*c*x^2
+ c)/a

Fricas [A]
time = 0.39, size = 134, normalized size = 1.56

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{-c}\,\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4\,a},\ \frac{\sqrt{-a^2cx^2+c}\,(ax+4)+3\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x + 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x + 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.40, size = 62, normalized size = 0.72

$$\frac{1}{2}\,\sqrt{-a^2cx^2+c}\,\left(x+\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\,\right|\right)}{2\,\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x + 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.677 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=75

$$\sqrt{c-a^2cx^2}-2\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)^
(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6286, 1823, 858, 223, 209, 272, 65, 214}

$$-2\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c-a^2cx^2}+\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx \\
&= -\left(c\int \frac{(1+ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \sqrt{c-a^2cx^2} + \frac{\int \frac{-a^2c-2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2} \\
&= \sqrt{c-a^2cx^2} - c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - (2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx \\
&= \sqrt{c-a^2cx^2} - \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - (2ac)\text{Subst}\left(\int \frac{1}{1+a
\end{aligned}
$$

$$
\begin{aligned}
&= \sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-
}{a^2} \\
&= \sqrt{c-a^2cx^2} - 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 1.29

$$
\sqrt{c-a^2cx^2} + 2\sqrt{c}\,\text{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) - \sqrt{c}\log(x) + \sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(61) = 122$.
time = 0.19, size = 129, normalized size = 1.72

method	result
default	$-\sqrt{-a^2cx^2+c}+\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2cx^2+c}}{x}\right)+2\sqrt{-a^2c\left(x-\frac{1}{a}\right)^2-2\left(x-\frac{1}{a}\right)ac}\ -\ \frac{2ac\,\mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(-a^2*c*x^2+c)^(1/2)+c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)+2*
(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(
1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2))
```

Maxima [A]
time = 0.47, size = 90, normalized size = 1.20

$$-a^2\left(\frac{\sqrt{c}\,\arcsin\left(ax\right)}{a^2}-\frac{\sqrt{-a^2cx^2+c}}{a^2}\right)-a\left(\frac{\sqrt{c}\,\arcsin\left(ax\right)}{a}-\frac{\sqrt{c}\,\log\left(\frac{2\sqrt{-a^2cx^2+c}\,\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a^2*(sqrt(c)*arcsin(a*x)/a^2 - sqrt(-a^2*c*x^2 + c)/a^2) - a*(sqrt(c)*arcs
in(a*x)/a - sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x))
/a)
```

Fricas [A]
time = 0.35, size = 191, normalized size = 2.55

$$\left[2\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\,\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\,\sqrt{c}\,-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}\,,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-c}\,\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+\sqrt{-a^2cx^2+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*s
qrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt(
-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 -
 c)) + sqrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
+ sqrt(-a^2*c*x^2 + c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax+1\right)}{x\left(ax-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x*(a*x - 1)), x)
```

Giac [A]
time = 0.42, size = 95, normalized size = 1.27

$$-\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\,a\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="giac")
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) - 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.678 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+2*a*arctanh((-a^2*c*x^2
+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.24, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6286, 1821, 858, 223, 209, 272, 65, 214}

$$-a\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{\sqrt{c-a^2cx^2}}{x}+2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 + 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + \int \frac{-2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - (ac)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - \left(a^2c\right)\text{Subst}\left(\int \frac{}{1}\right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \frac{2\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c}\right.}{a}$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + 2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right.$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 1.27

$$\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\,\text{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) - 2a\sqrt{c}\,\log(x) + 2a\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] - 2*a*Sqrt[c]*Log[x] + 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(68) = 136$.
time = 0.21, size = 209, normalized size = 2.55

method	result
risch	$-\frac{(a^2x^2-1)c}{x\sqrt{-c(a^2x^2-1)}} - \left(\frac{a^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}} - \frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}\right)c$

default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx} + 2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2} + \frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right) - 2a\left(\sqrt{-a^2c\,x^2+c} - \sqrt{c}\,\ln\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] 1/c/x*(-a^2*c*x^2+c)^(3/2)+2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))-2*a*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x))+2*a*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^2), x)

Fricas [A]
time = 0.38, size = 209, normalized size = 2.55

$$\left[\frac{a\sqrt{c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+a\sqrt{c}\,x\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\,\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x}, \frac{4a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)+a\sqrt{-c}\,x\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+2\sqrt{-a^2cx^2+c}}{2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="fricas")

[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt(-a^2*c*x^2 + c))/x, 1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c) + 2*sqrt(-a^2*c*x^2 + c))/x]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**2*(a*x - 1)), x)
```

Giac [A]
time = 0.43, size = 134, normalized size = 1.63

$$-\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}\,c}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="giac")
[Out] -4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
 a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a
^2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^2\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

3.679 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(1
/2)/x^2+2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.22, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6286, 1821, 821, 272, 65, 214}

$$\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) + (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$

$$= -\left(c\int \frac{(1+ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{1}{2}\int \frac{-4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{2}(3a^2c)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{4}(3a^2c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.97

$$\frac{1}{2}\left(\frac{(1+4ax)\sqrt{c-a^2cx^2}}{x^2} - 3a^2\sqrt{c}\,\log(x) + 3a^2\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
[Out] (((1 + 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(64) = 128.
time = 0.20, size = 239, normalized size = 3.06

method	result
risch	$-\frac{(4a^3x^3+a^2x^2-4ax-1)c}{2x^2\sqrt{-c(a^2x^2-1)}} + \frac{3a^2\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2}$

default	$-2a\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}-2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)\right)+\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{3a^2(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*(-1/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(
a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))+1/2/c/x^2*(-a^2
*c*x^2+c)^(3/2)-3/2*a^2*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a
^2*c*x^2+c)^(1/2))/x))+2*a^2*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^3), x)
```

Fricas [A]
time = 0.35, size = 148, normalized size = 1.90

$$\left[\frac{3\,a^2\sqrt{c}\,x^2\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\,\sqrt{c}-2c}{x^2}\right)+2\sqrt{-a^2cx^2+c}\,(4\,ax+1)}{4\,x^2},\ \frac{3\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2cx^2+c}\,(4\,ax+1)}{2\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(4*a*x + 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^
2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(-a^2*c*x^2 +
 c)*(4*a*x + 1))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**3*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(64) = 128.
time = 0.41, size = 200, normalized size = 2.56

$$-\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a^2c-4\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}\,c|a|+\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)a^2c^2+4\,a\sqrt{-c}\,c^2|a|}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c - 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 + 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^3\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

3.680 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{c-a^2cx^2}}{3x^3}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)/
x^3+a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.25, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6286, 1821, 849, 821, 272, 65, 214}

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}+\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) + (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) + a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{1}{3}\int \frac{-6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{\int \frac{10a^2c^2+6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - (a^3c)\int \frac{1}{x\sqrt{c-a^2cx^2}}$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - \frac{1}{2}(a^3c)\,\mathrm{Subst}\left(\int \frac{}{x\sqrt{c}}\right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + a\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx\right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + a^3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.$$

Mathematica [**A**]
time = 0.08, size = 82, normalized size = 0.83

$$\frac{(1+3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3} - a^3\sqrt{c}\,\log(x) + a^3\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
[Out] ((1 + 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) - a^3*Sqrt[c]*Log[x]
+ a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(83) = 166$.
time = 0.21, size = 317, normalized size = 3.20

method	result
risch	$-\frac{(5a^4x^4+3a^3x^3-4a^2x^2-3ax-1)c}{3x^3\sqrt{-c(a^2x^2-1)}} + a^3\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)$

default	$-2a^2\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{cx}-2a^2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)\right)-2a\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2*(-1/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c
/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))-2*a*(-1/2/c/x
^2*(-a^2*c*x^2+c)^(3/2)-1/2*a^2*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(
1/2)*(-a^2*c*x^2+c)^(1/2))/x)))-2*a^3*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c
+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x))+2*a^3*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c
)^(1/2)-a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a
)*a*c)^(1/2)))+1/3/c/x^3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^4), x)
```

Fricas [A]
time = 0.37, size = 164, normalized size = 1.66

$$\left[\frac{3\,a^3\sqrt{c}\,x^3\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)+2\,\sqrt{-a^2cx^2+c}\,(5\,a^2x^2+3\,ax+1)}{6\,x^3},\ \frac{3\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-a^2cx^2+c}\,(5\,a^2x^2+3\,ax+1)}{3\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3, 1/3*(3*a^3
*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) + sqrt(
-a^2*c*x^2 + c)*(5*a^2*x^2 + 3*a*x + 1))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**4*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(83) = 166.

time = 0.41, size = 250, normalized size = 2.53

$$-\frac{2\,a^{3}c\arctan\left(-\frac{\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\left(3\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{5}a^{3}c-3\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{4}a^{2}\sqrt{-c}\,c|a|+12\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{2}a^{2}\sqrt{-c}\,c^{2}|a|-3\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)a^{3}c^{3}-5\,a^{2}\sqrt{-c}\,c^{3}|a|\right)}{3\left(\left(\sqrt{-a^{2}c}\,x-\sqrt{-a^{2}cx^{2}+c}\right)^{2}-c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c - 3*(sqrt(-a^2*c)
*x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) + 12*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-
a^2*c*x^2 + c))*a^3*c^3 - 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqr
t(-a^2*c*x^2 + c))^2 - c)^3
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^4\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

3.681 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(1
/2)/x^4+2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2+4/3
*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.27, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6286, 1821, 849, 821, 272, 65, 214}

$$\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\sqrt{c-a^2cx^2}}{4x^4}+\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)+\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) + (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$
$$= -\left(c\int \frac{(1+ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{1}{4}\int \frac{-8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{\int \frac{21a^2c^2+16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{\int \frac{-32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}(7$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{16}($$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{8}(7$$
$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{7}{8}a^4$$

Mathematica [A]
time = 0.09, size = 95, normalized size = 0.73

$$\frac{\sqrt{c-a^2cx^2}\,(6+16ax+21a^2x^2+32a^3x^3)}{24x^4} - \frac{7}{8}a^4\sqrt{c}\,\log(x) + \frac{7}{8}a^4\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(106) = 212.
time = 0.21, size = 341, normalized size = 2.62

method	result

risch	$-\frac{\left(32 a^{5} x^{5}+21 a^{4} x^{4}-16 a^{3} x^{3}-15 a^{2} x^{2}-16 a x-6\right) c}{24 x^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{7 a^{4} \sqrt{c}\, \ln \left(\frac{2 c+2 \sqrt{c}\, \sqrt{-a^{2} c\, x^{2}+c}}{x}\right)}{8}$
default	$\frac{\left(-a^{2} c\, x^{2}+c\right)^{\frac{3}{2}}}{4 c\, x^{4}}-\frac{9 a^{2} \left(-\frac{\left(-a^{2} c\, x^{2}+c\right)^{\frac{3}{2}}}{2 c\, x^{2}}-\frac{a^{2} \left(\sqrt{-a^{2} c\, x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c+2 \sqrt{c}\, \sqrt{-a^{2} c\, x^{2}+c}}{x}\right)\right)}{2}\right)}{4}-2 a^{3} \left(-\frac{(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)-9/4*a^2*(-1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)-1/2
*a^2*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/
x)))-2*a^3*(-1/c/x*(-a^2*c*x^2+c)^(3/2)-2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1
/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))-2*a^4*((-
a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x))+2*a^
4*((-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)-a*c/(a^2*c)^(1/2)*arctan((a^2*c)^
(1/2)*x/(-a^2*c*(x-1/a)^2-2*(x-1/a)*a*c)^(1/2)))+2/3*a/c/x^3*(-a^2*c*x^2+c)
^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)/((a*x - 1)*x^5), x)
```

Fricas [A]
time = 0.38, size = 180, normalized size = 1.38

$$\left[\frac{21 \, a^{4} \sqrt{c} x^{4} \log\left(-\frac{a^{2} c x^{2} - 2 \, \sqrt{-a^{2} c x^{2} + c} \sqrt{c} - 2 \, c}{x^{2}}\right) + 2 \, {\left(32 \, a^{3} x^{3} + 21 \, a^{2} x^{2} + 16 \, a x + 6\right)} \sqrt{-a^{2} c x^{2} + c}}{48 \, x^{4}}, \frac{21 \, a^{4} \sqrt{-c} x^{4} \arctan\left(\frac{\sqrt{-a^{2} c x^{2} + c} \sqrt{-c}}{a^{2} c x^{2} - c}\right) + {\left(32 \, a^{3} x^{3} + 21 \, a^{2} x^{2} + 16 \, a x + 6\right)} \sqrt{-a^{2} c x^{2} + c}}{24 \, x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c)
```

```
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) + (32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax+1)}{x^5\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(x**5*(a*x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(106) = 212.
time = 0.41, size = 324, normalized size = 2.49

$$-\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{21\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2+96\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^4a^3\sqrt{-c}\,c^2|a|-45\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a^4c^3-128\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a^3\sqrt{-c}\,c^3|a|+21\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)a^4c^4+32\,a^3\sqrt{-c}\,c^4|a|}{12\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 + 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 - 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 + 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{x^5\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

3.682 $\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=228

$$\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^3
/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3
/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^2/x^
2)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) + (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*
x^2]*Log[1 - a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\frac{x^3(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(\frac{4}{a^3}+\frac{4x}{a^2}+\frac{4x^2}{a}+3x^3+ax^4+\frac{4}{a^3(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^3}+\frac{2x^2}{a^2}+\frac{4x^3}{3a}+\frac{3x^4}{4}+\frac{ax^5}{5}+\frac{4\log(1-ax)}{a^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^3*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 + (2*x^2)/a^2 + (4*x^3)/(3*a) + (3*x^4)/4 +
 (a*x^5)/5 + (4*Log[1 - a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 92, normalized size = 0.40

method	result	size
default	$\frac{\left(12 a^{5} x^{5}+45 a^{4} x^{4}+80 a^{3} x^{3}+120 a^{2} x^{2}+240 a x +240 \ln \left(a x -1\right)\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x -1\right)}{60 a^{4} \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/60*(12*a^5*x^5+45*a^4*x^4+80*a^3*x^3+120*a^2*x^2+240*a*x+240*ln(a*x-1))*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 58, normalized size = 0.25

$$\frac{\left(12\,a^{5}x^{5}+45\,a^{4}x^{4}+80\,a^{3}x^{3}+120\,a^{2}x^{2}+240\,ax+240\,\log\left(ax-1\right)\right)\sqrt{-a^{2}c}}{60\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/60*(12*a^5*x^5 + 45*a^4*x^4 + 80*a^3*x^3 + 120*a^2*x^2 + 240*a*x + 240*lo
g(a*x - 1))*sqrt(-a^2*c)/a^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x^3*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.683 $\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=186

$$\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^2
/(1-1/a^2/x^2)^(1/2)+x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3
*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)
/a^4/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*x
^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/
(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt
[c - a^2*c*x^2]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4}{\ }
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.40

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^2}+\frac{2x^2}{a}+x^3+\frac{ax^4}{4}+\frac{4\log(1-ax)}{a^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^2*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^2 + (2*x^2)/a + x^3 + (a*x^4)/4 + (4*Log[1 -
a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 83, normalized size = 0.45

method	result	size

default	$\frac{(a^4x^4+4a^3x^3+8a^2x^2+16ax+16\ln(ax-1))\sqrt{-c\,(a^2x^2-1)}\,(ax-1)}{4a^3(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2
)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 49, normalized size = 0.26

$$\frac{(a^4x^4 + 4\,a^3x^3 + 8\,a^2x^2 + 16\,ax + 16\,\log\left(ax-1\right))\sqrt{-a^2c}}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(-a^2*
c)/a^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.684 $\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=152

$$\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x*(-a^2*c*x^2+c)^(1/2)/a
/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(
-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 78}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [**A**]
time = 0.03, size = 66, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\,(ax(24+9ax+2a^2x^2)+24\log(1-ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [**A**]
time = 0.10, size = 76, normalized size = 0.50

method	result	size

default	$\frac{(2a^3x^3+9a^2x^2+24ax+24\ln(ax-1))\sqrt{-c(a^2x^2-1)}\,(ax-1)}{6a^2(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1
)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 42, normalized size = 0.28

$$\frac{(2\,a^3x^3+9\,a^2x^2+24\,ax+24\,\log\left(ax-1\right))\sqrt{-a^2c}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(-a^2*c)/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-c\,(ax-1)\,(ax+1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(-a^2*c*x^2+c)^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.685 $\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=113

$$\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6328, 45}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2])
/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a^2*Sqrt
[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(3+ax+\frac{4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(6+ax)+8\log(1-ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/(a
^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 67, normalized size = 0.59

method	result	size
default	$\frac{\left(a^2x^2+6ax+8\ln(ax-1)\right)\sqrt{-c\left(a^2x^2-1\right)}\,(ax-1)}{2a(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)/a/(a*x+1)^2/
((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 33, normalized size = 0.29

$$\frac{(a^2x^2 + 6\,ax + 8\,\log{(ax - 1)})\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(-a^2*c)/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax - 1)\,(ax + 1)}}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.686 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 84}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{4\sqrt{c-a^2cx^2}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.46

$$\frac{\sqrt{c-a^2cx^2}\,(ax-\log(x)+4\log(1-ax))}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x,x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x)
```

Maple [A]
time = 0.10, size = 59, normalized size = 0.52

method	result	size
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(-ax+\ln(x)-4\ln(ax-1))(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x,method=_RETURNVERBOS
E)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x)-4*ln(a*x-1))*(a*x-1)/(a*x+1)^2/((a*x-1)
/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.36, size = 28, normalized size = 0.25

$$\frac{\sqrt{-a^2c}\,(ax+4\,\log{(ax-1)}-\log{(x)})}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/(x*((a*x - 1)/(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.687 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=114

$$\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/2)
/x/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A]
time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*Sqrt[c - a^2*c*x^2]*
Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(S
qrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.48

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{x}-3a\log(x)+4a\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^2,x]
[Out] (Sqrt[c - a^2*c*x^2]*(x^(-1) - 3*a*Log[x] + 4*a*Log[1 - a*x]))/(a*Sqrt[1 -
1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.57

method	result	size

default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, (3 a \ln (x) x-4 x \ln (a x-1) a-1) (a x-1)}{x (a x+1)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x,method=_RETURNVERB
OSE)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(3*a*ln(x)*x-4*x*ln(a*x-1)*a-1)*(a*x-1)/x/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.40, size = 33, normalized size = 0.29

$$\frac{\sqrt{-a^{2} c}\, (4 \, a x \log (a x-1)-3 \, a x \log (x)+1)}{a x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
fricas")
```

```
[Out] sqrt(-a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^2,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.688 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)/x
^2/(1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
+4*a*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*x^2
])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 -
 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*
x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-a^2c}}{\sqrt{1-}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{2x^2}+\frac{3a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^3,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(2*x^2) + (3*a)/x - 4*a^2*Log[x] + 4*a^2*Log[1 - a*
x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 77, normalized size = 0.50

method	result	size

default	$-\frac{\left(8a^{2}\ln\left(x\right)x^{2}-8x^{2}\ln\left(ax-1\right)a^{2}-6ax-1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{2x^{2}\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*x^2*ln(a*x-1)*a^2-6*a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x-1)/x^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 90, normalized size = 0.59

$$\frac{8\,a^{3}\sqrt{-c}\,x^{2}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx+\sqrt{-a^{2}c}\,\left(2\,ax-1\right)\sqrt{-c}\,+ac}{ax^{2}-x}\right)+\sqrt{-a^{2}c}\,\left(6\,ax+1\right)}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
fricas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
- 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + sqrt(-a^2*c)*(6*a*x + 1))/(a*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^3,x, algorithm="
giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - a^2*c*x^2)^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.689 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^2\sqrt{c-a^2cx^2}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/2)
/x^3/(1-1/a^2/x^2)^(1/2)+4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4
*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(-a^
2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*x^2
])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x
) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^4(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\,\mathrm{lo}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.41

$$
\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{3x^3}+\frac{3a}{2x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^4,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(3*x^3) + (3*a)/(2*x^2) + (4*a^2)/x - 4*a^3*Log[x]
+ 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.44

method	result	size

default	$-\frac{(24\ln(x)a^3x^3-24x^3\ln(ax-1)a^3-24a^2x^2-9ax-2)\sqrt{-c\,(a^2x^2-1)}\,(ax-1)}{6x^3(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x,method=_RETURNVERB
OSE)
```

```
[Out] -1/6*(24*ln(x)*a^3*x^3-24*x^3*ln(a*x-1)*a^3-24*a^2*x^2-9*a*x-2)*(-c*(a^2*x^
2-1))^(1/2)*(a*x-1)/x^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.36, size = 98, normalized size = 0.51

$$\frac{24\,a^4\sqrt{-c}\,x^3\log\left(\frac{2\,a^3cx^2-2\,a^2cx+\sqrt{-a^2c}\,(2\,ax-1)\sqrt{-c}+ac}{ax^2-x}\right)+(24\,a^2x^2+9\,ax+2)\sqrt{-a^2c}}{6\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
fricas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(-a^2*c))
/(a*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**4,x)
```

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^4,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.690 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=228

$$\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^3\sqrt{c-a}}{\sqrt{1}}$$

```
[Out] 1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x^4
/(1-1/a^2/x^2)^(1/2)+2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*a^2
*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(-a^2*c*x^2+c)^(1
/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/
x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-a^2cx^2}\,\log(1-ax)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2]/(
Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2
)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a^3*
Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a^3*Sqrt[c - a^2
*c*x^2]*Log[1 - a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 84, normalized size = 0.37

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{4x^4}+\frac{a}{x^3}+\frac{2a^2}{x^2}+\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2])/x^5,x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1/(4*x^4) + a/x^3 + (2*a^2)/x^2 + (4*a^3)/x - 4*a^4*L
og[x] + 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 93, normalized size = 0.41

method	result	size
default	$-\frac{\left(16\ln\left(x\right)a^{4}x^{4}-16x^{4}\ln\left(ax-1\right)a^{4}-16a^{3}x^{3}-8a^{2}x^{2}-4ax-1\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax-1\right)}{4x^{4}\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x,method=_RETURNVERB
OSE)
```

```
[Out] -1/4*(16*ln(x)*a^4*x^4-16*x^4*ln(a*x-1)*a^4-16*a^3*x^3-8*a^2*x^2-4*a*x-1)*(
-c*(a^2*x^2-1))^(1/2)*(a*x-1)/x^4/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.37, size = 106, normalized size = 0.46

$$\frac{16\,a^{5}\sqrt{-c}\,x^{4}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx+\sqrt{-a^{2}c}\,(2\,ax-1)\sqrt{-c}\,+ac}{ax^{2}-x}\right)+\left(16\,a^{3}x^{3}+8\,a^{2}x^{2}+4\,ax+1\right)\sqrt{-a^{2}c}}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
fricas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 - 1)*sqrt(-c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*s
qrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^(1/2)/x^5,x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.691 $\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=211

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3}}{4a^2\left(c\right.}$$

```
[Out] (1-1/a^2/x^2)^(3/2)*x^4/a/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^5/
(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a^2/(-a*x+1)/(-a^2*c*x^2+c
)^(3/2)+7/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)+1/4
*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a^2/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{a\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a^2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{7x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a^2\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a^2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(a*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2)
)^(3/2)*x^5)/(2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*a
^2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (7*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
- a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1
+ a*x])/(4*a^2*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}x}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{(c-a^2cx^2)^{3/2}}\\ &= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^4}{(-1+ax)^2(1+ax)}\,dx}{(c-a^2cx^2)^{3/2}}\\ &= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{a^4}+\frac{x}{a^3}+\frac{1}{2a^4(-1+ax)^2}+\frac{7}{4a^4(-1+ax)}+\frac{1}{4a^4(1+ax)}\right)dx}{(c-a^2cx^2)^{3/2}}\\ &= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{a\,(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}{2\,(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a^2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{7\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{4a^2\,(c-a^2c}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 77, normalized size = 0.36

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(2\left(2ax+a^2x^2+\frac{1}{1-ax}\right)+7\log(1-ax)+\log(1+ax)\right)}{4a^2\,(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(3/2),x]
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(2*(2*a*x + a^2*x^2 + (1 - a*x)^(-1)) + 7*Log[
1 - a*x] + Log[1 + a*x]))/(4*a^2*(c - a^2*c*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 106, normalized size = 0.50

method	result	size
default	$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(2a^3x^3+2a^2x^2+\ln(ax+1)ax+7x\ln(ax-1)a-4ax-\ln(ax+1)-7\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a^5}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(2*a^3*x^3+2*a^2*x^2+ln(
a*x+1)*a*x+7*x*ln(a*x-1)*a-4*a*x-ln(a*x+1)-7*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 76, normalized size = 0.36

$$\frac{\left(2\,a^{3}x^{3}+2\,a^{2}x^{2}-4\,ax+(ax-1)\log\left(ax+1\right)+7\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{-a^{2}c}}{4\,(a^{7}c^{2}x-a^{6}c^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*a^3*x^3 + 2*a^2*x^2 - 4*a*x + (a*x - 1)*log(a*x + 1) + 7*(a*x - 1)*l
og(a*x - 1) - 2)*sqrt(-a^2*c)/(a^7*c^2*x - a^6*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{4}}{\sqrt{\frac{ax-1}{ax+1}}\;(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^4/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.692 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\,(c-a^2cx^2)^{3/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4a\,(c-a^2cx^2)^{3/2}}$$

```
[Out] (1-1/a^2/x^2)^(3/2)*x^4/(-a^2*c*x^2+c)^(3/2)+1/2*(1-1/a^2/x^2)^(3/2)*x^3/a/
(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+5/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/a/(-a^
2*c*x^2+c)^(3/2)-1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/a/(-a^2*c*x^2+c)^(3/
2)
```

Rubi [A]
time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{(c-a^2cx^2)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2a(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4a\,(c-a^2cx^2)^{3/2}}-\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4a\,(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^4)/(c - a^2*c*x^2)^(3/2) + ((1 - 1/(a^2*x^2))^(3
/2)*x^3)/(2*a*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (5*(1 - 1/(a^2*x^2))^(3/2)
*x^3*Log[1 - a*x])/(4*a*(c - a^2*c*x^2)^(3/2)) - ((1 - 1/(a^2*x^2))^(3/2)*x
^3*Log[1 + a*x])/(4*a*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}{(c-a^2cx^2)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2a(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4a\,(c-a^2cx^2)^{3/2}}-\frac{(1-}{}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.41

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(1+ax)\right)}{4a\,(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(3/2),x]
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1
+ a*x]))/(4*a*(c - a^2*c*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 98, normalized size = 0.57

method	result	size
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(-4a^2x^2+\ln(ax+1)ax-5x\ln(ax-1)a+4ax-\ln(ax+1)+5\ln(ax-1)+2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^2x^2-1\right)c^2a^4}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(-4*a^2*x^2+ln(a*x+1)*a*x-5*x*ln(a*x-1)*a+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^4

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)

Fricas [A]
time = 0.36, size = 69, normalized size = 0.40

$$\frac{\left(4 a^{2} x^{2}-4 a x-(a x-1) \log (a x+1)+5(a x-1) \log (a x-1)-2\right) \sqrt{-a^{2} c}}{4\left(a^{6} c^{2} x-a^{5} c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1) - 2)*sqrt(-a^2*c)/(a^6*c^2*x - a^5*c^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{\frac{a x-1}{a x+1}}\,(-c(a x-1)(a x+1))^{\frac{3}{2}}}\, d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(3/2),x)

[Out] Integral(x**3/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(c-a^2\,c\,x^2\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^3/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.693 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+ax)}{4\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+3/4*(1-1/a^2/x^2)
^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a
*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (3*(1 -
 1/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(
a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x^2}{(-1+ax)^2(1+ax)}\,dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a^2(-1+ax)^2}+\frac{3}{4a^2(-1+ax)}+\frac{1}{4a^2(1+ax)}\right)dx}{(c-a^2cx^2)^{3/2}}\\
&= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1+a}{4\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 75, normalized size = 0.58

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+3(-1+ax)\log(1-ax)+(-1+ax)\log(1+ax))}{4a^2c(-1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + 3*(-1 + a*x)*Log[1 - a*x] + (-1 + a*x)*
Log[1 + a*x]))/(a^2*c*(-1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.66

method	result	size
default	$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(\ln(ax+1)ax+3x\ln(ax-1)a-\ln(ax+1)-3\ln(ax-1)-2)}{4\sqrt{\frac{ax-1}{ax+1}}\,(a^2x^2-1)c^2a^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a*x+3*x*ln(a*
x-1)*a-ln(a*x+1)-3*ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 56, normalized size = 0.43

$$\frac{\sqrt{-a^2c}\left((ax-1)\log\left(ax+1\right)+3\left(ax-1\right)\log\left(ax-1\right)-2\right)}{4\left(a^5c^2x-a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 2)/(a
^5*c^2*x - a^4*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{\frac{ax-1}{ax+1}}\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^2/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.694 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/2*a*(1-1/a^2/
x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]

time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6327, 6328, 78, 213}

$$\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{ax^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) - (a*(1
 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}x}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{x}{(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2a(-1+ax)^2}+\frac{1}{2a\left(-1+a^2x^2\right)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{\left(a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2\left(c-a^2cx^2\right)^{3/2}}\\
&= \frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 59, normalized size = 0.68

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(1+(-1+ax)\tanh^{-1}(ax)\right)}{2ac(-1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(3/2),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(1 + (-1 + a*x)*ArcTanh[a*x]))/(2*a*c*(-1 + a*x)*S
qrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 84, normalized size = 0.97

method	result	size
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(\ln \left(a x +1\right) a x -x \ln \left(a x -1\right) a -\ln \left(a x +1\right)+\ln \left(a x -1\right)+2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} a^{2}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a*x-x*ln(a*x
-1)*a-ln(a*x+1)+ln(a*x-1)+2)/(a^2*x^2-1)/c^2/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.34, size = 86, normalized size = 0.99

$$-\frac{(a^2x-a)\sqrt{-c}\,\log\left(\frac{a^2cx^2-2\,\sqrt{-a^2c}\,\sqrt{-c}\,x+c}{a^2x^2-1}\right)+2\,\sqrt{-a^2c}}{4\,(a^4c^2x-a^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^4*c^2*x - a^3*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{ax-1}{ax+1}}\,\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
[Out] integrate(x/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(x/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.695 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] 1/2*a^2*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+1/2*a^2*(1-1/
a^2/x^2)^(3/2)*x^3*arctanh(a*x)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6327, 6328, 46, 213}

$$\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^2x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\tanh^{-1}(ax)}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (a^2*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
2*(1 - 1/(a^2*x^2))^(3/2)*x^3*ArcTanh[a*x])/(2*(c - a^2*c*x^2)^(3/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{(-1+ax)^2(1+ax)}\,dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{2(-1+ax)^2}-\frac{1}{2(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{3/2}}$$
$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(c-a^2cx^2)^{3/2}} - \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{-1+a^2x^2}\,dx}{2(c-a^2cx^2)^{3/2}}$$
$$= \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(c-a^2cx^2)^{3/2}} + \frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\tanh^{-1}(ax)}{2(c-a^2cx^2)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.62

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+(-1+ax)\tanh^{-1}(ax))}{2c(-1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + (-1 + a*x)*ArcTanh[a*x]))/(c*(-1 + a*x)
*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 84, normalized size = 0.92

method	result	size

default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, \left(\ln \left(a x +1\right) a x -x \ln \left(a x -1\right) a -\ln \left(a x +1\right)+\ln \left(a x -1\right)-2\right)}{4 \sqrt{\frac{a x -1}{a x +1}}\, \left(a^{2} x^{2}-1\right) c^{2} a}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a*x-x*ln(a*x-
1)*a-ln(a*x+1)+ln(a*x-1)-2)/(a^2*x^2-1)/c^2/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.37, size = 86, normalized size = 0.95

$$-\frac{{\left(a^{2} x - a\right)} \sqrt{-c} \log\left(\frac{a^{2} c x^{2} + 2 \, \sqrt{-a^{2} c} \sqrt{-c} x + c}{a^{2} x^{2} - 1}\right) + 2 \, \sqrt{-a^{2} c}}{4 \, {\left(a^{3} c^{2} x - a^{2} c^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/4*((a^2*x - a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/
(a^2*x^2 - 1)) + 2*sqrt(-a^2*c))/(a^3*c^2*x - a^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a x - 1}{a x + 1}} \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac
")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.696 $\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log}{4\left(c-a^2cx^2\right)^{3}}$$

```
[Out] 1/2*a^3*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/
x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-3/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*l
n(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^3*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-
a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 84}

$$\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{3a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^
3*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (3*a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^3*(1 - 1
/(a^2*x^2))^(3/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x}+\frac{a}{2(-1+ax)^2}-\frac{3a}{4(-1+ax)}-\frac{a}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}\\ &= \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{3a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-a}{4\left(c-a^2cx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.38

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(\frac{1}{2-2ax}+\log(x)-\frac{3}{4}\log(1-ax)-\frac{1}{4}\log(1+ax)\right)}{\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*((2 - 2*a*x)^(-1) + Log[x] - (3*Log[1 - a*
x])/4 - Log[1 + a*x]/4))/(c - a^2*c*x^2)^(3/2)
```

Maple [A]
time = 0.10, size = 93, normalized size = 0.53

method	result	size
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax+1)ax-4a\ln(x)x+3x\ln(ax-1)a-\ln(ax+1)+4\ln(x)-3\ln(ax-1)+2\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^2x^2-1\right)c^2}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a*x-4*a*ln(x
)*x+3*x*ln(a*x-1)*a-ln(a*x+1)+4*ln(x)-3*ln(a*x-1)+2)/(a^2*x^2-1)/c^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [**A**]
time = 0.36, size = 63, normalized size = 0.36

$$-\frac{\sqrt{-a^2c}\,((ax-1)\log{(ax+1)}+3\,(ax-1)\log{(ax-1)}-4\,(ax-1)\log{(x)}+2)}{4\,(a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/4*sqrt(-a^2*c)*((a*x - 1)*log(a*x + 1) + 3*(a*x - 1)*log(a*x - 1) - 4*(a
*x - 1)*log(x) + 2)/(a^2*c^2*x - a*c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/(x*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.697 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{(c-a^2cx^2)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{(c-a^2cx^2)^{3/2}}-\frac{5a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(1-ax)}{4\,(c-a^2cx^2)^{3/2}}$$

```
[Out] -a^3*(1-1/a^2/x^2)^(3/2)*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^4*(1-1/a^2/x^2)^(3/
2)*x^3/(-a*x+1)/(-a^2*c*x^2+c)^(3/2)+a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^
2*c*x^2+c)^(3/2)-5/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^
(3/2)+1/4*a^4*(1-1/a^2/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]

time = 0.17, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\,(c-a^2cx^2)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{(c-a^2cx^2)^{3/2}}-\frac{5a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\,(c-a^2cx^2)^{3/2}}+\frac{a^4x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\,(c-a^2cx^2)^{3/2}}-\frac{a^3x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*
x^2))^(3/2)*x^3*Log[x])/(c - a^2*c*x^2)^(3/2) - (5*a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 - a*x])/(4*(c - a^2*c*x^2)^(3/2)) + (a^4*(1 - 1/(a^2*x^2))^(3
/2)*x^3*Log[1 + a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^5}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^2(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^2}+\frac{a}{x}+\frac{a^2}{2(-1+ax)^2}-\frac{5a^2}{4(-1+ax)}+\frac{a^2}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$

$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{5a^4(}{}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 0.37

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{4}{x}+\frac{2a}{1-ax}+4a\log(x)-5a\log(1-ax)+a\log(1+ax)\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(3/2)),x]
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-4/x + (2*a)/(1 - a*x) + 4*a*Log[x] - 5*a
*Log[1 - a*x] + a*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 118, normalized size = 0.55

method	result	size
default	$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax+1)a^2x^2+4a^2\ln(x)x^2-5x^2\ln(ax-1)a^2-\ln(ax+1)ax-4a\ln(x)x+5x\ln(ax-1)a-6ax+4\right)}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^2x^2-1\right)c^2x}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a^2*x^2+4*a^2
*ln(x)*x^2-5*x^2*ln(a*x-1)*a^2-ln(a*x+1)*a*x-4*a*ln(x)*x+5*x*ln(a*x-1)*a-6*
a*x+4)/(a^2*x^2-1)/c^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.37, size = 92, normalized size = 0.43

$$-\frac{\sqrt{-a^2c}\,(6\,ax - (a^2x^2 - ax)\log{(ax + 1)} + 5\,(a^2x^2 - ax)\log{(ax - 1)} - 4\,(a^2x^2 - ax)\log{(x)} - 4)}{4\,(a^2c^2x^2 - ac^2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*sqrt(-a^2*c)*(6*a*x - (a^2*x^2 - a*x)*log(a*x + 1) + 5*(a^2*x^2 - a*x)
*log(a*x - 1) - 4*(a^2*x^2 - a*x)*log(x) - 4)/(a^2*c^2*x^2 - a*c^2*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x^2*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.698 $\int \frac{e^{\coth^{-1}(ax)}}{x^3(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=252

$$-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{7a^5\left(1-\frac{1}{a^2x^2}\right.}{4\left(c\right.}$$

```
[Out] -1/2*a^3*(1-1/a^2/x^2)^(3/2)*x/(-a^2*c*x^2+c)^(3/2)-a^4*(1-1/a^2/x^2)^(3/2)
*x^2/(-a^2*c*x^2+c)^(3/2)+1/2*a^5*(1-1/a^2/x^2)^(3/2)*x^3/(-a*x+1)/(-a^2*c*
x^2+c)^(3/2)+2*a^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(x)/(-a^2*c*x^2+c)^(3/2)-7/4*a
^5*(1-1/a^2/x^2)^(3/2)*x^3*ln(-a*x+1)/(-a^2*c*x^2+c)^(3/2)-1/4*a^5*(1-1/a^2
/x^2)^(3/2)*x^3*ln(a*x+1)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]

time = 0.18, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(x)}{\left(c-a^2cx^2\right)^{3/2}}-\frac{7a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(1-ax)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^5x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\log(ax+1)}{4\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4x^2\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x\left(1-\frac{1}{a^2x^2}\right)^{3/2}}{2\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -1/2*(a^3*(1 - 1/(a^2*x^2))^(3/2)*x)/(c - a^2*c*x^2)^(3/2) - (a^4*(1 - 1/(a
^2*x^2))^(3/2)*x^2)/(c - a^2*c*x^2)^(3/2) + (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^
3)/(2*(1 - a*x)*(c - a^2*c*x^2)^(3/2)) + (2*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3
*Log[x])/(c - a^2*c*x^2)^(3/2) - (7*a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 -
 a*x])/(4*(c - a^2*c*x^2)^(3/2)) - (a^5*(1 - 1/(a^2*x^2))^(3/2)*x^3*Log[1 +
 a*x])/(4*(c - a^2*c*x^2)^(3/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^3\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{1}{x^3(-1+ax)^2(1+ax)}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{2a^2}{x}+\frac{a^3}{2(-1+ax)^2}-\frac{7a^3}{4(-1+ax)}-\frac{a^3}{4(1+ax)}\right)dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x}{2\left(c-a^2cx^2\right)^{3/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^2}{\left(c-a^2cx^2\right)^{3/2}}+\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)\left(c-a^2cx^2\right)^{3/2}}+\frac{2a^5\left(1-\frac{1}{a^2x\cdots}\right.}{\left(c-a\cdots\right.}$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 0.37

$$\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{2}{x^2}-\frac{4a}{x}+\frac{2a^2}{1-ax}+8a^2\log(x)-7a^2\log(1-ax)-a^2\log(1+ax)\right)}{4\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^3*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (a^3*(1 - 1/(a^2*x^2))^(3/2)*x^3*(-2/x^2 - (4*a)/x + (2*a^2)/(1 - a*x) + 8*
a^2*Log[x] - 7*a^2*Log[1 - a*x] - a^2*Log[1 + a*x]))/(4*(c - a^2*c*x^2)^(3/
2))
```

Maple [A]
time = 0.10, size = 138, normalized size = 0.55

method	result
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(\ln(ax+1)a^3x^3-8\ln(x)a^3x^3+7x^3\ln(ax-1)a^3-\ln(ax+1)a^2x^2+8a^2\ln(x)x^2-7x^2\ln(ax-1)a^2+6a^2x^2-2ax\cdots\right.}{4\sqrt{\frac{ax-1}{ax+1}}\,\left(a^2x^2-1\right)c^2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a^3*x^3-8*ln
(x)*a^3*x^3+7*x^3*ln(a*x-1)*a^3-ln(a*x+1)*a^2*x^2+8*a^2*ln(x)*x^2-7*x^2*ln(
a*x-1)*a^2+6*a^2*x^2-2*a*x-2)/(a^2*x^2-1)/c^2/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.37, size = 113, normalized size = 0.45

$$-\frac{\left(6\,a^2x^2-2\,ax+\left(a^3x^3-a^2x^2\right)\log\left(ax+1\right)+7\left(a^3x^3-a^2x^2\right)\log\left(ax-1\right)-8\left(a^3x^3-a^2x^2\right)\log\left(x\right)-2\right)\sqrt{-a^2c}}{4\left(a^2c^2x^3-ac^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/4*(6*a^2*x^2 - 2*a*x + (a^3*x^3 - a^2*x^2)*log(a*x + 1) + 7*(a^3*x^3 - a
^2*x^2)*log(a*x - 1) - 8*(a^3*x^3 - a^2*x^2)*log(x) - 2)*sqrt(-a^2*c)/(a^2*
c^2*x^3 - a*c^2*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*x^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(c-a^2\,c\,x^2\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x^3*(c - a^2*c*x^2)^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.699 $\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=262

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{(c-a^2cx^2)^{5/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2\,(c-a^2cx^2)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)\,(c-a^2cx^2)^{5/2}}-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1+ax)\,(c-a^2cx^2)^{5/2}}+\frac{23(1-}{}$$

```
[Out] (1-1/a^2/x^2)^(5/2)*x^6/(-a^2*c*x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/
(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+(1-1/a^2/x^2)^(5/2)*x^5/a/(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)-1/8*(1-1/a^2/x^2)^(5/2)*x^5/a/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+23/
16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/a/(-a^2*c*x^2+c)^(5/2)-7/16*(1-1/a^2/
x^2)^(5/2)*x^5*ln(a*x+1)/a/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.15, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\frac{x^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{(c-a^2cx^2)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{a(1-ax)\,(c-a^2cx^2)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(ax+1)\,(c-a^2cx^2)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8a(1-ax)^2\,(c-a^2cx^2)^{5/2}}+\frac{23x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16a\,(c-a^2cx^2)^{5/2}}-\frac{7x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16a\,(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^6)/(c - a^2*c*x^2)^(5/2) - ((1 - 1/(a^2*x^2))^(5
/2)*x^5)/(8*a*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + ((1 - 1/(a^2*x^2))^(5/2)
*x^5)/(a*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - ((1 - 1/(a^2*x^2))^(5/2)*x^5)/(
8*a*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (23*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log[
1 - a*x])/(16*a*(c - a^2*c*x^2)^(5/2)) - (7*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log
[1 + a*x])/(16*a*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^5}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{(c-a^2cx^2)^{5/2}}$$
$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}$$
$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{}{16a^5}\right.}{(c-a^2cx^2)^{5/2}}$$
$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}{(c-a^2cx^2)^{5/2}} - \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8a(1-ax)^2(c-a^2cx^2)^{5/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{a(1-ax)(c-a^2cx^2)^{5/2}} - \frac{(1-}{8a(1+a}$$

Mathematica [A]
time = 0.08, size = 89, normalized size = 0.34

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(16ax+\frac{16}{1-ax}-\frac{2}{(-1+ax)^2}-\frac{2}{1+ax}+23\log(1-ax)-7\log(1+ax)\right)}{16a\,(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^5)/(c - a^2*c*x^2)^(5/2),x]
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*(16*a*x + 16/(1 - a*x) - 2/(-1 + a*x)^2 - 2/(1
 + a*x) + 23*Log[1 - a*x] - 7*Log[1 + a*x]))/(16*a*(c - a^2*c*x^2)^(5/2))
```

Maple [A]
time = 0.10, size = 185, normalized size = 0.71

method	result
default	$\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(-16a^4x^4+7\ln(ax+1)a^3x^3-23x^3\ln(ax-1)a^3+16a^3x^3-7\ln(ax+1)a^2x^2+23x^2\ln(ax-1)a^2+34a^2x^2-7\ln(ax\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^6(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(-16*a^4*x^4+7*
ln(a*x+1)*a^3*x^3-23*x^3*ln(a*x-1)*a^3+16*a^3*x^3-7*ln(a*x+1)*a^2*x^2+23*x^
2*ln(a*x-1)*a^2+34*a^2*x^2-7*ln(a*x+1)*a*x+23*x*ln(a*x-1)*a-18*a*x+7*ln(a*x
+1)-23*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/a^6/(a*x+1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^5/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [**A**]
time = 0.36, size = 138, normalized size = 0.53

$$-\frac{\left(16\,a^{4}x^{4}-16\,a^{3}x^{3}-34\,a^{2}x^{2}+18\,ax-7\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\log\left(ax+1\right)+23\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\log\left(ax-1\right)+12\right)\sqrt{-a^{2}c}}{16\left(a^{10}c^{3}x^{3}-a^{9}c^{3}x^{2}-a^{8}c^{3}x+a^{7}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
 - a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) +
12)*sqrt(-a^2*c)/(a^10*c^3*x^3 - a^9*c^3*x^2 - a^8*c^3*x + a^7*c^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**5/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^5/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^5/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.700 $\int \frac{e^{\coth^{-1}(ax)}x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{11\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(1-}{16\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+3/4*(1-1/a^2/x
^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*(1-1/a^2/x^2)^(5/2)*x^5/(a*
x+1)/(-a^2*c*x^2+c)^(5/2)+11/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*
x^2+c)^(5/2)+5/16*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.19, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$\frac{3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{11x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*((1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) + (3
*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + ((1 - 1
/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (11*(1 - 1/(a^
2*x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*(1 - 1/(a^2
*x^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}\,dx}{(c-a^2cx^2)^{5/2}}\\ &= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^4}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}\\ &= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^4(-1+ax)^3}+\frac{3}{4a^4(-1+ax)^2}+\frac{11}{16a^4(-1+ax)}-\frac{1}{8a^4(1+ax)^2}+\frac{5}{16a^4(1+a}\right.}{(c-a^2cx^2)^{5/2}}\\ &= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}+\end{aligned}$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 0.39

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2(-6+3ax+5a^2x^2)}{(-1+ax)^2(1+ax)}+11\log(1-ax)+5\log(1+ax)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^4)/(c - a^2*c*x^2)^(5/2),x]
[Out] ((1 - 1/(a^2*x^2))^(5/2)*x^5*((-2*(-6 + 3*a*x + 5*a^2*x^2))/((-1 + a*x)^2*(
1 + a*x)) + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5/2))
```

Maple [A]
time = 0.10, size = 169, normalized size = 0.78

method	result
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(5\ln(ax+1)a^3x^3+11x^3\ln(ax-1)a^3-5\ln(ax+1)a^2x^2-11x^2\ln(ax-1)a^2-10a^2x^2-5\ln(ax+1)ax-11x\ln(a}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^5(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(5*ln(a*x+1)*a^3*x^3+11*x^3*ln(a*x-1)*a^3-5*ln(a*x+1)*a^2*x^2-11*x^2*ln(a*x-1)*a^2-10*a^2*x^2-5*ln(a*x+1)*a*x-11*x*ln(a*x-1)*a-6*a*x+5*ln(a*x+1)+11*ln(a*x-1)+12)/(a^2*x^2-1)/c^3/a^5/(a*x+1)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)

Fricas [**A**]
time = 0.35, size = 122, normalized size = 0.56

$$\frac{\left(10\,a^2x^2+6\,ax-5\,(a^3x^3-a^2x^2-ax+1)\log\left(ax+1\right)-11\,(a^3x^3-a^2x^2-ax+1)\log\left(ax-1\right)-12\right)\sqrt{-a^2c}}{16\,(a^9c^3x^3-a^8c^3x^2-a^7c^3x+a^6c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] 1/16*(10*a^2*x^2 + 6*a*x - 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) - 11*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 12)*sqrt(-a^2*c)/(a^9*c^3*x^3 - a^8*c^3*x^2 - a^7*c^3*x + a^6*c^3)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**4/(-a**2*c*x**2+c)**(5/2),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^4/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^4/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.701 $\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/2*a*(1-1/a
^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a*(1-1/a^2/x^2)^(5/2)*x
^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a*(1-1/a^2/x^2)^(5/2)*x^5*arctanh(a*x)/
(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.18, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6327, 6328, 90, 213}

$$\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) +
(a*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (a*(1
 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a*(1 -
1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^3}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^3(-1+ax)^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{1}{8a^3(1+ax)^2}+\frac{3}{8a^3(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}\\
&= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)(c-a^2cx^2)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}+\\
&= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)(c-a^2cx^2)^{5/2}}-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}-
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.49

$$-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(-2-ax+5a^2x^2+3(-1+ax)^2(1+ax)\tanh^{-1}(ax)\right)}{8a^3c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(-2 - a*x + 5*a^2*x^2 + 3*(-1 + a*x)^2*(1 + a
*x)*ArcTanh[a*x]))/(a^3*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 169, normalized size = 0.96

method	result
default	$\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\,\left(3 \ln (a x+1) a^{3} x^{3}-3 x^{3} \ln (a x-1) a^{3}-3 \ln (a x+1) a^{2} x^{2}+3 x^{2} \ln (a x-1) a^{2}+10 a^{2} x^{2}-3 \ln (a x+1) a x+3 x \ln (a x-1)\right.}{16 \sqrt{\frac{a x-1}{a x+1}}\,(a x-1)\left(a^{2} x^{2}-1\right) c^{3} a^{4}(a x+1)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*a^3*x^3-3*x^3*ln(a*x-1)*a^3-3*ln(a*x+1)*a^2*x^2+3*x^2*ln(a*x-1)*a^2+10*a^2*x^2-3*ln(a*x+1)*a*x+3*x*ln(a*x-1)*a-2*a*x+3*ln(a*x+1)-3*ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^4/(a*x+1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")

[Out] integrate(x^3/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)

Fricas [A]
time = 0.34, size = 136, normalized size = 0.77

$$-\frac{3\left(a^{4} x^{3}-a^{3} x^{2}-a^{2} x+a\right) \sqrt{-c}\, \log \left(\frac{a^{2} c x^{2}+2 \sqrt{-a^{2} c}\, \sqrt{-c}\, x+c}{a^{2} x^{2}-1}\right)-2\left(5 a^{2} x^{2}-a x-2\right) \sqrt{-a^{2} c}}{16\left(a^{8} c^{3} x^{3}-a^{7} c^{3} x^{2}-a^{6} c^{3} x+a^{5} c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(5*a^2*x^2 - a*x - 2)*sqrt(-a^2*c))/(a^8*c^3*x^3 - a^7*c^3*x^2 - a^6*c^3*x + a^5*c^3)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(c-a^2\,c\,x^2\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^3/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.702 $\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)+1/4*a^2*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^2*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.18, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6327, 6328, 90, 213}

$$\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
+ (a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (
a^2*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (a^2
*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^3}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x^2}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a^2(-1+ax)^3}+\frac{1}{4a^2(-1+ax)^2}-\frac{1}{8a^2(1+ax)^2}-\frac{1}{8a^2(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}-$$

$$= -\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^2\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}+$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 0.46

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2-3ax-a^2x^2+(-1+ax)^2(1+ax)\tanh^{-1}(ax)\right)}{8a^2c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x*(2 - 3*a*x - a^2*x^2 + (-1 + a*x)^2*(1 + a*x)*ArcT
anh[a*x]))/(8*a^2*c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 164, normalized size = 0.89

method	result
default	$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,(\ln(ax+1)a^{3}x^{3}-x^{3}\ln(ax-1)a^{3}-\ln(ax+1)a^{2}x^{2}+x^{2}\ln(ax-1)a^{2}-2a^{2}x^{2}-\ln(ax+1)ax+x\ln(ax-1)a-6ax}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^{2}x^{2}-1)c^{3}a^{3}(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a^3
*x^3-x^3*ln(a*x-1)*a^3-ln(a*x+1)*a^2*x^2+x^2*ln(a*x-1)*a^2-2*a^2*x^2-ln(a*x
+1)*a*x+x*ln(a*x-1)*a-6*a*x+ln(a*x+1)-ln(a*x-1)+4)/(a^2*x^2-1)/c^3/a^3/(a*x
+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 134, normalized size = 0.73

$$-\frac{\left(a^{4}x^{3}-a^{3}x^{2}-a^{2}x+a\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{2}-2\sqrt{-a^{2}c}\,\sqrt{-c}\,x+c}{a^{2}x^{2}-1}\right)-2\left(a^{2}x^{2}+3\,ax-2\right)\sqrt{-a^{2}c}}{16\left(a^{7}c^{3}x^{3}-a^{6}c^{3}x^{2}-a^{5}c^{3}x+a^{4}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 + 3*a*x - 2)*sqrt(-a^2*c))
/(a^7*c^3*x^3 - a^6*c^3*x^2 - a^5*c^3*x + a^4*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(x^2/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x^2/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.703 $\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^3*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/8*a^3*(1
-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^3*(1-1/a^2/x^2)^(5
/2)*x^5*arctanh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6327, 6328, 78, 213}

$$-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}+\frac{a^3x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
- (a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) + (
a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{x}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4a(-1+ax)^3}+\frac{1}{8a(1+ax)^2}-\frac{1}{8a(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}}-\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{-1+a}\cdots}{8\,(c-a^2cx^2)^{5/2}}\\
&= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}(a\cdots}{8\,(c-a^2cx^2)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 0.44

$$-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(\frac{1}{(-1+ax)^2}+\frac{1}{1+ax}-\tanh^{-1}(ax)\right)}{8\,(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^3*(1 - 1/(a^2*x^2))^(5/2)*x^5*((-1 + a*x)^(-2) + (1 + a*x)^(-1) - A
rcTanh[a*x]))/(c - a^2*c*x^2)^(5/2)
```

Maple [A]
time = 0.09, size = 164, normalized size = 1.20

method	result
default	$-\frac{\sqrt{-c\left(a^{2} x^{2}-1\right)}\, (\ln(ax+1)a^3x^3-x^3\ln(ax-1)a^3-\ln(ax+1)a^2x^2+x^2\ln(ax-1)a^2-2a^2x^2-\ln(ax+1)ax+x\ln(ax-1)a+2ax}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2-1)c^3a^2(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(ln(a*x+1)*a^3
*x^3-x^3*ln(a*x-1)*a^3-ln(a*x+1)*a^2*x^2+x^2*ln(a*x-1)*a^2-2*a^2*x^2-ln(a*x
+1)*a*x+x*ln(a*x-1)*a+2*a*x+ln(a*x+1)-ln(a*x-1)-4)/(a^2*x^2-1)/c^3/a^2/(a*x
+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.34, size = 134, normalized size = 0.98

$$-\frac{\left(a^{4} x^{3}-a^{3} x^{2}-a^{2} x+a\right) \sqrt{-c}\, \log\left(\frac{a^{2} c x^{2}-2 \sqrt{-a^{2} c}\, \sqrt{-c}\, x+c}{a^{2} x^{2}-1}\right)-2\left(a^{2} x^{2}-a x+2\right) \sqrt{-a^{2} c}}{16\left(a^{6} c^{3} x^{3}-a^{5} c^{3} x^{2}-a^{4} c^{3} x+a^{3} c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*((a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 - 2*sqrt(-a^
2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) - 2*(a^2*x^2 - a*x + 2)*sqrt(-a^2*c))/(
a^6*c^3*x^3 - a^5*c^3*x^2 - a^4*c^3*x + a^3*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate(x/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(x/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.704 $\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\tanh^{-1}}{8\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/4*a^4*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^4*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-3/8*a^4*(1-1/a^2/x^2)^(5/2)*x^5*arcta
nh(a*x)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6327, 6328, 46, 213}

$$-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}}+\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^4x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\tanh^{-1}(ax)}{8\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
 - (a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (
a^4*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (3*a
^4*(1 - 1/(a^2*x^2))^(5/2)*x^5*ArcTanh[a*x])/(8*(c - a^2*c*x^2)^(5/2))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{(-1+ax)^3(1+ax)^2}\,dx}{(c-a^2cx^2)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(\frac{1}{4(-1+ax)^3}-\frac{1}{4(-1+ax)^2}-\frac{1}{8(1+ax)^2}+\frac{3}{8(-1+a^2x^2)}\right)dx}{(c-a^2cx^2)^{5/2}}\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}+\\
&= -\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(c-a^2cx^2)^{5/2}}-\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)(c-a^2cx^2)^{5/2}}+\frac{a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)(c-a^2cx^2)^{5/2}}-
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 83, normalized size = 0.45

$$
-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x\left(2+3ax-3a^2x^2+3(-1+ax)^2(1+ax)\tanh^{-1}(ax)\right)}{8c^2(-1+ax)^2(1+ax)\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/8*(Sqrt[1 - 1/(a^2*x^2)]*x*(2 + 3*a*x - 3*a^2*x^2 + 3*(-1 + a*x)^2*(1 +
a*x)*ArcTanh[a*x]))/(c^2*(-1 + a*x)^2*(1 + a*x)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 169, normalized size = 0.92

method	result
default	$\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(3\ln(ax+1)a^{3}x^{3}-3x^{3}\ln(ax-1)a^{3}-3\ln(ax+1)a^{2}x^{2}+3x^{2}\ln(ax-1)a^{2}-6a^{2}x^{2}-3\ln(ax+1)ax+3x\ln(ax-1)a\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^{2}x^{2}-1)c^{3}(ax+1)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(3*ln(a*x+1)*a^
3*x^3-3*x^3*ln(a*x-1)*a^3-3*ln(a*x+1)*a^2*x^2+3*x^2*ln(a*x-1)*a^2-6*a^2*x^2
-3*ln(a*x+1)*a*x+3*x*ln(a*x-1)*a+6*a*x+3*ln(a*x+1)-3*ln(a*x-1)+4)/(a^2*x^2-
1)/c^3/(a*x+1)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.37, size = 136, normalized size = 0.74

$$-\frac{3\left(a^{4}x^{3}-a^{3}x^{2}-a^{2}x+a\right)\sqrt{-c}\,\log\left(\frac{a^{2}cx^{2}+2\sqrt{-a^{2}c}\,\sqrt{-c}\,x+c}{a^{2}x^{2}-1}\right)+2\left(3\,a^{2}x^{2}-3\,ax-2\right)\sqrt{-a^{2}c}}{16\left(a^{5}c^{3}x^{3}-a^{4}c^{3}x^{2}-a^{3}c^{3}x+a^{2}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fric
as")
```

```
[Out] -1/16*(3*(a^4*x^3 - a^3*x^2 - a^2*x + a)*sqrt(-c)*log((a^2*c*x^2 + 2*sqrt(-
a^2*c)*sqrt(-c)*x + c)/(a^2*x^2 - 1)) + 2*(3*a^2*x^2 - 3*a*x - 2)*sqrt(-a^2
*c))/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac
")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.705 $\int \frac{e^{\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\log(x)}{\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/8*a^5*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-1/2*a^5*(1
-1/a^2/x^2)^(5/2)*x^5/(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-1/8*a^5*(1-1/a^2/x^2)^(
5/2)*x^5/(a*x+1)/(-a^2*c*x^2+c)^(5/2)-a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a
^2*c*x^2+c)^(5/2)+11/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+
c)^(5/2)+5/16*a^5*(1-1/a^2/x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.18, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{\left(c-a^2cx^2\right)^{5/2}}+\frac{11a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\left(c-a^2cx^2\right)^{5/2}}+\frac{5a^5x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -1/8*(a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/((1 - a*x)^2*(c - a^2*c*x^2)^(5/2))
- (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(2*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) - (
a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^5
*(1 - 1/(a^2*x^2))^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (11*a^5*(1 - 1
/(a^2*x^2))^(5/2)*x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) + (5*a^5*(1
- 1/(a^2*x^2))^(5/2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx &= \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}\\
&= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{x}+\frac{a}{4(-1+ax)^3}-\frac{a}{2(-1+ax)^2}+\frac{11a}{16(-1+ax)}+\frac{a}{8(1+ax)^2}+\frac{5a}{16(1+ax}\right.}{\left(c-a^2cx^2\right)^{5/2}}\\
&= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{2(1-ax)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\left(c-a^2cx^2\right)^{5/2}}-
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 88, normalized size = 0.32

$$
\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(-\frac{2}{(-1+ax)^2}+\frac{8}{-1+ax}-\frac{2}{1+ax}-16\log(x)+11\log(1-ax)+5\log(1+ax)\right)}{16\left(c-a^2cx^2\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(-2/(-1 + a*x)^2 + 8/(-1 + a*x) - 2/(1 + a
*x) - 16*Log[x] + 11*Log[1 - a*x] + 5*Log[1 + a*x]))/(16*(c - a^2*c*x^2)^(5
/2))
```

Maple [A]
time = 0.10, size = 196, normalized size = 0.72

method	result
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(5\ln(ax+1)a^3x^3-16\ln(x)a^3x^3+11x^3\ln(ax-1)a^3-5\ln(ax+1)a^2x^2+16a^2\ln(x)x^2-11x^2\ln(ax-1)a^2+6a^2x\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)\left(a^2x^2-1\right)c^3(ax+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(5*ln(a*x+1)*a
^3*x^3-16*ln(x)*a^3*x^3+11*x^3*ln(a*x-1)*a^3-5*ln(a*x+1)*a^2*x^2+16*a^2*ln(
x)*x^2-11*x^2*ln(a*x-1)*a^2+6*a^2*x^2-5*ln(a*x+1)*a*x+16*a*ln(x)*x-11*x*ln(
a*x-1)*a+2*a*x+5*ln(a*x+1)-16*ln(x)+11*ln(a*x-1)-12)/(a^2*x^2-1)/c^3/(a*x+1
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.37, size = 145, normalized size = 0.54

$$-\frac{\left(6\,a^2x^2+2\,ax+5\,(a^3x^3-a^2x^2-ax+1)\log{(ax+1)}+11\,(a^3x^3-a^2x^2-ax+1)\log{(ax-1)}-16\,(a^3x^3-a^2x^2-ax+1)\log{(x)}-12\right)\sqrt{-a^2c}}{16\,(a^4c^3x^3-a^3c^3x^2-a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fr
icas")
```

```
[Out] -1/16*(6*a^2*x^2 + 2*a*x + 5*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1) + 1
1*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) - 16*(a^3*x^3 - a^2*x^2 - a*x
+ 1)*log(x) - 12)*sqrt(-a^2*c)/(a^4*c^3*x^3 - a^3*c^3*x^2 - a^2*c^3*x + a*c
^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="gi
ac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/(x*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.706 $\int \frac{e^{\coth^{-1}(ax)}}{x^2(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=307

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{(c-a^2cx^2)^{5/2}}-\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\,(c-a^2cx^2)^{5/2}}+\frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1+ax)\,(c-a^2cx^2)^{5/2}}-\frac{a^6\left(1-\right.}{(c}$$

```
[Out] a^5*(1-1/a^2/x^2)^(5/2)*x^4/(-a^2*c*x^2+c)^(5/2)-1/8*a^6*(1-1/a^2/x^2)^(5/2
)*x^5/(-a*x+1)^2/(-a^2*c*x^2+c)^(5/2)-3/4*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(-a*x
+1)/(-a^2*c*x^2+c)^(5/2)+1/8*a^6*(1-1/a^2/x^2)^(5/2)*x^5/(a*x+1)/(-a^2*c*x^
2+c)^(5/2)-a^6*(1-1/a^2/x^2)^(5/2)*x^5*ln(x)/(-a^2*c*x^2+c)^(5/2)+23/16*a^6
*(1-1/a^2/x^2)^(5/2)*x^5*ln(-a*x+1)/(-a^2*c*x^2+c)^(5/2)-7/16*a^6*(1-1/a^2/
x^2)^(5/2)*x^5*ln(a*x+1)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]
time = 0.21, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 90}

$$-\frac{3a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{4(1-ax)\,(c-a^2cx^2)^{5/2}}+\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(ax+1)\,(c-a^2cx^2)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{8(1-ax)^2\,(c-a^2cx^2)^{5/2}}-\frac{a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(x)}{(c-a^2cx^2)^{5/2}}+\frac{23a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(1-ax)}{16\,(c-a^2cx^2)^{5/2}}-\frac{7a^6x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\log(ax+1)}{16\,(c-a^2cx^2)^{5/2}}+\frac{a^5x^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}}{(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^4)/(c - a^2*c*x^2)^(5/2) - (a^6*(1 - 1/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(c - a^2*c*x^2)^(5/2)) - (3*a^6*(1 - 1/(a^2
*x^2))^(5/2)*x^5)/(4*(1 - a*x)*(c - a^2*c*x^2)^(5/2)) + (a^6*(1 - 1/(a^2*x^
2))^(5/2)*x^5)/(8*(1 + a*x)*(c - a^2*c*x^2)^(5/2)) - (a^6*(1 - 1/(a^2*x^2))
^(5/2)*x^5*Log[x])/(c - a^2*c*x^2)^(5/2) + (23*a^6*(1 - 1/(a^2*x^2))^(5/2)*
x^5*Log[1 - a*x])/(16*(c - a^2*c*x^2)^(5/2)) - (7*a^6*(1 - 1/(a^2*x^2))^(5/
2)*x^5*Log[1 + a*x])/(16*(c - a^2*c*x^2)^(5/2))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
```

```
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{x^2\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^7}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{1}{x^2(-1+ax)^3(1+ax)^2}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{\left(a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \left(-\frac{1}{x^2}-\frac{a}{x}+\frac{a^2}{4(-1+ax)^3}-\frac{3a^2}{4(-1+ax)^2}+\frac{23a^2}{16(-1+ax)}-\frac{a^2}{8(1+ax)^2}-\right.}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= \frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^4}{\left(c-a^2cx^2\right)^{5/2}} - \frac{a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^6\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5}{4(1-ax)\left(c-a^2cx^2\right)^{5/2}} + \frac{a^6}{8(1+}$$

Mathematica [A]
time = 0.07, size = 99, normalized size = 0.32

$$\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\left(\frac{16}{x}-\frac{2a}{(-1+ax)^2}+\frac{12a}{-1+ax}+\frac{2a}{1+ax}-16a\log(x)+23a\log(1-ax)-7a\log(1+ax)\right)}{16\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(x^2*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (a^5*(1 - 1/(a^2*x^2))^(5/2)*x^5*(16/x - (2*a)/(-1 + a*x)^2 + (12*a)/(-1 +
a*x) + (2*a)/(1 + a*x) - 16*a*Log[x] + 23*a*Log[1 - a*x] - 7*a*Log[1 + a*x]
))/(16*(c - a^2*c*x^2)^(5/2))
```

Maple [A]
time = 0.10, size = 225, normalized size = 0.73

method	result

default	$\frac{\sqrt{-c(a^2x^2-1)}\,(7\ln(ax+1)a^4x^4+16\ln(x)a^4x^4-23x^4\ln(ax-1)a^4-7\ln(ax+1)a^3x^3-16\ln(x)a^3x^3+23x^3\ln(ax-1)a^3-30a^3}{16\sqrt{\frac{ax-1}{ax+1}}\,(ax-1)(a^2x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/16/((a*x-1)/(a*x+1))^(1/2)/(a*x-1)*(-c*(a^2*x^2-1))^(1/2)*(7*ln(a*x+1)*a^
4*x^4+16*ln(x)*a^4*x^4-23*x^4*ln(a*x-1)*a^4-7*ln(a*x+1)*a^3*x^3-16*ln(x)*a^
3*x^3+23*x^3*ln(a*x-1)*a^3-30*a^3*x^3-7*ln(a*x+1)*a^2*x^2-16*a^2*ln(x)*x^2+
23*x^2*ln(a*x-1)*a^2+22*a^2*x^2+7*ln(a*x+1)*a*x+16*a*ln(x)*x-23*x*ln(a*x-1)
*a+28*a*x-16)/(a^2*x^2-1)/c^3/x/(a*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.37, size = 174, normalized size = 0.57

$$-\frac{\left(30\,a^3x^3-22\,a^2x^2-28\,ax-7\,(a^4x^4-a^3x^3-a^2x^2+ax)\log(ax+1)+23\,(a^4x^4-a^3x^3-a^2x^2+ax)\log(ax-1)-16\,(a^4x^4-a^3x^3-a^2x^2+ax)\log(x)+16\right)\sqrt{-a^2c}}{16\,(a^4c^3x^4-a^3c^3x^3-a^2c^3x^2+ac^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/16*(30*a^3*x^3 - 22*a^2*x^2 - 28*a*x - 7*(a^4*x^4 - a^3*x^3 - a^2*x^2 +
a*x)*log(a*x + 1) + 23*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(a*x - 1) - 1
6*(a^4*x^4 - a^3*x^3 - a^2*x^2 + a*x)*log(x) + 16)*sqrt(-a^2*c)/(a^4*c^3*x^
4 - a^3*c^3*x^3 - a^2*c^3*x^2 + a*c^3*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/x**2/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(c-a^2\,c\,x^2\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/(x^2*(c - a^2*c*x^2)^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.707 $\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=76

$$-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/3*x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^3*(-a^2*c*x^2+c)^
(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 45}

$$\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -1/3*(x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x^3*Sqrt[c - a^
2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int x^2(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int\left(-x^2+ax^3\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{x^2\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.59

$$\frac{x^2(-4+3ax)\sqrt{c-a^2cx^2}}{12a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
[Out] (x^2*(-4 + 3*a*x)*Sqrt[c - a^2*c*x^2])/(12*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.63

method	result	size
gosper	$\frac{x^3(3ax-4)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{12ax-12}$	47
default	$\frac{(3ax-4)x^3\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{12ax-12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/12*(3*a*x-4)*x^3*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.34, size = 25, normalized size = 0.33

$$\frac{(3\,ax^4-4\,x^3)\sqrt{-a^2c}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*a*x^4 - 4*x^3)*sqrt(-a^2*c)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.708 $\int e^{-\coth^{-1}(ax)} x\sqrt{c-a^2cx^2}\,dx$

<u>**Optimal**</u>. Leaf size=74

$$-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*x*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6327, 6328, 45}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(x*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*
c*x^2])/(3*Sqrt[1 - 1/(a^2*x^2)])
```

<u>Rule 45</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

<u>Rule 6327</u>

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

<u>Rule 6328</u>

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\left(-x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= -\frac{x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.58

$$\frac{x(-3+2ax)\sqrt{c-a^2cx^2}}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
[Out] (x*(-3 + 2*a*x)*Sqrt[c - a^2*c*x^2])/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.65

method	result	size
gosper	$\frac{x^2(2ax-3)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	47
default	$\frac{(2ax-3)x^2\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*a*x-3)*x^2*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 25, normalized size = 0.34

$$\frac{(2\,ax^3 - 3\,x^2)\sqrt{-a^2c}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(-a^2*c)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.709 $\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6327, 6328}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)])) + (x*Sqrt[c - a^2*c*x^2])/
(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.59

$$\frac{(-2+ax)\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^ArcCoth[a*x],x]
[Out] ((-2 + a*x)*Sqrt[c - a^2*c*x^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.09, size = 45, normalized size = 0.65

method	result	size
gosper	$\frac{x(ax-2)\sqrt{-a^2c\,x^2+c}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	44
default	$\frac{(ax-2)x\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x-2)*x*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.32

$$\frac{\sqrt{-a^2c}\,(ax^2-2\,x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*sqrt(-a^2*c)*(a*x^2 - 2*x)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.710 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 45}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(a-\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.63

$$\frac{\sqrt{c-a^2cx^2}\,(ax-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 46, normalized size = 0.66

method	result	size
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(-ax+\ln(x))\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+ln(x))*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [A]
time = 0.38, size = 20, normalized size = 0.29

$$\frac{\sqrt{-a^2c}\,(ax-\log{(x)})}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="fric
as")
```

```
[Out] sqrt(-a^2*c)*(a*x - log(x))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.711 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/x
/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 45}

$$\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (Sqrt[c - a^2*c*x^2]*Lo
g[x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.61

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{1}{x}+a\log(x)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^ArcCoth[a*x]*x^2),x]
[Out] (Sqrt[c - a^2*c*x^2]*(x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.11, size = 48, normalized size = 0.67

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{\frac{ax-1}{ax+1}}}{(ax-1)x}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOS
E)
```

```
[Out] (a*ln(x)*x+1)*(-c*(a^2*x^2-1))^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A]
time = 0.38, size = 22, normalized size = 0.31

$$\frac{\sqrt{-a^2c}\,(ax\log(x)+1)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(a*x*log(x) + 1)/(a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\,(ax-1)\,(ax+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(a*x - 1)*(a*x + 1))/x**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2, x)
```

3.712 $\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}+\frac{3\sqrt{c}\,\mathrm{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}$$

```
[Out] 3/4*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^4+3/5*x^2*(-a^2*c*x^
2+c)^(1/2)/a^2-1/2*x^3*(-a^2*c*x^2+c)^(1/2)/a+1/5*x^4*(-a^2*c*x^2+c)^(1/2)+
3/20*(-5*a*x+8)*(-a^2*c*x^2+c)^(1/2)/a^4
```

Rubi [A]
time = 0.29, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6287, 1823, 847, 794, 223, 209}

$$\frac{3x^2\sqrt{c-a^2cx^2}}{5a^2}+\frac{1}{5}x^4\sqrt{c-a^2cx^2}-\frac{x^3\sqrt{c-a^2cx^2}}{2a}+\frac{3\sqrt{c}\,\mathrm{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{4a^4}+\frac{3(8-5ax)\sqrt{c-a^2cx^2}}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (3*x^2*Sqrt[c - a^2*c*x^2])/(5*a^2) - (x^3*Sqrt[c - a^2*c*x^2])/(2*a) + (x^
4*Sqrt[c - a^2*c*x^2])/5 + (3*(8 - 5*a*x)*Sqrt[c - a^2*c*x^2])/(20*a^4) + (
3*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(4*a^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^3(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x^3(-9a^2c+10a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{5a^2}$$
$$= -\frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} - \frac{\int \frac{x^2(-30a^3c^2+36a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{20a^4c}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{\int \frac{x(-72a^4c^3+90a^5c}{\sqrt{c-a^2cx^2}}}{60a^6c^2}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8-5ax)\sqrt{c-}}{20a^4}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8-5ax)\sqrt{c-}}{20a^4}$$
$$= \frac{3x^2\sqrt{c-a^2cx^2}}{5a^2} - \frac{x^3\sqrt{c-a^2cx^2}}{2a} + \frac{1}{5}x^4\sqrt{c-a^2cx^2} + \frac{3(8-5ax)\sqrt{c-}}{20a^4}$$

Mathematica [A]
time = 0.09, size = 96, normalized size = 0.70

$$\frac{\sqrt{c-a^2cx^2}\,(24-15ax+12a^2x^2-10a^3x^3+4a^4x^4)-15\sqrt{c}\,\mathrm{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(24 - 15*a*x + 12*a^2*x^2 - 10*a^3*x^3 + 4*a^4*x^4) -
15*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(20*
a^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(113) = 226$.
time = 0.20, size = 261, normalized size = 1.91

method	result

risch	$-\frac{\left(4 a^{4} x^{4}-10 a^{3} x^{3}+12 a^{2} x^{2}-15 a x +24\right) \left(a^{2} x^{2}-1\right) c}{20 a^{4} \sqrt{-c \left(a^{2} x^{2}-1\right)}}+\frac{3 \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right) c}{4 a^{3} \sqrt{a^{2} c}}$
default	$-\frac{x^{2} \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 a^{2} c}-\frac{4 \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{5 c \,a^{4}}-\frac{2 \left(-\frac{x \left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{4 a^{2} c}+\frac{\frac{x \sqrt{-a^{2} c \,x^{2}+c}}{2}+\frac{c \arctan \left(\frac{\sqrt{a^{2} c}\, x}{\sqrt{-a^{2} c \,x^{2}+c}}\right)}{2 \sqrt{a^{2} c}}}{4 a^{2}}\right)}{a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*x^2*(-a^2*c*x^2+c)^(3/2)/a^2/c-4/5/c/a^4*(-a^2*c*x^2+c)^(3/2)-2/a*(-1/
4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+1/4/a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a
^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))-2/a^3*(1/2*x*(-a
^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)
^(1/2)))+2/a^4*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*ar
ctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.48, size = 117, normalized size = 0.85

$$-\frac{{\left(-a^{2} c x^{2} + c\right)}^{\frac{3}{2}} x^{2}}{5 \, a^{2} c} - \frac{5 \, \sqrt{-a^{2} c x^{2} + c} x}{4 \, a^{3}} + \frac{{\left(-a^{2} c x^{2} + c\right)}^{\frac{3}{2}} x}{2 \, a^{3} c} + \frac{3 \, \sqrt{c} \arcsin\left(a x\right)}{4 \, a^{4}} + \frac{2 \, \sqrt{-a^{2} c x^{2} + c}}{a^{4}} - \frac{4 \, {\left(-a^{2} c x^{2} + c\right)}^{\frac{3}{2}}}{5 \, a^{4} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*c*x^2 + c)^(3/2)*x^2/(a^2*c) - 5/4*sqrt(-a^2*c*x^2 + c)*x/a^3 +
1/2*(-a^2*c*x^2 + c)^(3/2)*x/(a^3*c) + 3/4*sqrt(c)*arcsin(a*x)/a^4 + 2*sqrt
(-a^2*c*x^2 + c)/a^4 - 4/5*(-a^2*c*x^2 + c)^(3/2)/(a^4*c)
```

Fricas [A]
time = 0.36, size = 184, normalized size = 1.34

$$\left[\frac{2 \, {\left(4 \, a^{4} x^{4} - 10 \, a^{3} x^{3} + 12 \, a^{2} x^{2} - 15 \, a x + 24\right)} \sqrt{-a^{2} c x^{2} + c} + 15 \, \sqrt{-c} \log\left(2 \, a^{2} c x^{2} + 2 \, \sqrt{-a^{2} c x^{2} + c} a \sqrt{-c} x - c\right)}{40 \, a^{4}}, \frac{{\left(4 \, a^{4} x^{4} - 10 \, a^{3} x^{3} + 12 \, a^{2} x^{2} - 15 \, a x + 24\right)} \sqrt{-a^{2} c x^{2} + c} - 15 \, \sqrt{c} \arctan\left(\frac{\sqrt{-a^{2} c x^{2} + c} a \sqrt{c} x}{a^{2} c x^{2} - c}\right)}{20 \, a^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/40*(2*(4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-a^2*c*x^
2 + c) + 15*sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x
```

```
- c))/a^4, 1/20*((4*a^4*x^4 - 10*a^3*x^3 + 12*a^2*x^2 - 15*a*x + 24)*sqrt(-
a^2*c*x^2 + c) - 15*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*
x^2 - c)))/a^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^3*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.713 $\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}-\frac{7\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}$$

```
[Out] -7/8*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^3-2/3*x^2*(-a^2*c*x
^2+c)^(1/2)/a+1/4*x^3*(-a^2*c*x^2+c)^(1/2)-1/24*(-21*a*x+32)*(-a^2*c*x^2+c)
^(1/2)/a^3
```

Rubi [A]
time = 0.27, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6287, 1823, 847, 794, 223, 209}

$$-\frac{2x^2\sqrt{c-a^2cx^2}}{3a}+\frac{1}{4}x^3\sqrt{c-a^2cx^2}-\frac{7\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{8a^3}-\frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-2*x^2*Sqrt[c - a^2*c*x^2])/(3*a) + (x^3*Sqrt[c - a^2*c*x^2])/4 - ((32 - 2
1*a*x)*Sqrt[c - a^2*c*x^2])/(24*a^3) - (7*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt
[c - a^2*c*x^2]])/(8*a^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx = -\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$$
$$= -\left(c\int \frac{x^2(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{1}{4}x^3\sqrt{c-a^2cx^2} + \frac{\int \frac{x^2(-7a^2c+8a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{4a^2}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{\int \frac{x(-16a^3c^2+21a^4c^2x)}{\sqrt{c-a^2cx^2}}\,dx}{12a^4c}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c}{}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{(7c}{}$$
$$= -\frac{2x^2\sqrt{c-a^2cx^2}}{3a} + \frac{1}{4}x^3\sqrt{c-a^2cx^2} - \frac{(32-21ax)\sqrt{c-a^2cx^2}}{24a^3} - \frac{7\sqrt{}}{}$$

Mathematica [A]
time = 0.07, size = 88, normalized size = 0.79

$$\frac{\sqrt{c-a^2cx^2}\,(-32+21ax-16a^2x^2+6a^3x^3)+21\sqrt{c}\,\text{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6*a^3*x^3) + 21*Sqrt[c]*A
rcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(24*a^3)
```

Maple [A]
time = 0.20, size = 176, normalized size = 1.57

method	result
risch	$-\frac{(6a^3x^3-16a^2x^2+21ax-32)(a^2x^2-1)c}{24a^3\sqrt{-c(a^2x^2-1)}} - \frac{7\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{8a^2\sqrt{a^2c}}$

default	$-\frac{x\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{4 a^{2} c}+\frac{\frac{9 x \sqrt{-a^{2} c\,x^{2}+c}}{8}+\frac{9 c \arctan\left(\frac{\sqrt{a^{2} c}\,x}{\sqrt{-a^{2} c\,x^{2}+c}}\right)}{8 \sqrt{a^{2} c}}}{a^{2}}+\frac{2\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}{3 a^{3} c}-\frac{2\left(\sqrt{-a^{2} c \left(x+\frac{1}{a}\right)^{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x*(-a^2*c*x^2+c)^(3/2)/a^2/c+9/4/a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c
/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))+2/3/a^3*(-a^2*
c*x^2+c)^(3/2)/c-2/a^3*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^
(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.46, size = 93, normalized size = 0.83

$$\frac{9\sqrt{-a^2cx^2+c}\,x}{8\,a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}x}{4\,a^2c}-\frac{7\sqrt{c}\,\arcsin(ax)}{8\,a^3}-\frac{2\sqrt{-a^2cx^2+c}}{a^3}+\frac{2\,(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(-a^2*c*x^2 + c)*x/a^2 - 1/4*(-a^2*c*x^2 + c)^(3/2)*x/(a^2*c) - 7/8
*sqrt(c)*arcsin(a*x)/a^3 - 2*sqrt(-a^2*c*x^2 + c)/a^3 + 2/3*(-a^2*c*x^2 + c
)^(3/2)/(a^3*c)
```

Fricas [A]
time = 0.36, size = 168, normalized size = 1.50

$$\left[\frac{2\,(6\,a^3x^3-16\,a^2x^2+21\,ax-32)\sqrt{-a^2cx^2+c}+21\sqrt{-c}\,\log\left(2\,a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{48\,a^3},\ \frac{(6\,a^3x^3-16\,a^2x^2+21\,ax-32)\sqrt{-a^2cx^2+c}+21\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{24\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*s
qrt(-c)*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^3, 1/
24*((6*a^3*x^3 - 16*a^2*x^2 + 21*a*x - 32)*sqrt(-a^2*c*x^2 + c) + 21*sqrt(c
)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**2*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A]
time = 0.43, size = 84, normalized size = 0.75

$$\frac{1}{24}\sqrt{-a^2cx^2+c}\left(\left(2\left(3x-\frac{8}{a}\right)x+\frac{21}{a^2}\right)x-\frac{32}{a^3}\right)+\frac{7c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{8a^2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(-a^2*c*x^2 + c)*((2*(3*x - 8/a)*x + 21/a^2)*x - 32/a^3) + 7/8*c*l
og(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/(a^2*sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^2*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.714 $\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}$$

```
[Out] arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a^2+1/3*x^2*(-a^2*c*x^2+c)
^(1/2)+1/3*(-3*a*x+5)*(-a^2*c*x^2+c)^(1/2)/a^2
```

Rubi [A]
time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6302, 6287, 1823, 794, 223, 209}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}+\frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^2*Sqrt[c - a^2*c*x^2])/3 + ((5 - 3*a*x)*Sqrt[c - a^2*c*x^2])/(3*a^2) + (
Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/a^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{x(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{\int \frac{x(-5a^2c+6a^3cx)}{\sqrt{c-a^2cx^2}}\,dx}{3a^2}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{c\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,x,\frac{}{\sqrt{c}}\right.}{a}\\
&= \frac{1}{3}x^2\sqrt{c-a^2cx^2}+\frac{(5-3ax)\sqrt{c-a^2cx^2}}{3a^2}+\frac{\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{a^2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 79, normalized size = 0.94

$$\frac{(5-3ax+a^2x^2)\sqrt{c-a^2cx^2}-3\sqrt{c}\,\text{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] ((5 - 3*a*x + a^2*x^2)*Sqrt[c - a^2*c*x^2] - 3*Sqrt[c]*ArcTan[(a*x*Sqrt[c -
 a^2*c*x^2])/(Sqrt[c]*(-1 + a^2*x^2))])/(3*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(70) = 140$.
time = 0.19, size = 154, normalized size = 1.83

method	result
risch	$-\frac{(a^2x^2-3ax+5)(a^2x^2-1)c}{3a^2\sqrt{-c(a^2x^2-1)}}+\frac{\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)c}{a\sqrt{a^2c}}$
default	$-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{3a^2c}-\frac{2\left(\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}\right)}{a}+\frac{2\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-a^2*c*x^2+c)^(3/2)/a^2/c-2/a*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*
c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2)))+2/a^2*((-a^2*c*(x+1/
a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*
(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))
```

Maxima [A]
time = 0.46, size = 70, normalized size = 0.83

$$-\frac{\sqrt{-a^2cx^2+c}\,x}{a}+\frac{\sqrt{c}\,\arcsin(ax)}{a^2}+\frac{2\sqrt{-a^2cx^2+c}}{a^2}-\frac{(-a^2cx^2+c)^{\frac{3}{2}}}{3\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*x/a + sqrt(c)*arcsin(a*x)/a^2 + 2*sqrt(-a^2*c*x^2 + c
)/a^2 - 1/3*(-a^2*c*x^2 + c)^(3/2)/(a^2*c)
```

Fricas [A]
time = 0.35, size = 150, normalized size = 1.79

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(a^2x^2-3\,ax+5)+3\sqrt{-c}\,\log\left(2\,a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{6\,a^2},\ \frac{\sqrt{-a^2cx^2+c}\,(a^2x^2-3\,ax+5)-3\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{3\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(-a^2*c*x^2 + c)*(a^2*x^2 - 3*a*x + 5) + 3*sqrt(-c)*log(2*a^2*c
*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a^2, 1/3*(sqrt(-a^2*c*x^2
+ c)*(a^2*x^2 - 3*a*x + 5) - 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c
)*x/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A]
time = 0.41, size = 73, normalized size = 0.87

$$\frac{1}{3}\sqrt{-a^2cx^2+c}\left(\left(x-\frac{3}{a}\right)x+\frac{5}{a^2}\right)-\frac{c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\,\right|\right)}{a\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(-a^2*c*x^2 + c)*((x - 3/a)*x + 5/a^2) - c*log(abs(-sqrt(-a^2*c)*x
+ sqrt(-a^2*c*x^2 + c)))/(a*sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.715 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=87

$$-\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

```
[Out] -3/2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)/a-3/2*(-a^2*c*x^2+c)^
(1/2)/a-1/2*(-a*x+1)*(-a^2*c*x^2+c)^(1/2)/a
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6302, 6277, 685, 655, 223, 209}

$$-\frac{3\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c-a^2cx^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(2*a) - ((1 - a*x)*Sqrt[c - a^2*c*x^2])/(2*a) - (3
*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]])/(2*a)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
```

```
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int \frac{(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= -\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1-ax}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx\\
&= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{1}{2}(3c)\text{Subst}\left(\int \frac{1}{1+a^2cx^2}\,dx,
\end{aligned}$$

$$= -\frac{3\sqrt{c-a^2cx^2}}{2a}-\frac{(1-ax)\sqrt{c-a^2cx^2}}{2a}-\frac{3\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)}{2a}$$

Mathematica [A]
time = 0.04, size = 100, normalized size = 1.15

$$\frac{\sqrt{c-a^2cx^2}\left(-\sqrt{1+ax}\,(4-5ax+a^2x^2)+6\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(2*ArcCoth[a*x]),x]
```

[Out] (Sqrt[c - a^2*c*x^2]*(-(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2)) + 6*Sqrt[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2])

Maple [A]
time = 0.19, size = 127, normalized size = 1.46

method	result
risch	$-\frac{(ax-4)(a^2x^2-1)c}{2a\sqrt{-c(a^2x^2-1)}}-\frac{3c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}$
default	$\frac{x\sqrt{-a^2c\,x^2+c}}{2}+\frac{c\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{2\sqrt{a^2c}}-\frac{2\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{}{\sqrt{-a^2c}}\right.}{}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)

[Out] 1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))-2/a*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))

Maxima [A]
time = 0.47, size = 47, normalized size = 0.54

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\,x-\frac{3\sqrt{c}\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2cx^2+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")

[Out] 1/2*sqrt(-a^2*c*x^2 + c)*x - 3/2*sqrt(c)*arcsin(a*x)/a - 2*sqrt(-a^2*c*x^2 + c)/a

Fricas [A]
time = 0.40, size = 134, normalized size = 1.54

$$\left[\frac{2\sqrt{-a^2cx^2+c}\,(ax-4)+3\sqrt{-c}\,\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)}{4a},\frac{\sqrt{-a^2cx^2+c}\,(ax-4)+3\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

```
[Out] [1/4*(2*sqrt(-a^2*c*x^2 + c)*(a*x - 4) + 3*sqrt(-c)*log(2*a^2*c*x^2 - 2*sqr
t(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c))/a, 1/2*(sqrt(-a^2*c*x^2 + c)*(a*x - 4)
 + 3*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [A]
time = 0.41, size = 62, normalized size = 0.71

$$\frac{1}{2}\sqrt{-a^2cx^2+c}\left(x-\frac{4}{a}\right)+\frac{3\,c\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{2\sqrt{-c}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-a^2*c*x^2 + c)*(x - 4/a) + 3/2*c*log(abs(-sqrt(-a^2*c)*x + sqrt(-
a^2*c*x^2 + c)))/(sqrt(-c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,(a\,x-1)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.716 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=75

$$\sqrt{c-a^2cx^2}+2\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 2*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)+arctanh((-a^2*c*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6287, 1823, 858, 223, 209, 272, 65, 214}

$$2\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\sqrt{c-a^2cx^2}+\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] + 2*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]] +
 Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \sqrt{c-a^2cx^2} + \frac{\int \frac{-a^2c+2a^3cx}{x\sqrt{c-a^2cx^2}}\,dx}{a^2}$$
$$= \sqrt{c-a^2cx^2} - c\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx + (2ac)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \sqrt{c-a^2cx^2} - \frac{1}{2}c\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) + (2ac)\text{Subst}\left(\int \frac{}{1+}\right.$$
$$= \sqrt{c-a^2cx^2} + 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-}\right.}{a^2}$$
$$= \sqrt{c-a^2cx^2} + 2\sqrt{c}\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) + \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 1.29

$$\sqrt{c-a^2cx^2} - 2\sqrt{c}\,\text{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) - \sqrt{c}\,\log(x) + \sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2] - 2*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]*(
-1 + a^2*x^2))] - Sqrt[c]*Log[x] + Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x
^2]]
```

Maple [A]
time = 0.19, size = 121, normalized size = 1.61

method	result
default	$2\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac} + \frac{2ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}} - \sqrt{-a^2c\,x^2+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+2*a*c/(a^2*c)^(1/2)*arctan((a^2*c)
^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2))-(-a^2*c*x^2+c)^(1/2)+c^(1/
2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x)
```

Maxima [**A**]
time = 0.49, size = 86, normalized size = 1.15

$$a^2\left(\frac{\sqrt{c}\,\arcsin\left(ax\right)}{a^2}+\frac{\sqrt{-a^2cx^2+c}}{a^2}\right)+a\left(\frac{\sqrt{c}\,\arcsin\left(ax\right)}{a}+\frac{\sqrt{c}\,\log\left(\frac{2\sqrt{-a^2cx^2+c}\,\sqrt{c}}{|x|}+\frac{2c}{|x|}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] a^2*(sqrt(c)*arcsin(a*x)/a^2 + sqrt(-a^2*c*x^2 + c)/a^2) + a*(sqrt(c)*arcsi
n(a*x)/a + sqrt(c)*log(2*sqrt(-a^2*c*x^2 + c)*sqrt(c)/abs(x) + 2*c/abs(x))/
a)
```

Fricas [**A**]
time = 0.34, size = 191, normalized size = 2.55

$$\left[-2\sqrt{c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\,\log\left(-\frac{a^2cx^2-2\sqrt{-a^2cx^2+c}\,\sqrt{c}\,-2c}{x^2}\right)+\sqrt{-a^2cx^2+c},\sqrt{-c}\,\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)+\sqrt{-c}\,\log\left(2a^2cx^2+2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)+\sqrt{-a^2cx^2+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(c)*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + 1/2*
sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sqrt
(-a^2*c*x^2 + c), sqrt(-c)*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2
- c)) + sqrt(-c)*log(2*a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*a*sqrt(-c)*x - c)
 + sqrt(-a^2*c*x^2 + c)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\left(ax-1\right)}{x\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x*(a*x + 1)), x)
```

Giac [**A**]
time = 0.42, size = 95, normalized size = 1.27

$$-\frac{2\,c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{2\,a\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}+\sqrt{-a^2cx^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
[Out] -2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) + 2
*a*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) + sqrt(
-a^2*c*x^2 + c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.717 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{c-a^2cx^2}}{x}-a\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)-2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a*arctan(a*x*c^(1/2)/(-a^2*c*x^2+c)^(1/2))*c^(1/2)-2*a*arctanh((-a^2*c*x^2
+c)^(1/2)/c^(1/2))*c^(1/2)+(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.23, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {6302, 6287, 1821, 858, 223, 209, 272, 65, 214}

$$-a\sqrt{c}\,\text{ArcTan}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right)+\frac{\sqrt{c-a^2cx^2}}{x}-2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x - a*Sqrt[c]*ArcTan[(a*Sqrt[c]*x)/Sqrt[c - a^2*c*x^2]]
 - 2*a*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x^2\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + \int \frac{2ac-a^2cx}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + (2ac)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx - \left(a^2c\right)\int \frac{1}{\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} + (ac)\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right) - \left(a^2c\right)\text{Subst}\left(\int\right.$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - \frac{2\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{\right.}}{a}$$
$$= \frac{\sqrt{c-a^2cx^2}}{x} - a\sqrt{c}\,\tan^{-1}\left(\frac{a\sqrt{c}\,x}{\sqrt{c-a^2cx^2}}\right) - 2a\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx}}{\sqrt{c}}\right.$$

Mathematica [A]
time = 0.06, size = 104, normalized size = 1.27

$$\frac{\sqrt{c-a^2cx^2}}{x} + a\sqrt{c}\,\text{ArcTan}\left(\frac{ax\sqrt{c-a^2cx^2}}{\sqrt{c}\,(-1+a^2x^2)}\right) + 2a\sqrt{c}\,\log(x) - 2a\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/x + a*Sqrt[c]*ArcTan[(a*x*Sqrt[c - a^2*c*x^2])/(Sqrt[c]
*(-1 + a^2*x^2))] + 2*a*Sqrt[c]*Log[x] - 2*a*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c
 - a^2*c*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(68) = 136$.
time = 0.23, size = 200, normalized size = 2.44

method	result
risch	$-\frac{(a^2x^2-1)c}{x\sqrt{-c(a^2x^2-1)}} - \left(\frac{a^2\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\,x^2+c}}\right)}{\sqrt{a^2c}} + \frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{\sqrt{c}}\right)c$

default	$-2a\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)+\frac{(-a^2cx^2\ldots}{cx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*arctan((a^2*
c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))+1/c/x*(-a^2*c*x^2+c)^(3
/2)+2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^2*c)^(1
/2)*x/(-a^2*c*x^2+c)^(1/2)))+2*a*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^
(1/2)*(-a^2*c*x^2+c)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^2), x)
```

Fricas [A]
time = 0.37, size = 210, normalized size = 2.56

$$\left[\frac{a\sqrt{c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,a\sqrt{c}\,x}{a^2cx^2-c}\right)+a\sqrt{c}\,x\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\,\sqrt{c}-2c}{x^2}\right)+\sqrt{-a^2cx^2+c}}{x}, -\frac{4a\sqrt{-c}\,x\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)-a\sqrt{-c}\,x\log\left(2a^2cx^2-2\sqrt{-a^2cx^2+c}\,a\sqrt{-c}\,x-c\right)-2\sqrt{-a^2cx^2+c}}{2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [(a*sqrt(c)*x*arctan(sqrt(-a^2*c*x^2 + c)*a*sqrt(c)*x/(a^2*c*x^2 - c)) + a*
sqrt(c)*x*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) - 2*c)/x^2) + sq
rt(-a^2*c*x^2 + c))/x, -1/2*(4*a*sqrt(-c)*x*arctan(sqrt(-a^2*c*x^2 + c)*sqr
t(-c)/(a^2*c*x^2 - c)) - a*sqrt(-c)*x*log(2*a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 +
 c)*a*sqrt(-c)*x - c) - 2*sqrt(-a^2*c*x^2 + c))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax-1\right)}{x^2\left(ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**2*(a*x + 1)), x)
```

Giac [A]
time = 0.42, size = 134, normalized size = 1.63

$$\frac{4\,ac\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{a^2\sqrt{-c}\,\log\left(\left|-\sqrt{-a^2c}\,x+\sqrt{-a^2cx^2+c}\right|\right)}{|a|}-\frac{2\,a^2\sqrt{-c}\,c}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
[Out] 4*a*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c) -
a^2*sqrt(-c)*log(abs(-sqrt(-a^2*c)*x + sqrt(-a^2*c*x^2 + c)))/abs(a) - 2*a^
2*sqrt(-c)*c/(((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^2\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.718 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{c-a^2cx^2}}{2x^2}-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 3/2*a^2*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/2*(-a^2*c*x^2+c)^(1
/2)/x^2-2*a*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.23, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6287, 1821, 821, 272, 65, 214}

$$-\frac{2a\sqrt{c-a^2cx^2}}{x}+\frac{\sqrt{c-a^2cx^2}}{2x^2}+\frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(2*x^2) - (2*a*Sqrt[c - a^2*c*x^2])/x + (3*a^2*Sqrt[c]*
ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$$
$$= -\left(c\int \frac{(1-ax)^2}{x^3\sqrt{c-a^2cx^2}}\,dx\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} + \frac{1}{2}\int \frac{4ac-3a^2cx}{x^2\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{2}(3a^2c)\int \frac{1}{x\sqrt{c-a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} - \frac{1}{4}(3a^2c)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c-a^2cx}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}}\,dx, x, \sqrt{c-a^2cx^2}\right)$$
$$= \frac{\sqrt{c-a^2cx^2}}{2x^2} - \frac{2a\sqrt{c-a^2cx^2}}{x} + \frac{3}{2}a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.97

$$\frac{1}{2}\left(\frac{(1-4ax)\sqrt{c-a^2cx^2}}{x^2} - 3a^2\sqrt{c}\,\log(x) + 3a^2\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (((1 - 4*a*x)*Sqrt[c - a^2*c*x^2])/x^2 - 3*a^2*Sqrt[c]*Log[x] + 3*a^2*Sqrt[
c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(64) = 128$.
time = 0.22, size = 230, normalized size = 2.95

method	result
risch	$\frac{(4a^3x^3-a^2x^2-4ax+1)c}{2x^2\sqrt{-c(a^2x^2-1)}} + \frac{3a^2\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{-a^2c\,x^2+c}}{x}\right)}{2}$

default	$2a^2\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)+2a\Bigg(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*arctan((a^2
*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))+2*a*(-1/c/x*(-a^2*c*x^
2+c)^(3/2)-2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan((a^
2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))+1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)-3/2*a^2
*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^3), x)
```

Fricas [A]
time = 0.40, size = 149, normalized size = 1.91

$$\left[\frac{3\,a^2\sqrt{c}\,x^2\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-2\,\sqrt{-a^2cx^2+c}\,(4\,ax-1)}{4\,x^2},\ \frac{3\,a^2\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2cx^2+c}\,(4\,ax-1)}{2\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*sqrt(c)*x^2*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) - 2*sqrt(-a^2*c*x^2 + c)*(4*a*x - 1))/x^2, 1/2*(3*a^2*sqrt(-c)*x^
2*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt(-a^2*c*x^2 +
 c)*(4*a*x - 1))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**3*(a*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(64) = 128.
time = 0.42, size = 200, normalized size = 2.56

$$-\frac{3\,a^2c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a^2c+4\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a\sqrt{-c}\,c|a|+\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)a^2c^2-4\,a\sqrt{-c}\,c^2|a|}{\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -3*a^2*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
 + ((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^3*a^2*c + 4*(sqrt(-a^2*c)*x - s
qrt(-a^2*c*x^2 + c))^2*a*sqrt(-c)*c*abs(a) + (sqrt(-a^2*c)*x - sqrt(-a^2*c*
x^2 + c))*a^2*c^2 - 4*a*sqrt(-c)*c^2*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c
*x^2 + c))^2 - c)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.719 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt{c-a^2cx^2}}{3x^3}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] -a^3*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/3*(-a^2*c*x^2+c)^(1/2)
/x^3-a*(-a^2*c*x^2+c)^(1/2)/x^2+5/3*a^2*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.25, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6287, 1821, 849, 821, 272, 65, 214}

$$\frac{5a^2\sqrt{c-a^2cx^2}}{3x}-\frac{a\sqrt{c-a^2cx^2}}{x^2}+\frac{\sqrt{c-a^2cx^2}}{3x^3}+a^3\left(-\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(3*x^3) - (a*Sqrt[c - a^2*c*x^2])/x^2 + (5*a^2*Sqrt[c -
 a^2*c*x^2])/(3*x) - a^3*Sqrt[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$$

$$= -\left(c\int \frac{(1-ax)^2}{x^4\sqrt{c-a^2cx^2}}\,dx\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} + \frac{1}{3}\int \frac{6ac-5a^2cx}{x^3\sqrt{c-a^2cx^2}}\,dx$$

$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} - \frac{\int \frac{10a^2c^2-6a^3c^2x}{x^2\sqrt{c-a^2cx^2}}\,dx}{6c}$$

$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + \left(a^3c\right)\int \frac{1}{x\sqrt{c-a^2cx^2}} [...]$$

$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} + \frac{1}{2}\left(a^3c\right)\text{Subst}\left(\int \frac{}{x\sqrt{}} [...]\right.$$

$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2c}} [...]\right.$$

$$= \frac{\sqrt{c-a^2cx^2}}{3x^3} - \frac{a\sqrt{c-a^2cx^2}}{x^2} + \frac{5a^2\sqrt{c-a^2cx^2}}{3x} - a^3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-}}{} [...]\right.$$

Mathematica [A]
time = 0.08, size = 82, normalized size = 0.81

$$\frac{(1-3ax+5a^2x^2)\sqrt{c-a^2cx^2}}{3x^3} + a^3\sqrt{c}\log(x) - a^3\sqrt{c}\log\left(c+\sqrt{c}\sqrt{c-a^2cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^4),x]
[Out] ((1 - 3*a*x + 5*a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*x^3) + a^3*Sqrt[c]*Log[x]
- a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(85) = 170$.
time = 0.21, size = 308, normalized size = 3.05

method	result
risch	$-\frac{(5a^4x^4-3a^3x^3-4a^2x^2+3ax-1)c}{3x^3\sqrt{-c(a^2x^2-1)}} - a^3\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2c\,x^2+c}}{x}\right)$

default	$-2a^3\left(\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}+\frac{ac\arctan\left(\frac{\sqrt{a^2c}\,x}{\sqrt{-a^2c\left(x+\frac{1}{a}\right)^2+2\left(x+\frac{1}{a}\right)ac}}\right)}{\sqrt{a^2c}}\right)-2a^2\Bigg(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^3*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*arctan((a^
2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))-2*a^2*(-1/c/x*(-a^2*c
*x^2+c)^(3/2)-2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arctan(
(a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))+2*a*(-1/2/c/x^2*(-a^2*c*x^2+c)^(3/2
)-1/2*a^2*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1
/2))/x)))+2*a^3*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2
+c)^(1/2))/x))+1/3/c/x^3*(-a^2*c*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^4), x)
```

Fricas [A]
time = 0.35, size = 165, normalized size = 1.63

$$\left[\frac{3\,a^3\sqrt{c}\,x^3\log\left(-\frac{a^2cx^2+2\sqrt{-a^2cx^2+c}\sqrt{c}-2\,c}{x^2}\right)+2\sqrt{-a^2cx^2+c}\,(5\,a^2x^2-3\,ax+1)}{6\,x^3},-\frac{3\,a^3\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{-a^2cx^2+c}\sqrt{-c}}{a^2cx^2-c}\right)-\sqrt{-a^2cx^2+c}\,(5\,a^2x^2-3\,ax+1)}{3\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(-a^2*c*x^2 + c)*sqrt(c) -
2*c)/x^2) + 2*sqrt(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3, -1/3*(3*a^
3*sqrt(-c)*x^3*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*c*x^2 - c)) - sqrt
(-a^2*c*x^2 + c)*(5*a^2*x^2 - 3*a*x + 1))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^4\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**4*(a*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(85) = 170.
time = 0.42, size = 250, normalized size = 2.48

$$\frac{2\,a^3c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2\left(3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^3c+3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^4a^2\sqrt{-c}\,c|a|-12\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a^2\sqrt{-c}\,c^2|a|-3\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)a^3c^3+5\,a^2\sqrt{-c}\,c^3|a|\right)}{3\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^3*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-c)
- 2/3*(3*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^3*c + 3*(sqrt(-a^2*c)*
x - sqrt(-a^2*c*x^2 + c))^4*a^2*sqrt(-c)*c*abs(a) - 12*(sqrt(-a^2*c)*x - sq
rt(-a^2*c*x^2 + c))^2*a^2*sqrt(-c)*c^2*abs(a) - 3*(sqrt(-a^2*c)*x - sqrt(-a
^2*c*x^2 + c))*a^3*c^3 + 5*a^2*sqrt(-c)*c^3*abs(a))/((sqrt(-a^2*c)*x - sqrt
(-a^2*c*x^2 + c))^2 - c)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^4\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.720 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)$$

```
[Out] 7/8*a^4*arctanh((-a^2*c*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/4*(-a^2*c*x^2+c)^(1
/2)/x^4-2/3*a*(-a^2*c*x^2+c)^(1/2)/x^3+7/8*a^2*(-a^2*c*x^2+c)^(1/2)/x^2-4/3
*a^3*(-a^2*c*x^2+c)^(1/2)/x
```

Rubi [A]
time = 0.27, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6302, 6287, 1821, 849, 821, 272, 65, 214}

$$\frac{7a^2\sqrt{c-a^2cx^2}}{8x^2}+\frac{\sqrt{c-a^2cx^2}}{4x^4}-\frac{2a\sqrt{c-a^2cx^2}}{3x^3}+\frac{7}{8}a^4\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c-a^2cx^2}}{\sqrt{c}}\right)-\frac{4a^3\sqrt{c-a^2cx^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/(4*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(3*x^3) + (7*a^2*Sq
rt[c - a^2*c*x^2])/(8*x^2) - (4*a^3*Sqrt[c - a^2*c*x^2])/(3*x) + (7*a^4*Sqr
t[c]*ArcTanh[Sqrt[c - a^2*c*x^2]/Sqrt[c]])/8
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$$

$$= -\left(c\int \frac{(1-ax)^2}{x^5\sqrt{c-a^2cx^2}}\,dx\right)$$

$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} + \frac{1}{4}\int \frac{8ac-7a^2cx}{x^4\sqrt{c-a^2cx^2}}\,dx$$

$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} - \frac{\int \frac{21a^2c^2-16a^3c^2x}{x^3\sqrt{c-a^2cx^2}}\,dx}{12c}$$

$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} + \frac{\int \frac{32a^3c^3-21a^4c^3x}{x^2\sqrt{c-a^2cx^2}}\,dx}{24c^2}$$

$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{8}($$

$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} - \frac{1}{16}$$

$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{1}{8}($$

$$= \frac{\sqrt{c-a^2cx^2}}{4x^4} - \frac{2a\sqrt{c-a^2cx^2}}{3x^3} + \frac{7a^2\sqrt{c-a^2cx^2}}{8x^2} - \frac{4a^3\sqrt{c-a^2cx^2}}{3x} + \frac{7}{8}c$$

Mathematica [**A**]
time = 0.09, size = 95, normalized size = 0.73

$$\frac{\sqrt{c-a^2cx^2}\,(6-16ax+21a^2x^2-32a^3x^3)}{24x^4} - \frac{7}{8}a^4\sqrt{c}\,\log(x) + \frac{7}{8}a^4\sqrt{c}\,\log\left(c+\sqrt{c}\,\sqrt{c-a^2cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(6 - 16*a*x + 21*a^2*x^2 - 32*a^3*x^3))/(24*x^4) - (7*
a^4*Sqrt[c]*Log[x])/8 + (7*a^4*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c - a^2*c*x^2]]
)/8
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(106) = 212.
time = 0.21, size = 332, normalized size = 2.55

method	result

risch	$\frac{(32a^5x^5-21a^4x^4-16a^3x^3+15a^2x^2-16ax+6)c}{24x^4\sqrt{-c(a^2x^2-1)}}+\frac{7a^4\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2c\,x^2+c}}{x}\right)}{8}$
default	$\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{4c\,x^4}-\frac{9a^2\left(-\frac{(-a^2c\,x^2+c)^{\frac{3}{2}}}{2c\,x^2}-\frac{a^2\left(\sqrt{-a^2c\,x^2+c}-\sqrt{c}\ \ln\left(\frac{2c+2\sqrt{c}\ \sqrt{-a^2c\,x^2+c}}{x}\right)\right)}{2}\right)}{4}+2a^4\left(\sqrt{\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/x^4*(-a^2*c*x^2+c)^(3/2)-9/4*a^2*(-1/2/c/x^2*(-a^2*c*x^2+c)^(3/2)-1/2
*a^2*((-a^2*c*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/
x)))+2*a^4*((-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)+a*c/(a^2*c)^(1/2)*arctan
((a^2*c)^(1/2)*x/(-a^2*c*(x+1/a)^2+2*(x+1/a)*a*c)^(1/2)))+2*a^3*(-1/c/x*(-a
^2*c*x^2+c)^(3/2)-2*a^2*(1/2*x*(-a^2*c*x^2+c)^(1/2)+1/2*c/(a^2*c)^(1/2)*arc
tan((a^2*c)^(1/2)*x/(-a^2*c*x^2+c)^(1/2))))-2*a^4*((-a^2*c*x^2+c)^(1/2)-c^(
1/2)*ln((2*c+2*c^(1/2)*(-a^2*c*x^2+c)^(1/2))/x))-2/3*a/c/x^3*(-a^2*c*x^2+c)
^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)/((a*x + 1)*x^5), x)
```

Fricas [A]
time = 0.36, size = 181, normalized size = 1.39

$$\left[\frac{21\,a^4\sqrt{c}\,x^4\log\left(-\frac{a^2cx^2-2\,\sqrt{-a^2cx^2+c}\,\sqrt{c}-2\,c}{x^2}\right)-2\,(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{-a^2cx^2+c}}{48\,x^4},\frac{21\,a^4\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{-a^2cx^2+c}\,\sqrt{-c}}{a^2cx^2-c}\right)-(32\,a^3x^3-21\,a^2x^2+16\,ax-6)\sqrt{-a^2cx^2+c}}{24\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^4*sqrt(c)*x^4*log(-(a^2*c*x^2 - 2*sqrt(-a^2*c*x^2 + c)*sqrt(c)
- 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c)
```

```
)/x^4, 1/24*(21*a^4*sqrt(-c)*x^4*arctan(sqrt(-a^2*c*x^2 + c)*sqrt(-c)/(a^2*
c*x^2 - c)) - (32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqrt(-a^2*c*x^2 + c))/
x^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(x**5*(a*x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(106) = 212.
time = 0.42, size = 324, normalized size = 2.49

$$-\frac{7\,a^4c\arctan\left(-\frac{\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}}{\sqrt{-c}}\right)}{4\,\sqrt{-c}}+\frac{21\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^7a^4c-45\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^5a^4c^2-96\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^4a^3\sqrt{-c}\,c^2|a|-45\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^3a^4c^3+128\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2a^3\sqrt{-c}\,c^3|a|+21\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)a^4c^4-32\,a^3\sqrt{-c}\,c^4|a|}{12\left(\left(\sqrt{-a^2c}\,x-\sqrt{-a^2cx^2+c}\right)^2-c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] -7/4*a^4*c*arctan(-(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))/sqrt(-c))/sqrt(-
c) + 1/12*(21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^7*a^4*c - 45*(sqrt(-a
^2*c)*x - sqrt(-a^2*c*x^2 + c))^5*a^4*c^2 - 96*(sqrt(-a^2*c)*x - sqrt(-a^2*
c*x^2 + c))^4*a^3*sqrt(-c)*c^2*abs(a) - 45*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^
2 + c))^3*a^4*c^3 + 128*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2*a^3*sqrt(
-c)*c^3*abs(a) + 21*(sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))*a^4*c^4 - 32*a^
3*sqrt(-c)*c^4*abs(a))/((sqrt(-a^2*c)*x - sqrt(-a^2*c*x^2 + c))^2 - c)^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(a\,x-1\right)}{x^5\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.721 $\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=227

$$\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\,\log(1}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^4/(1-1/a^2/x^2)^(1/2)-2*x*(-a^2*c*x^2+c)^(1/2)/a^3
/(1-1/a^2/x^2)^(1/2)+4/3*x^2*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3
/4*x^3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/5*x^4*(-a^2*c*x^2+c)^(1
/2)/(1-1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^5/x/(1-1/a^2/x^2
)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-a^2cx^2}}{5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a^5x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^4*Sqrt[1 - 1/(a^2*x^2)]) - (2*x*Sqrt[c - a^2*c*x
^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^2*Sqrt[c - a^2*c*x^2])/(3*a^2*Sqrt[
1 - 1/(a^2*x^2)]) - (3*x^3*Sqrt[c - a^2*c*x^2])/(4*a*Sqrt[1 - 1/(a^2*x^2)])
 + (x^4*Sqrt[c - a^2*c*x^2])/(5*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - a^2*c*
x^2]*Log[1 + a*x])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}x^3\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^4\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{\sqrt{c-a^2cx^2}\int\frac{x^3(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{\sqrt{c-a^2cx^2}\int\left(\frac{4}{a^3}-\frac{4x}{a^2}+\frac{4x^2}{a}-3x^3+ax^4-\frac{4}{a^3(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{4\sqrt{c-a^2cx^2}}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2x\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^2\sqrt{c-a^2cx^2}}{3a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^3\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.04, size = 87, normalized size = 0.38

$$\frac{\sqrt{c-a^2cx^2}\left(\frac{4x}{a^3}-\frac{2x^2}{a^2}+\frac{4x^3}{3a}-\frac{3x^4}{4}+\frac{ax^5}{5}-\frac{4\log(1+ax)}{a^4}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*((4*x)/a^3 - (2*x^2)/a^2 + (4*x^3)/(3*a) - (3*x^4)/4 +
 (a*x^5)/5 - (4*Log[1 + a*x])/a^4))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.09, size = 92, normalized size = 0.41

method	result	size
default	$-\frac{\left(-12a^{5}x^{5}+45a^{4}x^{4}-80a^{3}x^{3}+120a^{2}x^{2}-240ax+240\ln\left(ax+1\right)\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{60a^{4}\left(ax-1\right)^{2}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/60*(-12*a^5*x^5+45*a^4*x^4-80*a^3*x^3+120*a^2*x^2-240*a*x+240*ln(a*x+1))
*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^4/(a*x-1)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [**A**]
time = 0.37, size = 58, normalized size = 0.26

$$\frac{\left(12\,a^{5}x^{5}-45\,a^{4}x^{4}+80\,a^{3}x^{3}-120\,a^{2}x^{2}+240\,ax-240\,\log\left(ax+1\right)\right)\sqrt{-a^{2}c}}{60\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(12*a^5*x^5 - 45*a^4*x^4 + 80*a^3*x^3 - 120*a^2*x^2 + 240*a*x - 240*lo
g(a*x + 1))*sqrt(-a^2*c)/a^5
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^3*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.722 $\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=186

$$-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] -4*(-a^2*c*x^2+c)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x*(-a^2*c*x^2+c)^(1/2)/a^
2/(1-1/a^2/x^2)^(1/2)-x^2*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^
3*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)
/a^4/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a^4x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - a^2*c*x^2])/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*x*Sqrt[c - a^2*c*
x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (x^2*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1
/(a^2*x^2)]) + (x^3*Sqrt[c - a^2*c*x^2])/(4*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqr
t[c - a^2*c*x^2]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}x^2\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \frac{x^2(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= -\frac{4\sqrt{c-a^2cx^2}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2x\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-a^2cx^2}}{4\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 72, normalized size = 0.39

$$\frac{\sqrt{c-a^2cx^2}\left(ax(-16+8ax-4a^2x^2+a^3x^3)+16\log(1+ax)\right)}{4a^4\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-16 + 8*a*x - 4*a^2*x^2 + a^3*x^3) + 16*Log[1 +
a*x]))/(4*a^4*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [**A**]
time = 0.11, size = 83, normalized size = 0.45

method	result	size

default	$\frac{(a^4x^4-4a^3x^3+8a^2x^2-16ax+16\ln(ax+1))\sqrt{-c\,(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4a^3(ax-1)^2}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2
)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 49, normalized size = 0.26

$$\frac{(a^4x^4-4\,a^3x^3+8\,a^2x^2-16\,ax+16\,\log\left(ax+1\right))\sqrt{-a^2c}}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(-a^2*
c)/a^4
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int(x^2*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.723 $\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=151

$$\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 4*(-a^2*c*x^2+c)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x*(-a^2*c*x^2+c)^(1/2)/a
/(1-1/a^2/x^2)^(1/2)+1/3*x^2*(-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*ln(
a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^3/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 78}

$$\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a^3x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - a^2*c*x^2])/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*x*Sqrt[c - a^2*c*x
^2])/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (x^2*Sqrt[c - a^2*c*x^2])/(3*Sqrt[1 - 1/
(a^2*x^2)]) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^3*Sqrt[1 - 1/(a^2*x^2
)]*x)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}x\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{4\sqrt{c-a^2cx^2}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^2\sqrt{c-a^2cx^2}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\log(1}{a^3\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 0.43

$$\frac{\sqrt{c-a^2cx^2}\,(ax(24-9ax+2a^2x^2)-24\log(1+ax))}{6a^3\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6*a
^3*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.50

method	result	size

default	$-\frac{\left(-2a^{3}x^{3}+9a^{2}x^{2}-24ax+24\ln\left(ax+1\right)\right)\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6a^{2}\left(ax-1\right)^{2}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x
+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.34, size = 42, normalized size = 0.28

$$\frac{\left(2\,a^{3}x^{3}-9\,a^{2}x^{2}+24\,ax-24\,\log\left(ax+1\right)\right)\sqrt{-a^{2}c}}{6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(-a^2*c)/a^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.724 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] -3*(-a^2*c*x^2+c)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x*(-a^2*c*x^2+c)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6327, 6328, 45}

$$\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - a^2*c*x^2])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (x*Sqrt[c - a^2*c*x^2]
)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a^2*Sqr
t[1 - 1/(a^2*x^2)]*x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= \frac{\sqrt{c-a^2cx^2}\int\left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} \\
&= -\frac{3\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.50

$$\frac{\sqrt{c-a^2cx^2}\,(ax(-6+ax)+8\log(1+ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - a^2*c*x^2]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1/(
a^2*x^2)]*x)
```

Maple [A]
time = 0.09, size = 67, normalized size = 0.60

method	result	size
default	$\frac{(a^2x^2-6ax+8\ln(ax+1))\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2a(ax-1)^2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/a/(a*x-1)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [**A**]
time = 0.33, size = 33, normalized size = 0.29

$$\frac{(a^2x^2 - 6\,ax + 8\,\log{(ax+1)})\sqrt{-a^2c}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(-a^2*c)/a^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.725 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] (-a^2*c*x^2+c)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-
1/a^2/x^2)^(1/2)-4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 84}

$$\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-a^2cx^2}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - a^2*c*x^2]/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - a^2*c*x^2]*Log[x])/(a
*Sqrt[1 - 1/(a^2*x^2)]*x) - (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(a*Sqrt[1
- 1/(a^2*x^2)]*x)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-a^2cx^2}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{4\sqrt{c-a^2cx^2}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\,(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^
2)]*x)
```

Maple [A]
time = 0.10, size = 61, normalized size = 0.54

method	result	size
default	$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,(-ax+4\ln(ax+1)-\ln(x))(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-a*x+4*ln(a*x+1)-ln(x))*(a*x+1)*((a*x-1)/(a*x+1))^
(3/2)/(a*x-1)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Fricas [**A**]
time = 0.42, size = 26, normalized size = 0.23

$$\frac{\sqrt{-a^2c}\,(ax-4\,\log(ax+1)+\log(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="fric
as")
```

```
[Out] sqrt(-a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac
")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.726 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] -(-a^2*c*x^2+c)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(-a^2*c*x^2+c)^(1/2
)/x/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A]
time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6327, 6328, 90}

$$-\frac{\sqrt{c-a^2cx^2}}{ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)) - (3*Sqrt[c - a^2*c*x^
2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])
/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{\sqrt{c-a^2cx^2}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4\sqrt{c-a^2cx^2}\,\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.49

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{x}-3a\log(x)+4a\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^2),x]
[Out] (Sqrt[c - a^2*c*x^2]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1 -
 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.56

method	result	size
default	$\frac{(4\ln(ax+1)ax-3a\ln(x)x-1)\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{x(ax-1)^2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOS
E)
```

```
[Out] (4*ln(a*x+1)*a*x-3*a*ln(x)*x-1)*(-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*
x+1))^(3/2)/x/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A]
time = 0.39, size = 33, normalized size = 0.29

$$\frac{\sqrt{-a^2c}\,(4\,ax\log(ax+1)-3\,ax\log(x)-1)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fr
icas")
```

```
[Out] sqrt(-a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.727 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-a^2cx^2}\,\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] -1/2*(-a^2*c*x^2+c)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3*(-a^2*c*x^2+c)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2
)-4*a*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{3\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{2ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -1/2*Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - a^2*c*
x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[
1 - 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a
^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^3}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$= -\frac{\sqrt{c-a^2cx^2}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-a^2cx^2}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-a^2cx^2}\cdots}{\sqrt{\cdots}}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{2x^2}+\frac{3a}{x}+4a^2\log(x)-4a^2\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 + a
*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.09, size = 77, normalized size = 0.51

method	result	size

default	$-\frac{\left(8 \ln \left(a x+1\right) a^{2} x^{2}-8 a^{2} \ln \left(x \right) x^{2}-6 a x+1\right) \sqrt{-c \left(a^{2} x^{2}-1\right)}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{2 \left(a x-1\right)^{2} x^{2}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2*(8*ln(a*x+1)*a^2*x^2-8*a^2*ln(x)*x^2-6*a*x+1)*(-c*(a^2*x^2-1))^(1/2)*(
a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A]
time = 0.35, size = 88, normalized size = 0.58

$$\frac{8\, a^{3} \sqrt{-c}\, x^{2} \log \left(\frac{2\, a^{3} c x^{2} + 2\, a^{2} c x + \sqrt{-a^{2} c}\, (2\, a x + 1) \sqrt{-c} + a c}{a x^{2} + x}\right) + \sqrt{-a^{2} c}\, (6\, a x - 1)}{2\, a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fr
icas")
```

```
[Out] 1/2*(8*a^3*sqrt(-c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
+ 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + sqrt(-a^2*c)*(6*a*x - 1))/(a*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.728 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx$

Optimal. Leaf size=193

$$-\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^2\sqrt{c-a^2cx^2}\,\log(1+a}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] -1/3*(-a^2*c*x^2+c)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+3/2*(-a^2*c*x^2+c)^(1/2
)/x^3/(1-1/a^2/x^2)^(1/2)-4*a*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-
4*a^2*ln(x)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(-a^
2*c*x^2+c)^(1/2)/x/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$-\frac{4a\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{3ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-a^2cx^2}}{2x^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] -1/3*Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (3*Sqrt[c - a^2*c*
x^2])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (4*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1
/(a^2*x^2)]*x^2) - (4*a^2*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)
]*x) + (4*a^2*Sqrt[c - a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^4}\,dx &= \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{\sqrt{c-a^2cx^2}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{3\sqrt{c-a^2cx^2}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{4a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x}}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.40

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{3x^3}+\frac{3a}{2x^2}-\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[x]
 + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.44

method	result	size

default	$\frac{(24\ln(ax+1)a^3x^3-24\ln(x)a^3x^3-24a^2x^2+9ax-2)\sqrt{-c(a^2x^2-1)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{6(ax-1)^2x^3}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*(24*ln(a*x+1)*a^3*x^3-24*ln(x)*a^3*x^3-24*a^2*x^2+9*a*x-2)*(-c*(a^2*x^2
-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A]
time = 0.36, size = 98, normalized size = 0.51

$$\frac{24\,a^4\sqrt{-c}\,x^3\log\left(\frac{2\,a^3cx^2+2\,a^2cx-\sqrt{-a^2c}\,(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)-(24\,a^2x^2-9\,ax+2)\sqrt{-a^2c}}{6\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fr
icas")
```

```
[Out] 1/6*(24*a^4*sqrt(-c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(-a^2*c))
/(a*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.729 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^3\sqrt{c-a^2cx^2}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^3\sqrt{c-}}{\sqrt{}}$$

```
[Out] -1/4*(-a^2*c*x^2+c)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+(-a^2*c*x^2+c)^(1/2)/x^
4/(1-1/a^2/x^2)^(1/2)-2*a*(-a^2*c*x^2+c)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+4*a^
2*(-a^2*c*x^2+c)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(-a^2*c*x^2+c)^(
1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(-a^2*c*x^2+c)^(1/2)/x/(1-1/a^2/
x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 90}

$$\frac{4a^2\sqrt{c-a^2cx^2}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-a^2cx^2}}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-a^2cx^2}}{x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-a^2cx^2}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-a^2cx^2}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-a^2cx^2}\,\log(ax+1)}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/4*Sqrt[c - a^2*c*x^2]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + Sqrt[c - a^2*c*x^2
]/(Sqrt[1 - 1/(a^2*x^2)]*x^4) - (2*a*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*
x^2)]*x^3) + (4*a^2*Sqrt[c - a^2*c*x^2])/(Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a
^3*Sqrt[c - a^2*c*x^2]*Log[x])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^3*Sqrt[c -
a^2*c*x^2]*Log[1 + a*x])/(Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-a^2cx^2}}{x^5}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{\sqrt{c-a^2cx^2}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= -\frac{\sqrt{c-a^2cx^2}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{2a\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4a^2\sqrt{c-a^2cx^2}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 0.37

$$\frac{\sqrt{c-a^2cx^2}\left(-\frac{1}{4x^4}+\frac{a}{x^3}-\frac{2a^2}{x^2}+\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^4*
Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 93, normalized size = 0.41

method	result	size
default	$-\frac{\left(16\ln(ax+1)a^4x^4-16\ln(x)a^4x^4-16a^3x^3+8a^2x^2-4ax+1\right)\sqrt{-c\left(a^2x^2-1\right)}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4(ax-1)^2x^4}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4*(16*ln(a*x+1)*a^4*x^4-16*ln(x)*a^4*x^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*(
-c*(a^2*x^2-1))^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Fricas [A]
time = 0.33, size = 104, normalized size = 0.46

$$\frac{16\,a^5\sqrt{-c}\,x^4\log\left(\frac{2\,a^3cx^2+2\,a^2cx+\sqrt{-a^2c}\,(2\,ax+1)\sqrt{-c}+ac}{ax^2+x}\right)+\left(16\,a^3x^3-8\,a^2x^2+4\,ax-1\right)\sqrt{-a^2c}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fr
icas")
```

```
[Out] 1/4*(16*a^5*sqrt(-c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(-a^2*c)*(2*a*x
 + 1)*sqrt(-c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*s
qrt(-a^2*c))/(a*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
```

```
[Out] int(((c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.730 $\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=136

$$\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2+m;ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+
c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)-4*x^m*hypergeom([1, 1+m],[2+m],a*x)*(-a^
2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6327, 6328, 90, 66}

$$-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*
Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) - (4*x^m*Sqrt[c - a^2*
c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, a*x])/(a*(1 + m)*Sqrt[1 - 1/(a^2*
x^2)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{\sqrt{c-a^2cx^2}\int\frac{x^m(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{\sqrt{c-a^2cx^2}\int\left(3x^m+ax^{1+m}+\frac{4x^m}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int\frac{x^m}{-1+ax}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$=\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,1}{a(1+m)\sqrt{1-}}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.54

$$\frac{x^m\sqrt{c-a^2cx^2}\,(6+ax+m(3+ax)-4(2+m)\,{}_2F_1(1,1+m;2+m;ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(6 + a*x + m*(3 + a*x) - 4*(2 + m)*Hypergeometric2
F1[1, 1 + m, 2 + m, a*x]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \sqrt{-a^{2} c\,x^{2}+c}}{\left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a^2*x^2 + 2*a*x + 1)*x^m*sqrt((a*x - 1)/(a*x
 + 1))/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.731 $\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}-\frac{2acx^{2+m}\sqrt{1-a^2x^2}\ {}_2F_1\left(\frac{1}{2},\frac{2+m}{2};4+\right.}{(2+m)\sqrt{c-a^2cx^2}}$$

```
[Out] -c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^
2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)-2*a*c*x^(2+m)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)+x^
(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A]
time = 0.26, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6286, 1823, 822, 372, 371}

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}-\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) - (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6286

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[c^(n/2), Int[x^m*(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x]
/; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] && !(IntegerQ[p] || GtQ[
c, 0]) && IGtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= -\int e^{2\tanh^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx\\
&= -\left(c\int\frac{x^m(1+ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right)\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}+\frac{\int\frac{x^m\left(-a^2c(3+2m)-2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-(2ac)\int\frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx-\frac{(c(3+2m))\int\frac{x^m}{\sqrt{c-a^2cx}}}{2+m}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{\left(2ac\sqrt{1-a^2x^2}\right)\int\frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}-\frac{\left(c(3+2m)\sqrt{\right.}}{(2}\\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}-
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.15, size = 129, normalized size = 0.75

$$\frac{x^{1+m}\left(\frac{2\sqrt{1-ax}\sqrt{-c(1+ax)}\,F_1\left(1+m;\frac{1}{2},-\frac{1}{2};2+m;ax,-ax\right)}{\sqrt{-1+ax}\sqrt{1+ax}}+\frac{\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^(1 + m)*((2*Sqrt[1 - a*x]*Sqrt[-(c*(1 + a*x))]*AppellF1[1 + m, 1/2, -1/2
, 2 + m, a*x, -(a*x)])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[c - a^2*c*x^2
]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]
))/(1 + m)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\,x^m\sqrt{-a^2c\,x^2+c}}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] int(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x + 1)*x^m/(a*x - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-c(ax-1)(ax+1)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x + 1)/(a*x - 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.732 $\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=82

$$\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c)
^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6327, 6328, 45}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)*Sq
rt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int x^m(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{\sqrt{c-a^2cx^2}\int\left(x^m+ax^{1+m}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\ &= \frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.68

$$\frac{x^m(2+m+ax+amx)\sqrt{c-a^2cx^2}}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*x^m*Sqrt[c - a^2*c*x^2],x]
[Out] (x^m*(2 + m + a*x + a*m*x)*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*(2 + m)*Sqrt[1 -
 1/(a^2*x^2)])
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.76

method	result	size
gosper	$\frac{x^{1+m}(amx+ax+m+2)\sqrt{-a^2cx^2+c}}{(2+m)(1+m)(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	62

risch	$-\frac{\sqrt{-\frac{c(a^2x^2-1)}{(ax+1)(ax-1)}}\,(ax-1)c(amx+ax+m+2)x\,x^m}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{-c}\,(2+m)(1+m)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] x^(1+m)*(a*m*x+a*x+m+2)*(-a^2*c*x^2+c)^(1/2)/(2+m)/(1+m)/(a*x+1)/((a*x-1)/(
a*x+1))^(1/2)
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.66

$$\frac{\left(a\sqrt{-c}\,(m+1)x^2+\sqrt{-c}\,(m+2)x\right)(ax+1)x^m}{(m^2+3\,m+2)ax+m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 + sqrt(-c)*(m + 2)*x)*(a*x + 1)*x^m/((m^2 + 3*m + 2
)*a*x + m^2 + 3*m + 2)
```

Fricas [A]
time = 0.38, size = 74, normalized size = 0.90

$$-\frac{\sqrt{-a^2cx^2+c}\,((am+a)x^2+(m+2)x)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-(am^2+3\,am+2\,a)x+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 + (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x +
 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^m\sqrt{-c\,(ax-1)\,(ax+1)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(-a^2*c*x^2+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B]
time = 1.59, size = 93, normalized size = 1.13

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(m+1\right)}{m^2+3\,m+2}+\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(m+2\right)}{a\,\left(m^2+3\,m+2\right)}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
+ m^2 + 2) + (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/(x
 - 1/a)
```

3.733 $\int e^{-\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\, dx$

Optimal. Leaf size=83

$$-\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2+c
)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6327, 6328, 45}

$$\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
```

```
[Out] -((x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)])) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx=\frac{\sqrt{c-a^2cx^2}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{\sqrt{c-a^2cx^2}\int x^m(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=\frac{\sqrt{c-a^2cx^2}\int\left(-x^m+ax^{1+m}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}$$
$$=-\frac{x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.70

$$\frac{x^m\sqrt{c-a^2cx^2}\,(-2+ax+m(-1+ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^ArcCoth[a*x],x]
[Out] (x^m*Sqrt[c - a^2*c*x^2]*(-2 + a*x + m*(-1 + a*x)))/(a*(1 + m)*(2 + m)*Sqrt
[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.77

method	result	size
gosper	$\frac{x^{1+m}(amx+ax-m-2)\sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\frac{ax-1}{ax+1}}}{(2+m)(1+m)(ax-1)}$	64

risch	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-\frac{c(a^2x^2-1)}{(ax+1)(ax-1)}}\,(ax+1)c(amx+ax-m-2)x\,x^m}{\sqrt{-c\,(a^2x^2-1)}\,\sqrt{-c}\,(2+m)(1+m)}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] x^(1+m)*(a*m*x+a*x-m-2)*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(2+m)/
(1+m)/(a*x-1)
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.69

$$\frac{\left(a\sqrt{-c}\,(m+1)x^2-\sqrt{-c}\,(m+2)x\right)(ax-1)x^m}{(m^2+3\,m+2)ax-m^2-3\,m-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="ma
xima")
```

```
[Out] (a*sqrt(-c)*(m + 1)*x^2 - sqrt(-c)*(m + 2)*x)*(a*x - 1)*x^m/((m^2 + 3*m + 2
)*a*x - m^2 - 3*m - 2)
```

Fricas [A]
time = 0.37, size = 75, normalized size = 0.90

$$-\frac{\sqrt{-a^2cx^2+c}\,\left((am+a)x^2-(m+2)x\right)x^m\sqrt{\frac{ax-1}{ax+1}}}{m^2-(am^2+3\,am+2\,a)x+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -sqrt(-a^2*c*x^2 + c)*((a*m + a)*x^2 - (m + 2)*x)*x^m*sqrt((a*x - 1)/(a*x +
 1))/(m^2 - (a*m^2 + 3*a*m + 2*a)*x + 3*m + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**B**]
time = 1.45, size = 94, normalized size = 1.13

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}\,\left(\frac{x^m\,x^2\,\sqrt{c-a^2\,c\,x^2}\,\left(m+1\right)}{m^2+3\,m+2}-\frac{x\,x^m\,\sqrt{c-a^2\,c\,x^2}\,\left(m+2\right)}{a\,\left(m^2+3\,m+2\right)}\right)}{x-\frac{1}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (((a*x - 1)/(a*x + 1))^(1/2)*((x^m*x^2*(c - a^2*c*x^2)^(1/2)*(m + 1))/(3*m
+ m^2 + 2) - (x*x^m*(c - a^2*c*x^2)^(1/2)*(m + 2))/(a*(3*m + m^2 + 2))))/(x
 - 1/a)
```

3.734 $\int e^{-2\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=172

$$\frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m}-\frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}+\frac{2acx^{2+m}\sqrt{1-a^2x^2}\ {}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+}{}\right.}{(2+m)\sqrt{c-a^2cx^2}}$$

```
[Out] -c*(3+2*m)*x^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)*(-a^2*x^
2+1)^(1/2)/(m^2+3*m+2)/(-a^2*c*x^2+c)^(1/2)+2*a*c*x^(2+m)*hypergeom([1/2, 1
+1/2*m],[2+1/2*m],a^2*x^2)*(-a^2*x^2+1)^(1/2)/(2+m)/(-a^2*c*x^2+c)^(1/2)+x^
(1+m)*(-a^2*c*x^2+c)^(1/2)/(2+m)
```

Rubi [A]
time = 0.25, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6287, 1823, 822, 372, 371}

$$-\frac{c(2m+3)\sqrt{1-a^2x^2}\,x^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{(m+1)(m+2)\sqrt{c-a^2cx^2}}+\frac{2ac\sqrt{1-a^2x^2}\,x^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};a^2x^2\right)}{(m+2)\sqrt{c-a^2cx^2}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*Sqrt[c - a^2*c*x^2])/(2 + m) - (c*(3 + 2*m)*x^(1 + m)*Sqrt[1 - a
^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x^2])/((1 + m)*(2
+ m)*Sqrt[c - a^2*c*x^2]) + (2*a*c*x^(2 + m)*Sqrt[1 - a^2*x^2]*Hypergeomet
ric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/((2 + m)*Sqrt[c - a^2*c*x^2])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
```

```
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6287

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_
Symbol] :> Dist[1/c^(n/2), Int[x^m*((c + d*x^2)^(p + n/2)/(1 - a*x)^n), x],
 x] /; FreeQ[{a, c, d, m, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] ||
GtQ[c, 0]) && ILtQ[n/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx &= -\int e^{-2\tanh^{-1}(ax)} x^m \sqrt{c-a^2cx^2}\,dx \\
&= -\left(c\int \frac{x^m(1-ax)^2}{\sqrt{c-a^2cx^2}}\,dx\right) \\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\int \frac{x^m\left(-a^2c(3+2m)+2a^3c(2+m)x\right)}{\sqrt{c-a^2cx^2}}\,dx}{a^2(2+m)} \\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + (2ac)\int \frac{x^{1+m}}{\sqrt{c-a^2cx^2}}\,dx - \frac{(c(3+2m))\int \frac{x^m}{\sqrt{c-a^2c}}}{2+m} \\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} + \frac{\left(2ac\sqrt{1-a^2x^2}\right)\int \frac{x^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}} - \frac{\left(c(3+2m)\right.}{(2} \\
&= \frac{x^{1+m}\sqrt{c-a^2cx^2}}{2+m} - \frac{c(3+2m)x^{1+m}\sqrt{1-a^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{(1+m)(2+m)\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.20, size = 110, normalized size = 0.64

$$\frac{x^{1+m}\left(-\frac{2\sqrt{c-acx}\,F_1\left(1+m;\frac{1}{2},-\frac{1}{2};2+m;-ax,ax\right)}{\sqrt{1-ax}}+\frac{\sqrt{c-a^2cx^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{\sqrt{1-a^2x^2}}\right)}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (x^(1 + m)*((-2*Sqrt[c - a*c*x]*AppellF1[1 + m, 1/2, -1/2, 2 + m, -(a*x), a
*x])/Sqrt[1 - a*x] + (Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2
, (3 + m)/2, a^2*x^2])/Sqrt[1 - a^2*x^2]))/(1 + m)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-a^2c\,x^2+c}\,\left(ax-1\right)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m/(a*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}\sqrt{-c\left(ax-1\right)\left(ax+1\right)}\left(ax-1\right)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**m*sqrt(-c*(a*x - 1)*(a*x + 1))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m}\,\sqrt{c-a^{2}\,c\,x^{2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^m*(c - a^2*c*x^2)^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.735 $\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=137

$$-\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,1+m;2+m;-ax)}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*x^m*(-a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(-a^2*c*x^2
+c)^(1/2)/(2+m)/(1-1/a^2/x^2)^(1/2)+4*x^m*hypergeom([1, 1+m],[2+m],-a*x)*(-
a^2*c*x^2+c)^(1/2)/a/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {6327, 6328, 90, 66}

$$\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(1,m+1;m+2;-ax)}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{m+1}\sqrt{c-a^2cx^2}}{(m+2)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x^m\sqrt{c-a^2cx^2}}{a(m+1)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*x^m*Sqrt[c - a^2*c*x^2])/(a*(1 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (x^(1 + m)
*Sqrt[c - a^2*c*x^2])/((2 + m)*Sqrt[1 - 1/(a^2*x^2)]) + (4*x^m*Sqrt[c - a^2
*c*x^2]*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)])/(a*(1 + m)*Sqrt[1 - 1/(
a^2*x^2)])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
```

```
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}x^m\sqrt{c-a^2cx^2}\,dx &= \frac{\sqrt{c-a^2cx^2}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^{1+m}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \frac{x^m(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= \frac{\sqrt{c-a^2cx^2}\int \left(-3x^m+ax^{1+m}+\frac{4x^m}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\left(4\sqrt{c-a^2cx^2}\right)\int \frac{x^{\cdot}}{1+\cdot}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\\
&= -\frac{3x^m\sqrt{c-a^2cx^2}}{a(1+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^{1+m}\sqrt{c-a^2cx^2}}{(2+m)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x^m\sqrt{c-a^2cx^2}\,{}_2F_1(}{a(1+m)\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 0.55

$$\frac{x^m\sqrt{c-a^2cx^2}\,(-6+ax+m(-3+ax)+4(2+m)\,{}_2F_1(1,1+m;2+m;-ax))}{a(1+m)(2+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*Sqrt[c - a^2*c*x^2])/E^(3*ArcCoth[a*x]),x]
```

[Out] (x^m*Sqrt[c - a^2*c*x^2]*(-6 + a*x + m*(-3 + a*x) + 4*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -(a*x)]))/(a*(1 + m)*(2 + m)*Sqrt[1 - 1/(a^2*x^2)])

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^{m} \sqrt{-a^{2} c \,x^{2}+c}\, \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)

[Out] int(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*c*x^2 + c)*x^m*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-a^2*c*x^2 + c)*(a*x - 1)*x^m*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1), x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(-a**2*c*x**2+c)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-a^2*c*x^2+c)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^m*(c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.736 $\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx$

Optimal. Leaf size=81

$$-\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\,{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

```
[Out] -256*c^3*(1-1/a/x)^(4-1/2*n)*(1+1/a/x)^(-4+1/2*n)*hypergeom([8, 4-1/2*n],[5
-1/2*n],(a-1/x)/(a+1/x))/a/(8-n)
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {6326, 6330, 133}

$$-\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-8}{2}}\,{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] (-256*c^3*(1 - 1/(a*x))^(4 - n/2)*(1 + 1/(a*x))^((-8 + n)/2)*Hypergeometric
2F1[8, 4 - n/2, 5 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(8 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
```

```
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^3\,dx &= -\left(\left(a^6c^3\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^3x^6\,dx\right)\\ &= \left(a^6c^3\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{3+\frac{n}{2}}}{x^8}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{256c^3\left(1-\frac{1}{ax}\right)^{4-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-8+n)}\,{}_2F_1\left(8,4-\frac{n}{2};5-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(8-n)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(81) = 162.
time = 1.52, size = 267, normalized size = 3.30

$$-\frac{c^3e^{n\coth^{-1}(ax)}\left(-912n+58n^3-n^5-5040ax+912an^2x-58an^4x+an^6x+1368a^2nx^2-64a^2n^3x^2+a^2n^5x^2+5040a^3x^3-152a^3n^2x^3+2a^3n^4x^3-576a^4nx^4+6a^4n^3x^4-3024a^5x^5+24a^5n^2x^5+120a^6nx^6+720a^7x^7+e^{2\coth^{-1}(ax)}n(-1152+576n+104n^2-52n^3-2n^4+n^5)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(-2304+784n^2-56n^4+n^6)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{5040a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/5040*(c^3*E^(n*ArcCoth[a*x])*(-912*n + 58*n^3 - n^5 - 5040*a*x + 912*a*n
^2*x - 58*a*n^4*x + a*n^6*x + 1368*a^2*n*x^2 - 64*a^2*n^3*x^2 + a^2*n^5*x^2
 + 5040*a^3*x^3 - 152*a^3*n^2*x^3 + 2*a^3*n^4*x^3 - 576*a^4*n*x^4 + 6*a^4*n
^3*x^4 - 3024*a^5*x^5 + 24*a^5*n^2*x^5 + 120*a^6*n*x^6 + 720*a^7*x^7 + E^(2
*ArcCoth[a*x])*n*(-1152 + 576*n + 104*n^2 - 52*n^3 - 2*n^4 + n^5)*Hypergeom
etric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (-2304 + 784*n^2 - 56*n
^4 + n^6)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/a
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*((a*x + 1)/(a
*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int 3a^2x^2e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-3a^4x^4e^{n\operatorname{acoth}(ax)}\right)dx+\int a^6x^6e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-e^{n\operatorname{acoth}(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**3,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-3*a**4*x**4*e
xp(n*acoth(a*x)), x) + Integral(a**6*x**6*exp(n*acoth(a*x)), x) + Integral(
-exp(n*acoth(a*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3,x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^3, x)
```

3.737 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^2\,dx$

Optimal. Leaf size=81

$$\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}\,{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

```
[Out] 64*c^2*(1-1/a/x)^(3-1/2*n)*(1+1/a/x)^(-3+1/2*n)*hypergeom([6, 3-1/2*n],[4-1
/2*n],(a-1/x)/(a+1/x))/a/(6-n)
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6326, 6330, 133}

$$\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-6}{2}}\,{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (64*c^2*(1 - 1/(a*x))^(3 - n/2)*(1 + 1/(a*x))^((-6 + n)/2)*Hypergeometric2F
1[6, 3 - n/2, 4 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(6 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
```

```
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^2\,dx&=\left(a^4c^2\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^2x^4\,dx\\&=-\left(\left(a^4c^2\right)\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{2+\frac{n}{2}}}{x^6}\,dx,x,\frac{1}{x}\right)\right)\\&=\frac{64c^2\left(1-\frac{1}{ax}\right)^{3-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-6+n)}{}_2F_1\left(6,3-\frac{n}{2};4-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(6-n)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.
time = 1.50, size = 179, normalized size = 2.21

$$\frac{c^2e^{n\coth^{-1}(ax)}\left(22n-n^3+120ax-22an^2x+an^4x-28a^2nx^2+a^2n^3x^2-80a^3x^3+2a^3n^2x^3+6a^4nx^4+24a^5x^5+e^{2\coth^{-1}(ax)}n(32-16n-2n^2+n^3)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(64-20n^2+n^4)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{120a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^2,x]
```

```
[Out] (c^2*E^(n*ArcCoth[a*x])*(22*n - n^3 + 120*a*x - 22*a*n^2*x + a*n^4*x - 28*a
^2*n*x^2 + a^2*n^3*x^2 - 80*a^3*x^3 + 2*a^3*n^2*x^3 + 6*a^4*n*x^4 + 24*a^5*
x^5 + E^(2*ArcCoth[a*x])*n*(32 - 16*n - 2*n^2 + n^3)*Hypergeometric2F1[1, 1
 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (64 - 20*n^2 + n^4)*Hypergeometric2F
1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))/(120*a)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*((a*x + 1)/(a*x - 1))^(1/2*n),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \left(-2a^2x^2e^{n\,\mathrm{acoth}\,(ax)}\right)\,dx + \int a^4x^4e^{n\,\mathrm{acoth}\,(ax)}\,dx + \int e^{n\,\mathrm{acoth}\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2*exp(n*acoth(a*x)), x) + Integral(a**4*x**4*exp(
n*acoth(a*x)), x) + Integral(exp(n*acoth(a*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,{\left(c-a^2\,c\,x^2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^2, x)
```

3.738 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)\,dx$

Optimal. Leaf size=79

$$-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}\,{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

```
[Out] -16*c*(1-1/a/x)^(2-1/2*n)*(1+1/a/x)^(-2+1/2*n)*hypergeom([4, 2-1/2*n],[3-1/
2*n],(a-1/x)/(a+1/x))/a/(4-n)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6326, 6330, 133}

$$-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-4}{2}}\,{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
```

```
[Out] (-16*c*(1 - 1/(a*x))^(2 - n/2)*(1 + 1/(a*x))^((-4 + n)/2)*Hypergeometric2F1
[4, 2 - n/2, 3 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(4 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6326

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[u*x^(2*p)*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x],
x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && Int
egerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
```

```
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)\,dx=-\left(\left(a^2c\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\,dx\right)$$
$$=\left(a^2c\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^4}\,dx,x,\frac{1}{x}\right)$$
$$=-\frac{16c\left(1-\frac{1}{ax}\right)^{2-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-4+n)}{}_2F_1\left(4,2-\frac{n}{2};3-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(4-n)}$$

Mathematica [A]
time = 0.50, size = 111, normalized size = 1.41

$$-\frac{ce^{n\coth^{-1}(ax)}\left(-n-6ax+an^2x+a^2nx^2+2a^3x^3+e^{2\coth^{-1}(ax)}(-2+n)n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+\left(-4+n^2\right){}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2),x]
[Out] -1/6*(c*E^(n*ArcCoth[a*x])*(-n - 6*a*x + a*n^2*x + a^2*n*x^2 + 2*a^3*x^3 +
E^(2*ArcCoth[a*x])*(-2 + n)*n*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*A
rcCoth[a*x])] + (-4 + n^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[
a*x])]))/a
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int a^{2}x^{2}e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-e^{n\operatorname{acoth}(ax)}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c),x)
```

```
[Out] -c*(Integral(a**2*x**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x)),
 x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-a^2\,c\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2), x)
```

3.739 $\int e^{n\coth^{-1}(ax)}\,dx$

Optimal. Leaf size=78

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

```
[Out] 4*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*n]
,(a-1/x)/(a+1/x))/a/(2-n)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6305, 133}

$$\frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x]),x]
```

```
[Out] (4*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2,
1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n))
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6305

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_)), x_Symbol] :> -Subst[Int[(1 + x/a)^(n/2)/(
x^2*(1 - x/a)^(n/2)), x], x, 1/x] /; FreeQ[{a, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\,dx &= -\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-n/2}\left(1+\frac{x}{a}\right)^{n/2}}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{4\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 82, normalized size = 1.05

$$\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(ax+{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{a(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x]),x]
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n*Hypergeometric2F1[1, 1 + n/2, 2 +
 n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(a*x + Hypergeometric2F1[1, n/2, 1 + n/
2, E^(2*ArcCoth[a*x])])))/(a*(2 + n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x)),x)
[Out] int(exp(n*arccoth(a*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x)),x)
[Out] Integral(exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x)),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x)),x)
[Out] int(exp(n*acoth(a*x)), x)
```

3.740 $\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx$

Optimal. Leaf size=18

$$\frac{e^{n\coth^{-1}(ax)}}{acn}$$

```
[Out] exp(n*arccoth(a*x))/a/c/n
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6318}

$$\frac{e^{n\coth^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
[Out] E^(n*ArcCoth[a*x])/(a*c*n)
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx = \frac{e^{n\coth^{-1}(ax)}}{acn}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.00

$$\frac{e^{n\coth^{-1}(ax)}}{acn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2),x]
[Out] E^(n*ArcCoth[a*x])/(a*c*n)
```

Maple [A]
time = 0.09, size = 18, normalized size = 1.00

method	result	size
gosper	$\frac{e^{n \,\mathrm{arccoth}(ax)}}{acn}$	18
risch	$\frac{(ax-1)^{-\frac{n}{2}}(ax+1)^{\frac{n}{2}}}{can}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x,method=_RETURNVERBOSE)
[Out] exp(n*arccoth(a*x))/a/c/n
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.67

$$\frac{e^{\left(\frac{1}{2}\,n\log(ax+1)-\frac{1}{2}\,n\log(ax-1)\right)}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="maxima")
[Out] e^(1/2*n*log(a*x + 1) - 1/2*n*log(a*x - 1))/(a*c*n)
```

Fricas [A]
time = 0.35, size = 27, normalized size = 1.50

$$\frac{\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="fricas")
[Out] ((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c*n)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 49, normalized size = 2.72

$$\begin{cases} \frac{x}{c} & \text{for}\: a = 0 \wedge n = 0 \\ -\frac{\log\left(x-\frac{1}{a}\right)}{2ac} + \frac{\log\left(x+\frac{1}{a}\right)}{2ac} & \text{for}\: n = 0 \\ \frac{xe^{\frac{i\pi n}{2}}}{c} & \text{for}\: a = 0 \\ \frac{e^{n\,\mathrm{acoth}\,(ax)}}{acn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c),x)
```

```
[Out] Piecewise((x/c, Eq(a, 0) & Eq(n, 0)), (-log(x - 1/a)/(2*a*c) + log(x + 1/a)
/(2*a*c), Eq(n, 0)), (x*exp(I*pi*n/2)/c, Eq(a, 0)), (exp(n*acoth(a*x))/(a*c
*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Mupad [B]
time = 1.45, size = 39, normalized size = 2.17

$$\frac{\left(\frac{1}{a\,x}+1\right)^{n/2}}{a\,c\,n\,\left(1-\frac{1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2),x)
```

```
[Out] (1/(a*x) + 1)^(n/2)/(a*c*n*(1 - 1/(a*x))^(n/2))
```

3.741 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=72

$$\frac{2e^{n\coth^{-1}(ax)}}{ac^2n\left(4-n^2\right)}-\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

```
[Out] 2*exp(n*arccoth(a*x))/a/c^2/n/(-n^2+4)-exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^2
/(-n^2+4)/(-a^2*x^2+1)
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6320, 6318}

$$\frac{2e^{n\coth^{-1}(ax)}}{ac^2n\left(4-n^2\right)}-\frac{(n-2ax)e^{n\coth^{-1}(ax)}}{ac^2\left(4-n^2\right)\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
```

```
[Out] (2*E^(n*ArcCoth[a*x]))/(a*c^2*n*(4 - n^2)) - (E^(n*ArcCoth[a*x])*(n - 2*a*x
))/(a*c^2*(4 - n^2)*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] && !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] && !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] || !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2(4-n^2)(1-a^2x^2)} + \frac{2\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c(4-n^2)}$$
$$= \frac{2e^{n\coth^{-1}(ax)}}{ac^2n(4-n^2)} - \frac{e^{n\coth^{-1}(ax)}(n-2ax)}{ac^2(4-n^2)(1-a^2x^2)}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.76

$$-\frac{e^{n\coth^{-1}(ax)}(-2+n^2-2anx+2a^2x^2)}{ac^2n(-4+n^2)(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^2,x]
[Out] -((E^(n*ArcCoth[a*x])*(-2 + n^2 - 2*a*n*x + 2*a^2*x^2))/(a*c^2*n*(-4 + n^2)
*(-1 + a^2*x^2)))
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.76

method	result	size
gosper	$-\frac{e^{n\,\mathrm{arccoth}(ax)}(2a^2x^2-2xan+n^2-2)}{(a^2x^2-1)c^2an(n^2-4)}$	55
risch	$-\frac{(2a^2x^2-2xan+n^2-2)(ax-1)^{-\frac{n}{2}}(ax+1)^{\frac{n}{2}}}{(a^2x^2-1)c^2an(n^2-4)}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] -exp(n*arccoth(a*x))*(2*a^2*x^2-2*a*n*x+n^2-2)/(a^2*x^2-1)/c^2/a/n/(n^2-4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Fricas [A]
time = 0.35, size = 79, normalized size = 1.10

$$\frac{\left(2\,a^{2}x^{2}-2\,anx+n^{2}-2\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{2}n^{3}-4\,ac^{2}n-\left(a^{3}c^{2}n^{3}-4\,a^{3}c^{2}n\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*x^2 - 2*a*n*x + n^2 - 2)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^3 -
4*a*c^2*n - (a^3*c^2*n^3 - 4*a^3*c^2*n)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{xe^{\frac{i\pi n}{2}}}{c^{2}} & \text{for}\: a = 0 \\ \tilde{\infty}xe^{-\infty n} & \text{for}\: a = -\frac{1}{x} \\ \tilde{\infty}xe^{\infty n} & \text{for}\: a = \frac{1}{x} \\ -\frac{a^{2}x^{2}\operatorname{acoth}{\left(ax\right)}}{4a^{3}c^{2}x^{2}e^{2\operatorname{acoth}{\left(ax\right)}}-4ac^{2}e^{2\operatorname{acoth}{\left(ax\right)}}} - \frac{2ax\operatorname{acoth}{\left(ax\right)}}{4a^{3}c^{2}x^{2}e^{2\operatorname{acoth}{\left(ax\right)}}-4ac^{2}e^{2\operatorname{acoth}{\left(ax\right)}}} + \frac{ax}{4a^{3}c^{2}x^{2}e^{2\operatorname{acoth}{\left(ax\right)}}-4ac^{2}e^{2\operatorname{acoth}{\left(ax\right)}}} - \frac{\operatorname{acoth}{\left(ax\right)}}{4a^{3}c^{2}x^{2}e^{2\operatorname{acoth}{\left(ax\right)}}-4ac^{2}e^{2\operatorname{acoth}{\left(ax\right)}}} + \frac{2}{4a^{3}c^{2}x^{2}e^{2\operatorname{acoth}{\left(ax\right)}}-4ac^{2}e^{2\operatorname{acoth}{\left(ax\right)}}} & \text{for}\: n = -2 \\ -\frac{a^{2}x^{2}\log{\left(x-\frac{1}{a}\right)}}{4a^{3}c^{2}x^{2}-4ac^{2}} + \frac{a^{2}x^{2}\log{\left(x+\frac{1}{a}\right)}}{4a^{3}c^{2}x^{2}-4ac^{2}} - \frac{2ax}{4a^{3}c^{2}x^{2}-4ac^{2}} + \frac{\log{\left(x-\frac{1}{a}\right)}}{4a^{3}c^{2}x^{2}-4ac^{2}} - \frac{\log{\left(x+\frac{1}{a}\right)}}{4a^{3}c^{2}x^{2}-4ac^{2}} & \text{for}\: n = 0 \\ \frac{\int \frac{e^{2\operatorname{acoth}{\left(ax\right)}}}{a^{4}x^{4}-2a^{2}x^{2}+1}\, dx}{c^{2}} & \text{for}\: n = 2 \\ -\frac{2a^{2}x^{2}e^{n\operatorname{acoth}{\left(ax\right)}}}{a^{3}c^{2}n^{3}x^{2}-4a^{3}c^{2}nx^{2}-ac^{2}n^{3}+4ac^{2}n} + \frac{2anxe^{n\operatorname{acoth}{\left(ax\right)}}}{a^{3}c^{2}n^{3}x^{2}-4a^{3}c^{2}nx^{2}-ac^{2}n^{3}+4ac^{2}n} - \frac{n^{2}e^{n\operatorname{acoth}{\left(ax\right)}}}{a^{3}c^{2}n^{3}x^{2}-4a^{3}c^{2}nx^{2}-ac^{2}n^{3}+4ac^{2}n} + \frac{2e^{n\operatorname{acoth}{\left(ax\right)}}}{a^{3}c^{2}n^{3}x^{2}-4a^{3}c^{2}nx^{2}-ac^{2}n^{3}+4ac^{2}n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Piecewise((x*exp(I*pi*n/2)/c**2, Eq(a, 0)), (zoo*x*exp(-oo*n), Eq(a, -1/x))
, (zoo*x*exp(oo*n), Eq(a, 1/x)), (-a**2*x**2*acoth(a*x)/(4*a**3*c**2*x**2*e
xp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - 2*a*x*acoth(a*x)/(4*a**3*c
**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) + a*x/(4*a**3*c**2
*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) - acoth(a*x)/(4*a**3*
c**2*x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))) + 2/(4*a**3*c**2*
x**2*exp(2*acoth(a*x)) - 4*a*c**2*exp(2*acoth(a*x))), Eq(n, -2)), (-a**2*x*
*2*log(x - 1/a)/(4*a**3*c**2*x**2 - 4*a*c**2) + a**2*x**2*log(x + 1/a)/(4*a
**3*c**2*x**2 - 4*a*c**2) - 2*a*x/(4*a**3*c**2*x**2 - 4*a*c**2) + log(x - 1
/a)/(4*a**3*c**2*x**2 - 4*a*c**2) - log(x + 1/a)/(4*a**3*c**2*x**2 - 4*a*c*
*2), Eq(n, 0)), (Integral(exp(2*acoth(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1),
x)/c**2, Eq(n, 2)), (-2*a**2*x**2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 -
4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n) + 2*a*n*x*exp(n*acoth(a*x))/
(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3 + 4*a*c**2*n) - n**
2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x**2 - a*c**2*n**3
 + 4*a*c**2*n) + 2*exp(n*acoth(a*x))/(a**3*c**2*n**3*x**2 - 4*a**3*c**2*n*x
**2 - a*c**2*n**3 + 4*a*c**2*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^2, x)
```

Mupad [B]
time = 1.59, size = 106, normalized size = 1.47

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{2\,x^2}{a\,c^2\,n\,\left(n^2-4\right)}-\frac{2\,x}{a^2\,c^2\,\left(n^2-4\right)}+\frac{n^2-2}{a^3\,c^2\,n\,\left(n^2-4\right)}\right)}{\left(\frac{1}{a^2}-x^2\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^2,x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((2*x^2)/(a*c^2*n*(n^2 - 4)) - (2*x)/(a^2*c^2*(n^2
 - 4)) + (n^2 - 2)/(a^3*c^2*n*(n^2 - 4))))/((1/a^2 - x^2)*((a*x - 1)/(a*x))
^(n/2))
```

3.742 $\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=127

$$\frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(64-20n^2+n^4\right)}-\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}$$

```
[Out] 24*exp(n*arccoth(a*x))/a/c^3/n/(n^4-20*n^2+64)-exp(n*arccoth(a*x))*(-4*a*x+
n)/a/c^3/(-n^2+16)/(-a^2*x^2+1)^2-12*exp(n*arccoth(a*x))*(-2*a*x+n)/a/c^3/(
n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {6320, 6318}

$$-\frac{(n-4ax)e^{n\coth^{-1}(ax)}}{ac^3\left(16-n^2\right)\left(1-a^2x^2\right)^2}-\frac{12(n-2ax)e^{n\coth^{-1}(ax)}}{ac^3\left(4-n^2\right)\left(16-n^2\right)\left(1-a^2x^2\right)}+\frac{24e^{n\coth^{-1}(ax)}}{ac^3n\left(n^4-20n^2+64\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (24*E^(n*ArcCoth[a*x]))/(a*c^3*n*(64 - 20*n^2 + n^4)) - (E^(n*ArcCoth[a*x])
*(n - 4*a*x))/(a*c^3*(16 - n^2)*(1 - a^2*x^2)^2) - (12*E^(n*ArcCoth[a*x])*(
n - 2*a*x))/(a*c^3*(4 - n^2)*(16 - n^2)*(1 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} + \frac{12\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}\,dx}{c(16-n^2)}$$
$$= -\frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)} + \frac{24\int \frac{e^{n\coth^{-1}(ax)}}{c-a^2cx^2}\,dx}{c^2(64-20n^2+n^4)}$$
$$= \frac{24e^{n\coth^{-1}(ax)}}{ac^3n(64-20n^2+n^4)} - \frac{e^{n\coth^{-1}(ax)}(n-4ax)}{ac^3(16-n^2)(1-a^2x^2)^2} - \frac{12e^{n\coth^{-1}(ax)}(n-2ax)}{ac^3(4-n^2)(16-n^2)(1-a^2x^2)}$$

Mathematica [A]
time = 0.08, size = 97, normalized size = 0.76

$$\frac{e^{n\coth^{-1}(ax)}\left(n^4-4an^3x+24(-1+a^2x^2)^2-8anx(-5+3a^2x^2)+4n^2(-4+3a^2x^2)\right)}{ac^3n(-16+n^2)(-4+n^2)(-1+a^2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^3,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n^4 - 4*a*n^3*x + 24*(-1 + a^2*x^2)^2 - 8*a*n*x*(-5 +
3*a^2*x^2) + 4*n^2*(-4 + 3*a^2*x^2)))/(a*c^3*n*(-16 + n^2)*(-4 + n^2)*(-1 +
 a^2*x^2)^2)
```

Maple [A]
time = 0.10, size = 101, normalized size = 0.80

method	result	size
gosper	$\frac{\left(24a^4x^4-24a^3x^3n+12a^2n^2x^2-4a\,n^3x-48a^2x^2+n^4+40xan-16n^2+24\right)e^{n\,\mathrm{arccoth}(ax)}}{\left(a^2x^2-1\right)^2c^3a\left(n^2-16\right)\left(n^2-4\right)n}$	101
risch	$\frac{\left(24a^4x^4-24a^3x^3n+12a^2n^2x^2-4a\,n^3x-48a^2x^2+n^4+40xan-16n^2+24\right)(ax-1)^{-\frac{n}{2}}(ax+1)^{\frac{n}{2}}}{\left(a^2x^2-1\right)^2c^3a\left(n^2-16\right)\left(n^2-4\right)n}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (24*a^4*x^4-24*a^3*n*x^3+12*a^2*n^2*x^2-4*a*n^3*x-48*a^2*x^2+n^4+40*a*n*x-1
6*n^2+24)*exp(n*arccoth(a*x))/(a^2*x^2-1)^2/c^3/a/(n^2-16)/(n^2-4)/n
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
[Out] -integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

Fricas [A]
time = 0.34, size = 174, normalized size = 1.37

$$\frac{(24\,a^4x^4-24\,a^3nx^3+n^4+12\,(a^2n^2-4\,a^2)x^2-16\,n^2-4\,(an^3-10\,an)x+24)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^3n^5-20\,ac^3n^3+64\,ac^3n+(a^5c^3n^5-20\,a^5c^3n^3+64\,a^5c^3n)x^4-2\,(a^3c^3n^5-20\,a^3c^3n^3+64\,a^3c^3n)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
[Out] (24*a^4*x^4 - 24*a^3*n*x^3 + n^4 + 12*(a^2*n^2 - 4*a^2)*x^2 - 16*n^2 - 4*(a
*n^3 - 10*a*n)*x + 24)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^5 - 20*a*c^3*
n^3 + 64*a*c^3*n + (a^5*c^3*n^5 - 20*a^5*c^3*n^3 + 64*a^5*c^3*n)*x^4 - 2*(a
^3*c^3*n^5 - 20*a^3*c^3*n^3 + 64*a^3*c^3*n)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^3,x, algorithm="giac")
[Out] integrate(-((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^3, x)
```

Mupad [B]
time = 1.73, size = 192, normalized size = 1.51

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{24\,x^4}{a\,c^3\,n\,(n^4-20\,n^2+64)}-\frac{4\,x\,(n^2-10)}{a^4\,c^3\,(n^4-20\,n^2+64)}-\frac{24\,x^3}{a^2\,c^3\,(n^4-20\,n^2+64)}+\frac{n^4-16\,n^2+24}{a^5\,c^3\,n\,(n^4-20\,n^2+64)}+\frac{x^2\,(12\,n^2-48)}{a^3\,c^3\,n\,(n^4-20\,n^2+64)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{1}{a^4}+x^4-\frac{2\,x^2}{a^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^3,x)
[Out] (((a*x + 1)/(a*x))^(n/2)*((24*x^4)/(a*c^3*n*(n^4 - 20*n^2 + 64)) - (4*x*(n^
2 - 10))/(a^4*c^3*(n^4 - 20*n^2 + 64)) - (24*x^3)/(a^2*c^3*(n^4 - 20*n^2 +
64)) + (n^4 - 16*n^2 + 24)/(a^5*c^3*n*(n^4 - 20*n^2 + 64)) + (x^2*(12*n^2 -
 48))/(a^3*c^3*n*(n^4 - 20*n^2 + 64))))/(((a*x - 1)/(a*x))^(n/2)*(1/a^4 + x
^4 - (2*x^2)/a^2))
```

3.743 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx$

Optimal. Leaf size=197

$$\frac{720e^{n\coth^{-1}(ax)}}{ac^4n\left(36-n^2\right)\left(64-20n^2+n^4\right)}-\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}-\frac{3}{ac^4\left(4-n\right.}$$

```
[Out] 720*exp(n*arccoth(a*x))/a/c^4/n/(-n^2+36)/(n^4-20*n^2+64)-exp(n*arccoth(a*x
))*(-6*a*x+n)/a/c^4/(-n^2+36)/(-a^2*x^2+1)^3-30*exp(n*arccoth(a*x))*(-4*a*x
+n)/a/c^4/(n^4-52*n^2+576)/(-a^2*x^2+1)^2-360*exp(n*arccoth(a*x))*(-2*a*x+n
)/a/c^4/(-n^2+36)/(n^4-20*n^2+64)/(-a^2*x^2+1)
```

Rubi [A]
time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6320, 6318}

$$-\frac{(n-6ax)e^{n\coth^{-1}(ax)}}{ac^4\left(36-n^2\right)\left(1-a^2x^2\right)^3}-\frac{360(n-2ax)e^{n\coth^{-1}(ax)}}{ac^4\left(4-n^2\right)\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)}-\frac{30(n-4ax)e^{n\coth^{-1}(ax)}}{ac^4\left(16-n^2\right)\left(36-n^2\right)\left(1-a^2x^2\right)^2}+\frac{720e^{n\coth^{-1}(ax)}}{ac^4n\left(36-n^2\right)\left(n^4-20n^2+64\right)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] (720*E^(n*ArcCoth[a*x]))/(a*c^4*n*(36 - n^2)*(64 - 20*n^2 + n^4)) - (E^(n*A
rcCoth[a*x])*(n - 6*a*x))/(a*c^4*(36 - n^2)*(1 - a^2*x^2)^3) - (30*E^(n*Arc
Coth[a*x])*(n - 4*a*x))/(a*c^4*(16 - n^2)*(36 - n^2)*(1 - a^2*x^2)^2) - (36
0*E^(n*ArcCoth[a*x])*(n - 2*a*x))/(a*c^4*(4 - n^2)*(16 - n^2)*(36 - n^2)*(1
 - a^2*x^2))
```

Rule 6318

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[
E^(n*ArcCoth[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d,
0] &&  !IntegerQ[n/2]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^4}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} + \frac{30\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^3}\,dx}{c(36-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2} + \frac{360\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^2}}{c^2(576-52n^2+}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{30e^{n\coth^{-1}(ax)}(n-4ax)}{ac^4(16-n^2)(36-n^2)(1-a^2x^2)^2} - \frac{360e^{n\,\mathrm{cot}}}{ac^4(4-n^2)(576}$$

$$= \frac{720e^{n\coth^{-1}(ax)}}{ac^4n(4-n^2)(576-52n^2+n^4)} - \frac{e^{n\coth^{-1}(ax)}(n-6ax)}{ac^4(36-n^2)(1-a^2x^2)^3} - \frac{30e^{n\coth^{-1}(ax)}(n}{ac^4(16-n^2)(36-n^2}$$

Mathematica [A]
time = 0.11, size = 152, normalized size = 0.77

$$-\frac{e^{n\coth^{-1}(ax)}\left(n^6-6an^5x-120an^3x(-2+a^2x^2)+720(-1+a^2x^2)^3+10n^4(-5+3a^2x^2)-48anx(33-40a^2x^2+15a^4x^4)+8n^2(68-105a^2x^2+45a^4x^4)\right)}{ac^4n(-36+n^2)(-16+n^2)(-4+n^2)(-1+a^2x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^4,x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n^6 - 6*a*n^5*x - 120*a*n^3*x*(-2 + a^2*x^2) + 720*(
-1 + a^2*x^2)^3 + 10*n^4*(-5 + 3*a^2*x^2) - 48*a*n*x*(33 - 40*a^2*x^2 + 15*
a^4*x^4) + 8*n^2*(68 - 105*a^2*x^2 + 45*a^4*x^4)))/(a*c^4*n*(-36 + n^2)*(-1
6 + n^2)*(-4 + n^2)*(-1 + a^2*x^2)^3))
```

Maple [A]
time = 0.10, size = 167, normalized size = 0.85

method	result
gosper	$-\frac{(720a^6x^6-720a^5x^5n+360a^4n^2x^4-120a^3n^3x^3-2160a^4x^4+30a^2n^4x^2+1920a^3x^3n-6a\,n^5x-840a^2n^2x^2+n^6+240a\,n^3x+2160a^2x}{(a^2x^2-1)^3c^4an(n^6-56n^4+784n^2-2304)}$
risch	$-\frac{(720a^6x^6-720a^5x^5n+360a^4n^2x^4-120a^3n^3x^3-2160a^4x^4+30a^2n^4x^2+1920a^3x^3n-6a\,n^5x-840a^2n^2x^2+n^6+240a\,n^3x+2160a^2x}{c^4(n^2-36)(n^2-16)(n^2-4)an(a^2x^2-1)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(720*a^6*x^6-720*a^5*n*x^5+360*a^4*n^2*x^4-120*a^3*n^3*x^3-2160*a^4*x^4+30
*a^2*n^4*x^2+1920*a^3*n*x^3-6*a*n^5*x-840*a^2*n^2*x^2+n^6+240*a*n^3*x+2160*
a^2*x^2-50*n^4-1584*a*n*x+544*n^2-720)*exp(n*arccoth(a*x))/(a^2*x^2-1)^3/c^
4/a/n/(n^6-56*n^4+784*n^2-2304)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

Fricas [A]
time = 0.37, size = 309, normalized size = 1.57

$$\frac{\left(720\,a^{6}x^{6}-720\,a^{5}nx^{5}+n^{6}+360\,(a^{4}n^{2}-6\,a^{4})x^{4}-50\,n^{4}-120\,(a^{3}n^{3}-16\,a^{3}n)x^{3}+30\,(a^{2}n^{4}-28\,a^{2}n^{2}+72\,a^{2})x^{2}+544\,n^{2}-6\,(an^{5}-40\,an^{3}+264\,an)x-720\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{4}n^{7}-56\,ac^{4}n^{5}+784\,ac^{4}n^{3}-(a^{7}c^{4}n^{7}-56\,a^{7}c^{4}n^{5}+784\,a^{7}c^{4}n^{3}-2304\,a^{7}c^{4}n)x^{6}-2304\,ac^{4}n+3\,(a^{5}c^{4}n^{7}-56\,a^{5}c^{4}n^{5}+784\,a^{5}c^{4}n^{3}-2304\,a^{5}c^{4}n)x^{4}-3\,(a^{3}c^{4}n^{7}-56\,a^{3}c^{4}n^{5}+784\,a^{3}c^{4}n^{3}-2304\,a^{3}c^{4}n)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] (720*a^6*x^6 - 720*a^5*n*x^5 + n^6 + 360*(a^4*n^2 - 6*a^4)*x^4 - 50*n^4 - 1
20*(a^3*n^3 - 16*a^3*n)*x^3 + 30*(a^2*n^4 - 28*a^2*n^2 + 72*a^2)*x^2 + 544*
n^2 - 6*(a*n^5 - 40*a*n^3 + 264*a*n)*x - 720)*((a*x + 1)/(a*x - 1))^(1/2*n)
/(a*c^4*n^7 - 56*a*c^4*n^5 + 784*a*c^4*n^3 - (a^7*c^4*n^7 - 56*a^7*c^4*n^5
+ 784*a^7*c^4*n^3 - 2304*a^7*c^4*n)*x^6 - 2304*a*c^4*n + 3*(a^5*c^4*n^7 - 5
6*a^5*c^4*n^5 + 784*a^5*c^4*n^3 - 2304*a^5*c^4*n)*x^4 - 3*(a^3*c^4*n^7 - 56
*a^3*c^4*n^5 + 784*a^3*c^4*n^3 - 2304*a^3*c^4*n)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c)^4, x)
```

Mupad [B]
time = 1.76, size = 314, normalized size = 1.59

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{n^6-50\,n^4+544\,n^2-720}{a^7\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{720\,x^5}{a^2\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{x^3\,\left(120\,n^2-1920\right)}{a^4\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}+\frac{720\,x^6}{a\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2-2304\right)}-\frac{6\,x\,\left(n^4-40\,n^2+264\right)}{a^6\,c^4\,\left(n^6-56\,n^4+784\,n^2-2304\right)}+\frac{x^2\,\left(30\,n^4-840\,n^2+2160\right)}{a^5\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2-2304\right)}+\frac{x^4\,\left(360\,n^2-2160\right)}{a^3\,c^4\,n\,\left(n^6-56\,n^4+784\,n^2-2304\right)}\right)}{{\left(\frac{a\,x-1}{a\,x}\right)}^{n/2}\,\left(\frac{1}{a^6}-x^6+\frac{3\,x^4}{a^2}-\frac{3\,x^2}{a^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^4,x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((544*n^2 - 50*n^4 + n^6 - 720)/(a^7*c^4*n*(784*n^
2 - 56*n^4 + n^6 - 2304)) - (720*x^5)/(a^2*c^4*(784*n^2 - 56*n^4 + n^6 - 23
04)) - (x^3*(120*n^2 - 1920))/(a^4*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (
720*x^6)/(a*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) - (6*x*(n^4 - 40*n^2 + 2
64))/(a^6*c^4*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^2*(30*n^4 - 840*n^2 + 2
160))/(a^5*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304)) + (x^4*(360*n^2 - 2160))/
(a^3*c^4*n*(784*n^2 - 56*n^4 + n^6 - 2304))))/(((a*x - 1)/(a*x))^(n/2)*(1/a
^6 - x^6 + (3*x^4)/a^2 - (3*x^2)/a^4))
```

3.744 $\int e^{n \coth^{-1}(ax)}(c-a^2cx^2)^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}(c-a^2cx^2)^{3/2}\,{}_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] 32*(1-1/a/x)^(5/2-1/2*n)*(1+1/a/x)^(-5/2+1/2*n)*(-a^2*c*x^2+c)^(3/2)*hyperg
eom([5, 5/2-1/2*n],[7/2-1/2*n],(a-1/x)/(a+1/x))/a^4/(5-n)/(1-1/a^2/x^2)^(3/
2)/x^3
```

Rubi [A]
time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6330, 133}

$$\frac{32(c-a^2cx^2)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-5}{2}}\,{}_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (32*(1 - 1/(a*x))^((5 - n)/2)*(1 + 1/(a*x))^((-5 + n)/2)*(c - a^2*c*x^2)^(3
/2)*Hypergeometric2F1[5, (5 - n)/2, (7 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^4*(5 - n)*(1 - 1/(a^2*x^2))^(3/2)*x^3)
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^{3/2}\,dx&=\frac{\left(c-a^2cx^2\right)^{3/2}\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\,dx}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\&=-\frac{\left(c-a^2cx^2\right)^{3/2}\operatorname{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{3}{2}+\frac{n}{2}}}{x^5}\,dx,x,\frac{1}{x}\right)}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\\&=\frac{32\left(1-\frac{1}{ax}\right)^{\frac{5-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-5+n)}\left(c-a^2cx^2\right)^{3/2}{}_2F_1\left(5,\frac{5-n}{2};\frac{7-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^4(5-n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 280 vs. $2(116)=232$.
time = 1.55, size = 280, normalized size = 2.41

$$\frac{c^2\left(96a^3c\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x(n+ax)+2e^{(1+n)\coth^{-1}(ax)}(-1+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)\right)-c(-1+a^2x^2)\left(2e^{n\coth^{-1}(ax)}(-1+a^2x^2)^2\left(-a(-21+n^2)x+2n\left(1-n^2+(3+n^2)\cosh\left(2\coth^{-1}(ax)\right)\right)+a(3+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)\right)+16ae^{(1+n)\coth^{-1}(ax)}(-3+3n-n^2+n^3)\sqrt{1-\frac{1}{a^2x^2}}\,x\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{192a\left(c-a^2cx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (c^2*(96*a^3*c*(1 - 1/(a^2*x^2))^(3/2)*x^3*(a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1
/(a^2*x^2)]*x*(n + a*x) + 2*E^((1 + n)*ArcCoth[a*x])*(-1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]) - c*(-1 + a^2*x^2)*(2*E^
(n*ArcCoth[a*x])*(-1 + a^2*x^2)^2*(-(a*(-21 + n^2)*x) + 2*n*(1 - n^2 + (3 +
 n^2)*Cosh[2*ArcCoth[a*x]]) + a*(3 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*Ar
cCoth[a*x]]) + 16*a*E^((1 + n)*ArcCoth[a*x])*(-3 + 3*n - n^2 + n^3)*Sqrt[1
- 1/(a^2*x^2)]*x*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*
x])])))/(192*a*(c - a^2*c*x^2)^(3/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c(ax-1)\,(ax+1)\right)^{\frac{3}{2}} e^{n\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*exp(n*acoth(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a x)} \left(c - a^2 c x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(3/2), x)
```

3.745 $\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx$

Optimal. Leaf size=116

$$\frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}\sqrt{c-a^2cx^2}\;{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

```
[Out] 8*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*hypergeom([3, 3/2-1/2*n],[5/
2-1/2*n],(a-1/x)/(a+1/x))*(-a^2*c*x^2+c)^(1/2)/a^2/(3-n)/x/(1-1/a^2/x^2)^(1
/2)
```

Rubi [A]

time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6327, 6330, 133}

$$\frac{8\sqrt{c-a^2cx^2}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\;{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (8*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*Sqrt[c - a^2*c*x^2]
*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, (a - x^(-1))/(a + x^(-1))])/(a^
2*(3 - n)*Sqrt[1 - 1/(a^2*x^2)]*x)
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] && !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,dx = \frac{\sqrt{c-a^2cx^2}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= -\frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^3}\,dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

$$= \frac{8\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}\sqrt{c-a^2cx^2}\,{}_2F_1\left(3,\frac{3-n}{2};\frac{5-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2(3-n)\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A]
time = 0.40, size = 101, normalized size = 0.87

$$-\frac{ce^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(n+ax)+2e^{\coth^{-1}(ax)}(-1+n)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2],x]
```

```
[Out] -1/2*(c*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*(a*Sqrt[1 - 1/(a^2*x^2)]
*x*(n + a*x) + 2*E^ArcCoth[a*x]*(-1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3
 + n)/2, E^(2*ArcCoth[a*x])]))/Sqrt[c - a^2*c*x^2]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\sqrt{-a^2c\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-c\,(ax-1)\,(ax+1)}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*exp(n*acoth(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^(1/2), x)
```

3.746 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=111

$$\frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] 2*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],[
3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6327, 6330, 133}

$$\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/
2)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])
/((1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx = \frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{\sqrt{c-a^2cx^2}}$$

$$= -\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\mathrm{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{\sqrt{c-a^2cx^2}}$$

$$= \frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{(1-n)\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.13, size = 81, normalized size = 0.73

$$-\frac{2e^{(1+n)\coth^{-1}(ax)}\sqrt{c-a^2cx^2}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)}{\sqrt{1-\frac{1}{a^2x^2}}\,(a^2cx+a^2cnx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*E^((1 + n)*ArcCoth[a*x])*Sqrt[c - a^2*c*x^2]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(Sqrt[1 - 1/(a^2*x^2)]*(a^2*c*x + a^2
*c*n*x))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\sqrt{-a^2c\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
[Out] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(-a^2*c*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(1/2), x)
```

3.747 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=46

$$-\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6319}

$$-\frac{(n-ax)e^{n \coth^{-1}(ax)}}{ac\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx = -\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.13, size = 43, normalized size = 0.93

$$\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac\,(-1+n^2)\,\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])

Maple [A]
time = 0.10, size = 49, normalized size = 1.07

method	result	size
gosper	$\frac{(ax+1)(ax-1)(ax-n)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{(n^2-1)a(-a^2c\,x^2+c)^{\frac{3}{2}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] (a*x+1)*(a*x-1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")

[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)

Fricas [A]
time = 0.36, size = 80, normalized size = 1.74

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")

[Out] -sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 - a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{n\operatorname{acoth}(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)

[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B]
time = 1.49, size = 78, normalized size = 1.70

$$-\frac{\left(\frac{x}{c\,(n^2-1)}-\frac{n}{a\,c\,(n^2-1)}\right)\,\left(\frac{a\,x+1}{a\,x}\right)^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] -((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2*
c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.748 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*a
rccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6320, 6319}

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac(9-n^2)(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}} + \frac{6\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{c(9-n^2)}$$
$$= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}} - \frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.43, size = 110, normalized size = 1.08

$$\frac{e^{n\coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)\right)}{4ac^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 84, normalized size = 0.82

method	result	size
gosper	$\frac{(ax+1)(ax-1)(6a^3x^3-6a^2n\,x^2+3a\,n^2x-n^3-9ax+7n)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+1)*(a*x-1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 0.37, size = 165, normalized size = 1.62

$$-\frac{\left(6\,a^{3}x^{3}-6\,a^{2}nx^{2}-n^{3}+3\,(an^{2}-3\,a)x+7\,n\right)\sqrt{-a^{2}cx^{2}+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{3}n^{4}-10\,ac^{3}n^{2}+(a^{5}c^{3}n^{4}-10\,a^{5}c^{3}n^{2}+9\,a^{5}c^{3})x^{4}+9\,ac^{3}-2\,(a^{3}c^{3}n^{4}-10\,a^{3}c^{3}n^{2}+9\,a^{3}c^{3})x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] -(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B]
time = 1.60, size = 173, normalized size = 1.70

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^{3}}{c^{2}\,(n^{4}-10\,n^{2}+9)}+\frac{7\,n-n^{3}}{a^{3}\,c^{2}\,(n^{4}-10\,n^{2}+9)}+\frac{3\,x\,(n^{2}-3)}{a^{2}\,c^{2}\,(n^{4}-10\,n^{2}+9)}-\frac{6\,n\,x^{2}}{a\,c^{2}\,(n^{4}-10\,n^{2}+9)}\right)}{\left(\frac{\sqrt{c-a^{2}\,c\,x^{2}}}{a^{2}}-x^{2}\,\sqrt{c-a^{2}\,c\,x^{2}}\right)\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
[Out] -(((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/(
a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9))
- (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*
(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.749 $\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}-\frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}}-\frac{120e^{n\coth^{-1}(ax)}(n-ax)}{ac^3\,(1-n^2)\,(9-n^2)\,(25-n^2)\,\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-5*a*x+n)/a/c/(-n^2+25)/(-a^2*c*x^2+c)^(5/2)-20*exp(n
*arccoth(a*x))*(-3*a*x+n)/a/c^2/(n^4-34*n^2+225)/(-a^2*c*x^2+c)^(3/2)-120*e
xp(n*arccoth(a*x))*(-a*x+n)/a/c^3/(-n^2+25)/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(
1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6320, 6319}

$$-\frac{120(n-ax)e^{n\coth^{-1}(ax)}}{ac^3\,(1-n^2)\,(9-n^2)\,(25-n^2)\,\sqrt{c-a^2cx^2}}-\frac{20(n-3ax)e^{n\coth^{-1}(ax)}}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}}-\frac{(n-5ax)e^{n\coth^{-1}(ax)}}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c*(25 - n^2)*(c - a^2*c*x^2)^(5/2)))
- (20*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^2*(9 - n^2)*(25 - n^2)*(c - a^2*
c*x^2)^(3/2)) - (120*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^3*(1 - n^2)*(9 - n^
2)*(25 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}} + \frac{20\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx}{c\,(25-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{120\int \frac{e^{n\coth^{-1}}}{(c-a^2c}}{c^2\,(9-n^2)\,(2}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-5ax)}{ac\,(25-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{20e^{n\coth^{-1}(ax)}(n-3ax)}{ac^2\,(9-n^2)\,(25-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{12}{ac^3\,(1-n^2)}$$

Mathematica [A]
time = 1.03, size = 299, normalized size = 1.80

$$-\frac{a^2e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\left(-\frac{10(225-34n^2+n^4)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2250n}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{340n^3}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{10n^5}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+15(25-26n^2+n^4)\cosh(3\coth^{-1}(ax))-45\cosh(5\coth^{-1}(ax))+50n^2\cosh(5\coth^{-1}(ax))-5n^4\cosh(5\coth^{-1}(ax))-125n\sinh(3\coth^{-1}(ax))+130n^3\sinh(3\coth^{-1}(ax))-5n^5\sinh(3\coth^{-1}(ax))+9n\sinh(5\coth^{-1}(ax))-10n^3\sinh(5\coth^{-1}(ax))+n^5\sinh(5\coth^{-1}(ax))\right)}{16c^2(-5+n)(-3+n)(-1+n)(1+n)(3+n)(5+n)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/16*(a^2*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))^(3/2)*x^3*((-10*(225 - 34*n
^2 + n^4))/Sqrt[1 - 1/(a^2*x^2)] + (2250*n)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (
340*n^3)/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (10*n^5)/(a*Sqrt[1 - 1/(a^2*x^2)]*x)
 + 15*(25 - 26*n^2 + n^4)*Cosh[3*ArcCoth[a*x]] - 45*Cosh[5*ArcCoth[a*x]] +
50*n^2*Cosh[5*ArcCoth[a*x]] - 5*n^4*Cosh[5*ArcCoth[a*x]] - 125*n*Sinh[3*Arc
Coth[a*x]] + 130*n^3*Sinh[3*ArcCoth[a*x]] - 5*n^5*Sinh[3*ArcCoth[a*x]] + 9*
n*Sinh[5*ArcCoth[a*x]] - 10*n^3*Sinh[5*ArcCoth[a*x]] + n^5*Sinh[5*ArcCoth[a
*x]]))/(c^2*(-5 + n)*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(5 + n)*(c - a^2*c*x
^2)^(3/2))
```

Maple [A]
time = 0.09, size = 140, normalized size = 0.84

method	result
gosper	$\frac{(ax+1)(ax-1)(120a^5x^5-120n\,a^4x^4+60a^3n^2x^3-20a^2n^3x^2-300a^3x^3+5a\,n^4x+260a^2n\,x^2-n^5-110a\,n^2x+30n^3+225ax-149n)\mathrm{e}^{n\,a}}{a(n^6-35n^4+259n^2-225)(-a^2c\,x^2+c)^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+1)*(a*x-1)*(120*a^5*x^5-120*a^4*n*x^4+60*a^3*n^2*x^3-20*a^2*n^3*x^2-30
0*a^3*x^3+5*a*n^4*x+260*a^2*n*x^2-n^5-110*a*n^2*x+30*n^3+225*a*x-149*n)*exp
(n*arccoth(a*x))/a/(n^6-35*n^4+259*n^2-225)/(-a^2*c*x^2+c)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Fricas [A]
time = 0.37, size = 291, normalized size = 1.75

$$-\frac{\left(120\,a^{5}x^{5}-120\,a^{4}nx^{4}-n^{5}+60\,(a^{3}n^{2}-5\,a^{3})x^{3}+30\,n^{3}-20\,(a^{2}n^{3}-13\,a^{2}n)x^{2}+5\,(an^{4}-22\,an^{2}+45\,a)x-149\,n\right)\sqrt{-a^{2}cx^{2}+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{4}n^{6}-35\,ac^{4}n^{4}+259\,ac^{4}n^{2}-(a^{7}c^{4}n^{6}-35\,a^{7}c^{4}n^{4}+259\,a^{7}c^{4}n^{2}-225\,a^{7}c^{4})x^{6}-225\,ac^{4}+3\,(a^{5}c^{4}n^{6}-35\,a^{5}c^{4}n^{4}+259\,a^{5}c^{4}n^{2}-225\,a^{5}c^{4})x^{4}-3\,(a^{3}c^{4}n^{6}-35\,a^{3}c^{4}n^{4}+259\,a^{3}c^{4}n^{2}-225\,a^{3}c^{4})x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -(120*a^5*x^5 - 120*a^4*n*x^4 - n^5 + 60*(a^3*n^2 - 5*a^3)*x^3 + 30*n^3 - 2
0*(a^2*n^3 - 13*a^2*n)*x^2 + 5*(a*n^4 - 22*a*n^2 + 45*a)*x - 149*n)*sqrt(-a
^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^4*n^6 - 35*a*c^4*n^4 + 259
*a*c^4*n^2 - (a^7*c^4*n^6 - 35*a^7*c^4*n^4 + 259*a^7*c^4*n^2 - 225*a^7*c^4)
*x^6 - 225*a*c^4 + 3*(a^5*c^4*n^6 - 35*a^5*c^4*n^4 + 259*a^5*c^4*n^2 - 225*
a^5*c^4)*x^4 - 3*(a^3*c^4*n^6 - 35*a^3*c^4*n^4 + 259*a^3*c^4*n^2 - 225*a^3*
c^4)*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(7/2), x)
```

Mupad [B]
time = 1.74, size = 289, normalized size = 1.74

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{120\,x^5}{c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{120\,n\,x^4}{a\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}+\frac{x^3\,\left(60\,n^2-300\right)}{a^2\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{n\,\left(n^4-30\,n^2+149\right)}{a^5\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}+\frac{5\,x\,\left(n^4-22\,n^2+45\right)}{a^4\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}-\frac{20\,n\,x^2\,\left(n^2-13\right)}{a^3\,c^3\,\left(n^6-35\,n^4+259\,n^2-225\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^4}+x^4\,\sqrt{c-a^2\,c\,x^2}-\frac{2\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] -((((a*x + 1)/(a*x))^(n/2)*((120*x^5)/(c^3*(259*n^2 - 35*n^4 + n^6 - 225)) -
 (120*n*x^4)/(a*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) + (x^3*(60*n^2 - 300))/
(a^2*c^3*(259*n^2 - 35*n^4 + n^6 - 225)) - (n*(n^4 - 30*n^2 + 149))/(a^5*c^
3*(259*n^2 - 35*n^4 + n^6 - 225)) + (5*x*(n^4 - 22*n^2 + 45))/(a^4*c^3*(259
*n^2 - 35*n^4 + n^6 - 225)) - (20*n*x^2*(n^2 - 13))/(a^3*c^3*(259*n^2 - 35*
n^4 + n^6 - 225))))/(((c - a^2*c*x^2)^(1/2)/a^4 + x^4*(c - a^2*c*x^2)^(1/2)
 - (2*x^2*(c - a^2*c*x^2)^(1/2))/a^2)*((a*x - 1)/(a*x))^(n/2))
```

3.750 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{e^{n \coth^{-1}(ax)}(n-7ax)}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}-\frac{42e^{n \coth^{-1}(ax)}(n-5ax)}{ac^2\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{840e^{n \coth^{-1}(ax)}(n-3ax)}{ac^3\left(9-n^2\right)\left(25-n^2\right)\left(49-n^2\right)(c-a}$$

```
[Out] -exp(n*arccoth(a*x))*(-7*a*x+n)/a/c/(-n^2+49)/(-a^2*c*x^2+c)^(7/2)-42*exp(n
*arccoth(a*x))*(-5*a*x+n)/a/c^2/(n^4-74*n^2+1225)/(-a^2*c*x^2+c)^(5/2)-840*
exp(n*arccoth(a*x))*(-3*a*x+n)/a/c^3/(-n^2+49)/(n^4-34*n^2+225)/(-a^2*c*x^2
+c)^(3/2)-5040*exp(n*arccoth(a*x))*(-a*x+n)/a/c^4/(n^4-74*n^2+1225)/(n^4-10
*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6320, 6319}

$$-\frac{5040(n-ax)e^{n \coth^{-1}(ax)}}{ac^4\left(1-n^2\right)\left(9-n^2\right)\left(25-n^2\right)\left(49-n^2\right)\sqrt{c-a^2cx^2}}-\frac{840(n-3ax)e^{n \coth^{-1}(ax)}}{ac^3\left(9-n^2\right)\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{3/2}}-\frac{42(n-5ax)e^{n \coth^{-1}(ax)}}{ac^2\left(25-n^2\right)\left(49-n^2\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{(n-7ax)e^{n \coth^{-1}(ax)}}{ac\left(49-n^2\right)\left(c-a^2cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 7*a*x))/(a*c*(49 - n^2)*(c - a^2*c*x^2)^(7/2)))
- (42*E^(n*ArcCoth[a*x])*(n - 5*a*x))/(a*c^2*(25 - n^2)*(49 - n^2)*(c - a^2
*c*x^2)^(5/2)) - (840*E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c^3*(9 - n^2)*(25
- n^2)*(49 - n^2)*(c - a^2*c*x^2)^(3/2)) - (5040*E^(n*ArcCoth[a*x])*(n - a*
x))/(a*c^4*(1 - n^2)*(9 - n^2)*(25 - n^2)*(49 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{9/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\,(49-n^2)\,(c-a^2cx^2)^{7/2}} + \frac{42\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{7/2}}\,dx}{c\,(49-n^2)}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\,(49-n^2)\,(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{5/2}} + \frac{840\int \frac{e^{n\cot}}{(c-a}}{c^2\,(25-n^2}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\,(49-n^2)\,(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{8}{ac^3\,(9-n^2}$$

$$= -\frac{e^{n\coth^{-1}(ax)}(n-7ax)}{ac\,(49-n^2)\,(c-a^2cx^2)^{7/2}} - \frac{42e^{n\coth^{-1}(ax)}(n-5ax)}{ac^2\,(25-n^2)\,(49-n^2)\,(c-a^2cx^2)^{5/2}} - \frac{8}{ac^3\,(9-n^2}$$

Mathematica [A]
time = 1.45, size = 260, normalized size = 1.09

$$\frac{ae^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)x^2\left(-\frac{35n}{-1+n^2}+\frac{35ax}{-1+n^2}-\frac{63a\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh(3\coth^{-1}(ax))}{-9+n^2}+\frac{35a\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh(5\coth^{-1}(ax))}{-25+n^2}-\frac{7a\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh(7\coth^{-1}(ax))}{-49+n^2}+\frac{21an\sqrt{1-\frac{1}{a^2x^2}}\,x\sinh(3\coth^{-1}(ax))}{-9+n^2}-\frac{7an\sqrt{1-\frac{1}{a^2x^2}}\,x\sinh(5\coth^{-1}(ax))}{-25+n^2}+\frac{an\sqrt{1-\frac{1}{a^2x^2}}\,x\sinh(7\coth^{-1}(ax))}{-49+n^2}\right)}{64c^3\,(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(9/2),x]
```

```
[Out] (a*E^(n*ArcCoth[a*x])*(1 - 1/(a^2*x^2))*x^2*((-35*n)/(-1 + n^2) + (35*a*x)/
(-1 + n^2) - (63*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]])/(-9 + n^2)
 + (35*a*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[5*ArcCoth[a*x]])/(-25 + n^2) - (7*a*S
qrt[1 - 1/(a^2*x^2)]*x*Cosh[7*ArcCoth[a*x]])/(-49 + n^2) + (21*a*n*Sqrt[1 -
 1/(a^2*x^2)]*x*Sinh[3*ArcCoth[a*x]])/(-9 + n^2) - (7*a*n*Sqrt[1 - 1/(a^2*x
^2)]*x*Sinh[5*ArcCoth[a*x]])/(-25 + n^2) + (a*n*Sqrt[1 - 1/(a^2*x^2)]*x*Sin
h[7*ArcCoth[a*x]])/(-49 + n^2)))/(64*c^3*(c - a^2*c*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 218, normalized size = 0.91

method	result
gosper	$\frac{(ax+1)(ax-1)(5040a^7x^7-5040n\,a^6x^6+2520a^5n^2x^5-840a^4n^3x^4-17640a^5x^5+210a^3n^4x^3+15960n\,a^4x^4-42a^2n^5x^2-7140a^3n^2x^3-}{a(n^8-84n^6+1974n^4-12916n^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+1)*(a*x-1)*(5040*a^7*x^7-5040*a^6*n*x^6+2520*a^5*n^2*x^5-840*a^4*n^3*x
^4-17640*a^5*x^5+210*a^3*n^4*x^3+15960*a^4*n*x^4-42*a^2*n^5*x^2-7140*a^3*n^
```

```
2*x^3+7*a*n^6*x+2100*a^2*n^3*x^2-n^7+22050*a^3*x^3-455*a*n^4*x-17178*a^2*n*
x^2+77*n^5+6433*a*n^2*x-1519*n^3-11025*a*x+6483*n)*exp(n*arccoth(a*x))/a/(n
^8-84*n^6+1974*n^4-12916*n^2+11025)/(-a^2*c*x^2+c)^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

Fricas [A]
time = 0.36, size = 453, normalized size = 1.90

$$-\frac{\left(5040\,a^{7}x^{7}-5040\,a^{6}nx^{6}-n^{7}+2520\,\left(a^{5}n^{2}-7\,a^{5}\right)x^{5}+77\,n^{5}-840\,\left(a^{4}n^{3}-19\,a^{4}n\right)x^{4}+210\,\left(a^{3}n^{4}-34\,a^{3}n^{2}+105\,a^{3}\right)x^{3}-1519\,n^{3}-42\,\left(a^{2}n^{5}-50\,a^{2}n^{3}+409\,a^{2}n\right)x^{2}+7\,\left(an^{6}-65\,an^{4}+919\,an^{2}-1575\,a\right)x+6483\,n\right)\sqrt{-a^{2}cx^{2}+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{5}n^{8}-84\,ac^{5}n^{6}+1974\,ac^{5}n^{4}+\left(a^{9}c^{5}n^{8}-84\,a^{9}c^{5}n^{6}+1974\,a^{9}c^{5}n^{4}-12916\,a^{9}c^{5}n^{2}+11025\,a^{9}c^{5}\right)x^{8}-12916\,ac^{5}n^{2}-4\,\left(a^{7}c^{5}n^{8}-84\,a^{7}c^{5}n^{6}+1974\,a^{7}c^{5}n^{4}-12916\,a^{7}c^{5}n^{2}+11025\,a^{7}c^{5}\right)x^{6}+11025\,ac^{5}+6\,\left(a^{5}c^{5}n^{8}-84\,a^{5}c^{5}n^{6}+1974\,a^{5}c^{5}n^{4}-12916\,a^{5}c^{5}n^{2}+11025\,a^{5}c^{5}\right)x^{4}-4\,\left(a^{3}c^{5}n^{8}-84\,a^{3}c^{5}n^{6}+1974\,a^{3}c^{5}n^{4}-12916\,a^{3}c^{5}n^{2}+11025\,a^{3}c^{5}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="fricas")
[Out] -(5040*a^7*x^7 - 5040*a^6*n*x^6 - n^7 + 2520*(a^5*n^2 - 7*a^5)*x^5 + 77*n^5
 - 840*(a^4*n^3 - 19*a^4*n)*x^4 + 210*(a^3*n^4 - 34*a^3*n^2 + 105*a^3)*x^3
- 1519*n^3 - 42*(a^2*n^5 - 50*a^2*n^3 + 409*a^2*n)*x^2 + 7*(a*n^6 - 65*a*n^
4 + 919*a*n^2 - 1575*a)*x + 6483*n)*sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x -
1))^(1/2*n)/(a*c^5*n^8 - 84*a*c^5*n^6 + 1974*a*c^5*n^4 + (a^9*c^5*n^8 - 84*
a^9*c^5*n^6 + 1974*a^9*c^5*n^4 - 12916*a^9*c^5*n^2 + 11025*a^9*c^5)*x^8 - 1
2916*a*c^5*n^2 - 4*(a^7*c^5*n^8 - 84*a^7*c^5*n^6 + 1974*a^7*c^5*n^4 - 12916
*a^7*c^5*n^2 + 11025*a^7*c^5)*x^6 + 11025*a*c^5 + 6*(a^5*c^5*n^8 - 84*a^5*c
^5*n^6 + 1974*a^5*c^5*n^4 - 12916*a^5*c^5*n^2 + 11025*a^5*c^5)*x^4 - 4*(a^3
*c^5*n^8 - 84*a^3*c^5*n^6 + 1974*a^3*c^5*n^4 - 12916*a^3*c^5*n^2 + 11025*a^
3*c^5)*x^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(9/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(9/2), x)
```

Mupad [B]
time = 1.98, size = 441, normalized size = 1.85

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\,\left(\frac{5040\,x^7}{c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}+\frac{-n^7+77\,n^5-1519\,n^3+6483\,n}{a^7\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}-\frac{5040\,n\,x^6}{a\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}+\frac{x^5\,\left(2520\,n^2-17640\right)}{a^2\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}+\frac{x^3\,\left(210\,n^4-7140\,n^2+22050\right)}{a^4\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}+\frac{7\,x\,\left(n^6-65\,n^4+919\,n^2-1575\right)}{a^6\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}-\frac{840\,n\,x^4\,\left(n^2-19\right)}{a^3\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}-\frac{42\,n\,x^2\,\left(n^4-50\,n^2+409\right)}{a^5\,c^4\,\left(n^8-84\,n^6+1974\,n^4-12916\,n^2+11025\right)}\right)}{\left(\frac{a\,x-1}{a\,x}\right)^{n/2}\,\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^6}-x^6\,\sqrt{c-a^2\,c\,x^2}+\frac{3\,x^4\,\sqrt{c-a^2\,c\,x^2}}{a^2}-\frac{3\,x^2\,\sqrt{c-a^2\,c\,x^2}}{a^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(9/2),x)
[Out] -((((a*x + 1)/(a*x))^(n/2)*((5040*x^7)/(c^4*(1974*n^4 - 12916*n^2 - 84*n^6 +
 n^8 + 11025)) + (6483*n - 1519*n^3 + 77*n^5 - n^7)/(a^7*c^4*(1974*n^4 - 12
916*n^2 - 84*n^6 + n^8 + 11025)) - (5040*n*x^6)/(a*c^4*(1974*n^4 - 12916*n^
2 - 84*n^6 + n^8 + 11025)) + (x^5*(2520*n^2 - 17640))/(a^2*c^4*(1974*n^4 -
12916*n^2 - 84*n^6 + n^8 + 11025)) + (x^3*(210*n^4 - 7140*n^2 + 22050))/(a^
4*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) + (7*x*(919*n^2 - 65*n
^4 + n^6 - 1575))/(a^6*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 11025)) -
 (840*n*x^4*(n^2 - 19))/(a^3*c^4*(1974*n^4 - 12916*n^2 - 84*n^6 + n^8 + 110
25)) - (42*n*x^2*(n^4 - 50*n^2 + 409))/(a^5*c^4*(1974*n^4 - 12916*n^2 - 84*
n^6 + n^8 + 11025))))/(((a*x - 1)/(a*x))^(n/2)*((c - a^2*c*x^2)^(1/2)/a^6 -
 x^6*(c - a^2*c*x^2)^(1/2) + (3*x^4*(c - a^2*c*x^2)^(1/2))/a^2 - (3*x^2*(c
- a^2*c*x^2)^(1/2))/a^4))
```

3.751 $\int \frac{e^{n\coth^{-1}(ax)}x^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=359

$$-\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)\left(c-a^2cx^2\right)^{3/2}}+\frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)}{a(1-n)(1+n)\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -(2+n)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^
3/a/(1+n)/(-a^2*c*x^2+c)^(3/2)+(n^2+2*n+2)*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1
/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/a/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)+(1-1/a
^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^4/(-a^2*c*x^2
+c)^(3/2)-2*n*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2
*n)*x^3*hypergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/a/(1-n)/(-a^
2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.22, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6327, 6329, 105, 160, 12, 133}

$$-\frac{2nx^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{a(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{(n^2+2n+2)\,x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{a(1-n)(n+1)\left(c-a^2cx^2\right)^{3/2}}+\frac{x^4\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{\left(c-a^2cx^2\right)^{3/2}}-\frac{(n+2)x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{a(n+1)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(((2 + n)*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3)/(a*(1 + n)*(c - a^2*c*x^2)^(3/2))) + ((2 + 2*n + n^2)*(1
 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*
x^3)/(a*(1 - n)*(1 + n)*(c - a^2*c*x^2)^(3/2)) + ((1 - 1/(a^2*x^2))^(3/2)*(
1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^4)/(c - a^2*c*x^2)^(
3/2) - (2*n*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))
^((-1 + n)/2)*x^3*Hypergeometric2F1[1, (-1 + n)/2, (1 + n)/2, (a + x^(-1))/
(a - x^(-1))])/(a*(1 - n)*(c - a^2*c*x^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{(c-a^2cx^2)^{3/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{(c-a^2cx^2)^{3/2}}$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^4}{(c-a^2cx^2)^{3/2}} + \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\right.}{}\right.}{(c-a^2}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)(c-a^2cx^2)^{3/2}} + \frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(}}{(c-a^2}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)(c-a^2cx^2)^{3/2}} + \frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right.}{a(1-n)(}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)(c-a^2cx^2)^{3/2}} + \frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right.}{a(1-n)(}$$

$$= -\frac{(2+n)\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{a(1+n)(c-a^2cx^2)^{3/2}} + \frac{(2+2n+n^2)\left(1-\frac{1}{a^2x^2}\right.}{a(1-n)(}$$

Mathematica [A]
time = 0.41, size = 133, normalized size = 0.37

$$\frac{\frac{ce^{n\coth^{-1}(ax)}(-1+anx)}{-1+n^2} - \frac{c(-1+a^2x^2)\left(e^{n\coth^{-1}(ax)}(1+n)+\frac{2e^{(1+n)\coth^{-1}(ax)}n\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}\right)}{1+n}}{a^4c^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] ((c*E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(-1 + n^2) - (c*(-1 + a^2*x^2)*(E^(n*A
rcCoth[a*x])*(1 + n) + (2*E^((1 + n)*ArcCoth[a*x])*n*Hypergeometric2F1[1, (
1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(a*Sqrt[1 - 1/(a^2*x^2)]*x)))/(1
+ n))/(a^4*c^2*Sqrt[c - a^2*c*x^2])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{e^{n \,\mathrm{arccoth}(ax)} x^{3}}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
[Out] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} e^{n \operatorname{acoth}{\left(a x \right)}}}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \,\mathrm{e}^{n \,\mathrm{acoth}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

3.752 $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a^3/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)-2*(1-1/a/
x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x*hypergeom([1, 1/2-1/2*n],[3/2-1/2*n
],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a^2/c/(1-n)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6324, 6327, 6330, 133}

$$-\frac{2x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a^2c(1-n)\sqrt{c-a^2cx^2}}-\frac{(n-ax)e^{n\coth^{-1}(ax)}}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a^3*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])) - (
2*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2
)*x*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(a + x^(-1))])/
(a^2*c*(1 - n)*Sqrt[c - a^2*c*x^2])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6324

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
```

```
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ
[n])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{n\coth^{-1}(ax)}x^2}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx}{a^2c}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}\,dx}{a^2c\sqrt{c-a^2cx^2}}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}+\frac{\left(\sqrt{1-\frac{1}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x}\,dx,x,\right.}{a^2c\sqrt{c-a^2cx^2}}\\
&= -\frac{e^{n\coth^{-1}(ax)}(n-ax)}{a^3c\left(1-n^2\right)\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x\,{}_2F_1\left(1,\frac{1-n}{2}\right.}{a^2c(1-n)\sqrt{c-a^2cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 127, normalized size = 0.77

$$
-\frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-n+ax)+2e^{\coth^{-1}(ax)}(-1+n)\left(-1+a^2x^2\right)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{a^4c(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-n + a*x) + 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2
*ArcCoth[a*x])]))/(a^4*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c -
a^2*c*x^2]))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n \,\mathrm{arccoth}(a x)} x^{2}}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} e^{n \operatorname{acoth}{\left(a x \right)}}}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2), x)
```

3.753 $\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{e^{n\coth^{-1}(ax)}(1-anx)}{a^2c\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

```
[Out] exp(n*arccoth(a*x))*(-a*n*x+1)/a^2/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {6321}

$$\frac{(1-anx)e^{n\coth^{-1}(ax)}}{a^2c\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(1 - a*n*x))/(a^2*c*(1 - n^2)*Sqrt[c - a^2*c*x^2])
```

Rule 6321

```
Int[(E^(ArcCoth[(a_.)*(x_)]*(n_))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(E^(n*ArcCoth[a*x])/(a^2*c*(n^2 - 1)*Sqrt[c + d*
x^2])), x] /; FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{3/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(1-anx)}{a^2c\,(1-n^2)\,\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.13, size = 43, normalized size = 0.93

$$\frac{e^{n\coth^{-1}(ax)}(-1+anx)}{a^2c\,(-1+n^2)\,\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-1 + a*n*x))/(a^2*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 49, normalized size = 1.07

method	result	size
gosper	$-\frac{(ax+1)(ax-1)(xan-1)\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a^2(n^2-1)(-a^2cx^2+c)^{\frac{3}{2}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(a*x+1)*(a*x-1)*(a*n*x-1)*exp(n*arccoth(a*x))/a^2/(n^2-1)/(-a^2*c*x^2+c)^(
3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 0.34, size = 82, normalized size = 1.78

$$\frac{\sqrt{-a^2cx^2+c}\,(anx-1)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{a^2c^2n^2-a^2c^2-(a^4c^2n^2-a^4c^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] sqrt(-a^2*c*x^2 + c)*(a*n*x - 1)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^2*n^2
 - a^2*c^2 - (a^4*c^2*n^2 - a^4*c^2)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{xe^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B]
time = 1.43, size = 81, normalized size = 1.76

$$-\frac{\left(\frac{1}{a^2\,c\,\left(n^2-1\right)}-\frac{n\,x}{a\,c\,\left(n^2-1\right)}\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x}\right)}^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(3/2),x)
[Out] -((1/(a^2*c*(n^2 - 1)) - (n*x)/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((
c - a^2*c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.754 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx$

Optimal. Leaf size=46

$$-\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-a*x+n)/a/c/(-n^2+1)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6319}

$$-\frac{(n-ax)e^{n \coth^{-1}(ax)}}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(1 - n^2)*Sqrt[c - a^2*c*x^2]))
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} \, dx = -\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac(1-n^2)\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.13, size = 43, normalized size = 0.93

$$\frac{e^{n \coth^{-1}(ax)}(n-ax)}{ac(-1+n^2)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(n - a*x))/(a*c*(-1 + n^2)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 49, normalized size = 1.07

method	result	size
gosper	$\frac{(ax+1)(ax-1)(ax-n)e^{n \operatorname{arccoth}(ax)}}{(n^2-1)a(-a^2c\,x^2+c)^{\frac{3}{2}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] (a*x+1)*(a*x-1)*(a*x-n)*exp(n*arccoth(a*x))/(n^2-1)/a/(-a^2*c*x^2+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Fricas [A]
time = 0.35, size = 80, normalized size = 1.74

$$-\frac{\sqrt{-a^2cx^2+c}\,(ax-n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}n}}{ac^2n^2-ac^2-(a^3c^2n^2-a^3c^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] -sqrt(-a^2*c*x^2 + c)*(a*x - n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^2*n^2 -
a*c^2 - (a^3*c^2*n^2 - a^3*c^2)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n\operatorname{acoth}(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(3/2), x)
```

Mupad [B]
time = 0.00, size = 78, normalized size = 1.70

$$-\frac{\left(\frac{x}{c\,\left(n^2-1\right)}-\frac{n}{a\,c\,\left(n^2-1\right)}\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^{n/2}}{\sqrt{c-a^2\,c\,x^2}\,{\left(\frac{a\,x-1}{a\,x}\right)}^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(3/2),x)
[Out] -((x/(c*(n^2 - 1)) - n/(a*c*(n^2 - 1)))*((a*x + 1)/(a*x))^(n/2))/((c - a^2*
c*x^2)^(1/2)*((a*x - 1)/(a*x))^(n/2))
```

3.755 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1-n^2)\left(c-a^2cx^2\right)^{3/2}}-\frac{2^{\frac{1+n}{2}}\cdots}{\cdots}$$

```
[Out] -a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^3/
(1+n)/(-a^2*c*x^2+c)^(3/2)+a^3*(1-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*(1
+1/a/x)^(-1/2+1/2*n)*x^3/(-n^2+1)/(-a^2*c*x^2+c)^(3/2)-2^(1/2+1/2*n)*a^3*(1
-1/a^2/x^2)^(3/2)*(1-1/a/x)^(1/2-1/2*n)*x^3*hypergeom([1/2-1/2*n, 1/2-1/2*n
],[3/2-1/2*n],1/2*(a-1/x)/a)/(1-n)/(-a^2*c*x^2+c)^(3/2)
```

Rubi [A]
time = 0.24, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {6327, 6330, 91, 80, 71}

$$-\frac{a^3 2^{\frac{n+1}{2}} x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{(1-n)\left(c-a^2cx^2\right)^{3/2}}+\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(1-n^2)\left(c-a^2cx^2\right)^{3/2}}-\frac{a^3x^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n+1)\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] -((a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-
1 + n)/2)*x^3)/((1 + n)*(c - a^2*c*x^2)^(3/2))) + (a^3*(1 - 1/(a^2*x^2))^(3
/2)*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^3)/((1 - n^2)*(c
 - a^2*c*x^2)^(3/2)) - (2^((1 + n)/2)*a^3*(1 - 1/(a^2*x^2))^(3/2)*(1 - 1/(a
*x))^((1 - n)/2)*x^3*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (a
- x^(-1))/(2*a)])/((1 - n)*(c - a^2*c*x^2)^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
```

```
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{3/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^4}\,dx}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}\left(\int x^2\left(1-\frac{x}{a}\right)^{-\frac{3}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{3}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{3/2}}$$
$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{\left(a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}x^3\right)\text{Subst}}{}$$
$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-}{2}}}{(1-n^2)\left(c-a\right.}$$
$$= -\frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}x^3}{(1+n)\left(c-a^2cx^2\right)^{3/2}} + \frac{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(1-\frac{1}{ax}\right)^{\frac{1-}{2}}}{(1-n^2)\left(c-a\right.}$$

Mathematica [A]
time = 0.59, size = 127, normalized size = 0.46

$$\frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1+anx)-2e^{\coth^{-1}(ax)}(-1+n)\left(-1+a^2x^2\right)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-e^{2\coth^{-1}(ax)}\right)\right)}{ac(-1+n)(1+n)\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(3/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1 + a*n*x) - 2*E^ArcCoth[a
*x]*(-1 + n)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -E^(
2*ArcCoth[a*x])]))/(a*c*(-1 + n)*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x*Sqrt[c - a
^2*c*x^2])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\text{arccoth}(ax)}}{x\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^5 -
2*a^2*c^2*x^3 + c^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{x\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(3/2)), x)
```

3.756 $\int \frac{e^{n \coth^{-1}(ax)} x^4}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=463

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x}{(1+n)(3+n)\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3+n
)/(-a^2*c*x^2+c)^(5/2)-(6+n)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+(n^2+6*n+15)*(1-1/
a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2
+n+3)/(-a^2*c*x^2+c)^(5/2)-(-n^3-2*n^2+7*n+18)*(1-1/a^2/x^2)^(5/2)*(1-1/a/x
)^(3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2
)-2*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5*hy
pergeom([1, -1/2+1/2*n],[1/2+1/2*n],(a+1/x)/(a-1/x))/(1-n)/(-a^2*c*x^2+c)^(
5/2)
```

Rubi [A]
time = 0.36, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6327, 6330, 136, 160, 12, 133}

$$-\frac{2x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{n-1}{2};\frac{n+1}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{(1-n)\left(c-a^2cx^2\right)^{5/2}}+\frac{\left(n^2+6n+15\right)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(1-n)(n+1)(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{\left(-n^3-2n^2+7n+18\right)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{\left(n^4-10n^2+9\right)\left(c-a^2cx^2\right)^{5/2}}-\frac{x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{(n+6)x^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n+1)(n+3)\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(((1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3 +
n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - ((6 + n)*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) + ((15 + 6*n + n^2)*(1 - 1/(a^2*x^2))^(5/2)*(1
- 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((1 - n)*(1 + n)*(3
+ n)*(c - a^2*c*x^2)^(5/2)) - ((18 + 7*n - 2*n^2 - n^3)*(1 - 1/(a^2*x^2))^(
5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2
 + n^4)*(c - a^2*c*x^2)^(5/2)) - (2*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^(
(1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5*Hypergeometric2F1[1, (-1 + n)/2,
(1 + n)/2, (a + x^(-1))/(a - x^(-1))])/((1 - n)*(c - a^2*c*x^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 136

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimp
lerQ[p, 1]))
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^4}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} + \frac{\left(a\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\Big(\int \cdots}{(3+\cdots}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\cdots}}{(1+n)(3+n)\,(c\cdots}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\cdots}}{(1+n)(3+n)\,(c\cdots}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\cdots}}{(1+n)(3+n)\,(c\cdots}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\cdots}}{(1+n)(3+n)\,(c\cdots}$$

$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{(6+n)\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\cdots}}{(1+n)(3+n)\,(c\cdots}$$

Mathematica [A]

time = 1.43, size = 201, normalized size = 0.43

$$\frac{(-1+a^2x^2)\left(\frac{8e^{n\coth^{-1}(ax)}(n-ax)}{-1+n^2}+\frac{e^{n\coth^{-1}(ax)}\left(26n-2n^3-27ax+3an^2x+2n(-1+n^2)\cosh(2\coth^{-1}(ax))-3a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh(3\coth^{-1}(ax))\right)}{9-10n^2+n^4}-\frac{8ae^{(1+n)\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)}{1+n}\right)}{4a^5c\,(c-a^2cx^2)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^4)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] ((-1 + a^2*x^2)*((8*E^(n*ArcCoth[a*x])*(n - a*x))/(-1 + n^2) + (E^(n*ArcCot
h[a*x])*(26*n - 2*n^3 - 27*a*x + 3*a*n^2*x + 2*n*(-1 + n^2)*Cosh[2*ArcCoth[
```

```
a*x]] - 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))/(9 -
10*n^2 + n^4) - (8*a*E^((1 + n)*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*x*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])])/(1 + n)))/(4*a^5
*c*(c - a^2*c*x^2)^(3/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{e^{n \,\mathrm{arccoth}(a x)} x^{4}}{\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x
^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} e^{n \operatorname{acoth}{\left(a x \right)}}}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**4/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(x**4*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^4/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
[Out] int((x^4*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2), x)
```

3.757 $\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}+\frac{6a}{}$$

```
[Out] -a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(3
+n)/(-a^2*c*x^2+c)^(5/2)-3*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a*(1-1/a^2/x^2)^
(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(-a
^2*c*x^2+c)^(5/2)-6*a*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+1/a/x)^(
-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]

time = 0.24, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6327, 6330, 47, 37}

$$-\frac{3ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}+\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)(1-n^2)\left(c-a^2cx^2\right)^{5/2}}-\frac{6ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{(n^4-10n^2+9)\left(c-a^2cx^2\right)^{5/2}}-\frac{ax^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{\frac{n-3}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-3
+ n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a*(1 - 1/(a^2*x^2))^(5/2
)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n + n^
2)*(c - a^2*c*x^2)^(5/2)) + (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((1
- n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x^2)^
(5/2)) - (6*a*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1 + 1/(a*x
))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
```

```
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^3}{(c-a^2cx^2)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^2}\,dx}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\operatorname{Subst}\left(\int \left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx, x, \frac{1}{x}\right)}{(c-a^2cx^2)^{5/2}}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{\left(3\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\operatorname{Subst}\left(\int\right.}{(3+n}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1}}{(3+4n+n^2)\,(c-}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1}}{(3+4n+n^2)\,(c-}$$

$$= -\frac{a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\,(c-a^2cx^2)^{5/2}} - \frac{3a\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1}}{(3+4n+n^2)\,(c-}$$

Mathematica [A]

time = 0.49, size = 110, normalized size = 0.33

$$-\frac{e^{n\coth^{-1}(ax)}\left(3(10-2n^2-9anx+an^3x)-6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+an(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)\right)}{4a^4c^2\left(9-10n^2+n^4\right)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^3)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/4*(E^(n*ArcCoth[a*x])*(3*(10 - 2*n^2 - 9*a*n*x + a*n^3*x) - 6*(-1 + n^2)
*Cosh[2*ArcCoth[a*x]] + a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCo
th[a*x]]))/(a^4*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 93, normalized size = 0.28

method	result	size
gosper	$-\frac{(ax+1)(ax-1)\left(a^{3} n^{3} x^{3}-7 a^{3} x^{3} n-3 a^{2} n^{2} x^{2}+9 a^{2} x^{2}+6 x a n-6\right) \mathrm{e}^{n \,\mathrm{arccoth}(a x)}}{a^{4}\left(n^{4}-10 n^{2}+9\right)\left(-a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x+1)*(a*x-1)*(a^3*n^3*x^3-7*a^3*n*x^3-3*a^2*n^2*x^2+9*a^2*x^2+6*a*n*x-6
)*exp(n*arccoth(a*x))/a^4/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

```
[Out] integrate(x^3*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 0.35, size = 175, normalized size = 0.53

$$\frac{\sqrt{-a^2cx^2+c}\,\left((a^3n^3-7\,a^3n)x^3+6\,anx-3\,(a^2n^2-3\,a^2)x^2-6\right)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3+(a^8c^3n^4-10\,a^8c^3n^2+9\,a^8c^3)x^4-2\,(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas
")
```

```
[Out] sqrt(-a^2*c*x^2 + c)*((a^3*n^3 - 7*a^3*n)*x^3 + 6*a*n*x - 3*(a^2*n^2 - 3*a^
2)*x^2 - 6)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^3*n^4 - 10*a^4*c^3*n^2 + 9
*a^4*c^3 + (a^8*c^3*n^4 - 10*a^8*c^3*n^2 + 9*a^8*c^3)*x^4 - 2*(a^6*c^3*n^4
- 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 e^{n \operatorname{acoth}(ax)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x**3/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 1.56, size = 175, normalized size = 0.53

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6}{a^6\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{6\,n\,x}{a^5\,c^2\,\left(n^4-10\,n^2+9\right)}+\frac{x^2\,\left(3\,n^2-9\right)}{a^4\,c^2\,\left(n^4-10\,n^2+9\right)}-\frac{n\,x^3\,\left(n^2-7\right)}{a^3\,c^2\,\left(n^4-10\,n^2+9\right)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*(6/(a^6*c^2*(n^4 - 10*n^2 + 9)) - (6*n*x)/(a^5*c^
2*(n^4 - 10*n^2 + 9)) + (x^2*(3*n^2 - 9))/(a^4*c^2*(n^4 - 10*n^2 + 9)) - (n
*x^3*(n^2 - 7))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2
- x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.758 $\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c\,(9-n^2)\,(c-a^2cx^2)^{3/2}}+\frac{e^{n\coth^{-1}(ax)}\,(3-n^2)\,(n-ax)}{a^3c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a^3/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+exp(n*a
rccoth(a*x))*(-n^2+3)*(-a*x+n)/a^3/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {6324, 6319}

$$\frac{(3-n^2)\,(n-ax)e^{n\coth^{-1}(ax)}}{a^3c^2\,(n^4-10n^2+9)\,\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{a^3c\,(9-n^2)\,(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a^3*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)))
 + (E^(n*ArcCoth[a*x])*(3 - n^2)*(n - a*x))/(a^3*c^2*(9 - 10*n^2 + n^4)*Sqr
t[c - a^2*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6324

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)^2*((c_) + (d_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(n + 2*(p + 1)*a*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^
3*c*(n^2 - 4*(p + 1)^2))), x] - Dist[(n^2 + 2*(p + 1))/(a^2*c*(n^2 - 4*(p +
 1)^2)), Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
 d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[n^2
 + 2*(p + 1), 0] && NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ
[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x^2}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{(3-n^2)\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{a^2c\,(9-n^2)}$$
$$= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{a^3c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{e^{n\coth^{-1}(ax)}(3-n^2)\,(n-ax)}{a^3c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.51, size = 109, normalized size = 1.07

$$\frac{e^{n\coth^{-1}(ax)}\left(10n-2n^3-9ax+an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)\right)}{4a^3c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x^2)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(10*n - 2*n^3 - 9*a*x + a*n^2*x - 2*n*(-1 + n^2)*Cosh[2
*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]
]))/(4*a^3*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.09, size = 96, normalized size = 0.94

method	result	size
gosper	$\frac{(ax+1)(ax-1)(a^3n^2x^3-a^2n^3x^2-3a^3x^3+3a^2n\,x^2+2a\,n^2x-2n)\,e^{n\,\mathrm{arccoth}(ax)}}{(n^4-10n^2+9)a^3(-a^2c\,x^2+c)^{\frac{5}{2}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+1)*(a*x-1)*(a^3*n^2*x^3-a^2*n^3*x^2-3*a^3*x^3+3*a^2*n*x^2+2*a*n^2*x-2*
n)*exp(n*arccoth(a*x))/(n^4-10*n^2+9)/a^3/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima
")
```

[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)

Fricas [A]
time = 0.35, size = 180, normalized size = 1.76

$$-\frac{\sqrt{-a^2cx^2+c}\,(2\,an^2x+(a^3n^2-3\,a^3)x^3-(a^2n^3-3\,a^2n)x^2-2\,n)\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^3c^3n^4-10\,a^3c^3n^2+9\,a^3c^3+(a^7c^3n^4-10\,a^7c^3n^2+9\,a^7c^3)x^4-2\,(a^5c^3n^4-10\,a^5c^3n^2+9\,a^5c^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")

[Out] -sqrt(-a^2*c*x^2 + c)*(2*a*n^2*x + (a^3*n^2 - 3*a^3)*x^3 - (a^2*n^3 - 3*a^2*n)*x^2 - 2*n)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^3*c^3*n^4 - 10*a^3*c^3*n^2 + 9*a^3*c^3 + (a^7*c^3*n^4 - 10*a^7*c^3*n^2 + 9*a^7*c^3)*x^4 - 2*(a^5*c^3*n^4 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*acoth(a*x))*x**2/(-a**2*c*x**2+c)**(5/2),x)

[Out] Integral(x**2*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*arccoth(a*x))*x^2/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)

Mupad [B]
time = 1.56, size = 175, normalized size = 1.72

$$\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{2\,n}{a^5\,c^2\,(n^4-10\,n^2+9)}-\frac{x^3\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{2\,n^2\,x}{a^4\,c^2\,(n^4-10\,n^2+9)}+\frac{n\,x^2\,(n^2-3)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] (((a*x + 1)/(a*x))^(n/2)*((2*n)/(a^5*c^2*(n^4 - 10*n^2 + 9)) - (x^3*(n^2 -
3))/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n^2*x)/(a^4*c^2*(n^4 - 10*n^2 + 9)) +
 (n*x^2*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a
^2 - x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.759 $\int \frac{e^{n \coth^{-1}(ax)} x}{(c-a^2cx^2)^{5/2}} dx$

Optimal. Leaf size=97

$$\frac{e^{n \coth^{-1}(ax)}(3-anx)}{a^2c(9-n^2)(c-a^2cx^2)^{3/2}}+\frac{2e^{n \coth^{-1}(ax)}n(n-ax)}{a^2c^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

```
[Out] exp(n*arccoth(a*x))*(-a*n*x+3)/a^2/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)+2*exp(n*
arccoth(a*x))*n*(-a*x+n)/a^2/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {6322, 6319}

$$\frac{2n(n-ax)e^{n \coth^{-1}(ax)}}{a^2c^2(n^4-10n^2+9)\sqrt{c-a^2cx^2}}+\frac{(3-anx)e^{n \coth^{-1}(ax)}}{a^2c(9-n^2)(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(3 - a*n*x))/(a^2*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2)) +
(2*E^(n*ArcCoth[a*x])*n*(n - a*x))/(a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2
*c*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6322

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(x_)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(2*(p + 1) + a*n*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a^2*
c*(n^2 - 4*(p + 1)^2))), x] - Dist[n*((2*p + 3)/(a*c*(n^2 - 4*(p + 1)^2))),
 Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n},
x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LeQ[p, -1] && NeQ[p, -3/2] &&
 NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}x}{(c-a^2cx^2)^{5/2}}\,dx = \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} - \frac{(2n)\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{ac\,(9-n^2)}$$
$$= \frac{e^{n\coth^{-1}(ax)}(3-anx)}{a^2c\,(9-n^2)\,(c-a^2cx^2)^{3/2}} + \frac{2e^{n\coth^{-1}(ax)}n(n-ax)}{a^2c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.45, size = 108, normalized size = 1.11

$$\frac{e^{n\coth^{-1}(ax)}\left(6+2n^2-9anx+an^3x+6(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)-an(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)\right)}{4a^2c^2\,(9-10n^2+n^4)\,\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(n*ArcCoth[a*x])*x)/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(6 + 2*n^2 - 9*a*n*x + a*n^3*x + 6*(-1 + n^2)*Cosh[2*Ar
cCoth[a*x]] - a*n*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[a*x]]))
/(4*a^2*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.89

method	result	size
gosper	$-\frac{(ax+1)(ax-1)(2a^3x^3n-2a^2n^2x^2+a\,n^3x-3xan-n^2+3)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a^2(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x+1)*(a*x-1)*(2*a^3*n*x^3-2*a^2*n^2*x^2+a*n^3*x-3*a*n*x-n^2+3)*exp(n*ar
ccoth(a*x))/a^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 0.35, size = 171, normalized size = 1.76

$$\frac{(2\,a^3nx^3-2\,a^2n^2x^2-n^2+(an^3-3\,an)x+3)\sqrt{-a^2cx^2+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{a^2c^3n^4-10\,a^2c^3n^2+9\,a^2c^3+(a^6c^3n^4-10\,a^6c^3n^2+9\,a^6c^3)x^4-2\,(a^4c^3n^4-10\,a^4c^3n^2+9\,a^4c^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] (2*a^3*n*x^3 - 2*a^2*n^2*x^2 - n^2 + (a*n^3 - 3*a*n)*x + 3)*sqrt(-a^2*c*x^2
 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c^3*n^4 - 10*a^2*c^3*n^2 + 9*a^2*c
^3 + (a^6*c^3*n^4 - 10*a^6*c^3*n^2 + 9*a^6*c^3)*x^4 - 2*(a^4*c^3*n^4 - 10*a
^4*c^3*n^2 + 9*a^4*c^3)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{xe^{n\,\mathrm{acoth}\,(ax)}}{(-c\,(ax-1)\,(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(x*exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B]
time = 1.55, size = 176, normalized size = 1.81

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{n^2-3}{a^4\,c^2\,(n^4-10\,n^2+9)}+\frac{2\,n^2\,x^2}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{2\,n\,x^3}{a\,c^2\,(n^4-10\,n^2+9)}-\frac{n\,x\,(n^2-3)}{a^3\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(n*acoth(a*x)))/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] -(((a*x + 1)/(a*x))^(n/2)*((n^2 - 3)/(a^4*c^2*(n^4 - 10*n^2 + 9)) + (2*n^2*
x^2)/(a^2*c^2*(n^4 - 10*n^2 + 9)) - (2*n*x^3)/(a*c^2*(n^4 - 10*n^2 + 9)) -
(n*x*(n^2 - 3))/(a^3*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2
- x^2*(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.760 $\int \frac{e^{n \coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac\,(9-n^2)\,(c-a^2cx^2)^{3/2}}-\frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2\,(1-n^2)\,(9-n^2)\sqrt{c-a^2cx^2}}$$

```
[Out] -exp(n*arccoth(a*x))*(-3*a*x+n)/a/c/(-n^2+9)/(-a^2*c*x^2+c)^(3/2)-6*exp(n*a
rccoth(a*x))*(-a*x+n)/a/c^2/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6320, 6319}

$$-\frac{6(n-ax)e^{n\coth^{-1}(ax)}}{ac^2\,(1-n^2)\,(9-n^2)\sqrt{c-a^2cx^2}}-\frac{(n-3ax)e^{n\coth^{-1}(ax)}}{ac\,(9-n^2)\,(c-a^2cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((E^(n*ArcCoth[a*x])*(n - 3*a*x))/(a*c*(9 - n^2)*(c - a^2*c*x^2)^(3/2))) -
 (6*E^(n*ArcCoth[a*x])*(n - a*x))/(a*c^2*(1 - n^2)*(9 - n^2)*Sqrt[c - a^2*c
*x^2])
```

Rule 6319

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n - a*x)*(E^(n*ArcCoth[a*x])/(a*c*(n^2 - 1)*Sqrt[c + d*x^2])), x] /;
FreeQ[{a, c, d, n}, x] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n]
```

Rule 6320

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(n + 2*a*(p + 1)*x)*(c + d*x^2)^(p + 1)*(E^(n*ArcCoth[a*x])/(a*c*(n^2
- 4*(p + 1)^2))), x] - Dist[2*(p + 1)*((2*p + 3)/(c*(n^2 - 4*(p + 1)^2))),
Int[(c + d*x^2)^(p + 1)*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n}, x
] && EqQ[a^2*c + d, 0] &&  !IntegerQ[n/2] && LtQ[p, -1] && NeQ[p, -3/2] &&
NeQ[n^2 - 4*(p + 1)^2, 0] && (IntegerQ[p] ||  !IntegerQ[n])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx = -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}} + \frac{6\int \frac{e^{n\coth^{-1}(ax)}}{(c-a^2cx^2)^{3/2}}\,dx}{c(9-n^2)}$$
$$= -\frac{e^{n\coth^{-1}(ax)}(n-3ax)}{ac(9-n^2)(c-a^2cx^2)^{3/2}} - \frac{6e^{n\coth^{-1}(ax)}(n-ax)}{ac^2(1-n^2)(9-n^2)\sqrt{c-a^2cx^2}}$$

Mathematica [A]
time = 0.15, size = 110, normalized size = 1.08

$$\frac{e^{n\coth^{-1}(ax)}\left(-26n+2n^3+27ax-3an^2x-2n(-1+n^2)\cosh\left(2\coth^{-1}(ax)\right)+3a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(3\coth^{-1}(ax)\right)\right)}{4ac^2(9-10n^2+n^4)\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-26*n + 2*n^3 + 27*a*x - 3*a*n^2*x - 2*n*(-1 + n^2)*Co
sh[2*ArcCoth[a*x]] + 3*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[3*ArcCoth[
a*x]]))/(4*a*c^2*(9 - 10*n^2 + n^4)*Sqrt[c - a^2*c*x^2])
```

Maple [A]
time = 0.10, size = 84, normalized size = 0.82

method	result	size
gosper	$\frac{(ax+1)(ax-1)(6a^3x^3-6a^2n\,x^2+3a\,n^2x-n^3-9ax+7n)\,\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{a(n^4-10n^2+9)(-a^2c\,x^2+c)^{\frac{5}{2}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+1)*(a*x-1)*(6*a^3*x^3-6*a^2*n*x^2+3*a*n^2*x-n^3-9*a*x+7*n)*exp(n*arcco
th(a*x))/a/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Fricas [A]
time = 0.35, size = 165, normalized size = 1.62

$$-\frac{\left(6\,a^{3}x^{3}-6\,a^{2}nx^{2}-n^{3}+3\,(an^{2}-3\,a)x+7\,n\right)\sqrt{-a^{2}cx^{2}+c}\,\left(\frac{ax+1}{ax-1}\right)^{\frac{1}{2}\,n}}{ac^{3}n^{4}-10\,ac^{3}n^{2}+(a^{5}c^{3}n^{4}-10\,a^{5}c^{3}n^{2}+9\,a^{5}c^{3})x^{4}+9\,ac^{3}-2\,(a^{3}c^{3}n^{4}-10\,a^{3}c^{3}n^{2}+9\,a^{3}c^{3})x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
[Out] -(6*a^3*x^3 - 6*a^2*n*x^2 - n^3 + 3*(a*n^2 - 3*a)*x + 7*n)*sqrt(-a^2*c*x^2
+ c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a*c^3*n^4 - 10*a*c^3*n^2 + (a^5*c^3*n^4
 - 10*a^5*c^3*n^2 + 9*a^5*c^3)*x^4 + 9*a*c^3 - 2*(a^3*c^3*n^4 - 10*a^3*c^3*
n^2 + 9*a^3*c^3)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{n\,\mathrm{acoth}\,(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(exp(n*acoth(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(-a^2*c*x^2 + c)^(5/2), x)
```

Mupad [B]
time = 0.00, size = 173, normalized size = 1.70

$$-\frac{\left(\frac{a\,x+1}{a\,x}\right)^{n/2}\left(\frac{6\,x^3}{c^2\,(n^4-10\,n^2+9)}+\frac{7\,n-n^3}{a^3\,c^2\,(n^4-10\,n^2+9)}+\frac{3\,x\,(n^2-3)}{a^2\,c^2\,(n^4-10\,n^2+9)}-\frac{6\,n\,x^2}{a\,c^2\,(n^4-10\,n^2+9)}\right)}{\left(\frac{\sqrt{c-a^2\,c\,x^2}}{a^2}-x^2\,\sqrt{c-a^2\,c\,x^2}\right)\,\left(\frac{a\,x-1}{a\,x}\right)^{n/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - a^2*c*x^2)^(5/2),x)
[Out] -((((a*x + 1)/(a*x))^(n/2)*((6*x^3)/(c^2*(n^4 - 10*n^2 + 9)) + (7*n - n^3)/(
a^3*c^2*(n^4 - 10*n^2 + 9)) + (3*x*(n^2 - 3))/(a^2*c^2*(n^4 - 10*n^2 + 9))
- (6*n*x^2)/(a*c^2*(n^4 - 10*n^2 + 9))))/(((c - a^2*c*x^2)^(1/2)/a^2 - x^2*
(c - a^2*c*x^2)^(1/2))*((a*x - 1)/(a*x))^(n/2))
```

3.761 $\int \frac{e^{n\coth^{-1}(ax)}}{x(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=944

$$-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-1-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+4n+n^2)\left(c-a^2cx^2\right)^{5/2}}+$$

```
[Out] -a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/
(3+n)/(-a^2*c*x^2+c)^(5/2)-3*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)
*(1+1/a/x)^(-3/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+6*a^5*(1-1/a^2
/x^2)^(5/2)*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-3/2+1/2*n)*x^5/(-n^3-3*n^2+n+
3)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(3/2-1/2*n)*(1+
1/a/x)^(-3/2+1/2*n)*x^5/(n^4-10*n^2+9)/(-a^2*c*x^2+c)^(5/2)+4*a^5*(1-1/a^2/
x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(3+n)/(-a^2*c*
x^2+c)^(5/2)+8*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-1/2-1/2*n)*(1+1/a/x)^(-1
/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)-8*a^5*(1-1/a^2/x^2)^(5/2)*(1
-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*x^5/(-n^3-3*n^2+n+3)/(-a^2*c*x^2
+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(1/2+1
/2*n)*x^5/(3+n)/(-a^2*c*x^2+c)^(5/2)-6*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-
1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x^5/(n^2+4*n+3)/(-a^2*c*x^2+c)^(5/2)+4*a^5
*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2-1/2*n)*(1+1/a/x)^(3/2+1/2*n)*x^5/(3+n)
/(-a^2*c*x^2+c)^(5/2)-2^(5/2+1/2*n)*a^5*(1-1/a^2/x^2)^(5/2)*(1-1/a/x)^(-3/2
-1/2*n)*x^5*hypergeom([-3/2-1/2*n, -3/2-1/2*n],[-1/2-1/2*n],1/2*(a-1/x)/a)/
(3+n)/(-a^2*c*x^2+c)^(5/2)
```

Rubi [A]

time = 0.42, antiderivative size = 944, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6327, 6330, 128, 47, 37, 71}

$$-\frac{2^{\frac{n+5}{2}}a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\,{}_2F_1\left(\frac{1}{2}(-n-3),\frac{1}{2}(-n-3);\frac{1}{2}(-n-1);\frac{a-\frac{1}{x}}{2a}\right)\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}+\frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n+3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-3)}}{(n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}+\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n+1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-n-1)}}{(n^2+4n+3)\left(c-a^2cx^2\right)^{5/2}}+\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)(1-n^2)\left(c-a^2cx^2\right)^{5/2}}-\frac{8a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-1}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{(n+3)(1-n^2)\left(c-a^2cx^2\right)^{5/2}}-\frac{6a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{\frac{n-3}{2}}x^5\left(1-\frac{1}{ax}\right)^{\frac{3-n}{2}}}{(n^4-10n^2+9)\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] -((a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-
3 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2))) - (3*a^5*(1 - 1/(a^2*x^2))^
(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + 4*n
+ n^2)*(c - a^2*c*x^2)^(5/2)) + (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x)
)^((1 - n)/2)*(1 + 1/(a*x))^((-3 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c
*x^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((3 - n)/2)*(1
+ 1/(a*x))^((-3 + n)/2)*x^5)/((9 - 10*n^2 + n^4)*(c - a^2*c*x^2)^(5/2)) + (
4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((-1
```

```
 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) + (8*a^5*(1 - 1/(a^2*x^2))^(5
/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + 4*n +
n^2)*(c - a^2*c*x^2)^(5/2)) - (8*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^
((1 - n)/2)*(1 + 1/(a*x))^((-1 + n)/2)*x^5)/((3 + n)*(1 - n^2)*(c - a^2*c*x
^2)^(5/2)) - (6*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-3 - n)/2)*(1 +
 1/(a*x))^((1 + n)/2)*x^5)/((3 + n)*(c - a^2*c*x^2)^(5/2)) - (6*a^5*(1 - 1/
(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*x^5)/
((3 + 4*n + n^2)*(c - a^2*c*x^2)^(5/2)) + (4*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1
 - 1/(a*x))^((-3 - n)/2)*(1 + 1/(a*x))^((3 + n)/2)*x^5)/((3 + n)*(c - a^2*c
*x^2)^(5/2)) - (2^((5 + n)/2)*a^5*(1 - 1/(a^2*x^2))^(5/2)*(1 - 1/(a*x))^((-
3 - n)/2)*x^5*Hypergeometric2F1[(-3 - n)/2, (-3 - n)/2, (-1 - n)/2, (a - x^
(-1))/(2*a)])/((3 + n)*(c - a^2*c*x^2)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 128

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (IGtQ[m, 0] || (ILtQ[
m, 0] && ILtQ[n, 0]))
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6330

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_.), x
_Symbol] :> Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x
^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0
] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p +
n/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{x\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\int \frac{e^{n\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^6}\,dx}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int x^4\left(1-\frac{x}{a}\right)^{-\frac{5}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}}$$

$$= -\frac{\left(\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}-4a^4\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\right.}{}$$

$$= -\frac{\left(a^4\left(1-\frac{1}{a^2x^2}\right)^{5/2}x^5\right)\text{Subst}\left(\int\left(1-\frac{x}{a}\right)^{\frac{1}{2}(-5-n)}\left(1+\frac{x}{a}\right)^{-\frac{5}{2}+\frac{n}{2}}\,dx,x,\frac{1}{x}\right)}{\left(c-a^2cx^2\right)^{5/2}} - \frac{\left(a^4\left(1-\right.\right.}{}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} + \frac{4a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(}}{(3+n)\left(c-\right.}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(}}{\left(3+4n+n^2\right)\left(c\right.}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(}}{\left(3+4n+n^2\right)\left(c\right.}$$

$$= -\frac{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(-3-n)}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-3+n)}x^5}{(3+n)\left(c-a^2cx^2\right)^{5/2}} - \frac{3a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}(}}{\left(3+4n+n^2\right)\left(c\right.}$$

Mathematica [**A**]

time = 1.64, size = 220, normalized size = 0.23

$$\frac{e^{n\coth^{-1}(ax)}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x(42-2n^2-45anx+5an^3x)+6a(-1+n^2)\sqrt{1-\frac{1}{a^2x^2}}\,x\cosh\left(2\coth^{-1}(ax)\right)-n(-1+n^2)\left(-1+a^2x^2\right)\cosh\left(3\coth^{-1}(ax)\right)\right)-8e^{(1+n)\coth^{-1}(ax)}(9-9n-n^2+n^3)\left(-1+a^2x^2\right){}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-e^{2\coth^{-1}(ax)}\right)}{4ac^2(-1+n)(1+n)\left(-9+n^2\right)\sqrt{1-\frac{1}{a^2x^2}}\,x\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(x*(c - a^2*c*x^2)^(5/2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(a*Sqrt[1 - 1/(a^2*x^2)]*x*(42 - 2*n^2 - 45*a*n*x + 5*a
*n^3*x) + 6*a*(-1 + n^2)*Sqrt[1 - 1/(a^2*x^2)]*x*Cosh[2*ArcCoth[a*x]] - n*(
-1 + n^2)*(-1 + a^2*x^2)*Cosh[3*ArcCoth[a*x]]) - 8*E^((1 + n)*ArcCoth[a*x])
*(9 - 9*n - n^2 + n^3)*(-1 + a^2*x^2)*Hypergeometric2F1[1, (1 + n)/2, (3 +
n)/2, -E^(2*ArcCoth[a*x])])/(4*a*c^2*(-1 + n)*(1 + n)*(-9 + n^2)*Sqrt[1 - 1
/(a^2*x^2)]*x*Sqrt[c - a^2*c*x^2])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{arccoth}(ax)}}{x\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^6*c^3*x^7 -
 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 - c^3*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{x\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/x/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(exp(n*acoth(a*x))/(x*(-c*(a*x - 1)*(a*x + 1))**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/x/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/((-a^2*c*x^2 + c)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n \operatorname{acoth}(a\,x)}}{x\,{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(exp(n*acoth(a*x))/(x*(c - a^2*c*x^2)^(5/2)), x)
```

3.762 $\int e^{n\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=127

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x(c-a^2cx^2)^p\,{}_2F_1\left(-1-2p,\frac{1}{2}(n-2p);-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2*n-p)*(1-1/a/x)^(-1/2*n+p)*(1+1/a/x)^(1+1/2*n+p)*x*(-
a^2*c*x^2+c)^p*hypergeom([-1-2*p, 1/2*n-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-
1/a^2/x^2)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6331, 134}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}(c-a^2cx^2)^p\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{p-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}\,{}_2F_1\left(-2p-1,\frac{1}{2}(n-2p);-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^((n - 2*p)/2)*(1 - 1/(a*x))^(-1/2*n + p)*(1 +
1/(a*x))^(1 + n/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, (n -
 2*p)/2, -2*p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
```

```
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(\right.\right.\right.\right.$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}(n-2p)}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}+p}\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\right.}{1+2p}$$

Mathematica [A]
time = 0.13, size = 83, normalized size = 0.65

$$-\frac{e^{(-2+n)\coth^{-1}(ax)}\left(-1+e^{2\coth^{-1}(ax)}\right)\left(-1+a^2x^2\right)\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1,-\frac{n}{2}-p;2-\frac{n}{2}+p;e^{-2\coth^{-1}(ax)}\right)}{a(n-2(1+p))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] -((E^((-2 + n)*ArcCoth[a*x])*(-1 + E^(2*ArcCoth[a*x]))*(-1 + a^2*x^2)*(c -
a^2*c*x^2)^p*Hypergeometric2F1[1, -1/2*n - p, 2 - n/2 + p, E^(-2*ArcCoth[a*
x])])/(a*(n - 2*(1 + p))))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \mathrm{e}^{n\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
[Out] int(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-c(ax-1)(ax+1))^p e^{n \operatorname{acoth}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n \operatorname{acoth}(a x)} \left(c - a^2 c x^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - a^2*c*x^2)^p, x)
```

3.763 $\int e^{2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=51

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}$$

```
[Out] (1+1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {6327, 6331, 37}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}(c-a^2cx^2)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] ((1 + 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}(c-a^2cx^2)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$
$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}(c-a^2cx^2)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1+\frac{x}{a}\right)^{2p}\,dx\right.\right.$$
$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1+\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 0.71

$$\frac{e^{2p\coth^{-1}(ax)}(1+ax)(c-a^2cx^2)^p}{a+2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] (E^(2*p*ArcCoth[a*x])*(1 + a*x)*(c - a^2*c*x^2)^p)/(a + 2*a*p)
```

Maple [A]
time = 0.12, size = 38, normalized size = 0.75

method	result
gosper	$\frac{(ax+1)e^{2p\,\mathrm{arccoth}(ax)}\left(-a^2c\,x^2+c\right)^p}{a(1+2p)}$
risch	$\frac{(ax+1)(ax+1)^p(ax-1)^{-p}e^{\frac{p\left(-i\pi\mathrm{csgn}(i(ax-1)(ax+1))^3+i\pi\mathrm{csgn}(i(ax-1)(ax+1))^2\mathrm{csgn}(i(ax-1))+i\pi\mathrm{csgn}(i(ax-1)(ax+1))^2\mathrm{csgn}(i(ax+1))-i\pi\,\mathrm{c}}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x,method=_RETURNVERBOSE)
[Out] (a*x+1)/a/(1+2*p)*exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p
```

Maxima [A]
time = 0.27, size = 34, normalized size = 0.67

$$\frac{(a(-c)^p\,x+(-c)^p)(ax+1)^{2\,p}}{a(2\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x + (-c)^p)*(a*x + 1)^(2*p)/(a*(2*p + 1))
```

Fricas [A]
time = 0.37, size = 42, normalized size = 0.82

$$\frac{(ax+1)(-a^2cx^2+c)^p\left(\frac{ax+1}{ax-1}\right)^p}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] (a*x + 1)*(-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p/(2*a*p + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{ix}{\sqrt{c}} & \text{for}\ a=0 \wedge p=-\frac{1}{2} \\ c^p x e^{i\pi p} & \text{for}\ a=0 \\ \int \frac{e^{-\operatorname{acoth}(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx & \text{for}\ p=-\frac{1}{2} \\ \frac{ax(-a^2cx^2+c)^p e^{2p\operatorname{acoth}(ax)}}{2ap+a}+\frac{(-a^2cx^2+c)^p e^{2p\operatorname{acoth}(ax)}}{2ap+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Piecewise((-I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(I*pi*p), Eq(a
, 0)), (Integral(exp(-acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, -
1/2)), (a*x*(-a**2*c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a) + (-a**2*
c*x**2 + c)**p*exp(2*p*acoth(a*x))/(2*a*p + a), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [B]
time = 1.34, size = 59, normalized size = 1.16

$$\frac{{\left(c-a^2\,c\,x^2\right)}^p\,\left(a\,x+1\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^p}{a\,\left(2\,p+1\right)\,{\left(\frac{a\,x-1}{a\,x}\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

```
[Out] ((c - a^2*c*x^2)^p*(a*x + 1)*((a*x + 1)/(a*x))^p)/(a*(2*p + 1)*((a*x - 1)/(
a*x))^p)
```

3.764 $\int e^{-2p\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=52

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x(c-a^2cx^2)^p}{1+2p}$$

```
[Out] (1-1/a/x)^(1+2*p)*x*(-a^2*c*x^2+c)^p/(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {6327, 6331, 37}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}(c-a^2cx^2)^p}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((1 - 1/(a*x))^(1 + 2*p)*x*(c - a^2*c*x^2)^p)/((1 + 2*p)*(1 - 1/(a^2*x^2))^
p)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-2p\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$
$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{2p}\right.\right.$$
$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{1+2p}x\left(c-a^2cx^2\right)^p}{1+2p}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.69

$$\frac{e^{-2p\coth^{-1}(ax)}(-1+ax)\left(c-a^2cx^2\right)^p}{a+2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*p*ArcCoth[a*x]),x]
[Out] ((-1 + a*x)*(c - a^2*c*x^2)^p)/(E^(2*p*ArcCoth[a*x])*(a + 2*a*p))
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.77

method	result
gosper	$\frac{(ax-1)\left(-a^2c\,x^2+c\right)^p \mathrm{e}^{-2p\,\mathrm{arccoth}(ax)}}{a(1+2p)}$
risch	$(ax-1)(ax+1)^{-p}(ax-1)^p\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}(i(ax-1)(ax+1))^3+i\pi\mathrm{csgn}(i(ax-1)(ax+1))^2\mathrm{csgn}(i(ax-1))+i\pi\mathrm{csgn}(i(ax-1)(ax+1))^2\mathrm{csgn}(i(ax+1))-i\pi\,\mathrm{c}}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x,method=_RETURNVERBOSE)
[Out] (a*x-1)/a/(1+2*p)*(-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x))
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.69

$$\frac{\left(a(-c)^p\,x-(-c)^p\right)(ax-1)^{2\,p}}{a(2\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
```

```
[Out] (a*(-c)^p*x - (-c)^p)*(a*x - 1)^(2*p)/(a*(2*p + 1))
```

Fricas [A]
time = 0.34, size = 44, normalized size = 0.85

$$\frac{(ax-1)(-a^2cx^2+c)^p}{(2\,ap+a)\left(\frac{ax+1}{ax-1}\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
```

```
[Out] (a*x - 1)*(-a^2*c*x^2 + c)^p/((2*a*p + a)*((a*x + 1)/(a*x - 1))^p)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ix}{\sqrt{c}} & \text{for}\ a = 0 \wedge p = -\frac{1}{2} \\ c^p x e^{-i\pi p} & \text{for}\ a = 0 \\ \int \frac{e^{\operatorname{acoth}(ax)}}{\sqrt{-c\,(ax-1)\,(ax+1)}}\,dx & \text{for}\ p = -\frac{1}{2} \\ \frac{ax(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}} - \frac{(-a^2cx^2+c)^p}{2ape^{2p\operatorname{acoth}(ax)}+ae^{2p\operatorname{acoth}(ax)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p/exp(2*p*acoth(a*x)),x)
```

```
[Out] Piecewise((I*x/sqrt(c), Eq(a, 0) & Eq(p, -1/2)), (c**p*x*exp(-I*pi*p), Eq(a
, 0)), (Integral(exp(acoth(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x), Eq(p, -1
/2)), (a*x*(-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*aco
th(a*x))) - (-a**2*c*x**2 + c)**p/(2*a*p*exp(2*p*acoth(a*x)) + a*exp(2*p*ac
oth(a*x))), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [B]
time = 1.30, size = 59, normalized size = 1.13

$$\frac{{\left(c-a^2\,c\,x^2\right)}^p\,\left(a\,x-1\right)\,{\left(\frac{a\,x-1}{a\,x}\right)}^p}{a\,\left(2\,p+1\right)\,{\left(\frac{a\,x+1}{a\,x}\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*acoth(a*x))*(c - a^2*c*x^2)^p,x)
```

```
[Out] ((c - a^2*c*x^2)^p*(a*x - 1)*((a*x - 1)/(a*x))^p)/(a*(2*p + 1)*((a*x + 1)/(
a*x))^p)
```

3.765 $\int e^{4\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=63

$$\frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\ {}_2F_1\left(-2-p,-1+p;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

```
[Out] 2^(2+p)*c*(a*x+1)^(1-p)*(-a^2*c*x^2+c)^(-1+p)*hypergeom([-1+p, -2-p],[p],-1
/2*a*x+1/2)/a/(1-p)
```

Rubi [A]
time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6276, 692, 71}

$$\frac{c2^{p+2}(ax+1)^{1-p}\left(c-a^2cx^2\right)^{p-1}\ {}_2F_1\left(-p-2,p-1;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(2 + p)*c*(1 + a*x)^(1 - p)*(c - a^2*c*x^2)^(-1 + p)*Hypergeometric2F1[-
2 - p, -1 + p, p, (1 - a*x)/2])/(a*(1 - p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx\\
&= c^2\int (1+ax)^4\left(c-a^2cx^2\right)^{-2+p}\,dx\\
&= \left(c^2(1+ax)^{1-p}(c-acx)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\right)\int (1+ax)^{2+p}(c-acx)^{-2+p}\\
&= \frac{2^{2+p}c(1+ax)^{1-p}\left(c-a^2cx^2\right)^{-1+p}\,{}_2F_1\left(-2-p,-1+p;p;\frac{1}{2}(1-ax)\right)}{a(1-p)}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 1.14

$$
-\frac{2^{2+p}(1-ax)^{-1+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-2-p,-1+p;p;\frac{1}{2}(1-ax)\right)}{a(-1+p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] -((2^(2 + p)*(1 - a*x)^(-1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-2 - p,
 -1 + p, p, (1 - a*x)/2])/(a*(-1 + p)*(1 - a^2*x^2)^p))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$
\int \frac{(ax+1)^2\left(-a^2c\,x^2+c\right)^p}{(ax-1)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
[Out] int(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1)(ax+1))^p (ax+1)^2}{(ax-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p*(a*x + 1)**2/(a*x - 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)^2*(-a^2*c*x^2 + c)^p/(a*x - 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c-a^2\,c\,x^2)^p\,(a\,x+1)^2}{(a\,x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1)^2)/(a*x - 1)^2, x)
```

3.766 $\int e^{3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=118

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{5}{2}+p}x(c-a^2cx^2)^p\,{}_2F_1\left(-1-2p,\frac{3}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(3/2-p)*(1-1/a/x)^(-3/2+p)*(1+1/a/x)^(5/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, 3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2)
^p)
```

Rubi [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6331, 134}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{5}{2}}(c-a^2cx^2)^p\,{}_2F_1\left(-2p-1,\frac{3}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(3/2 - p)*(1 - 1/(a*x))^(-3/2 + p)*(1 + 1/(a*x
))^(5/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 3/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
```

```
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{3}{2}+p}\left(\right.\right.\right.$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{5}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-\right.}{1+2p}$$

Mathematica [A]
time = 0.17, size = 122, normalized size = 1.03

$$-\frac{4^{1+p}e^{5\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{5}{2}+p,2+2p;\frac{7}{2}+p;e^{2\coth^{-1}(ax)}\right)}{5a+2ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] -((4^(1 + p)*E^(5*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[5/
2 + p, 2 + 2*p, 7/2 + p, E^(2*ArcCoth[a*x])])/((5*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
[Out] int(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 + 2*a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))
/(a^2*x^2 - 2*a*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1)(ax+1))^p}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p/((a*x - 1)/(a*x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c-a^2\,c\,x^2)^p}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.767 $\int e^{2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=54

$$\frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1-ax)\right)}{ap}$$

```
[Out] 2^(1+p)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[1+p],-1/2*a*x+1/2)/a/p/((a*x+
1)^p)
```

Rubi [A]

time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6276, 692, 71}

$$\frac{2^{p+1}(ax+1)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
```

```
[Out] (2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 - a*x)/
2])/(a*p*(1 + a*x)^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6276

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[c^(n/2), Int[(c + d*x^2)^(p - n/2)*(1 + a*x)^n, x], x] /; FreeQ[{a, c,
 d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && IGtQ[n/2
, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx\\ &= -\left(c\int(1+ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)\\ &= -\left(\left(c(1+ax)^{-p}(c-acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1+ax)^{1+p}(c-acx)^{-1+p}\,dx\right)\\ &= \frac{2^{1+p}(1+ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1-ax)\right)}{ap}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.24

$$\frac{2^{1+p}(1-ax)^p\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1-ax)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - a^2*c*x^2)^p,x]
[Out] (2^(1 + p)*(1 - a*x)^p*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p
, (1 - a*x)/2])/(a*p*(1 - a^2*x^2)^p)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)\left(-a^2c\,x^2+c\right)^p}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
[Out] int(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.83, size = 651, normalized size = 12.06

$$a\left(\begin{cases} \frac{0^p x}{a} - \frac{0^p \log\left(\frac{1}{a^2x^2}\right)}{2a^2} + \frac{0^p \log\left(-1+\frac{1}{a^2x^2}\right)}{2a^2} - \frac{0^p \operatorname{acoth}\left(\frac{1}{ax}\right)}{a^2} + \frac{c^p x^2 \Gamma(p)\Gamma(1-p)\,{}_3F_2\left(\left.\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\right| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} - \frac{a^{2p}c^p p x x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix} 1-p,-p-\frac{1}{2} \\ \frac{1}{2}-p \end{matrix}\right| \frac{1}{a^2x^2}\right)}{2a\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ \frac{0^p x}{a} - \frac{0^p \log\left(\frac{1}{a^2x^2}\right)}{2a^2} + \frac{0^p \log\left(1-\frac{1}{a^2x^2}\right)}{2a^2} - \frac{0^p \operatorname{atanh}\left(\frac{1}{ax}\right)}{a^2} + \frac{c^p x^2 \Gamma(p)\Gamma(1-p)\,{}_3F_2\left(\left.\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\right| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} - \frac{a^{2p}c^p p x x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix} 1-p,-p-\frac{1}{2} \\ \frac{1}{2}-p \end{matrix}\right| \frac{1}{a^2x^2}\right)}{2a\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} & \text{otherwise} \end{cases}\right) + \begin{cases} \frac{0^p \log\left(a^2x^2-1\right)}{2a} - \frac{0^p \operatorname{acoth}(ax)}{a} + \frac{a c^p x^2 \Gamma(p)\Gamma(1-p)\,{}_3F_2\left(\left.\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\right| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} - \frac{a^{2p}c^p p x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\left.\begin{matrix} 1-p,\frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\right| \frac{1}{a^2x^2}\right)}{2a^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} & \text{for } |a^2x^2| > 1 \\ \frac{0^p \log\left(-a^2x^2+1\right)}{2a} - \frac{0^p \operatorname{atanh}(ax)}{a} + \frac{a c^p x^2 \Gamma(p)\Gamma(1-p)\,{}_3F_2\left(\left.\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\right| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} - \frac{a^{2p}c^p p x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\left.\begin{matrix} 1-p,\frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\right| \frac{1}{a^2x^2}\right)}{2a^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a**2*c*x**2+c)**p,x)
```

```
[Out] a*Piecewise((0**p*x/a - 0**p*log(1/(a**2*x**2))/(2*a**2) + 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 + c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/2
 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a - 0**p*log(1/(a**2*x*
*2))/(2*a**2) + 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))/
a**2 + c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x*
*2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,)
, 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) + Piecewise((0**
p*log(a**2*x**2 - 1)/(2*a) - 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2
 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/
2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
- 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p)
, (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p
)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((a*x + 1)*(-a^2*c*x^2 + c)^p/(a*x - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c-a^2\,c\,x^2)^p\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - a^2*c*x^2)^p*(a*x + 1))/(a*x - 1), x)
```

3.768 $\int e^{\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=118

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x(c-a^2cx^2)^p\ {}_2F_1\left(-1-2p,\frac{1}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(1/2-p)*(1-1/a/x)^(-1/2+p)*(1+1/a/x)^(3/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, 1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2)
^p)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6327, 6331, 134}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{p-\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{3}{2}}(c-a^2cx^2)^p\ {}_2F_1\left(-2p-1,\frac{1}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(1/2 - p)*(1 - 1/(a*x))^(-1/2 + p)*(1 + 1/(a*x
))^(3/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, 1/2 - p, -2*p,
 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] && !IntegerQ[n/2] && !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
```

```
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\operatorname{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{-\frac{1}{2}+p}\right.\right.$$

$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{-\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{3}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-\right.}{1+2p}$$

Mathematica [A]
time = 0.16, size = 122, normalized size = 1.03

$$-\frac{4^{1+p}e^{3\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{3}{2}+p,2+2p;\frac{5}{2}+p;e^{2\coth^{-1}(ax)}\right)}{3a+2ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - a^2*c*x^2)^p,x]
```

```
[Out] -((4^(1 + p)*E^(3*ArcCoth[a*x])*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a
*x]/(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[3/
2 + p, 2 + 2*p, 5/2 + p, E^(2*ArcCoth[a*x])])/((3*a + 2*a*p)*(a*Sqrt[1 - 1/
(a^2*x^2)]*x)^(2*p)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

```
[Out] int(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="fricas")
[Out] integral((a*x + 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x - 1),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(ax-1)(ax+1))^p}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(-a**2*c*x**2+c)**p,x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(-a^2*c*x^2+c)^p,x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^p}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^p/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.769 $\int e^{-\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=118

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}+p}x(c-a^2cx^2)^p\,{}_2F_1\left(-1-2p,-\frac{1}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(-1/2-p)*(1-1/a/x)^(1/2+p)*(1+1/a/x)^(1/2+p)*x*(-a^2*c*x^
2+c)^p*hypergeom([-1-2*p, -1/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^2
)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6331, 134}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{1}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{1}{2}}\left(\frac{1}{ax}+1\right)^{p+\frac{1}{2}}(c-a^2cx^2)^p\,{}_2F_1\left(-2p-1,-p-\frac{1}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-1/2 - p)*(1 - 1/(a*x))^(1/2 + p)*(1 + 1/(a*x
))^(1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -1/2 - p, -2*p
, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
```

```
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{1}{2}+p}\right.\right.\right.$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{1}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{1}{2}+p}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1\right.}{1+2p}$$

Mathematica [A]
time = 0.18, size = 118, normalized size = 1.00

$$-\frac{4^{1+p}e^{\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(\frac{1}{2}+p,2+2p;\frac{3}{2}+p;e^{2\coth^{-1}(ax)}\right)}{a+2ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^ArcCoth[a*x],x]
```

```
[Out] -((4^(1 + p)*E^ArcCoth[a*x]*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/
(-1 + E^(2*ArcCoth[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1/2 +
p, 2 + 2*p, 3/2 + p, E^(2*ArcCoth[a*x])])/((a + 2*a*p)*(a*Sqrt[1 - 1/(a^2*x
^2)]*x)^(2*p)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(-a^2c\,x^2+c\right)^p\sqrt{\frac{ax-1}{ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
[Out] integral((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{ax-1}{ax+1}}\,(-c(ax-1)\,(ax+1))^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*(-c*(a*x - 1)*(a*x + 1))**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-a^2\,c\,x^2\right)}^p\,\sqrt{\frac{a\,x-1}{a\,x+1}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.770 $\int e^{-2\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=55

$$-\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1+ax)\right)}{ap}$$

```
[Out] -2^(1+p)*(-a^2*c*x^2+c)^p*hypergeom([p, -1-p],[1+p],1/2*a*x+1/2)/a/p/((-a*x
+1)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6277, 692, 71}

$$-\frac{2^{p+1}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-p-1,p;p+1;\frac{1}{2}(ax+1)\right)}{ap}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((2^(1 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - p, p, 1 + p, (1 + a*x
)/2])/(a*p*(1 - a*x)^p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 692

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[
d^(m - 1)*((a + c*x^2)^(p + 1)/((1 + e*(x/d))^(p + 1)*(a/d + (c*x)/e)^(p +
1))), Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^p, x], x] /; FreeQ[{a, c, d
, e, m}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && (IntegerQ[m] || Gt
Q[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 6277

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :>
Dist[1/c^(n/2), Int[(c + d*x^2)^(p + n/2)/(1 - a*x)^n, x], x] /; FreeQ[{a,
c, d, p}, x] && EqQ[a^2*c + d, 0] &&  !(IntegerQ[p] || GtQ[c, 0]) && ILtQ[n
/2, 0]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{-2\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx&=-\int e^{-2\tanh^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx\\&=-\left(c\int(1-ax)^2\left(c-a^2cx^2\right)^{-1+p}\,dx\right)\\&=-\left(\left(c(1-ax)^{-p}(c+acx)^{-p}\left(c-a^2cx^2\right)^p\right)\int(1-ax)^{1+p}(c+acx)^{-1+p}\,dx\right)\\&=-\frac{2^{1+p}(1-ax)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1-p,p;1+p;\frac{1}{2}(1+ax)\right)}{ap}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.33

$$\frac{2^{-1+p}(1-ax)^{2+p}\left(1-a^2x^2\right)^{-p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(1-p,2+p;3+p;\frac{1}{2}(1-ax)\right)}{a(2+p)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(2*ArcCoth[a*x]),x]
[Out] (2^(-1 + p)*(1 - a*x)^(2 + p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[1 - p, 2
+ p, 3 + p, (1 - a*x)/2])/(a*(2 + p)*(1 - a^2*x^2)^p)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int\frac{\left(-a^2c\,x^2+c\right)^p(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
[Out] int((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.93, size = 651, normalized size = 11.84

$$a\left(\begin{cases} \frac{0^p x}{a} + \frac{0^p \log\left(\frac{1}{a^2x^2}\right)}{2a^2} - \frac{0^p \log\left(-1+\frac{1}{a^2x^2}\right)}{2a^2} - \frac{0^p \operatorname{acoth}\left(\frac{1}{ax}\right)}{a^2} - \frac{c^p x^2 \Gamma(p)\Gamma(1-p) {}_{3}F_{2}\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} - \frac{a^{2p}c^p p x x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} 1-p,-p-\frac{1}{2} \\ \frac{1}{2}-p \end{matrix}\middle| \frac{1}{a^2x^2}\right)}{2a\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ \frac{0^p x}{a} + \frac{0^p \log\left(\frac{1}{a^2x^2}\right)}{2a^2} - \frac{0^p \log\left(1-\frac{1}{a^2x^2}\right)}{2a^2} - \frac{0^p \operatorname{atanh}\left(\frac{1}{ax}\right)}{a^2} - \frac{c^p x^2 \Gamma(p)\Gamma(1-p) {}_{3}F_{2}\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} - \frac{a^{2p}c^p p x x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(-p-\frac{1}{2}\right) {}_{2}F_{1}\left(\begin{matrix} 1-p,-p-\frac{1}{2} \\ \frac{1}{2}-p \end{matrix}\middle| \frac{1}{a^2x^2}\right)}{2a\Gamma\left(\frac{1}{2}-p\right)\Gamma(p+1)} & \text{otherwise} \end{cases}\right) - \begin{cases} \frac{0^p \log\left(a^2x^2-1\right)}{2a} + \frac{0^p \operatorname{acoth}(ax)}{a} + \frac{ac^p x^2 \Gamma(p)\Gamma(1-p) {}_{3}F_{2}\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} + \frac{a^{2p}c^p p x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right) {}_{2}F_{1}\left(\begin{matrix} 1-p,\frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{1}{a^2x^2}\right)}{2a^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} & \text{for } |a^2x^2| > 1 \\ \frac{0^p \log\left(-a^2x^2+1\right)}{2a} + \frac{0^p \operatorname{atanh}(ax)}{a} + \frac{ac^p x^2 \Gamma(p)\Gamma(1-p) {}_{3}F_{2}\left(\begin{matrix} 2,1,1-p \\ 2,2 \end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\Gamma(-p)\Gamma(p+1)} + \frac{a^{2p}c^p p x^{2p} e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right) {}_{2}F_{1}\left(\begin{matrix} 1-p,\frac{1}{2}-p \\ \frac{3}{2}-p \end{matrix}\middle| \frac{1}{a^2x^2}\right)}{2a^2x\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*(a*x-1)/(a*x+1),x)
```

```
[Out] a*Piecewise((0**p*x/a + 0**p*log(1/(a**2*x**2))/(2*a**2) - 0**p*log(-1 + 1/
(a**2*x**2))/(2*a**2) - 0**p*acoth(1/(a*x))/a**2 - c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,), 1/(a**2*x**2))/(2*a*gamma(1/2
 - p)*gamma(p + 1)), 1/Abs(a**2*x**2) > 1), (0**p*x/a + 0**p*log(1/(a**2*x*
*2))/(2*a**2) - 0**p*log(1 - 1/(a**2*x**2))/(2*a**2) - 0**p*atanh(1/(a*x))/
a**2 - c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x*
*2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) - a**(2*p)*c**p*p*x*x**(2*
p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((1 - p, -p - 1/2), (1/2 - p,)
, 1/(a**2*x**2))/(2*a*gamma(1/2 - p)*gamma(p + 1)), True)) - Piecewise((0**
p*log(a**2*x**2 - 1)/(2*a) + 0**p*acoth(a*x)/a + a*c**p*x**2*gamma(p)*gamma
(1 - p)*hyper((2, 1, 1 - p), (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(
-p)*gamma(p + 1)) + a**(2*p)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2
 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/
2 - p)*gamma(p + 1)), Abs(a**2*x**2) > 1), (0**p*log(-a**2*x**2 + 1)/(2*a)
+ 0**p*atanh(a*x)/a + a*c**p*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p)
, (2, 2), a**2*x**2*exp_polar(2*I*pi))/(2*gamma(-p)*gamma(p + 1)) + a**(2*p
)*c**p*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 - p, 1/2 - p
), (3/2 - p,), 1/(a**2*x**2))/(2*a**2*x*gamma(3/2 - p)*gamma(p + 1)), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] integrate((a*x - 1)*(-a^2*c*x^2 + c)^p/(a*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^p\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - a^2*c*x^2)^p*(a*x - 1))/(a*x + 1), x)
```

3.771 $\int e^{-3\coth^{-1}(ax)}(c-a^2cx^2)^p\,dx$

Optimal. Leaf size=118

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+p}x(c-a^2cx^2)^p\,{}_2F_1\left(-1-2p,-\frac{3}{2}-p;-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{1+2p}$$

```
[Out] ((a-1/x)/(a+1/x))^(-3/2-p)*(1-1/a/x)^(3/2+p)*(1+1/a/x)^(-1/2+p)*x*(-a^2*c*x
^2+c)^p*hypergeom([-1-2*p, -3/2-p],[-2*p],2/(a+1/x)/x)/(1+2*p)/((1-1/a^2/x^
2)^p)
```

Rubi [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6327, 6331, 134}

$$\frac{x\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-p-\frac{3}{2}}\left(1-\frac{1}{ax}\right)^{p+\frac{3}{2}}\left(\frac{1}{ax}+1\right)^{p-\frac{1}{2}}(c-a^2cx^2)^p\,{}_2F_1\left(-2p-1,-p-\frac{3}{2};-2p;\frac{2}{\left(a+\frac{1}{x}\right)x}\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (((a - x^(-1))/(a + x^(-1)))^(-3/2 - p)*(1 - 1/(a*x))^(3/2 + p)*(1 + 1/(a*x
))^(-1/2 + p)*x*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1 - 2*p, -3/2 - p, -2*
p, 2/((a + x^(-1))*x)])/((1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 6327

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_)^2)^(p_), x_Symbo
l] :> Dist[(c + d*x^2)^p/(x^(2*p)*(1 - 1/(a^2*x^2))^p), Int[u*x^(2*p)*(1 -
1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c, d, n, p}, x] && E
qQ[a^2*c + d, 0] &&  !IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6331

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.)*(x_)^(m_), x_
Symbol] :> Dist[(-c^p)*x^m*(1/x)^m, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a
```

```
)^(p + n/2)/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, c, d, m, n, p}, x] &&
EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !Int
egersQ[2*p, p + n/2] &&  !IntegerQ[m]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-a^2cx^2\right)^p\,dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}x^{-2p}\left(c-a^2cx^2\right)^p\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p x^{2p}\,dx$$

$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{x}\right)^{2p}\left(c-a^2cx^2\right)^p\right)\text{Subst}\left(\int x^{-2-2p}\left(1-\frac{x}{a}\right)^{\frac{3}{2}+p}\left(\right.\right.\right.$$

$$=\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)^{-\frac{3}{2}-p}\left(1-\frac{1}{ax}\right)^{\frac{3}{2}+p}\left(1+\frac{1}{ax}\right)^{-\frac{1}{2}+p}x\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-1\right.}{1+2p}$$

Mathematica [A]
time = 0.19, size = 119, normalized size = 1.01

$$\frac{4^{1+p}e^{-\coth^{-1}(ax)}\left(1-e^{2\coth^{-1}(ax)}\right)^{2p}\left(\frac{e^{\coth^{-1}(ax)}}{-1+e^{2\coth^{-1}(ax)}}\right)^{2p}\left(a\sqrt{1-\frac{1}{a^2x^2}}\,x\right)^{-2p}\left(c-a^2cx^2\right)^p\,{}_2F_1\left(-\frac{1}{2}+p,2+2p;\frac{1}{2}+p;e^{2\coth^{-1}(ax)}\right)}{a-2ap}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^p/E^(3*ArcCoth[a*x]),x]
[Out] (4^(1 + p)*(1 - E^(2*ArcCoth[a*x]))^(2*p)*(E^ArcCoth[a*x]/(-1 + E^(2*ArcCot
h[a*x])))^(2*p)*(c - a^2*c*x^2)^p*Hypergeometric2F1[-1/2 + p, 2 + 2*p, 1/2
+ p, E^(2*ArcCoth[a*x])])/(E^ArcCoth[a*x]*(a - 2*a*p)*(a*Sqrt[1 - 1/(a^2*x^
2)]*x)^(2*p))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int\left(-a^2c\,x^2+c\right)^p\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
[Out] int((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x - 1)*(-a^2*c*x^2 + c)^p*sqrt((a*x - 1)/(a*x + 1))/(a*x + 1),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**p*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^p*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-a^2\,c\,x^2\right)^p\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^p*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.772 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=342

$$-\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a}-\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a}-\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\right.}{280a}$$

```
[Out] 47/42*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)/a+8/7*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(9/2)/a+c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(9/2)*x+35/16*c^4*arccsc(a*x)/a+c^
4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-67/48*c^4*(1+1/a/x)^(3/2)*(1-1
/a/x)^(1/2)/a-91/120*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-131/280*c^4*(1+1
/a/x)^(7/2)*(1-1/a/x)^(1/2)/a+61/70*c^4*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a-5
1/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.17, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{9/2}}{7a}+\frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}{42a}+c^4x\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{280a}-\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{120a}-\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{48a}-\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{16a}+\frac{35c^4\csc^{-1}(ax)}{16a}+\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-51*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (67*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(48*a) - (91*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(120*a) - (131*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) +
 (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (47*c^4*(1 - 1/(a*
x))^(3/2)*(1 + 1/(a*x))^(9/2))/(42*a) + (8*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(
a*x))^(9/2))/(7*a) + c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(9/2)*x + (35*c^
4*ArcCsc[a*x])/(16*a) + (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/
a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx = -\left(c^4\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - c^4\text{Subst}\left(\int\frac{\left(\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/}}{x}\right.$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{7}\left(ac^4\right)\text{Subs}$$

$$= \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{9/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{7/2}$$

$$= \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}{42a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{5/2}}{7a}$$

$$= -\frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{47c^4\left(1-\frac{1}{ax}\right)}{4}$$

$$= -\frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} + \frac{61c^4\sqrt{1-\frac{1}{ax}}}{70}$$

$$= -\frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{120a} - \frac{131c^4\sqrt{1-\frac{1}{ax}}}{280}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}}{120a}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}}{120a}$$

$$= -\frac{51c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{67c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{48a} - \frac{91c^4\sqrt{1-\frac{1}{ax}}}{120a}$$

Mathematica [A]
time = 0.16, size = 120, normalized size = 0.35

$$\frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(240+280ax-1056a^2x^2-1330a^3x^3+1952a^4x^4+3045a^5x^5-2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\text{ArcSin}\left(\frac{1}{ax}\right)+1680a^6\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{1680a^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(240 + 280*a*x - 1056*a^2*x^2 - 1330*a^3*x^3 +
 1952*a^4*x^4 + 3045*a^5*x^5 - 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^6
*ArcSin[1/(a*x)] + 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7)
```

Maple [A]
time = 0.09, size = 320, normalized size = 0.94

method	result
risch	$-\frac{(ax-1)(2816a^6x^6-3045a^5x^5-1952a^4x^4+1330a^3x^3+1056a^2x^2-280ax-240)c^4}{1680x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^7\sqrt{(ax+1)(ax-1)}+\frac{a^8\ln\left(\frac{a^2x \ldots}{\sqrt{a \ldots}}\right.}{\ldots}\right.}{\ldots}$ …
default	$\frac{(ax-1)c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6+3675a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}+3675a^7x^7\sqrt{a^2}\,\mathrm{arc}\ldots\right.}{\ldots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680*(a*x-1)*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^8*x^8+1680*(a^2*x
^2-1)^(3/2)*(a^2)^(1/2)*a^6*x^6+3675*a^7*x^7*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)+
3675*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+1680*ln((a^2*x+(a^2*x^
2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^8*x^7-1995*(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)*a^5*x^5-1136*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4+1050*(a^2*x^2-1)^(3
/2)*(a^2)^(1/2)*a^3*x^3+816*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-280*(a^2)
^(1/2)*(a^2*x^2-1)^(3/2)*a*x-240*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a
*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 380, normalized size = 1.11

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{5355\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+31465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}+72051\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+71801\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+4569\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+17619\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+10185\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+1995\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{6(ax-1)a^2}{ax+1}+\frac{14(ax-1)^2a^2}{(ax+1)^2}+\frac{14(ax-1)^3a^2}{(ax+1)^3}-\frac{14(ax-1)^5a^2}{(ax+1)^5}-\frac{14(ax-1)^6a^2}{(ax+1)^6}-\frac{6(ax-1)^7a^2}{(ax+1)^7}-\frac{(ax-1)^8a^2}{(ax+1)^8}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (5355*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 31465*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 72051*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 71801*c^4*((a*x - 1)/(
a*x + 1))^(9/2) + 4569*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 17619*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 10185*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 1995*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [**A**]
time = 0.37, size = 201, normalized size = 0.59

$$-\frac{7350\,a^{7}c^{4}x^{7}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(1680\,a^{8}c^{4}x^{8}-1136\,a^{7}c^{4}x^{7}+229\,a^{6}c^{4}x^{6}+4997\,a^{5}c^{4}x^{5}+622\,a^{4}c^{4}x^{4}-2386\,a^{3}c^{4}x^{3}-776\,a^{2}c^{4}x^{2}+520\,ac^{4}x+240\,c^{4}\right)\sqrt{\frac{ax-1}{ax+1}}}{1680\,a^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 - 1136*a^7*c^4*x^7 + 229*a^6*c^4*x^6
+ 4997*a^5*c^4*x^5 + 622*a^4*c^4*x^4 - 2386*a^3*c^4*x^3 - 776*a^2*c^4*x^2 +
 520*a*c^4*x + 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{4}\left(\int\frac{a^{8}}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^{8}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^{2}}{x^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{6a^{4}}{x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{4a^{6}}{x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] c**4*(Integral(a**8/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-4*a**2/(x**6*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x) + Integral(6*a**4/(x**4*sqrt(a*x/(a*x + 1) -
1/(a*x + 1))), x) + Integral(-4*a**6/(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)), x))/a**8
```

Giac [**A**]
time = 0.44, size = 461, normalized size = 1.35

$$-\frac{35\,c^{4}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)}{8\,a\operatorname{sgn}\left(ax+1\right)}-\frac{c^{4}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)}{|a|\operatorname{sgn}\left(ax+1\right)}+\frac{\sqrt{a^{2}x^{2}-1}\,c^{4}}{a\operatorname{sgn}\left(ax+1\right)}-\frac{3045\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{13}c^{4}|a|+6720\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{12}ac^{4}+6860\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{11}c^{4}|a|+20160\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{10}ac^{4}+9065\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{9}c^{4}|a|+49280\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{8}ac^{4}+49280\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{6}ac^{4}-9065\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{5}c^{4}|a|+38976\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{4}ac^{4}-6860\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{3}c^{4}|a|+12992\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}ac^{4}-3045\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)c^{4}|a|+2816\,ac^{4}}{840\left(\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}+1\right)^{7}a|a|\operatorname{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^4*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 -
1)*c^4/(a*sgn(a*x + 1)) - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4
*abs(a) + 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 + 6860*(x*abs(a) - s
qrt(a^2*x^2 - 1))^11*c^4*abs(a) + 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a
*c^4 + 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) + 49280*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^8*a*c^4 + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4
- 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) + 38976*(x*abs(a) - sqrt
(a^2*x^2 - 1))^4*a*c^4 - 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a) +
 12992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 3045*(x*abs(a) - sqrt(a^2*x
^2 - 1))*c^4*abs(a) + 2816*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^7
*a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.42, size = 332, normalized size = 0.97

$$\frac{\frac{19\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{97\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8}+\frac{839\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{1523\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{280}+\frac{71801\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{840}+\frac{3431\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}+\frac{899\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{24}+\frac{51\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/2}}{8}}{a+\frac{6\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{14\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{14\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{14\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{14\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{6\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}-\frac{a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}}-\frac{35\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((19*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (97*c^4*((a*x - 1)/(a*x + 1))^(3/
2))/8 + (839*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (1523*c^4*((a*x - 1)/(a*
x + 1))^(7/2))/280 + (71801*c^4*((a*x - 1)/(a*x + 1))^(9/2))/840 + (3431*c^
4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (899*c^4*((a*x - 1)/(a*x + 1))^(13/2))
/24 + (51*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x +
1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (14*
a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x - 1
)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/(8*a) + (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.773 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=268

$$-\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\right.}{20a}$$

```
[Out] 6/5*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+c^3*(1-1/a/x)^(5/2)*(1+1/a/x)^(7/
2)*x+15/8*c^3*arccsc(a*x)/a+c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-
31/24*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-43/60*c^3*(1+1/a/x)^(5/2)*(1-1/
a/x)^(1/2)/a+23/20*c^3*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-23/8*c^3*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.13, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}{5a}+c^3x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}+\frac{23c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{60a}-\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{24a}-\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{8a}+\frac{15c^3\csc^{-1}(ax)}{8a}+\frac{c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-23*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (31*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(24*a) - (43*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^
(5/2))/(60*a) + (23*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*
c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(5*a) + c^3*(1 - 1/(a*x))^(5/2
)*(1 + 1/(a*x))^(7/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) + (c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x - c^3\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/}}{x}\right.$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{5}\left(ac^3\right)\text{Subs}$$

$$= \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{5/2}$$

$$= -\frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{3/2}}{5a}$$

$$= -\frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{60a} + \frac{23c^3\sqrt{1-\frac{1}{ax}}}{20a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}}{60a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}}{60a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}}{60a}$$

$$= -\frac{23c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{31c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} - \frac{43c^3\sqrt{1-\frac{1}{ax}}}{60a}$$

Mathematica [A]
time = 0.13, size = 104, normalized size = 0.39

$$\frac{c^3\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(-24-30ax+88a^2x^2+135a^3x^3-184a^4x^4+120a^5x^5)}{x^4}+225a^4\text{ArcSin}\left(\frac{1}{ax}\right)+120a^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*((Sqrt[1 - 1/(a^2*x^2)]*(-24 - 30*a*x + 88*a^2*x^2 + 135*a^3*x^3 - 184
*a^4*x^4 + 120*a^5*x^5))/x^4 + 225*a^4*ArcSin[1/(a*x)] + 120*a^4*Log[(1 + S
qrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5)
```

Maple [A]
time = 0.08, size = 272, normalized size = 1.01

method	result
risch	$-\frac{(ax-1)(184a^4x^4-135a^3x^3-88a^2x^2+30ax+24)c^3}{120x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^5\sqrt{(ax+1)(ax-1)}+\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^5\,a}{}\right.}{a^6\sqrt{\frac{ax-1}{ax+1}}\,(ax+1}$
default	$\frac{(ax-1)c^3\left(-120\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^6x^6+120\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4+225\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5+225\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{}}\right.\right.}{120\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(a*x-1)*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a^6*x^6+120*(a^2)^(1/
2)*(a^2*x^2-1)^(3/2)*a^4*x^4+225*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^5*x^5+225*
(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x^5+120*ln((a^2*x+(a^2*x^2-1)^(
1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5-105*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^
3*x^3-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+30*(a^2)^(1/2)*(a^2*x^2-1)^(
3/2)*a*x+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)
*(a*x-1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 302, normalized size = 1.13

$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+1345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+1654\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+86\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+305\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+105\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4(ax-1)a^2}{ax+1}+\frac{5(ax-1)^2a^2}{(ax+1)^2}-\frac{5(ax-1)^4a^2}{(ax+1)^4}-\frac{4(ax-1)^5a^2}{(ax+1)^5}-\frac{(ax-1)^6a^2}{(ax+1)^6}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")

[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (345*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(9/2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 86*c^3*((a*x - 1)/(a*x + 1))^(5/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 105*c^3*sqrt((a*x - 1)/(a*x + 1)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x + 1)^6 + a^2))*a

Fricas [A]
time = 0.34, size = 179, normalized size = 0.67

$$-\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(120\,a^6c^3x^6-64\,a^5c^3x^5-49\,a^4c^3x^4+223\,a^3c^3x^3+58\,a^2c^3x^2-54\,ac^3x-24\,c^3)\sqrt{\frac{ax-1}{ax+1}}}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")

[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (120*a^6*c^3*x^6 - 64*a^5*c^3*x^5 - 49*a^4*c^3*x^4 + 223*a^3*c^3*x^3 + 58*a^2*c^3*x^2 - 54*a*c^3*x - 24*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^6*x^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{a^6}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx+\int\frac{3a^2}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{3a^4}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**3,x)

[Out] c**3*(Integral(a**6/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(3*a**2/(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-3*a**4/(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x))/a**6

Giac [A]
time = 0.43, size = 355, normalized size = 1.32

$$-\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\mathrm{sgn}\left(ax+1\right)}-\frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^3}{a\mathrm{sgn}\left(ax+1\right)}-\frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|+360\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3+150\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|+720\left(x|a|-\sqrt{a^2x^2-1}\right)^6ac^3+1120\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^3-150\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|+560\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3-135\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|+184\,ac^3}{60\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^5a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^3*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 -
1)*c^3/(a*sgn(a*x + 1)) - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*ab
s(a) + 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 + 150*(x*abs(a) - sqrt(a^
2*x^2 - 1))^7*c^3*abs(a) + 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3 + 112
0*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 - 150*(x*abs(a) - sqrt(a^2*x^2 - 1
))^3*c^3*abs(a) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 135*(x*abs(a
) - sqrt(a^2*x^2 - 1))*c^3*abs(a) + 184*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 -
 1))^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.32, size = 258, normalized size = 0.96

$$\frac{\frac{7\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{61\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{43\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{30}+\frac{827\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{30}+\frac{269\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{12}+\frac{23\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}-\frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+\frac{2\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((7*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (61*c^3*((a*x - 1)/(a*x + 1))^(3/2
))/12 + (43*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (827*c^3*((a*x - 1)/(a*x
+ 1))^(7/2))/30 + (269*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (23*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/(4*a) + (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.774 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=194

$$-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right.$$

```
[Out] c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)*x+3/2*c^2*arccsc(a*x)/a+c^2*arctanh((1-
1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-7/6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+4
/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/x)^
(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^2x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}+\frac{3c^2\csc^{-1}(ax)}{2a}+\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (7*c^2*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(6*a) + (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(3*a) + c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) + (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\left(c^2\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x-c^2\text{Subst}\left(\int\frac{\left(\frac{1}{a}-\frac{4x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x}\right.$$

$$=\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}x-\frac{1}{3}\left(ac^2\right)\text{Subst}$$

$$=-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{3/2}\Big($$

$$=-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1-\right.}{3a}$$

$$=-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1-\right.}{3a}$$

$$=-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1-\right.}{3a}$$

$$=-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{7c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}+\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1-\right.}{3a}$$

Mathematica [A]
time = 0.08, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(2+3ax-8a^2x^2+6a^3x^3\right)+9a^2x^2\text{ArcSin}\left(\frac{1}{ax}\right)+6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 + 3*a*x - 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^2
*ArcSin[1/(a*x)] + 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x^
2)
```

Maple [A]
time = 0.07, size = 224, normalized size = 1.15

method	result
risch	$-\frac{(ax-1)(8a^2x^2-3ax-2)c^2}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^3\sqrt{(ax+1)(ax-1)}+\frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right.}{a^4\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c^2\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{}}{\sqrt{a^2}}\right.\right.}{6\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x-1)*c^2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+6*(a^2*x^2-1)^(3/
2)*(a^2)^(1/2)*a^2*x^2+9*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3+6*ln((a^2*x+
(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^3+9*a^3*x^3*(a^2)^(1/2)*a
rctan(1/(a^2*x^2-1)^(1/2))-3*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-2*(a^2*x^2-1
)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a^4/x^
3/(a^2)^(1/2)
```

Maxima [A]
time = 0.46, size = 223, normalized size = 1.15

$$-\frac{1}{3}\,a\left(\frac{9\,c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{15\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+29\,c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+3\,c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{2\,(ax-1)a^2}{ax+1}-\frac{2\,(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 + 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (1
5*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 29*c^2*((a*x - 1)/(a*x + 1))^(5/2) + c^
2*((a*x - 1)/(a*x + 1))^(3/2) + 3*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
```

```
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))
```

Fricas [A]
time = 0.38, size = 157, normalized size = 0.81

$$-\frac{18\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+6\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^4c^2x^4-2\,a^3c^2x^3-5\,a^2c^2x^2+5\,ac^2x+2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) + 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^2*x^4 - 2*a^3*c^2*x^3 - 5*a^2*c^2*x^2 + 5*a*c^2*x + 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\frac{a^4}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\frac{1}{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{2a^2}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(a**4/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(1/(x**
4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x) + Integral(-2*a**2/(x**2*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x))/a**4
```

Giac [A]
time = 0.44, size = 249, normalized size = 1.28

$$-\frac{3\,c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\,\mathrm{sgn}\left(ax+1\right)}-\frac{c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^2}{a\,\mathrm{sgn}\left(ax+1\right)}-\frac{3\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2+12\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-3\left(x|a|-\sqrt{a^2x^2-1}\right)c^2|a|+8\,ac^2}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c^2*log(abs
(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*
c^2/(a*sgn(a*x + 1)) - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a) +
 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 + 12*(x*abs(a) - sqrt(a^2*x^2 -
1))^2*a*c^2 - 3*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) + 8*a*c^2)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.33, size = 183, normalized size = 0.94

$$\frac{c^2\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+5\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (c^2*((a*x - 1)/(a*x + 1))^(1/2) + (c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 + (2
9*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (2*c^2*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.775 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=107

$$-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a+c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a/x)^(3
/2)*x*(1-1/a/x)^(1/2)-2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {6329, 99, 159, 21, 132, 41, 222, 94, 214}

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{a}+\frac{c\csc^{-1}(ax)}{a}+\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*Sqrt[1 - 1/(a*x)]*(1 + 1/(
a*x))^(3/2)*x + (c*ArcCsc[a*x])/a + (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1
/(a*x)]])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx = -\left(c\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - c\text{Subst}\left(\int \frac{\left(\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}dx,x,\frac{1}{x}\right)$$

$$= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + (ac)\text{Subst}\left(\int \frac{}{x\sqrt{}}\right.$$

$$= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{c\text{Subst}\left(\int \frac{\sqrt{1-}}{x\sqrt{1+}}\right.}{a}$$

$$= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{c\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{}$$

$$= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{1-}}\right.}{a^2}$$

$$= -\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a} + c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x + \frac{c\csc^{-1}(ax)}{a} + \frac{c\,\text{ta}}{}$$

Mathematica [A]

time = 0.05, size = 53, normalized size = 0.50

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}\,(-1+ax)+\text{ArcSin}\left(\frac{1}{ax}\right)+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) + ArcSin[1/(a*x)] + Log[(1 + Sqrt[1 - 
1/(a^2*x^2)])*x]))/a
```

Maple [A]
time = 0.06, size = 163, normalized size = 1.52

method	result
risch	$-\frac{(ax-1)c}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a\sqrt{(ax+1)(ax-1)}+\frac{a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{(ax+}{a^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$\frac{(ax-1)c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2x+\sqrt{}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(ax-1)}\,a^2x\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x-1)*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+(a^2*x^2-1)^(3/2)*(a^2)^(
1/2)+(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1
/2))/(a^2)^(1/2))*a^2*x+(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x)/((a*x-
1)/(a*x+1))^(1/2)/((a*x+1)*(a*x-1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A]
time = 0.46, size = 117, normalized size = 1.09

$$-\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [A]
time = 0.36, size = 104, normalized size = 0.97

$$-\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(a^2cx^2-c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) + a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 - c)*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{a^2}{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx+\int\left(-\frac{1}{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2/sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-1/(x**2*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x))/a**2
```

Giac [A]
time = 0.40, size = 130, normalized size = 1.21

$$-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\operatorname{sgn}(ax+1)}-\frac{c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\operatorname{sgn}(ax+1)}+\frac{\sqrt{a^2x^2-1}\,c}{a\operatorname{sgn}(ax+1)}-\frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\operatorname{sgn}(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] -2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - c*log(abs(-x*
abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/(a
*sgn(a*x + 1)) - 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn(a*x
 + 1))
```

Mupad [B]
time = 0.07, size = 84, normalized size = 0.79

$$\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (2*c*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

3.776 $\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=104

$$-\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-2*(1+1/a/x)^(1/2)/a/c/(1-1/a/x
)^(1/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6329, 105, 21, 96, 94, 214}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\sqrt{1-\frac{1}{ax}}}-\frac{2\sqrt{\frac{1}{ax}+1}}{ac\sqrt{1-\frac{1}{ax}}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 1/(a*x)])/(a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*S
qrt[1 - 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= -\frac{2\sqrt{1+\frac{1}{ax}}}{ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\sqrt{1-\frac{1}{ax}}} + \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.54

$$\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(-2+ax)}{-1+ax} + \frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2)),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(-2 + a*x))/(-1 + a*x) + Log[(1 + Sqrt[1 - 1/(a^2
*x^2)])*x]/a)/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(92) = 184$.
time = 0.14, size = 250, normalized size = 2.40

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^{2}x}{\sqrt{a^{2}}}+\sqrt{a^{2}x^{2}-1}\right)}{a^{2}\sqrt{a^{2}}}-\frac{\sqrt{a^{2}\left(x-\frac{1}{a}\right)^{2}+2a\left(x-\frac{1}{a}\right)}}{a^{4}\left(x-\frac{1}{a}\right)}\right)a^{2}\sqrt{(ax+1)(ax-1)}}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-3\sqrt{(ax+1)(ax-1)}\,\sqrt{a^{2}}\,a^{2}x^{2}-2\ln\left(\frac{a^{2}x+\sqrt{a^{2}}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^{2}}}\right)a^{3}x^{2}+((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^{2}}+6}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-3*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-2*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x+1)*(a*x-1))^(3/2)*(a^
2)^(1/2)+6*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+4*ln((a^2*x+(a^2)^(1/2)*
((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-3*(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(1/2)-2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(
a^2)^(1/2)/(a*x-1)/c/((a*x+1)*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.27, size = 116, normalized size = 1.12

$$-a\left(\frac{\frac{3\,(ax-1)}{ax+1}-1}{a^{2}c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^{2}c\sqrt{\frac{ax-1}{ax+1}}}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^{2}c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^{2}c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*((3*(a*x - 1)/(a*x + 1) - 1)/(a^2*c*((a*x - 1)/(a*x + 1))^(3/2) - a^2*c*
sqrt((a*x - 1)/(a*x + 1))) - log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) + l
og(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c))
```

Fricas [A]
time = 0.33, size = 93, normalized size = 0.89

$$\frac{(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-(ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2x^2-ax-2)\sqrt{\frac{ax-1}{ax+1}}}{a^2cx-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] ((a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - (a*x - 1)*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) + (a^2*x^2 - a*x - 2)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c*x
 - a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2}{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.08, size = 62, normalized size = 0.60

$$\frac{2\,a\,x+4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4}{2\,a\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (2*a*x + 4*atanh(((a*x - 1)/(a*x + 1))^(1/2))*((a*x - 1)/(a*x + 1))^(1/2) -
 4)/(2*a*c*((a*x - 1)/(a*x + 1))^(1/2))
```

3.777 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=180

$$-\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}+\frac{\tanh$$

```
[Out] arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-4/3/a/c^2/(1-1/a/x)^(3/2)/(1
+1/a/x)^(1/2)+x/c^2/(1-1/a/x)^(3/2)/(1+1/a/x)^(1/2)-11/3/a/c^2/(1-1/a/x)^(1
/2)/(1+1/a/x)^(1/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}-\frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -4/(3*a*c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) - 11/(3*a*c^2*Sqrt[1 - 1
/(a*x)]*Sqrt[1 + 1/(a*x)]) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*x
)]) + x/(c^2*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]) + ArcTanh[Sqrt[1 - 1/(a
*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\text{Subst}\left(\int \frac{\frac{3}{a^2}+\frac{8x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/}}\right.}{3c^2}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{}{c^2\left(1-\right.}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{}{c^2\left(1-\right.}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{}{c^2\left(1-\right.}$$

$$= -\frac{4}{3ac^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}} - \frac{11}{3ac^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{}{c^2\left(1-\right.}$$

Mathematica [A]
time = 0.08, size = 83, normalized size = 0.46

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(8-5ax-7a^2x^2+3a^3x^3)}{3(-1+ax)^2(1+ax)}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(8 - 5*a*x - 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 + a
*x)^2*(1 + a*x)) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(154) = 308.
time = 0.15, size = 530, normalized size = 2.94

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{6a^7\left(x-\frac{1}{a}\right)^2}-\frac{19\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{12a^6\left(x-\frac{1}{a}\right)}\right.}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-45\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^5x^5-24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^6x^5+21((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-45*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5-24*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+21*((a*x+1)*(a*x-1))^(3/
2)*(a^2)^(1/2)*a^3*x^3+45*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+24*ln
((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+11*(a^2)^
(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2+90*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
)*a^3*x^3+48*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^
4*x^3-5*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-90*((a*x+1)*(a*x-1))^(1/2)*
(a^2)^(1/2)*a^2*x^2-48*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)
^(1/2))*a^3*x^2-19*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-45*((a*x+1)*(a*x-1))
^(1/2)*(a^2)^(1/2)*a*x-24*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a
^2)^(1/2))*a^2*x+45*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+24*a*ln((a^2*x+(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a*x-1)^2/(a^2)^(1/2)/(a*x
+1)^2/c^2/((a*x+1)*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.27, size = 160, normalized size = 0.89

$$\frac{1}{12}\,a\left(\frac{\frac{17\,(ax-1)}{ax+1}-\frac{42\,(ax-1)^2}{(ax+1)^2}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}+\frac{3\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*a*((17*(a*x - 1)/(a*x + 1) - 42*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c^2*
((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)) + 12*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)
) - 1)/(a^2*c^2) + 3*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^2))
```

Fricas [A]
time = 0.37, size = 134, normalized size = 0.74

$$\frac{3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-7\,a^2x^2-5\,ax+8)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 7*a^2*x^2
- 5*a*x + 8)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 2*a**2*x*
*2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))),
x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 0.07, size = 128, normalized size = 0.71

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}-\frac{\frac{17\,\left(a\,x-1\right)}{3\,\left(a\,x+1\right)}-\frac{14\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{1}{3}}{4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}+\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(4*a*c^2) - ((17*(a*x - 1))/(3*(a*x + 1)) - (14
*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(3/2) - 4*a
*c^2*((a*x - 1)/(a*x + 1))^(5/2)) + (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/
(a*c^2)
```

3.778 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} dx$

Optimal. Leaf size=254

$$-\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/}}$$

```
[Out] -6/5/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-29/15/a/c^3/(1-1/a/x)^(3/2)/(1+1
/a/x)^(3/2)+x/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)+arctanh((1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2))/a/c^3-34/5/a/c^3/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+21/5*(1-1/
a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{21\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -6/(5*a*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)) - 29/(15*a*c^3*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)) - 34/(5*a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(3/2)) + (21*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sq
rt[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(5/2)*(
1 + 1/(a*x))^(3/2)) + ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{\frac{5}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{\cdots}}\right.}{5c^3}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\right.}$$

$$= -\frac{6}{5ac^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{15ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{34}{5ac^3\sqrt{1-\frac{1}{ax}}\left(1+\right.}$$

$$6 \qquad 29 \qquad 34$$

Mathematica [A]
time = 0.09, size = 99, normalized size = 0.39

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-48+33ax+87a^2x^2-52a^3x^3-38a^4x^4+15a^5x^5)}{15(-1+ax)^3(1+ax)^2}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-48 + 33*a*x + 87*a^2*x^2 - 52*a^3*x^3 - 38*a^
4*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^3*(1 + a*x)^2) + Log[(1 + Sqrt[1 - 1/(a
^2*x^2)])*x])/(a*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(216) = 432.
time = 0.18, size = 714, normalized size = 2.81

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{23\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{60a^9\left(x-\frac{1}{a}\right)^2}-\frac{493\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\right.}}{240a^8\left(x-\frac{1}{a}\right)}\right.}{}$
default	$-\frac{-525\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^7x^7-240\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^8x^7+285((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-525*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^7*x^7-240*ln((a^2*x+(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^8*x^7+285*((a*x+1)*(a*x-1))
^(3/2)*(a^2)^(1/2)*a^5*x^5+525*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^6*x^6+
240*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^7*x^6+83*
((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4+1575*((a*x+1)*(a*x-1))^(1/2)*(a
^2)^(1/2)*a^5*x^5+720*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^
(1/2))*a^6*x^5-218*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3-1575*(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-720*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4-342*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a
^2*x^2-1575*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-720*ln((a^2*x+(a^2)
^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-3*(a^2)^(1/2)*((a*x+1)
```

```
*(a*x-1))^(3/2)*a*x+1575*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+720*ln
((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+243*((a*x
+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+525*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+
240*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-525*(
a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-240*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(1/2))/(a^2)^(1/2)))/a/(a*x-1)^3/(a^2)^(1/2)/(a*x+1)^3/c^3/((a*x+1)*(
a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]
time = 0.26, size = 194, normalized size = 0.76

$$\frac{1}{240}\,a\left(\frac{\frac{37\,(ax-1)}{ax+1}+\frac{410\,(ax-1)^2}{(ax+1)^2}-\frac{930\,(ax-1)^3}{(ax+1)^3}+3}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+24\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^3}+\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{240\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*((37*(a*x - 1)/(a*x + 1) + 410*(a*x - 1)^2/(a*x + 1)^2 - 930*(a*x -
 1)^3/(a*x + 1)^3 + 3)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^3*((a*x
 - 1)/(a*x + 1))^(5/2)) + 5*(((a*x - 1)/(a*x + 1))^(3/2) + 24*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^3)
- 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A]
time = 0.37, size = 178, normalized size = 0.70

$$\frac{15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(15\,a^5x^5-38\,a^4x^4-52\,a^3x^3+87\,a^2x^2+33\,ax-48)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) +
1) - 15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1
) + (15*a^5*x^5 - 38*a^4*x^4 - 52*a^3*x^3 + 87*a^2*x^2 + 33*a*x - 48)*sqrt(
(a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\int\frac{x^6}{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 3*a**4*x**
*4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^3*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 0.10, size = 171, normalized size = 0.67

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}-\frac{\frac{82\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{62\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{37\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{48\,a\,c^3}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(2*a*c^3) - ((82*(a*x - 1)^2)/(3*(a*x + 1)^2) -
 (62*(a*x - 1)^3)/(a*x + 1)^3 + (37*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(16*a*
c^3*((a*x - 1)/(a*x + 1))^(5/2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + (
(a*x - 1)/(a*x + 1))^(3/2)/(48*a*c^3) - (atan(((a*x - 1)/(a*x + 1))^(1/2)*1
i)*2i)/(a*c^3)
```

3.779 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=328

$$-\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{26}{21ac^4\sqrt{1-\frac{1}{a}}}$$

```
[Out] -8/7/a/c^4/(1-1/a/x)^(7/2)/(1+1/a/x)^(5/2)-11/7/a/c^4/(1-1/a/x)^(5/2)/(1+1/
a/x)^(5/2)-62/21/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^4/(1-1/a/x)^(7/2
)/(1+1/a/x)^(5/2)+arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-269/21/a/c
^4/(1+1/a/x)^(5/2)/(1-1/a/x)^(1/2)+262/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(
5/2)+163/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/2)/a/
c^4/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{163\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{262\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{269}{21ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -8/(7*a*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) - 11/(7*a*c^4*(1 - 1/(
a*x))^(5/2)*(1 + 1/(a*x))^(5/2)) - 62/(21*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/
(a*x))^(5/2)) - 269/(21*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (262
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (163*Sqrt[1 - 1/(a*x)]
)/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(5/2)) + ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a\text{Subst}\left(\int \frac{\frac{7}{a^2}+\frac{48x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)}\right.}{7c^4}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right.}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.}$$

$$= -\frac{8}{7ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{11}{7ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{62}{21ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\right.}$$

8 11 62

Mathematica [A]
time = 0.12, size = 115, normalized size = 0.35

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(384-279ax-1065a^2x^2+715a^3x^3+965a^4x^4-559a^5x^5-281a^6x^6+105a^7x^7)}{105(-1+ax)^4(1+ax)^3}+\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(384 - 279*a*x - 1065*a^2*x^2 + 715*a^3*x^3 + 9
65*a^4*x^4 - 559*a^5*x^5 - 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^4*(1
 + a*x)^3) + Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(278) = 556.
time = 0.18, size = 898, normalized size = 2.74

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{211\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{336a^{11}\left(x-\frac{1}{a}\right)^2}-\frac{1657\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{672a^{10}\left(x-\frac{1}{a}\right)}\right.}{}$
default	$-\frac{53760\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3-13440\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^2x+7705((ax+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13440*(53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))
*a^4*x^3-13440*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*
a^2*x+7705*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3-198450*((a*x+1)*(a*x
-1))^(1/2)*(a^2)^(1/2)*a^5*x^5+37095*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^
2*x^2+2637*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+198450*(a^2)^(1/2)*((a*x
+1)*(a*x-1))^(1/2)*a^4*x^4+132300*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x
^3-132300*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-33075*((a*x+1)*(a*x-1
))^(1/2)*(a^2)^(1/2)*a*x-16077*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+13440*a*
ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))+33075*(a^2)^(1/
2)*((a*x+1)*(a*x-1))^(1/2)-80640*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1
/2))/(a^2)^(1/2))*a^6*x^5-53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/
2))/(a^2)^(1/2))*a^3*x^2+80640*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
```

```
))/(a^2)^(1/2))*a^5*x^4-53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)
)/(a^2)^(1/2))*a^7*x^6+53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))
/(a^2)^(1/2))*a^8*x^7+132300*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^7*x^7-27
673*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^5*x^5-132300*((a*x+1)*(a*x-1))^(1
/2)*(a^2)^(1/2)*a^6*x^6-24295*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4+2
893*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^6*x^6-13440*ln((a^2*x+(a^2)^(1/2)
*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^10*x^9+13440*ln((a^2*x+(a^2)^(1/2)
*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^9*x^8-33075*((a*x+1)*(a*x-1))^(1/2
)*(a^2)^(1/2)*a^9*x^9+19635*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^7*x^7+330
75*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^8*x^8)/a/(a*x-1)^4/(a^2)^(1/2)/(a*
x+1)^4/c^4/((a*x+1)*(a*x-1))^(1/2)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [A]

time = 0.27, size = 230, normalized size = 0.70

$$\frac{1}{6720}\,a\left(\frac{5\left(\frac{39\,(ax-1)}{ax+1}+\frac{287\,(ax-1)^2}{(ax+1)^2}+\frac{2611\,(ax-1)^3}{(ax+1)^3}-\frac{5628\,(ax-1)^4}{(ax+1)^4}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{7\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+50\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+705\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{6720\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{6720\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(5*(39*(a*x - 1)/(a*x + 1) + 287*(a*x - 1)^2/(a*x + 1)^2 + 2611*(a
*x - 1)^3/(a*x + 1)^3 - 5628*(a*x - 1)^4/(a*x + 1)^4 + 3)/(a^2*c^4*((a*x -
1)/(a*x + 1))^(9/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + 7*(3*((a*x - 1
)/(a*x + 1))^(5/2) + 50*((a*x - 1)/(a*x + 1))^(3/2) + 705*sqrt((a*x - 1)/(a
*x + 1)))/(a^2*c^4) + 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4) - 6
720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A]

time = 0.38, size = 274, normalized size = 0.84

$$\frac{105\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(105\,a^7x^7-281\,a^6x^6-559\,a^5x^5+965\,a^4x^4+715\,a^3x^3-1065\,a^2x^2-279\,ax+384)\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1
)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 +
 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (105
*a^7*x^7 - 281*a^6*x^6 - 559*a^5*x^5 + 965*a^4*x^4 + 715*a^3*x^3 - 1065*a^2
*x^2 - 279*a*x + 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 2*a^6*c^4*x
^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^8\int\frac{x^8}{a^8x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+6a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}-4a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}+\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8/(a**8*x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) - 4*a**6*x*
*6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + 6*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(
a*x + 1)) - 4*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)) + sqrt(a*x/(a*x +
 1) - 1/(a*x + 1))), x)/c**4
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^4*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [**B**]
time = 0.07, size = 210, normalized size = 0.64

$$\frac{47\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{41\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}+\frac{373\,{\left(a\,x-1\right)}^3}{3\,{\left(a\,x+1\right)}^3}-\frac{268\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}+\frac{13\,\left(a\,x-1\right)}{7\,\left(a\,x+1\right)}+\frac{1}{7}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}-64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}+\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{96\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{320\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] (47*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((41*(a*x - 1)^2)/(3*(a*x + 1
)^2) + (373*(a*x - 1)^3)/(3*(a*x + 1)^3) - (268*(a*x - 1)^4)/(a*x + 1)^4 +
(13*(a*x - 1))/(7*(a*x + 1)) + 1/7)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2) -
 64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2)) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(9
6*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(320*a*c^4) - (atan(((a*x - 1)/(a*x
+ 1))^(1/2)*1i)*2i)/(a*c^4)
```

3.780 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=127

$$-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

```
[Out] -1/9*c^5/a^10/x^9-1/4*c^5/a^9/x^8+3/7*c^5/a^8/x^7+4/3*c^5/a^7/x^6-2/5*c^5/a
^6/x^5-3*c^5/a^5/x^4-2/3*c^5/a^4/x^3+4*c^5/a^3/x^2+3*c^5/a^2/x+c^5*x+2*c^5*
ln(x)/a
```

Rubi [A]
time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+\frac{2c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -1/9*c^5/(a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^
7*x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4
*c^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx=-\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$$
$$=\frac{c^5\int\frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^5}{x^{10}}\,dx}{a^{10}}$$
$$=\frac{c^5\int\frac{(1-ax)^4(1+ax)^6}{x^{10}}\,dx}{a^{10}}$$
$$=\frac{c^5\int\left(a^{10}+\frac{1}{x^{10}}+\frac{2a}{x^9}-\frac{3a^2}{x^8}-\frac{8a^3}{x^7}+\frac{2a^4}{x^6}+\frac{12a^5}{x^5}+\frac{2a^6}{x^4}-\frac{8a^7}{x^3}-\frac{3a^8}{x^2}+\frac{2a^9}{x}\right)dx}{a^{10}}$$
$$=-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}$$

Mathematica [A]
time = 0.06, size = 127, normalized size = 1.00

$$-\frac{c^5}{9a^{10}x^9}-\frac{c^5}{4a^9x^8}+\frac{3c^5}{7a^8x^7}+\frac{4c^5}{3a^7x^6}-\frac{2c^5}{5a^6x^5}-\frac{3c^5}{a^5x^4}-\frac{2c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}+\frac{3c^5}{a^2x}+c^5x+\frac{2c^5\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] -1/9*c^5/(a^10*x^9) - c^5/(4*a^9*x^8) + (3*c^5)/(7*a^8*x^7) + (4*c^5)/(3*a^
7*x^6) - (2*c^5)/(5*a^6*x^5) - (3*c^5)/(a^5*x^4) - (2*c^5)/(3*a^4*x^3) + (4
*c^5)/(a^3*x^2) + (3*c^5)/(a^2*x) + c^5*x + (2*c^5*Log[x])/a
```

Maple [A]
time = 0.34, size = 88, normalized size = 0.69

method	result
default	$\frac{c^5\left(a^{10}x-\frac{3a^5}{x^4}-\frac{a}{4x^8}+\frac{3a^2}{7x^7}-\frac{1}{9x^9}+\frac{3a^8}{x}+\frac{4a^7}{x^2}+2a^9\ln(x)-\frac{2a^6}{3x^3}+\frac{4a^3}{3x^6}-\frac{2a^4}{5x^5}\right)}{a^{10}}$
risch	$c^5x+\frac{3a^8c^5x^8+4a^7c^5x^7-\frac{2}{3}a^6c^5x^6-3a^5c^5x^5-\frac{2}{5}a^4c^5x^4+\frac{4}{3}a^3c^5x^3+\frac{3}{7}a^2c^5x^2-\frac{1}{4}a\,c^5x-\frac{1}{9}c^5}{a^{10}x^9}+\frac{2c^5\ln(x)}{a}$
norman	$\frac{a^9c^5x^{10}-\frac{c^5}{9a}-\frac{c^5x}{4}+\frac{3a\,c^5x^2}{7}-3a^4c^5x^5-\frac{2a^5c^5x^6}{3}+4a^6c^5x^7+3a^7c^5x^8+\frac{4c^5a^2x^3}{3}-\frac{2c^5a^3x^4}{5}}{a^9x^9}+\frac{2c^5\ln(x)}{a}$
meijerg	$-\frac{c^5(-ax-\ln(-ax+1))}{a}+\frac{5c^5(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{10c^5\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+\frac{10c^5\left(-\ln(-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^5/a^10*(a^10*x-3*a^5/x^4-1/4*a/x^8+3/7*a^2/x^7-1/9/x^9+3*a^8/x+4*a^7/x^2+
2*a^9*ln(x)-2/3*a^6/x^3+4/3*a^3/x^6-2/5*a^4/x^5)
```

Maxima [A]
time = 0.26, size = 114, normalized size = 0.90

$$c^5x + \frac{2\,c^5\log(x)}{a} + \frac{3780\,a^8c^5x^8 + 5040\,a^7c^5x^7 - 840\,a^6c^5x^6 - 3780\,a^5c^5x^5 - 504\,a^4c^5x^4 + 1680\,a^3c^5x^3 + 540\,a^2c^5x^2 - 315\,ac^5x - 140\,c^5}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 2*c^5*log(x)/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 - 840*
a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 + 540*a
^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Fricas [A]
time = 0.35, size = 122, normalized size = 0.96

$$\frac{1260\,a^{10}c^5x^{10} + 2520\,a^9c^5x^9\log(x) + 3780\,a^8c^5x^8 + 5040\,a^7c^5x^7 - 840\,a^6c^5x^6 - 3780\,a^5c^5x^5 - 504\,a^4c^5x^4 + 1680\,a^3c^5x^3 + 540\,a^2c^5x^2 - 315\,ac^5x - 140\,c^5}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/1260*(1260*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) + 3780*a^8*c^5*x^8 + 5
040*a^7*c^5*x^7 - 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 16
80*a^3*c^5*x^3 + 540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Sympy [A]
time = 0.41, size = 124, normalized size = 0.98

$$\frac{a^{10}c^5x + 2a^9c^5\log(x) + \frac{3780a^8c^5x^8+5040a^7c^5x^7-840a^6c^5x^6-3780a^5c^5x^5-504a^4c^5x^4+1680a^3c^5x^3+540a^2c^5x^2-315ac^5x-140c^5}{1260x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 2*a**9*c**5*log(x) + (3780*a**8*c**5*x**8 + 5040*a**7*c**5*
x**7 - 840*a**6*c**5*x**6 - 3780*a**5*c**5*x**5 - 504*a**4*c**5*x**4 + 1680
*a**3*c**5*x**3 + 540*a**2*c**5*x**2 - 315*a*c**5*x - 140*c**5)/(1260*x**9)
)/a**10
```

Giac [A]
time = 0.40, size = 115, normalized size = 0.91

$$c^5x+\frac{2\,c^5\log\left(|x|\right)}{a}+\frac{3780\,a^8c^5x^8+5040\,a^7c^5x^7-840\,a^6c^5x^6-3780\,a^5c^5x^5-504\,a^4c^5x^4+1680\,a^3c^5x^3+540\,a^2c^5x^2-315\,ac^5x-140\,c^5}{1260\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] c^5*x + 2*c^5*log(abs(x))/a + 1/1260*(3780*a^8*c^5*x^8 + 5040*a^7*c^5*x^7 -
 840*a^6*c^5*x^6 - 3780*a^5*c^5*x^5 - 504*a^4*c^5*x^4 + 1680*a^3*c^5*x^3 +
540*a^2*c^5*x^2 - 315*a*c^5*x - 140*c^5)/(a^10*x^9)
```

Mupad [B]
time = 0.10, size = 89, normalized size = 0.70

$$\frac{c^5\,\left(\frac{3\,a^2\,x^2}{7}-\frac{a\,x}{4}+\frac{4\,a^3\,x^3}{3}-\frac{2\,a^4\,x^4}{5}-3\,a^5\,x^5-\frac{2\,a^6\,x^6}{3}+4\,a^7\,x^7+3\,a^8\,x^8+a^{10}\,x^{10}+2\,a^9\,x^9\,\ln\left(x\right)-\frac{1}{9}\right)}{a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^5*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^5*((3*a^2*x^2)/7 - (a*x)/4 + (4*a^3*x^3)/3 - (2*a^4*x^4)/5 - 3*a^5*x^5 -
 (2*a^6*x^6)/3 + 4*a^7*x^7 + 3*a^8*x^8 + a^10*x^10 + 2*a^9*x^9*log(x) - 1/9
))/(a^10*x^9)
```

3.781 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=90

$$\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

```
[Out] 1/7*c^4/a^8/x^7+1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5-3/2*c^4/a^5/x^4+3*c^4/a^3/x
^2+2*c^4/a^2/x+c^4*x+2*c^4*ln(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx \\
&= -\frac{c^4\int \frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^4}{x^8}\,dx}{a^8} \\
&= -\frac{c^4\int \frac{(1-ax)^3(1+ax)^5}{x^8}\,dx}{a^8} \\
&= -\frac{c^4\int \left(-a^8+\frac{1}{x^8}+\frac{2a}{x^7}-\frac{2a^2}{x^6}-\frac{6a^3}{x^5}+\frac{6a^5}{x^3}+\frac{2a^6}{x^2}-\frac{2a^7}{x}\right) dx}{a^8} \\
&= \frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 1.00

$$\frac{c^4}{7a^8x^7}+\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}-\frac{3c^4}{2a^5x^4}+\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x+\frac{2c^4\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] c^4/(7*a^8*x^7) + c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) - (3*c^4)/(2*a^5*x^
4) + (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x + (2*c^4*Log[x])/a
```

Maple [A]
time = 0.27, size = 64, normalized size = 0.71

method	result
default	$\frac{c^4\left(a^8x-\frac{3a^3}{2x^4}+\frac{1}{7x^7}+\frac{2a^6}{x}+\frac{3a^5}{x^2}+2a^7\ln(x)+\frac{a}{3x^6}-\frac{2a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{2a^6c^4x^6+3a^5c^4x^5-\frac{3}{2}a^3c^4x^3-\frac{2}{5}a^2c^4x^2+\frac{1}{3}a\,c^4x+\frac{1}{7}c^4}{a^8x^7}+\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^7c^4x^8+\frac{c^4}{7a}+\frac{c^4x}{3}-\frac{3a^2c^4x^3}{2}+3a^4c^4x^5+2a^5c^4x^6-\frac{2c^4a\,x^2}{5}}{a^7x^7}+\frac{2c^4\ln(x)}{a}$
meijerg	$-\frac{c^4(-ax-\ln(-ax+1))}{a}+\frac{4c^4(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{6c^4\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+\frac{4c^4\left(-\ln(-a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^4/a^8*(a^8*x-3/2*a^3/x^4+1/7/x^7+2*a^6/x+3*a^5/x^2+2*a^7*ln(x)+1/3*a/x^6-
2/5*a^2/x^5)
```

Maxima [A]
time = 0.26, size = 81, normalized size = 0.90

$$c^4x+\frac{2\,c^4\log\left(x\right)}{a}+\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Fricas [A]
time = 0.35, size = 89, normalized size = 0.99

$$\frac{210\,a^8c^4x^8+420\,a^7c^4x^7\log\left(x\right)+420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/210*(210*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 + 630*a^5
*c^4*x^5 - 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7
)
```

Sympy [A]
time = 0.26, size = 88, normalized size = 0.98

$$\frac{a^8c^4x+2a^7c^4\log\left(x\right)+\frac{420a^6c^4x^6+630a^5c^4x^5-315a^3c^4x^3-84a^2c^4x^2+70ac^4x+30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 + 630*a**5*c**4*x**
5 - 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 + 70*a*c**4*x + 30*c**4)/(210*x*
*7))/a**8
```

Giac [A]
time = 0.41, size = 82, normalized size = 0.91

$$c^4x+\frac{2\,c^4\log\left(|x|\right)}{a}+\frac{420\,a^6c^4x^6+630\,a^5c^4x^5-315\,a^3c^4x^3-84\,a^2c^4x^2+70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] c^4*x + 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 + 630*a^5*c^4*x^5 - 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 + 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Mupad [B]
time = 1.24, size = 65, normalized size = 0.72

$$\frac{c^4\left(\frac{a\,x}{3}-\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5+2\,a^6\,x^6+a^8\,x^8+2\,a^7\,x^7\,\ln\left(x\right)+\frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^4*((a*x)/3 - (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 + 2*a^6*x^6 + a^8
*x^8 + 2*a^7*x^7*log(x) + 1/7))/(a^8*x^7)
```

3.782 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=76

$$-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

```
[Out] -1/5*c^3/a^6/x^5-1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3+2*c^3/a^3/x^2+c^3/a^2/x+c^
3*x+2*c^3*ln(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] -1/5*c^3/(a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx\\ &= \frac{c^3\int \frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^3}{x^6}\,dx}{a^6}\\ &= \frac{c^3\int \frac{(1-ax)^2(1+ax)^4}{x^6}\,dx}{a^6}\\ &= \frac{c^3\int \left(a^6+\frac{1}{x^6}+\frac{2a}{x^5}-\frac{a^2}{x^4}-\frac{4a^3}{x^3}-\frac{a^4}{x^2}+\frac{2a^5}{x}\right)dx}{a^6}\\ &= -\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 1.00

$$-\frac{c^3}{5a^6x^5}-\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}+\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x+\frac{2c^3\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
[Out] -1/5*c^3/(a^6*x^5) - c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) + (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x + (2*c^3*Log[x])/a
```

Maple [A]
time = 0.23, size = 55, normalized size = 0.72

method	result
default	$\frac{c^3\left(a^6x-\frac{a}{2x^4}+\frac{a^4}{x}+\frac{2a^3}{x^2}+2a^5\ln(x)+\frac{a^2}{3x^3}-\frac{1}{5x^5}\right)}{a^6}$
risch	$c^3x+\frac{a^4c^3x^4+2a^3c^3x^3+\frac{1}{3}a^2c^3x^2-\frac{1}{2}a\,c^3x-\frac{1}{5}c^3}{a^6x^5}+\frac{2c^3\ln(x)}{a}$
norman	$\frac{a^3c^3x^4+a^5c^3x^6-\frac{c^3}{5a}-\frac{c^3x}{2}+2a^2c^3x^3+\frac{c^3a\,x^2}{3}}{a^5x^5}+\frac{2c^3\ln(x)}{a}$
meijerg	$-\frac{c^3(-ax-\ln(-ax+1))}{a}+\frac{3c^3(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{3c^3\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+\frac{c^3\left(-\ln(-ax\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^6*(a^6*x-1/2*a/x^4+a^4/x+2*a^3/x^2+2*a^5*ln(x)+1/3*a^2/x^3-1/5/x^5)
```

Maxima [A]
time = 0.26, size = 70, normalized size = 0.92

$$c^3x+\frac{2\,c^3\log\left(x\right)}{a}+\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Fricas [A]
time = 0.34, size = 78, normalized size = 1.03

$$\frac{30\,a^6c^3x^6+60\,a^5c^3x^5\log\left(x\right)+30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 + 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A]
time = 0.19, size = 76, normalized size = 1.00

$$\frac{a^6c^3x+2a^5c^3\log\left(x\right)+\frac{30a^4c^3x^4+60a^3c^3x^3+10a^2c^3x^2-15ac^3x-6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 + 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 - 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Giac [A]
time = 0.38, size = 71, normalized size = 0.93

$$c^3x+\frac{2\,c^3\log\left(|x|\right)}{a}+\frac{30\,a^4c^3x^4+60\,a^3c^3x^3+10\,a^2c^3x^2-15\,ac^3x-6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] c^3*x + 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 + 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 - 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Mupad [B]
time = 1.22, size = 56, normalized size = 0.74

$$\frac{c^3\left(\frac{a^2\,x^2}{3}-\frac{a\,x}{2}+2\,a^3\,x^3+a^4\,x^4+a^6\,x^6+2\,a^5\,x^5\,\ln\left(x\right)-\frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^3*((a^2*x^2)/3 - (a*x)/2 + 2*a^3*x^3 + a^4*x^4 + a^6*x^6 + 2*a^5*x^5*log
(x) - 1/5))/(a^6*x^5)
```

3.783 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=39

$$\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

[Out] 1/3*c^2/a^4/x^3+c^2/a^3/x^2+c^2*x+2*c^2*ln(x)/a

Rubi [A]
time = 0.09, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6292, 6285, 76}

$$\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]

[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$$
$$= -\frac{c^2\int\frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)^2}{x^4}\,dx}{a^4}$$
$$= -\frac{c^2\int\frac{(1-ax)(1+ax)^3}{x^4}\,dx}{a^4}$$
$$= -\frac{c^2\int\left(-a^4+\frac{1}{x^4}+\frac{2a}{x^3}-\frac{2a^3}{x}\right)dx}{a^4}$$
$$= \frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{c^2}{3a^4x^3}+\frac{c^2}{a^3x^2}+c^2x+\frac{2c^2\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
[Out] c^2/(3*a^4*x^3) + c^2/(a^3*x^2) + c^2*x + (2*c^2*Log[x])/a
```

Maple [A]
time = 0.18, size = 31, normalized size = 0.79

method	result
default	$\frac{c^2\left(a^4x+\frac{a}{x^2}+2a^3\ln(x)+\frac{1}{3x^3}\right)}{a^4}$
risch	$c^2x+\frac{a\,c^2x+\frac{1}{3}c^2}{a^4x^3}+\frac{2c^2\ln(x)}{a}$
norman	$\frac{c^2x+a^3c^2x^4+\frac{c^2}{3a}}{a^3x^3}+\frac{2c^2\ln(x)}{a}$
meijerg	$-\frac{c^2(-ax-\ln(-ax+1))}{a}+\frac{2c^2(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}-\frac{c^2\left(-\ln(-ax+1)+\ln(x)+\ln(-a)-\frac{1}{2a^2x^2}-\frac{1}{ax}\right)}{a}+\frac{c^2\ln(-ax+1)}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
[Out] c^2/a^4*(a^4*x+a/x^2+2*a^3*ln(x)+1/3/x^3)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.90

$$c^2x+\frac{2\,c^2\log(x)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 2*c^2*log(x)/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Fricas [A]
time = 0.36, size = 43, normalized size = 1.10

$$\frac{3\,a^4c^2x^4+6\,a^3c^2x^3\log\left(x\right)+3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 6*a^3*c^2*x^3*log(x) + 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A]
time = 0.11, size = 39, normalized size = 1.00

$$\frac{a^4c^2x+2a^3c^2\log\left(x\right)+\frac{3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 2*a**3*c**2*log(x) + (3*a*c**2*x + c**2)/(3*x**3))/a**4
```

Giac [A]
time = 0.41, size = 36, normalized size = 0.92

$$c^2x+\frac{2\,c^2\log\left(|x|\right)}{a}+\frac{3\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] c^2*x + 2*c^2*log(abs(x))/a + 1/3*(3*a*c^2*x + c^2)/(a^4*x^3)
```

Mupad [B]
time = 0.05, size = 32, normalized size = 0.82

$$\frac{c^2\,\left(a\,x+a^4\,x^4+2\,a^3\,x^3\,\ln\left(x\right)+\frac{1}{3}\right)}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c^2*(a*x + a^4*x^4 + 2*a^3*x^3*log(x) + 1/3))/(a^4*x^3)
```

3.784 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=21

$$-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

```
[Out] -c/a^2/x+c*x+2*c*ln(x)/a
```

Rubi [A]
time = 0.06, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6302, 6292, 6285, 45}

$$-\frac{c}{a^2x}+\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx &= -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx\\
&= \frac{c\int \frac{e^{2\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}\,dx}{a^2}\\
&= \frac{c\int \frac{(1+ax)^2}{x^2}\,dx}{a^2}\\
&= \frac{c\int \left(a^2+\frac{1}{x^2}+\frac{2a}{x}\right)\,dx}{a^2}\\
&= -\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$-\frac{c}{a^2x}+cx+\frac{2c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] -(c/(a^2*x)) + c*x + (2*c*Log[x])/a
```

Maple [A]
time = 0.14, size = 22, normalized size = 1.05

method	result	size
default	$\frac{c\left(a^2x-\frac{1}{x}+2a\ln(x)\right)}{a^2}$	22
risch	$-\frac{c}{a^2x}+cx+\frac{2c\ln(x)}{a}$	22
norman	$\frac{acx^2-\frac{c}{a}}{ax}+\frac{2c\ln(x)}{a}$	30
meijerg	$-\frac{c(-ax-\ln(-ax+1))}{a}+\frac{c(-\ln(-ax+1)+\ln(x)+\ln(-a))}{a}+\frac{c\ln(-ax+1)}{a}-\frac{c\left(\ln(-ax+1)-\ln(x)-\ln(-a)+\frac{1}{ax}\right)}{a}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] c/a^2*(a^2*x-1/x+2*a*ln(x))
```

Maxima [A]
time = 0.26, size = 21, normalized size = 1.00

$$cx+\frac{2\,c\log\left(x\right)}{a}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] c*x + 2*c*log(x)/a - c/(a^2*x)
```

Fricas [A]
time = 0.33, size = 26, normalized size = 1.24

$$\frac{a^2cx^2 + 2\,acx\log\left(x\right) - c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^2*c*x^2 + 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.95

$$\frac{a^2cx + 2ac\log\left(x\right) - \frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2),x)
```

```
[Out] (a**2*c*x + 2*a*c*log(x) - c/x)/a**2
```

Giac [A]
time = 0.40, size = 22, normalized size = 1.05

$$cx + \frac{2\,c\log\left(|x|\right)}{a} - \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] c*x + 2*c*log(abs(x))/a - c/(a^2*x)
```

Mupad [B]
time = 0.04, size = 23, normalized size = 1.10

$$\frac{c\left(a^2\,x^2 + 2\,a\,x\,\ln\left(x\right) - 1\right)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1))/(a*x - 1),x)
```

```
[Out] (c*(a^2*x^2 + 2*a*x*log(x) - 1))/(a^2*x)
```

3.785 $\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}$$

```
[Out] x/c+1/a/c/(-a*x+1)+2*ln(-a*x+1)/a/c
```

Rubi [A]

time = 0.10, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 45}

$$\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c + 1/(a*c*(1 - a*x)) + (2*Log[1 - a*x])/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx\\
&= \frac{a^2\int \frac{e^{2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}\\
&= \frac{a^2\int \frac{x^2}{(1-ax)^2}\,dx}{c}\\
&= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(-1+ax)^2}+\frac{2}{a^2(-1+ax)}\right)dx}{c}\\
&= \frac{x}{c}+\frac{1}{ac(1-ax)}+\frac{2\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.78

$$\frac{x+\frac{1}{a-a^2x}+\frac{2\log(1-ax)}{a}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
[Out] (x + (a - a^2*x)^(-1) + (2*Log[1 - a*x])/a)/c
```

Maple [A]
time = 0.10, size = 37, normalized size = 1.03

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac(ax-1)}+\frac{2\ln(ax-1)}{ac}$	36
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{1}{a^3(ax-1)}+\frac{2\ln(ax-1)}{a^3}\right)}{c}$	37
norman	$\frac{\frac{a\,x^2}{c}-\frac{2x}{c}}{ax-1}+\frac{2\ln(ax-1)}{ac}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] a^2/c*(x/a^2-1/a^3/(a*x-1)+2/a^3*ln(a*x-1))
```

Maxima [A]

time = 0.26, size = 35, normalized size = 0.97

$$\frac{x}{c} - \frac{1}{a^2cx - ac} + \frac{2 \log(ax - 1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] x/c - 1/(a^2*c*x - a*c) + 2*log(a*x - 1)/(a*c)
```

Fricas [A]

time = 0.34, size = 40, normalized size = 1.11

$$\frac{a^2x^2 - ax + 2(ax - 1)\log(ax - 1) - 1}{a^2cx - ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*x^2 - a*x + 2*(a*x - 1)*log(a*x - 1) - 1)/(a^2*c*x - a*c)
```

Sympy [A]

time = 0.07, size = 36, normalized size = 1.00

$$a^2\left(-\frac{1}{a^4cx - a^3c} + \frac{x}{a^2c} + \frac{2\log(ax - 1)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2),x)
[Out] a**2*(-1/(a**4*c*x - a**3*c) + x/(a**2*c) + 2*log(a*x - 1)/(a**3*c))
```

Giac [A]

time = 0.39, size = 36, normalized size = 1.00

$$\frac{x}{c} + \frac{2 \log(|ax - 1|)}{ac} - \frac{1}{(ax - 1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] x/c + 2*log(abs(a*x - 1))/(a*c) - 1/((a*x - 1)*a*c)
```

Mupad [B]

time = 0.05, size = 33, normalized size = 0.92

$$\frac{x}{c} + \frac{1}{a(c - a c x)} + \frac{2 \ln(a x - 1)}{a c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))*(a*x - 1)),x)
[Out] x/c + 1/(a*(c - a*c*x)) + (2*log(a*x - 1))/(a*c)
```

3.786 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=75

$$\frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

```
[Out] x/c^2-1/4/a/c^2/(-a*x+1)^2+7/4/a/c^2/(-a*x+1)+17/8*ln(-a*x+1)/a/c^2-1/8*ln(
a*x+1)/a/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{7}{4ac^2(1-ax)}-\frac{1}{4ac^2(1-ax)^2}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx\\ &= -\frac{a^4\int \frac{e^{2\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}\\ &= -\frac{a^4\int \frac{x^4}{(1-ax)^3(1+ax)}\,dx}{c^2}\\ &= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{2a^4(-1+ax)^3}-\frac{7}{4a^4(-1+ax)^2}-\frac{17}{8a^4(-1+ax)}+\frac{1}{8a^4(1+ax)}\right)dx}{c^2}\\ &= \frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 75, normalized size = 1.00

$$\frac{x}{c^2}-\frac{1}{4ac^2(1-ax)^2}+\frac{7}{4ac^2(1-ax)}+\frac{17\log(1-ax)}{8ac^2}-\frac{\log(1+ax)}{8ac^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
[Out] x/c^2 - 1/(4*a*c^2*(1 - a*x)^2) + 7/(4*a*c^2*(1 - a*x)) + (17*Log[1 - a*x])
/(8*a*c^2) - Log[1 + a*x]/(8*a*c^2)
```

Maple [A]
time = 0.11, size = 60, normalized size = 0.80

method	result	size
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{\ln(ax+1)}{8a^5}+\frac{17\ln(ax-1)}{8a^5}-\frac{1}{4a^5(ax-1)^2}-\frac{7}{4a^5(ax-1)}\right)}{c^2}$	60
risch	$\frac{x}{c^2}+\frac{-\frac{7c^2x}{4}+\frac{3c^2}{2a}}{c^4(ax-1)^2}+\frac{17\ln(-ax+1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	62
norman	$\frac{\frac{a^3x^4}{c}+\frac{9x}{4c}-\frac{5a\,x^2}{4c}-\frac{5a^2x^3}{2c}}{(ax-1)^2(ax+1)c}+\frac{17\ln(ax-1)}{8a\,c^2}-\frac{\ln(ax+1)}{8a\,c^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^2*(x/a^4-1/8/a^5*ln(a*x+1)+17/8/a^5*ln(a*x-1)-1/4/a^5/(a*x-1)^2-7/4/a
^5/(a*x-1))
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.92

$$-\frac{7\,ax-6}{4\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}+\frac{x}{c^2}-\frac{\log(ax+1)}{8\,ac^2}+\frac{17\,\log(ax-1)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x - 6)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2) + x/c^2 - 1/8*log(a*x
+ 1)/(a*c^2) + 17/8*log(a*x - 1)/(a*c^2)
```

Fricas [A]
time = 0.33, size = 93, normalized size = 1.24

$$\frac{8\,a^3x^3-16\,a^2x^2-6\,ax-(a^2x^2-2\,ax+1)\log(ax+1)+17\,(a^2x^2-2\,ax+1)\log(ax-1)+12}{8\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 - 16*a^2*x^2 - 6*a*x - (a^2*x^2 - 2*a*x + 1)*log(a*x + 1) +
17*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 12)/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*
c^2)
```

Sympy [A]
time = 0.22, size = 73, normalized size = 0.97

$$a^4\left(\frac{-7ax+6}{4a^7c^2x^2-8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{17\log\left(x-\frac{1}{a}\right)}{8}-\frac{\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-7*a*x + 6)/(4*a**7*c**2*x**2 - 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (17*log(x - 1/a)/8 - log(x + 1/a)/8)/(a**5*c**2))
```

Giac [A]
time = 0.39, size = 57, normalized size = 0.76

$$\frac{x}{c^2}-\frac{\log(|ax+1|)}{8\,ac^2}+\frac{17\,\log(|ax-1|)}{8\,ac^2}-\frac{7\,ax-6}{4\,(ax-1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 1/8*log(abs(a*x + 1))/(a*c^2) + 17/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x - 6)/((a*x - 1)^2*a*c^2)
```

Mupad [B]
time = 0.09, size = 68, normalized size = 0.91

$$\frac{x}{c^2} - \frac{\frac{7\,x}{4} - \frac{3}{2\,a}}{a^2\,c^2\,x^2 - 2\,a\,c^2\,x + c^2} + \frac{17\,\ln\left(a\,x - 1\right)}{8\,a\,c^2} - \frac{\ln\left(a\,x + 1\right)}{8\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^2*(a*x - 1)),x)
```

```
[Out] x/c^2 - ((7*x)/4 - 3/(2*a))/(c^2 + a^2*c^2*x^2 - 2*a*c^2*x) + (17*log(a*x -
 1))/(8*a*c^2) - log(a*x + 1)/(8*a*c^2)
```

3.787 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=110

$$\frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(1+ax)}{4ac^3}$$

```
[Out] x/c^3+1/12/a/c^3/(-a*x+1)^3-5/8/a/c^3/(-a*x+1)^2+39/16/a/c^3/(-a*x+1)-1/16/
a/c^3/(a*x+1)+9/4*ln(-a*x+1)/a/c^3-1/4*ln(a*x+1)/a/c^3
```

Rubi [A]
time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(ax+1)}-\frac{5}{8ac^3(1-ax)^2}+\frac{1}{12ac^3(1-ax)^3}+\frac{9\log(1-ax)}{4ac^3}-\frac{\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 + 1/(12*a*c^3*(1 - a*x)^3) - 5/(8*a*c^3*(1 - a*x)^2) + 39/(16*a*c^3*(
1 - a*x)) - 1/(16*a*c^3*(1 + a*x)) + (9*Log[1 - a*x])/(4*a*c^3) - Log[1 + a
*x]/(4*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx\\ &= \frac{a^6\int \frac{e^{2\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3}\\ &= \frac{a^6\int \frac{x^6}{(1-ax)^4(1+ax)^2}\,dx}{c^3}\\ &= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{4a^6(-1+ax)^4}+\frac{5}{4a^6(-1+ax)^3}+\frac{39}{16a^6(-1+ax)^2}+\frac{9}{4a^6(-1+ax)}+\frac{1}{16a^6(1+ax)^2}-\frac{1}{4a^6(1+ax)}\right.}{c^3}\\ &= \frac{x}{c^3}+\frac{1}{12ac^3(1-ax)^3}-\frac{5}{8ac^3(1-ax)^2}+\frac{39}{16ac^3(1-ax)}-\frac{1}{16ac^3(1+ax)}+\frac{9\log(1-a}{4ac^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 82, normalized size = 0.75

$$\frac{\frac{2(-11+7ax+24a^2x^2-15a^3x^3-12a^4x^4+6a^5x^5)}{(-1+ax)^3(1+ax)}+27\log(1-ax)-3\log(1+ax)}{12ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((2*(-11 + 7*a*x + 24*a^2*x^2 - 15*a^3*x^3 - 12*a^4*x^4 + 6*a^5*x^5))/((-1
+ a*x)^3*(1 + a*x)) + 27*Log[1 - a*x] - 3*Log[1 + a*x])/(12*a*c^3)
```

Maple [A]
time = 0.12, size = 84, normalized size = 0.76

method	result	size
default	$\frac{a^6\left(\frac{x}{a^6}-\frac{1}{16a^7(ax+1)}-\frac{\ln(ax+1)}{4a^7}-\frac{1}{12a^7(ax-1)^3}-\frac{5}{8a^7(ax-1)^2}-\frac{39}{16a^7(ax-1)}+\frac{9\ln(ax-1)}{4a^7}\right)}{c^3}$	84
risch	$\frac{x}{c^3}+\frac{-\frac{5a^2c^3x^3}{2}+2c^3a\,x^2+\frac{13c^3x}{6}-\frac{11c^3}{6a}}{c^6(ax-1)^2(a^2x^2-1)}-\frac{\ln(ax+1)}{4a\,c^3}+\frac{9\ln(-ax+1)}{4a\,c^3}$	93
norman	$\frac{\frac{a^5x^6}{c}-\frac{5x}{2c}+\frac{3a\,x^2}{2c}+\frac{31a^2x^3}{6c}-\frac{8a^3x^4}{3c}-\frac{17a^4x^5}{6c}}{(ax-1)^3(ax+1)^2c^2}+\frac{9\ln(ax-1)}{4c^3a}-\frac{\ln(ax+1)}{4a\,c^3}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^6/c^3*(1/a^6*x-1/16/a^7/(a*x+1)-1/4/a^7*ln(a*x+1)-1/12/a^7/(a*x-1)^3-5/8/
a^7/(a*x-1)^2-39/16/a^7/(a*x-1)+9/4/a^7*ln(a*x-1))
```

Maxima [A]
time = 0.26, size = 97, normalized size = 0.88

$$-\frac{15\,a^3x^3-12\,a^2x^2-13\,ax+11}{6\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{\log(ax+1)}{4\,ac^3}+\frac{9\,\log(ax-1)}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 +
 2*a^2*c^3*x - a*c^3) + x/c^3 - 1/4*log(a*x + 1)/(a*c^3) + 9/4*log(a*x - 1)
/(a*c^3)
```

Fricas [A]
time = 0.35, size = 137, normalized size = 1.25

$$\frac{12\,a^5x^5-24\,a^4x^4-30\,a^3x^3+48\,a^2x^2+14\,ax-3\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax+1)+27\,(a^4x^4-2\,a^3x^3+2\,ax-1)\log(ax-1)-22}{12\,(a^5c^3x^4-2\,a^4c^3x^3+2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*a^5*x^5 - 24*a^4*x^4 - 30*a^3*x^3 + 48*a^2*x^2 + 14*a*x - 3*(a^4*x
^4 - 2*a^3*x^3 + 2*a*x - 1)*log(a*x + 1) + 27*(a^4*x^4 - 2*a^3*x^3 + 2*a*x
- 1)*log(a*x - 1) - 22)/(a^5*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3)
```

Sympy [A]
time = 0.54, size = 102, normalized size = 0.93

$$a^6\left(\frac{-15a^3x^3+12a^2x^2+13ax-11}{6a^{11}c^3x^4-12a^{10}c^3x^3+12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{9\log\left(x-\frac{1}{a}\right)}{4}-\frac{\log\left(x+\frac{1}{a}\right)}{4}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-15*a**3*x**3 + 12*a**2*x**2 + 13*a*x - 11)/(6*a**11*c**3*x**4 - 12*
a**10*c**3*x**3 + 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (9*log(x
- 1/a)/4 - log(x + 1/a)/4)/(a**7*c**3))
```

Giac [A]
time = 0.40, size = 80, normalized size = 0.73

$$\frac{x}{c^3} - \frac{\log(|ax+1|)}{4\,ac^3} + \frac{9\,\log(|ax-1|)}{4\,ac^3} - \frac{15\,a^3x^3 - 12\,a^2x^2 - 13\,ax + 11}{6\,(ax+1)(ax-1)^3ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 1/4*log(abs(a*x + 1))/(a*c^3) + 9/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 - 12*a^2*x^2 - 13*a*x + 11)/((a*x + 1)*(a*x - 1)^3*a*c^3)
```

Mupad [B]
time = 1.28, size = 94, normalized size = 0.85

$$\frac{x}{c^3} - \frac{\frac{13\,x}{6} + 2\,a\,x^2 - \frac{11}{6\,a} - \frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4 + 2\,a^3\,c^3\,x^3 - 2\,a\,c^3\,x + c^3} + \frac{9\,\ln(a\,x-1)}{4\,a\,c^3} - \frac{\ln(a\,x+1)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^3*(a*x - 1)),x)
```

```
[Out] x/c^3 - ((13*x)/6 + 2*a*x^2 - 11/(6*a) - (5*a^2*x^3)/2)/(c^3 + 2*a^3*c^3*x^
3 - a^4*c^3*x^4 - 2*a*c^3*x) + (9*log(a*x - 1))/(4*a*c^3) - log(a*x + 1)/(4
*a*c^3)
```

3.788 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=145

$$\frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)}+\frac{1}{64ac^4(1+ax)^2}-\frac{11}{64ac^4(1+ax)}+\frac{30}{}$$

```
[Out] x/c^4-1/32/a/c^4/(-a*x+1)^4+13/48/a/c^4/(-a*x+1)^3-35/32/a/c^4/(-a*x+1)^2+9
9/32/a/c^4/(-a*x+1)+1/64/a/c^4/(a*x+1)^2-11/64/a/c^4/(a*x+1)+303/128*ln(-a*
x+1)/a/c^4-47/128*ln(a*x+1)/a/c^4
```

Rubi [A]
time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{99}{32ac^4(1-ax)}-\frac{11}{64ac^4(ax+1)}-\frac{35}{32ac^4(1-ax)^2}+\frac{1}{64ac^4(ax+1)^2}+\frac{13}{48ac^4(1-ax)^3}-\frac{1}{32ac^4(1-ax)^4}+\frac{303\log(1-ax)}{128ac^4}-\frac{47\log(ax+1)}{128ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 - 1/(32*a*c^4*(1 - a*x)^4) + 13/(48*a*c^4*(1 - a*x)^3) - 35/(32*a*c^4
*(1 - a*x)^2) + 99/(32*a*c^4*(1 - a*x)) + 1/(64*a*c^4*(1 + a*x)^2) - 11/(64
*a*c^4*(1 + a*x)) + (303*Log[1 - a*x])/(128*a*c^4) - (47*Log[1 + a*x])/(128
*a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx\\ &= -\frac{a^8\int \frac{e^{2\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}\\ &= -\frac{a^8\int \frac{x^8}{(1-ax)^5(1+ax)^3}\,dx}{c^4}\\ &= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{8a^8(-1+ax)^5}-\frac{13}{16a^8(-1+ax)^4}-\frac{35}{16a^8(-1+ax)^3}-\frac{99}{32a^8(-1+ax)^2}-\frac{303}{128a^8(-1+ax)}+\right.}{c^4}\\ &= \frac{x}{c^4}-\frac{1}{32ac^4(1-ax)^4}+\frac{13}{48ac^4(1-ax)^3}-\frac{35}{32ac^4(1-ax)^2}+\frac{99}{32ac^4(1-ax)}+\frac{1}{64ac^4(1}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 98, normalized size = 0.68

$$\frac{\frac{2(400-275ax-1258a^2x^2+866a^3x^3+1254a^4x^4-819a^5x^5-384a^6x^6+192a^7x^7)}{(-1+ax)^4(1+ax)^2}+909\log(1-ax)-141\log(1+ax)}{384ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
[Out] ((2*(400 - 275*a*x - 1258*a^2*x^2 + 866*a^3*x^3 + 1254*a^4*x^4 - 819*a^5*x^
5 - 384*a^6*x^6 + 192*a^7*x^7))/((-1 + a*x)^4*(1 + a*x)^2) + 909*Log[1 - a*
x] - 141*Log[1 + a*x])/(384*a*c^4)
```

Maple [A]
time = 0.12, size = 108, normalized size = 0.74

method	result	size
default	$\frac{a^8\left(\frac{x}{a^8}+\frac{1}{64a^9(ax+1)^2}-\frac{11}{64a^9(ax+1)}-\frac{47\ln(ax+1)}{128a^9}-\frac{1}{32a^9(ax-1)^4}-\frac{13}{48a^9(ax-1)^3}-\frac{35}{32a^9(ax-1)^2}-\frac{99}{32a^9(ax-1)}+\frac{303\ln(ax-1)}{128a^9}\right)}{c^4}$	108

risch	$\frac{x}{c^4}+\frac{-\frac{209a^4c^4x^5}{64}+\frac{81a^3c^4x^4}{32}+\frac{529a^2c^4x^3}{96}-\frac{437c^4a\,x^2}{96}-\frac{467c^4x}{192}+\frac{25c^4}{12a}}{c^8(ax-1)^2(a^2x^2-1)^2}+\frac{303\ln(-ax+1)}{128a\,c^4}-\frac{47\ln(ax+1)}{128a\,c^4}$	115
norman	$\frac{\frac{a^7x^8}{c}+\frac{175x}{64c}-\frac{111a\,x^2}{64c}-\frac{199a^2x^3}{24c}+\frac{115a^3x^4}{24c}+\frac{545a^4x^5}{64c}-\frac{803a^5x^6}{192c}-\frac{37a^6x^7}{12c}}{(ax-1)^4(ax+1)^3c^3}+\frac{303\ln(ax-1)}{128c^4a}-\frac{47\ln(ax+1)}{128a\,c^4}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^8/c^4*(1/a^8*x+1/64/a^9/(a*x+1)^2-11/64/a^9/(a*x+1)-47/128/a^9*ln(a*x+1)-
1/32/a^9/(a*x-1)^4-13/48/a^9/(a*x-1)^3-35/32/a^9/(a*x-1)^2-99/32/a^9/(a*x-1
)+303/128/a^9*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 145, normalized size = 1.00

$$-\frac{627\,a^5x^5-486\,a^4x^4-1058\,a^3x^3+874\,a^2x^2+467\,ax-400}{192\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}+\frac{x}{c^4}-\frac{47\,\log{(ax+1)}}{128\,ac^4}+\frac{303\,\log{(ax-1)}}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x -
400)/(a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x
^2 - 2*a^2*c^4*x + a*c^4) + x/c^4 - 47/128*log(a*x + 1)/(a*c^4) + 303/128*l
og(a*x - 1)/(a*c^4)
```

Fricas [A]
time = 0.37, size = 233, normalized size = 1.61

$$\frac{384\,a^7x^7-768\,a^6x^6-1638\,a^5x^5+2508\,a^4x^4+1732\,a^3x^3-2516\,a^2x^2-550\,ax-141\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\log{(ax+1)}+909\,(a^6x^6-2\,a^5x^5-a^4x^4+4\,a^3x^3-a^2x^2-2\,ax+1)\log{(ax-1)}+800}{384\,(a^7c^4x^6-2\,a^6c^4x^5-a^5c^4x^4+4\,a^4c^4x^3-a^3c^4x^2-2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 - 768*a^6*x^6 - 1638*a^5*x^5 + 2508*a^4*x^4 + 1732*a^3*x
^3 - 2516*a^2*x^2 - 550*a*x - 141*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^
3 - a^2*x^2 - 2*a*x + 1)*log(a*x + 1) + 909*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4
+ 4*a^3*x^3 - a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 800)/(a^7*c^4*x^6 - 2*a^6
*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x + a*c^4)
```

Sympy [A]
time = 0.73, size = 156, normalized size = 1.08

$$a^8\left(\frac{-627a^5x^5+486a^4x^4+1058a^3x^3-874a^2x^2-467ax+400}{192a^{15}c^4x^6-384a^{14}c^4x^5-192a^{13}c^4x^4+768a^{12}c^4x^3-192a^{11}c^4x^2-384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{303\log{\left(x-\frac{1}{a}\right)}}{128}-\frac{47\log{\left(x+\frac{1}{a}\right)}}{128}}{a^9c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-627*a**5*x**5 + 486*a**4*x**4 + 1058*a**3*x**3 - 874*a**2*x**2 - 46
7*a*x + 400)/(192*a**15*c**4*x**6 - 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 + 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 - 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (303*log(x - 1/a)/128 - 47*log(x + 1/a)/128)/(a**
9*c**4))
```

Giac [A]
time = 0.41, size = 96, normalized size = 0.66

$$\frac{x}{c^4} - \frac{47 \log (|ax+1|)}{128\,ac^4} + \frac{303 \log (|ax-1|)}{128\,ac^4} - \frac{627\,a^5x^5 - 486\,a^4x^4 - 1058\,a^3x^3 + 874\,a^2x^2 + 467\,ax - 400}{192\,(ax+1)^2(ax-1)^4ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 47/128*log(abs(a*x + 1))/(a*c^4) + 303/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 - 486*a^4*x^4 - 1058*a^3*x^3 + 874*a^2*x^2 + 467*a*x
 - 400)/((a*x + 1)^2*(a*x - 1)^4*a*c^4)
```

Mupad [B]
time = 1.45, size = 142, normalized size = 0.98

$$\frac{x}{c^4} + \frac{\frac{467\,x}{192} + \frac{437\,a\,x^2}{96} - \frac{25}{12\,a} - \frac{529\,a^2\,x^3}{96} - \frac{81\,a^3\,x^4}{32} + \frac{209\,a^4\,x^5}{64}}{-a^6\,c^4\,x^6 + 2\,a^5\,c^4\,x^5 + a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 + a^2\,c^4\,x^2 + 2\,a\,c^4\,x - c^4} + \frac{303\,\ln(a\,x-1)}{128\,a\,c^4} - \frac{47\,\ln(a\,x+1)}{128\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^4*(a*x - 1)),x)
```

```
[Out] x/c^4 + ((467*x)/192 + (437*a*x^2)/96 - 25/(12*a) - (529*a^2*x^3)/96 - (81*
a^3*x^4)/32 + (209*a^4*x^5)/64)/(a^2*c^4*x^2 - c^4 - 4*a^3*c^4*x^3 + a^4*c^
4*x^4 + 2*a^5*c^4*x^5 - a^6*c^4*x^6 + 2*a*c^4*x) + (303*log(a*x - 1))/(128*
a*c^4) - (47*log(a*x + 1))/(128*a*c^4)
```

3.789 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$-\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}-\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}-\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\right.}{280a}$$

```
[Out] 8/7*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(11/2)/a+c^4*(1-1/a/x)^(5/2)*(1+1/a/x)^(1
1/2)*x+15/16*c^4*arccsc(a*x)/a+3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2
))/a-37/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-61/40*c^4*(1+1/a/x)^(5/2)*
(1-1/a/x)^(1/2)/a-303/280*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-57/70*c^4*(
1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a+15/14*c^4*(1+1/a/x)^(11/2)*(1-1/a/x)^(1/2)
/a-63/16*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.16, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$\frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{11/2}}{7a}+c^4x\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{11/2}+\frac{15c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{11/2}}{14a}-\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{70a}-\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{280a}-\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{40a}-\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{16a}-\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{16a}+\frac{15c^4\csc^{-1}(ax)}{16a}+\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (-63*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (37*c^4*Sqrt[1 - 1/(
a*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (61*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))
^(5/2))/(40*a) - (303*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(280*a) -
(57*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(70*a) + (15*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(11/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*
x))^(11/2))/(7*a) + c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(11/2)*x + (15*c^
4*ArcCsc[a*x])/(16*a) + (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]
)/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}{x^2}dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x - c^4\text{Subst}\left(\int\frac{\left(\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right.}{x}\right.$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{11/2}x - \frac{1}{7}\left(ac^4\right)\text{Su}$$

$$= \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{11/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{5}$$

$$= -\frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{11/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{3}}{}$$

$$= -\frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{70a} + \frac{15c^4\sqrt{1-}}{}$$

$$= -\frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{280a} - \frac{57c^4\sqrt{1-}}{7}$$

$$= -\frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} - \frac{303c^4\sqrt{1-}}{28}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}}{40}$$

$$= -\frac{63c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{37c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{61c^4\sqrt{1-\frac{1}{ax}}}{40}$$

Mathematica [A]

time = 0.13, size = 126, normalized size = 0.37

$$\frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}\left(-80-280ax-96a^2x^2+770a^3x^3+992a^4x^4-525a^5x^5-2496a^6x^6+560a^7x^7\right)+525a^6x^6\text{ArcSin}\left(\frac{1}{ax}\right)+1680a^6x^6\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{560a^7x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(-80 - 280*a*x - 96*a^2*x^2 + 770*a^3*x^3 + 992
*a^4*x^4 - 525*a^5*x^5 - 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1
/(a*x)] + 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

Maple [A]

time = 0.09, size = 329, normalized size = 0.96

method	result
risch	$\frac{(ax-1)(560a^7x^7-2496a^6x^6-525a^5x^5+992a^4x^4+770a^3x^3-96a^2x^2-280ax-80)c^4}{560x^7a^8\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{15a^7\operatorname{arc}}{}\right.}{a^8(a}$
default	$\frac{(ax-1)^2c^4\left(-1680\sqrt{a^2x^2-1}\sqrt{a^2}a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}a^6x^6+525a^7x^7\sqrt{a^2}\sqrt{a^2x^2-1}+525a^7x^7\sqrt{a^2}\operatorname{arct}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/560*(a*x-1)^2*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^8*x^8+1680*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2)*a^6*x^6+525*a^7*x^7*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)+
525*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+1680*ln((a^2*x+(a^2*x^2
-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^8*x^7+35*(a^2*x^2-1)^(3/2)*(a^2)^(1/2
)*a^5*x^5-816*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-490*(a^2*x^2-1)^(3/2)*(
a^2)^(1/2)*a^3*x^3+176*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+280*(a^2)^(1/2
)*(a^2*x^2-1)^(3/2)*a*x+80*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))
^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

Maxima [A]

time = 0.47, size = 380, normalized size = 1.11

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{2205\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+13615\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}+33621\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+39071\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+12799\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-20811\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-7665\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-1155\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{6(ax-1)a^2}{ax+1}+\frac{14(ax-1)^2a^2}{(ax+1)^2}+\frac{14(ax-1)^3a^2}{(ax+1)^3}-\frac{14(ax-1)^5a^2}{(ax+1)^5}-\frac{14(ax-1)^6a^2}{(ax+1)^6}-\frac{6(ax-1)^7a^2}{(ax+1)^7}-\frac{(ax-1)^8a^2}{(ax+1)^8}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 + 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 - (2205*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 13615*c^4*((a*x - 1)/(a*x + 1)
)^(13/2) + 33621*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 39071*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 12799*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 20811*c^4*((a*x -
1)/(a*x + 1))^(5/2) - 7665*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 1155*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [A]
time = 0.37, size = 201, normalized size = 0.59

$$-\frac{1050\,a^{7}c^{4}x^{7}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(560\,a^{8}c^{4}x^{8}-1936\,a^{7}c^{4}x^{7}-3021\,a^{6}c^{4}x^{6}+467\,a^{5}c^{4}x^{5}+1762\,a^{4}c^{4}x^{4}+674\,a^{3}c^{4}x^{3}-376\,a^{2}c^{4}x^{2}-360\,ac^{4}x-80\,c^{4}\right)\sqrt{\frac{ax-1}{ax+1}}}{560\,a^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) - 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) + 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 - 1936*a^7*c^4*x^7 - 3021*a^6*c^4*x^6 +
 467*a^5*c^4*x^5 + 1762*a^4*c^4*x^4 + 674*a^3*c^4*x^3 - 376*a^2*c^4*x^2 - 3
60*a*c^4*x - 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{4}\left(\int\left(-\frac{4a^{2}}{\frac{ax^{7}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{6a^{4}}{\frac{ax^{5}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{4a^{6}}{\frac{ax^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^{8}}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^{9}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^{8}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**4,x)
```

```
[Out] c**4*(Integral(-4*a**2/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(6*a**4/(
a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-4*a**6/(a*x**3*sqrt(a*x/(a*x +
1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1)), x) + Integral(a**8/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(1/(a*x**9*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**8*sqrt(a*x/(a*x + 1) - 1/(a*x
+ 1))/(a*x + 1)), x))/a**8
```

Giac [A]
time = 0.44, size = 461, normalized size = 1.34

$$-\frac{15\,c^4\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{8\,a\,\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c^4\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^4}{a\,\mathrm{sgn}\left(ax+1\right)}+\frac{525\left(x|a|-\sqrt{a^2x^2-1}\right)^{13}c^4|a|-4480\left(x|a|-\sqrt{a^2x^2-1}\right)^{12}ac^4-980\left(x|a|-\sqrt{a^2x^2-1}\right)^{11}c^4|a|-20160\left(x|a|-\sqrt{a^2x^2-1}\right)^{10}ac^4+945\left(x|a|-\sqrt{a^2x^2-1}\right)^{9}c^4|a|-38080\left(x|a|-\sqrt{a^2x^2-1}\right)^{8}ac^4-49280\left(x|a|-\sqrt{a^2x^2-1}\right)^{6}ac^4-945\left(x|a|-\sqrt{a^2x^2-1}\right)^{5}c^4|a|-32256\left(x|a|-\sqrt{a^2x^2-1}\right)^{4}ac^4+980\left(x|a|-\sqrt{a^2x^2-1}\right)^{3}c^4|a|-12992\left(x|a|-\sqrt{a^2x^2-1}\right)^{2}ac^4-525\left(x|a|-\sqrt{a^2x^2-1}\right)c^4|a|-2496\,ac^4}{280\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^7a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^4*lo
g(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2
- 1)*c^4/(a*sgn(a*x + 1)) + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^
4*abs(a) - 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4 - 980*(x*abs(a) - s
qrt(a^2*x^2 - 1))^11*c^4*abs(a) - 20160*(x*abs(a) - sqrt(a^2*x^2 - 1))^10*a
*c^4 + 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^4*abs(a) - 38080*(x*abs(a) -
sqrt(a^2*x^2 - 1))^8*a*c^4 - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4 -
 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a) - 32256*(x*abs(a) - sqrt(a
^2*x^2 - 1))^4*a*c^4 + 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^4*abs(a) - 12
992*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^4 - 525*(x*abs(a) - sqrt(a^2*x^2 -
 1))*c^4*abs(a) - 2496*a*c^4)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^7*a*a
bs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.38, size = 332, normalized size = 0.97

$$\frac{\frac{12799\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{280}-\frac{219\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{8}-\frac{2973\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{40}-\frac{33\,c^4\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{39071\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{280}+\frac{4803\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{11/2}}{40}+\frac{389\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{13/2}}{8}+\frac{63\,c^4\left(\frac{a\,x-1}{a\,x+1}\right)^{15/2}}{8}}{a+\frac{6\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{14\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{14\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{14\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{14\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{6\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}-\frac{a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}}-\frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}+\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((12799*c^4*((a*x - 1)/(a*x + 1))^(7/2))/280 - (219*c^4*((a*x - 1)/(a*x + 1
))^(3/2))/8 - (2973*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 - (33*c^4*((a*x - 1
)/(a*x + 1))^(1/2))/8 + (39071*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (4803
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (389*c^4*((a*x - 1)/(a*x + 1))^(13/
2))/8 + (63*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x
+ 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (1
4*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x -
 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((a*x - 1)
/(a*x + 1))^(1/2)))/(8*a) + (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.790 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\right.}{20a}$$

```
[Out] c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(9/2)*x+3/8*c^3*arccsc(a*x)/a+3*c^3*arctanh((
1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-17/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/
a-29/20*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-21/20*c^3*(1+1/a/x)^(7/2)*(1-
1/a/x)^(1/2)/a+6/5*c^3*(1+1/a/x)^(9/2)*(1-1/a/x)^(1/2)/a-27/8*c^3*(1-1/a/x)
^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.13, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^3x\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}+\frac{6c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}{5a}-\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}{20a}-\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{20a}-\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{8a}-\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{8a}+\frac{3c^3\csc^{-1}(ax)}{8a}+\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (-27*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) - (17*c^3*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(8*a) - (29*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(20*a) - (21*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(20*a) + (6*c
^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(9/2))/(5*a) + c^3*(1 - 1/(a*x))^(3/2)*(
1 + 1/(a*x))^(9/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) + (3*c^3*ArcTanh[Sqrt[1 -
1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x - c^3\text{Subst}\left(\int \frac{\left(\frac{3}{a}-\frac{6x}{a^2}\right)\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/}}{x}\right.$$

$$= \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}x - \frac{1}{5}\left(ac^3\right)\text{Subs}$$

$$= -\frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{3/}$$

$$= -\frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{20a} + \frac{6c^3\sqrt{1-\frac{1}{ax}}}{5a}$$

$$= -\frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{20a} - \frac{21c^3\sqrt{1-\frac{1}{a}}}{20}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}}{20}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}}{20}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}}{20}$$

$$= -\frac{27c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} - \frac{17c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} - \frac{29c^3\sqrt{1-\frac{1}{ax}}}{20}$$

Mathematica [A]
time = 0.11, size = 110, normalized size = 0.41

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\,(8+30ax+24a^2x^2-55a^3x^3-152a^4x^4+40a^5x^5)+15a^4x^4\text{ArcSin}\left(\frac{1}{ax}\right)+120a^4x^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(8 + 30*a*x + 24*a^2*x^2 - 55*a^3*x^3 - 152*a^4
*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] + 120*a^4*x^4*Log[(1 + Sqrt
[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Maple [A]
time = 0.08, size = 281, normalized size = 1.04

method	result
risch	$-\frac{(ax-1)(152a^4x^4+55a^3x^3-24a^2x^2-30ax-8)c^3}{40x^5a^6\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^5\sqrt{(ax+1)(ax-1)}+\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+\frac{3a^5\text{arct}}{}\right.}{a^6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$\frac{(ax-1)^2c^3\left(-120\sqrt{a^2}\sqrt{a^2x^2-1}\,a^6x^6+120\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4+15\sqrt{a^2x^2-1}\sqrt{a^2}\,a^5x^5+15\sqrt{a^2}\,\arctan\left(\frac{}{\sqrt{a}}\right.\right.}{40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(a*x-1)^2*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a^6*x^6+120*(a^2)^(1
/2)*(a^2*x^2-1)^(3/2)*a^4*x^4+15*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^5*x^5+15*(
a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x^5+120*ln((a^2*x+(a^2*x^2-1)^(1
/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5+25*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*
x^3-32*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-30*(a^2)^(1/2)*(a^2*x^2-1)^(3/
2)*a*x-8*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a
*x+1)*(a*x-1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A]
time = 0.47, size = 302, normalized size = 1.12

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{135\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+575\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+842\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+298\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-465\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-105\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4(ax-1)a^2}{ax+1}+\frac{5(ax-1)^2a^2}{(ax+1)^2}-\frac{5(ax-1)^4a^2}{(ax+1)^4}-\frac{4(ax-1)^5a^2}{(ax+1)^5}-\frac{(ax-1)^6a^2}{(ax+1)^6}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 + 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
(135*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 575*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 + 842*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 298*c^3*((a*x - 1)/(a*x + 1))^(5/2
) - 465*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 105*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^
4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x +
1)^6 + a^2))*a
```

Fricas [**A**]
time = 0.35, size = 179, normalized size = 0.67

$$-\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(40\,a^6c^3x^6-112\,a^5c^3x^5-207\,a^4c^3x^4-31\,a^3c^3x^3+54\,a^2c^3x^2+38\,ac^3x+8\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) - 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) + 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 - 112*a^5*c^3*x^5 - 207*a^4*c^3*x^4 - 31*a^3*
c^3*x^3 + 54*a^2*c^3*x^2 + 38*a*c^3*x + 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(
a^6*x^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{3a^2}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{3a^4}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^6}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**3,x)
```

```
[Out] c**3*(Integral(3*a**2/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-3*a**4/(
a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1
) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**6/(a*x*sqrt(a*x/(a*x + 1) - 1
/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) +
Integral(-1/(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**6*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**6
```

Giac [**A**]
time = 0.42, size = 355, normalized size = 1.32

$$-\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{4\,a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^3}{a\mathrm{sgn}\left(ax+1\right)}+\frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|-200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3-10\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|-720\left(x|a|-\sqrt{a^2x^2-1}\right)^6ac^3-800\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^3+10\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|-560\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3-55\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|-152\,ac^3}{20\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^5a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^3*log
(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 -
 1)*c^3/(a*sgn(a*x + 1)) + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*ab
s(a) - 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3 - 10*(x*abs(a) - sqrt(a^2
*x^2 - 1))^7*c^3*abs(a) - 720*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^3 - 800*
(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^3 + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^
3*c^3*abs(a) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3 - 55*(x*abs(a) -
sqrt(a^2*x^2 - 1))*c^3*abs(a) - 152*a*c^3)/(((x*abs(a) - sqrt(a^2*x^2 - 1))
^2 + 1)^5*a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 1.41, size = 258, normalized size = 0.96

$$\frac{\frac{149\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}-\frac{21\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{421\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{10}+\frac{115\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}+\frac{27\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}+\frac{6\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^3/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((149*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(
3/2))/4 - (21*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (421*c^3*((a*x - 1)/(a*x
 + 1))^(7/2))/10 + (115*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 + (27*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2
)))/(4*a) + (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.791 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}-\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{6a}-\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{3a}+c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)$$

```
[Out] -1/2*c^2*arccsc(a*x)/a+3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a-11/
6*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-4/3*c^2*(1+1/a/x)^(5/2)*(1-1/a/x)^(
1/2)/a+c^2*(1+1/a/x)^(7/2)*x*(1-1/a/x)^(1/2)-5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a
/x)^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^2x\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}-\frac{4c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}{3a}-\frac{11c^2\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}{6a}-\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}+\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) - (11*c^2*Sqrt[1 - 1/(a*
x)]*(1 + 1/(a*x))^(3/2))/(6*a) - (4*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(3*a) + c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)*x - (c^2*ArcCsc[a*x])
/(2*a) + (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\left(c^2\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{7/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x - c^2\text{Subst}\left(\int \frac{\left(\frac{3}{a}-\frac{4x}{a^2}\right)\left(1+\frac{x}{a}\right)^{5/2}}{x\sqrt{1-\frac{x}{a}}}dx, x, \frac{1}{x}\right)$$

$$= -\frac{4c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}x + \frac{1}{3}\left(ac^2\right)\text{Subst}$$

$$= -\frac{11c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{5/2}}{3a} + c^2\sqrt{1-\frac{1}{ax}}\left(\right.$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\,\left(\right.}{3a}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\,\left(\right.}{3a}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\,\left(\right.}{3a}$$

$$= -\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} - \frac{11c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}}{6a} - \frac{4c^2\sqrt{1-\frac{1}{ax}}\,\left(\right.}{3a}$$

Mathematica [A]

time = 0.08, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\,(-2-9ax-16a^2x^2+6a^3x^3)-3a^2x^2\text{ArcSin}\left(\frac{1}{ax}\right)+18a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 - 9*a*x - 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x
^2*ArcSin[1/(a*x)] + 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3
*x^2)
```

Maple [A]
time = 0.06, size = 233, normalized size = 1.19

method	result
risch	$-\frac{(ax-1)(16a^2x^2+9ax+2)c^2}{6x^3a^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a^3\sqrt{(ax+1)(ax-1)}+\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{a^3\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)}{2}\right.}{a^4(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c^2\left(18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+3a^3x^3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)-18(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-\right.}{6\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(ax+1)(ax-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(a*x-1)^2*c^2*(18*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^4*x^4+3*a^3*x^3*(a^2
)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-18*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^
2+3*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^3*x^3-18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a
^2)^(1/2))/(a^2)^(1/2))*a^4*x^3-9*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-2*(a^2*
x^2-1)^(3/2)*(a^2)^(1/2))/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1)*(a*x-1))
^(1/2)/a^4/x^3/(a^2)^(1/2)
```

Maxima [A]
time = 0.46, size = 223, normalized size = 1.14

$$\frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{15c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+37c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+17c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-21c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 - 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 + (15
*c^2*((a*x - 1)/(a*x + 1))^(7/2) + 37*c^2*((a*x - 1)/(a*x + 1))^(5/2) + 17*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 21*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))
```

Fricas [A]
time = 0.36, size = 156, normalized size = 0.80

$$\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^2x^4-10\,a^3c^2x^3-25\,a^2c^2x^2-11\,ac^2x-2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) - 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^2*x^4 - 10*a^3*c^2*x^3 - 25*a^2*c^2*x^2 - 11*a*c^2*x - 2*
c^2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{2a^2}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx+\int\frac{a^4}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\frac{1}{\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2/(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
- x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(a**4/(a*
x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*x/(a*x + 1) - 1/(a*x
 + 1))/(a*x + 1)), x) + Integral(1/(a*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/(a*x + 1) - x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x))/a**4
```

Giac [A]
time = 0.43, size = 248, normalized size = 1.27

$$\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c^2}{a\mathrm{sgn}\left(ax+1\right)}+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|-12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2-36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2-9\left(x|a|-\sqrt{a^2x^2-1}\right)c^2|a|-16\,ac^2}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c^2*log(abs(
-x*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c
^2/(a*sgn(a*x + 1)) + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a) -
12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2 - 36*(x*abs(a) - sqrt(a^2*x^2 - 1
))^2*a*c^2 - 9*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^2*abs(a) - 16*a*c^2)/(((x*a
bs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^3*a*abs(a)*sgn(a*x + 1))
```

Mupad [B]
time = 0.14, size = 183, normalized size = 0.94

$$\frac{\frac{17\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{3}-7\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{3}+5\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{a+\frac{2\,a\,(a\,x-1)}{a\,x+1}-\frac{2\,a\,(a\,x-1)^3}{(a\,x+1)^3}-\frac{a\,(a\,x-1)^4}{(a\,x+1)^4}}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((17*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(1/2)
 + (37*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + 5*c^2*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c^2
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.792 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=76

$$c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] -3*c*arccsc(a*x)/a+3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1+1/a/
x)^(3/2)*x*(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {6329, 100, 12, 132, 41, 222, 94, 214}

$$cx\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)*x - (3*c*ArcCsc[a*x])/a + (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\left(c\text{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x+c\text{Subst}\left(\int-\frac{3\sqrt{1+\frac{x}{a}}}{ax\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{(3c)\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{(3c)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2}-$$

$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{(3c)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2}+\frac{(3c)\text{Sub}}{}$$

$$=c\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}x-\frac{3c\csc^{-1}(ax)}{a}+\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\cdots}\right.}{a}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.75

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}\,(1+ax)-3\text{ArcSin}\left(\frac{1}{ax}\right)+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) - 3*ArcSin[1/(a*x)] + 3*Log[(1 + Sqrt[1
 - 1/(a^2*x^2)])*x]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(68) = 136$.

time = 0.10, size = 235, normalized size = 3.09

method	result
risch	$\frac{(ax-1)c}{x\,a^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(a\sqrt{(ax+1)(ax-1)}+\frac{3a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{(ax+}}{a^2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$
default	$-\frac{(ax-1)^2c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}+3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}\,\sqrt{a^2}}{\sqrt{a^2}}\right)a^2\right.}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{(a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x-1)^2*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)+3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^
2)^(1/2))/(a^2)^(1/2))*a^2*x+3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x-
4*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-4*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x)/((a*x-1)/(a*x+1))^(3/2)/(a*x+1)/((a*x+1
)*(a*x-1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A]

time = 0.47, size = 118, normalized size = 1.55

$$-a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]

time = 0.35, size = 106, normalized size = 1.39

$$\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2+2\,acx+c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) - 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 + 2*
a*c*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{a^2}{\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx+\int\left(-\frac{1}{\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\right)dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2/(a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x) + Integral(-1/(a*x**3*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1) - x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(
a*x + 1)), x))/a**2
```

Giac [A]
time = 0.43, size = 130, normalized size = 1.71

$$\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)}{a\mathrm{sgn}\left(ax+1\right)}-\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)}{|a|\mathrm{sgn}\left(ax+1\right)}+\frac{\sqrt{a^2x^2-1}\,c}{a\mathrm{sgn}\left(ax+1\right)}+\frac{2\,c}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|\mathrm{sgn}\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))/(a*sgn(a*x + 1)) - 3*c*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))/(abs(a)*sgn(a*x + 1)) + sqrt(a^2*x^2 - 1)*c/(
a*sgn(a*x + 1)) + 2*c/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)*sgn(a*
x + 1))
```

Mupad [B]
time = 0.09, size = 84, normalized size = 1.11

$$\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a + (6*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

3.793 $\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c-5/3*(1+1/a/x)^(1/2)/a/c/(1-1
/a/x)^(3/2)+x*(1+1/a/x)^(1/2)/c/(1-1/a/x)^(3/2)-14/3*(1+1/a/x)^(1/2)/a/c/(1
-1/a/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 101, 157, 12, 94, 214}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{14\sqrt{\frac{1}{ax}+1}}{3ac\sqrt{1-\frac{1}{ax}}}-\frac{5\sqrt{\frac{1}{ax}+1}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (-5*Sqrt[1 + 1/(a*x)])/(3*a*c*(1 - 1/(a*x))^(3/2)) - (14*Sqrt[1 + 1/(a*x)])
/(3*a*c*Sqrt[1 - 1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c*(1 - 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x^2\left(1-\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{3}{a}+\frac{2x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{a\text{Subst}\left(\int \frac{-\frac{9}{a^2}-\frac{5x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{3c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{a^2\text{Subst}\left(\int \frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\right.}{3c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx\right.}{ac}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\right.}{a^2c}$$

$$= -\frac{5\sqrt{1+\frac{1}{ax}}}{3ac\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{14\sqrt{1+\frac{1}{ax}}}{3ac\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A]

time = 0.20, size = 69, normalized size = 0.48

$$\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(14-19ax+3a^2x^2)}{(-1+ax)^2}+\frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]

[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 - 19*a*x + 3*a^2*x^2))/(-1 + a*x)^2 + (9*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(124) = 248.
time = 0.11, size = 346, normalized size = 2.40

method	result
risch	$\frac{ax-1}{ac\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{2\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{3a^5\left(x-\frac{1}{a}\right)^2}-\frac{13\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\right.}}{3a^4\left(x-\frac{1}{a}\right)}\right.}{c\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-9\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3-9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3+6\sqrt{a^2}\,((ax+1)(ax-1))^{\frac{3}{2}}ax}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)

[Out] -1/3*(-9*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-9*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+6*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+27*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+27*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-5*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-27*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-27*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+9*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+9*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)/c/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)

Maxima [A]

time = 0.25, size = 133, normalized size = 0.92

$$\frac{1}{3}\,a\left(\frac{\frac{11\,(ax-1)}{ax+1}-\frac{18\,(ax-1)^2}{(ax+1)^2}+1}{a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] 1/3*a*((11*(a*x - 1)/(a*x + 1) - 18*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*c*((a
*x - 1)/(a*x + 1))^(5/2) - a^2*c*((a*x - 1)/(a*x + 1))^(3/2)) + 9*log(sqrt(
(a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a
^2*c))
```

Fricas [A]
time = 0.34, size = 128, normalized size = 0.89

$$\frac{9\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(a^2x^2-2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(3\,a^3x^3-16\,a^2x^2-5\,ax+14)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3cx^2-2\,a^2cx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] 1/3*(9*(a^2*x^2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a^2*x^
2 - 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (3*a^3*x^3 - 16*a^2*x^2
 - 5*a*x + 14)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2}{\frac{a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2/(a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)
), x)/c
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.35, size = 100, normalized size = 0.69

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}-\frac{\frac{11\,(a\,x-1)}{3\,(a\,x+1)}-\frac{6\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-a\,c\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c) - ((11*(a*x - 1))/(3*(a*x + 1)
) - (6*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(a*c*((a*x - 1)/(a*x + 1))^(3/2) - a
*c*((a*x - 1)/(a*x + 1))^(5/2))
```

3.794 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=181

$$-\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}}+\frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-6/5*(1+1/a/x)^(1/2)/a/c^2/
(1-1/a/x)^(5/2)-9/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(3/2)+x*(1+1/a/x)^(1/2)
/c^2/(1-1/a/x)^(5/2)-24/5*(1+1/a/x)^(1/2)/a/c^2/(1-1/a/x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 105, 21, 101, 157, 12, 94, 214}

$$\frac{x\sqrt{\frac{1}{ax}+1}}{c^2\left(1-\frac{1}{ax}\right)^{5/2}}-\frac{24\sqrt{\frac{1}{ax}+1}}{5ac^2\sqrt{1-\frac{1}{ax}}}-\frac{9\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}}-\frac{6\sqrt{\frac{1}{ax}+1}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (-6*Sqrt[1 + 1/(a*x)])/(5*a*c^2*(1 - 1/(a*x))^(5/2)) - (9*Sqrt[1 + 1/(a*x)]
)/(5*a*c^2*(1 - 1/(a*x))^(3/2)) - (24*Sqrt[1 + 1/(a*x)])/(5*a*c^2*Sqrt[1 -
1/(a*x)]) + (Sqrt[1 + 1/(a*x)]*x)/(c^2*(1 - 1/(a*x))^(5/2)) + (3*ArcTanh[Sq
rt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{1+\frac{x}{a}}}{x\left(1-\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{6\text{Subst}\left(\int \frac{-\frac{5}{2}-\frac{2x}{a}}{x\left(1-\frac{x}{a}\right)^{5/2}\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{5ac^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{2\text{Subst}\left(\int \frac{\frac{15}{2a}+\frac{9x}{2a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}\right.}{5c^2}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{(2a)\text{Subst}\left(\right.}{}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{3\text{Subst}\left(\int\right.}{}$$

$$= -\frac{6\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{5/2}} - \frac{9\sqrt{1+\frac{1}{ax}}}{5ac^2\left(1-\frac{1}{ax}\right)^{3/2}} - \frac{24\sqrt{1+\frac{1}{ax}}}{5ac^2\sqrt{1-\frac{1}{ax}}} + \frac{\sqrt{1+\frac{1}{ax}}\,x}{c^2\left(1-\frac{1}{ax}\right)^{5/2}} + \frac{3\text{Subst}\left(\int\right.}{}$$

$$6\sqrt{1+\frac{1}{}} \quad 9\sqrt{1+\frac{1}{}} \quad 24\sqrt{1+\frac{1}{}} \quad \sqrt{1+\frac{1}{}}\,x \quad 3\tanh^{-1}\left(\right.$$

Mathematica [A]
time = 0.09, size = 78, normalized size = 0.43

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-24+57ax-39a^2x^2+5a^3x^3)}{5(-1+ax)^3}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-24 + 57*a*x - 39*a^2*x^2 + 5*a^3*x^3))/(5*(-1
 + a*x)^3) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(155) = 310.
time = 0.16, size = 438, normalized size = 2.42

method	result
risch	$\frac{ax-1}{a\,c^2\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{6\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{5a^7\left(x-\frac{1}{a}\right)^2}-\frac{24\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{5a^6\left(x-\frac{1}{a}\right)}\right.}{c^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$
default	$-\frac{-125\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^4x^4-120\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^5x^4+85\sqrt{a^2}\,((ax+1)(ax-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(-125*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-120*ln((a^2*x+(a^2)
^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+85*(a^2)^(1/2)*((a*x+1
)*(a*x-1))^(3/2)*a^2*x^2+500*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+48
0*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-148*(
a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-750*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1
/2)*a^2*x^2-720*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))
*a^3*x^2+67*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+500*((a*x+1)*(a*x-1))^(1/2)
*(a^2)^(1/2)*a*x+480*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(
1/2))*a^2*x-125*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-120*a*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a^2)^(1/2)/(a*x-1)^2/c^2/((a
*x+1)*(a*x-1))^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.26, size = 153, normalized size = 0.85

$$\frac{1}{20}\,a\left(\frac{\frac{9\,(ax-1)}{ax+1}+\frac{75\,(ax-1)^2}{(ax+1)^2}-\frac{125\,(ax-1)^3}{(ax+1)^3}+1}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/20*a*((9*(a*x - 1)/(a*x + 1) + 75*(a*x - 1)^2/(a*x + 1)^2 - 125*(a*x - 1)
^3/(a*x + 1)^3 + 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^2*((a*x -
1)/(a*x + 1))^(5/2)) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^2) - 60
*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A]
time = 0.35, size = 170, normalized size = 0.94

$$\frac{15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(5\,a^4x^4-34\,a^3x^3+18\,a^2x^2+33\,ax-24)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/5*(15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1
) - 15*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1)
 + (5*a^4*x^4 - 34*a^3*x^3 + 18*a^2*x^2 + 33*a*x - 24)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x - a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4}{\frac{a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{2a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{2a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4/(a**5*x**5*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**4*x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 2*a**3*x**3*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 2*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + a*x*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1)), x)/c**2
```

Giac [A]
time = 0.43, size = 63, normalized size = 0.35

$$-\frac{3 \log \left(\left|-x|a| + \sqrt{a^2x^2 - 1}\right|\right)}{c^2|a|\mathrm{sgn}\,(ax+1)} + \frac{\sqrt{a^2x^2 - 1}}{ac^2\mathrm{sgn}\,(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))/(c^2*abs(a)*sgn(a*x + 1)) + sqrt
(a^2*x^2 - 1)/(a*c^2*sgn(a*x + 1))
```

Mupad [B]
time = 0.09, size = 121, normalized size = 0.67

$$\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^2} - \frac{\frac{15\,(a\,x-1)^2}{(a\,x+1)^2} - \frac{25\,(a\,x-1)^3}{(a\,x+1)^3} + \frac{9\,(a\,x-1)}{5\,(a\,x+1)} + \frac{1}{5}}{4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2} - 4\,a\,c^2\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c^2) - ((15*(a*x - 1)^2)/(a*x + 1
)^2 - (25*(a*x - 1)^3)/(a*x + 1)^3 + (9*(a*x - 1))/(5*(a*x + 1)) + 1/5)/(4*
a*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 4*a*c^2*((a*x - 1)/(a*x + 1))^(7/2))
```

3.795 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=255

$$-\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}}$$

```
[Out] 3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-8/7/a/c^3/(1-1/a/x)^(7/2)/
(1+1/a/x)^(1/2)-53/35/a/c^3/(1-1/a/x)^(5/2)/(1+1/a/x)^(1/2)-88/35/a/c^3/(1-
1/a/x)^(3/2)/(1+1/a/x)^(1/2)+x/c^3/(1-1/a/x)^(7/2)/(1+1/a/x)^(1/2)-281/35/a
/c^3/(1-1/a/x)^(1/2)/(1+1/a/x)^(1/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)
^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}-\frac{281}{35ac^3\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}}-\frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{\frac{1}{ax}+1}}-\frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{\frac{1}{ax}+1}}-\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{\frac{1}{ax}+1}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] -8/(7*a*c^3*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]) - 53/(35*a*c^3*(1 - 1/(a
*x))^(5/2)*Sqrt[1 + 1/(a*x)]) - 88/(35*a*c^3*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1
/(a*x)]) - 281/(35*a*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]) + (176*Sqrt[1
 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(7/2)*Sqrt
[1 + 1/(a*x)]) + (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{a\text{Subst}\left(\int \frac{\frac{21}{a^2}+\frac{32x}{a^3}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{3/2}}\right.}{7c^3}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+}}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+}}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+}}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+}}$$

$$= -\frac{8}{7ac^3\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}} - \frac{53}{35ac^3\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}} - \frac{88}{35ac^3\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+}}$$

$$8 \qquad 53 \qquad 88$$

Mathematica [A]
time = 0.23, size = 101, normalized size = 0.40

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(176-423ax+125a^2x^2+368a^3x^3-286a^4x^4+35a^5x^5)}{35(-1+ax)^4(1+ax)}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(176 - 423*a*x + 125*a^2*x^2 + 368*a^3*x^3 - 28
6*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)^4*(1 + a*x)) + 3*Log[(1 + Sqrt[1 -
1/(a^2*x^2)])*x])/(a*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(217) = 434$.
time = 0.17, size = 714, normalized size = 2.80

method	result
risch	$\frac{ax-1}{a\,c^3\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{477\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{280a^9\left(x-\frac{1}{a}\right)^2}-\frac{2931\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a}}{560a^8\left(x-\frac{1}{a}\right)}\right.}{}$
default	$-\frac{-3675\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^7x^7-3360\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^8x^7+2555((ax+1)(ax-1)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1120*(-3675*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^7*x^7-3360*ln((a^2*x+(
a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^8*x^7+2555*((a*x+1)*(a*x
-1))^(3/2)*(a^2)^(1/2)*a^5*x^5+11025*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^
6*x^6+10080*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^7
*x^6-1873*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4-3675*((a*x+1)*(a*x-1)
)^(1/2)*(a^2)^(1/2)*a^5*x^5-3360*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1
/2))/(a^2)^(1/2))*a^6*x^5-4426*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3-
18375*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-16800*ln((a^2*x+(a^2)^(1/
2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+3350*(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(3/2)*a^2*x^2+18375*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+16
800*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+251
```

```
1*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+3675*((a*x+1)*(a*x-1))^(1/2)*(a^2
)^(1/2)*a^2*x^2+3360*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(
1/2))*a^3*x^2-1957*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-11025*((a*x+1)*(a*x-
1))^(1/2)*(a^2)^(1/2)*a*x-10080*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/
2))/(a^2)^(1/2))*a^2*x+3675*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+3360*a*ln((
a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a*x-1)^3/(a^2)^
(1/2)/c^3/((a*x+1)*(a*x-1))^(1/2)/(a*x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.26, size = 192, normalized size = 0.75

$$\frac{1}{560}\,a\left(\frac{\frac{51\,(ax-1)}{ax+1}+\frac{294\,(ax-1)^2}{(ax+1)^2}+\frac{2170\,(ax-1)^3}{(ax+1)^3}-\frac{3640\,(ax-1)^4}{(ax+1)^4}+5}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}}+\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}+\frac{35\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/560*a*((51*(a*x - 1)/(a*x + 1) + 294*(a*x - 1)^2/(a*x + 1)^2 + 2170*(a*x
- 1)^3/(a*x + 1)^3 - 3640*(a*x - 1)^4/(a*x + 1)^4 + 5)/(a^2*c^3*((a*x - 1)/
(a*x + 1))^(9/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(7/2)) + 1680*log(sqrt((a*
x - 1)/(a*x + 1)) + 1)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c^3) + 35*sqrt((a*x - 1)/(a*x + 1))/(a^2*c^3))
```

Fricas [A]
time = 0.35, size = 204, normalized size = 0.80

$$\frac{105\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(35\,a^5x^5-286\,a^4x^4+368\,a^3x^3+125\,a^2x^2-423\,ax+176)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/35*(105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqrt((a*x - 1)/
(a*x + 1)) + 1) - 105*(a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(sqr
t((a*x - 1)/(a*x + 1)) - 1) + (35*a^5*x^5 - 286*a^4*x^4 + 368*a^3*x^3 + 125
*a^2*x^2 - 423*a*x + 176)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 - 4*a^4*c
^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^6\int\frac{x^6}{\frac{a^7x^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{a^6x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^5x^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^4x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{3a^3x^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{3a^2x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}-\frac{ax\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}+\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}}\,dx\Big/c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6/(a**7*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) -
 a**6*x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - 3*a**5*x**5*sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + 3*a**4*x**4*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1) + 3*a**3*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x +
 1) - 3*a**2*x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) - a*x*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1) + sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a
*x + 1)), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B]
time = 1.41, size = 160, normalized size = 0.63

$$\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^3}-\frac{\frac{42\,{\left(a\,x-1\right)}^2}{5\,{\left(a\,x+1\right)}^2}+\frac{62\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{104\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}+\frac{51\,\left(a\,x-1\right)}{35\,\left(a\,x+1\right)}+\frac{1}{7}}{16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}+\frac{6\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] ((a*x - 1)/(a*x + 1))^(1/2)/(16*a*c^3) - ((42*(a*x - 1)^2)/(5*(a*x + 1)^2)
+ (62*(a*x - 1)^3)/(a*x + 1)^3 - (104*(a*x - 1)^4)/(a*x + 1)^4 + (51*(a*x -
 1))/(35*(a*x + 1)) + 1/7)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 16*a*c^3
*((a*x - 1)/(a*x + 1))^(9/2)) + (6*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c
^3)
```

3.796 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=329

$$-\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{}{315ac^4\left(1-\right.}$$

```
[Out] -10/9/a/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)-29/21/a/c^4/(1-1/a/x)^(7/2)/(1+
1/a/x)^(3/2)-208/105/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(3/2)-1147/315/a/c^4/(
1-1/a/x)^(3/2)/(1+1/a/x)^(3/2)+x/c^4/(1-1/a/x)^(9/2)/(1+1/a/x)^(3/2)+3*arct
anh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-1462/105/a/c^4/(1+1/a/x)^(3/2)/(
1-1/a/x)^(1/2)+2609/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315*(1-1
/a/x)^(1/2)/a/c^4/(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{2609\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1462}{105ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{1147}{315ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] -10/(9*a*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2)) - 29/(21*a*c^4*(1 - 1
/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2)) - 208/(105*a*c^4*(1 - 1/(a*x))^(5/2)*(1
+ 1/(a*x))^(3/2)) - 1147/(315*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2)
) - 1462/(105*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (2609*Sqrt[1 -
 1/(a*x)])/(315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*
a*c^4*Sqrt[1 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(3/2))
+ (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{\frac{27}{a^2}+\frac{60x}{a^3}}{x\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{5/}}\right.}{9c^4}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}(1-}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}(1-}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}(1-}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}(1-}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}(1-}$$

$$= -\frac{10}{9ac^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{29}{21ac^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{208}{105ac^4\left(1-\frac{1}{ax}\right)^{5/2}(1-}$$

Mathematica [A]
time = 0.12, size = 117, normalized size = 0.36

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-1664+4047ax+339a^2x^2-7399a^3x^3+4029a^4x^4+2967a^5x^5-2669a^6x^6+315a^7x^7)}{315(-1+ax)^5(1+ax)^2}+3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]

[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-1664 + 4047*a*x + 339*a^2*x^2 - 7399*a^3*x^3 + 4029*a^4*x^4 + 2967*a^5*x^5 - 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x)^5*(1 + a*x)^2) + 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(279) = 558.
time = 0.20, size = 766, normalized size = 2.33

method	result
risch	$\frac{ax-1}{a\,c^4\sqrt{\frac{ax-1}{ax+1}}}+\frac{\left(\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{691\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{315a^{11}\left(x-\frac{1}{a}\right)^2}-\frac{113591\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a}}{20160a^{10}\left(x-\frac{1}{a}\right)}\right.}{}$
default	$-\frac{-138915\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^9x^9-120960\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax+1)}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)

[Out] -1/40320*(-138915*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^9*x^9-120960*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^10*x^9+98595*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^7*x^7+416745*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^8*x^8+362880*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^9*x^8-75113*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^6*x^6-240861*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^5*x^5-1111320*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^6*x^6-967680*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^7*x^6+178863*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4+833490*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5+725760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+252497*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3+833490*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+725760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4-1823

```
07*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2-1111320*(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(1/2)*a^3*x^3-967680*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
))/(a^2)^(1/2))*a^4*x^3-101271*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+7407
7*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+416745*((a*x+1)*(a*x-1))^(1/2)*(a^2)^
(1/2)*a*x+362880*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)
)*a^2*x-138915*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-120960*a*ln((a^2*x+(a^2)
^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a/(a*x-1)^4/(a^2)^(1/2)/c^4/(
(a*x+1)*(a*x-1))^(1/2)/(a*x+1)^4/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [A]
time = 0.27, size = 226, normalized size = 0.69

$$\frac{1}{20160}a\left(\frac{\frac{415\,(ax-1)}{ax+1}+\frac{2511\,(ax-1)^2}{(ax+1)^2}+\frac{11739\,(ax-1)^3}{(ax+1)^3}+\frac{80745\,(ax-1)^4}{(ax+1)^4}-\frac{135765\,(ax-1)^5}{(ax+1)^5}+35}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}}+\frac{105\left(\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+30\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}+\frac{60480\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}-\frac{60480\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/20160*a*((415*(a*x - 1)/(a*x + 1) + 2511*(a*x - 1)^2/(a*x + 1)^2 + 11739*
(a*x - 1)^3/(a*x + 1)^3 + 80745*(a*x - 1)^4/(a*x + 1)^4 - 135765*(a*x - 1)^
5/(a*x + 1)^5 + 35)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(11/2) - a^2*c^4*((a*x -
 1)/(a*x + 1))^(9/2)) + 105*(((a*x - 1)/(a*x + 1))^(3/2) + 30*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4
) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A]
time = 0.34, size = 248, normalized size = 0.75

$$\frac{945\,(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\,(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(315\,a^7x^7-2669\,a^6x^6+2967\,a^5x^5+4029\,a^4x^4-7399\,a^3x^3+339\,a^2x^2+4047\,ax-1664)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2*x^2 + 4*a*x - 1)*log(sq
rt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*a^2
*x^2 + 4*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (315*a^7*x^7 - 2669*
a^6*x^6 + 2967*a^5*x^5 + 4029*a^4*x^4 - 7399*a^3*x^3 + 339*a^2*x^2 + 4047*a
*x - 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*
c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
[Out] integrate(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [B]
time = 0.11, size = 203, normalized size = 0.62

$$\frac{5\sqrt{\frac{a\,x-1}{a\,x+1}}}{32\,a\,c^4}-\frac{\frac{279\,{\left(a\,x-1\right)}^2}{35\,{\left(a\,x+1\right)}^2}+\frac{559\,{\left(a\,x-1\right)}^3}{15\,{\left(a\,x+1\right)}^3}+\frac{769\,{\left(a\,x-1\right)}^4}{3\,{\left(a\,x+1\right)}^4}-\frac{431\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}+\frac{83\,\left(a\,x-1\right)}{63\,\left(a\,x+1\right)}+\frac{1}{9}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}-64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{192\,a\,c^4}-\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] (5*((a*x - 1)/(a*x + 1))^(1/2))/(32*a*c^4) - ((279*(a*x - 1)^2)/(35*(a*x +
1)^2) + (559*(a*x - 1)^3)/(15*(a*x + 1)^3) + (769*(a*x - 1)^4)/(3*(a*x + 1)
^4) - (431*(a*x - 1)^5)/(a*x + 1)^5 + (83*(a*x - 1))/(63*(a*x + 1)) + 1/9)/
(64*a*c^4*((a*x - 1)/(a*x + 1))^(9/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(11/
2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(192*a*c^4) - (atan(((a*x - 1)/(a*x + 1))
^(1/2)*1i)*6i)/(a*c^4)
```

3.797 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5\,dx$

Optimal. Leaf size=116

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}$$

```
[Out] 1/9*c^5/a^10/x^9+1/2*c^5/a^9/x^8+3/7*c^5/a^8/x^7-4/3*c^5/a^7/x^6-14/5*c^5/a
^6/x^5+14/3*c^5/a^4/x^3+4*c^5/a^3/x^2-3*c^5/a^2/x+c^5*x+4*c^5*ln(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+\frac{4c^5\log(x)}{a}+c^5x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5dx&=\int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^5dx\\&=-\frac{c^5\int\frac{e^{4\tanh^{-1}(ax)}(1-a^2x^2)^5}{x^{10}}\,dx}{a^{10}}\\&=-\frac{c^5\int\frac{(1-ax)^3(1+ax)^7}{x^{10}}\,dx}{a^{10}}\\&=-\frac{c^5\int\left(-a^{10}+\frac{1}{x^{10}}+\frac{4a}{x^9}+\frac{3a^2}{x^8}-\frac{8a^3}{x^7}-\frac{14a^4}{x^6}+\frac{14a^6}{x^4}+\frac{8a^7}{x^3}-\frac{3a^8}{x^2}-\frac{4a^9}{x}\right)dx}{a^{10}}\\&=\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x\end{aligned}$$

Mathematica [A]
time = 0.03, size = 116, normalized size = 1.00

$$\frac{c^5}{9a^{10}x^9}+\frac{c^5}{2a^9x^8}+\frac{3c^5}{7a^8x^7}-\frac{4c^5}{3a^7x^6}-\frac{14c^5}{5a^6x^5}+\frac{14c^5}{3a^4x^3}+\frac{4c^5}{a^3x^2}-\frac{3c^5}{a^2x}+c^5x+\frac{4c^5\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^5,x]
```

```
[Out] c^5/(9*a^10*x^9) + c^5/(2*a^9*x^8) + (3*c^5)/(7*a^8*x^7) - (4*c^5)/(3*a^7*x
^6) - (14*c^5)/(5*a^6*x^5) + (14*c^5)/(3*a^4*x^3) + (4*c^5)/(a^3*x^2) - (3*
c^5)/(a^2*x) + c^5*x + (4*c^5*Log[x])/a
```

Maple [A]
time = 0.42, size = 80, normalized size = 0.69

method	result
default	$\frac{c^5\left(a^{10}x+\frac{a}{2x^8}+\frac{3a^2}{7x^7}+\frac{1}{9x^9}-\frac{3a^8}{x}+\frac{4a^7}{x^2}+4a^9\ln(x)+\frac{14a^6}{3x^3}-\frac{4a^3}{3x^6}-\frac{14a^4}{5x^5}\right)}{a^{10}}$
risch	$c^5x+\frac{-3a^8c^5x^8+4a^7c^5x^7+\frac{14}{3}a^6c^5x^6-\frac{14}{5}a^4c^5x^4-\frac{4}{3}a^3c^5x^3+\frac{3}{7}a^2c^5x^2+\frac{1}{2}a\,c^5x+\frac{1}{9}c^5}{a^{10}x^9}+\frac{4c^5\ln(x)}{a}$
norman	$\frac{-4a^9c^5x^{10}+a^{10}c^5x^{11}-\frac{c^5}{9a}-\frac{7c^5x}{18}+\frac{a\,c^5x^2}{14}-\frac{14a^4c^5x^5}{5}-\frac{14a^5c^5x^6}{3}+\frac{2a^6c^5x^7}{3}+7a^7c^5x^8+\frac{37c^5a^2x^3}{21}+\frac{22c^5a^3x^4}{15}}{(ax-1)a^9x^9}+\frac{4c^5\ln(x)}{a}$
meijerg	$-\frac{c^5\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{4c^5x}{-ax+1}-\frac{5c^5\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{5c^5\left(-\frac{7ax}{-7ax+7}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^5/a^10*(a^10*x+1/2*a/x^8+3/7*a^2/x^7+1/9/x^9-3*a^8/x+4*a^7/x^2+4*a^9*ln(x
)+14/3*a^6/x^3-4/3*a^3/x^6-14/5*a^4/x^5)
```

Maxima [A]
time = 0.28, size = 103, normalized size = 0.89

$$c^5x+\frac{4\,c^5\log\left(x\right)}{a}-\frac{1890\,a^8c^5x^8-2520\,a^7c^5x^7-2940\,a^6c^5x^6+1764\,a^4c^5x^4+840\,a^3c^5x^3-270\,a^2c^5x^2-315\,ac^5x-70\,c^5}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="maxima")
```

```
[Out] c^5*x + 4*c^5*log(x)/a - 1/630*(1890*a^8*c^5*x^8 - 2520*a^7*c^5*x^7 - 2940*
a^6*c^5*x^6 + 1764*a^4*c^5*x^4 + 840*a^3*c^5*x^3 - 270*a^2*c^5*x^2 - 315*a*
c^5*x - 70*c^5)/(a^10*x^9)
```

Fricas [A]
time = 0.38, size = 111, normalized size = 0.96

$$\frac{630\,a^{10}c^5x^{10}+2520\,a^9c^5x^9\log\left(x\right)-1890\,a^8c^5x^8+2520\,a^7c^5x^7+2940\,a^6c^5x^6-1764\,a^4c^5x^4-840\,a^3c^5x^3+270\,a^2c^5x^2+315\,ac^5x+70\,c^5}{630\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="fricas")
```

```
[Out] 1/630*(630*a^10*c^5*x^10 + 2520*a^9*c^5*x^9*log(x) - 1890*a^8*c^5*x^8 + 252
0*a^7*c^5*x^7 + 2940*a^6*c^5*x^6 - 1764*a^4*c^5*x^4 - 840*a^3*c^5*x^3 + 270
*a^2*c^5*x^2 + 315*a*c^5*x + 70*c^5)/(a^10*x^9)
```

Sympy [A]
time = 0.38, size = 112, normalized size = 0.97

$$\frac{a^{10}c^5x+4a^9c^5\log\left(x\right)+\frac{-1890a^8c^5x^8+2520a^7c^5x^7+2940a^6c^5x^6-1764a^4c^5x^4-840a^3c^5x^3+270a^2c^5x^2+315ac^5x+70c^5}{630x^9}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**5,x)
```

```
[Out] (a**10*c**5*x + 4*a**9*c**5*log(x) + (-1890*a**8*c**5*x**8 + 2520*a**7*c**5
*x**7 + 2940*a**6*c**5*x**6 - 1764*a**4*c**5*x**4 - 840*a**3*c**5*x**3 + 27
0*a**2*c**5*x**2 + 315*a*c**5*x + 70*c**5)/(630*x**9))/a**10
```

Giac [A]
time = 0.40, size = 184, normalized size = 1.59

$$-\frac{4\,c^{5}\log\left(\frac{|ax-1|}{(ax-1)^{2}|a|}\right)}{a}+\frac{4\,c^{5}\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(630\,c^{5}+\frac{4049\,c^{5}}{ax-1}+\frac{6201\,c^{5}}{(ax-1)^{2}}-\frac{18036\,c^{5}}{(ax-1)^{3}}-\frac{89124\,c^{5}}{(ax-1)^{4}}-\frac{160146\,c^{5}}{(ax-1)^{5}}-\frac{153090\,c^{5}}{(ax-1)^{6}}-\frac{80220\,c^{5}}{(ax-1)^{7}}-\frac{21420\,c^{5}}{(ax-1)^{8}}-\frac{2520\,c^{5}}{(ax-1)^{9}}\right)(ax-1)}{630\,a\left(\frac{1}{ax-1}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^5,x, algorithm="giac")
```

```
[Out] -4*c^5*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^5*log(abs(-1/(a*x - 1
) - 1))/a + 1/630*(630*c^5 + 4049*c^5/(a*x - 1) + 6201*c^5/(a*x - 1)^2 - 18
036*c^5/(a*x - 1)^3 - 89124*c^5/(a*x - 1)^4 - 160146*c^5/(a*x - 1)^5 - 1530
90*c^5/(a*x - 1)^6 - 80220*c^5/(a*x - 1)^7 - 21420*c^5/(a*x - 1)^8 - 2520*c
^5/(a*x - 1)^9)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^9)
```

Mupad [B]
time = 0.09, size = 81, normalized size = 0.70

$$\frac{c^{5}\left(\frac{a\,x}{2}+\frac{3\,a^{2}\,x^{2}}{7}-\frac{4\,a^{3}\,x^{3}}{3}-\frac{14\,a^{4}\,x^{4}}{5}+\frac{14\,a^{6}\,x^{6}}{3}+4\,a^{7}\,x^{7}-3\,a^{8}\,x^{8}+a^{10}\,x^{10}+4\,a^{9}\,x^{9}\,\ln(x)+\frac{1}{9}\right)}{a^{10}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^5*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^5*((a*x)/2 + (3*a^2*x^2)/7 - (4*a^3*x^3)/3 - (14*a^4*x^4)/5 + (14*a^6*x^
6)/3 + 4*a^7*x^7 - 3*a^8*x^8 + a^10*x^10 + 4*a^9*x^9*log(x) + 1/9))/(a^10*x
^9)
```

3.798 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=100

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

```
[Out] -1/7*c^4/a^8/x^7-2/3*c^4/a^7/x^6-4/5*c^4/a^6/x^5+c^4/a^5/x^4+10/3*c^4/a^4/x
^3+2*c^4/a^3/x^2-4*c^4/a^2/x+c^4*x+4*c^4*ln(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+\frac{4c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x
^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x +
(4*c^4*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx\\ &= \frac{c^4\int \frac{e^{4\tanh^{-1}(ax)}(1-a^2x^2)^4}{x^8}\,dx}{a^8}\\ &= \frac{c^4\int \frac{(1-ax)^2(1+ax)^6}{x^8}\,dx}{a^8}\\ &= \frac{c^4\int \left(a^8+\frac{1}{x^8}+\frac{4a}{x^7}+\frac{4a^2}{x^6}-\frac{4a^3}{x^5}-\frac{10a^4}{x^4}-\frac{4a^5}{x^3}+\frac{4a^6}{x^2}+\frac{4a^7}{x}\right)dx}{a^8}\\ &= -\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(}{a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 100, normalized size = 1.00

$$-\frac{c^4}{7a^8x^7}-\frac{2c^4}{3a^7x^6}-\frac{4c^4}{5a^6x^5}+\frac{c^4}{a^5x^4}+\frac{10c^4}{3a^4x^3}+\frac{2c^4}{a^3x^2}-\frac{4c^4}{a^2x}+c^4x+\frac{4c^4\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^4,x]
```

```
[Out] -1/7*c^4/(a^8*x^7) - (2*c^4)/(3*a^7*x^6) - (4*c^4)/(5*a^6*x^5) + c^4/(a^5*x
^4) + (10*c^4)/(3*a^4*x^3) + (2*c^4)/(a^3*x^2) - (4*c^4)/(a^2*x) + c^4*x +
(4*c^4*Log[x])/a
```

Maple [A]
time = 0.32, size = 71, normalized size = 0.71

method	result
default	$\frac{c^4\left(a^8x+\frac{a^3}{x^4}-\frac{1}{7x^7}-\frac{4a^6}{x}+\frac{2a^5}{x^2}+4a^7\ln(x)+\frac{10a^4}{3x^3}-\frac{2a}{3x^6}-\frac{4a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{-4a^6c^4x^6+2a^5c^4x^5+\frac{10}{3}a^4c^4x^4+a^3c^4x^3-\frac{4}{5}a^2c^4x^2-\frac{2}{3}a\,c^4x-\frac{1}{7}c^4}{a^8x^7}+\frac{4c^4\ln(x)}{a}$
norman	$\frac{-5a^7c^4x^8+a^8c^4x^9+\frac{c^4}{7a}+\frac{11c^4x}{21}-\frac{9a^2c^4x^3}{5}-\frac{7a^3c^4x^4}{3}+\frac{4a^4c^4x^5}{3}+6a^5c^4x^6+\frac{2c^4a\,x^2}{15}}{(ax-1)a^7x^7}+\frac{4c^4\ln(x)}{a}$
meijerg	$-\frac{c^4\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{3c^4x}{-ax+1}-\frac{2c^4\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}-\frac{2c^4\left(-\frac{5ax}{-5ax+5}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^4/a^8*(a^8*x+a^3/x^4-1/7/x^7-4*a^6/x+2*a^5/x^2+4*a^7*ln(x)+10/3*a^4/x^3-2
/3*a/x^6-4/5*a^2/x^5)
```

Maxima [A]
time = 0.27, size = 92, normalized size = 0.92

$$c^4x + \frac{4\,c^4\log(x)}{a} - \frac{420\,a^6c^4x^6 - 210\,a^5c^4x^5 - 350\,a^4c^4x^4 - 105\,a^3c^4x^3 + 84\,a^2c^4x^2 + 70\,ac^4x + 15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] c^4*x + 4*c^4*log(x)/a - 1/105*(420*a^6*c^4*x^6 - 210*a^5*c^4*x^5 - 350*a^4
*c^4*x^4 - 105*a^3*c^4*x^3 + 84*a^2*c^4*x^2 + 70*a*c^4*x + 15*c^4)/(a^8*x^7
)
```

Fricas [A]
time = 0.35, size = 100, normalized size = 1.00

$$\frac{105\,a^8c^4x^8 + 420\,a^7c^4x^7\log(x) - 420\,a^6c^4x^6 + 210\,a^5c^4x^5 + 350\,a^4c^4x^4 + 105\,a^3c^4x^3 - 84\,a^2c^4x^2 - 70\,ac^4x - 15\,c^4}{105\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*a^8*c^4*x^8 + 420*a^7*c^4*x^7*log(x) - 420*a^6*c^4*x^6 + 210*a^5
*c^4*x^5 + 350*a^4*c^4*x^4 + 105*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x
- 15*c^4)/(a^8*x^7)
```

Sympy [A]
time = 0.31, size = 100, normalized size = 1.00

$$\frac{a^8c^4x + 4a^7c^4\log(x) + \frac{-420a^6c^4x^6 + 210a^5c^4x^5 + 350a^4c^4x^4 + 105a^3c^4x^3 - 84a^2c^4x^2 - 70ac^4x - 15c^4}{105x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**4,x)
```

```
[Out] (a**8*c**4*x + 4*a**7*c**4*log(x) + (-420*a**6*c**4*x**6 + 210*a**5*c**4*x*
*5 + 350*a**4*c**4*x**4 + 105*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**
4*x - 15*c**4)/(105*x**7))/a**8
```

Giac [A]
time = 0.41, size = 160, normalized size = 1.60

$$-\frac{4\,c^{4}\log\left(\frac{|ax-1|}{(ax-1)^{2}|a|}\right)}{a}+\frac{4\,c^{4}\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(105\,c^{4}+\frac{659\,c^{4}}{ax-1}+\frac{1253\,c^{4}}{(ax-1)^{2}}-\frac{231\,c^{4}}{(ax-1)^{3}}-\frac{3885\,c^{4}}{(ax-1)^{4}}-\frac{5250\,c^{4}}{(ax-1)^{5}}-\frac{2730\,c^{4}}{(ax-1)^{6}}-\frac{420\,c^{4}}{(ax-1)^{7}}\right)(ax-1)}{105\,a\left(\frac{1}{ax-1}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] -4*c^4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^4*log(abs(-1/(a*x - 1
) - 1))/a + 1/105*(105*c^4 + 659*c^4/(a*x - 1) + 1253*c^4/(a*x - 1)^2 - 231
*c^4/(a*x - 1)^3 - 3885*c^4/(a*x - 1)^4 - 5250*c^4/(a*x - 1)^5 - 2730*c^4/(
a*x - 1)^6 - 420*c^4/(a*x - 1)^7)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^7)
```

Mupad [B]
time = 1.29, size = 72, normalized size = 0.72

$$\frac{c^{4}\left(a^{3}\,x^{3}-\frac{4\,a^{2}\,x^{2}}{5}-\frac{2\,a\,x}{3}+\frac{10\,a^{4}\,x^{4}}{3}+2\,a^{5}\,x^{5}-4\,a^{6}\,x^{6}+a^{8}\,x^{8}+4\,a^{7}\,x^{7}\,\ln\left(x\right)-\frac{1}{7}\right)}{a^{8}\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^4*(a^3*x^3 - (4*a^2*x^2)/5 - (2*a*x)/3 + (10*a^4*x^4)/3 + 2*a^5*x^5 - 4*
a^6*x^6 + a^8*x^8 + 4*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

3.799 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=63

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

```
[Out] 1/5*c^3/a^6/x^5+c^3/a^5/x^4+5/3*c^3/a^4/x^3-5*c^3/a^2/x+c^3*x+4*c^3*ln(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6292, 6285, 76}

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+\frac{4c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
```

```
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx\\
&= -\frac{c^3\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^3}{x^6}\,dx}{a^6}\\
&= -\frac{c^3\int \frac{(1-ax)(1+ax)^5}{x^6}\,dx}{a^6}\\
&= -\frac{c^3\int \left(-a^6+\frac{1}{x^6}+\frac{4a}{x^5}+\frac{5a^2}{x^4}-\frac{5a^4}{x^2}-\frac{4a^5}{x}\right)dx}{a^6}\\
&= \frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 1.00

$$\frac{c^3}{5a^6x^5}+\frac{c^3}{a^5x^4}+\frac{5c^3}{3a^4x^3}-\frac{5c^3}{a^2x}+c^3x+\frac{4c^3\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^3,x]
[Out] c^3/(5*a^6*x^5) + c^3/(a^5*x^4) + (5*c^3)/(3*a^4*x^3) - (5*c^3)/(a^2*x) + c
^3*x + (4*c^3*Log[x])/a
```

Maple [A]
time = 0.31, size = 47, normalized size = 0.75

method	result
default	$\frac{c^3\left(a^6x+\frac{a}{x^4}-\frac{5a^4}{x}+4a^5\ln(x)+\frac{5a^2}{3x^3}+\frac{1}{5x^5}\right)}{a^6}$
risch	$c^3x+\frac{-5a^4c^3x^4+\frac{5}{3}a^2c^3x^2+a\,c^3x+\frac{1}{5}c^3}{a^6x^5}+\frac{4c^3\ln(x)}{a}$
norman	$\frac{-6a^5c^3x^6+c^3a^6x^7-\frac{c^3}{5a}-\frac{4c^3x}{5}+\frac{5a^2c^3x^3}{3}+5a^3c^3x^4-\frac{2c^3a\,x^2}{3}}{(ax-1)a^5x^5}+\frac{4c^3\ln(x)}{a}$
meijerg	$-\frac{c^3\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{2c^3x}{-ax+1}-\frac{2c^3\left(-\frac{5ax}{-5ax+5}+4\ln(-ax+1)-1-4\ln(x)-4\ln(-a)+\frac{1}{3x^3a^3}+\frac{1}{a^2x^2}+\frac{3}{ax}\right)}{a}+\frac{2c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
[Out] c^3/a^6*(a^6*x+a/x^4-5*a^4/x+4*a^5*ln(x)+5/3*a^2/x^3+1/5/x^5)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.94

$$c^3x + \frac{4\,c^3\log\left(x\right)}{a} - \frac{75\,a^4c^3x^4 - 25\,a^2c^3x^2 - 15\,ac^3x - 3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] c^3*x + 4*c^3*log(x)/a - 1/15*(75*a^4*c^3*x^4 - 25*a^2*c^3*x^2 - 15*a*c^3*x
 - 3*c^3)/(a^6*x^5)
```

Fricas [A]
time = 0.58, size = 67, normalized size = 1.06

$$\frac{15\,a^6c^3x^6 + 60\,a^5c^3x^5\log\left(x\right) - 75\,a^4c^3x^4 + 25\,a^2c^3x^2 + 15\,ac^3x + 3\,c^3}{15\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a^6*c^3*x^6 + 60*a^5*c^3*x^5*log(x) - 75*a^4*c^3*x^4 + 25*a^2*c^3*
x^2 + 15*a*c^3*x + 3*c^3)/(a^6*x^5)
```

Sympy [A]
time = 0.17, size = 65, normalized size = 1.03

$$\frac{a^6c^3x + 4a^5c^3\log\left(x\right) + \frac{-75a^4c^3x^4+25a^2c^3x^2+15ac^3x+3c^3}{15x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**3,x)
```

```
[Out] (a**6*c**3*x + 4*a**5*c**3*log(x) + (-75*a**4*c**3*x**4 + 25*a**2*c**3*x**2
 + 15*a*c**3*x + 3*c**3)/(15*x**5))/a**6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.
time = 0.39, size = 136, normalized size = 2.16

$$-\frac{4\,c^3\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} + \frac{4\,c^3\log\left(\left|-\frac{1}{ax-1} - 1\right|\right)}{a} + \frac{\left(15\,c^3 + \frac{107\,c^3}{ax-1} + \frac{235\,c^3}{(ax-1)^2} + \frac{170\,c^3}{(ax-1)^3} - \frac{30\,c^3}{(ax-1)^4} - \frac{60\,c^3}{(ax-1)^5}\right)(ax-1)}{15\,a\left(\frac{1}{ax-1} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] -4*c^3*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^3*log(abs(-1/(a*x - 1
) - 1))/a + 1/15*(15*c^3 + 107*c^3/(a*x - 1) + 235*c^3/(a*x - 1)^2 + 170*c^
3/(a*x - 1)^3 - 30*c^3/(a*x - 1)^4 - 60*c^3/(a*x - 1)^5)*(a*x - 1)/(a*(1/(a
*x - 1) + 1)^5)
```

Mupad [B]
time = 0.06, size = 48, normalized size = 0.76

$$\frac{c^3\,\left(a\,x+\frac{5\,a^2\,x^2}{3}-5\,a^4\,x^4+a^6\,x^6+4\,a^5\,x^5\,\ln\left(x\right)+\frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x + 1)^2)/(a*x - 1)^2,x)
```

```
[Out] (c^3*(a*x + (5*a^2*x^2)/3 - 5*a^4*x^4 + a^6*x^6 + 4*a^5*x^5*log(x) + 1/5))/
(a^6*x^5)
```

3.800 $\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}$$

```
[Out] -1/3*c^2/a^4/x^3-2*c^2/a^3/x^2-6*c^2/a^2/x+c^2*x+4*c^2*ln(x)/a
```

Rubi [A]
time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 45}

$$-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+\frac{4c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*L
og[x])/a
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx\\
&= \frac{c^2\int \frac{e^{4\tanh^{-1}(ax)}\left(1-a^2x^2\right)^2}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int \frac{(1+ax)^4}{x^4}\,dx}{a^4}\\
&= \frac{c^2\int \left(a^4+\frac{1}{x^4}+\frac{4a}{x^3}+\frac{6a^2}{x^2}+\frac{4a^3}{x}\right)\,dx}{a^4}\\
&= -\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 1.00

$$
-\frac{c^2}{3a^4x^3}-\frac{2c^2}{a^3x^2}-\frac{6c^2}{a^2x}+c^2x+\frac{4c^2\log(x)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2))^2,x]
```

```
[Out] -1/3*c^2/(a^4*x^3) - (2*c^2)/(a^3*x^2) - (6*c^2)/(a^2*x) + c^2*x + (4*c^2*L
og[x])/a
```

Maple [A]
time = 0.24, size = 40, normalized size = 0.78

method	result
default	$\frac{c^2\left(a^4x-\frac{6a^2}{x}-\frac{2a}{x^2}+4a^3\ln(x)-\frac{1}{3x^3}\right)}{a^4}$
risch	$c^2x+\frac{-6a^2c^2x^2-2a\,c^2x-\frac{1}{3}c^2}{a^4x^3}+\frac{4c^2\ln(x)}{a}$
norman	$\frac{-7a^3c^2x^4+c^2a^4x^5+\frac{c^2}{3a}+\frac{5c^2x}{3}+4a\,c^2x^2}{(ax-1)a^3x^3}+\frac{4c^2\ln(x)}{a}$
meijerg	$-\frac{c^2\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}-\frac{c^2x}{-ax+1}+\frac{c^2\left(-\frac{3ax}{-3ax+3}+2\ln(-ax+1)-1-2\ln(x)-2\ln(-a)+\frac{1}{ax}\right)}{a}+\frac{2c^2\left(\frac{ax}{-ax+1}+\ln(-\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2/a^4*(a^4*x-6*a^2/x-2*a/x^2+4*a^3*ln(x)-1/3/x^3)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.90

$$c^2x+\frac{4\,c^2\log\left(x\right)}{a}-\frac{18\,a^2c^2x^2+6\,ac^2x+c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] c^2*x + 4*c^2*log(x)/a - 1/3*(18*a^2*c^2*x^2 + 6*a*c^2*x + c^2)/(a^4*x^3)
```

Fricas [A]
time = 0.61, size = 56, normalized size = 1.10

$$\frac{3\,a^4c^2x^4+12\,a^3c^2x^3\log\left(x\right)-18\,a^2c^2x^2-6\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*c^2*x^4 + 12*a^3*c^2*x^3*log(x) - 18*a^2*c^2*x^2 - 6*a*c^2*x - c
^2)/(a^4*x^3)
```

Sympy [A]
time = 0.11, size = 53, normalized size = 1.04

$$\frac{a^4c^2x+4a^3c^2\log\left(x\right)+\frac{-18a^2c^2x^2-6ac^2x-c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2)**2,x)
```

```
[Out] (a**4*c**2*x + 4*a**3*c**2*log(x) + (-18*a**2*c**2*x**2 - 6*a*c**2*x - c**2
)/(3*x**3))/a**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.
time = 0.41, size = 112, normalized size = 2.20

$$-\frac{4\,c^2\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a}+\frac{4\,c^2\log\left(\left|-\frac{1}{ax-1}-1\right|\right)}{a}+\frac{\left(3\,c^2+\frac{34\,c^2}{ax-1}+\frac{66\,c^2}{(ax-1)^2}+\frac{36\,c^2}{(ax-1)^3}\right)(ax-1)}{3\,a\left(\frac{1}{ax-1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c^2*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a + 4*c^2*log(abs(-1/(a*x - 1
) - 1))/a + 1/3*(3*c^2 + 34*c^2/(a*x - 1) + 66*c^2/(a*x - 1)^2 + 36*c^2/(a*
x - 1)^3)*(a*x - 1)/(a*(1/(a*x - 1) + 1)^3)
```

Mupad [B]
time = 0.06, size = 43, normalized size = 0.84

$$-\frac{c^2\,\left(6\,a\,x+18\,a^2\,x^2-3\,a^4\,x^4-12\,a^3\,x^3\,\ln\left(x\right)+1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] -(c^2*(6*a*x + 18*a^2*x^2 - 3*a^4*x^4 - 12*a^3*x^3*log(x) + 1))/(3*a^4*x^3)
```

3.801 $\int e^{4 \coth^{-1}(ax)} \left(c - \frac{c}{a^2 x^2}\right) \, dx$

Optimal. Leaf size=33

$$\frac{c}{a^2 x} + cx - \frac{4c \log(x)}{a} + \frac{8c \log(1 - ax)}{a}$$

```
[Out] c/a^2/x+c*x-4*c*ln(x)/a+8*c*ln(-a*x+1)/a
```

Rubi [A]
time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6302, 6292, 6285, 90}

$$\frac{c}{a^2 x} - \frac{4c \log(x)}{a} + \frac{8c \log(1 - ax)}{a} + cx$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{4\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx &= \int e^{4\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx\\
&= -\frac{c\int \frac{e^{4\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}\,dx}{a^2}\\
&= -\frac{c\int \frac{(1+ax)^3}{x^2(1-ax)}\,dx}{a^2}\\
&= -\frac{c\int \left(-a^2+\frac{1}{x^2}+\frac{4a}{x}-\frac{8a^2}{-1+ax}\right)\,dx}{a^2}\\
&= \frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{c}{a^2x}+cx-\frac{4c\log(x)}{a}+\frac{8c\log(1-ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
[Out] c/(a^2*x) + c*x - (4*c*Log[x])/a + (8*c*Log[1 - a*x])/a
```

Maple [A]
time = 0.19, size = 29, normalized size = 0.88

method	result
default	$\frac{c\left(a^2x+\frac{1}{x}-4a\ln(x)+8a\ln(ax-1)\right)}{a^2}$
risch	$\frac{c}{a^2x}+cx-\frac{4c\ln(x)}{a}+\frac{8c\ln(-ax+1)}{a}$
norman	$\frac{c\,a^2x^3-\frac{c}{a}}{xa(ax-1)}-\frac{4c\ln(x)}{a}+\frac{8c\ln(ax-1)}{a}$
meijerg	$-\frac{c\left(-\frac{ax(-3ax+6)}{3(-ax+1)}-2\ln(-ax+1)\right)}{a}+\frac{2c\left(\frac{ax}{-ax+1}+\ln(-ax+1)\right)}{a}-\frac{2c\left(\frac{2ax}{-2ax+2}-\ln(-ax+1)+1+\ln(x)+\ln(-a)\right)}{a}+\frac{c\left(-\frac{3ax}{-3ax+3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] c/a^2*(a^2*x+1/x-4*a*ln(x)+8*a*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 32, normalized size = 0.97

$$cx + \frac{8\,c\log\left(ax-1\right)}{a} - \frac{4\,c\log\left(x\right)}{a} + \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="maxima")
[Out] c*x + 8*c*log(a*x - 1)/a - 4*c*log(x)/a + c/(a^2*x)
```

Fricas [A]
time = 0.40, size = 35, normalized size = 1.06

$$\frac{a^2cx^2 + 8\,acx\log\left(ax-1\right) - 4\,acx\log\left(x\right) + c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*c*x^2 + 8*a*c*x*log(a*x - 1) - 4*a*c*x*log(x) + c)/(a^2*x)
```

Sympy [A]
time = 0.15, size = 26, normalized size = 0.79

$$cx + \frac{4c\left(-\log\left(x\right) + 2\log\left(x - \frac{1}{a}\right)\right)}{a} + \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2*(c-c/a**2/x**2),x)
[Out] c*x + 4*c*(-log(x) + 2*log(x - 1/a))/a + c/(a**2*x)
```

Giac [A]
time = 0.41, size = 66, normalized size = 2.00

$$-\frac{4\,c\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{a} - \frac{4\,c\log\left(\left|-\frac{1}{ax-1} - 1\right|\right)}{a} + \frac{(ax-1)c}{a\left(\frac{1}{ax-1} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2*(c-c/a^2/x^2),x, algorithm="giac")
[Out] -4*c*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/a - 4*c*log(abs(-1/(a*x - 1) -
1))/a + (a*x - 1)*c/(a*(1/(a*x - 1) + 1))
```

Mupad [B]
time = 0.07, size = 32, normalized size = 0.97

$$c\,x+\frac{c}{a^2\,x}-\frac{4\,c\,\ln\left(x\right)}{a}+\frac{8\,c\,\ln\left(a\,x-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x + 1)^2)/(a*x - 1)^2,x)
[Out] c*x + c/(a^2*x) - (4*c*log(x))/a + (8*c*log(a*x - 1))/a
```

3.802 $\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

```
[Out] x/c-1/a/c/(-a*x+1)^2+5/a/c/(-a*x+1)+4*ln(-a*x+1)/a/c
```

Rubi [A]
time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 78}

$$\frac{5}{ac(1-ax)}-\frac{1}{ac(1-ax)^2}+\frac{4\log(1-ax)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{4\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx \\
&= -\frac{a^2\int \frac{e^{4\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c} \\
&= -\frac{a^2\int \frac{x^2(1+ax)}{(1-ax)^3}\,dx}{c} \\
&= -\frac{a^2\int \left(-\frac{1}{a^2}-\frac{2}{a^2(-1+ax)^3}-\frac{5}{a^2(-1+ax)^2}-\frac{4}{a^2(-1+ax)}\right)dx}{c} \\
&= \frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.00

$$\frac{x}{c}-\frac{1}{ac(1-ax)^2}+\frac{5}{ac(1-ax)}+\frac{4\log(1-ax)}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
[Out] x/c - 1/(a*c*(1 - a*x)^2) + 5/(a*c*(1 - a*x)) + (4*Log[1 - a*x])/(a*c)
```

Maple [A]
time = 0.20, size = 49, normalized size = 0.92

method	result	size
risch	$\frac{x}{c}+\frac{-5cx+\frac{4c}{a}}{c^2(ax-1)^2}+\frac{4\ln(ax-1)}{ac}$	43
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{5}{a^3(ax-1)}-\frac{1}{a^3(ax-1)^2}+\frac{4\ln(ax-1)}{a^3}\right)}{c}$	49
norman	$\frac{\frac{a^2x^3}{c}-\frac{6a\,x^2}{c}+\frac{4x}{c}}{(ax-1)^2}+\frac{4\ln(ax-1)}{ac}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/c*(x/a^2-5/a^3/(a*x-1)-1/a^3/(a*x-1)^2+4/a^3*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.92

$$-\frac{5\,ax-4}{a^3cx^2-2\,a^2cx+ac}+\frac{x}{c}+\frac{4\,\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -(5*a*x - 4)/(a^3*c*x^2 - 2*a^2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Fricas [A]
time = 0.42, size = 64, normalized size = 1.21

$$\frac{a^3x^3-2\,a^2x^2-4\,ax+4\,(a^2x^2-2\,ax+1)\log\left(ax-1\right)+4}{a^3cx^2-2\,a^2cx+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - 2*a^2*x^2 - 4*a*x + 4*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) + 4)/(a
^3*c*x^2 - 2*a^2*c*x + a*c)
```

Sympy [A]
time = 0.13, size = 41, normalized size = 0.77

$$\frac{-5ax+4}{a^3cx^2-2a^2cx+ac}+\frac{x}{c}+\frac{4\log\left(ax-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2),x)
```

```
[Out] (-5*a*x + 4)/(a**3*c*x**2 - 2*a**2*c*x + a*c) + x/c + 4*log(a*x - 1)/(a*c)
```

Giac [A]
time = 0.41, size = 74, normalized size = 1.40

$$\frac{ax-1}{ac}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac}-\frac{\frac{5\,a^3c}{ax-1}+\frac{a^3c}{(ax-1)^2}}{a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c) - (5*a^3*c
/(a*x - 1) + a^3*c/(a*x - 1)^2)/(a^4*c^2)
```

Mupad [B]
time = 1.26, size = 48, normalized size = 0.91

$$\frac{x}{c}-\frac{5\,x-\frac{4}{a}}{c\,a^2\,x^2-2\,c\,a\,x+c}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))*(a*x - 1)^2),x)
[Out] x/c - (5*x - 4/a)/(c + a^2*c*x^2 - 2*a*c*x) + (4*log(a*x - 1))/(a*c)
```

3.803 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}$$

```
[Out] x/c^2+1/3/a/c^2/(-a*x+1)^3-2/a/c^2/(-a*x+1)^2+6/a/c^2/(-a*x+1)+4*ln(-a*x+1)
/a/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 45}

$$\frac{6}{ac^2(1-ax)}-\frac{2}{ac^2(1-ax)^2}+\frac{1}{3ac^2(1-ax)^3}+\frac{4\log(1-ax)}{ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] x/c^2 + 1/(3*a*c^2*(1 - a*x)^3) - 2/(a*c^2*(1 - a*x)^2) + 6/(a*c^2*(1 - a*x
)) + (4*Log[1 - a*x])/(a*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx\\ &= \frac{a^4\int \frac{e^{4\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}\\ &= \frac{a^4\int \frac{x^4}{(1-ax)^4}\,dx}{c^2}\\ &= \frac{a^4\int \left(\frac{1}{a^4}+\frac{1}{a^4(-1+ax)^4}+\frac{4}{a^4(-1+ax)^3}+\frac{6}{a^4(-1+ax)^2}+\frac{4}{a^4(-1+ax)}\right)dx}{c^2}\\ &= \frac{x}{c^2}+\frac{1}{3ac^2(1-ax)^3}-\frac{2}{ac^2(1-ax)^2}+\frac{6}{ac^2(1-ax)}+\frac{4\log(1-ax)}{ac^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.89

$$\frac{-13+27ax-9a^2x^2-9a^3x^3+3a^4x^4+12(-1+ax)^3\log(1-ax)}{3ac^2(-1+ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
[Out] (-13 + 27*a*x - 9*a^2*x^2 - 9*a^3*x^3 + 3*a^4*x^4 + 12*(-1 + a*x)^3*Log[1 -
 a*x])/(3*a*c^2*(-1 + a*x)^3)
```

Maple [A]
time = 0.14, size = 61, normalized size = 0.86

method	result	size
risch	$\frac{x}{c^2}+\frac{-6a\,c^2x^2+10c^2x-\frac{13c^2}{3a}}{c^4(ax-1)^3}+\frac{4\ln(ax-1)}{a\,c^2}$	56
default	$\frac{a^4\left(\frac{x}{a^4}-\frac{6}{a^5(ax-1)}-\frac{1}{3a^5(ax-1)^3}-\frac{2}{a^5(ax-1)^2}+\frac{4\ln(ax-1)}{a^5}\right)}{c^2}$	61
norman	$\frac{\frac{a^4x^5}{c}+\frac{6a\,x^2}{c}-\frac{4x}{c}+\frac{8a^2x^3}{3c}-\frac{19a^3x^4}{3c}}{(ax-1)^3(ax+1)c}+\frac{4\ln(ax-1)}{a\,c^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^2*(x/a^4-6/a^5/(a*x-1)-1/3/a^5/(a*x-1)^3-2/a^5/(a*x-1)^2+4/a^5*ln(a*x
-1))
```

Maxima [A]
time = 0.27, size = 75, normalized size = 1.06

$$-\frac{18\,a^2x^2-30\,ax+13}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}+\frac{x}{c^2}+\frac{4\,\log{(ax-1)}}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*x^2 - 30*a*x + 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x
- a*c^2) + x/c^2 + 4*log(a*x - 1)/(a*c^2)
```

Fricas [A]
time = 0.46, size = 100, normalized size = 1.41

$$\frac{3\,a^4x^4-9\,a^3x^3-9\,a^2x^2+27\,ax+12\,(a^3x^3-3\,a^2x^2+3\,ax-1)\log{(ax-1)}-13}{3\,(a^4c^2x^3-3\,a^3c^2x^2+3\,a^2c^2x-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*x^4 - 9*a^3*x^3 - 9*a^2*x^2 + 27*a*x + 12*(a^3*x^3 - 3*a^2*x^2 +
 3*a*x - 1)*log(a*x - 1) - 13)/(a^4*c^2*x^3 - 3*a^3*c^2*x^2 + 3*a^2*c^2*x -
 a*c^2)
```

Sympy [A]
time = 0.19, size = 83, normalized size = 1.17

$$a^4\left(\frac{-18a^2x^2+30ax-13}{3a^8c^2x^3-9a^7c^2x^2+9a^6c^2x-3a^5c^2}+\frac{x}{a^4c^2}+\frac{4\log{(ax-1)}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-18*a**2*x**2 + 30*a*x - 13)/(3*a**8*c**2*x**3 - 9*a**7*c**2*x**2 +
9*a**6*c**2*x - 3*a**5*c**2) + x/(a**4*c**2) + 4*log(a*x - 1)/(a**5*c**2))
```

Giac [A]
time = 0.42, size = 93, normalized size = 1.31

$$\frac{ax-1}{ac^2}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^2}-\frac{\frac{18\,a^5c^4}{ax-1}+\frac{6\,a^5c^4}{(ax-1)^2}+\frac{a^5c^4}{(ax-1)^3}}{3\,a^6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^2) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^2) - 1/3*
(18*a^5*c^4/(a*x - 1) + 6*a^5*c^4/(a*x - 1)^2 + a^5*c^4/(a*x - 1)^3)/(a^6*c
^6)
```

Mupad [B]
time = 1.27, size = 71, normalized size = 1.00

$$\frac{6\,a\,x^2-10\,x+\frac{13}{3\,a}}{-a^3\,c^2\,x^3+3\,a^2\,c^2\,x^2-3\,a\,c^2\,x+c^2}+\frac{x}{c^2}+\frac{4\,\ln\left(a\,x-1\right)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^2*(a*x - 1)^2),x)
```

```
[Out] (6*a*x^2 - 10*x + 13/(3*a))/(c^2 + 3*a^2*c^2*x^2 - a^3*c^2*x^3 - 3*a*c^2*x)
 + x/c^2 + (4*log(a*x - 1))/(a*c^2)
```

3.804 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3} dx$

Optimal. Leaf size=111

$$\frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(1+ax)}{32ac^3}$$

```
[Out] x/c^3-1/8/a/c^3/(-a*x+1)^4+11/12/a/c^3/(-a*x+1)^3-49/16/a/c^3/(-a*x+1)^2+11
1/16/a/c^3/(-a*x+1)+129/32*ln(-a*x+1)/a/c^3-1/32*ln(a*x+1)/a/c^3
```

Rubi [A]
time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{111}{16ac^3(1-ax)}-\frac{49}{16ac^3(1-ax)^2}+\frac{11}{12ac^3(1-ax)^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{129\log(1-ax)}{32ac^3}-\frac{\log(ax+1)}{32ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] x/c^3 - 1/(8*a*c^3*(1 - a*x)^4) + 11/(12*a*c^3*(1 - a*x)^3) - 49/(16*a*c^3*
(1 - a*x)^2) + 111/(16*a*c^3*(1 - a*x)) + (129*Log[1 - a*x])/(32*a*c^3) - L
og[1 + a*x]/(32*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx \\ &= -\frac{a^6\int \frac{e^{4\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3} \\ &= -\frac{a^6\int \frac{x^6}{(1-ax)^5(1+ax)}\,dx}{c^3} \\ &= -\frac{a^6\int \left(-\frac{1}{a^6}-\frac{1}{2a^6(-1+ax)^5}-\frac{11}{4a^6(-1+ax)^4}-\frac{49}{8a^6(-1+ax)^3}-\frac{111}{16a^6(-1+ax)^2}-\frac{129}{32a^6(-1+ax)}+\frac{}{32a}\right.}{c^3} \\ &= \frac{x}{c^3}-\frac{1}{8ac^3(1-ax)^4}+\frac{11}{12ac^3(1-ax)^3}-\frac{49}{16ac^3(1-ax)^2}+\frac{111}{16ac^3(1-ax)}+\frac{129\log(1}{32ac}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 0.80

$$\frac{2(224-701ax+660a^2x^2-45a^3x^3-192a^4x^4+48a^5x^5)+387(-1+ax)^4\log(1-ax)-3(-1+ax)^4\log(1+ax)}{96ac^3(-1+ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^3,x]
```

```
[Out] (2*(224 - 701*a*x + 660*a^2*x^2 - 45*a^3*x^3 - 192*a^4*x^4 + 48*a^5*x^5) +
387*(-1 + a*x)^4*Log[1 - a*x] - 3*(-1 + a*x)^4*Log[1 + a*x])/(96*a*c^3*(-1
+ a*x)^4)
```

Maple [A]
time = 0.15, size = 84, normalized size = 0.76

method	result	size
risch	$\frac{x}{c^3}+\frac{-\frac{111a^2c^3x^3}{16}+\frac{71c^3a\,x^2}{4}-\frac{749c^3x}{48}+\frac{14c^3}{3a}}{c^6(ax-1)^4}+\frac{129\ln(-ax+1)}{32a\,c^3}-\frac{\ln(ax+1)}{32a\,c^3}$	82
default	$\frac{a^6\left(\frac{x}{a^6}-\frac{\ln(ax+1)}{32a^7}-\frac{1}{8a^7(ax-1)^4}-\frac{11}{12a^7(ax-1)^3}-\frac{49}{16a^7(ax-1)^2}-\frac{111}{16a^7(ax-1)}+\frac{129\ln(ax-1)}{32a^7}\right)}{c^3}$	84

norman	$\frac{\frac{a^6x^7}{c}+\frac{65x}{16c}-\frac{49a\,x^2}{8c}-\frac{161a^2x^3}{24c}+\frac{301a^3x^4}{24c}+\frac{67a^4x^5}{48c}-\frac{20a^5x^6}{3c}}{(ax-1)^4c^2(ax+1)^2}+\frac{129\ln(ax-1)}{32c^3a}-\frac{\ln(ax+1)}{32a\,c^3}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^6/c^3*(1/a^6*x-1/32/a^7*ln(a*x+1)-1/8/a^7/(a*x-1)^4-11/12/a^7/(a*x-1)^3-4
9/16/a^7/(a*x-1)^2-111/16/a^7/(a*x-1)+129/32/a^7*ln(a*x-1))
```

Maxima [A]
time = 0.28, size = 107, normalized size = 0.96

$$-\frac{333\,a^3x^3-852\,a^2x^2+749\,ax-224}{48\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}+\frac{x}{c^3}-\frac{\log(ax+1)}{32\,ac^3}+\frac{129\,\log(ax-1)}{32\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(333*a^3*x^3 - 852*a^2*x^2 + 749*a*x - 224)/(a^5*c^3*x^4 - 4*a^4*c^3*
x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3) + x/c^3 - 1/32*log(a*x + 1)/(a*c
^3) + 129/32*log(a*x - 1)/(a*c^3)
```

Fricas [A]
time = 0.71, size = 163, normalized size = 1.47

$$\frac{96\,a^5x^5-384\,a^4x^4-90\,a^3x^3+1320\,a^2x^2-1402\,ax-3\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log(ax+1)+387\,(a^4x^4-4\,a^3x^3+6\,a^2x^2-4\,ax+1)\log(ax-1)+448}{96\,(a^5c^3x^4-4\,a^4c^3x^3+6\,a^3c^3x^2-4\,a^2c^3x+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(96*a^5*x^5 - 384*a^4*x^4 - 90*a^3*x^3 + 1320*a^2*x^2 - 1402*a*x - 3*(
a^4*x^4 - 4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x + 1) + 387*(a^4*x^4 -
4*a^3*x^3 + 6*a^2*x^2 - 4*a*x + 1)*log(a*x - 1) + 448)/(a^5*c^3*x^4 - 4*a^4
*c^3*x^3 + 6*a^3*c^3*x^2 - 4*a^2*c^3*x + a*c^3)
```

Sympy [A]
time = 0.37, size = 114, normalized size = 1.03

$$a^6\left(\frac{-333a^3x^3+852a^2x^2-749ax+224}{48a^{11}c^3x^4-192a^{10}c^3x^3+288a^9c^3x^2-192a^8c^3x+48a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{129\log\left(x-\frac{1}{a}\right)}{32}-\frac{\log\left(x+\frac{1}{a}\right)}{32}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*((-333*a**3*x**3 + 852*a**2*x**2 - 749*a*x + 224)/(48*a**11*c**3*x**4
- 192*a**10*c**3*x**3 + 288*a**9*c**3*x**2 - 192*a**8*c**3*x + 48*a**7*c**3
) + x/(a**6*c**3) + (129*log(x - 1/a)/32 - log(x + 1/a)/32)/(a**7*c**3))
```

Giac [A]
time = 0.39, size = 130, normalized size = 1.17

$$\frac{ax-1}{ac^3}-\frac{4\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^3}-\frac{\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{32\,ac^3}-\frac{\frac{333\,a^{11}c^9}{ax-1}+\frac{147\,a^{11}c^9}{(ax-1)^2}+\frac{44\,a^{11}c^9}{(ax-1)^3}+\frac{6\,a^{11}c^9}{(ax-1)^4}}{48\,a^{12}c^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] (a*x - 1)/(a*c^3) - 4*log(abs(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^3) - 1/32
*log(abs(-2/(a*x - 1) - 1))/(a*c^3) - 1/48*(333*a^11*c^9/(a*x - 1) + 147*a^
11*c^9/(a*x - 1)^2 + 44*a^11*c^9/(a*x - 1)^3 + 6*a^11*c^9/(a*x - 1)^4)/(a^1
2*c^12)
```

Mupad [B]
time = 0.11, size = 104, normalized size = 0.94

$$\frac{x}{c^3}-\frac{\frac{749\,x}{48}-\frac{71\,a\,x^2}{4}-\frac{14}{3\,a}+\frac{111\,a^2\,x^3}{16}}{a^4\,c^3\,x^4-4\,a^3\,c^3\,x^3+6\,a^2\,c^3\,x^2-4\,a\,c^3\,x+c^3}+\frac{129\,\ln\left(a\,x-1\right)}{32\,a\,c^3}-\frac{\ln\left(a\,x+1\right)}{32\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^3*(a*x - 1)^2),x)
```

```
[Out] x/c^3 - ((749*x)/48 - (71*a*x^2)/4 - 14/(3*a) + (111*a^2*x^3)/16)/(c^3 + 6*
a^2*c^3*x^2 - 4*a^3*c^3*x^3 + a^4*c^3*x^4 - 4*a*c^3*x) + (129*log(a*x - 1))
/(32*a*c^3) - log(a*x + 1)/(32*a*c^3)
```

3.805 $\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=146

$$\frac{x}{c^4}+\frac{1}{20ac^4(1-ax)^5}-\frac{7}{16ac^4(1-ax)^4}+\frac{83}{48ac^4(1-ax)^3}-\frac{67}{16ac^4(1-ax)^2}+\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(1+ax)}+\frac{26}{}$$

```
[Out] x/c^4+1/20/a/c^4/(-a*x+1)^5-7/16/a/c^4/(-a*x+1)^4+83/48/a/c^4/(-a*x+1)^3-67
/16/a/c^4/(-a*x+1)^2+501/64/a/c^4/(-a*x+1)-1/64/a/c^4/(a*x+1)+261/64*ln(-a*
x+1)/a/c^4-5/64*ln(a*x+1)/a/c^4
```

Rubi [A]
time = 0.15, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{501}{64ac^4(1-ax)}-\frac{1}{64ac^4(ax+1)}-\frac{67}{16ac^4(1-ax)^2}+\frac{83}{48ac^4(1-ax)^3}-\frac{7}{16ac^4(1-ax)^4}+\frac{1}{20ac^4(1-ax)^5}+\frac{261\log(1-ax)}{64ac^4}-\frac{5\log(ax+1)}{64ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] x/c^4 + 1/(20*a*c^4*(1 - a*x)^5) - 7/(16*a*c^4*(1 - a*x)^4) + 83/(48*a*c^4*
(1 - a*x)^3) - 67/(16*a*c^4*(1 - a*x)^2) + 501/(64*a*c^4*(1 - a*x)) - 1/(64
*a*c^4*(1 + a*x)) + (261*Log[1 - a*x])/(64*a*c^4) - (5*Log[1 + a*x])/(64*a*
c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{4\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = \int \frac{e^{4\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$$
$$= \frac{a^8 \int \frac{e^{4\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}$$
$$= \frac{a^8 \int \frac{x^8}{(1-ax)^6(1+ax)^2}\,dx}{c^4}$$
$$= \frac{a^8 \int \left(\frac{1}{a^8} + \frac{1}{4a^8(-1+ax)^6} + \frac{7}{4a^8(-1+ax)^5} + \frac{83}{16a^8(-1+ax)^4} + \frac{67}{8a^8(-1+ax)^3} + \frac{501}{64a^8(-1+ax)^2} + \frac{261}{64a^8(-1\ldots}\right.}{c^4}$$
$$= \frac{x}{c^4} + \frac{1}{20ac^4(1-ax)^5} - \frac{7}{16ac^4(1-ax)^4} + \frac{83}{48ac^4(1-ax)^3} - \frac{67}{16ac^4(1-ax)^2} + \frac{50\ldots}{64ac^4(1\ldots}$$

Mathematica [A]
time = 0.07, size = 98, normalized size = 0.67

$$\frac{\frac{2(-2384+7541ax-4900a^2x^2-6800a^3x^3+9300a^4x^4-1365a^5x^5-1920a^6x^6+480a^7x^7)}{(-1+ax)^5(1+ax)} + 3915\log(1-ax) - 75\log(1+ax)}{960ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*ArcCoth[a*x])/(c - c/(a^2*x^2))^4,x]
```

```
[Out] ((2*(-2384 + 7541*a*x - 4900*a^2*x^2 - 6800*a^3*x^3 + 9300*a^4*x^4 - 1365*a
^5*x^5 - 1920*a^6*x^6 + 480*a^7*x^7))/((-1 + a*x)^5*(1 + a*x)) + 3915*Log[1
 - a*x] - 75*Log[1 + a*x])/(960*a*c^4)
```

Maple [A]
time = 0.17, size = 108, normalized size = 0.74

method	result	size
default	$\frac{a^8\left(\frac{x}{a^8}-\frac{1}{64a^9(ax+1)}-\frac{5\ln(ax+1)}{64a^9}-\frac{1}{20a^9(ax-1)^5}-\frac{7}{16a^9(ax-1)^4}-\frac{83}{48a^9(ax-1)^3}-\frac{67}{16a^9(ax-1)^2}-\frac{501}{64a^9(ax-1)}+\frac{261\ln(ax-1)}{64a^9}\right)}{c^4}$	108

risch	$\frac{x}{c^4}+\frac{-\frac{251a^4c^4x^5}{32}+\frac{155a^3c^4x^4}{8}-\frac{55a^2c^4x^3}{6}-\frac{341c^4a\,x^2}{24}+\frac{8021c^4x}{480}-\frac{149c^4}{30a}}{c^8(ax-1)^4(a^2x^2-1)}+\frac{261\ln(-ax+1)}{64a\,c^4}-\frac{5\ln(ax+1)}{64a\,c^4}$	115
norman	$\frac{\frac{a^8x^9}{c}-\frac{115a^3x^4}{6c}-\frac{133x}{32c}+\frac{101a\,x^2}{16c}+\frac{1049a^2x^3}{96c}-\frac{3869a^4x^5}{480c}+\frac{4709a^5x^6}{240c}+\frac{43a^6x^7}{480c}-\frac{209a^7x^8}{30c}}{(ax+1)^3(ax-1)^5c^3}+\frac{261\ln(ax-1)}{64c^4a}-\frac{5\ln(ax+1)}{64a\,c^4}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^8/c^4*(1/a^8*x-1/64/a^9/(a*x+1)-5/64/a^9*ln(a*x+1)-1/20/a^9/(a*x-1)^5-7/1
6/a^9/(a*x-1)^4-83/48/a^9/(a*x-1)^3-67/16/a^9/(a*x-1)^2-501/64/a^9/(a*x-1)+
261/64/a^9*ln(a*x-1))
```

Maxima [A]
time = 0.28, size = 135, normalized size = 0.92

$$-\frac{3765\,a^5x^5-9300\,a^4x^4+4400\,a^3x^3+6820\,a^2x^2-8021\,ax+2384}{480\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)}+\frac{x}{c^4}-\frac{5\,\log\left(ax+1\right)}{64\,ac^4}+\frac{261\,\log\left(ax-1\right)}{64\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/480*(3765*a^5*x^5 - 9300*a^4*x^4 + 4400*a^3*x^3 + 6820*a^2*x^2 - 8021*a*
x + 2384)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 + 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*
a^2*c^4*x - a*c^4) + x/c^4 - 5/64*log(a*x + 1)/(a*c^4) + 261/64*log(a*x - 1
)/(a*c^4)
```

Fricas [A]
time = 0.52, size = 207, normalized size = 1.42

$$\frac{960\,a^7x^7-3840\,a^6x^6-2730\,a^5x^5+18600\,a^4x^4-13600\,a^3x^3-9800\,a^2x^2+15082\,ax-75\,(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1)\log\left(ax+1\right)+3915\,(a^6x^6-4\,a^5x^5+5\,a^4x^4-5\,a^2x^2+4\,ax-1)\log\left(ax-1\right)-4768}{960\,(a^7c^4x^6-4\,a^6c^4x^5+5\,a^5c^4x^4-5\,a^3c^4x^2+4\,a^2c^4x-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/960*(960*a^7*x^7 - 3840*a^6*x^6 - 2730*a^5*x^5 + 18600*a^4*x^4 - 13600*a^
3*x^3 - 9800*a^2*x^2 + 15082*a*x - 75*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 - 5*
a^2*x^2 + 4*a*x - 1)*log(a*x + 1) + 3915*(a^6*x^6 - 4*a^5*x^5 + 5*a^4*x^4 -
 5*a^2*x^2 + 4*a*x - 1)*log(a*x - 1) - 4768)/(a^7*c^4*x^6 - 4*a^6*c^4*x^5 +
 5*a^5*c^4*x^4 - 5*a^3*c^4*x^2 + 4*a^2*c^4*x - a*c^4)
```

Sympy [A]
time = 0.54, size = 144, normalized size = 0.99

$$a^8\left(\frac{-3765a^5x^5+9300a^4x^4-4400a^3x^3-6820a^2x^2+8021ax-2384}{480a^{15}c^4x^6-1920a^{14}c^4x^5+2400a^{13}c^4x^4-2400a^{11}c^4x^2+1920a^{10}c^4x-480a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{261\log\left(x-\frac{1}{a}\right)}{64}-\frac{5\log\left(x+\frac{1}{a}\right)}{64}}{a^9c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)**2*(a*x+1)**2/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-3765*a**5*x**5 + 9300*a**4*x**4 - 4400*a**3*x**3 - 6820*a**2*x**2 +
 8021*a*x - 2384)/(480*a**15*c**4*x**6 - 1920*a**14*c**4*x**5 + 2400*a**13*
c**4*x**4 - 2400*a**11*c**4*x**2 + 1920*a**10*c**4*x - 480*a**9*c**4) + x/(
a**8*c**4) + (261*log(x - 1/a)/64 - 5*log(x + 1/a)/64)/(a**9*c**4))
```

Giac [A]
time = 0.40, size = 170, normalized size = 1.16

$$\frac{(ax-1)\left(\frac{257}{ax-1}+128\right)}{128\,ac^4\left(\frac{2}{ax-1}+1\right)}-\frac{4\,\log\left(\frac{|ax-1|}{(ax-1)^2|a|}\right)}{ac^4}-\frac{5\,\log\left(\left|-\frac{2}{ax-1}-1\right|\right)}{64\,ac^4}-\frac{\frac{7515\,a^{19}c^{16}}{ax-1}+\frac{4020\,a^{19}c^{16}}{(ax-1)^2}+\frac{1660\,a^{19}c^{16}}{(ax-1)^3}+\frac{420\,a^{19}c^{16}}{(ax-1)^4}+\frac{48\,a^{19}c^{16}}{(ax-1)^5}}{960\,a^{20}c^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)^2*(a*x+1)^2/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] 1/128*(a*x - 1)*(257/(a*x - 1) + 128)/(a*c^4*(2/(a*x - 1) + 1)) - 4*log(abs
(a*x - 1)/((a*x - 1)^2*abs(a)))/(a*c^4) - 5/64*log(abs(-2/(a*x - 1) - 1))/(
a*c^4) - 1/960*(7515*a^19*c^16/(a*x - 1) + 4020*a^19*c^16/(a*x - 1)^2 + 166
0*a^19*c^16/(a*x - 1)^3 + 420*a^19*c^16/(a*x - 1)^4 + 48*a^19*c^16/(a*x - 1
)^5)/(a^20*c^20)
```

Mupad [B]
time = 0.15, size = 131, normalized size = 0.90

$$\frac{\frac{341\,a\,x^2}{24}-\frac{8021\,x}{480}+\frac{149}{30\,a}+\frac{55\,a^2\,x^3}{6}-\frac{155\,a^3\,x^4}{8}+\frac{251\,a^4\,x^5}{32}}{-a^6\,c^4\,x^6+4\,a^5\,c^4\,x^5-5\,a^4\,c^4\,x^4+5\,a^2\,c^4\,x^2-4\,a\,c^4\,x+c^4}+\frac{x}{c^4}+\frac{261\,\ln\left(a\,x-1\right)}{64\,a\,c^4}-\frac{5\,\ln\left(a\,x+1\right)}{64\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^2/((c - c/(a^2*x^2))^4*(a*x - 1)^2),x)
```

```
[Out] ((341*a*x^2)/24 - (8021*x)/480 + 149/(30*a) + (55*a^2*x^3)/6 - (155*a^3*x^4
)/8 + (251*a^4*x^5)/32)/(c^4 + 5*a^2*c^4*x^2 - 5*a^4*c^4*x^4 + 4*a^5*c^4*x^
5 - a^6*c^4*x^6 - 4*a*c^4*x) + x/c^4 + (261*log(a*x - 1))/(64*a*c^4) - (5*l
og(a*x + 1))/(64*a*c^4)
```

3.806 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$-\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}-\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}+\frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a}+\frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/}}{40a}$$

```
[Out] 29/30*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(7/2)/a+7/6*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(7/2)/a+8/7*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(7/2)/a+c^4*(1-1/a/x)^(9/2)*(1
+1/a/x)^(7/2)*x+35/16*c^4*arccsc(a*x)/a-c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2))/a-1/16*c^4*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+7/40*c^4*(1+1/a/x)^(
5/2)*(1-1/a/x)^(1/2)/a+19/40*c^4*(1+1/a/x)^(7/2)*(1-1/a/x)^(1/2)/a-19/16*c^
4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]

time = 0.17, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^4x\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{7/2}}{7a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{5/2}}{6a}+\frac{29c^4\left(\frac{1}{ax}+1\right)^{7/2}\left(1-\frac{1}{ax}\right)^{3/2}}{30a}+\frac{19c^4\left(\frac{1}{ax}+1\right)^{7/2}\sqrt{1-\frac{1}{ax}}}{40a}+\frac{7c^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{40a}-\frac{c^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{16a}-\frac{19c^4\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{16a}+\frac{35c^4\csc^{-1}(ax)}{16a}-\frac{c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

```
[Out] (-19*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) - (c^4*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(16*a) + (7*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/
2))/(40*a) + (19*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2))/(40*a) + (29*c^
4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(7/2))/(30*a) + (7*c^4*(1 - 1/(a*x))^(5
/2)*(1 + 1/(a*x))^(7/2))/(6*a) + (8*c^4*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(
7/2))/(7*a) + c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x))^(7/2)*x + (35*c^4*ArcCs
c[a*x])/(16*a) - (c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{7/2}}{x^2}dx, x, \frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x - c^4\text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right.}{x}\right.$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{7/2}x - \frac{1}{7}(ac^4)\,\text{Subs}$$

$$= \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{7/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{9/2}$$

$$= \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a} + \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}{6a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{7/2}}{7a}$$

$$= \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} + \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}{30a} + \frac{7c^4\left(1-\frac{1}{ax}\right)^{5/2}}{6a}$$

$$= \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + \frac{19c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}{40a} + \frac{29c^4\left(1-\frac{1}{ax}\right)^{3/2}(}{30a}$$

$$= -\frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{40a} + \frac{19c^4\sqrt{1-\frac{1}{ax}}\,(1}{40a}$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\,(1-}{40a}$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\,(1-}{40a}$$

$$= -\frac{19c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} - \frac{c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} + \frac{7c^4\sqrt{1-\frac{1}{ax}}\,(1-}{40a}$$

Mathematica [A]

time = 0.16, size = 120, normalized size = 0.35

$$\frac{c^4\left(\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(-240+280ax+1056a^2x^2-1330a^3x^3-1952a^4x^4+3045a^5x^5+2816a^6x^6+1680a^7x^7)}{x^6}+3675a^6\operatorname{ArcSin}\left(\frac{1}{ax}\right)-1680a^6\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{1680a^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^ArcCoth[a*x],x]
```

```
[Out] (c^4*((Sqrt[1 - 1/(a^2*x^2)]*(-240 + 280*a*x + 1056*a^2*x^2 - 1330*a^3*x^3
- 1952*a^4*x^4 + 3045*a^5*x^5 + 2816*a^6*x^6 + 1680*a^7*x^7))/x^6 + 3675*a^
6*ArcSin[1/(a*x)] - 1680*a^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(1680*a^7
)
```

Maple [A]

time = 0.09, size = 320, normalized size = 0.93

method	result
risch	$\frac{(ax+1)(2816a^6x^6+3045a^5x^5-1952a^4x^4-1330a^3x^3+1056a^2x^2+280ax-240)c^4\sqrt{\frac{ax-1}{ax+1}}}{1680x^7a^8}+\frac{\left(a^7\sqrt{(ax+1)(ax-1)}-\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-3675a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}-3675\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)^
(1/2)*a^8*x^8+1680*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^6*x^6-3675*a^7*x^7*(a^2)
^(1/2)*(a^2*x^2-1)^(1/2)-3675*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2
))+1680*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^8*x^7+1995*
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^5*x^5-1136*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^
4*x^4-1050*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+816*(a^2*x^2-1)^(3/2)*(a^2
)^(1/2)*a^2*x^2+280*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-240*(a^2*x^2-1)^(3/2)
*(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

Maxima [A]

time = 0.48, size = 380, normalized size = 1.11

$$-\frac{1}{840}\left(\frac{3675\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{1995\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+10185\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}+17619\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+4569\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+71801\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+72051\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+31465\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+5355\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{6(ax-1)a^2}{ax+1}+\frac{14(ax-1)^2a^2}{(ax+1)^2}+\frac{14(ax-1)^3a^2}{(ax+1)^3}-\frac{14(ax-1)^5a^2}{(ax+1)^5}-\frac{14(ax-1)^6a^2}{(ax+1)^6}-\frac{6(ax-1)^7a^2}{(ax+1)^7}-\frac{(ax-1)^8a^2}{(ax+1)^8}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/840*(3675*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((
a*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a
^2 - (1995*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 10185*c^4*((a*x - 1)/(a*x + 1
))^(13/2) + 17619*c^4*((a*x - 1)/(a*x + 1))^(11/2) + 4569*c^4*((a*x - 1)/(a
*x + 1))^(9/2) + 71801*c^4*((a*x - 1)/(a*x + 1))^(7/2) + 72051*c^4*((a*x -
1)/(a*x + 1))^(5/2) + 31465*c^4*((a*x - 1)/(a*x + 1))^(3/2) + 5355*c^4*sqrt
((a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
 + 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
 14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)
^8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [A]
time = 0.40, size = 201, normalized size = 0.59

$$-\frac{7350\,a^{7}c^{4}x^{7}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(1680\,a^{8}c^{4}x^{8}+4496\,a^{7}c^{4}x^{7}+5861\,a^{6}c^{4}x^{6}+1093\,a^{5}c^{4}x^{5}-3282\,a^{4}c^{4}x^{4}-274\,a^{3}c^{4}x^{3}+1336\,a^{2}c^{4}x^{2}+40\,ac^{4}x-240\,c^{4}\right)\sqrt{\frac{ax-1}{ax+1}}}{1680\,a^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(7350*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*
x^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1
)/(a*x + 1)) - 1) - (1680*a^8*c^4*x^8 + 4496*a^7*c^4*x^7 + 5861*a^6*c^4*x^6
 + 1093*a^5*c^4*x^5 - 3282*a^4*c^4*x^4 - 274*a^3*c^4*x^3 + 1336*a^2*c^4*x^2
 + 40*a*c^4*x - 240*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{4}\left(\int a^{8}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{8}}\,dx+\int\left(-\frac{4a^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{6}}\right)dx+\int\frac{6a^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{4}}\,dx+\int\left(-\frac{4a^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{2}}\right)dx\right)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**4*(Integral(a**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**8, x) + Integral(-4*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**6, x) + Integral(6*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))
/x**4, x) + Integral(-4*a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a*
*8
```

Giac [A]
time = 0.43, size = 524, normalized size = 1.53

$$-\frac{35\,c^{4}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)\mathrm{sgn}(ax+1)}{8\,a}+\frac{c^{4}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^{2}x^{2}-1}\,c^{4}\mathrm{sgn}(ax+1)}{a}-\frac{3045\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{13}c^{4}|a|\mathrm{sgn}(ax+1)-6720\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{12}ac^{4}\mathrm{sgn}(ax+1)+6860\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{11}c^{4}|a|\mathrm{sgn}(ax+1)-20160\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{10}ac^{4}\mathrm{sgn}(ax+1)+9065\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{9}c^{4}|a|\mathrm{sgn}(ax+1)-49280\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{8}ac^{4}\mathrm{sgn}(ax+1)-49280\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{6}ac^{4}\mathrm{sgn}(ax+1)-9065\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{5}c^{4}|a|\mathrm{sgn}(ax+1)-38976\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{4}ac^{4}\mathrm{sgn}(ax+1)-6860\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{3}c^{4}|a|\mathrm{sgn}(ax+1)-12992\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}ac^{4}\mathrm{sgn}(ax+1)-3045\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)c^{4}|a|\mathrm{sgn}(ax+1)-2816\,ac^{4}\mathrm{sgn}(ax+1)}{840\left(\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}+1\right)^{7}a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -35/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^4*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^4*sgn(a*x + 1)/a - 1/840*(3045*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(a
)*sgn(a*x + 1) - 6720*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
+ 6860*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) - 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 9065*(x*abs(a) - sqrt(
a^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 -
1))^8*a*c^4*sgn(a*x + 1) - 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn
(a*x + 1) - 9065*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) -
 38976*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) - 6860*(x*abs(a)
 - sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) - 12992*(x*abs(a) - sqrt(a^
2*x^2 - 1))^2*a*c^4*sgn(a*x + 1) - 3045*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*
abs(a)*sgn(a*x + 1) - 2816*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 -
 1))^2 + 1)^7*a*abs(a))
```

Mupad [B]
time = 1.37, size = 332, normalized size = 0.97

$$\frac{\frac{51\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{899\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{24}+\frac{3431\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{71801\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{840}+\frac{1523\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{280}+\frac{839\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}+\frac{97\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{8}+\frac{19\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/2}}{8}}{a+\frac{6\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{14\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{14\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{14\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{14\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{6\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}-\frac{a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}}-\frac{35\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{2\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] ((51*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (899*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/24 + (3431*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (71801*c^4*((a*x - 1)
/(a*x + 1))^(7/2))/840 + (1523*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 + (839*
c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 + (97*c^4*((a*x - 1)/(a*x + 1))^(13/2)
)/8 + (19*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x +
1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (14*
a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x - 1
)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (35*c^4*atan(((a*x - 1)/(
a*x + 1))^(1/2)))/(8*a) - (2*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.807 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$-\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}+\frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a}+\frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a}+\frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a}$$

```
[Out] 5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+6/5*c^3*(1-1/a/x)^(5/2)*(1+1/a/x)
^(5/2)/a+c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)*x+15/8*c^3*arccsc(a*x)/a-c^3*a
rctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+1/24*c^3*(1+1/a/x)^(3/2)*(1-1/a/x
)^(1/2)/a+11/12*c^3*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a-7/8*c^3*(1-1/a/x)^(1/
2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.12, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^3x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{5a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{11c^3\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{12a}+\frac{c^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{24a}-\frac{7c^3\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{8a}+\frac{15c^3\csc^{-1}(ax)}{8a}-\frac{c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

```
[Out] (-7*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (c^3*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(24*a) + (11*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(5/2
))/(12*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(4*a) + (6*c^3*
(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(5*a) + c^3*(1 - 1/(a*x))^(7/2)*(1
 + 1/(a*x))^(5/2)*x + (15*c^3*ArcCsc[a*x])/(8*a) - (c^3*ArcTanh[Sqrt[1 - 1/
(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx = -\left(c^3\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^3\text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right.}{x}\right.$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{5}\left(ac^3\right)\text{Subs}$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(\right.$$

$$= \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{4a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(\right.}{5a}$$

$$= \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{12a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1-\right.}{4a}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1-\right.}{12a}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1-\right.}{12a}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1-\right.}{12a}$$

$$= -\frac{7c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{24a} + \frac{11c^3\sqrt{1-\frac{1}{ax}}\left(1-\right.}{12a}$$

Mathematica [A]

time = 0.23, size = 110, normalized size = 0.41

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\,(24-30ax-88a^2x^2+135a^3x^3+184a^4x^4+120a^5x^5)+225a^4x^4\text{ArcSin}\left(\frac{1}{ax}\right)-120a^4x^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{120a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^ArcCoth[a*x],x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(24 - 30*a*x - 88*a^2*x^2 + 135*a^3*x^3 + 184*a
^4*x^4 + 120*a^5*x^5) + 225*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 +
Sqrt[1 - 1/(a^2*x^2)])*x]))/(120*a^5*x^4)
```

Maple [A]
time = 0.08, size = 272, normalized size = 1.01

method	result
risch	$\frac{(ax+1)(184a^4x^4+135a^3x^3-88a^2x^2-30ax+24)c^3\sqrt{\frac{ax-1}{ax+1}}}{120x^5a^6}+\frac{\left(a^5\sqrt{(ax+1)(ax-1)}-\frac{a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right.}{\sqrt{a^2}}\right.}{}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^3\left(-120\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^6x^6+120\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4-225\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5-225\sqrt{a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(1
/2)*a^6*x^6+120*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-225*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*a^5*x^5-225*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x^5+12
0*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5+105*(a^2*x^
2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3-64*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-30*
(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+24*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x+1
)*(a*x-1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 302, normalized size = 1.12

$$-\frac{1}{60}\left(\frac{225\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+305\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+86\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+1654\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+1345\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+345\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4(ax-1)a^2}{ax+1}+\frac{5(ax-1)^2a^2}{(ax+1)^2}-\frac{5(ax-1)^4a^2}{(ax+1)^4}-\frac{4(ax-1)^5a^2}{(ax+1)^5}-\frac{(ax-1)^6a^2}{(ax+1)^6}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/60*(225*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
 - 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 -
 (105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 305*c^3*((a*x - 1)/(a*x + 1))^(9/2
) + 86*c^3*((a*x - 1)/(a*x + 1))^(7/2) + 1654*c^3*((a*x - 1)/(a*x + 1))^(5/
2) + 1345*c^3*((a*x - 1)/(a*x + 1))^(3/2) + 345*c^3*sqrt((a*x - 1)/(a*x + 1
)))/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1
)^4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x
+ 1)^6 + a^2))*a
```

<u>Fricas</u> [A]
time = 0.46, size = 179, normalized size = 0.67

$$-\frac{450\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(120\,a^6c^3x^6+304\,a^5c^3x^5+319\,a^4c^3x^4+47\,a^3c^3x^3-118\,a^2c^3x^2-6\,ac^3x+24\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{120\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(450*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5
*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (120*a^6*c^3*x^6 + 304*a^5*c^3*x^5 + 319*a^4*c^3*x^4 + 47*a
^3*c^3*x^3 - 118*a^2*c^3*x^2 - 6*a*c^3*x + 24*c^3)*sqrt((a*x - 1)/(a*x + 1)
))/(a^6*x^5)
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int a^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^6}\right)dx+\int\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^4}\,dx+\int\left(-\frac{3a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^2}\right)dx\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**3*(Integral(a**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/x**6, x) + Integral(3*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**4, x) + Integral(-3*a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)
)/x**2, x))/a**6
```

<u>Giac</u> [A]
time = 0.43, size = 394, normalized size = 1.46

$$-\frac{15\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{4\,a}+\frac{c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^3\mathrm{sgn}(ax+1)}{a}-\frac{135\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|\mathrm{sgn}(ax+1)-360\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3\mathrm{sgn}(ax+1)+150\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|\mathrm{sgn}(ax+1)-720\left(x|a|-\sqrt{a^2x^2-1}\right)^6ac^3\mathrm{sgn}(ax+1)-1120\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^3\mathrm{sgn}(ax+1)-150\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|\mathrm{sgn}(ax+1)-560\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3\mathrm{sgn}(ax+1)-135\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|\mathrm{sgn}(ax+1)-184\,ac^3\mathrm{sgn}(ax+1)}{60\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^5a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

[Out] -15/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^3*log(ab
s(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c
^3*sgn(a*x + 1)/a - 1/60*(135*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) - 360*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) + 150
*(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) - 720*(x*abs(a) -
 sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) - 1120*(x*abs(a) - sqrt(a^2*x^2 -
1))^4*a*c^3*sgn(a*x + 1) - 150*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*
sgn(a*x + 1) - 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 13
5*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) - 184*a*c^3*sgn(a*
x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))

Mupad [B]
time = 0.11, size = 258, normalized size = 0.96

$$\frac{\frac{23\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{269\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12}+\frac{827\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{30}+\frac{43\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{30}+\frac{61\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{12}+\frac{7\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}-\frac{15\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{2\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(1/2),x)

[Out] ((23*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (269*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/12 + (827*c^3*((a*x - 1)/(a*x + 1))^(5/2))/30 + (43*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/30 + (61*c^3*((a*x - 1)/(a*x + 1))^(9/2))/12 + (7*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (15*c^3*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/(4*a) - (2*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a

3.808 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$-\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)$$

```
[Out] 4/3*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+c^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(3/
2)*x+3/2*c^2*arccsc(a*x)/a-c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+3
/2*c^2*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a-1/2*c^2*(1-1/a/x)^(1/2)*(1+1/a/x)^
(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^2x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}+\frac{4c^2\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{3a}+\frac{3c^2\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}+\frac{3c^2\csc^{-1}(ax)}{2a}-\frac{c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + (3*c^2*Sqrt[1 - 1/(a*x)]
*(1 + 1/(a*x))^(3/2))/(2*a) + (4*c^2*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3/2
))/(3*a) + c^2*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2)*x + (3*c^2*ArcCsc[a*
x])/(2*a) - (c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx = -\left(c^2\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x - c^2\text{Subst}\left(\int \frac{\left(-\frac{1}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\cdots}}{x}\right.$$

$$= \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{3}\left(ac^2\right)\text{Subs}\cdots$$

$$= \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{3a} + c^2\left(1-\frac{1}{ax}\right)^{5/2}\left(\cdots\right.$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1-\cdots\right.}{3a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1-\cdots\right.}{3a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1-\cdots\right.}{3a}$$

$$= -\frac{c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a} + \frac{3c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{2a} + \frac{4c^2\left(1-\frac{1}{ax}\right)^{3/2}\left(1-\cdots\right.}{3a}$$

Mathematica [A]
time = 0.16, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\left(-2+3ax+8a^2x^2+6a^3x^3\right)+9a^2x^2\text{ArcSin}\left(\frac{1}{ax}\right)-6a^2x^2\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(-2 + 3*a*x + 8*a^2*x^2 + 6*a^3*x^3) + 9*a^2*x^
2*ArcSin[1/(a*x)] - 6*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*x
^2)
```

Maple [A]
time = 0.06, size = 224, normalized size = 1.15

method	result
risch	$\frac{(ax+1)(8a^2x^2+3ax-2)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4} + \frac{\left(a^3\sqrt{(ax+1)(ax-1)} - \frac{a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}} + \frac{3a^3\arctan\left(\frac{1}{\sqrt{a^2x^2 \cdots}}\right)}{2} \cdots\right.}{a^4(ax-1) \cdots}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c^2\left(-6\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+6(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2-9\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+6\ln\left(\frac{a^2x+\sqrt{\cdots}}{\cdots}\right.\right.}{6\sqrt{(ax+1)(ax-\cdots)}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c^2*(-6*(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*
a^4*x^4+6*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2-9*(a^2*x^2-1)^(1/2)*(a^2)^(
1/2)*a^3*x^3+6*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^4*x^
3-9*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))+3*(a^2)^(1/2)*(a^2*x^2-
1)^(3/2)*a*x-2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/((a*x+1)*(a*x-1))^(1/2)/a^4/x
^3/(a^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 223, normalized size = 1.14

$$-\frac{1}{3}a\left(\frac{9c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{3c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+29c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+15c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(9*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c^2*log(sqrt((a*x -
 1)/(a*x + 1)) + 1)/a^2 - 3*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (3
*c^2*((a*x - 1)/(a*x + 1))^(7/2) + c^2*((a*x - 1)/(a*x + 1))^(5/2) + 29*c^2
*((a*x - 1)/(a*x + 1))^(3/2) + 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x -
1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x + 1
)^4 + a^2))
```

Fricas [A]
time = 0.44, size = 156, normalized size = 0.80

$$-\frac{18\,a^{3}c^{2}x^{3}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+6\,a^{3}c^{2}x^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-6\,a^{3}c^{2}x^{3}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(6\,a^{4}c^{2}x^{4}+14\,a^{3}c^{2}x^{3}+11\,a^{2}c^{2}x^{2}+ac^{2}x-2\,c^{2}\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) + 6*a^3*c^2*x^3*log(
sqrt((a*x - 1)/(a*x + 1)) + 1) - 6*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) - (6*a^4*c^2*x^4 + 14*a^3*c^2*x^3 + 11*a^2*c^2*x^2 + a*c^2*x - 2*c^2
)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{2}\left(\int a^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{4}}\,dx+\int\left(-\frac{2a^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{2}}\right)dx\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c**2*(Integral(a**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(sqrt(a
*x/(a*x + 1) - 1/(a*x + 1))/x**4, x) + Integral(-2*a**2*sqrt(a*x/(a*x + 1)
- 1/(a*x + 1))/x**2, x))/a**4
```

Giac [A]
time = 0.44, size = 264, normalized size = 1.35

$$-\frac{3\,c^{2}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)\mathrm{sgn}(ax+1)}{a}+\frac{c^{2}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^{2}x^{2}-1}\,c^{2}\mathrm{sgn}(ax+1)}{a}-\frac{3\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{5}c^{2}|a|\mathrm{sgn}(ax+1)-12\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{4}ac^{2}\mathrm{sgn}(ax+1)-12\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}ac^{2}\mathrm{sgn}(ax+1)-3\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)c^{2}|a|\mathrm{sgn}(ax+1)-8\,ac^{2}\mathrm{sgn}(ax+1)}{3\left(\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}+1\right)^{3}a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -3*c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c^2*log(abs(-
x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*
sgn(a*x + 1)/a - 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
 + 1) - 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) - 12*(x*abs(
a) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 3*(x*abs(a) - sqrt(a^2*x^2 -
 1))*c^2*abs(a)*sgn(a*x + 1) - 8*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B]
time = 1.30, size = 183, normalized size = 0.94

$$\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{29\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}+c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}-\frac{3\,c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (29*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 + c^2*((a*x - 1)/(a*x + 1))^(7/2))/(
a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*x - 1
)^4)/(a*x + 1)^4) - (3*c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c^2*at
anh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.809 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=108

$$\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x+\frac{c\csc^{-1}(ax)}{a}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] c*arccsc(a*x)/a-c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a/x)^(3
/2)*x*(1+1/a/x)^(1/2)+2*c*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6329, 99, 159, 21, 132, 41, 222, 94, 214}

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}+\frac{2c\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{a}+\frac{c\csc^{-1}(ax)}{a}-\frac{c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

```
[Out] (2*c*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/a + c*(1 - 1/(a*x))^(3/2)*Sqrt[1
+ 1/(a*x)]*x + (c*ArcCsc[a*x])/a - (c*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/
(a*x)]])/a
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\left(c\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{3/2}\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x-c\text{Subst}\left(\int\frac{\left(-\frac{1}{a}-\frac{2x}{a^2}\right)\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x-(ac)\text{Subst}\left(\int\frac{}{x\sqrt{1}}\right.$$

$$=\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x+\frac{c\text{Subst}\left(\int\frac{\sqrt{1+\frac{x}{a}}}{x\sqrt{1-\frac{x}{a}}}\right.}{a}$$

$$=\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x+\frac{c\text{Subst}\left(\int\frac{}{\sqrt{1-\frac{x}{a}}}\right.}{a}$$

$$=\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x+\frac{c\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right.}{a^2}$$

$$=\frac{2c\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{a}+c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x+\frac{c\csc^{-1}(ax)}{a}-\frac{c\tanh}{}$$

Mathematica [A]

time = 0.05, size = 55, normalized size = 0.51

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}\,(1+ax)+\text{ArcSin}\left(\frac{1}{ax}\right)-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))/E^ArcCoth[a*x],x]
```

```
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(1 + a*x) + ArcSin[1/(a*x)] - Log[(1 + Sqrt[1 - 1
/(a^2*x^2)])*x]))/a
```

Maple [A]
time = 0.06, size = 166, normalized size = 1.54

method	result
risch	$\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(a\sqrt{(ax+1)(ax-1)}-\frac{a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}+a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{\frac{ax-}{ax+}}}{a^2(ax-1)}$
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax+\ln\left(\frac{a^2x+\sqrt{a^2x^2-1}}{\sqrt{a^2}}\right.\right.}{\sqrt{(ax+1)(ax-1)}\,a^2x\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2+
(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x+ln((a^2*x+(
a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-(a^2)^(1/2)*arctan(1/(a^2*
x^2-1)^(1/2))*a*x)/((a*x+1)*(a*x-1))^(1/2)/a^2/x/(a^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 117, normalized size = 1.08

$$-a\left(\frac{4\,c\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}+\frac{2\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -a*(4*c*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) + 2*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + c*log(sqrt((a*x - 1)/(a*x + 1)) +
1)/a^2 - c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)
```

Fricas [A]
time = 0.36, size = 107, normalized size = 0.99

$$-\frac{2\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(a^{2}cx^{2}+2\,acx+c\right)\sqrt{\frac{ax-1}{ax+1}}}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) + a*c*x*log(sqrt((a*x - 1)/(a*x
 + 1)) + 1) - a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (a^2*c*x^2 + 2*a*c
*x + c)*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int a^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}\,dx+\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{x^{2}}\right)dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] c*(Integral(a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1)), x) + Integral(-sqrt(a*x
/(a*x + 1) - 1/(a*x + 1))/x**2, x))/a**2
```

Giac [A]
time = 0.43, size = 121, normalized size = 1.12

$$-\frac{2\,c\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)\mathrm{sgn}\left(ax+1\right)}{a}+\frac{c\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\mathrm{sgn}\left(ax+1\right)}{|a|}+\frac{\sqrt{a^{2}x^{2}-1}\,c\,\mathrm{sgn}\left(ax+1\right)}{a}+\frac{2\,c\,\mathrm{sgn}\left(ax+1\right)}{\left(\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] -2*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + c*log(abs(-x*ab
s(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a*
x + 1)/a + 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a))
```

Mupad [B]
time = 1.28, size = 84, normalized size = 0.78

$$\frac{4\,c\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}-\frac{2\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{2\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (4*c*((a*x - 1)/(a*x + 1))^(1/2))/(a - (a*(a*x - 1)^2)/(a*x + 1)^2) - (2*c*
atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a - (2*c*atan(((a*x - 1)/(a*x + 1))^(1/
2)))/a
```

3.810 $\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=105

$$\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] -arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+2*(1-1/a/x)^(1/2)/a/c/(1+1/a/
x)^(1/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 105, 21, 96, 94, 214}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\sqrt{\frac{1}{ax}+1}}+\frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{\frac{1}{ax}+1}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] (2*Sqrt[1 - 1/(a*x)])/(a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*Sq
rt[1 + 1/(a*x)]) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{x}{a^2}}{x\sqrt{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx, x, \sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\right)}{a^2c}$$

$$= \frac{2\sqrt{1-\frac{1}{ax}}}{ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\sqrt{1+\frac{1}{ax}}} - \frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.54

$$\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(2+ax)}{1+ax} - \frac{\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(2 + a*x))/(1 + a*x) - Log[(1 + Sqrt[1 - 1/(a^2*x
^2)])*x]/a)/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(93) = 186$.
time = 0.14, size = 250, normalized size = 2.38

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}+\frac{\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{a^4\left(x+\frac{1}{a}\right)}\right)a^2\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{(ax+1)(}}{c(ax-1)}$
default	$-\frac{\left(-3\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^2x^2+2\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^3x^2+((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-3*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+2*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+((a*x+1)*(a*x-1))^(3/2)*(a^
2)^(1/2)-6*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+4*ln((a^2*x+(a^2)^(1/2)*
((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-3*(a^2)^(1/2)*((a*x+1)*(a*x-1))
^(1/2)+2*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*(
(a*x-1)/(a*x+1))^(1/2)/(a^2)^(1/2)/(a*x+1)/c/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]
time = 0.27, size = 121, normalized size = 1.15

$$-a\left(\frac{2\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}+\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}-\frac{\sqrt{\frac{ax-1}{ax+1}}}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -a*(2*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) + log(s
qrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - log(sqrt((a*x - 1)/(a*x + 1)) - 1)/
(a^2*c) - sqrt((a*x - 1)/(a*x + 1))/(a^2*c))
```

Fricas [A]
time = 0.37, size = 67, normalized size = 0.64

$$\frac{(ax+2)\sqrt{\frac{ax-1}{ax+1}} - \log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right) + \log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] ((a*x + 2)*sqrt((a*x - 1)/(a*x + 1)) - log(sqrt((a*x - 1)/(a*x + 1)) + 1) +
 log(sqrt((a*x - 1)/(a*x + 1)) - 1))/(a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \int \frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**2*x**2 - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [B]
time = 0.05, size = 86, normalized size = 0.82

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,(a\,x-1)}{a\,x+1}}+\frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}-\frac{2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + ((a*x -
 1)/(a*x + 1))^(1/2)/(a*c) - (2*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/(a*c)
```

3.811 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=179

$$-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}}+\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\right.}{c}$$

```
[Out] -arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2-2/a/c^2/(1+1/a/x)^(3/2)/(1-
1/a/x)^(1/2)+x/c^2/(1+1/a/x)^(3/2)/(1-1/a/x)^(1/2)+5/3*(1-1/a/x)^(1/2)/a/c^
2/(1+1/a/x)^(3/2)+8/3*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^2\sqrt{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] -2/(a*c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) + (5*Sqrt[1 - 1/(a*x)])/(3
*a*c^2*(1 + 1/(a*x))^(3/2)) + (8*Sqrt[1 - 1/(a*x)])/(3*a*c^2*Sqrt[1 + 1/(a*
x)]) + x/(c^2*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(3/2)) - ArcTanh[Sqrt[1 - 1/(
a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{3x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{1}{a^2}+\frac{4x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/}}\right.}{c^2}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\text{Subst}}{}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\,(1+}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\,(1+}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\,(1+}$$

$$= -\frac{2}{ac^2\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{5\sqrt{1-\frac{1}{ax}}}{3ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{8\sqrt{1-\frac{1}{ax}}}{3ac^2\sqrt{1+\frac{1}{ax}}} + \frac{x}{c^2\sqrt{1-\frac{1}{ax}}\,(1+}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.47

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-8-5ax+7a^2x^2+3a^3x^3)}{3(-1+ax)(1+ax)^2}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^2),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-8 - 5*a*x + 7*a^2*x^2 + 3*a^3*x^3))/(3*(-1 +
a*x)*(1 + a*x)^2) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(155) = 310$.
time = 0.16, size = 530, normalized size = 2.96

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{4a^6\left(x-\frac{1}{a}\right)}+\frac{19\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x\right.}}{12a^6\left(x+\frac{1}{a}\right)}\right.}{c^2(ax-1)}$
default	$-\frac{\left(-45\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^5x^5+24\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^6x^5+21((ax+1)(ax-1))^{\frac{3}{2}}\sqrt{a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-45*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5+24*ln((a^2*x+(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+21*((a*x+1)*(a*x-1))^(3/
2)*(a^2)^(1/2)*a^3*x^3-45*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+24*ln
((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4-11*(a^2)^
(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2+90*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
)*a^3*x^3-48*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^
4*x^3-5*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x+90*((a*x+1)*(a*x-1))^(1/2)*
(a^2)^(1/2)*a^2*x^2-48*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)
^(1/2))*a^3*x^2+19*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-45*((a*x+1)*(a*x-1))
^(1/2)*(a^2)^(1/2)*a*x+24*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a
^2)^(1/2))*a^2*x-45*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+24*a*ln((a^2*x+(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(1/2)/(a
*x-1)^2/(a^2)^(1/2)/(a*x+1)^2/c^2/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]
time = 0.27, size = 163, normalized size = 0.91

$$-\frac{1}{12}\,a\left(\frac{3\left(\frac{9\,(ax-1)}{ax+1}-1\right)}{a^2c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^2\sqrt{\frac{ax-1}{ax+1}}}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+18\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{12\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*a*(3*(9*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^2*((a*x - 1)/(a*x + 1))^(3/2)
 - a^2*c^2*sqrt((a*x - 1)/(a*x + 1))) - (((a*x - 1)/(a*x + 1))^(3/2) + 18*s
qrt((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 12*log(sqrt((a*x - 1)/(a*x + 1)) + 1)
/(a^2*c^2) - 12*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A]
time = 0.47, size = 119, normalized size = 0.66

$$-\frac{3\,(a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-3\,(a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(3\,a^3x^3+7\,a^2x^2-5\,ax-8)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^3c^2x^2-ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 3*(a^2*x^2 - 1)*
log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (3*a^3*x^3 + 7*a^2*x^2 - 5*a*x - 8)*sq
rt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 - a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\int\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**4*x**4 - 2*a**2*x*
*2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^2, x)
```

Mupad [B]
time = 1.30, size = 137, normalized size = 0.77

$$\frac{\frac{9\,(a\,x-1)}{a\,x+1}-1}{4\,a\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}-4\,a\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}+\frac{3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^2}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{12\,a\,c^2}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^2,x)
```

```
[Out] ((9*(a*x - 1))/(a*x + 1) - 1)/(4*a*c^2*((a*x - 1)/(a*x + 1))^(1/2) - 4*a*c^
2*((a*x - 1)/(a*x + 1))^(3/2)) + (3*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^2)
+ ((a*x - 1)/(a*x + 1))^(3/2)/(12*a*c^2) + (atan(((a*x - 1)/(a*x + 1))^(1/2
)*1i)*2i)/(a*c^2)
```

3.812 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=255

$$-\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{16\sqrt{1-}}{5ac^3\sqrt{1-}}$$

```
[Out] -4/3/a/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)^(5/2)+x/c^3/(1-1/a/x)^(3/2)/(1+1/a/x)^
(5/2)-arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-13/3/a/c^3/(1+1/a/x)^(
5/2)/(1-1/a/x)^(1/2)+14/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+11/5*(1-1/a
/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+16/5*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{16\sqrt{1-\frac{1}{ax}}}{5ac^3\sqrt{\frac{1}{ax}+1}}+\frac{11\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -4/(3*a*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2)) - 13/(3*a*c^3*Sqrt[1 -
 1/(a*x)]*(1 + 1/(a*x))^(5/2)) + (14*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*
x))^(5/2)) + (11*Sqrt[1 - 1/(a*x)])/(5*a*c^3*(1 + 1/(a*x))^(3/2)) + (16*Sqr
t[1 - 1/(a*x)])/(5*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*(1 - 1/(a*x))^(3/2)*(1
 + 1/(a*x))^(5/2)) - ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{7/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{16x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)\cdots}\right.}{3c^3}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5\cdots}}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\cdots}{c^3\left(\cdots\right.}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\cdots}{5ac\cdots}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\cdots}{5ac\cdots}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\cdots}{5ac\cdots}$$

$$= -\frac{4}{3ac^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{13}{3ac^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{5ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{11\cdots}{5ac\cdots}$$

Mathematica [A]
time = 0.09, size = 101, normalized size = 0.40

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(48+33ax-87a^2x^2-52a^3x^3+38a^4x^4+15a^5x^5)}{15(-1+ax)^2(1+ax)^3}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(48 + 33*a*x - 87*a^2*x^2 - 52*a^3*x^3 + 38*a^4
*x^4 + 15*a^5*x^5))/(15*(-1 + a*x)^2*(1 + a*x)^3) - Log[(1 + Sqrt[1 - 1/(a^
2*x^2)])*x])/(a*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(217) = 434$.
time = 0.19, size = 714, normalized size = 2.80

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{24a^9\left(x-\frac{1}{a}\right)^2}-\frac{25\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x\right.}}{48a^8\left(x-\frac{1}{a}\right)}\right.}{}$
default	$-\frac{\left(-525\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^7x^7+240\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^8x^7+285((ax+1)(ax-1))^{\frac{3}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-525*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^7*x^7+240*ln((a^2*x+(a^2
)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^8*x^7+285*((a*x+1)*(a*x-1))
^(3/2)*(a^2)^(1/2)*a^5*x^5-525*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^6*x^6+
240*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^7*x^6-83*
((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4+1575*((a*x+1)*(a*x-1))^(1/2)*(a
^2)^(1/2)*a^5*x^5-720*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^
(1/2))*a^6*x^5-218*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3+1575*(a^2)^(
1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-720*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+342*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a
^2*x^2-1575*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+720*ln((a^2*x+(a^2)
^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3-3*(a^2)^(1/2)*((a*x+1)
*(a*x-1))^(3/2)*a*x-1575*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+720*ln
((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2-243*((a*x
```

```
+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+525*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x-
240*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x+525*(
a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-240*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*
x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)^3/(a^2)^(1/2)/
(a*x+1)^3/c^3/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]
time = 0.27, size = 197, normalized size = 0.77

$$\frac{1}{240}a\left(\frac{5\left(\frac{23\,(ax-1)}{ax+1}-\frac{120\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+40\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+450\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}-\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}+\frac{240\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/240*a*(5*(23*(a*x - 1)/(a*x + 1) - 120*(a*x - 1)^2/(a*x + 1)^2 + 1)/(a^2*
c^3*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (3
*((a*x - 1)/(a*x + 1))^(5/2) + 40*((a*x - 1)/(a*x + 1))^(3/2) + 450*sqrt((a
*x - 1)/(a*x + 1)))/(a^2*c^3) - 240*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2
*c^3) + 240*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A]
time = 0.36, size = 161, normalized size = 0.63

$$-\frac{15\,(a^4x^4-2\,a^2x^2+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^4x^4-2\,a^2x^2+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(15\,a^5x^5+38\,a^4x^4-52\,a^3x^3-87\,a^2x^2+33\,ax+48)\sqrt{\frac{ax-1}{ax+1}}}{15\,(a^5c^3x^4-2\,a^3c^3x^2+ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (15*a^5*x^5
 + 38*a^4*x^4 - 52*a^3*x^3 - 87*a^2*x^2 + 33*a*x + 48)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\int\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*Integral(x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^3, x)
```

Mupad [B]
time = 0.06, size = 178, normalized size = 0.70

$$\frac{15\sqrt{\frac{a\,x-1}{a\,x+1}}}{8\,a\,c^3}-\frac{\frac{23\,(a\,x-1)}{3\,(a\,x+1)}-\frac{40\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-16\,a\,c^3\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{6\,a\,c^3}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{80\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^3,x)
```

```
[Out] (15*((a*x - 1)/(a*x + 1))^(1/2))/(8*a*c^3) - ((23*(a*x - 1))/(3*(a*x + 1)) - (40*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(5/2)) + ((a*x - 1)/(a*x + 1))^(3/2)/(6*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)/(80*a*c^3) + (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*2i)/(a*c^3)
```

3.813 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=329

$$-\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}}-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(1+\frac{1}{ax}\right)}$$

```
[Out] -6/5/a/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-31/15/a/c^4/(1-1/a/x)^(3/2)/(1+1
/a/x)^(7/2)+x/c^4/(1-1/a/x)^(5/2)/(1+1/a/x)^(7/2)-arctanh((1-1/a/x)^(1/2)*(
1+1/a/x)^(1/2))/a/c^4-28/3/a/c^4/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+115/21*(1-
1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+122/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(
5/2)+93/35*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+128/35*(1-1/a/x)^(1/2)/a/c
^4/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{128\sqrt{1-\frac{1}{ax}}}{35ac^4\sqrt{\frac{1}{ax}+1}}+\frac{93\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{122\sqrt{1-\frac{1}{ax}}}{35ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{115\sqrt{1-\frac{1}{ax}}}{21ac^4\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -6/(5*a*c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - 31/(15*a*c^4*(1 - 1/
(a*x))^(3/2)*(1 + 1/(a*x))^(7/2)) - 28/(3*a*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a
*x))^(7/2)) + (115*Sqrt[1 - 1/(a*x)])/(21*a*c^4*(1 + 1/(a*x))^(7/2)) + (122
*Sqrt[1 - 1/(a*x)])/(35*a*c^4*(1 + 1/(a*x))^(5/2)) + (93*Sqrt[1 - 1/(a*x)])
/(35*a*c^4*(1 + 1/(a*x))^(3/2)) + (128*Sqrt[1 - 1/(a*x)])/(35*a*c^4*Sqrt[1
+ 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(7/2)) - ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]]/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{7/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{5}{a^2}+\frac{36x}{a^3}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)\cdots}\right.}{5c^4}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{a}\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\cdots\right.}$$

$$= -\frac{6}{5ac^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{31}{15ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{28}{3ac^4\sqrt{1-\frac{1}{ax}}\left(1+\cdots\right.}$$

Mathematica [A]
time = 0.11, size = 117, normalized size = 0.36

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-384-279ax+1065a^2x^2+715a^3x^3-965a^4x^4-559a^5x^5+281a^6x^6+105a^7x^7)}{105(-1+ax)^3(1+ax)^4}-\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-384 - 279*a*x + 1065*a^2*x^2 + 715*a^3*x^3 -
965*a^4*x^4 - 559*a^5*x^5 + 281*a^6*x^6 + 105*a^7*x^7))/(105*(-1 + a*x)^3*(
1 + a*x)^4) - Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 897 vs. $2(279) = 558$.
time = 0.19, size = 898, normalized size = 2.73

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{7\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(x-\frac{1}{a}\right)}}{60a^{11}\left(x-\frac{1}{a}\right)^2}-\frac{379\sqrt{a^2\left(x-\frac{1}{a}\right)^2+2a\left(\right.}}{480a^{10}\left(x-\frac{1}{a}\right)}\right.}{}$
default	$-\frac{\left(-53760\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3+13440\ln\left(\frac{a^2x+\sqrt{a^2}\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^2x+7705((a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13440*(-53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)
)*a^4*x^3+13440*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))
*a^2*x+7705*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3-198450*((a*x+1)*(a*
x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5-37095*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a
^2*x^2+2637*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-198450*(a^2)^(1/2)*((a*
x+1)*(a*x-1))^(1/2)*a^4*x^4+132300*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*
x^3+132300*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2-33075*((a*x+1)*(a*x-
1))^(1/2)*(a^2)^(1/2)*a*x+16077*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+13440*a
*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))-33075*(a^2)^(1
/2)*((a*x+1)*(a*x-1))^(1/2)+80640*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(
1/2))/(a^2)^(1/2))*a^6*x^5-53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1
/2))/(a^2)^(1/2))*a^3*x^2+80640*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/
2))/(a^2)^(1/2))*a^5*x^4-53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
```

```
))/(a^2)^(1/2))*a^7*x^6-53760*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)
)/(a^2)^(1/2))*a^8*x^7+132300*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^7*x^7-2
7673*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^5*x^5+132300*((a*x+1)*(a*x-1))^(
1/2)*(a^2)^(1/2)*a^6*x^6+24295*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4-
2893*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^6*x^6+13440*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^10*x^9+13440*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^9*x^8-33075*((a*x+1)*(a*x-1))^(1/
2)*(a^2)^(1/2)*a^9*x^9+19635*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^7*x^7-33
075*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^8*x^8)/a*((a*x-1)/(a*x+1))^(1/2)/
(a*x-1)^4/(a^2)^(1/2)/(a*x+1)^4/c^4/((a*x+1)*(a*x-1))^(1/2)
```

Maxima [A]
time = 0.27, size = 231, normalized size = 0.70

$$\frac{1}{6720}\,a\left(\frac{7\left(\frac{47\,(ax-1)}{ax+1}+\frac{655\,(ax-1)^2}{(ax+1)^2}-\frac{2625\,(ax-1)^3}{(ax+1)^3}+3\right)}{a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-a^2c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}}+\frac{5\left(3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+42\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+329\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2940\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2c^4}-\frac{6720\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}+\frac{6720\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/6720*a*(7*(47*(a*x - 1)/(a*x + 1) + 655*(a*x - 1)^2/(a*x + 1)^2 - 2625*(a
*x - 1)^3/(a*x + 1)^3 + 3)/(a^2*c^4*((a*x - 1)/(a*x + 1))^(7/2) - a^2*c^4*(
(a*x - 1)/(a*x + 1))^(5/2)) + 5*(3*((a*x - 1)/(a*x + 1))^(7/2) + 42*((a*x -
 1)/(a*x + 1))^(5/2) + 329*((a*x - 1)/(a*x + 1))^(3/2) + 2940*sqrt((a*x - 1
)/(a*x + 1)))/(a^2*c^4) - 6720*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c^4)
 + 6720*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A]
time = 0.51, size = 205, normalized size = 0.62

$$-\frac{105\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,(a^6x^6-3\,a^4x^4+3\,a^2x^2-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(105\,a^7x^7+281\,a^6x^6-559\,a^5x^5-965\,a^4x^4+715\,a^3x^3+1065\,a^2x^2-279\,ax-384)\sqrt{\frac{ax-1}{ax+1}}}{105\,(a^7c^4x^6-3\,a^5c^4x^4+3\,a^3c^4x^2-ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/105*(105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a*x +
 1)) + 1) - 105*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(sqrt((a*x - 1)/(a
*x + 1)) - 1) - (105*a^7*x^7 + 281*a^6*x^6 - 559*a^5*x^5 - 965*a^4*x^4 + 71
5*a^3*x^3 + 1065*a^2*x^2 - 279*a*x - 384)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c
^4*x^6 - 3*a^5*c^4*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^8\int\frac{x^8\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*Integral(x**8*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**8*x**8 - 4*a**6*x**6 + 6*a**4*x**4 - 4*a**2*x**2 + 1), x)/c**4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^4, x)
```

Mupad [B]
time = 1.28, size = 217, normalized size = 0.66

$$\frac{35\sqrt{\frac{a\,x-1}{a\,x+1}}}{16\,a\,c^4}-\frac{\frac{131\,{\left(a\,x-1\right)}^2}{3\,{\left(a\,x+1\right)}^2}-\frac{175\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}+\frac{47\,\left(a\,x-1\right)}{15\,\left(a\,x+1\right)}+\frac{1}{5}}{64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}-64\,a\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}+\frac{47\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{192\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{32\,a\,c^4}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{448\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^4,x)
```

```
[Out] (35*((a*x - 1)/(a*x + 1))^(1/2))/(16*a*c^4) - ((131*(a*x - 1)^2)/(3*(a*x + 1)^2) - (175*(a*x - 1)^3)/(a*x + 1)^3 + (47*(a*x - 1))/(15*(a*x + 1)) + 1/5)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(7/2)) + (47*((a*x - 1)/(a*x + 1))^(3/2))/(192*a*c^4) + ((a*x - 1)/(a*x + 1))^(5/2)/(32*a*c^4) + ((a*x - 1)/(a*x + 1))^(7/2)/(448*a*c^4) + (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*2i)/(a*c^4)
```

3.814 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=90

$$\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

```
[Out] 1/7*c^4/a^8/x^7-1/3*c^4/a^7/x^6-2/5*c^4/a^6/x^5+3/2*c^4/a^5/x^4-3*c^4/a^3/x
^2+2*c^4/a^2/x+c^4*x-2*c^4*ln(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}-\frac{2c^4\log(x)}{a}+c^4x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx\\
&= -\frac{c^4\int \frac{e^{-2\tanh^{-1}(ax)}\left(1-a^2x^2\right)^4}{x^8}\,dx}{a^8}\\
&= -\frac{c^4\int \frac{(1-ax)^5(1+ax)^3}{x^8}\,dx}{a^8}\\
&= -\frac{c^4\int \left(-a^8+\frac{1}{x^8}-\frac{2a}{x^7}-\frac{2a^2}{x^6}+\frac{6a^3}{x^5}-\frac{6a^5}{x^3}+\frac{2a^6}{x^2}+\frac{2a^7}{x}\right)dx}{a^8}\\
&= \frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 1.00

$$\frac{c^4}{7a^8x^7}-\frac{c^4}{3a^7x^6}-\frac{2c^4}{5a^6x^5}+\frac{3c^4}{2a^5x^4}-\frac{3c^4}{a^3x^2}+\frac{2c^4}{a^2x}+c^4x-\frac{2c^4\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(2*ArcCoth[a*x]),x]
```

```
[Out] c^4/(7*a^8*x^7) - c^4/(3*a^7*x^6) - (2*c^4)/(5*a^6*x^5) + (3*c^4)/(2*a^5*x^
4) - (3*c^4)/(a^3*x^2) + (2*c^4)/(a^2*x) + c^4*x - (2*c^4*Log[x])/a
```

Maple [A]
time = 0.31, size = 64, normalized size = 0.71

method	result
default	$\frac{c^4\left(a^8x+\frac{3a^3}{2x^4}+\frac{1}{7x^7}+\frac{2a^6}{x}-\frac{3a^5}{x^2}-2a^7\ln(x)-\frac{a}{3x^6}-\frac{2a^2}{5x^5}\right)}{a^8}$
risch	$c^4x+\frac{2a^6c^4x^6-3a^5c^4x^5+\frac{3}{2}a^3c^4x^3-\frac{2}{5}a^2c^4x^2-\frac{1}{3}a\,c^4x+\frac{1}{7}c^4}{a^8x^7}-\frac{2c^4\ln(x)}{a}$
norman	$\frac{a^7c^4x^8+\frac{c^4}{7a}-\frac{c^4x}{3}+\frac{3a^2c^4x^3}{2}-3a^4c^4x^5+2a^5c^4x^6-\frac{2c^4a\,x^2}{5}}{a^7x^7}-\frac{2c^4\ln(x)}{a}$
meijerg	$\frac{c^4(ax-\ln(ax+1))}{a}-\frac{c^4\ln(ax+1)}{a}-\frac{4c^4(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{4c^4\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\frac{6c^4\left(-\ln(ax+1)+\ln(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^4/a^8*(a^8*x+3/2*a^3/x^4+1/7/x^7+2*a^6/x-3*a^5/x^2-2*a^7*ln(x)-1/3*a/x^6-
2/5*a^2/x^5)
```

Maxima [A]
time = 0.27, size = 81, normalized size = 0.90

$$c^4x-\frac{2\,c^4\log\left(x\right)}{a}+\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^4*x - 2*c^4*log(x)/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 315*a^3
*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Fricas [A]
time = 0.37, size = 89, normalized size = 0.99

$$\frac{210\,a^8c^4x^8-420\,a^7c^4x^7\log\left(x\right)+420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/210*(210*a^8*c^4*x^8 - 420*a^7*c^4*x^7*log(x) + 420*a^6*c^4*x^6 - 630*a^5
*c^4*x^5 + 315*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7
)
```

Sympy [A]
time = 0.25, size = 88, normalized size = 0.98

$$\frac{a^8c^4x-2a^7c^4\log\left(x\right)+\frac{420a^6c^4x^6-630a^5c^4x^5+315a^3c^4x^3-84a^2c^4x^2-70ac^4x+30c^4}{210x^7}}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**8*c**4*x - 2*a**7*c**4*log(x) + (420*a**6*c**4*x**6 - 630*a**5*c**4*x**
5 + 315*a**3*c**4*x**3 - 84*a**2*c**4*x**2 - 70*a*c**4*x + 30*c**4)/(210*x*
*7))/a**8
```

Giac [A]
time = 0.40, size = 82, normalized size = 0.91

$$c^4x-\frac{2\,c^4\log\left(|x|\right)}{a}+\frac{420\,a^6c^4x^6-630\,a^5c^4x^5+315\,a^3c^4x^3-84\,a^2c^4x^2-70\,ac^4x+30\,c^4}{210\,a^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^4*x - 2*c^4*log(abs(x))/a + 1/210*(420*a^6*c^4*x^6 - 630*a^5*c^4*x^5 + 31
5*a^3*c^4*x^3 - 84*a^2*c^4*x^2 - 70*a*c^4*x + 30*c^4)/(a^8*x^7)
```

Mupad [B]
time = 1.32, size = 67, normalized size = 0.74

$$-\frac{c^4\left(\frac{a\,x}{3}+\frac{2\,a^2\,x^2}{5}-\frac{3\,a^3\,x^3}{2}+3\,a^5\,x^5-2\,a^6\,x^6-a^8\,x^8+2\,a^7\,x^7\,\ln\left(x\right)-\frac{1}{7}\right)}{a^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^4*(a*x - 1))/(a*x + 1),x)
```

```
[Out] -(c^4*((a*x)/3 + (2*a^2*x^2)/5 - (3*a^3*x^3)/2 + 3*a^5*x^5 - 2*a^6*x^6 - a^
8*x^8 + 2*a^7*x^7*log(x) - 1/7))/(a^8*x^7)
```

3.815 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=76

$$-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

```
[Out] -1/5*c^3/a^6/x^5+1/2*c^3/a^5/x^4+1/3*c^3/a^4/x^3-2*c^3/a^3/x^2+c^3/a^2/x+c^
3*x-2*c^3*ln(x)/a
```

Rubi [A]
time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}-\frac{2c^3\log(x)}{a}+c^3x$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/5*c^3/(a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx &= -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3 dx\\
&= \frac{c^3\int \frac{e^{-2\tanh^{-1}(ax)}(1-a^2x^2)^3}{x^6}\,dx}{a^6}\\
&= \frac{c^3\int \frac{(1-ax)^4(1+ax)^2}{x^6}\,dx}{a^6}\\
&= \frac{c^3\int \left(a^6+\frac{1}{x^6}-\frac{2a}{x^5}-\frac{a^2}{x^4}+\frac{4a^3}{x^3}-\frac{a^4}{x^2}-\frac{2a^5}{x}\right)dx}{a^6}\\
&= -\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 1.00

$$-\frac{c^3}{5a^6x^5}+\frac{c^3}{2a^5x^4}+\frac{c^3}{3a^4x^3}-\frac{2c^3}{a^3x^2}+\frac{c^3}{a^2x}+c^3x-\frac{2c^3\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/5*c^3/(a^6*x^5) + c^3/(2*a^5*x^4) + c^3/(3*a^4*x^3) - (2*c^3)/(a^3*x^2)
+ c^3/(a^2*x) + c^3*x - (2*c^3*Log[x])/a
```

Maple [A]
time = 0.26, size = 55, normalized size = 0.72

method	result
default	$\frac{c^3\left(a^6x+\frac{a}{2x^4}+\frac{a^4}{x}-\frac{2a^3}{x^2}-2a^5\ln(x)+\frac{a^2}{3x^3}-\frac{1}{5x^5}\right)}{a^6}$
risch	$c^3x+\frac{a^4c^3x^4-2a^3c^3x^3+\frac{1}{3}a^2c^3x^2+\frac{1}{2}a\,c^3x-\frac{1}{5}c^3}{a^6x^5}-\frac{2c^3\ln(x)}{a}$
norman	$\frac{a^3c^3x^4+a^5c^3x^6-\frac{c^3}{5a}+\frac{c^3x}{2}-2a^2c^3x^3+\frac{c^3a\,x^2}{3}}{a^5x^5}-\frac{2c^3\ln(x)}{a}$
meijerg	$\frac{c^3(ax-\ln(ax+1))}{a}-\frac{c^3\ln(ax+1)}{a}-\frac{3c^3(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{3c^3\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\frac{3c^3\left(-\ln(ax+1)+\ln(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] c^3/a^6*(a^6*x+1/2*a/x^4+a^4/x-2*a^3/x^2-2*a^5*ln(x)+1/3*a^2/x^3-1/5/x^5)
```

Maxima [A]
time = 0.26, size = 70, normalized size = 0.92

$$c^3x - \frac{2\,c^3\log\left(x\right)}{a} + \frac{30\,a^4c^3x^4 - 60\,a^3c^3x^3 + 10\,a^2c^3x^2 + 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] c^3*x - 2*c^3*log(x)/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^2*c^3
*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Fricas [A]
time = 0.48, size = 78, normalized size = 1.03

$$\frac{30\,a^6c^3x^6 - 60\,a^5c^3x^5\log\left(x\right) + 30\,a^4c^3x^4 - 60\,a^3c^3x^3 + 10\,a^2c^3x^2 + 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] 1/30*(30*a^6*c^3*x^6 - 60*a^5*c^3*x^5*log(x) + 30*a^4*c^3*x^4 - 60*a^3*c^3*
x^3 + 10*a^2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Sympy [A]
time = 0.18, size = 76, normalized size = 1.00

$$\frac{a^6c^3x - 2a^5c^3\log\left(x\right) + \frac{30a^4c^3x^4 - 60a^3c^3x^3 + 10a^2c^3x^2 + 15ac^3x - 6c^3}{30x^5}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*(a*x-1)/(a*x+1),x)
```

```
[Out] (a**6*c**3*x - 2*a**5*c**3*log(x) + (30*a**4*c**3*x**4 - 60*a**3*c**3*x**3
+ 10*a**2*c**3*x**2 + 15*a*c**3*x - 6*c**3)/(30*x**5))/a**6
```

Giac [A]
time = 0.41, size = 71, normalized size = 0.93

$$c^3x - \frac{2\,c^3\log\left(|x|\right)}{a} + \frac{30\,a^4c^3x^4 - 60\,a^3c^3x^3 + 10\,a^2c^3x^2 + 15\,ac^3x - 6\,c^3}{30\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] c^3*x - 2*c^3*log(abs(x))/a + 1/30*(30*a^4*c^3*x^4 - 60*a^3*c^3*x^3 + 10*a^
2*c^3*x^2 + 15*a*c^3*x - 6*c^3)/(a^6*x^5)
```

Mupad [B]
time = 1.25, size = 56, normalized size = 0.74

$$\frac{c^3\left(\frac{a\,x}{2}+\frac{a^2\,x^2}{3}-2\,a^3\,x^3+a^4\,x^4+a^6\,x^6-2\,a^5\,x^5\,\ln\left(x\right)-\frac{1}{5}\right)}{a^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^3*(a*x - 1))/(a*x + 1),x)
```

```
[Out] (c^3*((a*x)/2 + (a^2*x^2)/3 - 2*a^3*x^3 + a^4*x^4 + a^6*x^6 - 2*a^5*x^5*log
(x) - 1/5))/(a^6*x^5)
```

3.816 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=40

$$\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

[Out] 1/3*c^2/a^4/x^3-c^2/a^3/x^2+c^2*x-2*c^2*ln(x)/a

Rubi [A]
time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {6302, 6292, 6285, 76}

$$\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}-\frac{2c^2\log(x)}{a}+c^2x$$

Antiderivative was successfully verified.

[In] Int[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]

[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2 dx$$
$$= -\frac{c^2\int \frac{e^{-2\tanh^{-1}(ax)}(1-a^2x^2)^2}{x^4}dx}{a^4}$$
$$= -\frac{c^2\int \frac{(1-ax)^3(1+ax)}{x^4}dx}{a^4}$$
$$= -\frac{c^2\int \left(-a^4+\frac{1}{x^4}-\frac{2a}{x^3}+\frac{2a^3}{x}\right)dx}{a^4}$$
$$= \frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{c^2}{3a^4x^3}-\frac{c^2}{a^3x^2}+c^2x-\frac{2c^2\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(2*ArcCoth[a*x]),x]
[Out] c^2/(3*a^4*x^3) - c^2/(a^3*x^2) + c^2*x - (2*c^2*Log[x])/a
```

Maple [A]
time = 0.22, size = 32, normalized size = 0.80

method	result
default	$\frac{c^2\left(a^4x-\frac{a}{x^2}-2a^3\ln(x)+\frac{1}{3x^3}\right)}{a^4}$
risch	$c^2x+\frac{-a\,c^2x+\frac{1}{3}c^2}{a^4x^3}-\frac{2c^2\ln(x)}{a}$
norman	$\frac{a^3c^2x^4+\frac{c^2}{3a}-c^2x}{a^3x^3}-\frac{2c^2\ln(x)}{a}$
meijerg	$\frac{c^2(ax-\ln(ax+1))}{a}-\frac{c^2\ln(ax+1)}{a}-\frac{2c^2(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{2c^2\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}+\frac{c^2\left(-\ln(ax+1)+\ln(x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c^2/a^4*(a^4*x-a/x^2-2*a^3*ln(x)+1/3/x^3)
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.92

$$c^2x-\frac{2\,c^2\log\left(x\right)}{a}-\frac{3\,ac^2x-c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c^2*x - 2*c^2*log(x)/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Fricas [A]
time = 0.48, size = 43, normalized size = 1.08

$$\frac{3\,a^4c^2x^4 - 6\,a^3c^2x^3\log\left(x\right) - 3\,ac^2x + c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] 1/3*(3*a^4*c^2*x^4 - 6*a^3*c^2*x^3*log(x) - 3*a*c^2*x + c^2)/(a^4*x^3)
```

Sympy [A]
time = 0.08, size = 39, normalized size = 0.98

$$\frac{a^4c^2x - 2a^3c^2\log\left(x\right) + \frac{-3ac^2x+c^2}{3x^3}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*(a*x-1)/(a*x+1),x)
[Out] (a**4*c**2*x - 2*a**3*c**2*log(x) + (-3*a*c**2*x + c**2)/(3*x**3))/a**4
```

Giac [A]
time = 0.39, size = 38, normalized size = 0.95

$$c^2x - \frac{2\,c^2\log\left(|x|\right)}{a} - \frac{3\,ac^2x - c^2}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c^2*x - 2*c^2*log(abs(x))/a - 1/3*(3*a*c^2*x - c^2)/(a^4*x^3)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.88

$$-\frac{c^2\,\left(3\,a\,x - 3\,a^4\,x^4 + 6\,a^3\,x^3\,\ln\left(x\right) - 1\right)}{3\,a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^2*(a*x - 1))/(a*x + 1),x)
[Out] -(c^2*(3*a*x - 3*a^4*x^4 + 6*a^3*x^3*log(x) - 1))/(3*a^4*x^3)
```

3.817 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=21

$$-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

[Out] -c/a^2/x+c*x-2*c*ln(x)/a

Rubi [A]
time = 0.06, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6302, 6292, 6285, 45}

$$-\frac{c}{a^2x}-\frac{2c\log(x)}{a}+cx$$

Antiderivative was successfully verified.

[In] Int[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]

[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$$
$$=\frac{c\int\frac{e^{-2\tanh^{-1}(ax)}(1-a^2x^2)}{x^2}\,dx}{a^2}$$
$$=\frac{c\int\frac{(1-ax)^2}{x^2}\,dx}{a^2}$$
$$=\frac{c\int\left(a^2+\frac{1}{x^2}-\frac{2a}{x}\right)\,dx}{a^2}$$
$$=-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$-\frac{c}{a^2x}+cx-\frac{2c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(2*ArcCoth[a*x]),x]
[Out] -(c/(a^2*x)) + c*x - (2*c*Log[x])/a
```

Maple [A]
time = 0.16, size = 22, normalized size = 1.05

method	result	size
default	$\frac{c\left(a^2x-\frac{1}{x}-2a\ln(x)\right)}{a^2}$	22
risch	$-\frac{c}{a^2x}+cx-\frac{2c\ln(x)}{a}$	22
norman	$\frac{acx^2-\frac{c}{a}}{ax}-\frac{2c\ln(x)}{a}$	30
meijerg	$\frac{c(ax-\ln(ax+1))}{a}-\frac{c\ln(ax+1)}{a}-\frac{c(-\ln(ax+1)+\ln(x)+\ln(a))}{a}+\frac{c\left(\ln(ax+1)-\ln(x)-\ln(a)-\frac{1}{ax}\right)}{a}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
[Out] c/a^2*(a^2*x-1/x-2*a*ln(x))
```

Maxima [A]
time = 0.26, size = 21, normalized size = 1.00

$$cx-\frac{2\,c\log\left(x\right)}{a}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
[Out] c*x - 2*c*log(x)/a - c/(a^2*x)
```

Fricas [A]
time = 0.49, size = 26, normalized size = 1.24

$$\frac{a^2cx^2 - 2\,acx\log(x) - c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] (a^2*c*x^2 - 2*a*c*x*log(x) - c)/(a^2*x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.95

$$\frac{a^2cx - 2ac\log(x) - \frac{c}{x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*(a*x-1)/(a*x+1),x)
[Out] (a**2*c*x - 2*a*c*log(x) - c/x)/a**2
```

Giac [A]
time = 0.39, size = 22, normalized size = 1.05

$$cx - \frac{2\,c\log(|x|)}{a} - \frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] c*x - 2*c*log(abs(x))/a - c/(a^2*x)
```

Mupad [B]
time = 1.23, size = 25, normalized size = 1.19

$$-\frac{c\,(2\,a\,x\,\ln(x) - a^2\,x^2 + 1)}{a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))*(a*x - 1))/(a*x + 1),x)
[Out] -(c*(2*a*x*log(x) - a^2*x^2 + 1))/(a^2*x)
```

3.818 $\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}$$

```
[Out] x/c-1/a/c/(a*x+1)-2*ln(a*x+1)/a/c
```

Rubi [A]
time = 0.11, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 45}

$$-\frac{1}{ac(ax+1)}-\frac{2\log(ax+1)}{ac}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] x/c - 1/(a*c*(1 + a*x)) - (2*Log[1 + a*x])/(a*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$
$$= \frac{a^2\int \frac{e^{-2\tanh^{-1}(ax)}x^2}{1-a^2x^2}\,dx}{c}$$
$$= \frac{a^2\int \frac{x^2}{(1+ax)^2}\,dx}{c}$$
$$= \frac{a^2\int \left(\frac{1}{a^2}+\frac{1}{a^2(1+ax)^2}-\frac{2}{a^2(1+ax)}\right)dx}{c}$$
$$= \frac{x}{c}-\frac{1}{ac(1+ax)}-\frac{2\log(1+ax)}{ac}$$

Mathematica [**A**]
time = 0.02, size = 28, normalized size = 0.80

$$\frac{x-\frac{1}{a+a^2x}-\frac{2\log(1+ax)}{a}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
[Out] (x - (a + a^2*x)^(-1) - (2*Log[1 + a*x])/a)/c
```

Maple [**A**]
time = 0.12, size = 37, normalized size = 1.06

method	result	size
risch	$\frac{x}{c}-\frac{1}{ac(ax+1)}-\frac{2\ln(ax+1)}{ac}$	36
default	$\frac{a^2\left(\frac{x}{a^2}-\frac{1}{a^3(ax+1)}-\frac{2\ln(ax+1)}{a^3}\right)}{c}$	37
norman	$\frac{\frac{a\,x^2}{c}+\frac{2x}{c}}{ax+1}-\frac{2\ln(ax+1)}{ac}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
[Out] a^2/c*(x/a^2-1/a^3/(a*x+1)-2/a^3*ln(a*x+1))
```

Maxima [A]
time = 0.27, size = 34, normalized size = 0.97

$$\frac{x}{c} - \frac{1}{a^2cx + ac} - \frac{2\,\log\left(ax + 1\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="maxima")
[Out] x/c - 1/(a^2*c*x + a*c) - 2*log(a*x + 1)/(a*c)
```

Fricas [A]
time = 0.35, size = 38, normalized size = 1.09

$$\frac{a^2x^2 + ax - 2\left(ax + 1\right)\log\left(ax + 1\right) - 1}{a^2cx + ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="fricas")
[Out] (a^2*x^2 + a*x - 2*(a*x + 1)*log(a*x + 1) - 1)/(a^2*c*x + a*c)
```

Sympy [A]
time = 0.07, size = 36, normalized size = 1.03

$$a^2\left(-\frac{1}{a^4cx + a^3c} + \frac{x}{a^2c} - \frac{2\log\left(ax + 1\right)}{a^3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2),x)
[Out] a**2*(-1/(a**4*c*x + a**3*c) + x/(a**2*c) - 2*log(a*x + 1)/(a**3*c))
```

Giac [A]
time = 0.40, size = 36, normalized size = 1.03

$$\frac{x}{c} - \frac{2\,\log\left(|ax + 1|\right)}{ac} - \frac{1}{(ax + 1)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2),x, algorithm="giac")
[Out] x/c - 2*log(abs(a*x + 1))/(a*c) - 1/((a*x + 1)*a*c)
```

Mupad [B]
time = 0.05, size = 33, normalized size = 0.94

$$\frac{x}{c} - \frac{1}{a\,\left(c + a\,c\,x\right)} - \frac{2\,\ln\left(a\,x + 1\right)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))*(a*x + 1)),x)
[Out] x/c - 1/(a*(c + a*c*x)) - (2*log(a*x + 1))/(a*c)
```

3.819 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=73

$$\frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}$$

```
[Out] x/c^2+1/4/a/c^2/(a*x+1)^2-7/4/a/c^2/(a*x+1)+1/8*ln(-a*x+1)/a/c^2-17/8*ln(a*
x+1)/a/c^2
```

Rubi [A]

time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6302, 6292, 6285, 90}

$$-\frac{7}{4ac^2(ax+1)}+\frac{1}{4ac^2(ax+1)^2}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(ax+1)}{8ac^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] x/c^2 + 1/(4*a*c^2*(1 + a*x)^2) - 7/(4*a*c^2*(1 + a*x)) + Log[1 - a*x]/(8*a
*c^2) - (17*Log[1 + a*x])/(8*a*c^2)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx\\ &= -\frac{a^4\int \frac{e^{-2\tanh^{-1}(ax)}x^4}{(1-a^2x^2)^2}\,dx}{c^2}\\ &= -\frac{a^4\int \frac{x^4}{(1-ax)(1+ax)^3}\,dx}{c^2}\\ &= -\frac{a^4\int \left(-\frac{1}{a^4}-\frac{1}{8a^4(-1+ax)}+\frac{1}{2a^4(1+ax)^3}-\frac{7}{4a^4(1+ax)^2}+\frac{17}{8a^4(1+ax)}\right)dx}{c^2}\\ &= \frac{x}{c^2}+\frac{1}{4ac^2(1+ax)^2}-\frac{7}{4ac^2(1+ax)}+\frac{\log(1-ax)}{8ac^2}-\frac{17\log(1+ax)}{8ac^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.96

$$\frac{2(-6-3ax+8a^2x^2+4a^3x^3)+(1+ax)^2\log(1-ax)-17(1+ax)^2\log(1+ax)}{8a(c+acx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
[Out] (2*(-6 - 3*a*x + 8*a^2*x^2 + 4*a^3*x^3) + (1 + a*x)^2*Log[1 - a*x] - 17*(1
+ a*x)^2*Log[1 + a*x])/(8*a*(c + a*c*x)^2)
```

Maple [A]
time = 0.12, size = 60, normalized size = 0.82

method	result	size
default	$\frac{a^4\left(\frac{x}{a^4}+\frac{1}{4a^5(ax+1)^2}-\frac{7}{4a^5(ax+1)}-\frac{17\ln(ax+1)}{8a^5}+\frac{\ln(ax-1)}{8a^5}\right)}{c^2}$	60
risch	$\frac{x}{c^2}+\frac{-\frac{7c^2x}{4}-\frac{3c^2}{2a}}{c^4(ax+1)^2}-\frac{17\ln(ax+1)}{8a\,c^2}+\frac{\ln(-ax+1)}{8a\,c^2}$	62
norman	$\frac{\frac{a^3x^4}{c}-\frac{9x}{4c}-\frac{5a\,x^2}{4c}+\frac{5a^2x^3}{2c}}{(ax-1)(ax+1)^2c}+\frac{\ln(ax-1)}{8a\,c^2}-\frac{17\ln(ax+1)}{8a\,c^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/c^2*(x/a^4+1/4/a^5/(a*x+1)^2-7/4/a^5/(a*x+1)-17/8/a^5*ln(a*x+1)+1/8/a^5
*ln(a*x-1))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.95

$$-\frac{7\,ax+6}{4\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}+\frac{x}{c^2}-\frac{17\,\log\left(ax+1\right)}{8\,ac^2}+\frac{\log\left(ax-1\right)}{8\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(7*a*x + 6)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2) + x/c^2 - 17/8*log(a*x
 + 1)/(a*c^2) + 1/8*log(a*x - 1)/(a*c^2)
```

Fricas [A]
time = 0.39, size = 92, normalized size = 1.26

$$\frac{8\,a^3x^3+16\,a^2x^2-6\,ax-17\,(a^2x^2+2\,ax+1)\log\left(ax+1\right)+(a^2x^2+2\,ax+1)\log\left(ax-1\right)-12}{8\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^3*x^3 + 16*a^2*x^2 - 6*a*x - 17*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1)
 + (a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 12)/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*
c^2)
```

Sympy [A]
time = 0.22, size = 75, normalized size = 1.03

$$a^4\left(\frac{-7ax-6}{4a^7c^2x^2+8a^6c^2x+4a^5c^2}+\frac{x}{a^4c^2}+\frac{\frac{\log\left(x-\frac{1}{a}\right)}{8}-\frac{17\log\left(x+\frac{1}{a}\right)}{8}}{a^5c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*((-7*a*x - 6)/(4*a**7*c**2*x**2 + 8*a**6*c**2*x + 4*a**5*c**2) + x/(a*
*4*c**2) + (log(x - 1/a)/8 - 17*log(x + 1/a)/8)/(a**5*c**2))
```

Giac [A]
time = 0.40, size = 57, normalized size = 0.78

$$\frac{x}{c^2}-\frac{17\,\log\left(|ax+1|\right)}{8\,ac^2}+\frac{\log\left(|ax-1|\right)}{8\,ac^2}-\frac{7\,ax+6}{4\,(ax+1)^2ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] x/c^2 - 17/8*log(abs(a*x + 1))/(a*c^2) + 1/8*log(abs(a*x - 1))/(a*c^2) - 1/
4*(7*a*x + 6)/((a*x + 1)^2*a*c^2)
```

Mupad [B]
time = 1.31, size = 68, normalized size = 0.93

$$\frac{x}{c^2}-\frac{\frac{7\,x}{4}+\frac{3}{2\,a}}{a^2\,c^2\,x^2+2\,a\,c^2\,x+c^2}+\frac{\ln\left(a\,x-1\right)}{8\,a\,c^2}-\frac{17\,\ln\left(a\,x+1\right)}{8\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^2*(a*x + 1)),x)
```

```
[Out] x/c^2 - ((7*x)/4 + 3/(2*a))/(c^2 + a^2*c^2*x^2 + 2*a*c^2*x) + log(a*x - 1)/
(8*a*c^2) - (17*log(a*x + 1))/(8*a*c^2)
```

3.820 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=108

$$\frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2}-\frac{39}{16ac^3(1+ax)}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(1+ax)}{4ac^3}$$

```
[Out] x/c^3+1/16/a/c^3/(-a*x+1)-1/12/a/c^3/(a*x+1)^3+5/8/a/c^3/(a*x+1)^2-39/16/a/
c^3/(a*x+1)+1/4*ln(-a*x+1)/a/c^3-9/4*ln(a*x+1)/a/c^3
```

Rubi [A]
time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{1}{16ac^3(1-ax)}-\frac{39}{16ac^3(ax+1)}+\frac{5}{8ac^3(ax+1)^2}-\frac{1}{12ac^3(ax+1)^3}+\frac{\log(1-ax)}{4ac^3}-\frac{9\log(ax+1)}{4ac^3}+\frac{x}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] x/c^3 + 1/(16*a*c^3*(1 - a*x)) - 1/(12*a*c^3*(1 + a*x)^3) + 5/(8*a*c^3*(1 +
 a*x)^2) - 39/(16*a*c^3*(1 + a*x)) + Log[1 - a*x]/(4*a*c^3) - (9*Log[1 + a*
x])/(4*a*c^3)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
 /; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx \\
&= \frac{a^6\int \frac{e^{-2\tanh^{-1}(ax)}x^6}{(1-a^2x^2)^3}\,dx}{c^3} \\
&= \frac{a^6\int \frac{x^6}{(1-ax)^2(1+ax)^4}\,dx}{c^3} \\
&= \frac{a^6\int \left(\frac{1}{a^6}+\frac{1}{16a^6(-1+ax)^2}+\frac{1}{4a^6(-1+ax)}+\frac{1}{4a^6(1+ax)^4}-\frac{5}{4a^6(1+ax)^3}+\frac{39}{16a^6(1+ax)^2}-\frac{9}{4a^6(1+ax)}\right)}{c^3} \\
&= \frac{x}{c^3}+\frac{1}{16ac^3(1-ax)}-\frac{1}{12ac^3(1+ax)^3}+\frac{5}{8ac^3(1+ax)^2}-\frac{39}{16ac^3(1+ax)}+\frac{\log(1-a}{4ac^3}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 104, normalized size = 0.96

$$\frac{2(11+7ax-24a^2x^2-15a^3x^3+12a^4x^4+6a^5x^5)+3(-1+ax)(1+ax)^3\log(1-ax)-27(-1+ax)(1+ax)^3\log(1+ax)}{12a(-1+ax)(c+acx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
[Out] (2*(11 + 7*a*x - 24*a^2*x^2 - 15*a^3*x^3 + 12*a^4*x^4 + 6*a^5*x^5) + 3*(-1
 + a*x)*(1 + a*x)^3*Log[1 - a*x] - 27*(-1 + a*x)*(1 + a*x)^3*Log[1 + a*x])/(
12*a*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [A]
time = 0.12, size = 84, normalized size = 0.78

method	result	size
default	$\frac{a^6\left(\frac{x}{a^6}-\frac{1}{12a^7(ax+1)^3}+\frac{5}{8a^7(ax+1)^2}-\frac{39}{16a^7(ax+1)}-\frac{9\ln(ax+1)}{4a^7}-\frac{1}{16a^7(ax-1)}+\frac{\ln(ax-1)}{4a^7}\right)}{c^3}$	84
risch	$\frac{x}{c^3}+\frac{-\frac{5a^2c^3x^3}{2}-2c^3a\,x^2+\frac{13c^3x}{6}+\frac{11c^3}{6a}}{c^6(ax+1)^2(a^2x^2-1)}+\frac{\ln(-ax+1)}{4a\,c^3}-\frac{9\ln(ax+1)}{4a\,c^3}$	93
norman	$\frac{\frac{a^5x^6}{c}+\frac{5x}{2c}+\frac{3a\,x^2}{2c}-\frac{31a^2x^3}{6c}-\frac{8a^3x^4}{3c}+\frac{17a^4x^5}{6c}}{(ax-1)^2(ax+1)^3c^2}+\frac{\ln(ax-1)}{4c^3a}-\frac{9\ln(ax+1)}{4a\,c^3}$	107

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)

[Out] a^6/c^3*(1/a^6*x-1/12/a^7/(a*x+1)^3+5/8/a^7/(a*x+1)^2-39/16/a^7/(a*x+1)-9/4/a^7*ln(a*x+1)-1/16/a^7/(a*x-1)+1/4/a^7*ln(a*x-1))

Maxima [A]
time = 0.26, size = 97, normalized size = 0.90

$$-\frac{15\,a^3x^3+12\,a^2x^2-13\,ax-11}{6\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}+\frac{x}{c^3}-\frac{9\,\log{(ax+1)}}{4\,ac^3}+\frac{\log{(ax-1)}}{4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="maxima")

[Out] -1/6*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3) + x/c^3 - 9/4*log(a*x + 1)/(a*c^3) + 1/4*log(a*x - 1)/(a*c^3)

Fricas [A]
time = 0.35, size = 137, normalized size = 1.27

$$\frac{12\,a^5x^5+24\,a^4x^4-30\,a^3x^3-48\,a^2x^2+14\,ax-27\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log{(ax+1)}+3\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log{(ax-1)}+22}{12\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="fricas")

[Out] 1/12*(12*a^5*x^5 + 24*a^4*x^4 - 30*a^3*x^3 - 48*a^2*x^2 + 14*a*x - 27*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x + 1) + 3*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(a*x - 1) + 22)/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3)

Sympy [A]
time = 0.36, size = 102, normalized size = 0.94

$$a^6\left(\frac{-15a^3x^3-12a^2x^2+13ax+11}{6a^{11}c^3x^4+12a^{10}c^3x^3-12a^8c^3x-6a^7c^3}+\frac{x}{a^6c^3}+\frac{\frac{\log{(x-\frac{1}{a})}}{4}-\frac{9\log{(x+\frac{1}{a})}}{4}}{a^7c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**3,x)

[Out] a**6*((-15*a**3*x**3 - 12*a**2*x**2 + 13*a*x + 11)/(6*a**11*c**3*x**4 + 12*a**10*c**3*x**3 - 12*a**8*c**3*x - 6*a**7*c**3) + x/(a**6*c**3) + (log(x - 1/a)/4 - 9*log(x + 1/a)/4)/(a**7*c**3))

Giac [A]
time = 0.42, size = 80, normalized size = 0.74

$$\frac{x}{c^3} - \frac{9 \log\left(|ax+1|\right)}{4\,ac^3} + \frac{\log\left(|ax-1|\right)}{4\,ac^3} - \frac{15\,a^3x^3 + 12\,a^2x^2 - 13\,ax - 11}{6\,(ax+1)^3(ax-1)ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] x/c^3 - 9/4*log(abs(a*x + 1))/(a*c^3) + 1/4*log(abs(a*x - 1))/(a*c^3) - 1/6
*(15*a^3*x^3 + 12*a^2*x^2 - 13*a*x - 11)/((a*x + 1)^3*(a*x - 1)*a*c^3)
```

Mupad [B]
time = 0.12, size = 94, normalized size = 0.87

$$\frac{x}{c^3} - \frac{\frac{13\,x}{6} - 2\,a\,x^2 + \frac{11}{6\,a} - \frac{5\,a^2\,x^3}{2}}{-a^4\,c^3\,x^4 - 2\,a^3\,c^3\,x^3 + 2\,a\,c^3\,x + c^3} + \frac{\ln\left(a\,x-1\right)}{4\,a\,c^3} - \frac{9\,\ln\left(a\,x+1\right)}{4\,a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^3*(a*x + 1)),x)
```

```
[Out] x/c^3 - ((13*x)/6 - 2*a*x^2 + 11/(6*a) - (5*a^2*x^3)/2)/(c^3 - 2*a^3*c^3*x^
3 - a^4*c^3*x^4 + 2*a*c^3*x) + log(a*x - 1)/(4*a*c^3) - (9*log(a*x + 1))/(4
*a*c^3)
```

3.821 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=143

$$\frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{35}{32ac^4(1+ax)^2}-\frac{99}{32ac^4(1+ax)}+\frac{47}{}$$

```
[Out] x/c^4-1/64/a/c^4/(-a*x+1)^2+11/64/a/c^4/(-a*x+1)+1/32/a/c^4/(a*x+1)^4-13/48
/a/c^4/(a*x+1)^3+35/32/a/c^4/(a*x+1)^2-99/32/a/c^4/(a*x+1)+47/128*ln(-a*x+1
)/a/c^4-303/128*ln(a*x+1)/a/c^4
```

Rubi [A]
time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6302, 6292, 6285, 90}

$$\frac{11}{64ac^4(1-ax)}-\frac{99}{32ac^4(ax+1)}-\frac{1}{64ac^4(1-ax)^2}+\frac{35}{32ac^4(ax+1)^2}-\frac{13}{48ac^4(ax+1)^3}+\frac{1}{32ac^4(ax+1)^4}+\frac{47\log(1-ax)}{128ac^4}-\frac{303\log(ax+1)}{128ac^4}+\frac{x}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] x/c^4 - 1/(64*a*c^4*(1 - a*x)^2) + 11/(64*a*c^4*(1 - a*x)) + 1/(32*a*c^4*(1
 + a*x)^4) - 13/(48*a*c^4*(1 + a*x)^3) + 35/(32*a*c^4*(1 + a*x)^2) - 99/(32
*a*c^4*(1 + a*x)) + (47*Log[1 - a*x])/(128*a*c^4) - (303*Log[1 + a*x])/(128
*a*c^4)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6285

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(x_)^(m_.)*((c_) + (d_.)*(x_)^2)^(p_.), x
_Symbol] :> Dist[c^p, Int[x^m*(1 - a*x)^(p - n/2)*(1 + a*x)^(p + n/2), x],
x] /; FreeQ[{a, c, d, m, n, p}, x] && EqQ[a^2*c + d, 0] && (IntegerQ[p] ||
GtQ[c, 0])
```

Rule 6292

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[d^p, Int[(u/x^(2*p))*(1 - a^2*x^2)^p*E^(n*ArcTanh[a*x]), x], x]
```

```
/; FreeQ[{a, c, d, n}, x] && EqQ[c + a^2*d, 0] && IntegerQ[p]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx &= -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx\\ &= -\frac{a^8\int \frac{e^{-2\tanh^{-1}(ax)}x^8}{(1-a^2x^2)^4}\,dx}{c^4}\\ &= -\frac{a^8\int \frac{x^8}{(1-ax)^3(1+ax)^5}\,dx}{c^4}\\ &= -\frac{a^8\int \left(-\frac{1}{a^8}-\frac{1}{32a^8(-1+ax)^3}-\frac{11}{64a^8(-1+ax)^2}-\frac{47}{128a^8(-1+ax)}+\frac{1}{8a^8(1+ax)^5}-\frac{13}{16a^8(1+ax)^4}+\frac{}{16}\right.}{c^4}\\ &= \frac{x}{c^4}-\frac{1}{64ac^4(1-ax)^2}+\frac{11}{64ac^4(1-ax)}+\frac{1}{32ac^4(1+ax)^4}-\frac{13}{48ac^4(1+ax)^3}+\frac{3}{32ac^4(1}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 124, normalized size = 0.87

$$\frac{2(-400-275ax+1258a^2x^2+866a^3x^3-1254a^4x^4-819a^5x^5+384a^6x^6+192a^7x^7)+141(-1+ax)^2(1+ax)^4\log(1-ax)-909(-1+ax)^2(1+ax)^4\log(1+ax)}{384a(-1+ax)^2(c+acx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] (2*(-400 - 275*a*x + 1258*a^2*x^2 + 866*a^3*x^3 - 1254*a^4*x^4 - 819*a^5*x^
5 + 384*a^6*x^6 + 192*a^7*x^7) + 141*(-1 + a*x)^2*(1 + a*x)^4*Log[1 - a*x]
- 909*(-1 + a*x)^2*(1 + a*x)^4*Log[1 + a*x])/(384*a*(-1 + a*x)^2*(c + a*c*x
)^4)
```

Maple [A]
time = 0.12, size = 108, normalized size = 0.76

method	result	size
default	$\frac{a^8\left(\frac{x}{a^8}+\frac{1}{32a^9(ax+1)^4}-\frac{13}{48a^9(ax+1)^3}+\frac{35}{32a^9(ax+1)^2}-\frac{99}{32a^9(ax+1)}-\frac{303\ln(ax+1)}{128a^9}-\frac{1}{64a^9(ax-1)^2}-\frac{11}{64a^9(ax-1)}+\frac{47\ln(ax-1)}{128a^9}\right)}{c^4}$	108

risch	$\frac{x}{c^4}+\frac{-\frac{209a^4c^4x^5}{64}-\frac{81a^3c^4x^4}{32}+\frac{529a^2c^4x^3}{96}+\frac{437c^4a\,x^2}{96}-\frac{467c^4x}{192}-\frac{25c^4}{12a}}{c^8(ax+1)^2(a^2x^2-1)^2}+\frac{47\ln(-ax+1)}{128a\,c^4}-\frac{303\ln(ax+1)}{128a\,c^4}$	115
norman	$\frac{\frac{a^7x^8}{c}-\frac{175x}{64c}-\frac{111a\,x^2}{64c}+\frac{199a^2x^3}{24c}+\frac{115a^3x^4}{24c}-\frac{545a^4x^5}{64c}-\frac{803a^5x^6}{192c}+\frac{37a^6x^7}{12c}}{(ax+1)^4(ax-1)^3c^3}+\frac{47\ln(ax-1)}{128c^4a}-\frac{303\ln(ax+1)}{128a\,c^4}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^8/c^4*(1/a^8*x+1/32/a^9/(a*x+1)^4-13/48/a^9/(a*x+1)^3+35/32/a^9/(a*x+1)^2
-99/32/a^9/(a*x+1)-303/128/a^9*ln(a*x+1)-1/64/a^9/(a*x-1)^2-11/64/a^9/(a*x-
1)+47/128/a^9*ln(a*x-1))
```

Maxima [A]
time = 0.27, size = 145, normalized size = 1.01

$$-\frac{627\,a^5x^5+486\,a^4x^4-1058\,a^3x^3-874\,a^2x^2+467\,ax+400}{192\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}+\frac{x}{c^4}-\frac{303\,\log(ax+1)}{128\,ac^4}+\frac{47\,\log(ax-1)}{128\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x +
400)/(a^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x
^2 + 2*a^2*c^4*x + a*c^4) + x/c^4 - 303/128*log(a*x + 1)/(a*c^4) + 47/128*l
og(a*x - 1)/(a*c^4)
```

Fricas [A]
time = 0.41, size = 233, normalized size = 1.63

$$\frac{384\,a^7x^7+768\,a^6x^6-1638\,a^5x^5-2508\,a^4x^4+1732\,a^3x^3+2516\,a^2x^2-550\,ax-909\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\log(ax+1)+141\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\log(ax-1)-800}{384\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] 1/384*(384*a^7*x^7 + 768*a^6*x^6 - 1638*a^5*x^5 - 2508*a^4*x^4 + 1732*a^3*x
^3 + 2516*a^2*x^2 - 550*a*x - 909*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^
3 - a^2*x^2 + 2*a*x + 1)*log(a*x + 1) + 141*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(a*x - 1) - 800)/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [A]
time = 0.62, size = 156, normalized size = 1.09

$$a^8\left(\frac{-627a^5x^5-486a^4x^4+1058a^3x^3+874a^2x^2-467ax-400}{192a^{15}c^4x^6+384a^{14}c^4x^5-192a^{13}c^4x^4-768a^{12}c^4x^3-192a^{11}c^4x^2+384a^{10}c^4x+192a^9c^4}+\frac{x}{a^8c^4}+\frac{\frac{47\log\left(x-\frac{1}{a}\right)}{128}-\frac{303\log\left(x+\frac{1}{a}\right)}{128}}{a^9c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**4,x)
```

```
[Out] a**8*((-627*a**5*x**5 - 486*a**4*x**4 + 1058*a**3*x**3 + 874*a**2*x**2 - 46
7*a*x - 400)/(192*a**15*c**4*x**6 + 384*a**14*c**4*x**5 - 192*a**13*c**4*x*
*4 - 768*a**12*c**4*x**3 - 192*a**11*c**4*x**2 + 384*a**10*c**4*x + 192*a**
9*c**4) + x/(a**8*c**4) + (47*log(x - 1/a)/128 - 303*log(x + 1/a)/128)/(a**
9*c**4))
```

Giac [A]
time = 0.40, size = 96, normalized size = 0.67

$$\frac{x}{c^4} - \frac{303 \log(|ax+1|)}{128\,ac^4} + \frac{47 \log(|ax-1|)}{128\,ac^4} - \frac{627\,a^5x^5 + 486\,a^4x^4 - 1058\,a^3x^3 - 874\,a^2x^2 + 467\,ax + 400}{192\,(ax+1)^4(ax-1)^2ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] x/c^4 - 303/128*log(abs(a*x + 1))/(a*c^4) + 47/128*log(abs(a*x - 1))/(a*c^4
) - 1/192*(627*a^5*x^5 + 486*a^4*x^4 - 1058*a^3*x^3 - 874*a^2*x^2 + 467*a*x
 + 400)/((a*x + 1)^4*(a*x - 1)^2*a*c^4)
```

Mupad [B]
time = 0.15, size = 142, normalized size = 0.99

$$\frac{x}{c^4} - \frac{\frac{467\,x}{192} - \frac{437\,a\,x^2}{96} + \frac{25}{12\,a} - \frac{529\,a^2\,x^3}{96} + \frac{81\,a^3\,x^4}{32} + \frac{209\,a^4\,x^5}{64}}{a^6\,c^4\,x^6 + 2\,a^5\,c^4\,x^5 - a^4\,c^4\,x^4 - 4\,a^3\,c^4\,x^3 - a^2\,c^4\,x^2 + 2\,a\,c^4\,x + c^4} + \frac{47\,\ln(a\,x-1)}{128\,a\,c^4} - \frac{303\,\ln(a\,x+1)}{128\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^4*(a*x + 1)),x)
```

```
[Out] x/c^4 - ((467*x)/192 - (437*a*x^2)/96 + 25/(12*a) - (529*a^2*x^3)/96 + (81*
a^3*x^4)/32 + (209*a^4*x^5)/64)/(c^4 - a^2*c^4*x^2 - 4*a^3*c^4*x^3 - a^4*c^
4*x^4 + 2*a^5*c^4*x^5 + a^6*c^4*x^6 + 2*a*c^4*x) + (47*log(a*x - 1))/(128*a
*c^4) - (303*log(a*x + 1))/(128*a*c^4)
```

3.822 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4\,dx$

Optimal. Leaf size=343

$$\frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a}+\frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a}-\frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a}+\frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/}}{8a}$$

```
[Out] 5/8*c^4*(1-1/a/x)^(3/2)*(1+1/a/x)^(5/2)/a+11/10*c^4*(1-1/a/x)^(5/2)*(1+1/a/
x)^(5/2)/a+17/14*c^4*(1-1/a/x)^(7/2)*(1+1/a/x)^(5/2)/a+8/7*c^4*(1-1/a/x)^(9
/2)*(1+1/a/x)^(5/2)/a+c^4*(1-1/a/x)^(11/2)*(1+1/a/x)^(5/2)*x+15/16*c^4*arcc
sc(a*x)/a-3*c^4*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+27/16*c^4*(1+1/a
/x)^(3/2)*(1-1/a/x)^(1/2)/a-3/8*c^4*(1+1/a/x)^(5/2)*(1-1/a/x)^(1/2)/a+33/16
*c^4*(1-1/a/x)^(1/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]
time = 0.17, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^4x\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{11/2}+\frac{8c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{9/2}}{7a}+\frac{17c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{7/2}}{14a}+\frac{11c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{5/2}}{10a}+\frac{5c^4\left(\frac{1}{ax}+1\right)^{5/2}\left(1-\frac{1}{ax}\right)^{3/2}}{8a}-\frac{3c^4\left(\frac{1}{ax}+1\right)^{5/2}\sqrt{1-\frac{1}{ax}}}{8a}+\frac{27c^4\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{16a}+\frac{33c^4\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{16a}+\frac{15c^4\csc^{-1}(ax)}{16a}-\frac{3c^4\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (33*c^4*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(16*a) + (27*c^4*Sqrt[1 - 1/(a
*x)]*(1 + 1/(a*x))^(3/2))/(16*a) - (3*c^4*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(
5/2))/(8*a) + (5*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(5/2))/(8*a) + (11*c
^4*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(5/2))/(10*a) + (17*c^4*(1 - 1/(a*x))^
(7/2)*(1 + 1/(a*x))^(5/2))/(14*a) + (8*c^4*(1 - 1/(a*x))^(9/2)*(1 + 1/(a*x)
)^(5/2))/(7*a) + c^4*(1 - 1/(a*x))^(11/2)*(1 + 1/(a*x))^(5/2)*x + (15*c^4*A
rcCsc[a*x])/(16*a) - (3*c^4*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^4 dx = -\left(c^4\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{11/2}\left(1+\frac{x}{a}\right)^{5/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x - c^4\text{Subst}\left(\int\frac{\left(-\frac{3}{a}-\frac{8x}{a^2}\right)\left(1-\frac{x}{a}\right)^{9/2}\left(1+\right.}{x}\right.$$

$$= \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11/2}\left(1+\frac{1}{ax}\right)^{5/2}x - \frac{1}{7}\left(ac^4\right)\text{Sul}$$

$$= \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{5/2}}{7a} + c^4\left(1-\frac{1}{ax}\right)^{11}$$

$$= \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{5/2}}{14a} + \frac{8c^4\left(1-\frac{1}{ax}\right)^{9}}{7}$$

$$= \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{5/2}}{10a} + \frac{17c^4\left(1-\frac{1}{ax}\right)^{7}}{14}$$

$$= -\frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{11c^4\left(1-\frac{1}{ax}\right)^{5}}{10}$$

$$= \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{5/2}}{8a} + \frac{5c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\right.}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1\right.}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1\right.}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{16a} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{16a} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1\right.}{8a}$$

$$= \frac{33c^4\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{} + \frac{27c^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{} - \frac{3c^4\sqrt{1-\frac{1}{ax}}\left(1\right.}{}$$

Mathematica [A]
time = 0.14, size = 126, normalized size = 0.37

$$\frac{c^4\left(\sqrt{1-\frac{1}{a^2x^2}}\,(80-280ax+96a^2x^2+770a^3x^3-992a^4x^4-525a^5x^5+2496a^6x^6+560a^7x^7)+525a^6x^6\text{ArcSin}\left(\frac{1}{ax}\right)-1680a^6x^6\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{560a^7x^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^4/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^4*(Sqrt[1 - 1/(a^2*x^2)]*(80 - 280*a*x + 96*a^2*x^2 + 770*a^3*x^3 - 992*
a^4*x^4 - 525*a^5*x^5 + 2496*a^6*x^6 + 560*a^7*x^7) + 525*a^6*x^6*ArcSin[1/
(a*x)] - 1680*a^6*x^6*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(560*a^7*x^6)
```

Maple [A]
time = 0.08, size = 329, normalized size = 0.96

method	result
risch	$\frac{(ax+1)(560a^7x^7+2496a^6x^6-525a^5x^5-992a^4x^4+770a^3x^3+96a^2x^2-280ax+80)c^4\sqrt{\frac{ax-1}{ax+1}}}{560x^7a^8}+\frac{\left(-\frac{3a^8\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-\right.}{\sqrt{a^2}}\right.}{}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^4\left(-1680\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^8x^8+1680(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^6x^6-525a^7x^7\sqrt{a^2}\,\sqrt{a^2x^2-1}-525a^7\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/560*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^4*(-1680*(a^2*x^2-1)^(1/2)*(a^2)
^(1/2)*a^8*x^8+1680*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^6*x^6-525*a^7*x^7*(a^2)
^(1/2)*(a^2*x^2-1)^(1/2)-525*a^7*x^7*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2)
)+1680*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^8*x^7-35*(a^
2*x^2-1)^(3/2)*(a^2)^(1/2)*a^5*x^5-816*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^
4+490*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^3*x^3+176*(a^2*x^2-1)^(3/2)*(a^2)^(1/
2)*a^2*x^2-280*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x+80*(a^2*x^2-1)^(3/2)*(a^2)
^(1/2))/(a*x-1)/((a*x+1)*(a*x-1))^(1/2)/a^8/x^7/(a^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 379, normalized size = 1.10

$$-\frac{1}{280}\left(\frac{525\,c^4\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{840\,c^4\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{1155\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{15}{2}}+7665\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{13}{2}}+20811\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}-12799\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-39071\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-33621\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-13615\,c^4\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-2205\,c^4\sqrt{\frac{ax-1}{ax+1}}}{\frac{6\,(ax-1)a^2}{ax+1}+\frac{14\,(ax-1)^2a^2}{(ax+1)^2}+\frac{14\,(ax-1)^3a^2}{(ax+1)^3}-\frac{14\,(ax-1)^5a^2}{(ax+1)^5}-\frac{14\,(ax-1)^6a^2}{(ax+1)^6}-\frac{6\,(ax-1)^7a^2}{(ax+1)^7}-\frac{(ax-1)^8a^2}{(ax+1)^8}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/280*(525*c^4*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 840*c^4*log(sqrt((a
*x - 1)/(a*x + 1)) + 1)/a^2 - 840*c^4*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^
2 + (1155*c^4*((a*x - 1)/(a*x + 1))^(15/2) + 7665*c^4*((a*x - 1)/(a*x + 1))
^(13/2) + 20811*c^4*((a*x - 1)/(a*x + 1))^(11/2) - 12799*c^4*((a*x - 1)/(a*
x + 1))^(9/2) - 39071*c^4*((a*x - 1)/(a*x + 1))^(7/2) - 33621*c^4*((a*x - 1
)/(a*x + 1))^(5/2) - 13615*c^4*((a*x - 1)/(a*x + 1))^(3/2) - 2205*c^4*sqrt(
(a*x - 1)/(a*x + 1)))/(6*(a*x - 1)*a^2/(a*x + 1) + 14*(a*x - 1)^2*a^2/(a*x
+ 1)^2 + 14*(a*x - 1)^3*a^2/(a*x + 1)^3 - 14*(a*x - 1)^5*a^2/(a*x + 1)^5 -
14*(a*x - 1)^6*a^2/(a*x + 1)^6 - 6*(a*x - 1)^7*a^2/(a*x + 1)^7 - (a*x - 1)^
8*a^2/(a*x + 1)^8 + a^2))*a
```

Fricas [A]

time = 0.37, size = 201, normalized size = 0.59

$$-\frac{1050\,a^{7}c^{4}x^{7}\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-1680\,a^{7}c^{4}x^{7}\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(560\,a^{8}c^{4}x^{8}+3056\,a^{7}c^{4}x^{7}+1971\,a^{6}c^{4}x^{6}-1517\,a^{5}c^{4}x^{5}-222\,a^{4}c^{4}x^{4}+866\,a^{3}c^{4}x^{3}-184\,a^{2}c^{4}x^{2}-200\,ac^{4}x+80\,c^{4}\right)\sqrt{\frac{ax-1}{ax+1}}}{560\,a^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/560*(1050*a^7*c^4*x^7*arctan(sqrt((a*x - 1)/(a*x + 1))) + 1680*a^7*c^4*x
^7*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 1680*a^7*c^4*x^7*log(sqrt((a*x - 1)
/(a*x + 1)) - 1) - (560*a^8*c^4*x^8 + 3056*a^7*c^4*x^7 + 1971*a^6*c^4*x^6 -
 1517*a^5*c^4*x^5 - 222*a^4*c^4*x^4 + 866*a^3*c^4*x^3 - 184*a^2*c^4*x^2 - 2
00*a*c^4*x + 80*c^4)*sqrt((a*x - 1)/(a*x + 1)))/(a^8*x^7)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{4}\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{9}+x^{8}}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{8}+x^{7}}\,dx+\int\frac{4a^{2}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{7}+x^{6}}\,dx+\int\left(-\frac{4a^{3}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{6}+x^{5}}\right)dx+\int\left(-\frac{6a^{4}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{5}+x^{4}}\right)dx+\int\frac{6a^{5}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{4}+x^{3}}\,dx+\int\frac{4a^{6}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{3}+x^{2}}\,dx+\int\left(-\frac{4a^{7}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^{2}+x}\right)dx+\int\left(-\frac{a^{8}\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^{9}x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**4*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**4*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**9 + x**8), x) + Int
egral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**8 + x**7), x) + Integral(4*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Integral(-4*a*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(-6*a**4
*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(6*a**5*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(4*a**6*sqrt(
a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-4*a**7*sqrt(a*
x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**8*sqrt(a*x/(a*x
+ 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**9*x*sqrt(a*x/(a*x + 1) - 1/
(a*x + 1))/(a*x + 1), x))/a**8
```

Giac [A]
time = 0.44, size = 525, normalized size = 1.53

$$-\frac{15\,c^{4}\arctan\left(-x|a|+\sqrt{a^{2}x^{2}-1}\right)\operatorname{sgn}(ax+1)}{8\,a}+\frac{3\,c^{4}\log\left(\left|-x|a|+\sqrt{a^{2}x^{2}-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^{2}x^{2}-1}\,c^{4}\operatorname{sgn}(ax+1)}{a}+\frac{525\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{13}c^{4}|a|\operatorname{sgn}(ax+1)+4480\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{12}ac^{4}\operatorname{sgn}(ax+1)-980\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{11}c^{4}|a|\operatorname{sgn}(ax+1)+20160\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{10}ac^{4}\operatorname{sgn}(ax+1)+945\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{9}c^{4}|a|\operatorname{sgn}(ax+1)+38080\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{8}ac^{4}\operatorname{sgn}(ax+1)+49280\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{6}ac^{4}\operatorname{sgn}(ax+1)-945\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{5}c^{4}|a|\operatorname{sgn}(ax+1)+32256\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{4}ac^{4}\operatorname{sgn}(ax+1)+980\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{3}c^{4}|a|\operatorname{sgn}(ax+1)+12992\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}ac^{4}\operatorname{sgn}(ax+1)-525\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)c^{4}|a|\operatorname{sgn}(ax+1)+2496\,ac^{4}\operatorname{sgn}(ax+1)}{280\left(\left(x|a|-\sqrt{a^{2}x^{2}-1}\right)^{2}+1\right)^{7}a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^4*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -15/8*c^4*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^4*log(
abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)
*c^4*sgn(a*x + 1)/a + 1/280*(525*(x*abs(a) - sqrt(a^2*x^2 - 1))^13*c^4*abs(
a)*sgn(a*x + 1) + 4480*(x*abs(a) - sqrt(a^2*x^2 - 1))^12*a*c^4*sgn(a*x + 1)
 - 980*(x*abs(a) - sqrt(a^2*x^2 - 1))^11*c^4*abs(a)*sgn(a*x + 1) + 20160*(x
*abs(a) - sqrt(a^2*x^2 - 1))^10*a*c^4*sgn(a*x + 1) + 945*(x*abs(a) - sqrt(a
^2*x^2 - 1))^9*c^4*abs(a)*sgn(a*x + 1) + 38080*(x*abs(a) - sqrt(a^2*x^2 - 1
))^8*a*c^4*sgn(a*x + 1) + 49280*(x*abs(a) - sqrt(a^2*x^2 - 1))^6*a*c^4*sgn(
a*x + 1) - 945*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^4*abs(a)*sgn(a*x + 1) + 3
2256*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^4*sgn(a*x + 1) + 980*(x*abs(a) -
sqrt(a^2*x^2 - 1))^3*c^4*abs(a)*sgn(a*x + 1) + 12992*(x*abs(a) - sqrt(a^2*x
^2 - 1))^2*a*c^4*sgn(a*x + 1) - 525*(x*abs(a) - sqrt(a^2*x^2 - 1))*c^4*abs(
a)*sgn(a*x + 1) + 2496*a*c^4*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))
^2 + 1)^7*a*abs(a))
```

Mupad [B]
time = 0.17, size = 332, normalized size = 0.97

$$\frac{\frac{63\,c^4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{8}+\frac{389\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8}+\frac{4803\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{40}+\frac{39071\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{280}+\frac{12799\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{280}-\frac{2973\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{40}-\frac{219\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{13/2}}{8}-\frac{33\,c^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{15/2}}{8}}{a+\frac{6\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{14\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}+\frac{14\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{14\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{14\,a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}-\frac{6\,a\,{\left(a\,x-1\right)}^7}{{\left(a\,x+1\right)}^7}-\frac{a\,{\left(a\,x-1\right)}^8}{{\left(a\,x+1\right)}^8}}-\frac{15\,c^4\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{8\,a}-\frac{6\,c^4\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^4*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((63*c^4*((a*x - 1)/(a*x + 1))^(1/2))/8 + (389*c^4*((a*x - 1)/(a*x + 1))^(3
/2))/8 + (4803*c^4*((a*x - 1)/(a*x + 1))^(5/2))/40 + (39071*c^4*((a*x - 1)/
(a*x + 1))^(7/2))/280 + (12799*c^4*((a*x - 1)/(a*x + 1))^(9/2))/280 - (2973
*c^4*((a*x - 1)/(a*x + 1))^(11/2))/40 - (219*c^4*((a*x - 1)/(a*x + 1))^(13/
2))/8 - (33*c^4*((a*x - 1)/(a*x + 1))^(15/2))/8)/(a + (6*a*(a*x - 1))/(a*x
+ 1) + (14*a*(a*x - 1)^2)/(a*x + 1)^2 + (14*a*(a*x - 1)^3)/(a*x + 1)^3 - (1
4*a*(a*x - 1)^5)/(a*x + 1)^5 - (14*a*(a*x - 1)^6)/(a*x + 1)^6 - (6*a*(a*x -
 1)^7)/(a*x + 1)^7 - (a*(a*x - 1)^8)/(a*x + 1)^8) - (15*c^4*atan(((a*x - 1)
/(a*x + 1))^(1/2)))/(8*a) - (6*c^4*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.823 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx$

Optimal. Leaf size=269

$$\frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a}+\frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a}+\frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a}+\frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3}}{20a}$$

```
[Out] 5/4*c^3*(1-1/a/x)^(3/2)*(1+1/a/x)^(3/2)/a+27/20*c^3*(1-1/a/x)^(5/2)*(1+1/a/
x)^(3/2)/a+6/5*c^3*(1-1/a/x)^(7/2)*(1+1/a/x)^(3/2)/a+c^3*(1-1/a/x)^(9/2)*(1
+1/a/x)^(3/2)*x+3/8*c^3*arccsc(a*x)/a-3*c^3*arctanh((1-1/a/x)^(1/2)*(1+1/a/
x)^(1/2))/a+3/8*c^3*(1+1/a/x)^(3/2)*(1-1/a/x)^(1/2)/a+21/8*c^3*(1-1/a/x)^(1
/2)*(1+1/a/x)^(1/2)/a
```

Rubi [A]

time = 0.13, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^3x\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{9/2}+\frac{6c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{7/2}}{5a}+\frac{27c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{5/2}}{20a}+\frac{5c^3\left(\frac{1}{ax}+1\right)^{3/2}\left(1-\frac{1}{ax}\right)^{3/2}}{4a}+\frac{3c^3\left(\frac{1}{ax}+1\right)^{3/2}\sqrt{1-\frac{1}{ax}}}{8a}+\frac{21c^3\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{8a}+\frac{3c^3\csc^{-1}(ax)}{8a}-\frac{3c^3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (21*c^3*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(8*a) + (3*c^3*Sqrt[1 - 1/(a*x
)]*(1 + 1/(a*x))^(3/2))/(8*a) + (5*c^3*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(3
/2))/(4*a) + (27*c^3*(1 - 1/(a*x))^(5/2)*(1 + 1/(a*x))^(3/2))/(20*a) + (6*c
^3*(1 - 1/(a*x))^(7/2)*(1 + 1/(a*x))^(3/2))/(5*a) + c^3*(1 - 1/(a*x))^(9/2)
*(1 + 1/(a*x))^(3/2)*x + (3*c^3*ArcCsc[a*x])/(8*a) - (3*c^3*ArcTanh[Sqrt[1
- 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^3\,dx = -\left(c^3\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{9/2}\left(1+\frac{x}{a}\right)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - c^3\text{Subst}\left(\int\frac{\left(-\frac{3}{a}-\frac{6x}{a^2}\right)\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+}}{x}\right.$$

$$= \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/2}\left(1+\frac{1}{ax}\right)^{3/2}x - \frac{1}{5}\left(ac^3\right)\text{Sub}$$

$$= \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}\left(1+\frac{1}{ax}\right)^{3/2}}{5a} + c^3\left(1-\frac{1}{ax}\right)^{9/}$$

$$= \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}\left(1+\frac{1}{ax}\right)^{3/2}}{20a} + \frac{6c^3\left(1-\frac{1}{ax}\right)^{7/2}}{5a}$$

$$= \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{3/2}}{4a} + \frac{27c^3\left(1-\frac{1}{ax}\right)^{5/2}}{20a}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}(1}{4a}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}(1}{4a}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}(1}{4a}$$

$$= \frac{21c^3\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{8a} + \frac{3c^3\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{3/2}}{8a} + \frac{5c^3\left(1-\frac{1}{ax}\right)^{3/2}(1}{4a}$$

Mathematica [A]
time = 0.11, size = 110, normalized size = 0.41

$$\frac{c^3\left(\sqrt{1-\frac{1}{a^2x^2}}\,(-8+30ax-24a^2x^2-55a^3x^3+152a^4x^4+40a^5x^5)+15a^4x^4\,\mathrm{ArcSin}\left(\frac{1}{ax}\right)-120a^4x^4\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{40a^5x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^3/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*(Sqrt[1 - 1/(a^2*x^2)]*(-8 + 30*a*x - 24*a^2*x^2 - 55*a^3*x^3 + 152*a^
4*x^4 + 40*a^5*x^5) + 15*a^4*x^4*ArcSin[1/(a*x)] - 120*a^4*x^4*Log[(1 + Sqr
t[1 - 1/(a^2*x^2)])*x]))/(40*a^5*x^4)
```

Maple [A]
time = 0.08, size = 281, normalized size = 1.04

method	result
risch	$\frac{(ax+1)(152a^4x^4-55a^3x^3-24a^2x^2+30ax-8)c^3\sqrt{\frac{ax-1}{ax+1}}}{40x^5a^6}+\frac{\left(a^5\sqrt{(ax+1)(ax-1)}-\frac{3a^6\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}\right.}{a}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^3\left(-120\sqrt{a^2}\,\sqrt{a^2x^2-1}\,a^6x^6+120\sqrt{a^2}\,(a^2x^2-1)^{\frac{3}{2}}a^4x^4-15\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^5x^5-15\sqrt{a^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^3*(-120*(a^2)^(1/2)*(a^2*x^2-1)^(
1/2)*a^6*x^6+120*(a^2)^(1/2)*(a^2*x^2-1)^(3/2)*a^4*x^4-15*(a^2*x^2-1)^(1/2)
*(a^2)^(1/2)*a^5*x^5-15*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a^5*x^5+120
*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^6*x^5-25*(a^2*x^2-
1)^(3/2)*(a^2)^(1/2)*a^3*x^3-32*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+30*(a
^2)^(1/2)*(a^2*x^2-1)^(3/2)*a*x-8*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/(a*x-1)/((
a*x+1)*(a*x-1))^(1/2)/a^6/x^5/(a^2)^(1/2)
```

Maxima [A]
time = 0.48, size = 301, normalized size = 1.12

$$-\frac{1}{20}\left(\frac{15\,c^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{60\,c^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}+\frac{105\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{11}{2}}+465\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}-298\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-842\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-575\,c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-135\,c^3\sqrt{\frac{ax-1}{ax+1}}}{\frac{4(ax-1)a^2}{ax+1}+\frac{5(ax-1)^2a^2}{(ax+1)^2}-\frac{5(ax-1)^4a^2}{(ax+1)^4}-\frac{4(ax-1)^5a^2}{(ax+1)^5}-\frac{(ax-1)^6a^2}{(ax+1)^6}+a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/20*(15*c^3*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 60*c^3*log(sqrt((a*x
- 1)/(a*x + 1)) + 1)/a^2 - 60*c^3*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 +
(105*c^3*((a*x - 1)/(a*x + 1))^(11/2) + 465*c^3*((a*x - 1)/(a*x + 1))^(9/2)
 - 298*c^3*((a*x - 1)/(a*x + 1))^(7/2) - 842*c^3*((a*x - 1)/(a*x + 1))^(5/2
) - 575*c^3*((a*x - 1)/(a*x + 1))^(3/2) - 135*c^3*sqrt((a*x - 1)/(a*x + 1))
)/(4*(a*x - 1)*a^2/(a*x + 1) + 5*(a*x - 1)^2*a^2/(a*x + 1)^2 - 5*(a*x - 1)^
4*a^2/(a*x + 1)^4 - 4*(a*x - 1)^5*a^2/(a*x + 1)^5 - (a*x - 1)^6*a^2/(a*x +
1)^6 + a^2))*a
```

Fricas [A]
time = 0.38, size = 179, normalized size = 0.67

$$-\frac{30\,a^5c^3x^5\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)+120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-120\,a^5c^3x^5\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-\left(40\,a^6c^3x^6+192\,a^5c^3x^5+97\,a^4c^3x^4-79\,a^3c^3x^3+6\,a^2c^3x^2+22\,ac^3x-8\,c^3\right)\sqrt{\frac{ax-1}{ax+1}}}{40\,a^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/40*(30*a^5*c^3*x^5*arctan(sqrt((a*x - 1)/(a*x + 1))) + 120*a^5*c^3*x^5*l
og(sqrt((a*x - 1)/(a*x + 1)) + 1) - 120*a^5*c^3*x^5*log(sqrt((a*x - 1)/(a*x
 + 1)) - 1) - (40*a^6*c^3*x^6 + 192*a^5*c^3*x^5 + 97*a^4*c^3*x^4 - 79*a^3*c
^3*x^3 + 6*a^2*c^3*x^2 + 22*a*c^3*x - 8*c^3)*sqrt((a*x - 1)/(a*x + 1)))/(a^
6*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^3\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^7+x^6}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^6+x^5}\right)dx+\int\left(-\frac{3a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{3a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{3a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{3a^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^7x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**3*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**3*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**7 + x**6), x) + Inte
gral(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**6 + x**5), x) + Integral(-3
*a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Integral(3*a*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(3*a**4*
sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-3*a**5*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**6*sqrt(a*x/
(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**7*x*sqrt(a*x/(a*x + 1)
 - 1/(a*x + 1))/(a*x + 1), x))/a**6
```

Giac [A]
time = 0.44, size = 395, normalized size = 1.47

$$-\frac{3\,c^3\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\mathrm{sgn}(ax+1)}{4\,a}+\frac{3\,c^3\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^3\mathrm{sgn}(ax+1)}{a}+\frac{55\left(x|a|-\sqrt{a^2x^2-1}\right)^9c^3|a|\mathrm{sgn}(ax+1)+200\left(x|a|-\sqrt{a^2x^2-1}\right)^8ac^3\mathrm{sgn}(ax+1)-10\left(x|a|-\sqrt{a^2x^2-1}\right)^7c^3|a|\mathrm{sgn}(ax+1)+720\left(x|a|-\sqrt{a^2x^2-1}\right)^6ac^3\mathrm{sgn}(ax+1)+800\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^3\mathrm{sgn}(ax+1)+10\left(x|a|-\sqrt{a^2x^2-1}\right)^3c^3|a|\mathrm{sgn}(ax+1)+560\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^3\mathrm{sgn}(ax+1)-55\left(x|a|-\sqrt{a^2x^2-1}\right)c^3|a|\mathrm{sgn}(ax+1)+152\,ac^3\mathrm{sgn}(ax+1)}{20\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^5a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^3*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] -3/4*c^3*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^3*log(a
bs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*
c^3*sgn(a*x + 1)/a + 1/20*(55*(x*abs(a) - sqrt(a^2*x^2 - 1))^9*c^3*abs(a)*s
gn(a*x + 1) + 200*(x*abs(a) - sqrt(a^2*x^2 - 1))^8*a*c^3*sgn(a*x + 1) - 10*
(x*abs(a) - sqrt(a^2*x^2 - 1))^7*c^3*abs(a)*sgn(a*x + 1) + 720*(x*abs(a) -
sqrt(a^2*x^2 - 1))^6*a*c^3*sgn(a*x + 1) + 800*(x*abs(a) - sqrt(a^2*x^2 - 1)
)^4*a*c^3*sgn(a*x + 1) + 10*(x*abs(a) - sqrt(a^2*x^2 - 1))^3*c^3*abs(a)*sgn
(a*x + 1) + 560*(x*abs(a) - sqrt(a^2*x^2 - 1))^2*a*c^3*sgn(a*x + 1) - 55*(x
*abs(a) - sqrt(a^2*x^2 - 1))*c^3*abs(a)*sgn(a*x + 1) + 152*a*c^3*sgn(a*x +
1))/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)^5*a*abs(a))
```

Mupad [B]

time = 1.26, size = 258, normalized size = 0.96

$$\frac{\frac{27\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{4}+\frac{115\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{4}+\frac{421\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10}+\frac{149\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{10}-\frac{93\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{9/2}}{4}-\frac{21\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{11/2}}{4}}{a+\frac{4\,a\,\left(a\,x-1\right)}{a\,x+1}+\frac{5\,a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}-\frac{5\,a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}-\frac{4\,a\,{\left(a\,x-1\right)}^5}{{\left(a\,x+1\right)}^5}-\frac{a\,{\left(a\,x-1\right)}^6}{{\left(a\,x+1\right)}^6}}-\frac{3\,c^3\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{4\,a}-\frac{6\,c^3\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^3*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] ((27*c^3*((a*x - 1)/(a*x + 1))^(1/2))/4 + (115*c^3*((a*x - 1)/(a*x + 1))^(3
/2))/4 + (421*c^3*((a*x - 1)/(a*x + 1))^(5/2))/10 + (149*c^3*((a*x - 1)/(a*
x + 1))^(7/2))/10 - (93*c^3*((a*x - 1)/(a*x + 1))^(9/2))/4 - (21*c^3*((a*x
- 1)/(a*x + 1))^(11/2))/4)/(a + (4*a*(a*x - 1))/(a*x + 1) + (5*a*(a*x - 1)^
2)/(a*x + 1)^2 - (5*a*(a*x - 1)^4)/(a*x + 1)^4 - (4*a*(a*x - 1)^5)/(a*x + 1
)^5 - (a*(a*x - 1)^6)/(a*x + 1)^6) - (3*c^3*atan(((a*x - 1)/(a*x + 1))^(1/2
)))/(4*a) - (6*c^3*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.824 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx$

Optimal. Leaf size=195

$$\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}$$

```
[Out] -1/2*c^2*arccsc(a*x)/a-3*c^2*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+11/
6*c^2*(1-1/a/x)^(3/2)*(1+1/a/x)^(1/2)/a+4/3*c^2*(1-1/a/x)^(5/2)*(1+1/a/x)^(
1/2)/a+c^2*(1-1/a/x)^(7/2)*x*(1+1/a/x)^(1/2)+5/2*c^2*(1-1/a/x)^(1/2)*(1+1/a
/x)^(1/2)/a
```

Rubi [A]

time = 0.09, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 99, 159, 163, 41, 222, 94, 214}

$$c^2x\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{7/2}+\frac{4c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{5/2}}{3a}+\frac{11c^2\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}}{6a}+\frac{5c^2\sqrt{\frac{1}{ax}+1}\sqrt{1-\frac{1}{ax}}}{2a}-\frac{c^2\csc^{-1}(ax)}{2a}-\frac{3c^2\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (5*c^2*Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)])/(2*a) + (11*c^2*(1 - 1/(a*x))^(
3/2)*Sqrt[1 + 1/(a*x)])/(6*a) + (4*c^2*(1 - 1/(a*x))^(5/2)*Sqrt[1 + 1/(a*x)
])/(3*a) + c^2*(1 - 1/(a*x))^(7/2)*Sqrt[1 + 1/(a*x)]*x - (c^2*ArcCsc[a*x])/
(2*a) - (3*c^2*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^2\,dx=-\left(c^2\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{7/2}\sqrt{1+\frac{x}{a}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)$$

$$=c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x-c^2\text{Subst}\left(\int\frac{\left(-\frac{3}{a}-\frac{4x}{a^2}\right)\left(1-\frac{x}{a}\right)^{5/2}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{7/2}\sqrt{1+\frac{1}{ax}}\,x-\frac{1}{3}\left(ac^2\right)\text{Subst}\Bigg($$

$$=\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{1+\frac{1}{ax}}}{3a}+c^2\left(1-\frac{1}{ax}\right)^{7/2}$$

$$=\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{}}{3a}$$

$$=\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{}}{3a}$$

$$=\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{}}{3a}$$

$$=\frac{5c^2\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}}{2a}+\frac{11c^2\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}}{6a}+\frac{4c^2\left(1-\frac{1}{ax}\right)^{5/2}\sqrt{}}{3a}$$

Mathematica [A]

time = 0.09, size = 94, normalized size = 0.48

$$\frac{c^2\left(\sqrt{1-\frac{1}{a^2x^2}}\,(2-9ax+16a^2x^2+6a^3x^3)-3a^2x^2\operatorname{ArcSin}\left(\frac{1}{ax}\right)-18a^2x^2\log\left(\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)\right)}{6a^3x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^2/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^2*(Sqrt[1 - 1/(a^2*x^2)]*(2 - 9*a*x + 16*a^2*x^2 + 6*a^3*x^3) - 3*a^2*x^
2*ArcSin[1/(a*x)] - 18*a^2*x^2*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x]))/(6*a^3*
x^2)
```

Maple [A]
time = 0.07, size = 233, normalized size = 1.19

method	result
risch	$\frac{(ax+1)(16a^2x^2-9ax+2)c^2\sqrt{\frac{ax-1}{ax+1}}}{6x^3a^4}+\frac{\left(a^3\sqrt{(ax+1)(ax-1)}-\frac{3a^4\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-\frac{a^3\arctan\left(\frac{}{\sqrt{a^2x}}\right.}{2}\right.}{a^4(ax-1)}$
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c^2\left(-18\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^4x^4+18(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}\,a^2x^2+3\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^3x^3+18\ln\left(\frac{a^2x+}{}\right.\right.}{6(ax-1)\sqrt{(ax+1)(ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c^2*(-18*(a^2*x^2-1)^(1/2)*(a^2)^(1/
2)*a^4*x^4+18*(a^2*x^2-1)^(3/2)*(a^2)^(1/2)*a^2*x^2+3*(a^2*x^2-1)^(1/2)*(a^
2)^(1/2)*a^3*x^3+18*ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a
^4*x^3+3*a^3*x^3*(a^2)^(1/2)*arctan(1/(a^2*x^2-1)^(1/2))-9*(a^2)^(1/2)*(a^2
*x^2-1)^(3/2)*a*x+2*(a^2*x^2-1)^(3/2)*(a^2)^(1/2))/(a*x-1)/((a*x+1)*(a*x-1)
)^(1/2)/a^4/x^3/(a^2)^(1/2)
```

Maxima [A]
time = 0.46, size = 224, normalized size = 1.15

$$\frac{1}{3}a\left(\frac{3c^2\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}-\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}+\frac{9c^2\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}-\frac{21c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}-17c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-37c^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-15c^2\sqrt{\frac{ax-1}{ax+1}}}{\frac{2(ax-1)a^2}{ax+1}-\frac{2(ax-1)^3a^2}{(ax+1)^3}-\frac{(ax-1)^4a^2}{(ax+1)^4}+a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*c^2*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 - 9*c^2*log(sqrt((a*x -
1)/(a*x + 1)) + 1)/a^2 + 9*c^2*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2 - (21
*c^2*((a*x - 1)/(a*x + 1))^(7/2) - 17*c^2*((a*x - 1)/(a*x + 1))^(5/2) - 37*
c^2*((a*x - 1)/(a*x + 1))^(3/2) - 15*c^2*sqrt((a*x - 1)/(a*x + 1)))/(2*(a*x
 - 1)*a^2/(a*x + 1) - 2*(a*x - 1)^3*a^2/(a*x + 1)^3 - (a*x - 1)^4*a^2/(a*x
+ 1)^4 + a^2))
```

Fricas [A]
time = 0.38, size = 156, normalized size = 0.80

$$\frac{6\,a^3c^2x^3\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+18\,a^3c^2x^3\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+\left(6\,a^4c^2x^4+22\,a^3c^2x^3+7\,a^2c^2x^2-7\,ac^2x+2\,c^2\right)\sqrt{\frac{ax-1}{ax+1}}}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^3*c^2*x^3*arctan(sqrt((a*x - 1)/(a*x + 1))) - 18*a^3*c^2*x^3*log(s
qrt((a*x - 1)/(a*x + 1)) + 1) + 18*a^3*c^2*x^3*log(sqrt((a*x - 1)/(a*x + 1)
) - 1) + (6*a^4*c^2*x^4 + 22*a^3*c^2*x^3 + 7*a^2*c^2*x^2 - 7*a*c^2*x + 2*c^
2)*sqrt((a*x - 1)/(a*x + 1)))/(a^4*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int\left(-\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^5+x^4}\right)dx+\int\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^4+x^3}\,dx+\int\frac{2a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{2a^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^5x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**2*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c**2*(Integral(-sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**5 + x**4), x) + Int
egral(a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**4 + x**3), x) + Integral(2*
a**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integral(-2*a*
*3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**4*sqrt
(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**5*x*sqrt(a*x/(a*x
 + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**4
```

Giac [A]
time = 0.43, size = 264, normalized size = 1.35

$$\frac{c^2\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c^2\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c^2\operatorname{sgn}(ax+1)}{a}+\frac{9\left(x|a|-\sqrt{a^2x^2-1}\right)^5c^2|a|\operatorname{sgn}(ax+1)+12\left(x|a|-\sqrt{a^2x^2-1}\right)^4ac^2\operatorname{sgn}(ax+1)+36\left(x|a|-\sqrt{a^2x^2-1}\right)^2ac^2\operatorname{sgn}(ax+1)-9\left(x|a|-\sqrt{a^2x^2-1}\right)c^2|a|\operatorname{sgn}(ax+1)+16\,ac^2\operatorname{sgn}(ax+1)}{3\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)^3a|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^2*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c^2*log(abs(-x
*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c^2*s
gn(a*x + 1)/a + 1/3*(9*(x*abs(a) - sqrt(a^2*x^2 - 1))^5*c^2*abs(a)*sgn(a*x
+ 1) + 12*(x*abs(a) - sqrt(a^2*x^2 - 1))^4*a*c^2*sgn(a*x + 1) + 36*(x*abs(a
) - sqrt(a^2*x^2 - 1))^2*a*c^2*sgn(a*x + 1) - 9*(x*abs(a) - sqrt(a^2*x^2 -
1))*c^2*abs(a)*sgn(a*x + 1) + 16*a*c^2*sgn(a*x + 1))/(((x*abs(a) - sqrt(a^2
*x^2 - 1))^2 + 1)^3*a*abs(a))
```

Mupad [B]
time = 0.09, size = 183, normalized size = 0.94

$$\frac{5\,c^2\,\sqrt{\frac{a\,x-1}{a\,x+1}}+\frac{37\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3}+\frac{17\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{3}-7\,c^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{a+\frac{2\,a\,\left(a\,x-1\right)}{a\,x+1}-\frac{2\,a\,{\left(a\,x-1\right)}^3}{{\left(a\,x+1\right)}^3}-\frac{a\,{\left(a\,x-1\right)}^4}{{\left(a\,x+1\right)}^4}}+\frac{c^2\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c^2\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^2*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (5*c^2*((a*x - 1)/(a*x + 1))^(1/2) + (37*c^2*((a*x - 1)/(a*x + 1))^(3/2))/3
 + (17*c^2*((a*x - 1)/(a*x + 1))^(5/2))/3 - 7*c^2*((a*x - 1)/(a*x + 1))^(7/
2))/(a + (2*a*(a*x - 1))/(a*x + 1) - (2*a*(a*x - 1)^3)/(a*x + 1)^3 - (a*(a*
x - 1)^4)/(a*x + 1)^4) + (c^2*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c^2
*atanh(((a*x - 1)/(a*x + 1))^(1/2)))/a
```

3.825 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=76

$$c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{a}$$

```
[Out] -3*c*arccsc(a*x)/a-3*c*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a+c*(1-1/a/
x)^(3/2)*x*(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6329, 100, 12, 132, 41, 222, 94, 214}

$$cx\sqrt{\frac{1}{ax}+1}\left(1-\frac{1}{ax}\right)^{3/2}-\frac{3c\csc^{-1}(ax)}{a}-\frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
```

```
[Out] c*(1 - 1/(a*x))^(3/2)*Sqrt[1 + 1/(a*x)]*x - (3*c*ArcCsc[a*x])/a - (3*c*ArcT
anh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx = -\left(c\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{5/2}}{x^2\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)\right)$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + c\text{Subst}\left(\int \frac{3\sqrt{1-\frac{x}{a}}}{ax\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x + \frac{(3c)\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - \frac{(3c)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,x,\frac{1}{x}\right)}{a^2} +$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - \frac{(3c)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,\frac{1}{x}\right)}{a^2} - \frac{(3c)\text{Sub}}{}$$

$$= c\left(1-\frac{1}{ax}\right)^{3/2}\sqrt{1+\frac{1}{ax}}\,x - \frac{3c\csc^{-1}(ax)}{a} - \frac{3c\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\cdot}\right.}{a}$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.75

$$\frac{c\left(\sqrt{1-\frac{1}{a^2x^2}}\,(-1+ax)-3\text{ArcSin}\left(\frac{1}{ax}\right)-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)\right)}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))/E^(3*ArcCoth[a*x]),x]
[Out] (c*(Sqrt[1 - 1/(a^2*x^2)]*(-1 + a*x) - 3*ArcSin[1/(a*x)] - 3*Log[(1 + Sqrt[
1 - 1/(a^2*x^2)])*x]))/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(68) = 136$.

time = 0.11, size = 234, normalized size = 3.08

method	result
risch	$-\frac{(ax+1)c\sqrt{\frac{ax-1}{ax+1}}}{x\,a^2}+\frac{\left(a\sqrt{(ax+1)(ax-1)}-\frac{3a^2\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{\sqrt{a^2}}-3a\arctan\left(\frac{1}{\sqrt{a^2x^2-1}}\right)\right)c\sqrt{}}{a^2(ax-1)}$
default	$\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)^2c\left(-\sqrt{a^2x^2-1}\,\sqrt{a^2}\,a^2x^2+(a^2x^2-1)^{\frac{3}{2}}\sqrt{a^2}-3\sqrt{a^2}\,\sqrt{a^2x^2-1}\,ax-3\sqrt{a^2}\,\arctan\left(\frac{1}{\sqrt{a^2x^2}}\right.\right.}{(ax-1)\sqrt{(ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a*x-1)/(a*x+1))^(3/2)*(a*x+1)^2*c*(-(a^2*x^2-1)^(1/2)*(a^2)^(1/2)*a^2*x^2
+(a^2*x^2-1)^(3/2)*(a^2)^(1/2)-3*(a^2)^(1/2)*(a^2*x^2-1)^(1/2)*a*x-3*(a^2)^
(1/2)*arctan(1/(a^2*x^2-1)^(1/2))*a*x+4*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)
*a*x+ln((a^2*x+(a^2*x^2-1)^(1/2)*(a^2)^(1/2))/(a^2)^(1/2))*a^2*x-4*ln((a^2*
x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x)/(a*x-1)/((a*x+1)
*(a*x-1))^(1/2)/a^2/(a^2)^(1/2)/x
```

Maxima [A]

time = 0.48, size = 118, normalized size = 1.55

$$-\left(\frac{4\,c\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{\frac{(ax-1)^2a^2}{(ax+1)^2}-a^2}-\frac{6\,c\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)}{a^2}+\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2}-\frac{3\,c\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")
```

```
[Out] -(4*c*((a*x - 1)/(a*x + 1))^(3/2)/((a*x - 1)^2*a^2/(a*x + 1)^2 - a^2) - 6*c
*arctan(sqrt((a*x - 1)/(a*x + 1)))/a^2 + 3*c*log(sqrt((a*x - 1)/(a*x + 1))
+ 1)/a^2 - 3*c*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/a^2)*a
```

Fricas [A]

time = 0.35, size = 103, normalized size = 1.36

$$\frac{6\,acx\arctan\left(\sqrt{\frac{ax-1}{ax+1}}\right)-3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)+3\,acx\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)+(a^2cx^2-c)\sqrt{\frac{ax-1}{ax+1}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a*c*x*arctan(sqrt((a*x - 1)/(a*x + 1))) - 3*a*c*x*log(sqrt((a*x - 1)/(a*
x + 1)) + 1) + 3*a*c*x*log(sqrt((a*x - 1)/(a*x + 1)) - 1) + (a^2*c*x^2 - c)
*sqrt((a*x - 1)/(a*x + 1)))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int\frac{\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^3+x^2}\,dx+\int\left(-\frac{a\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax^2+x}\right)dx+\int\left(-\frac{a^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\right)dx+\int\frac{a^3x\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{ax+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] c*(Integral(sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**3 + x**2), x) + Integra
l(-a*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x**2 + x), x) + Integral(-a**2*sq
rt(a*x/(a*x + 1) - 1/(a*x + 1))/(a*x + 1), x) + Integral(a**3*x*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a*x + 1), x))/a**2
```

Giac [A]
time = 0.43, size = 122, normalized size = 1.61

$$\frac{6\,c\arctan\left(-x|a|+\sqrt{a^2x^2-1}\right)\operatorname{sgn}(ax+1)}{a}+\frac{3\,c\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\operatorname{sgn}(ax+1)}{|a|}+\frac{\sqrt{a^2x^2-1}\,c\operatorname{sgn}(ax+1)}{a}-\frac{2\,c\operatorname{sgn}(ax+1)}{\left(\left(x|a|-\sqrt{a^2x^2-1}\right)^2+1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] 6*c*arctan(-x*abs(a) + sqrt(a^2*x^2 - 1))*sgn(a*x + 1)/a + 3*c*log(abs(-x*a
bs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/abs(a) + sqrt(a^2*x^2 - 1)*c*sgn(a
*x + 1)/a - 2*c*sgn(a*x + 1)/(((x*abs(a) - sqrt(a^2*x^2 - 1))^2 + 1)*abs(a)
)
```

Mupad [B]
time = 0.06, size = 84, normalized size = 1.11

$$\frac{6\,c\,\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}-\frac{6\,c\,\mathrm{atanh}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\right)}{a}+\frac{4\,c\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{a-\frac{a\,{\left(a\,x-1\right)}^2}{{\left(a\,x+1\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] (6*c*atan(((a*x - 1)/(a*x + 1))^(1/2)))/a - (6*c*atanh(((a*x - 1)/(a*x + 1)
)^(1/2)))/a + (4*c*((a*x - 1)/(a*x + 1))^(3/2))/(a - (a*(a*x - 1)^2)/(a*x +
 1)^2)
```

3.826 $\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=144

$$\frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c+5/3*(1-1/a/x)^(1/2)/a/c/(1+
1/a/x)^(3/2)+x*(1-1/a/x)^(1/2)/c/(1+1/a/x)^(3/2)+14/3*(1-1/a/x)^(1/2)/a/c/(
1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6329, 101, 157, 12, 94, 214}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{\frac{1}{ax}+1}}+\frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(\frac{1}{ax}+1\right)^{3/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\,\sqrt{\frac{1}{ax}+1}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] (5*Sqrt[1 - 1/(a*x)])/(3*a*c*(1 + 1/(a*x))^(3/2)) + (14*Sqrt[1 - 1/(a*x)])/
(3*a*c*Sqrt[1 + 1/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c*(1 + 1/(a*x))^(3/2)) -
 (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x^2\left(1+\frac{x}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{a}+\frac{2x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{5x}{a^3}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{3c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{a^2\text{Subst}\Bigg(\int -\frac{9}{a^3x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}}{3c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{3\text{Subst}\Bigg(\int \frac{1}{x\sqrt{1-\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx,}{ac}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\text{Subst}\Bigg(\int \frac{1}{\frac{1}{a}-\frac{x^2}{a}}\,dx,x,\sqrt{1-\frac{1}{ax}}}{a^2c}$$

$$= \frac{5\sqrt{1-\frac{1}{ax}}}{3ac\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{14\sqrt{1-\frac{1}{ax}}}{3ac\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c\left(1+\frac{1}{ax}\right)^{3/2}} - \frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac}$$

<u>Mathematica</u> [A]

time = 0.17, size = 69, normalized size = 0.48

$$\frac{\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x(14+19ax+3a^2x^2)}{(1+ax)^2}-\frac{9\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{a}}{3c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))),x]
```

```
[Out] ((Sqrt[1 - 1/(a^2*x^2)]*x*(14 + 19*a*x + 3*a^2*x^2))/(1 + a*x)^2 - (9*Log[(
1 + Sqrt[1 - 1/(a^2*x^2)])*x])/a)/(3*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(124) = 248.
time = 0.11, size = 346, normalized size = 2.40

method	result
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}{ac}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^2\sqrt{a^2}}-\frac{2\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{3a^5\left(x+\frac{1}{a}\right)^2}+\frac{13\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{3a^4\left(x+\frac{1}{a}\right)}\right.}{c(ax-1)}$
default	$-\frac{\left(-9\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^3x^3+9\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^4x^3+6\sqrt{a^2}\,((ax+1)(ax-1))\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-9*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+9*ln((a^2*x+(a^2)^(1/2
)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+6*(a^2)^(1/2)*((a*x+1)*(a*x
-1))^(3/2)*a*x-27*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+27*ln((a^2*x+
(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+5*((a*x+1)*(a*x-1
))^(3/2)*(a^2)^(1/2)-27*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+27*ln((a^2*
x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-9*(a^2)^(1/2)*((a
*x+1)*(a*x-1))^(1/2)+9*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^
2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x+1)/c/((a*x+1)*(a*x-1)
)^(1/2)/(a*x-1)
```

Maxima [A]
time = 0.26, size = 140, normalized size = 0.97

$$-\frac{1}{3}a\left(\frac{6\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c}{ax+1}-a^2c}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+12\sqrt{\frac{ax-1}{ax+1}}}{a^2c}+\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c}-\frac{9\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(6*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c/(a*x + 1) - a^2*c) - (
((a*x - 1)/(a*x + 1))^(3/2) + 12*sqrt((a*x - 1)/(a*x + 1)))/(a^2*c) + 9*log
(sqrt((a*x - 1)/(a*x + 1)) + 1)/(a^2*c) - 9*log(sqrt((a*x - 1)/(a*x + 1)) -
 1)/(a^2*c))
```

Fricas [A]
time = 0.41, size = 96, normalized size = 0.67

$$-\frac{9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-9\,(ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(3\,a^2x^2+19\,ax+14)\sqrt{\frac{ax-1}{ax+1}}}{3\,(a^2cx+ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] -1/3*(9*(a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 9*(a*x + 1)*log(sqrt
((a*x - 1)/(a*x + 1)) - 1) - (3*a^2*x^2 + 19*a*x + 14)*sqrt((a*x - 1)/(a*x
+ 1)))/(a^2*c*x + a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\left(-\frac{x^2\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\right)dx+\int\frac{ax^3\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^3x^3+a^2x^2-ax-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2),x)
```

```
[Out] a**2*(Integral(-x**2*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3*x**3 + a**2*x*
*2 - a*x - 1), x) + Integral(a*x**3*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**3
*x**3 + a**2*x**2 - a*x - 1), x))/c
```

Giac [A]
time = 0.41, size = 59, normalized size = 0.41

$$\frac{3\,\log\left(\left|-x|a|+\sqrt{a^2x^2-1}\right|\right)\mathrm{sgn}(ax+1)}{c|a|}+\frac{\sqrt{a^2x^2-1}\,\mathrm{sgn}(ax+1)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c*abs(a)) + sqrt(a^
2*x^2 - 1)*sgn(a*x + 1)/(a*c)
```

Mupad [B]
time = 0.07, size = 114, normalized size = 0.79

$$\frac{2\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c-\frac{a\,c\,\left(a\,x-1\right)}{a\,x+1}}+\frac{4\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{3\,a\,c}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2)),x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c - (a*c*(a*x - 1))/(a*x + 1)) + (4*((a*
x - 1)/(a*x + 1))^(1/2))/(a*c) + ((a*x - 1)/(a*x + 1))^(3/2)/(3*a*c) + (ata
n(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c)
```

3.827 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$

Optimal. Leaf size=181

$$\frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}}+\frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{1+\frac{1}{ax}}\right)}{ac^2}$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^2+6/5*(1-1/a/x)^(1/2)/a/c^2
/(1+1/a/x)^(5/2)+9/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(3/2)+x*(1-1/a/x)^(1/2
)/c^2/(1+1/a/x)^(5/2)+24/5*(1-1/a/x)^(1/2)/a/c^2/(1+1/a/x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6329, 105, 21, 101, 157, 12, 94, 214}

$$\frac{x\sqrt{1-\frac{1}{ax}}}{c^2\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{\frac{1}{ax}+1}}+\frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(\frac{1}{ax}+1\right)^{5/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] (6*Sqrt[1 - 1/(a*x)])/(5*a*c^2*(1 + 1/(a*x))^(5/2)) + (9*Sqrt[1 - 1/(a*x)])
/(5*a*c^2*(1 + 1/(a*x))^(3/2)) + (24*Sqrt[1 - 1/(a*x)])/(5*a*c^2*Sqrt[1 + 1
/(a*x)]) + (Sqrt[1 - 1/(a*x)]*x)/(c^2*(1 + 1/(a*x))^(5/2)) - (3*ArcTanh[Sqr
t[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
```

```
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{3x}{a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{3\text{Subst}\left(\int \frac{\sqrt{1-\frac{x}{a}}}{x\left(1+\frac{x}{a}\right)^{7/2}}\,dx,x,\frac{1}{x}\right)}{ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{6\text{Subst}\left(\int \frac{-\frac{5}{2}+\frac{2x}{a}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{5/2}}\,dx,x,\frac{1}{x}\right)}{5ac^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{2\text{Subst}\left(\int \frac{-\frac{15}{2a}+\frac{9x}{2a^2}}{x\sqrt{1-\frac{x}{a}}\left(1+\frac{x}{a}\right)^{3/2}}\right.}{5c^2}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{(2a)\text{Subst}\left(\right.}{}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{3\text{Subst}\left(\int\right.}{}$$

$$= \frac{6\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{9\sqrt{1-\frac{1}{ax}}}{5ac^2\left(1+\frac{1}{ax}\right)^{3/2}} + \frac{24\sqrt{1-\frac{1}{ax}}}{5ac^2\sqrt{1+\frac{1}{ax}}} + \frac{\sqrt{1-\frac{1}{ax}}\,x}{c^2\left(1+\frac{1}{ax}\right)^{5/2}} - \frac{3\text{Subst}\left(\int\right.}{}$$

$$6\sqrt{1-\frac{1}{}} \quad 9\sqrt{1-\frac{1}{}} \quad 24\sqrt{1-\frac{1}{}} \quad \sqrt{1-\frac{1}{}}\,x \quad 3\tanh^{-1}\left(\right.$$

Mathematica [A]
time = 0.16, size = 78, normalized size = 0.43

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(24+57ax+39a^2x^2+5a^3x^3)}{5(1+ax)^3}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^2),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(24 + 57*a*x + 39*a^2*x^2 + 5*a^3*x^3))/(5*(1 + a*x)^3) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(155) = 310$.
time = 0.15, size = 438, normalized size = 2.42

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^2}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^4\sqrt{a^2}}-\frac{6\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{5a^7\left(x+\frac{1}{a}\right)^2}+\frac{24\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(\right.}}{5a^6\left(x+\frac{1}{a}\right)}\right.}{c^2(ax-1)}$
default	$-\frac{\left(-125\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}\,a^4x^4+120\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^5x^4+85\sqrt{a^2}\,((ax+1)(ax-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(-125*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+120*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+85*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2-500*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3+480*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+148*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-750*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^2*x^2+720*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^3*x^2+67*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-500*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a*x+480*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^2*x-125*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+120*a*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a^2)^(1/2)/(a*x+1)^2/c^2/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)
```

Maxima [A]
time = 0.27, size = 161, normalized size = 0.89

$$-\frac{1}{20}\,a\left(\frac{40\,\sqrt{\frac{ax-1}{ax+1}}}{\frac{(ax-1)a^2c^2}{ax+1}-a^2c^2}-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+10\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+85\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^2}+\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^2}-\frac{60\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/20*a*(40*sqrt((a*x - 1)/(a*x + 1))/((a*x - 1)*a^2*c^2/(a*x + 1) - a^2*c^
2) - (((a*x - 1)/(a*x + 1))^(5/2) + 10*((a*x - 1)/(a*x + 1))^(3/2) + 85*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^2) + 60*log(sqrt((a*x - 1)/(a*x + 1)) + 1)/(
a^2*c^2) - 60*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^2))
```

Fricas [A]
time = 0.36, size = 135, normalized size = 0.75

$$-\frac{15\,(a^2x^2+2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-15\,(a^2x^2+2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(5\,a^3x^3+39\,a^2x^2+57\,ax+24)\sqrt{\frac{ax-1}{ax+1}}}{5\,(a^3c^2x^2+2\,a^2c^2x+ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/5*(15*(a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 15*(a^2
*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (5*a^3*x^3 + 39*a^2*
x^2 + 57*a*x + 24)*sqrt((a*x - 1)/(a*x + 1)))/(a^3*c^2*x^2 + 2*a^2*c^2*x +
a*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4\left(\int\left(-\frac{x^4\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\right)dx+\int\frac{ax^5\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^5x^5+a^4x^4-2a^3x^3-2a^2x^2+ax+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*(Integral(-x**4*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x*
*4 - 2*a**3*x**3 - 2*a**2*x**2 + a*x + 1), x) + Integral(a*x**5*sqrt(a*x/(a
*x + 1) - 1/(a*x + 1))/(a**5*x**5 + a**4*x**4 - 2*a**3*x**3 - 2*a**2*x**2 +
 a*x + 1), x))/c**2
```

Giac [A]
time = 0.44, size = 59, normalized size = 0.33

$$\frac{3 \log \left(\left|-x|a| + \sqrt{a^2x^2 - 1}\right|\right) \operatorname{sgn}(ax + 1)}{c^2|a|} + \frac{\sqrt{a^2x^2 - 1} \operatorname{sgn}(ax + 1)}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] 3*log(abs(-x*abs(a) + sqrt(a^2*x^2 - 1)))*sgn(a*x + 1)/(c^2*abs(a)) + sqrt(
a^2*x^2 - 1)*sgn(a*x + 1)/(a*c^2)
```

Mupad [B]
time = 0.05, size = 141, normalized size = 0.78

$$\frac{2\sqrt{\frac{a\,x-1}{a\,x+1}}}{a\,c^2-\frac{a\,c^2\,(a\,x-1)}{a\,x+1}}+\frac{17\sqrt{\frac{a\,x-1}{a\,x+1}}}{4\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{2\,a\,c^2}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{20\,a\,c^2}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1\mathrm{i}\right)\,6\mathrm{i}}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^2,x)
```

```
[Out] (2*((a*x - 1)/(a*x + 1))^(1/2))/(a*c^2 - (a*c^2*(a*x - 1))/(a*x + 1)) + (17
*((a*x - 1)/(a*x + 1))^(1/2))/(4*a*c^2) + ((a*x - 1)/(a*x + 1))^(3/2)/(2*a*
c^2) + ((a*x - 1)/(a*x + 1))^(5/2)/(20*a*c^2) + (atan(((a*x - 1)/(a*x + 1))
^(1/2)*1i)*6i)/(a*c^2)
```

3.828 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx$

Optimal. Leaf size=253

$$-\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{3/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{1+\frac{1}{ax}}}+\cdots$$

```
[Out] -3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^3-2/a/c^3/(1+1/a/x)^(7/2)/(
1-1/a/x)^(1/2)+x/c^3/(1+1/a/x)^(7/2)/(1-1/a/x)^(1/2)+11/7*(1-1/a/x)^(1/2)/a
/c^3/(1+1/a/x)^(7/2)+54/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(5/2)+71/35*(1-1
/a/x)^(1/2)/a/c^3/(1+1/a/x)^(3/2)+176/35*(1-1/a/x)^(1/2)/a/c^3/(1+1/a/x)^(1
/2)
```

Rubi [A]
time = 0.12, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^3\sqrt{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{176\sqrt{1-\frac{1}{ax}}}{35ac^3\sqrt{\frac{1}{ax}+1}}+\frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(\frac{1}{ax}+1\right)^{7/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] -2/(a*c^3*Sqrt[1 - 1/(a*x)]*(1 + 1/(a*x))^(7/2)) + (11*Sqrt[1 - 1/(a*x)])/(
7*a*c^3*(1 + 1/(a*x))^(7/2)) + (54*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x
))^(5/2)) + (71*Sqrt[1 - 1/(a*x)])/(35*a*c^3*(1 + 1/(a*x))^(3/2)) + (176*Sq
rt[1 - 1/(a*x)])/(35*a*c^3*Sqrt[1 + 1/(a*x)]) + x/(c^3*Sqrt[1 - 1/(a*x)]*(1
 + 1/(a*x))^(7/2)) - (3*ArcTanh[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^
3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
```

```
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{5x}{a^2}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{9/2}}\,dx, x, \frac{1}{x}\right)}{c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{3}{a^2}+\frac{8x}{a^3}}{x\sqrt{1-\frac{x}{a}}\,\left(1+\frac{x}{a}\right)^{9}}\right.}{c^3}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} - \frac{a^2\text{Subst}}{}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{x}{c^3\sqrt{1-\frac{1}{ax}}\,\left(1\right.}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)}$$

$$= -\frac{2}{ac^3\sqrt{1-\frac{1}{ax}}\,\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{11\sqrt{1-\frac{1}{ax}}}{7ac^3\left(1+\frac{1}{ax}\right)^{7/2}} + \frac{54\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)^{5/2}} + \frac{71\sqrt{1-\frac{1}{ax}}}{35ac^3\left(1+\frac{1}{ax}\right)}$$

Mathematica [A]
time = 0.09, size = 101, normalized size = 0.40

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(-176-423ax-125a^2x^2+368a^3x^3+286a^4x^4+35a^5x^5)}{35(-1+ax)(1+ax)^4}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^3),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(-176 - 423*a*x - 125*a^2*x^2 + 368*a^3*x^3 + 2
86*a^4*x^4 + 35*a^5*x^5))/(35*(-1 + a*x)*(1 + a*x)^4) - 3*Log[(1 + Sqrt[1 -
 1/(a^2*x^2)])*x])/(a*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(217) = 434.
time = 0.16, size = 714, normalized size = 2.82

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^3}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^6\sqrt{a^2}}-\frac{477\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{280a^9\left(x+\frac{1}{a}\right)^2}+\frac{2931\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2}}{560a^8\left(x+\frac{1}{a}\right)}\right.}{}$
default	$-\frac{\left(-3675\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^7x^7+3360\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^8x^7+2555((ax+1)(ax-1)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1120*(-3675*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^7*x^7+3360*ln((a^2*x+(
a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^8*x^7+2555*((a*x+1)*(a*x
-1))^(3/2)*(a^2)^(1/2)*a^5*x^5-11025*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^
6*x^6+10080*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^7
*x^6+1873*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4-3675*((a*x+1)*(a*x-1)
)^(1/2)*(a^2)^(1/2)*a^5*x^5+3360*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1
/2))/(a^2)^(1/2))*a^6*x^5-4426*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^3*x^3+
18375*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4-16800*ln((a^2*x+(a^2)^(1/
2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4-3350*(a^2)^(1/2)*((a*x+1)*
(a*x-1))^(3/2)*a^2*x^2+18375*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^3*x^3-16
800*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^4*x^3+251
1*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-3675*((a*x+1)*(a*x-1))^(1/2)*(a^2
)^(1/2)*a^2*x^2+3360*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(
```

```
1/2))*a^3*x^2+1957*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)-11025*((a*x+1)*(a*x-
1))^(1/2)*(a^2)^(1/2)*a*x+10080*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/
2))/(a^2)^(1/2))*a^2*x-3675*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)+3360*a*ln((
a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1)
)^(3/2)/(a^2)^(1/2)/(a*x+1)^3/c^3/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)^3
```

Maxima [A]
time = 0.27, size = 199, normalized size = 0.79

$$-\frac{1}{560}\,a\left(\frac{35\left(\frac{33\,(ax-1)}{ax+1}-1\right)}{a^2c^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}-a^2c^3\sqrt{\frac{ax-1}{ax+1}}}-\frac{5\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+56\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+350\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+2520\sqrt{\frac{ax-1}{ax+1}}}{a^2c^3}+\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^3}-\frac{1680\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/560*a*(35*(33*(a*x - 1)/(a*x + 1) - 1)/(a^2*c^3*((a*x - 1)/(a*x + 1))^(3
/2) - a^2*c^3*sqrt((a*x - 1)/(a*x + 1))) - (5*((a*x - 1)/(a*x + 1))^(7/2) +
 56*((a*x - 1)/(a*x + 1))^(5/2) + 350*((a*x - 1)/(a*x + 1))^(3/2) + 2520*sq
rt((a*x - 1)/(a*x + 1)))/(a^2*c^3) + 1680*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^3) - 1680*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^3))
```

Fricas [A]
time = 0.43, size = 179, normalized size = 0.71

$$-\frac{105\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-105\,(a^4x^4+2\,a^3x^3-2\,ax-1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(35\,a^5x^5+286\,a^4x^4+368\,a^3x^3-125\,a^2x^2-423\,ax-176)\sqrt{\frac{ax-1}{ax+1}}}{35\,(a^5c^3x^4+2\,a^4c^3x^3-2\,a^2c^3x-ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
+ 1) - 105*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*log(sqrt((a*x - 1)/(a*x + 1))
- 1) - (35*a^5*x^5 + 286*a^4*x^4 + 368*a^3*x^3 - 125*a^2*x^2 - 423*a*x - 17
6)*sqrt((a*x - 1)/(a*x + 1)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x -
a*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^6\left(\int\left(-\frac{x^6\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\right)dx+\int\frac{ax^7\sqrt{\frac{ax}{ax+1}-\frac{1}{ax+1}}}{a^7x^7+a^6x^6-3a^5x^5-3a^4x^4+3a^3x^3+3a^2x^2-ax-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**3,x)
```

```
[Out] a**6*(Integral(-x**6*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a**5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x) + Integral(a*x**7*sqrt(a*x/(a*x + 1) - 1/(a*x + 1))/(a**7*x**7 + a**6*x**6 - 3*a**5*x**5 - 3*a**4*x**4 + 3*a**3*x**3 + 3*a**2*x**2 - a*x - 1), x))/c**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3, x)
```

Mupad [B]
time = 0.05, size = 183, normalized size = 0.72

$$\frac{\frac{33\,(a\,x-1)}{a\,x+1}-1}{16\,a\,c^3\,\sqrt{\frac{a\,x-1}{a\,x+1}}-16\,a\,c^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}+\frac{9\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,a\,c^3}+\frac{5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{8\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{5/2}}{10\,a\,c^3}+\frac{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{7/2}}{112\,a\,c^3}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^3,x)
```

```
[Out] ((33*(a*x - 1))/(a*x + 1) - 1)/(16*a*c^3*((a*x - 1)/(a*x + 1))^(1/2) - 16*a*c^3*((a*x - 1)/(a*x + 1))^(3/2)) + (9*((a*x - 1)/(a*x + 1))^(1/2))/(2*a*c^3) + (5*((a*x - 1)/(a*x + 1))^(3/2))/(8*a*c^3) + ((a*x - 1)/(a*x + 1))^(5/2)/(10*a*c^3) + ((a*x - 1)/(a*x + 1))^(7/2)/(112*a*c^3) + (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^3)
```

3.829 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx$

Optimal. Leaf size=327

$$-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(1+\frac{1}{ax}\right)^{7/2}}+\frac{202\sqrt{1}}{105ac^4\,(1}$$

```
[Out] -4/3/a/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^(9/2)+x/c^4/(1-1/a/x)^(3/2)/(1+1/a/x)^
(9/2)-3*arctanh((1-1/a/x)^(1/2)*(1+1/a/x)^(1/2))/a/c^4-5/a/c^4/(1+1/a/x)^(9
/2)/(1-1/a/x)^(1/2)+28/9*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(9/2)+139/63*(1-1/
a/x)^(1/2)/a/c^4/(1+1/a/x)^(7/2)+202/105*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(5
/2)+719/315*(1-1/a/x)^(1/2)/a/c^4/(1+1/a/x)^(3/2)+1664/315*(1-1/a/x)^(1/2)/
a/c^4/(1+1/a/x)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6329, 105, 157, 12, 94, 214}

$$\frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}+\frac{1664\sqrt{1-\frac{1}{ax}}}{315ac^4\sqrt{\frac{1}{ax}+1}}+\frac{719\sqrt{1-\frac{1}{ax}}}{315ac^4\left(\frac{1}{ax}+1\right)^{3/2}}+\frac{202\sqrt{1-\frac{1}{ax}}}{105ac^4\left(\frac{1}{ax}+1\right)^{5/2}}+\frac{139\sqrt{1-\frac{1}{ax}}}{63ac^4\left(\frac{1}{ax}+1\right)^{7/2}}+\frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(\frac{1}{ax}+1\right)^{9/2}}-\frac{3\tanh^{-1}\left(\sqrt{1-\frac{1}{ax}}\sqrt{\frac{1}{ax}+1}\right)}{ac^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] -4/(3*a*c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - 5/(a*c^4*Sqrt[1 - 1/
(a*x)]*(1 + 1/(a*x))^(9/2)) + (28*Sqrt[1 - 1/(a*x)])/(9*a*c^4*(1 + 1/(a*x))
^(9/2)) + (139*Sqrt[1 - 1/(a*x)])/(63*a*c^4*(1 + 1/(a*x))^(7/2)) + (202*Sqr
t[1 - 1/(a*x)])/(105*a*c^4*(1 + 1/(a*x))^(5/2)) + (719*Sqrt[1 - 1/(a*x)])/(
315*a*c^4*(1 + 1/(a*x))^(3/2)) + (1664*Sqrt[1 - 1/(a*x)])/(315*a*c^4*Sqrt[1
 + 1/(a*x)]) + x/(c^4*(1 - 1/(a*x))^(3/2)*(1 + 1/(a*x))^(9/2)) - (3*ArcTanh
[Sqrt[1 - 1/(a*x)]*Sqrt[1 + 1/(a*x)]])/(a*c^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^4}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^2\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{\text{Subst}\left(\int \frac{\frac{3}{a}-\frac{7x}{a^2}}{x\left(1-\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{11/2}}\,dx, x, \frac{1}{x}\right)}{c^4}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{a\text{Subst}\left(\int \frac{-\frac{9}{a^2}+\frac{24x}{a^3}}{x\left(1-\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right.}\right.}{3c^4}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{x}{c^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9}}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{}{c^4\left(1\right.}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{13}{63ac}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{13}{63ac}$$

$$= -\frac{4}{3ac^4\left(1-\frac{1}{ax}\right)^{3/2}\left(1+\frac{1}{ax}\right)^{9/2}} - \frac{5}{ac^4\sqrt{1-\frac{1}{ax}}\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{28\sqrt{1-\frac{1}{ax}}}{9ac^4\left(1+\frac{1}{ax}\right)^{9/2}} + \frac{13}{63ac}$$

$$= -\frac{4}{\quad} - \frac{5}{\quad} + \frac{28\sqrt{1-\frac{1}{ax}}}{\quad} + \frac{13}{\quad}$$

Mathematica [A]
time = 0.12, size = 117, normalized size = 0.36

$$\frac{\frac{a\sqrt{1-\frac{1}{a^2x^2}}\,x(1664+4047ax-339a^2x^2-7399a^3x^3-4029a^4x^4+2967a^5x^5+2669a^6x^6+315a^7x^7)}{315(-1+ax)^2(1+ax)^5}-3\log\left(\left(1+\sqrt{1-\frac{1}{a^2x^2}}\right)x\right)}{ac^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^4),x]
```

```
[Out] ((a*Sqrt[1 - 1/(a^2*x^2)]*x*(1664 + 4047*a*x - 339*a^2*x^2 - 7399*a^3*x^3 -
 4029*a^4*x^4 + 2967*a^5*x^5 + 2669*a^6*x^6 + 315*a^7*x^7))/(315*(-1 + a*x)
^2*(1 + a*x)^5) - 3*Log[(1 + Sqrt[1 - 1/(a^2*x^2)])*x])/(a*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(279) = 558.
time = 0.19, size = 766, normalized size = 2.34

method	result
risch	$\frac{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}{a\,c^4}+\frac{\left(-\frac{3\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2-1}\right)}{a^8\sqrt{a^2}}-\frac{691\sqrt{a^2\left(x+\frac{1}{a}\right)^2-2a\left(x+\frac{1}{a}\right)}}{315a^{11}\left(x+\frac{1}{a}\right)^2}+\frac{113591\sqrt{a^2\left(x+\frac{1}{a}\right)^2-}}{20160a^{10}\left(x+\frac{1}{a}\right.}$
default	$-\frac{\left(-138915\sqrt{(ax+1)(ax-1)}\,\sqrt{a^2}\,a^9x^9+120960\ln\left(\frac{a^2x+\sqrt{a^2}\,\sqrt{(ax+1)(ax-1)}}{\sqrt{a^2}}\right)a^{10}x^9+98595((ax+1)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40320*(-138915*((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^9*x^9+120960*ln((a^
2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^10*x^9+98595*((a*x+
1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^7*x^7-416745*((a*x+1)*(a*x-1))^(1/2)*(a^2)^
(1/2)*a^8*x^8+362880*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(
1/2))*a^9*x^8+75113*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^6*x^6-240861*((a*
x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^5*x^5+1111320*((a*x+1)*(a*x-1))^(1/2)*(a^
2)^(1/2)*a^6*x^6-967680*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2
)^(1/2))*a^7*x^6-178863*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)*a^4*x^4+833490*
((a*x+1)*(a*x-1))^(1/2)*(a^2)^(1/2)*a^5*x^5-725760*ln((a^2*x+(a^2)^(1/2)*((
a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^6*x^5+252497*((a*x+1)*(a*x-1))^(3/2)*
(a^2)^(1/2)*a^3*x^3-833490*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)*a^4*x^4+7257
60*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2))*a^5*x^4+1823
07*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a^2*x^2-1111320*(a^2)^(1/2)*((a*x+1)
```

```
*(a*x-1))^(1/2)*a^3*x^3+967680*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2
))/(a^2)^(1/2))*a^4*x^3-101271*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(3/2)*a*x-7407
7*((a*x+1)*(a*x-1))^(3/2)*(a^2)^(1/2)+416745*((a*x+1)*(a*x-1))^(1/2)*(a^2)^
(1/2)*a*x-362880*ln((a^2*x+(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)
)*a^2*x+138915*(a^2)^(1/2)*((a*x+1)*(a*x-1))^(1/2)-120960*a*ln((a^2*x+(a^2)
^(1/2)*((a*x+1)*(a*x-1))^(1/2))/(a^2)^(1/2)))/a*((a*x-1)/(a*x+1))^(3/2)/(a^
2)^(1/2)/(a*x+1)^4/c^4/((a*x+1)*(a*x-1))^(1/2)/(a*x-1)^4
```

Maxima [A]
time = 0.26, size = 231, normalized size = 0.71

$$\frac{1}{20160}\,a\left(\frac{105\,\left(\frac{29\,(ax-1)}{ax+1}-\frac{414\,(ax-1)^2}{(ax+1)^2}+1\right)}{a^2c^4\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}-a^2c^4\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}+\frac{35\,\left(\frac{ax-1}{ax+1}\right)^{\frac{9}{2}}+450\,\left(\frac{ax-1}{ax+1}\right)^{\frac{7}{2}}+2961\,\left(\frac{ax-1}{ax+1}\right)^{\frac{5}{2}}+14700\,\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}+95445\,\sqrt{\frac{ax-1}{ax+1}}}{a^2c^4}-\frac{60480\,\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)}{a^2c^4}+\frac{60480\,\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)}{a^2c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="maxima")
```

```
[Out] 1/20160*a*(105*(29*(a*x - 1)/(a*x + 1) - 414*(a*x - 1)^2/(a*x + 1)^2 + 1)/(
a^2*c^4*((a*x - 1)/(a*x + 1))^(5/2) - a^2*c^4*((a*x - 1)/(a*x + 1))^(3/2))
+ (35*((a*x - 1)/(a*x + 1))^(9/2) + 450*((a*x - 1)/(a*x + 1))^(7/2) + 2961*
((a*x - 1)/(a*x + 1))^(5/2) + 14700*((a*x - 1)/(a*x + 1))^(3/2) + 95445*sqr
t((a*x - 1)/(a*x + 1)))/(a^2*c^4) - 60480*log(sqrt((a*x - 1)/(a*x + 1)) + 1
)/(a^2*c^4) + 60480*log(sqrt((a*x - 1)/(a*x + 1)) - 1)/(a^2*c^4))
```

Fricas [A]
time = 0.44, size = 275, normalized size = 0.84

$$-\frac{945\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}+1\right)-945\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\log\left(\sqrt{\frac{ax-1}{ax+1}}-1\right)-(315\,a^7x^7+2669\,a^6x^6+2967\,a^5x^5-4029\,a^4x^4-7399\,a^3x^3-339\,a^2x^2+4047\,ax+1664)\sqrt{\frac{ax-1}{ax+1}}}{315\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="fricas")
```

```
[Out] -1/315*(945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*log(sqrt((a*x - 1)/(a*x + 1)) + 1) - 945*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4
- 4*a^3*x^3 - a^2*x^2 + 2*a*x + 1)*log(sqrt((a*x - 1)/(a*x + 1)) - 1) - (31
5*a^7*x^7 + 2669*a^6*x^6 + 2967*a^5*x^5 - 4029*a^4*x^4 - 7399*a^3*x^3 - 339
*a^2*x^2 + 4047*a*x + 1664)*sqrt((a*x - 1)/(a*x + 1)))/(a^7*c^4*x^6 + 2*a^6
*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x + a*c^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^4,x, algorithm="giac")
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4, x)
```

Mupad [B]
time = 0.06, size = 224, normalized size = 0.69

$$\frac{303\sqrt{\frac{a\,x-1}{a\,x+1}}}{64\,a\,c^4}-\frac{\frac{29\,(a\,x-1)}{3\,(a\,x+1)}-\frac{138\,(a\,x-1)^2}{(a\,x+1)^2}+\frac{1}{3}}{64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}-64\,a\,c^4\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}+\frac{35\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{48\,a\,c^4}+\frac{47\,\left(\frac{a\,x-1}{a\,x+1}\right)^{5/2}}{320\,a\,c^4}+\frac{5\,\left(\frac{a\,x-1}{a\,x+1}\right)^{7/2}}{224\,a\,c^4}+\frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{9/2}}{576\,a\,c^4}+\frac{\mathrm{atan}\left(\sqrt{\frac{a\,x-1}{a\,x+1}}\,1{}\mathrm{i}\right)\,6{}\mathrm{i}}{a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^4,x)
```

```
[Out] (303*((a*x - 1)/(a*x + 1))^(1/2))/(64*a*c^4) - ((29*(a*x - 1))/(3*(a*x + 1)
) - (138*(a*x - 1)^2)/(a*x + 1)^2 + 1/3)/(64*a*c^4*((a*x - 1)/(a*x + 1))^(3
/2) - 64*a*c^4*((a*x - 1)/(a*x + 1))^(5/2)) + (35*((a*x - 1)/(a*x + 1))^(3/
2))/(48*a*c^4) + (47*((a*x - 1)/(a*x + 1))^(5/2))/(320*a*c^4) + (5*((a*x -
1)/(a*x + 1))^(7/2))/(224*a*c^4) + ((a*x - 1)/(a*x + 1))^(9/2)/(576*a*c^4)
+ (atan(((a*x - 1)/(a*x + 1))^(1/2)*1i)*6i)/(a*c^4)
```

3.830 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=321

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3c^3\sqrt{\ }}{a^2\sqrt{1}}$$

```
[Out] 1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/x^
2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1
-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+3/2*c
^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)^(1/2
)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c
^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 90}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqrt[c
 - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^
4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1
- 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^
2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (c^3*Sqrt[c
- c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3(1+ax)^4}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7-\frac{1}{x^7}-\frac{a}{x^6}+\frac{3a^2}{x^5}+\frac{3a^3}{x^4}-\frac{3a^4}{x^3}-\frac{3a^5}{x^2}+\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\ldots}}{a^4\sqrt{1-\ldots}}$$

Mathematica [A]
time = 0.05, size = 96, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{1}{6x^6}+\frac{a}{5x^5}-\frac{3a^2}{4x^4}-\frac{a^3}{x^3}+\frac{3a^4}{2x^2}+\frac{3a^5}{x}+a^7x+a^6\log(x)\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(1/(6*x^6) + a/(5*x^5) - (3*a^2)/(4*x^4) - a^3/x^3
 + (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x + a^6*Log[x]))/(a^7*(1 - 1/(a^2*x^2)
)^(7/2))
```

Maple [A]
time = 0.06, size = 112, normalized size = 0.35

method	result	size
default	$\frac{\left(60 a^{7} x^{7}+60 a^{6} \ln \left(x \right) x^{6}+180 a^{5} x^{5}+90 a^{4} x^{4}-60 a^{3} x^{3}-45 a^{2} x^{2}+12 a x +10\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} x}{60 \left(a x +1\right) \left(a^{2} x^{2}-1\right)^{3} \sqrt{\frac{a x -1}{a x +1}}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(60*a^7*x^7+60*a^6*ln(x)*x^6+180*a^5*x^5+90*a^4*x^4-60*a^3*x^3-45*a^2*
x^2+12*a*x+10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)/(a^2*x^2-1)^3/((a*x-
1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.38, size = 96, normalized size = 0.30

$$\frac{\left(60 \, a^{7} c^{3} x^{7} + 60 \, a^{6} c^{3} x^{6} \log\left(x\right) + 180 \, a^{5} c^{3} x^{5} + 90 \, a^{4} c^{3} x^{4} - 60 \, a^{3} c^{3} x^{3} - 45 \, a^{2} c^{3} x^{2} + 12 \, a c^{3} x + 10 \, c^{3}\right)} \sqrt{a^{2} c}}{60 \, a^{8} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/60*(60*a^7*c^3*x^7 + 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 + 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 - 45*a^2*c^3*x^2 + 12*a*c^3*x + 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.831 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=236

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-}}{a\sqrt{1}}$$

```
[Out] -1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/x
^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/
a^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c
-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 90}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -1/4*(c^2*Sqrt[c - c/(a^2*x^2)])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqr
t[c - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^
2*x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a
^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2
*x^2)] + (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^2(1+ax)^3}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5+\frac{1}{x^5}+\frac{a}{x^4}-\frac{2a^2}{x^3}-\frac{2a^3}{x^2}+\frac{a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{2c^2\sqrt{c-}}{a^2\sqrt{1-}}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 0.32

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{1}{4x^4}-\frac{a}{3x^3}+\frac{a^2}{x^2}+\frac{2a^3}{x}+a^5x+a^4\log(x)\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(5/2)*(-1/4*1/x^4 - a/(3*x^3) + a^2/x^2 + (2*a^3)/x + a^
5*x + a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A]
time = 0.04, size = 96, normalized size = 0.41

method	result	size
default	$\frac{\left(12 a^{5} x^{5}+12 \ln \left(x \right) a^{4} x^{4}+24 a^{3} x^{3}+12 a^{2} x^{2}-4 a x -3\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} x}{12 \left(a x +1\right) \left(a^{2} x^{2}-1\right)^{2} \sqrt{\frac{a x -1}{a x +1}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/12*(12*a^5*x^5+12*ln(x)*a^4*x^4+24*a^3*x^3+12*a^2*x^2-4*a*x-3)*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*x/(a*x+1)/(a^2*x^2-1)^2/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.31

$$\frac{\left(12\,a^{5}c^{2}x^{5}+12\,a^{4}c^{2}x^{4}\log\left(x\right)+24\,a^{3}c^{2}x^{3}+12\,a^{2}c^{2}x^{2}-4\,ac^{2}x-3\,c^{2}\right)\sqrt{a^{2}c}}{12\,a^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
[Out] 1/12*(12*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 + 12*a^2*c^2*
x^2 - 4*a*c^2*x - 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.832 $\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=146

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2)
/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+c*ln
(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 76}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c - c
/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/Sq
rt[1 - 1/(a^2*x^2)] + (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x
^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)(1+ax)^2}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3-\frac{1}{x^3}-\frac{a}{x^2}+\frac{a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,\log}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.44

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{3a^2}{2}+\frac{1}{2x^2}+\frac{a}{x}+a^3x+a^2\log(x)\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2),x]
[Out] ((c - c/(a^2*x^2))^(3/2)*((3*a^2)/2 + 1/(2*x^2) + a/x + a^3*x + a^2*Log[x])
)/(a^3*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.04, size = 80, normalized size = 0.55

method	result	size
default	$\frac{\left(2 a^{3} x^{3}+2 a^{2} \ln \left(x \right) x^{2}+2 a x +1\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} x}{2 \left(a x +1\right) \left(a^{2} x^{2}-1\right) \sqrt{\frac{a x -1}{a x +1}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*a^3*x^3+2*a^2*ln(x)*x^2+2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.37, size = 42, normalized size = 0.29

$$\frac{\left(2\, a^{3} c x^{3} + 2\, a^{2} c x^{2} \log\left(x\right) + 2\, a c x + c\right)\sqrt{a^{2} c}}{2\, a^{4} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*c*x^3 + 2*a^2*c*x^2*log(x) + 2*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.833 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.58

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 50, normalized size = 0.75

method	result	size

default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.34, size = 17, normalized size = 0.25

$$\frac{\sqrt{a^2c}\,(ax+\log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(x))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.834 $\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

[In] Int[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]

[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(a*Sqrt[c - c/(a^2*x^2)])

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{-1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{1}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.61

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(ax+\log(1-ax))}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x + Log[1 - a*x]))/(a*Sqrt[c - c/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 57, normalized size = 0.79

method	result	size
default	$\frac{(ax-1)(ax+\ln(ax-1))}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\,a^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2*(a*x+
ln(a*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.36, size = 24, normalized size = 0.33

$$\frac{\sqrt{a^2c}\,(ax+\log(ax-1))}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
[Out] sqrt(a^2*c)*(a*x + log(a*x - 1))/(a^2*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
[Out] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.835 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-a
*x+1)/(c-c/a^2/x^2)^(1/2)+5/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x
^2)^(1/2)-1/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^2(1+ax)}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{2a^3(-1+ax)^2}+\frac{5}{4a^3(-1+ax)}-\frac{1}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{4ac\sqrt{c-}}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.39

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax+\frac{2}{1-ax}+5\log(1-ax)-\log(1+ax)\right)}{4a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(4*a*x + 2/(1 - a*x) + 5*Log[1 - a*x] - Log[1 + a*
x]))/(4*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.04, size = 102, normalized size = 0.59

method	result	size
default	$-\frac{(ax-1)\left(-4a^{2}x^{2}+\ln(ax+1)ax-5x\ln(ax-1)a+4ax-\ln(ax+1)+5\ln(ax-1)+2\right)(ax+1)}{4\sqrt{\frac{ax-1}{ax+1}}\,a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(-4*a^2*x^2+ln(a*x+1)*a*x-5*x*ln(a*x-1
)*a+4*a*x-ln(a*x+1)+5*ln(a*x-1)+2)*(a*x+1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^
(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.41, size = 68, normalized size = 0.39

$$\frac{\left(4\,a^{2}x^{2}-4\,ax-(ax-1)\log\left(ax+1\right)+5\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{a^{2}c}}{4\,\left(a^{3}c^{2}x-a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^2*x^2 - 4*a*x - (a*x - 1)*log(a*x + 1) + 5*(a*x - 1)*log(a*x - 1)
 - 2)*sqrt(a^2*c)/(a^3*c^2*x - a^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.836 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}+\frac{23\sqrt{1-}}{16a}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a^2
/x^2)^(1/2)-1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+23/16
*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-7/16*ln(a*x+1)*(1
-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + Sqrt[1 - 1/(a^2*x^2)]/(a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(8*a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*
c^2*Sqrt[c - c/(a^2*x^2)]) - (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^3(1+ax)^2}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{4a^5(-1+ax)^3}+\frac{1}{a^5(-1+ax)^2}+\frac{23}{16a^5(-1+ax)}+\frac{1}{8a^5(1+ax)^2}-\frac{7}{16a^5(1+a}\right.}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2} + \frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)} - \frac{\sqrt{1-}}{8ac^2\sqrt{c-}}$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 0.33

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(16ax+\frac{16}{1-ax}-\frac{2}{(-1+ax)^2}-\frac{2}{1+ax}+23\log(1-ax)-7\log(1+ax)\right)}{16a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(16*a*x + 16/(1 - a*x) - 2/(-1 + a*x)^2 - 2/(1 + a
*x) + 23*Log[1 - a*x] - 7*Log[1 + a*x]))/(16*a*(c - c/(a^2*x^2))^(5/2))
```

Maple [A]
time = 0.04, size = 175, normalized size = 0.67

method	result
default	$-\frac{(ax-1)(ax+1)\left(-16a^4x^4+7\ln(ax+1)a^3x^3-23x^3\ln(ax-1)a^3+16a^3x^3-7\ln(ax+1)a^2x^2+23x^2\ln(ax-1)a^2+34a^2x^2-7\ln(ax+1)\right.}{16\sqrt{\frac{ax-1}{ax+1}}\,a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-16*a^4*x^4+7*ln(a*x+1)*a^3*
x^3-23*x^3*ln(a*x-1)*a^3+16*a^3*x^3-7*ln(a*x+1)*a^2*x^2+23*x^2*ln(a*x-1)*a^
2+34*a^2*x^2-7*ln(a*x+1)*a*x+23*x*ln(a*x-1)*a-18*a*x+7*ln(a*x+1)-23*ln(a*x-
1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.36, size = 137, normalized size = 0.52

$$\frac{\left(16\,a^4x^4-16\,a^3x^3-34\,a^2x^2+18\,ax-7\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax+1\right)+23\left(a^3x^3-a^2x^2-ax+1\right)\log\left(ax-1\right)+12\right)\sqrt{a^2c}}{16\left(a^5c^3x^3-a^4c^3x^2-a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/16*(16*a^4*x^4 - 16*a^3*x^3 - 34*a^2*x^2 + 18*a*x - 7*(a^3*x^3 - a^2*x^2
- a*x + 1)*log(a*x + 1) + 23*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 - a^4*c^3*x^2 - a^3*c^3*x + a^2*c^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.837 $\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=359

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{}{32a}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/24*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^2
/(c-c/a^2/x^2)^(1/2)+3/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2)^(
1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-5/16*(1-1
/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+51/32*ln(-a*x+1)*(1-1/a^2
/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-19/32*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a
/c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(24*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (11*Sqrt[1 - 1/(a^2*x^2)]
)/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/
(2*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (5*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^4(1+ax)^3}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{8a^7(-1+ax)^4}+\frac{11}{16a^7(-1+ax)^3}+\frac{3}{2a^7(-1+ax)^2}+\frac{51}{32a^7(-1+ax)}-\frac{}{16a^7}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{3}{2ac^3\sqrt{}}$$

Mathematica [A]
time = 0.11, size = 104, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96ax+\frac{144}{1-ax}-\frac{4}{(-1+ax)^3}-\frac{33}{(-1+ax)^2}+\frac{3}{(1+ax)^2}-\frac{30}{1+ax}+153\log(1-ax)-57\log(1+ax)\right)}{96a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*a*x + 144/(1 - a*x) - 4/(-1 + a*x)^3 - 33/(-1
+ a*x)^2 + 3/(1 + a*x)^2 - 30/(1 + a*x) + 153*Log[1 - a*x] - 57*Log[1 + a*x
]))/(96*a*(c - c/(a^2*x^2))^(7/2))
```

Maple [A]
time = 0.04, size = 247, normalized size = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)\left(-96a^6x^6+57\ln(ax+1)a^5x^5-153x^5\ln(ax-1)a^5+96a^5x^5-57\ln(ax+1)a^4x^4+153x^4\ln(ax-1)a^4+366a^4x^4-114\ln\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96/((a*x-1)/(a*x+1))^(1/2)*(a*x-1)*(a*x+1)*(-96*a^6*x^6+57*ln(a*x+1)*a^5
*x^5-153*x^5*ln(a*x-1)*a^5+96*a^5*x^5-57*ln(a*x+1)*a^4*x^4+153*x^4*ln(a*x-1
)*a^4+366*a^4*x^4-114*ln(a*x+1)*a^3*x^3+306*x^3*ln(a*x-1)*a^3-222*a^3*x^3+1
14*ln(a*x+1)*a^2*x^2-306*x^2*ln(a*x-1)*a^2-338*a^2*x^2+57*ln(a*x+1)*a*x-153
*x*ln(a*x-1)*a+122*a*x-57*ln(a*x+1)+153*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1
)/a^2/x^2)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.41, size = 205, normalized size = 0.57

$$\frac{\left(96\,a^6x^6-96\,a^5x^5-366\,a^4x^4+222\,a^3x^3+338\,a^2x^2-122\,ax-57\,(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1)\log\left(ax+1\right)+153\,(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1)\log\left(ax-1\right)-88\right)\sqrt{a^2c}}{96\left(a^7c^4x^5-a^6c^4x^4-2\,a^5c^4x^3+2\,a^4c^4x^2+a^3c^4x-a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/96*(96*a^6*x^6 - 96*a^5*x^5 - 366*a^4*x^4 + 222*a^3*x^3 + 338*a^2*x^2 - 1
22*a*x - 57*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x +
 1) + 153*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x - 1
) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 - a^6*c^4*x^4 - 2*a^5*c^4*x^3 + 2*a^4*c^4*
x^2 + a^3*c^4*x - a^2*c^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.838 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=372

$$\frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{30(1-ax)^3}-\frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{24(1-ax)^3(1+ax)^2}+\frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{80(1-ax)^3(1+ax)}-\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)}{40(1-}$$

```
[Out] 11/30*a^3*(c-c/a^2/x^2)^(7/2)*x^4/(-a*x+1)^3-57/16*a^6*(c-c/a^2/x^2)^(7/2)*
x^7/(-a*x+1)^3/(a*x+1)^3+41/24*a^5*(c-c/a^2/x^2)^(7/2)*x^6/(-a*x+1)^3/(a*x+
1)^2+57/80*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)-13/40*a^2*(c-c/a^
2/x^2)^(7/2)*x^3*(a*x+1)/(-a*x+1)^3+1/15*a*(c-c/a^2/x^2)^(7/2)*x^2*(a*x+1)/
(-a*x+1)^2+1/6*(c-c/a^2/x^2)^(7/2)*x*(a*x+1)/(-a*x+1)+2*a^6*(c-c/a^2/x^2)^(
7/2)*x^7*arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/x^2)^(
7/2)*x^7*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

Rubi [A]
time = 0.36, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(1-ax)}-\frac{13a^2x^3(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{40(1-ax)^3}+\frac{2a^6x^7\text{ArcSin}(ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{(1-ax)^{7/2}(ax+1)^{7/2}}-\frac{57a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{25a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{16(1-ax)^{7/2}(ax+1)^{7/2}}+\frac{41a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{24(1-ax)^3(ax+1)^2}+\frac{57a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{80(1-ax)^3(ax+1)}+\frac{11a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{30(1-ax)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (11*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(30*(1 - a*x)^3) - (57*a^6*(c - c/(a^2
*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (41*a^5*(c - c/(a^2*x^2))^
(7/2)*x^6)/(24*(1 - a*x)^3*(1 + a*x)^2) + (57*a^4*(c - c/(a^2*x^2))^(7/2)*x
^5)/(80*(1 - a*x)^3*(1 + a*x)) - (13*a^2*(c - c/(a^2*x^2))^(7/2)*x^3*(1 + a
*x))/(40*(1 - a*x)^3) + (a*(c - c/(a^2*x^2))^(7/2)*x^2*(1 + a*x))/(15*(1 -
a*x)^2) + ((c - c/(a^2*x^2))^(7/2)*x*(1 + a*x))/(6*(1 - a*x)) + (2*a^6*(c -
 c/(a^2*x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (2
5*a^6*(c - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16
*(1 - a*x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2} dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2} dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7\right)\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7} dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7\right)\int \frac{(1-ax)^{5/2}(1+ax)^{9/2}}{x^7} dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2} x(1+ax)}{6(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7\right)\int \frac{(1-ax)^{3/2}(1+ax)^{7/2}(2a-7a^2x)}{x^6} dx}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^2(1+ax)}{15(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2} x(1+ax)}{6(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7\right)}{30(1}$$

$$= -\frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^3(1+ax)}{40(1-ax)^3} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^2(1+ax)}{15(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/}}{6(1-}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^4}{30(1-ax)^3} - \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^3(1+ax)}{40(1-ax)^3} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^2(1}{15(1-ax)^2}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^4}{30(1-ax)^3} + \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^5}{80(1-ax)^3(1+ax)} - \frac{13a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^3(1+a}{40(1-ax)^3}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^4}{30(1-ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^6}{24(1-ax)^3(1+ax)^2} + \frac{57a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^5}{80(1-ax)^3(1+ax)} - 13$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^6}{24(1-ax)^3(1+ax)^2} + \frac{5}{8}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^6}{24(1-ax)^3(1+ax)^2} + \frac{5}{8}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^6}{24(1-ax)^3(1+ax)^2} + \frac{5}{8}$$

$$= \frac{11a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^4}{30(1-ax)^3} - \frac{57a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^7}{16(1-ax)^3(1+ax)^3} + \frac{41a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2} x^6}{24(1-ax)^3(1+ax)^2} + \frac{5}{8}$$

Mathematica [A]

time = 0.11, size = 150, normalized size = 0.40

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(-40-96ax+70a^2x^2+352a^3x^3+105a^4x^4-736a^5x^5+240a^6x^6)+375a^6x^6\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+480a^6x^6\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{240a^6x^5\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 - 96*a*x + 70*a^2*x^2 +
 352*a^3*x^3 + 105*a^4*x^4 - 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] + 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 794 vs. 2(326) = 652.
time = 0.16, size = 795, normalized size = 2.14

method	result
risch	$-\frac{(736a^7x^7-105a^6x^6-1088a^5x^5+35a^4x^4+448a^3x^3+110a^2x^2-96ax-40)c^3\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{240x^5a^6(a^2x^2-1)}+\frac{\left(\frac{a^6\sqrt{c\,(a^2x^2-1)}}{c}+\frac{2a^7\ln}{}\right.}{}$
default	$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}x\left(-2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^9cx^7+2016\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\,a^9x^5+375\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(-2016*(c*(a^2*x^2-1)/a^2)^(7/2
)*(-c/a^2)^(1/2)*a^9*c*x^7+2016*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*a^
9*x^5+375*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^8*c*x^6-480*(c*(a*x+1)
*(a*x-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^8*c*x^6+105*(c*(a^2*x^2-1)/a^2)^(9/2)*
(-c/a^2)^(1/2)*a^8*x^4+2352*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^7*c^
2*x^7-560*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^7*c^2*x^7+224*(c*(
a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*a^7*x^3-525*(c*(a^2*x^2-1)/a^2)^(5/2)*
(-c/a^2)^(1/2)*a^6*c^2*x^6-2940*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^
5*c^3*x^7+700*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^5*c^3*x^7+630*
(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*a^6*x^2+875*(c*(a^2*x^2-1)/a^2)^(3
/2)*(-c/a^2)^(1/2)*a^4*c^3*x^6+672*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)
*a^5*x+4410*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^4*x^7-1050*(c*(a
*x+1)*(a*x-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^4*x^7+280*a^4*(c*(a^2*x^2-1)/
```

a^2)^(9/2)*(-c/a^2)^(1/2)-2625*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2
*c^4*x^6-4410*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*c^(9/2)*(-c/a^2)^(1/2
)*a*x^6+1050*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*c^(9/2
)*(-c/a^2)^(1/2)*a*x^6-2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*
a^2-c)/a^2/x)*c^5*x^6)/(c*(a^2*x^2-1)/a^2)^(7/2)/(-c/a^2)^(1/2)/c

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(7/2)/(a*x - 1), x)
```

Fricas [A]
time = 0.41, size = 438, normalized size = 1.18

$$\left[-\frac{960\,a^5\sqrt{-c}\,c^3x^5\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-375\,a^5\sqrt{-c}\,c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(240\,a^6c^3x^6-736\,a^5c^3x^5+105\,a^4c^3x^4+352\,a^3c^3x^3+70\,a^2c^3x^2-96\,ac^3x-40\,c^3)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{480\,a^6x^5},\ \frac{375\,a^5c^{\frac{7}{2}}x^5\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+240\,a^5c^{\frac{7}{2}}x^5\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(240\,a^6c^3x^6-736\,a^5c^3x^5+105\,a^4c^3x^4+352\,a^3c^3x^3+70\,a^2c^3x^2-96\,ac^3x-40\,c^3)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{240\,a^6x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2))/(a^2*c*x^2 - c)) - 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2
- 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(240*a^6*c
^3*x^6 - 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x
^2 - 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240
*(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a
^2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 - 736*a^5*c^3*x^5 + 1
05*a^4*c^3*x^4 + 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 - 96*a*c^3*x - 40*c^3)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] Result contains complex when optimal does not.
time = 25.01, size = 1059, normalized size = 2.85

$$c^3\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}+\frac{\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log(a^2x^2)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c^3\left(\begin{cases}-\frac{a\sqrt{c}\,x}{\sqrt{a^2x^2-1}}+\sqrt{c}\,\mathrm{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}\,x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\,\mathrm{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}-\frac{c^3\left(\begin{cases}\frac{ia\sqrt{c}\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}-\frac{4c^3\left(\begin{cases}0 & \text{for } c=0\\ \frac{a^2\left(c-\frac{c}{a^2x^2}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^3}-\frac{c^3\left(\begin{cases}\frac{ia^3\sqrt{c}\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{ia^2\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{3i\sqrt{c}}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{4a^2x^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^3\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{8}+\frac{a^2\sqrt{c}}{8x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c}}{8x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c}}{4a^2x^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^4}+\frac{2c^3\left(\begin{cases}\frac{2a^3\sqrt{c}\sqrt{a^2x^2-1}}{15x}+\frac{a\sqrt{c}\sqrt{a^2x^2-1}}{15x^3}-\frac{\sqrt{c}\sqrt{a^2x^2-1}}{5ax^5} & \text{for } |a^2x^2|>1\\ \frac{2ia^3\sqrt{c}\sqrt{-a^2x^2+1}}{15x}+\frac{ia\sqrt{c}\sqrt{-a^2x^2+1}}{15x^3}-\frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{5ax^5} & \text{otherwise}\end{cases}\right)}{a^5}+\frac{c^3\left(\begin{cases}\frac{ia^5\sqrt{c}\,\mathrm{acosh}\left(\frac{1}{ax}\right)}{16}-\frac{ia^4\sqrt{c}}{16x\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{ia^2\sqrt{c}}{48x^3\sqrt{-1+\frac{1}{a^2x^2}}}+\frac{5i\sqrt{c}}{24x^5\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{6a^2x^7\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a^5\sqrt{c}\,\mathrm{asin}\left(\frac{1}{ax}\right)}{16}+\frac{a^4\sqrt{c}}{16x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{a^2\sqrt{c}}{48x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5\sqrt{c}}{24x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c}}{6a^2x^7\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise}\end{cases}\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
```

```
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**2 - 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 + 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

Giac [A]
time = 24.15, size = 561, normalized size = 1.51

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}+\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-c}\,c^3\mathrm{sgn}(x)}{a^2}+\frac{105\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{11}c^4|a|\mathrm{sgn}(x)+1440\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{10}ac^{\frac{9}{2}}\mathrm{sgn}(x)+595\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{9}c^5|a|\mathrm{sgn}(x)+4320\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{8}ac^{\frac{11}{2}}\mathrm{sgn}(x)-150\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{7}c^6|a|\mathrm{sgn}(x)+7360\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{6}ac^{\frac{13}{2}}\mathrm{sgn}(x)+150\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{5}c^7|a|\mathrm{sgn}(x)+6720\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{4}ac^{\frac{15}{2}}\mathrm{sgn}(x)-595\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{3}c^8|a|\mathrm{sgn}(x)+2976\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{2}ac^{\frac{17}{2}}\mathrm{sgn}(x)-105\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)c^9|a|\mathrm{sgn}(x)+736\,ac^{\frac{19}{2}}\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^6a^2|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 + 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) + 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) + 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) + 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) + 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
```

```
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) + 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) + 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x + 1))/(a*x - 1), x)
```

3.839 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=294

$$-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2}+\frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)}+\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/}}{4(1-}$$

```
[Out] -5/8*a^2*(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)^2+25/8*a^4*(c-c/a^2/x^2)^(5/2)*x^
5/(-a*x+1)^2/(a*x+1)^2-17/12*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)
+1/6*a*(c-c/a^2/x^2)^(5/2)*x^2*(a*x+1)/(-a*x+1)^2+1/4*(c-c/a^2/x^2)^(5/2)*x
*(a*x+1)/(-a*x+1)-2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/
(a*x+1)^(5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1
)^(1/2))/(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A]
time = 0.32, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\frac{ax^2(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(1-ax)^2}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(1-ax)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2}-\frac{2a^4x^5\text{ArcSin}(ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^{5/2}(ax+1)^{5/2}}+\frac{25a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{9a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{8(1-ax)^{5/2}(ax+1)^{5/2}}-\frac{17a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{12(1-ax)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (-5*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(8*(1 - a*x)^2) + (25*a^4*(c - c/(a^2*
x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (17*a^3*(c - c/(a^2*x^2))^(5
/2)*x^4)/(12*(1 - a*x)^2*(1 + a*x)) + (a*(c - c/(a^2*x^2))^(5/2)*x^2*(1 + a
*x))/(6*(1 - a*x)^2) + ((c - c/(a^2*x^2))^(5/2)*x*(1 + a*x))/(4*(1 - a*x))
- (2*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x
)^(5/2)) - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1
+ a*x]])/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{3/2}(1+ax)^{7/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{\sqrt{1-ax}\,(1+ax)^{5/2}(2a-5a^2x)}{x^4}}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+ax)}{4(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)}{12(1-a}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1+a}{4(1-ax)}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2(1+ax)}{6(1-ax)^2}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a(}{}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a(}{}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a(}{}$$

$$= -\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{8(1-ax)^2} + \frac{25a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2} - \frac{17a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{12(1-ax)^2(1+ax)} + \frac{a(}{}$$

Mathematica [A]
time = 0.09, size = 134, normalized size = 0.46

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(6+16ax-3a^2x^2-64a^3x^3+24a^4x^4)+27a^4x^4\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+48a^4x^4\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^4x^3\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x - 3*a^2*x^2 - 64*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 48*a^4*x^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(256) = 512$.

time = 0.14, size = 625, normalized size = 2.13

method	result
risch	$-\frac{(64a^5x^5+3a^4x^4-80a^3x^3-9a^2x^2+16ax+6)c^2\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3a^4(a^2x^2-1)}+\frac{\left(\frac{a^4\sqrt{c\left(a^2x^2-1\right)}}{c}+\frac{2a^5\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-c}\right.}{\sqrt{a^2c}}\right.}{}$
default	$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}x\left(-80\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^7cx^5+80\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\,a^7x^3+27\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^6cx^4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^7*c*x^5+80*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^7*x^3+27*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^6*c*x^4+48*(-c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^6*c*x^4-75*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^6*x^2+100*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^5*c^2*x^5+60*(-c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^5*c^2*x^5-80*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^5*x-45*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^4*c^2*x^4-150*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^3*x^5-90*(-c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*c^3*x^5-30*a^4*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)+135*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2*c^3*x^4+150*(-c/a^2)^(1/2)*c^(7/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*a*x^4+90*(-c/a^2)^(1/2)*c^(7/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a*x^4+135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*c^4*x^4)/(c*(a^2*x^2-1)/a^2)^(5/2)/(-c/a^2)^(1/2)/c

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(5/2)/(a*x - 1), x)
```

Fricas [A]
time = 0.41, size = 394, normalized size = 1.34

$$\left[-\frac{96\,a^{3}\sqrt{-c}\,c^{2}x^{3}\arctan\left(\frac{a^{2}\sqrt{-c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)-27\,a^{3}\sqrt{-c}\,c^{2}x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}\,x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)-2\,(24\,a^{4}c^{2}x^{4}-64\,a^{3}c^{2}x^{3}-3\,a^{2}c^{2}x^{2}+16\,ac^{2}x+6\,c^{2})\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{48\,a^{4}x^{3}},\ \frac{27\,a^{3}c^{\frac{5}{2}}x^{3}\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+24\,a^{3}c^{\frac{5}{2}}x^{3}\log\left(2\,a^{2}cx^{2}+2\,a^{2}\sqrt{c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)+(24\,a^{4}c^{2}x^{4}-64\,a^{3}c^{2}x^{3}-3\,a^{2}c^{2}x^{2}+16\,ac^{2}x+6\,c^{2})\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{24\,a^{4}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2))/(a^2*c*x^2 - c)) - 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2
*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(24*a^4*c^2*x
^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2
*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c
^2*x^4 - 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 + 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x
^2 - c)/(a^2*x^2)))/(a^4*x^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.40, size = 500, normalized size = 1.70

$$c^{2}\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^{2}x^{2}-1}}{a}-\frac{i\sqrt{c}\log\left(ax\right)}{a}+\frac{i\sqrt{c}\log\left(a^{2}x^{2}\right)}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for}\ \left|a^{2}x^{2}\right|>1\\\frac{i\sqrt{c}\sqrt{-a^{2}x^{2}+1}}{a}+\frac{i\sqrt{c}\log\left(a^{2}x^{2}\right)}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^{2}x^{2}+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)+\frac{2c^{2}\left(\begin{cases}-\frac{a\sqrt{c}\,x}{\sqrt{a^{2}x^{2}-1}}+\sqrt{c}\operatorname{acosh}\left(ax\right)+\frac{\sqrt{c}}{ax\sqrt{a^{2}x^{2}-1}} & \text{for}\ \left|a^{2}x^{2}\right|>1\\\frac{ia\sqrt{c}\,x}{\sqrt{-a^{2}x^{2}+1}}-i\sqrt{c}\operatorname{asin}\left(ax\right)-\frac{i\sqrt{c}}{ax\sqrt{-a^{2}x^{2}+1}} & \text{otherwise}\end{cases}\right)}{a}-\frac{2c^{2}\left(\begin{cases}0 & \text{for}\ c=0\\\frac{a^{2}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^{3}}-\frac{c^{2}\left(\begin{cases}\frac{ia^{3}\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{ia^{2}\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^{2}x^{2}}}}+\frac{3i\sqrt{c}}{8x^{3}\sqrt{-1+\frac{1}{a^{2}x^{2}}}}-\frac{i\sqrt{c}}{4a^{2}x^{5}\sqrt{-1+\frac{1}{a^{2}x^{2}}}} & \text{for}\ \frac{1}{\left|a^{2}x^{2}\right|}>1\\-\frac{a^{3}\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{a^{2}\sqrt{c}}{8x\sqrt{1-\frac{1}{a^{2}x^{2}}}}-\frac{3\sqrt{c}}{8x^{3}\sqrt{1-\frac{1}{a^{2}x^{2}}}}+\frac{\sqrt{c}}{4a^{2}x^{5}\sqrt{1-\frac{1}{a^{2}x^{2}}}} & \text{otherwise}\end{cases}\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c**2*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - 2*c**2*Piecewis
e((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2
*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 +
 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c
)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqr
t(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(
c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2
*x**2))), True))/a**4
```

Giac [A]
time = 1.57, size = 416, normalized size = 1.41

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}\left(x\right)}{a^2}+\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}\left(x\right)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}\,c^2\mathrm{sgn}\left(x\right)}{a^2}-\frac{3\,\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7c^3|a|\mathrm{sgn}\left(x\right)-96\,\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^6ac^{\frac{7}{2}}\mathrm{sgn}\left(x\right)-21\,\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5c^4|a|\mathrm{sgn}\left(x\right)-192\,\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^4ac^{\frac{9}{2}}\mathrm{sgn}\left(x\right)+21\,\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3c^5|a|\mathrm{sgn}\left(x\right)-160\,\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2ac^{\frac{11}{2}}\mathrm{sgn}\left(x\right)-3\,\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)c^6|a|\mathrm{sgn}\left(x\right)-64\,ac^{\frac{13}{2}}\mathrm{sgn}\left(x\right)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^4a^2|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 + 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) - 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) - 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) - 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) - 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x+1\right)}{a\,x-1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x + 1))/(a*x - 1), x)
```

3.840 $\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=213

$$\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)}+\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\text{ArcSin}(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}}+\frac{a^2\left(c-\frac{c}{a^2x^2}\right.}{}$$

```
[Out] a*(c-c/a^2/x^2)^(3/2)*x^2/(-a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1)
/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(a*x+1)/(-a*x+1)+2*a^2*(c-c/a^2/x^2)^(3/
2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3/2)
*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.458$, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$\frac{2a^2x^3\text{ArcSin}(ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{(1-ax)^{3/2}(ax+1)^{3/2}}+\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{1-ax}+\frac{x(ax+1)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2(1-ax)^{3/2}(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 - a*x) - (5*a^2*(c - c/(a^2*x^2))^(3/2)*
x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 + a*x))/(2*(1
- a*x)) + (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/2)*
(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x]*S
qrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx = -\int e^{2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{e^{2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1-ax}\,(1+ax)^{5/2}}{x^3}\,dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{(1+ax)^{3/2}(2a-3a^2x)}{x^2\sqrt{1-ax}}\,dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int \frac{\sqrt{1+}}{x}}{2(1-ax)^{3/2}(1+}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{((c}{}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} - \frac{(a^2}{}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{(a^3}{}$$

$$= \frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1-ax} - \frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1+ax)}{2(1-ax)} + \frac{2a^2}{}$$

Mathematica [A]
time = 0.07, size = 115, normalized size = 0.54

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(-1-4ax+2a^2x^2)+a^2x^2\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+4a^2x^2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{2a^2x\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 - 4*a*x + 2*a^2*x^2) + a^2
*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] + 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]
))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(185) = 370.
time = 0.12, size = 454, normalized size = 2.13

method	result
risch	$\frac{(2a^4x^4-4a^3x^3-3a^2x^2+4ax+1)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x\,a^2(a^2x^2-1)}+\frac{\left(\frac{2a^3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right.}{2\sqrt{-c}}\right.}{a^2(a^2x^2-1)}$
default	$-\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x\left(-12\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^5cx^3+12\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^5x+\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^4cx^2-4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(-12*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c
/a^2)^(1/2)*a^5*c*x^3+12*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^5*x+(c*
(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^4*c*x^2-4*(-c/a^2)^(1/2)*(c*(a*x+1)
*(a*x-1)/a^2)^(3/2)*a^4*c*x^2+3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2
)+18*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^2*x^3-6*(-c/a^2)^(1/2)*
(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*c^2*x^3-3*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a
^2)^(1/2)*a^2*c^2*x^2-18*c^(5/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(-
c/a^2)^(1/2)*a*x^2+6*c^(5/2)*(-c/a^2)^(1/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/
2)*c^(1/2)+c*x)/c^(1/2))*a*x^2-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(
1/2)*a^2-c)/a^2/x)*c^3*x^2)/(c*(a^2*x^2-1)/a^2)^(3/2)/(-c/a^2)^(1/2)/c
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*(c - c/(a^2*x^2))^(3/2)/(a*x - 1), x)
```

Fricas [A]

time = 0.41, size = 317, normalized size = 1.49

$$\left[-\frac{8\,a\sqrt{-c}\,cx\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-a\sqrt{-c}\,cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(2\,a^2cx^2-4\,acx-c)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,a^2x},\ \frac{ac^{\frac{3}{2}}x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+2\,ac^{\frac{3}{2}}x\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(2\,a^2cx^2-4\,acx-c)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x
^2))/(a^2*c*x^2 - c)) - a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(2*a^2*c*x^2 - 4*a*c*x - c)*sq
rt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x
*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2
*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*
x^2 - 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 8.56, size = 376, normalized size = 1.77

$$c\left(\begin{cases} \frac{\sqrt{c}\sqrt{a^2x^2-1}}{a} - \frac{i\sqrt{c}\log(ax)}{a} + \frac{i\sqrt{c}\log(a^2x^2)}{2a} + \frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for } |a^2x^2| > 1 \\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a} + \frac{i\sqrt{c}\log(a^2x^2)}{2a} - \frac{i\sqrt{c}\log\left(\sqrt{-a^2x^2+1}+1\right)}{a} & \text{otherwise} \end{cases}\right) + \frac{2c\left(\begin{cases} -\frac{a\sqrt{c}x}{\sqrt{a^2x^2-1}} + \sqrt{c}\operatorname{acosh}(ax) + \frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2| > 1 \\ \frac{ia\sqrt{c}x}{\sqrt{-a^2x^2+1}} - i\sqrt{c}\operatorname{asin}(ax) - \frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise} \end{cases}\right)}{a} + \frac{c\left(\begin{cases} \frac{ia\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{2} + \frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ -\frac{a\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{2} - \frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) + 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

Giac [A]
time = 0.52, size = 266, normalized size = 1.25

$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2} + \frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|} - \frac{\sqrt{a^2cx^2-c}\,c\operatorname{sgn}(x)}{a^2} - \frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3c^2|a|\operatorname{sgn}(x) - 4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2ac^{\frac{5}{2}}\operatorname{sgn}(x) - \left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)c^3|a|\operatorname{sgn}(x) - 4\,ac^{\frac{7}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^2a^2|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 + 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) - 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) - 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1),x)
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x + 1))/(a*x - 1), x)
```

3.841 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$-\frac{2x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\right.}{}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-a\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.69

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}-\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A]
time = 0.10, size = 197, normalized size = 1.70

method	result

default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, x \left(2\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\, a^2 \sqrt{-\frac{c}{a^2}} + 2\sqrt{c}\, \ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\, \sqrt{c} + cx}{\sqrt{c}}\right) a \sqrt{-\frac{c}{a^2}} - \sqrt{-\frac{c}{a^2}}\, \sqrt{\frac{c(a^2x}{a}}\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\, a^2 \sqrt{-\frac{c}{a^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)+2*c^(1/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Fricas [A]
time = 0.37, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - 4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a},\ \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - \sqrt{c}\,\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.842 $\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=111

$$-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}+\frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\mathrm{ArcSin}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

```
[Out] -2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2)^(1/2)-(a*x+1)^2/a^2/x/(c-c/a^2/x^2)
^(1/2)+2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6294, 6264, 79, 52, 41, 222}

$$\frac{2\sqrt{1-ax}\,\sqrt{ax+1}\,\mathrm{ArcSin}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{(ax+1)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(1-ax)(ax+1)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (-2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[c - c/(a^2*x^2)]*x) - (1 + a*x)^2/(a^2*S
qrt[c - c/(a^2*x^2)]*x) + (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^2*
Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

$$= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{e^{2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{x\sqrt{1+ax}}{(1-ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{(1+ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} + \frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.61

$$\frac{-3-2ax+a^2x^2+2\sqrt{-1+a^2x^2}\,\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (-3 - 2*a*x + a^2*x^2 + 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A]

time = 0.12, size = 177, normalized size = 1.59

method	result
risch	$\frac{a^2x^2-1}{a^2\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x} + \frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a\sqrt{a^2c}} - \frac{2\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{a^3c\left(x-\frac{1}{a}\right)}\right)\sqrt{c\left(a^2x^2-1\right)}}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\left(\sqrt{c}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2x+2\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)acx-2a\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,\sqrt{c}-\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\,c^{\frac{3}{2}}a(ax-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2-1)/a^2)^(1/2)*(c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*x+2*ln(c^(
1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*a*c*x-2*a*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c
^(1/2)-(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)-2*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^
2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x-1)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*sqrt(c - c/(a^2*x^2))), x)
```

Fricas [A]

time = 0.38, size = 216, normalized size = 1.95

$$\left[\frac{(ax-1)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(a^2x^2-3\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac},\ -\frac{2\,(ax-1)\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(a^2x^2-3\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx-ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) + (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
```

```
*c*x - a*c), -(2*(a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (a^2*x^2 - 3*a*x)*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)))/(a^2*c*x - a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral((a*x + 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(1/2)*(a*x - 1)), x)
```

3.843 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\mathrm{ArcSin}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -1/3*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-2*a*x+5)*(-a*x+1)*(a*x+1)^2/
a^4/(c-c/a^2/x^2)^(3/2)/x^3-2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A]
time = 0.27, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6302, 6294, 6264, 100, 148, 41, 222}

$$-\frac{(ax+1)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}-\frac{2(1-ax)^{3/2}(ax+1)^{3/2}\mathrm{ArcSin}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(5-2ax)(1-ax)(ax+1)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -1/3*(1 + a*x)^2/(a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(5 - 2*a*x)*(1 - a*x)
*(1 + a*x)^2)/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) - (2*(1 - a*x)^(3/2)*(1 +
 a*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx \\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{(1-ax)^{5/2}\sqrt{1+ax}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2+4ax)}{(1-ax)^{3/2}\sqrt{1+ax}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \cdots}{a^3\left(c-\frac{c}{a^2x^2}\right)\cdots} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \cdots}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\cdots} \\
&= -\frac{(1+ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(5-2ax)(1-ax)(1+ax)^2}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}-\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}\cdots}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 0.77

$$
\frac{10-4ax-11a^2x^2+3a^3x^3+6(-1+ax)\sqrt{-1+a^2x^2}\,\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (10 - 4*a*x - 11*a^2*x^2 + 3*a^3*x^3 + 6*(-1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(109) = 218.
time = 0.16, size = 326, normalized size = 2.65

method	result

risch	$\frac{a^2x^2-1}{a^2cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^3\sqrt{a^2c}}-\frac{8\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{3a^5c\left(x-\frac{1}{a}\right)}-\frac{\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+}}{3a^6c\left(x-\right.}\right.}{cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$
default	$\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,a^3x^3-15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}+4c^{\frac{3}{2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2x^2+6\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{\frac{c(a\right.}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*c^(3/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*x^3-15*x^2*a^2*c^(3/2)*(c*
(a*x+1)*(a*x-1)/a^2)^(1/2)+4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*x^2+6*ln
(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*(c*(a^2
*x^2-1)/a^2)^(1/2)*a^2*c*x-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a*x-6*ln(c^(
1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*(c*(a^2*x^2
-1)/a^2)^(1/2)*a*c+12*c^(3/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)-2*(c*(a^2*x^2-1
)/a^2)^(1/2)*c^(3/2))*(a*x+1)/(c*(a*x+1)*(a*x-1)/a^2)^(1/2)/x^3/(c*(a^2*x^2
-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(3/2)), x)
```

Fricas [A]
time = 0.37, size = 280, normalized size = 2.28

$$\left[\frac{3\,(a^2x^2-2\,ax+1)\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(3\,a^3x^3-14\,a^2x^2+10\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)},-\frac{6\,(a^2x^2-2\,ax+1)\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(3\,a^3x^3-14\,a^2x^2+10\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\,(a^3c^2x^2-2\,a^2c^2x+ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*x^2 - 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*s
qrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqr
```

```
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2), -1/3*(6*
(a^2*x^2 - 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)
/(a^2*x^2))/(a^2*c*x^2 - c)) - (3*a^3*x^3 - 14*a^2*x^2 + 10*a*x)*sqrt((a^2*
c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 - 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x+1}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,(a\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(3/2)*(a*x - 1)), x)
```

3.844 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}-\frac{2(1-ax)^3(1+ax)^2(28+43ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}+\frac{2(1-$$

```
[Out] -1/5*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x+2/3*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^
2/x^2)^(5/2)/x^2-58/15*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2/1
5*(-a*x+1)^3*(a*x+1)^2*(43*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5+2*(-a*x+1)^(
5/2)*(a*x+1)^(5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A]
time = 0.30, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6294, 6264, 100, 155, 148, 41, 222}

$$-\frac{(ax+1)^2}{5a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\text{ArcSin}(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(43ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{58(ax+1)^2(1-ax)^2}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)^2(1-ax)}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] -1/5*(1 + a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x) + (2*(1 - a*x)*(1 + a*x)^2
)/(3*a^3*(c - c/(a^2*x^2))^(5/2)*x^2) - (58*(1 - a*x)^2*(1 + a*x)^2)/(15*a^
4*(c - c/(a^2*x^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 43*a*x))/
(15*a^6*(c - c/(a^2*x^2))^(5/2)*x^5) + (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*A
rcSin[a*x])/(a^6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx \\
&= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{7/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+6ax)}{(1-ax)^{5/2}(1+ax)^{3/2}}\,dx}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}+\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2(-30a-28a^2\cdots}{(1-ax)^{3/2}(1+ax\cdots}}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5} \\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}+\frac{\left((1-ax)^{5/2}(1\cdots}{15\cdots} \\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}-\frac{2(1-ax)^3(1+\cdots}{15a^6\left(c-\cdots\right.} \\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}-\frac{2(1-ax)^3(1+\cdots}{15a^6\left(c-\cdots\right.} \\
&= -\frac{(1+ax)^2}{5a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}+\frac{2(1-ax)(1+ax)^2}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}-\frac{58(1-ax)^2(1+ax)^2}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}-\frac{2(1-ax)^3(1+\cdots}{15a^6\left(c-\cdots\right.}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 105, normalized size = 0.52

$$\frac{-56+82ax+32a^2x^2-76a^3x^3+15a^4x^4+30(-1+ax)^2\sqrt{-1+a^2x^2}\,\log\left(ax+\sqrt{-1+a^2x^2}\right)}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (-56 + 82*a*x + 32*a^2*x^2 - 76*a^3*x^3 + 15*a^4*x^4 + 30*(-1 + a*x)^2*Sqrt
[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x
^2)]*x*(-1 + a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(181) = 362$.
time = 0.15, size = 462, normalized size = 2.28

method	result
risch	$\frac{a^2x^2-1}{a^2c^2x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^5\sqrt{a^2c}}-\frac{383\sqrt{a^2c\left(x-\frac{1}{a}\right)^2+2\left(x-\frac{1}{a}\right)ac}}{120a^7c\left(x-\frac{1}{a}\right)}+\frac{\sqrt{a^2c\left(x+\frac{1}{a}\right.}}{8a}\right.}{}$
default	$-\frac{\left(-15c^{\frac{5}{2}}\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{3}{2}}a^5x^5+16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^4x^4+45x^4c^{\frac{5}{2}}a^4\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{3}{2}}-16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^3x^3+60c^{\frac{5}{2}}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-15*c^(5/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^5*x^5+16*(c*(a^2*x^2-1)/
a^2)^(3/2)*c^(5/2)*a^4*x^4+45*x^4*c^(5/2)*a^4*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)
-16*(c*(a^2*x^2-1)/a^2)^(3/2)*c^(5/2)*a^3*x^3+60*c^(5/2)*(c*(a*x+1)*(a*x-1)
/a^2)^(3/2)*a^3*x^3-30*(c*(a^2*x^2-1)/a^2)^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1
)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^4*c*x-24*(c*(a^2*x^2-1)/a^2)^
(3/2)*c^(5/2)*a^2*x^2-90*c^(5/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^2*x^2+30*(
c*(a^2*x^2-1)/a^2)^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)
*(a*x-1)/a^2)^(3/2)*a^3*c+24*(c*(a^2*x^2-1)/a^2)^(3/2)*c^(5/2)*a*x-50*c^(5/
2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a*x+6*(c*(a^2*x^2-1)/a^2)^(3/2)*c^(5/2)+50
*c^(5/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2))*(a*x+1)/(c*(a*x+1)*(a*x-1)/a^2)^(3/
2)/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [A]
time = 0.36, size = 352, normalized size = 1.73

$$\left[\frac{15\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\sqrt{c}\,\log\left(2\,a^{2}cx^{2}+2\,a^{2}\sqrt{c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)+\left(15\,a^{5}x^{5}-76\,a^{4}x^{4}+32\,a^{3}x^{3}+82\,a^{2}x^{2}-56\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{15\left(a^{5}c^{3}x^{4}-2\,a^{4}c^{3}x^{3}+2\,a^{2}c^{3}x-ac^{3}\right)},\ -\frac{30\left(a^{4}x^{4}-2\,a^{3}x^{3}+2\,ax-1\right)\sqrt{-c}\,\arctan\left(\frac{a^{2}\sqrt{-c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)-\left(15\,a^{5}x^{5}-76\,a^{4}x^{4}+32\,a^{3}x^{3}+82\,a^{2}x^{2}-56\,ax\right)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{15\left(a^{5}c^{3}x^{4}-2\,a^{4}c^{3}x^{3}+2\,a^{2}c^{3}x-ac^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 - 2*a^3*x^3 + 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 - 76*a^4*x^
4 + 32*a^3*x^3 + 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 - 2*a^4*c^3*x^3 + 2*a^2*c^3*x - a*c^3), -1/15*(30*(a^4*x^4 - 2*a^3
*x^3 + 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^
2*x^2))/(a^2*c*x^2 - c)) - (15*a^5*x^5 - 76*a^4*x^4 + 32*a^3*x^3 + 82*a^2*x
^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 - 2*a^4*c^3*x^3
+ 2*a^2*c^3*x - a*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)),x)
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(5/2)*(a*x - 1)), x)
```

3.845 $\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(1+ax)^2}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}+\frac{142(1-ax)^4(1}{35a^6\left(c-\frac{c}{a^2x^2}\right.}$$

```
[Out] -1/7*(a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+2/5*(-a*x+1)*(a*x+1)^2/a^3/(c-c/a^
2/x^2)^(7/2)/x^2-124/105*(-a*x+1)^2*(a*x+1)^2/a^4/(c-c/a^2/x^2)^(7/2)/x^3+7
82/105*(-a*x+1)^3*(a*x+1)^2/a^5/(c-c/a^2/x^2)^(7/2)/x^4+142/35*(-a*x+1)^4*(
a*x+1)^2/a^6/(c-c/a^2/x^2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(107*a*x+72)
/a^8/(c-c/a^2/x^2)^(7/2)/x^7-2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8
/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A]
time = 0.33, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6294, 6264, 100, 155, 148, 41, 222}

$$-\frac{(ax+1)^2}{7a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{2(ax+1)^{7/2}(1-ax)^{7/2}\text{ArcSin}(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(107ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{142(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{782(ax+1)^2(1-ax)^3}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}-\frac{124(ax+1)^2(1-ax)^2}{105a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^2(1-ax)}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -1/7*(1 + a*x)^2/(a^2*(c - c/(a^2*x^2))^(7/2)*x) + (2*(1 - a*x)*(1 + a*x)^2
)/(5*a^3*(c - c/(a^2*x^2))^(7/2)*x^2) - (124*(1 - a*x)^2*(1 + a*x)^2)/(105*
a^4*(c - c/(a^2*x^2))^(7/2)*x^3) + (782*(1 - a*x)^3*(1 + a*x)^2)/(105*a^5*(
c - c/(a^2*x^2))^(7/2)*x^4) + (142*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/
(a^2*x^2))^(7/2)*x^5) + (2*(1 - a*x)^4*(1 + a*x)^3*(72 + 107*a*x))/(35*a^8*
(c - c/(a^2*x^2))^(7/2)*x^7) - (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*
x])/(a^8*(c - c/(a^2*x^2))^(7/2)*x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx &= -\int \frac{e^{2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx\\
&= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{9/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+8ax)}{(1-ax)^{7/2}(1+ax)^{5/2}}\,dx}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}+\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4(-70a-54a^2 \cdots}{(1-ax)^{5/2}(1+ax) \cdots}}{35a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{\left((1-ax)^{7/2}(1 \cdots\right.}{10 \cdots}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(\cdots}{105a^5\left(c-\frac{c}{a^2 \cdots}\right.}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(\cdots}{105a^5\left(c-\frac{c}{a^2 \cdots}\right.}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(\cdots}{105a^5\left(c-\frac{c}{a^2 \cdots}\right.}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(\cdots}{105a^5\left(c-\frac{c}{a^2 \cdots}\right.}\\
&= -\frac{(1+ax)^2}{7a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{2(1-ax)(1+ax)^2}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}-\frac{124(1-ax)^2(1+ax)^2}{105a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{782(1-ax)^3(\cdots}{105a^5\left(c-\frac{c}{a^2 \cdots}\right.}
\end{aligned}
$$

Mathematica [A]

time = 0.08, size = 133, normalized size = 0.47

$$\frac{432-654ax-636a^2x^2+1226a^3x^3+74a^4x^4-562a^5x^5+105a^6x^6+210(-1+ax)^3(1+ax)\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{105a^2c^3\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)^3(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (432 - 654*a*x - 636*a^2*x^2 + 1226*a^3*x^3 + 74*a^4*x^4 - 562*a^5*x^5 + 10
5*a^6*x^6 + 210*(-1 + a*x)^3*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*c^3*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)^3*(1 + a*x))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(253) = 506.

time = 0.18, size = 572, normalized size = 2.02

method	result
risch	$\frac{a^2x^2-1}{a^2c^3x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^7\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{48a^{10}c\left(x+\frac{1}{a}\right)^2}+\frac{7\sqrt{a^2c\left(x+\frac{1}{a}\right)^2}}{24a^9c}\right.}{}$
default	$\frac{\left(105c^{\frac{7}{2}}\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{5}{2}}a^7x^7+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^6x^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{5}{2}}-96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^5x^5-392c^{\frac{7}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(105*c^(7/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^7*x^7+96*(c*(a^2*x^2-1)/
a^2)^(5/2)*c^(7/2)*a^6*x^6-553*x^6*c^(7/2)*a^6*(c*(a*x+1)*(a*x-1)/a^2)^(5/2
)-96*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a^5*x^5-392*c^(7/2)*(c*(a*x+1)*(a*x-
1)/a^2)^(5/2)*a^5*x^5-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a^4*x^4+1540*c^
(7/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^4*x^4+210*(c*(a^2*x^2-1)/a^2)^(5/2)*l
n(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^6*c*
x+240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a^3*x^3+350*c^(7/2)*(c*(a*x+1)*(a*x
-1)/a^2)^(5/2)*a^3*x^3-210*(c*(a^2*x^2-1)/a^2)^(5/2)*ln(c^(1/2)*x+(c*(a^2*x
^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^5*c+180*(c*(a^2*x^2-1)/a^
2)^(5/2)*c^(7/2)*a^2*x^2-1470*c^(7/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^2*x^2
-180*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a*x-42*c^(7/2)*(c*(a*x+1)*(a*x-1)/a^
2)^(5/2)*a*x-30*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)+462*c^(7/2)*(c*(a*x+1)*(a
*x-1)/a^2)^(5/2))*(a*x+1)/(c*(a*x+1)*(a*x-1)/a^2)^(5/2)/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)/a^8/c^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)/((a*x - 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [A]
time = 0.44, size = 496, normalized size = 1.75

$$\left[\frac{105\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(105a^7x^7-562a^6x^6+74a^5x^5+1226a^4x^4-636a^3x^3-654a^2x^2+432ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{105\left(a^7c^4x^6-2a^6c^4x^5-a^5c^4x^4+4a^4c^4x^3-a^3c^4x^2-2a^2c^4x+ac^4\right)},\ -\frac{210\left(a^6x^6-2a^5x^5-a^4x^4+4a^3x^3-a^2x^2-2ax+1\right)\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-\left(105a^7x^7-562a^6x^6+74a^5x^5+1226a^4x^4-636a^3x^3-654a^2x^2+432ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{105\left(a^7c^4x^6-2a^6c^4x^5-a^5c^4x^4+4a^4c^4x^3-a^3c^4x^2-2a^2c^4x+ac^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2 - 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^4 - 636*a^3
*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2*a^2*c^4*x
+ a*c^4), -1/105*(210*(a^6*x^6 - 2*a^5*x^5 - a^4*x^4 + 4*a^3*x^3 - a^2*x^2
- 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2
))/(a^2*c*x^2 - c)) - (105*a^7*x^7 - 562*a^6*x^6 + 74*a^5*x^5 + 1226*a^4*x^
4 - 636*a^3*x^3 - 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(
a^7*c^4*x^6 - 2*a^6*c^4*x^5 - a^5*c^4*x^4 + 4*a^4*c^4*x^3 - a^3*c^4*x^2 - 2
*a^2*c^4*x + a*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax+1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x + 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x+1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,\left(a\,x-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)),x)
```

```
[Out] int((a*x + 1)/((c - c/(a^2*x^2))^(7/2)*(a*x - 1)), x)
```

3.846 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=322

$$\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\ldots$$

```
[Out] 1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/x^
2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5/(1
-1/a^2/x^2)^(1/2)-3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+2
*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+4*c^4*(c-c/a^2/x^2)^(1
/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/
2)+3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 90}

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

```
[Out] (c^4*Sqrt[c - c/(a^2*x^2)])/(8*a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*Sqrt
[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c/(a
^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (3*c^4*Sqrt[c - c/(a^2*x^2)])
/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*Sqr
t[1 - 1/(a^2*x^2)]*x^3) + (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^
2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^4
*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}dx=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3(1+ax)^6}{x^9}dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^9-\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}+\frac{6a^4}{x^5}-\frac{6a^5}{x^4}-\frac{8a^6}{x^3}+\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{3c^4\sqrt{c}}{2a^5\sqrt{1}}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(\frac{1}{8x^8}+\frac{3a}{7x^7}-\frac{8a^3}{5x^5}-\frac{3a^4}{2x^4}+\frac{2a^5}{x^3}+\frac{4a^6}{x^2}+a^9x+3a^8\log(x)\right)}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(9/2),x]
```

[Out] ((c - c/(a^2*x^2))^(9/2)*(1/(8*x^8) + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) - (3*a^4)/(2*x^4) + (2*a^5)/x^3 + (4*a^6)/x^2 + a^9*x + 3*a^8*Log[x]))/(a^9*(1 - 1/(a^2*x^2))^(9/2))

Maple [A]
time = 0.04, size = 112, normalized size = 0.35

method	result	size
default	$\frac{(280a^9x^9+840a^8\ln(x)x^8+1120a^6x^6+560a^5x^5-420a^4x^4-448a^3x^3+120ax+35)\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{9}{2}}x}{280(ax+1)^3(a^2x^2-1)^3\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	112

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/280*(280*a^9*x^9+840*a^8*ln(x)*x^8+1120*a^6*x^6+560*a^5*x^5-420*a^4*x^4-448*a^3*x^3+120*a*x+35)*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*x/(a*x+1)^3/(a^2*x^2-1)^3/((a*x-1)/(a*x+1))^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="maxima")

[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [A]
time = 0.36, size = 96, normalized size = 0.30

$$\frac{(280\,a^9c^4x^9+840\,a^8c^4x^8\log(x)+1120\,a^6c^4x^6+560\,a^5c^4x^5-420\,a^4c^4x^4-448\,a^3c^4x^3+120\,ac^4x+35\,c^4)\sqrt{a^2c}}{280\,a^{10}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="fricas")

[Out] 1/280*(280*a^9*c^4*x^9 + 840*a^8*c^4*x^8*log(x) + 1120*a^6*c^4*x^6 + 560*a^5*c^4*x^5 - 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x + 35*c^4)*sqrt(a^2*c)/(a^10*x^8)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{9/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(9/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.847 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=324

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{c^3\sqrt{}}{a^2\sqrt{}}$$

```
[Out] -1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)-1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(
1-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+
5/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(
1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2
)+3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] -1/6*(c^3*Sqrt[c - c/(a^2*x^2)])/(a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*S
qrt[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) - (c^3*Sqrt[c - c/(
a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)]
)/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*
Sqrt[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a
^2*x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c^3*
Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^2(1+ax)^5}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7+\frac{1}{x^7}+\frac{3a}{x^6}+\frac{a^2}{x^5}-\frac{5a^3}{x^4}-\frac{5a^4}{x^3}+\frac{a^5}{x^2}+\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{}}{3a^4\sqrt{}}$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 0.29

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{10+36ax+15a^2x^2-100a^3x^3-150a^4x^4+60a^5x^5-60a^7x^7}{60x^6}+3a^6\log(x)\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(-1/60*(10 + 36*a*x + 15*a^2*x^2 - 100*a^3*x^3 - 1
50*a^4*x^4 + 60*a^5*x^5 - 60*a^7*x^7)/x^6 + 3*a^6*Log[x]))/(a^7*(1 - 1/(a^2
*x^2))^(7/2))
```

Maple [A]
time = 0.04, size = 112, normalized size = 0.35

method	result	size
default	$\frac{\left(60a^7x^7+180a^6\ln(x)x^6-60a^5x^5+150a^4x^4+100a^3x^3-15a^2x^2-36ax-10\right)\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{7}{2}}x}{60(ax+1)^3(a^2x^2-1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/60*(60*a^7*x^7+180*a^6*ln(x)*x^6-60*a^5*x^5+150*a^4*x^4+100*a^3*x^3-15*a^
2*x^2-36*a*x-10)*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/(a*x+1)^3/(a^2*x^2-1)^2/((
a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.39, size = 96, normalized size = 0.30

$$\frac{\left(60\,a^7c^3x^7+180\,a^6c^3x^6\log\left(x\right)-60\,a^5c^3x^5+150\,a^4c^3x^4+100\,a^3c^3x^3-15\,a^2c^3x^2-36\,ac^3x-10\,c^3\right)\sqrt{a^2c}}{60\,a^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
[Out] 1/60*(60*a^7*c^3*x^7 + 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 + 150*a^4*c^
3*x^4 + 100*a^3*c^3*x^3 - 15*a^2*c^3*x^2 - 36*a*c^3*x - 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.848 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=234

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\sqrt{c-\frac{c}{a^2x}}}{a\sqrt{1-}}$$

```
[Out] 1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(
1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x
^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-c/a^
2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2
/x^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 76}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqrt[c
 - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (c^2*Sqrt[c - c/(a^2*x^2
)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*Sq
rt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)
] + (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)(1+ax)^4}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{2a^2}{x^3}+\frac{2a^3}{x^2}+\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-}}{a^2\sqrt{1-}}$$

Mathematica [A]
time = 0.04, size = 87, normalized size = 0.37

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{(1+ax)^5}{4x^4}+\frac{3}{4}a\left(-\frac{1}{3x^3}-\frac{2a}{x^2}-\frac{6a^2}{x}+a^4x+4a^3\log(x)\right)\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2),x]
[Out] ((c - c/(a^2*x^2))^(5/2)*((1 + a*x)^5/(4*x^4) + (3*a*(-1/3*1/x^3 - (2*a)/x^
2 - (6*a^2)/x + a^4*x + 4*a^3*Log[x]))/4))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A]
time = 0.04, size = 96, normalized size = 0.41

method	result	size
default	$\frac{\left(4 a^{5} x^{5}+12 \ln \left(x\right) a^{4} x^{4}-8 a^{3} x^{3}+4 a^{2} x^{2}+4 a x +1\right) \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} x}{4 \left(a x +1\right)^{3} \left(a^{2} x^{2}-1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(4*a^5*x^5+12*ln(x)*a^4*x^4-8*a^3*x^3+4*a^2*x^2+4*a*x+1)*(c*(a^2*x^2-1)
/a^2/x^2)^(5/2)*x/(a*x+1)^3/(a^2*x^2-1)/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 72, normalized size = 0.31

$$\frac{\left(4 \, a^{5} c^{2} x^{5} + 12 \, a^{4} c^{2} x^{4} \log\left(x\right) - 8 \, a^{3} c^{2} x^{3} + 4 \, a^{2} c^{2} x^{2} + 4 \, a c^{2} x + c^{2}\right) \sqrt{a^{2} c}}{4 \, a^{6} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^5*c^2*x^5 + 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 + 4*a^2*c^2*x^2
+ 4*a*c^2*x + c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(5/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.849 $\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(1
/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+3
*c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6332, 6328, 45}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[
c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*
x)/Sqrt[1 - 1/(a^2*x^2)] + (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1
/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(1+ax)^3}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3+\frac{1}{x^3}+\frac{3a}{x^2}+\frac{3a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-}}$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 0.40

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{1}{2x^2}-\frac{3a}{x}+a^3x+3a^2\log(x)\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2),x]
[Out] ((c - c/(a^2*x^2))^(3/2)*(-1/2*1/x^2 - (3*a)/x + a^3*x + 3*a^2*Log[x]))/(a^
3*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.03, size = 69, normalized size = 0.47

method	result	size
default	$\frac{\left(2a^{3}x^{3}+6a^{2}\ln\left(x\right)x^{2}-6ax-1\right)\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}x}{2\left(ax+1\right)^{3}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*a^3*x^3+6*a^2*ln(x)*x^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/(a*
x+1)^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.34, size = 44, normalized size = 0.30

$$\frac{\left(2\,a^{3}cx^{3}+6\,a^{2}cx^{2}\log\left(x\right)-6\,acx-c\right)\sqrt{a^{2}c}}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*c*x^3 + 6*a^2*c*x^2*log(x) - 6*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(3/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.850 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 84}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.46

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-\log(x)+4\log(1-ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A]
time = 0.04, size = 65, normalized size = 0.60

method	result	size
default	$-\frac{(-ax+\ln(x)-4\ln(ax-1))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax-1)}{(ax+1)^2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 27, normalized size = 0.25

$$\frac{\sqrt{a^2c}\,(ax+4\log(ax-1)-\log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.851 $\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)+2*(1-1/a^2/x^2)^(1/2)/a/(-a*x+1)/
(c-c/a^2/x^2)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 78}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] + (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(1+ax)}{(-1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(-1+ax)^2}+\frac{3}{a(-1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A]
time = 0.07, size = 56, normalized size = 0.49

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(ax+\frac{2}{1-ax}+3\log(1-ax)\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x + 2/(1 - a*x) + 3*Log[1 - a*x]))/(a*Sqrt[c - c/
(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 85, normalized size = 0.74

method	result	size
default	$\frac{(ax-1)\left(a^{2}x^{2}+3x\ln(ax-1)a-ax-3\ln(ax-1)-2\right)}{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}\, x\, a^{2}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)/(a*x+1)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a
^2*(a^2*x^2+3*x*ln(a*x-1)*a-a*x-3*ln(a*x-1)-2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 49, normalized size = 0.43

$$\frac{\left(a^{2}x^{2}-ax+3\,(ax-1)\log\left(ax-1\right)-2\right)\sqrt{a^{2}c}}{a^{3}cx-a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a^2*x^2 - a*x + 3*(a*x - 1)*log(a*x - 1) - 2)*sqrt(a^2*c)/(a^3*c*x - a^2*c
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.852 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(-a
*x+1)^2/(c-c/a^2/x^2)^(1/2)+3*(1-1/a^2/x^2)^(1/2)/a/c/(-a*x+1)/(c-c/a^2/x^2
)^(1/2)+3*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{a^3(-1+ax)^3}+\frac{3}{a^3(-1+ax)^2}+\frac{3}{a^3(-1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 64, normalized size = 0.37

$$
\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2ax+\frac{5-6ax}{(-1+ax)^2}+6\log(1-ax)\right)}{2a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(3/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(2*a*x + (5 - 6*a*x)/(-1 + a*x)^2 + 6*Log[1 - a*x]
))/(2*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.04, size = 102, normalized size = 0.60

method	result	size
default	$\frac{(ax-1)\left(2a^{3}x^{3}+6x^{2}\ln(ax-1)a^{2}-4a^{2}x^{2}-12x\ln(ax-1)a-4ax+6\ln(ax-1)+5\right)}{2\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(2*a^3*x^3+6*x^2*ln(a*x-1)*a^2-4*a^2*x^
2-12*x*ln(a*x-1)*a-4*a*x+6*ln(a*x-1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.47, size = 81, normalized size = 0.47

$$\frac{\left(2\,a^{3}x^{3}-4\,a^{2}x^{2}-4\,ax+6\,\left(a^{2}x^{2}-2\,ax+1\right)\log\left(ax-1\right)+5\right)\sqrt{a^{2}c}}{2\,\left(a^{4}c^{2}x^{2}-2\,a^{3}c^{2}x+a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a^3*x^3 - 4*a^2*x^2 - 4*a*x + 6*(a^2*x^2 - 2*a*x + 1)*log(a*x - 1) +
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 - 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.853 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=267

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{49\sqrt{1-\cdots}}{16a\cdots}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/6*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)^3/(c-c/a^2/x^2)^(1/2)-9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)^2/(c
-c/a^2/x^2)^(1/2)+31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(-a*x+1)/(c-c/a^2/x^2)^(1/
2)+49/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-1/16*ln(a
*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 -
 a*x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^4(1+ax)}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{2a^5(-1+ax)^4}+\frac{9}{4a^5(-1+ax)^3}+\frac{31}{8a^5(-1+ax)^2}+\frac{49}{16a^5(-1+ax)}-\frac{}{16a^5}\right.}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{31}{8ac^2\sqrt{c}}$$

Mathematica [A]
time = 0.14, size = 86, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48ax+\frac{186}{1-ax}-\frac{8}{(-1+ax)^3}-\frac{54}{(-1+ax)^2}+147\log(1-ax)-3\log(1+ax)\right)}{48a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(5/2),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*a*x + 186/(1 - a*x) - 8/(-1 + a*x)^3 - 54/(-1
+ a*x)^2 + 147*Log[1 - a*x] - 3*Log[1 + a*x]))/(48*a*(c - c/(a^2*x^2))^(5/2
))
```

Maple [A]
time = 0.04, size = 175, normalized size = 0.66

method	result
default	$-\frac{(ax-1)(ax+1)\left(-48a^4x^4+3\ln(ax+1)a^3x^3-147x^3\ln(ax-1)a^3+144a^3x^3-9\ln(ax+1)a^2x^2+441x^2\ln(ax-1)a^2+42a^2x^2+9\ln(ax+1\right.}{48\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-48*a^4*x^4+3*ln(a*x+1)*a^3*
x^3-147*x^3*ln(a*x-1)*a^3+144*a^3*x^3-9*ln(a*x+1)*a^2*x^2+441*x^2*ln(a*x-1)
*a^2+42*a^2*x^2+9*ln(a*x+1)*a*x-441*x*ln(a*x-1)*a-270*a*x-3*ln(a*x+1)+147*l
n(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.44, size = 138, normalized size = 0.52

$$\frac{\left(48\,a^4x^4-144\,a^3x^3-42\,a^2x^2+270\,ax-3\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax+1\right)+147\left(a^3x^3-3\,a^2x^2+3\,ax-1\right)\log\left(ax-1\right)-140\right)\sqrt{a^2c}}{48\left(a^5c^3x^3-3\,a^4c^3x^2+3\,a^3c^3x-a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/48*(48*a^4*x^4 - 144*a^3*x^3 - 42*a^2*x^2 + 270*a*x - 3*(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)*log(a*x + 1) + 147*(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 - 3*a^4*c^3*x^2 + 3*a^3*c^3*x - a^2
*c^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int(1/((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.854 $\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=360

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^4}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/16*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^4/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^3/(
c-c/a^2/x^2)^(1/2)-59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)^2/(c-c/a^2/x^2)
^(1/2)+75/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)-1/32*(1
-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+201/64*ln(-a*x+1)*(1-1/
a^2/x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-9/64*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)
/a/c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(1-ax)^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)^4\sqrt{c-\frac{c}{a^2x^2}}}+\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^4) + Sqrt[1 - 1/(a^2*x^2)]/(2*
a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^3) - (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*
c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(32*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64
*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^5(1+ax)^2}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{4a^7(-1+ax)^5}+\frac{3}{2a^7(-1+ax)^4}+\frac{59}{16a^7(-1+ax)^3}+\frac{75}{16a^7(-1+ax)^2}+\ldots\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^4}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^3}-\frac{\ldots}{32ac^3\ldots}$$

Mathematica [A]
time = 0.14, size = 140, normalized size = 0.39

$$\frac{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(\frac{x}{a^7}-\frac{1}{16a^8(1-ax)^4}+\frac{1}{2a^8(1-ax)^3}-\frac{59}{32a^8(1-ax)^2}+\frac{75}{16a^8(1-ax)}-\frac{1}{32a^8(1+ax)}+\frac{201\log(1-ax)}{64a^8}-\frac{9\log(1+ax)}{64a^8}\right)}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])/(c - c/(a^2*x^2))^(7/2),x]
```

```
[Out] (a^7*(1 - 1/(a^2*x^2))^(7/2)*(x/a^7 - 1/(16*a^8*(1 - a*x)^4) + 1/(2*a^8*(1
- a*x)^3) - 59/(32*a^8*(1 - a*x)^2) + 75/(16*a^8*(1 - a*x)) - 1/(32*a^8*(1
+ a*x)) + (201*Log[1 - a*x])/(64*a^8) - (9*Log[1 + a*x])/(64*a^8)))/(c - c/
(a^2*x^2))^(7/2)
```

Maple [A]
time = 0.04, size = 247, normalized size = 0.69

method	result
default	$-\frac{(ax-1)(ax+1)\left(-64a^6x^6+9\ln(ax+1)a^5x^5-201x^5\ln(ax-1)a^5+192a^5x^5-27\ln(ax+1)a^4x^4+603x^4\ln(ax-1)a^4+174a^4x^4+18\ln(a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-64*a^6*x^6+9*ln(a*x+1)*a^5*
x^5-201*x^5*ln(a*x-1)*a^5+192*a^5*x^5-27*ln(a*x+1)*a^4*x^4+603*x^4*ln(a*x-1
)*a^4+174*a^4*x^4+18*ln(a*x+1)*a^3*x^3-402*x^3*ln(a*x-1)*a^3-618*a^3*x^3+18
*ln(a*x+1)*a^2*x^2-402*x^2*ln(a*x-1)*a^2+118*a^2*x^2-27*ln(a*x+1)*a*x+603*x
*ln(a*x-1)*a+414*a*x+9*ln(a*x+1)-201*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.37, size = 207, normalized size = 0.58

$$\frac{\left(64\,a^{6}x^{6}-192\,a^{5}x^{5}-174\,a^{4}x^{4}+618\,a^{3}x^{3}-118\,a^{2}x^{2}-414\,ax-9\,(a^{5}x^{5}-3\,a^{4}x^{4}+2\,a^{3}x^{3}+2\,a^{2}x^{2}-3\,ax+1)\log\left(ax+1\right)+201\,(a^{5}x^{5}-3\,a^{4}x^{4}+2\,a^{3}x^{3}+2\,a^{2}x^{2}-3\,ax+1)\log\left(ax-1\right)+208\right)\sqrt{a^{2}c}}{64\,(a^{7}c^{4}x^{5}-3\,a^{6}c^{4}x^{4}+2\,a^{5}c^{4}x^{3}+2\,a^{4}c^{4}x^{2}-3\,a^{3}c^{4}x+a^{2}c^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="frica
s")
```

[Out] 1/64*(64*a^6*x^6 - 192*a^5*x^5 - 174*a^4*x^4 + 618*a^3*x^3 - 118*a^2*x^2 - 414*a*x - 9*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log(a*x + 1) + 201*(a^5*x^5 - 3*a^4*x^4 + 2*a^3*x^3 + 2*a^2*x^2 - 3*a*x + 1)*log(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 - 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 + 2*a^4*c^4*x^2 - 3*a^3*c^4*x + a^2*c^4)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)

[Out] Timed out

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sa

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)),x)

[Out] int(1/((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2)), x)

3.855 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=322

$$-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3c^3\sqrt{}}{a^2\sqrt{1}}$$

```
[Out] -1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)+1/5*c^3*(c-c/a^2/x
^2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+3/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(
1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-3/2*
c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+3*c^3*(c-c/a^2/x^2)^(1/
2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-
c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6332, 6328, 90}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/6*(c^3*Sqrt[c - c/(a^2*x^2)])/(a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) + (c^3*Sqr
t[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^3*Sqrt[c - c/(
a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^3*Sqrt[c - c/(a^2*x^2)])/
(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (3*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt
[1 - 1/(a^2*x^2)]*x^2) + (3*c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2
*x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (c^3*Sqrt
[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^4(1+ax)^3}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7+\frac{1}{x^7}-\frac{a}{x^6}-\frac{3a^2}{x^5}+\frac{3a^3}{x^4}+\frac{3a^4}{x^3}-\frac{3a^5}{x^2}-\frac{a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^3\sqrt{}}{a^4\sqrt{1}}$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(-\frac{1}{6x^6}+\frac{a}{5x^5}+\frac{3a^2}{4x^4}-\frac{a^3}{x^3}-\frac{3a^4}{2x^2}+\frac{3a^5}{x}+a^7x-a^6\log(x)\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^ArcCoth[a*x],x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*(-1/6*1/x^6 + a/(5*x^5) + (3*a^2)/(4*x^4) - a^3/x^
3 - (3*a^4)/(2*x^2) + (3*a^5)/x + a^7*x - a^6*Log[x]))/(a^7*(1 - 1/(a^2*x^2
))^(7/2))
```

Maple [A]
time = 0.04, size = 112, normalized size = 0.35

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} \sqrt{\frac{a x-1}{a x+1}}\, x\left(-60 a^{7} x^{7}+60 a^{6} \ln \left(x\right) x^{6}-180 a^{5} x^{5}+90 a^{4} x^{4}+60 a^{3} x^{3}-45 a^{2} x^{2}-12 a x+10\right)}{60\left(a x-1\right)\left(a^{2} x^{2}-1\right)^{3}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-60*a^7*x^7+
60*a^6*ln(x)*x^6-180*a^5*x^5+90*a^4*x^4+60*a^3*x^3-45*a^2*x^2-12*a*x+10)/(a
*x-1)/(a^2*x^2-1)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.47, size = 96, normalized size = 0.30

$$\frac{\left(60\,a^{7}c^{3}x^{7}-60\,a^{6}c^{3}x^{6}\log\left(x\right)+180\,a^{5}c^{3}x^{5}-90\,a^{4}c^{3}x^{4}-60\,a^{3}c^{3}x^{3}+45\,a^{2}c^{3}x^{2}+12\,ac^{3}x-10\,c^{3}\right)\sqrt{a^{2}c}}{60\,a^{8}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 60*a^6*c^3*x^6*log(x) + 180*a^5*c^3*x^5 - 90*a^4*c^3
*x^4 - 60*a^3*c^3*x^3 + 45*a^2*c^3*x^2 + 12*a*c^3*x - 10*c^3)*sqrt(a^2*c)/(
a^8*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.856 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2} dx$

Optimal. Leaf size=238

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x}}}{a\sqrt{1-}}$$

```
[Out] 1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)-1/3*c^2*(c-c/a^2/x^
2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a
^2/x^2)^(1/2)+2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-
c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a
^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (c^2*Sqrt[c
 - c/(a^2*x^2)])/(3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*x
^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2*
Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^
2)] - (c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx = \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \frac{(-1+ax)^3(1+ax)^2}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int \left(a^5-\frac{1}{x^5}+\frac{a}{x^4}+\frac{2a^2}{x^3}-\frac{2a^3}{x^2}-\frac{a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4} - \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3} - \frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2} + \frac{2c^2\sqrt{c-}}{a^2\sqrt{1-}}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.32

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(\frac{1}{4x^4}-\frac{a}{3x^3}-\frac{a^2}{x^2}+\frac{2a^3}{x}+a^5x-a^4\log(x)\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^ArcCoth[a*x],x]
[Out] ((c - c/(a^2*x^2))^(5/2)*(1/(4*x^4) - a/(3*x^3) - a^2/x^2 + (2*a^3)/x + a^5
*x - a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A]
time = 0.04, size = 96, normalized size = 0.40

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} \sqrt{\frac{a x-1}{a x+1}}\, x\left(-12 a^{5} x^{5}+12 \ln \left(x\right) a^{4} x^{4}-24 a^{3} x^{3}+12 a^{2} x^{2}+4 a x-3\right)}{12\left(a x-1\right)\left(a^{2} x^{2}-1\right)^{2}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-12*a^5*x^5+
12*ln(x)*a^4*x^4-24*a^3*x^3+12*a^2*x^2+4*a*x-3)/(a*x-1)/(a^2*x^2-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.45, size = 74, normalized size = 0.31

$$\frac{\left(12\, a^{5} c^{2} x^{5} - 12\, a^{4} c^{2} x^{4} \log\left(x\right) + 24\, a^{3} c^{2} x^{3} - 12\, a^{2} c^{2} x^{2} - 4\, a c^{2} x + 3\, c^{2}\right) \sqrt{a^{2} c}}{12\, a^{6} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/12*(12*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) + 24*a^3*c^2*x^3 - 12*a^2*c^2*
x^2 - 4*a*c^2*x + 3*c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(5/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.857 $\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=147

$$-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c*(c-c/a^2/x^2)^(1/2
)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-c*l
n(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {6332, 6328, 76}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(c*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (c*Sqrt[c
 - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)
/Sqrt[1 - 1/(a^2*x^2)] - (c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^
2*x^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^2(1+ax)}{x^3}dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3+\frac{1}{x^3}-\frac{a}{x^2}-\frac{a^2}{x}\right)dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c\sqrt{c-\frac{c}{a^2x}}}{a\sqrt{1-}}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.44

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(-\frac{3a^2}{2}-\frac{1}{2x^2}+\frac{a}{x}+a^3x-a^2\log(x)\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^ArcCoth[a*x],x]
[Out] ((c - c/(a^2*x^2))^(3/2)*((-3*a^2)/2 - 1/(2*x^2) + a/x + a^3*x - a^2*Log[x]
))/(a^3*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.54

method	result	size
default	$-\frac{\left(\frac{c\left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} \sqrt{\frac{a x-1}{a x+1}}\, x\left(-2 a^{3} x^{3}+2 a^{2} \ln\left(x\right) x^{2}-2 a x+1\right)}{2\left(a x-1\right)\left(a^{2} x^{2}-1\right)}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2)*x*(-2*a^3*x^3+2*
a^2*ln(x)*x^2-2*a*x+1)/(a*x-1)/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 44, normalized size = 0.30

$$\frac{\left(2\,a^{3}cx^{3}-2\,a^{2}cx^{2}\log\left(x\right)+2\,acx-c\right)\sqrt{a^{2}c}}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*c*x^3 - 2*a^2*c*x^2*log(x) + 2*a*c*x - c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(3/2)*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\sqrt{\frac{a\,x-1}{a\,x+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.858 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 52, normalized size = 0.76

method	result	size

default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.34, size = 19, normalized size = 0.28

$$\frac{\sqrt{a^2c}\,(ax-\log{(x)})}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.859 $\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(
c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (Sqrt[1 - 1/(a^2*x^2)]*Lo
g[1 + a*x])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x}{1+ax}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}-\frac{1}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.62

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,(ax-\log(1+ax))}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]),x]
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x - Log[1 + a*x]))/(a*Sqrt[c - c/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 59, normalized size = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(-ax+\ln(ax+1))}{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\,a^2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-a*x+ln(a*x+1))/(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)/x/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

Fricas [A]
time = 0.38, size = 26, normalized size = 0.36

$$\frac{\sqrt{a^2c}\,(ax-\log(ax+1))}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(a*x + 1))/(a^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/sqrt(c - c/(a^2*x^2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(1/2), x)
```

3.860 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\,\log(1+ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a*
x+1)/(c-c/a^2/x^2)^(1/2)+1/4*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^
2)^(1/2)-5/4*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}-\frac{5\sqrt{1-\frac{1}{a^2x^2}}\,\log(ax+1)}{4ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a
*x])/(4*a*c*Sqrt[c - c/(a^2*x^2)]) - (5*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])
/(4*a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(-1+ax)(1+ax)^2}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}+\frac{1}{4a^3(-1+ax)}+\frac{1}{2a^3(1+ax)^2}-\frac{5}{4a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)} + \frac{\sqrt{1-\frac{1}{a^2x^2}}\,\log(1-ax)}{4ac\sqrt{c-\frac{c}{a^2x^2}}} - \frac{5\sqrt{1-\frac{1}{a^2x^2}}}{4ac\sqrt{c}}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 0.38

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(4ax-\frac{2}{1+ax}+\log(1-ax)-5\log(1+ax)\right)}{4a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(3/2)),x]
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(4*a*x - 2/(1 + a*x) + Log[1 - a*x] - 5*Log[1 + a*
x]))/(4*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.04, size = 103, normalized size = 0.60

method	result	size
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(-4a^{2}x^{2}+5\ln\left(ax+1\right)ax-x\ln\left(ax-1\right)a-4ax+5\ln\left(ax+1\right)-\ln\left(ax-1\right)+2\right)\left(ax-1\right)}{4a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(-4*a^2*x^2+5*ln(a*x+1)*a*x-x*ln(a*x-1
)*a-4*a*x+5*ln(a*x+1)-ln(a*x-1)+2)*(a*x-1)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^
(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 66, normalized size = 0.38

$$\frac{\left(4\,a^{2}x^{2}+4\,ax-5\,(ax+1)\log\left(ax+1\right)+(ax+1)\log\left(ax-1\right)-2\right)\sqrt{a^{2}c}}{4\left(a^{3}c^{2}x+a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*a^2*x^2 + 4*a*x - 5*(a*x + 1)*log(a*x + 1) + (a*x + 1)*log(a*x - 1)
- 2)*sqrt(a^2*c)/(a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.861 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}+$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2
/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c-c/
a^2/x^2)^(1/2)-(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+7/16*l
n(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-23/16*ln(a*x+1)*(1-
1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{7\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{23\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(8*a*c
^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - Sqrt[1 - 1/(a^2*x^2)]/(a*c^2*Sqrt[c
 - c/(a^2*x^2)]*(1 + a*x)) + (7*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)]) - (23*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(16*a*c
^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)^2(1+ax)^3}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{8a^5(-1+ax)^2}+\frac{7}{16a^5(-1+ax)}-\frac{1}{4a^5(1+ax)^3}+\frac{1}{a^5(1+ax)^2}-\frac{23}{16a^5(1+\cdots}\right.}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}-\frac{\sqrt{1\cdots}}{ac^2\sqrt{c-\cdots}}$$

Mathematica [A]
time = 0.08, size = 85, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(2\left(8ax+\frac{1}{1-ax}+\frac{1}{(1+ax)^2}-\frac{8}{1+ax}\right)+7\log(1-ax)-23\log(1+ax)\right)}{16a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(2*(8*a*x + (1 - a*x)^(-1) + (1 + a*x)^(-2) - 8/(1
 + a*x)) + 7*Log[1 - a*x] - 23*Log[1 + a*x]))/(16*a*(c - c/(a^2*x^2))^(5/2)
)
```

Maple [A]
time = 0.04, size = 175, normalized size = 0.67

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(-16a^4x^4+23\ln(ax+1)a^3x^3-7x^3\ln(ax-1)a^3-16a^3x^3+23\ln(ax+1)a^2x^2-7x^2\ln(ax-1)a^2+34a^2x^2-23\right.}{16a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-16*a^4*x^4+23*ln(a*x+1)*a^3
*x^3-7*x^3*ln(a*x-1)*a^3-16*a^3*x^3+23*ln(a*x+1)*a^2*x^2-7*x^2*ln(a*x-1)*a^
2+34*a^2*x^2-23*ln(a*x+1)*a*x+7*x*ln(a*x-1)*a+18*a*x-23*ln(a*x+1)+7*ln(a*x-
1)-12)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(5/2), x)
```

Fricas [A]
time = 0.37, size = 135, normalized size = 0.51

$$\frac{\left(16\,a^4x^4+16\,a^3x^3-34\,a^2x^2-18\,ax-23\,(a^3x^3+a^2x^2-ax-1)\log(ax+1)+7\,(a^3x^3+a^2x^2-ax-1)\log(ax-1)+12\right)\sqrt{a^2c}}{16\,(a^5c^3x^3+a^4c^3x^2-a^3c^3x-a^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/16*(16*a^4*x^4 + 16*a^3*x^3 - 34*a^2*x^2 - 18*a*x - 23*(a^3*x^3 + a^2*x^2
 - a*x - 1)*log(a*x + 1) + 7*(a^3*x^3 + a^2*x^2 - a*x - 1)*log(a*x - 1) + 1
2)*sqrt(a^2*c)/(a^5*c^3*x^3 + a^4*c^3*x^2 - a^3*c^3*x - a^2*c^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.862 $\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=358

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^3}+$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)-1/32*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)^2/(c-c/a^2/x^2)^(1/2)+5/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(-a*x+1)/(c
-c/a^2/x^2)^(1/2)-1/24*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)^(1
/2)+11/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-3/2*(1-1/
a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+19/32*ln(-a*x+1)*(1-1/a^2/
x^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-51/32*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}+\frac{11\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{24ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{19\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{51\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2) + (5*Sqrt[1 - 1/(a^2*x^2)])
/(16*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) - Sqrt[1 - 1/(a^2*x^2)]/(24*a*c
^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (11*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^
3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(2*a*c^3*S
qrt[c - c/(a^2*x^2)]*(1 + a*x)) + (19*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)]) - (51*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(
32*a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^3(1+ax)^4}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{16a^7(-1+ax)^3}+\frac{5}{16a^7(-1+ax)^2}+\frac{19}{32a^7(-1+ax)}+\frac{1}{8a^7(1+ax)^4}-\frac{}{16a\cdots}\right.}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}+\frac{5\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}-\frac{}{24ac^3\cdots}$$

Mathematica [A]
time = 0.10, size = 104, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(96ax+\frac{30}{1-ax}-\frac{3}{(-1+ax)^2}-\frac{4}{(1+ax)^3}+\frac{33}{(1+ax)^2}-\frac{144}{1+ax}+57\log(1-ax)-153\log(1+ax)\right)}{96a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^ArcCoth[a*x]*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(96*a*x + 30/(1 - a*x) - 3/(-1 + a*x)^2 - 4/(1 + a
*x)^3 + 33/(1 + a*x)^2 - 144/(1 + a*x) + 57*Log[1 - a*x] - 153*Log[1 + a*x]
))/(96*a*(c - c/(a^2*x^2))^(7/2))
```

Maple [A]
time = 0.04, size = 247, normalized size = 0.69

method	result
default	$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)(ax-1)\left(-96a^6x^6+153\ln(ax+1)a^5x^5-57x^5\ln(ax-1)a^5-96a^5x^5+153\ln(ax+1)a^4x^4-57x^4\ln(ax-1)a^4+366a^4x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*((a*x-1)/(a*x+1))^(1/2)*(a*x+1)*(a*x-1)*(-96*a^6*x^6+153*ln(a*x+1)*a^
5*x^5-57*x^5*ln(a*x-1)*a^5-96*a^5*x^5+153*ln(a*x+1)*a^4*x^4-57*x^4*ln(a*x-1
)*a^4+366*a^4*x^4-306*ln(a*x+1)*a^3*x^3+114*x^3*ln(a*x-1)*a^3+222*a^3*x^3-3
06*ln(a*x+1)*a^2*x^2+114*x^2*ln(a*x-1)*a^2-338*a^2*x^2+153*ln(a*x+1)*a*x-57
*x*ln(a*x-1)*a-122*a*x+153*ln(a*x+1)-57*ln(a*x-1)+88)/a^8/x^7/(c*(a^2*x^2-1
)/a^2/x^2)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt((a*x - 1)/(a*x + 1))/(c - c/(a^2*x^2))^(7/2), x)
```

Fricas [A]
time = 0.39, size = 201, normalized size = 0.56

$$\frac{\left(96\,a^6x^6+96\,a^5x^5-366\,a^4x^4-222\,a^3x^3+338\,a^2x^2+122\,ax-153\left(a^5x^5+a^4x^4-2\,a^3x^3-2\,a^2x^2+ax+1\right)\log\left(ax+1\right)+57\left(a^5x^5+a^4x^4-2\,a^3x^3-2\,a^2x^2+ax+1\right)\log\left(ax-1\right)-88\right)\sqrt{a^2c}}{96\left(a^7c^4x^5+a^6c^4x^4-2\,a^5c^4x^3-2\,a^4c^4x^2+a^3c^4x+a^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/96*(96*a^6*x^6 + 96*a^5*x^5 - 366*a^4*x^4 - 222*a^3*x^3 + 338*a^2*x^2 + 1
22*a*x - 153*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x
+ 1) + 57*(a^5*x^5 + a^4*x^4 - 2*a^3*x^3 - 2*a^2*x^2 + a*x + 1)*log(a*x - 1
) - 88)*sqrt(a^2*c)/(a^7*c^4*x^5 + a^6*c^4*x^4 - 2*a^5*c^4*x^3 - 2*a^4*c^4*
x^2 + a^3*c^4*x + a^2*c^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(1/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(1/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a\,x-1}{a\,x+1}}}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(1/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.863 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=375

$$\frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3}+\frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)}-\frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)}-$$

```
[Out] 7/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7/(-a*x+1)^3/(a*x+1)^3+3/8*a^5*(c-c/a^2/x^2)
^(7/2)*x^6/(-a*x+1)^3/(a*x+1)^2-1/15*a*(c-c/a^2/x^2)^(7/2)*x^2/(a*x+1)-19/1
6*a^4*(c-c/a^2/x^2)^(7/2)*x^5/(-a*x+1)^3/(a*x+1)+2/3*a^3*(c-c/a^2/x^2)^(7/2
)*x^4/(-a*x+1)^2/(a*x+1)-23/120*a^2*(c-c/a^2/x^2)^(7/2)*x^3/(-a*x+1)/(a*x+1
)+1/6*(c-c/a^2/x^2)^(7/2)*x*(-a*x+1)/(a*x+1)-2*a^6*(c-c/a^2/x^2)^(7/2)*x^7*
arcsin(a*x)/(-a*x+1)^(7/2)/(a*x+1)^(7/2)+25/16*a^6*(c-c/a^2/x^2)^(7/2)*x^7*
arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(7/2)/(a*x+1)^(7/2)
```

Rubi [A]
time = 0.35, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{15(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{6(ax+1)}-\frac{23a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{120(1-ax)(ax+1)}-\frac{2a^6x^7\text{ArcSin}(ax)\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{(1-ax)^{7/2}(ax+1)^{7/2}}+\frac{7a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)^3}+\frac{25a^6x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{16(1-ax)^{7/2}(ax+1)^{7/2}}+\frac{3a^5x^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{8(1-ax)^3(ax+1)^2}-\frac{19a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{16(1-ax)^3(ax+1)}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}{3(1-ax)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (7*a^6*(c - c/(a^2*x^2))^(7/2)*x^7)/(16*(1 - a*x)^3*(1 + a*x)^3) + (3*a^5*(
c - c/(a^2*x^2))^(7/2)*x^6)/(8*(1 - a*x)^3*(1 + a*x)^2) - (a*(c - c/(a^2*x^
2))^(7/2)*x^2)/(15*(1 + a*x)) - (19*a^4*(c - c/(a^2*x^2))^(7/2)*x^5)/(16*(1
 - a*x)^3*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(7/2)*x^4)/(3*(1 - a*x)^2*(
1 + a*x)) - (23*a^2*(c - c/(a^2*x^2))^(7/2)*x^3)/(120*(1 - a*x)*(1 + a*x))
+ ((c - c/(a^2*x^2))^(7/2)*x*(1 - a*x))/(6*(1 + a*x)) - (2*a^6*(c - c/(a^2*
x^2))^(7/2)*x^7*ArcSin[a*x])/((1 - a*x)^(7/2)*(1 + a*x)^(7/2)) + (25*a^6*(c
 - c/(a^2*x^2))^(7/2)*x^7*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(16*(1 - a*
x)^(7/2)*(1 + a*x)^(7/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2} dx = -\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2} dx$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{7/2}(1+ax)^{7/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^{9/2}(1+ax)^{5/2}}{x^7}dx}{(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int \frac{(1-ax)^{7/2}(1+ax)^{3/2}(-2a-7a^2x \cdots}{x^6} \cdots}{6(1-ax)^{7/2}(1+ax)^{7/2}}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7\right)\int (1 \cdots}{30(1-ax \cdots}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x(1-ax)}{6(1+ax)} - \cdots$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}{120(1-ax)(1+ax)} + \frac{(c \cdots}{\cdots}$$

$$= -\frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}{3(1-ax)^2(1+ax)} - \frac{23a \cdots}{120 \cdots}$$

$$= \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^5}{16(1-ax)^3(1+ax)} + \frac{2a^3 \cdots}{3(1 \cdots}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a \cdots}{16(\cdots}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a \cdots}{16(\cdots}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a \cdots}{16(\cdots}$$

$$= \frac{7a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}{16(1-ax)^3(1+ax)^3} + \frac{3a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^6}{8(1-ax)^3(1+ax)^2} - \frac{a\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}{15(1+ax)} - \frac{19a \cdots}{16(\cdots}$$

Mathematica [A]

time = 0.11, size = 150, normalized size = 0.40

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\left(-40+96ax+70a^2x^2-352a^3x^3+105a^4x^4+736a^5x^5+240a^6x^6\right)+375a^6x^6\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-480a^6x^6\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{240a^6x^5\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-40 + 96*a*x + 70*a^2*x^2 -
 352*a^3*x^3 + 105*a^4*x^4 + 736*a^5*x^5 + 240*a^6*x^6) + 375*a^6*x^6*ArcTa
n[1/Sqrt[-1 + a^2*x^2]] - 480*a^6*x^6*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(240*
a^6*x^5*Sqrt[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(329) = 658$.

time = 0.15, size = 795, normalized size = 2.12

method	result
risch	$\frac{(736a^7x^7+105a^6x^6-1088a^5x^5-35a^4x^4+448a^3x^3-110a^2x^2-96ax+40)c^3\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{240x^5a^6(a^2x^2-1)} + \frac{\left(\frac{a^6\sqrt{c(a^2x^2-1)}}{c} - \frac{2a^7\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{7}{2}} x \left(-2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\, a^9cx^7+2016\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{9}{2}}\sqrt{-\frac{c}{a^2}}\, a^9x^5-375\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\, a^8 \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*x/a^2*(-2016*(c*(a^2*x^2-1)/a^2)^(7/2)
*(-c/a^2)^(1/2)*a^9*c*x^7+2016*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*a^9
*x^5-375*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^8*c*x^6+480*(c*(a*x+1)*
(a*x-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^8*c*x^6-105*(c*(a^2*x^2-1)/a^2)^(9/2)*(
-c/a^2)^(1/2)*a^8*x^4+2352*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^7*c^2
*x^7-560*(-c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^7*c^2*x^7+224*(c*(a
^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*a^7*x^3+525*(c*(a^2*x^2-1)/a^2)^(5/2)*(
-c/a^2)^(1/2)*a^6*c^2*x^6-2940*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^5
*c^3*x^7+700*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^5*c^3*x^7-630*(
c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*a^6*x^2-875*(c*(a^2*x^2-1)/a^2)^(3/
2)*(-c/a^2)^(1/2)*a^4*c^3*x^6+672*(c*(a^2*x^2-1)/a^2)^(9/2)*(-c/a^2)^(1/2)*
a^5*x+4410*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^4*x^7-1050*(-c/a^
2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*c^4*x^7-280*a^4*(c*(a^2*x^2-1)/a
^2)^(9/2)*(-c/a^2)^(1/2)+2625*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2*
c^4*x^6+1050*(-c/a^2)^(1/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/
c^(1/2))*c^(9/2)*a*x^6-4410*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*c^(9/2)
*(-c/a^2)^(1/2)*a*x^6+2625*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a
^2-c)/a^2/x)*c^5*x^6)/(c*(a^2*x^2-1)/a^2)^(7/2)/(-c/a^2)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(7/2)/(a*x + 1), x)
```

Fricas [A]
time = 0.37, size = 438, normalized size = 1.17

$$\left[\frac{960\,a^5\sqrt{-c}\,c^3x^5\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+375\,a^5\sqrt{-c}\,c^3x^5\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(240\,a^6c^3x^6+736\,a^5c^3x^5+105\,a^4c^3x^4-352\,a^3c^3x^3+70\,a^2c^3x^2+96\,ac^3x-40\,c^3)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{480\,a^6x^5},\ \frac{375\,a^5c^{\frac{7}{2}}x^5\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+240\,a^5c^{\frac{7}{2}}x^5\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(240\,a^6c^3x^6+736\,a^5c^3x^5+105\,a^4c^3x^4-352\,a^3c^3x^3+70\,a^2c^3x^2+96\,ac^3x-40\,c^3)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{240\,a^6x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/480*(960*a^5*sqrt(-c)*c^3*x^5*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 -
c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 375*a^5*sqrt(-c)*c^3*x^5*log(-(a^2*c*x^2 -
 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(240*a^6*c^
3*x^6 + 736*a^5*c^3*x^5 + 105*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^
2 + 96*a*c^3*x - 40*c^3)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5), 1/240*
(375*a^5*c^(7/2)*x^5*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)) + 240*a^5*c^(7/2)*x^5*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (240*a^6*c^3*x^6 + 736*a^5*c^3*x^5 + 10
5*a^4*c^3*x^4 - 352*a^3*c^3*x^3 + 70*a^2*c^3*x^2 + 96*a*c^3*x - 40*c^3)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^6*x^5)]
```

Sympy [C] Result contains complex when optimal does not.
time = 25.21, size = 1059, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c**3*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sq
rt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1),
(I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt
(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**3*Piecewise((-a*sqrt(c)*
x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 -
1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*a
sin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a - c**3*Piecewise(
```

```
(I*a*sqrt(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) -
I*sqrt(c)/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-
a*sqrt(c)*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a
**2 + 4*c**3*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c
), True))/a**3 - c**3*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*s
qrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a*
*2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x
**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a
**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*
x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4 - 2*c**3*Piecewise((2*a**3*sqrt(
c)*sqrt(a**2*x**2 - 1)/(15*x) + a*sqrt(c)*sqrt(a**2*x**2 - 1)/(15*x**3) - s
qrt(c)*sqrt(a**2*x**2 - 1)/(5*a*x**5), Abs(a**2*x**2) > 1), (2*I*a**3*sqrt(
c)*sqrt(-a**2*x**2 + 1)/(15*x) + I*a*sqrt(c)*sqrt(-a**2*x**2 + 1)/(15*x**3)
 - I*sqrt(c)*sqrt(-a**2*x**2 + 1)/(5*a*x**5), True))/a**5 + c**3*Piecewise(
(I*a**5*sqrt(c)*acosh(1/(a*x))/16 - I*a**4*sqrt(c)/(16*x*sqrt(-1 + 1/(a**2*
x**2))) + I*a**2*sqrt(c)/(48*x**3*sqrt(-1 + 1/(a**2*x**2))) + 5*I*sqrt(c)/(
24*x**5*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(6*a**2*x**7*sqrt(-1 + 1/(a**
2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**5*sqrt(c)*asin(1/(a*x))/16 + a**4*sq
rt(c)/(16*x*sqrt(1 - 1/(a**2*x**2))) - a**2*sqrt(c)/(48*x**3*sqrt(1 - 1/(a*
*2*x**2))) - 5*sqrt(c)/(24*x**5*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(6*a**2*
x**7*sqrt(1 - 1/(a**2*x**2))), True))/a**6
```

Giac [A]
time = 24.19, size = 561, normalized size = 1.50

$$-\frac{1}{120}\left(\frac{375\,c^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{240\,c^{\frac{7}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{120\,\sqrt{a^2cx^2-c}\,c^3\mathrm{sgn}(x)}{a^2}+\frac{105\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{11}c^4|a|\mathrm{sgn}(x)-1440\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{10}ac^{\frac{9}{2}}\mathrm{sgn}(x)+595\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{9}c^5|a|\mathrm{sgn}(x)-4320\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{8}ac^{\frac{11}{2}}\mathrm{sgn}(x)-150\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{7}c^6|a|\mathrm{sgn}(x)-7360\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{6}ac^{\frac{13}{2}}\mathrm{sgn}(x)+150\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{5}c^7|a|\mathrm{sgn}(x)-6720\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{4}ac^{\frac{15}{2}}\mathrm{sgn}(x)-595\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{3}c^8|a|\mathrm{sgn}(x)-2976\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^{2}ac^{\frac{17}{2}}\mathrm{sgn}(x)-105\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)c^9|a|\mathrm{sgn}(x)-736\,ac^{\frac{19}{2}}\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^6a^2|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/120*(375*c^(7/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*
sgn(x)/a^2 - 240*c^(7/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn
(x)/(a*abs(a)) - 120*sqrt(a^2*c*x^2 - c)*c^3*sgn(x)/a^2 + (105*(sqrt(a^2*c)
*x - sqrt(a^2*c*x^2 - c))^11*c^4*abs(a)*sgn(x) - 1440*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^10*a*c^(9/2)*sgn(x) + 595*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2
- c))^9*c^5*abs(a)*sgn(x) - 4320*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^8*a*
c^(11/2)*sgn(x) - 150*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*c^6*abs(a)*sg
n(x) - 7360*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^6*a*c^(13/2)*sgn(x) + 150
*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^7*abs(a)*sgn(x) - 6720*(sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(15/2)*sgn(x) - 595*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^3*c^8*abs(a)*sgn(x) - 2976*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2*a*c^(17/2)*sgn(x) - 105*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^9
*abs(a)*sgn(x) - 736*a*c^(19/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
 c))^2 + c)^6*a^2*abs(a)))*abs(a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2 x^2}\right)^{7/2} (a x-1)}{a x+1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a^2*x^2))^(7/2)*(a*x - 1))/(a*x + 1), x)
```

3.864 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=293

$$-\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax}{4(1+ax)}$$

```
[Out] -7/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5/(-a*x+1)^2/(a*x+1)^2-1/6*a*(c-c/a^2/x^2)^(
5/2)*x^2/(a*x+1)+2*a^3*(c-c/a^2/x^2)^(5/2)*x^4/(-a*x+1)^2/(a*x+1)-7/24*a^2*
(c-c/a^2/x^2)^(5/2)*x^3/(-a*x+1)/(a*x+1)+1/4*(c-c/a^2/x^2)^(5/2)*x*(-a*x+1)
/(a*x+1)+2*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arcsin(a*x)/(-a*x+1)^(5/2)/(a*x+1)^(
5/2)-9/8*a^4*(c-c/a^2/x^2)^(5/2)*x^5*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/
(-a*x+1)^(5/2)/(a*x+1)^(5/2)
```

Rubi [A]
time = 0.32, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{6(ax+1)}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{4(ax+1)}-\frac{7a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{24(1-ax)(ax+1)}+\frac{2a^4x^5\text{ArcSin}(ax)\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^{5/2}(ax+1)^{5/2}}-\frac{7a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{8(1-ax)^2(ax+1)^2}-\frac{9a^4x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{8(1-ax)^{5/2}(ax+1)^{5/2}}+\frac{2a^3x^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}}{(1-ax)^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*a^4*(c - c/(a^2*x^2))^(5/2)*x^5)/(8*(1 - a*x)^2*(1 + a*x)^2) - (a*(c -
c/(a^2*x^2))^(5/2)*x^2)/(6*(1 + a*x)) + (2*a^3*(c - c/(a^2*x^2))^(5/2)*x^4)
/((1 - a*x)^2*(1 + a*x)) - (7*a^2*(c - c/(a^2*x^2))^(5/2)*x^3)/(24*(1 - a*x
)*(1 + a*x)) + ((c - c/(a^2*x^2))^(5/2)*x*(1 - a*x))/(4*(1 + a*x)) + (2*a^4
*(c - c/(a^2*x^2))^(5/2)*x^5*ArcSin[a*x])/((1 - a*x)^(5/2)*(1 + a*x)^(5/2))
 - (9*a^4*(c - c/(a^2*x^2))^(5/2)*x^5*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])
/(8*(1 - a*x)^(5/2)*(1 + a*x)^(5/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx$$
$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{5/2}(1+ax)^{5/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{7/2}(1+ax)^{3/2}}{x^5}dx}{(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}\,(-2a-5a}{x^4}}{4(1-ax)^{5/2}(1+ax)^{5/2}}$$
$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5\right)\int\frac{(1}{}}{12(1-a}$$
$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}-\frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x(1-ax)}{4(1+ax)}-$$
$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}{24(1-ax)(1+ax)}+\frac{\left(c-\right.}{}$$
$$=-\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2}{24(}$$
$$=-\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2}{24(}$$
$$=-\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2}{24(}$$
$$=-\frac{7a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}{8(1-ax)^2(1+ax)^2}-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}{6(1+ax)}+\frac{2a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^4}{(1-ax)^2(1+ax)}-\frac{7a^2}{24(}$$

Mathematica [A]
time = 0.09, size = 134, normalized size = 0.46

$$\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(6-16ax-3a^2x^2+64a^3x^3+24a^4x^4)+27a^4x^4\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-48a^4x^4\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^4x^3\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (c^2*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 - 16*a*x - 3*a^2*x^2 + 64
*a^3*x^3 + 24*a^4*x^4) + 27*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 48*a^4*x
^4*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^4*x^3*Sqrt[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(257) = 514$.
time = 0.14, size = 625, normalized size = 2.13

method	result
risch	$\frac{(64a^5x^5-3a^4x^4-80a^3x^3+9a^2x^2+16ax-6)c^2\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3a^4(a^2x^2-1)} + \frac{\left(\frac{a^4\sqrt{c(a^2x^2-1)}}{c} - \frac{2a^5\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2cx^2-c}\right)}{\sqrt{a^2c}} + \cdots\right.}{\cdots}$
default	$-\frac{\left(\frac{c(a^2x^2-1)}{a^2x^2}\right)^{\frac{5}{2}} x \left(-80\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\, a^7cx^5+80\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{7}{2}}\sqrt{-\frac{c}{a^2}}\, a^7x^3-27\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\, a^6cx^4 \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(c*(a^2*x^2-1)/a^2/x^2)^(5/2)*x/a^2*(-80*(c*(a^2*x^2-1)/a^2)^(5/2)*(
-c/a^2)^(1/2)*a^7*c*x^5+80*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)^(1/2)*a^7*x^3
-27*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^6*c*x^4-48*(-c/a^2)^(1/2)*(c
*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^6*c*x^4+75*(c*(a^2*x^2-1)/a^2)^(7/2)*(-c/a^2)
^(1/2)*a^6*x^2+100*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^5*c^2*x^5+60*
(-c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^5*c^2*x^5-80*(c*(a^2*x^2-1)/
a^2)^(7/2)*(-c/a^2)^(1/2)*a^5*x+45*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)
*a^4*c^2*x^4-150*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^3*x^5-90*(-
c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*c^3*x^5+30*a^4*(c*(a^2*x^2-1
)/a^2)^(7/2)*(-c/a^2)^(1/2)-135*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^
2*c^3*x^4+90*(-c/a^2)^(1/2)*c^(7/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/
2)+c*x)/c^(1/2))*a*x^4+150*(-c/a^2)^(1/2)*c^(7/2)*ln(c^(1/2)*x+(c*(a^2*x^2-
1)/a^2)^(1/2))*a*x^4-135*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2
-c)/a^2/x)*c^4*x^4)/(c*(a^2*x^2-1)/a^2)^(5/2)/(-c/a^2)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(5/2)/(a*x + 1), x)

Fricas [A]
time = 0.36, size = 394, normalized size = 1.34

$$\left[\frac{96\,a^{3}\sqrt{-c}\,c^{2}x^{3}\arctan\left(\frac{a^{2}\sqrt{-c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+27\,a^{3}\sqrt{-c}\,c^{2}x^{3}\log\left(-\frac{a^{2}cx^{2}-2\,a\sqrt{-c}\,x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)+2\,(24\,a^{4}c^{2}x^{4}+64\,a^{3}c^{2}x^{3}-3\,a^{2}c^{2}x^{2}-16\,ac^{2}x+6\,c^{2})\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{48\,a^{4}x^{3}},\ \frac{27\,a^{3}c^{\frac{5}{2}}x^{3}\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+24\,a^{3}c^{\frac{5}{2}}x^{3}\log\left(2\,a^{2}cx^{2}-2\,a^{2}\sqrt{c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)+(24\,a^{4}c^{2}x^{4}+64\,a^{3}c^{2}x^{3}-3\,a^{2}c^{2}x^{2}-16\,ac^{2}x+6\,c^{2})\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{24\,a^{4}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")

[Out] [1/48*(96*a^3*sqrt(-c)*c^2*x^3*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 27*a^3*sqrt(-c)*c^2*x^3*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*c^2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3), 1/24*(27*a^3*c^(5/2)*x^3*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 24*a^3*c^(5/2)*x^3*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (24*a^4*c^2*x^4 + 64*a^3*c^2*x^3 - 3*a^2*c^2*x^2 - 16*a*c^2*x + 6*c^2)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^4*x^3)]

Sympy [C] Result contains complex when optimal does not.
time = 12.95, size = 500, normalized size = 1.71

$$c^{2}\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^{2}x^{2}-1}}{a}-\frac{i\sqrt{c}\log(ax)}{a}+\frac{i\sqrt{c}\log(a^{2}x^{2})}{2a}+\frac{\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{a} & \text{for}\ |a^{2}x^{2}|>1\\ \frac{i\sqrt{c}\sqrt{-a^{2}x^{2}+1}}{a}+\frac{i\sqrt{c}\log(a^{2}x^{2})}{2a}-\frac{i\sqrt{c}\log\left(\sqrt{-a^{2}x^{2}+1}+1\right)}{a} & \text{otherwise}\end{cases}\right)-\frac{2c^{2}\left(\begin{cases}-\frac{a\sqrt{c}\,x}{\sqrt{a^{2}x^{2}-1}}+\sqrt{c}\operatorname{acosh}(ax)+\frac{\sqrt{c}}{ax\sqrt{a^{2}x^{2}-1}} & \text{for}\ |a^{2}x^{2}|>1\\ \frac{ia\sqrt{c}\,x}{\sqrt{-a^{2}x^{2}+1}}-i\sqrt{c}\operatorname{asin}(ax)-\frac{i\sqrt{c}}{ax\sqrt{-a^{2}x^{2}+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{2c^{2}\left(\begin{cases}0 & \text{for}\ c=0\\ \frac{a^{2}\left(c-\frac{c}{a^{2}x^{2}}\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)}{a^{3}}-\frac{c^{2}\left(\begin{cases}\frac{ia^{3}\sqrt{c}\operatorname{acosh}\left(\frac{1}{ax}\right)}{8}-\frac{ia^{2}\sqrt{c}}{8x\sqrt{-1+\frac{1}{a^{2}x^{2}}}}+\frac{3i\sqrt{c}}{8x^{3}\sqrt{-1+\frac{1}{a^{2}x^{2}}}}-\frac{i\sqrt{c}}{4a^{2}x^{5}\sqrt{-1+\frac{1}{a^{2}x^{2}}}} & \text{for}\ \frac{1}{|a^{2}x^{2}|}>1\\ -\frac{a^{3}\sqrt{c}\operatorname{asin}\left(\frac{1}{ax}\right)}{8}+\frac{a^{2}\sqrt{c}}{8x\sqrt{1-\frac{1}{a^{2}x^{2}}}}-\frac{3\sqrt{c}}{8x^{3}\sqrt{1-\frac{1}{a^{2}x^{2}}}}+\frac{\sqrt{c}}{4a^{2}x^{5}\sqrt{1-\frac{1}{a^{2}x^{2}}}} & \text{otherwise}\end{cases}\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a**2/x**2)**(5/2)*(a*x-1)/(a*x+1),x)

[Out] c**2*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c**2*Piecewise((-a*sqrt(c)*x/sqrt(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), Abs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + 2*c**2*Piecewise((0, Eq(c, 0)), (a**2*(c - c/(a**2*x**2))**(3/2)/(3*c), True))/a**3 - c**2*Piecewise((I*a**3*sqrt(c)*acosh(1/(a*x))/8 - I*a**2*sqrt(c)/(8*x*sqrt(-1 + 1/(a**2*x**2))) + 3*I*sqrt(c)/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)/(4*a**2*x**5*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a**3*sqrt(c)*asin(1/(a*x))/8 + a**2*sqrt(c)/(8*x*sqrt(1 - 1/(a**2*x**2))) - 3*sqrt(c)/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + sqrt(c)/(4*a**2*x**5*sqrt(1 - 1/(a**2*x**2))), True))/a**4

Giac [A]
time = 1.69, size = 416, normalized size = 1.42

$$-\frac{1}{12}\left(\frac{27\,c^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a^2}-\frac{24\,c^{\frac{5}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a|a|}-\frac{12\,\sqrt{a^2cx^2-c}\,c^2\mathrm{sgn}(x)}{a^2}-\frac{3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7c^3|a|\mathrm{sgn}(x)+96\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^6ac^{\frac{7}{2}}\mathrm{sgn}(x)-21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5c^4|a|\mathrm{sgn}(x)+192\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^4ac^{\frac{9}{2}}\mathrm{sgn}(x)+21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3c^5|a|\mathrm{sgn}(x)+160\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2ac^{\frac{11}{2}}\mathrm{sgn}(x)-3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)c^6|a|\mathrm{sgn}(x)+64\,ac^{\frac{13}{2}}\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^4a^2|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -1/12*(27*c^(5/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x)/a^2 - 24*c^(5/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)
/(a*abs(a)) - 12*sqrt(a^2*c*x^2 - c)*c^2*sgn(x)/a^2 - (3*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^7*c^3*abs(a)*sgn(x) + 96*(sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^6*a*c^(7/2)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*c^4
*abs(a)*sgn(x) + 192*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^4*a*c^(9/2)*sgn(
x) + 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*c^5*abs(a)*sgn(x) + 160*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(11/2)*sgn(x) - 3*(sqrt(a^2*c)*x -
 sqrt(a^2*c*x^2 - c))*c^6*abs(a)*sgn(x) + 64*a*c^(13/2)*sgn(x))/(((sqrt(a^2
*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^4*a^2*abs(a)))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-\frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x-1\right)}{a\,x+1} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(5/2)*(a*x - 1))/(a*x + 1), x)
```

3.865 $\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=213

$$-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{2a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\text{ArcSin}(ax)}{(1-ax)^{3/2}(1+ax)^{3/2}}+\frac{a^2\left(c-\right.}{}$$

```
[Out] -a*(c-c/a^2/x^2)^(3/2)*x^2/(a*x+1)-5/2*a^2*(c-c/a^2/x^2)^(3/2)*x^3/(-a*x+1)
/(a*x+1)+1/2*(c-c/a^2/x^2)^(3/2)*x*(-a*x+1)/(a*x+1)-2*a^2*(c-c/a^2/x^2)^(3/
2)*x^3*arcsin(a*x)/(-a*x+1)^(3/2)/(a*x+1)^(3/2)+1/2*a^2*(c-c/a^2/x^2)^(3/2)
*x^3*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))/(-a*x+1)^(3/2)/(a*x+1)^(3/2)
```

Rubi [A]
time = 0.30, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {6302, 6294, 6264, 99, 154, 159, 163, 41, 222, 94, 214}

$$-\frac{2a^2x^3\text{ArcSin}(ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{(1-ax)^{3/2}(ax+1)^{3/2}}-\frac{ax^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{ax+1}+\frac{x(1-ax)\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(ax+1)}-\frac{5a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}{2(1-ax)(ax+1)}+\frac{a^2x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{2(1-ax)^{3/2}(ax+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -((a*(c - c/(a^2*x^2))^(3/2)*x^2)/(1 + a*x)) - (5*a^2*(c - c/(a^2*x^2))^(3/
2)*x^3)/(2*(1 - a*x)*(1 + a*x)) + ((c - c/(a^2*x^2))^(3/2)*x*(1 - a*x))/(2*
(1 + a*x)) - (2*a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcSin[a*x])/((1 - a*x)^(3/
2)*(1 + a*x)^(3/2)) + (a^2*(c - c/(a^2*x^2))^(3/2)*x^3*ArcTanh[Sqrt[1 - a*x
]*Sqrt[1 + a*x]])/(2*(1 - a*x)^(3/2)*(1 + a*x)^(3/2))
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
```

```
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
```

```
x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx=-\int e^{-2\tanh^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}dx$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{e^{-2\tanh^{-1}(ax)}(1-ax)^{3/2}(1+ax)^{3/2}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{5/2}\sqrt{1+ax}}{x^3}dx}{(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\frac{(1-ax)^{3/2}(-2a-3a^2x)}{x^2\sqrt{1+ax}}dx}{2(1-ax)^{3/2}(1+ax)^{3/2}}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{\left(\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3\right)\int\sqrt{1\ldots}}{2(1-ax)^{3/2}(1\ldots}$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\ldots$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\ldots$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}+\ldots$$

$$=-\frac{a\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^2}{1+ax}-\frac{5a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}{2(1-ax)(1+ax)}+\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x(1-ax)}{2(1+ax)}-\frac{2a\ldots}{}$$

Mathematica [**A**]
time = 0.08, size = 115, normalized size = 0.54

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(-1+4ax+2a^2x^2)+a^2x^2\operatorname{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-4a^2x^2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{2a^2x\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(2*ArcCoth[a*x]),x]
[Out] (c*Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-1 + 4*a*x + 2*a^2*x^2) + a^2
*x^2*ArcTan[1/Sqrt[-1 + a^2*x^2]] - 4*a^2*x^2*Log[a*x + Sqrt[-1 + a^2*x^2]]
))/(2*a^2*x*Sqrt[-1 + a^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(185) = 370.
time = 0.12, size = 455, normalized size = 2.14

method	result
risch	$\frac{(2a^4x^4+4a^3x^3-3a^2x^2-4ax+1)c\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x\,a^2(a^2x^2-1)}+\frac{\left(-\frac{2a^3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-}}{x}\right.}{2\sqrt{-c}}\right.}{a^2(a^2x^2-1)}$
default	$\frac{\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{3}{2}}x\left(-12\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^5cx^3+12\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}\sqrt{-\frac{c}{a^2}}\,a^5x-\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^4cx^2+4(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*x/a^2*(-12*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/
a^2)^(1/2)*a^5*c*x^3+12*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)*a^5*x-(c*(
a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^4*c*x^2+4*(c*(a*x+1)*(a*x-1)/a^2)^(3
/2)*(-c/a^2)^(1/2)*a^4*c*x^2-3*a^4*(c*(a^2*x^2-1)/a^2)^(5/2)*(-c/a^2)^(1/2)
+18*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^2*x^3-6*(c*(a*x+1)*(a*x-
1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c^2*x^3+3*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^
2)^(1/2)*a^2*c^2*x^2+6*c^(5/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*
x)/c^(1/2))*(-c/a^2)^(1/2)*a*x^2-18*c^(5/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2
)^(1/2))*(-c/a^2)^(1/2)*a*x^2+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1
/2)*a^2-c)/a^2/x)*c^3*x^2)/(c*(a^2*x^2-1)/a^2)^(3/2)/(-c/a^2)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*(c - c/(a^2*x^2))^(3/2)/(a*x + 1), x)
```

Fricas [A]
time = 0.36, size = 316, normalized size = 1.48

$$\left[\frac{8\,a\sqrt{-c}\,cx\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+a\sqrt{-c}\,cx\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(2\,a^2cx^2+4\,acx-c)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,a^2x},\ \frac{ac^{\frac{3}{2}}x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+2\,ac^{\frac{3}{2}}x\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(2\,a^2cx^2+4\,acx-c)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*sqrt(-c)*c*x*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c)) + a*sqrt(-c)*c*x*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt
((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(2*a^2*c*x^2 + 4*a*c*x - c)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x), 1/2*(a*c^(3/2)*x*arctan(a*sqrt(c)*x*
sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 2*a*c^(3/2)*x*log(2*a^2*
c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (2*a^2*c*x
^2 + 4*a*c*x - c)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 9.50, size = 376, normalized size = 1.77

$$c\left(\begin{cases}\frac{\sqrt{c}\sqrt{a^2x^2-1}}{a}-\frac{i\sqrt{c}\log{(ax)}}{a}+\frac{i\sqrt{c}\log{(a^2x^2)}}{2a}+\frac{\sqrt{c}\operatorname{asin}{\left(\frac{1}{ax}\right)}}{a} & \text{for } |a^2x^2|>1\\ \frac{i\sqrt{c}\sqrt{-a^2x^2+1}}{a}+\frac{i\sqrt{c}\log{(a^2x^2)}}{2a}-\frac{i\sqrt{c}\log{\left(\sqrt{-a^2x^2+1}+1\right)}}{a} & \text{otherwise}\end{cases}\right)-\frac{2c\left(\begin{cases}-\frac{a\sqrt{c}\,x}{\sqrt{a^2x^2-1}}+\sqrt{c}\operatorname{acosh}{(ax)}+\frac{\sqrt{c}}{ax\sqrt{a^2x^2-1}} & \text{for } |a^2x^2|>1\\ \frac{ia\sqrt{c}\,x}{\sqrt{-a^2x^2+1}}-i\sqrt{c}\operatorname{asin}{(ax)}-\frac{i\sqrt{c}}{ax\sqrt{-a^2x^2+1}} & \text{otherwise}\end{cases}\right)}{a}+\frac{c\left(\begin{cases}\frac{ia\sqrt{c}\operatorname{acosh}{\left(\frac{1}{ax}\right)}}{2}+\frac{i\sqrt{c}}{2x\sqrt{-1+\frac{1}{a^2x^2}}}-\frac{i\sqrt{c}}{2a^2x^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|}>1\\ -\frac{a\sqrt{c}\operatorname{asin}{\left(\frac{1}{ax}\right)}}{2}-\frac{\sqrt{c}\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] c*Piecewise((sqrt(c)*sqrt(a**2*x**2 - 1)/a - I*sqrt(c)*log(a*x)/a + I*sqrt(
c)*log(a**2*x**2)/(2*a) + sqrt(c)*asin(1/(a*x))/a, Abs(a**2*x**2) > 1), (I*
sqrt(c)*sqrt(-a**2*x**2 + 1)/a + I*sqrt(c)*log(a**2*x**2)/(2*a) - I*sqrt(c)
*log(sqrt(-a**2*x**2 + 1) + 1)/a, True)) - 2*c*Piecewise((-a*sqrt(c)*x/sqrt
(a**2*x**2 - 1) + sqrt(c)*acosh(a*x) + sqrt(c)/(a*x*sqrt(a**2*x**2 - 1)), A
bs(a**2*x**2) > 1), (I*a*sqrt(c)*x/sqrt(-a**2*x**2 + 1) - I*sqrt(c)*asin(a*
x) - I*sqrt(c)/(a*x*sqrt(-a**2*x**2 + 1)), True))/a + c*Piecewise((I*a*sqrt
(c)*acosh(1/(a*x))/2 + I*sqrt(c)/(2*x*sqrt(-1 + 1/(a**2*x**2))) - I*sqrt(c)
/(2*a**2*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (-a*sqrt(c)
*asin(1/(a*x))/2 - sqrt(c)*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/a**2
```

Giac [A]
time = 0.55, size = 266, normalized size = 1.25

$$-\left(\frac{c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a^2}-\frac{2\,c^{\frac{3}{2}}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a|a|}-\frac{\sqrt{a^2cx^2-c}\,c\operatorname{sgn}(x)}{a^2}-\frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3c^2|a|\operatorname{sgn}(x)+4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2ac^{\frac{5}{2}}\operatorname{sgn}(x)-\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)c^3|a|\operatorname{sgn}(x)+4\,ac^{\frac{7}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^2a^2|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] -(c^(3/2)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a^2
 - 2*c^(3/2)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a*abs(a
)) - sqrt(a^2*c*x^2 - c)*c*sgn(x)/a^2 - ((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 -
c))^3*c^2*abs(a)*sgn(x) + 4*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2*a*c^(5/
2)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*c^3*abs(a)*sgn(x) + 4*a*c
^(7/2)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)^2*a^2*abs(a))
)*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1),x)
[Out] int(((c - c/(a^2*x^2))^(3/2)*(a*x - 1))/(a*x + 1), x)
```

3.866 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\mathrm{ArcSin}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$\frac{2x\mathrm{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}dx}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}dx}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\frac{c}{a}}\right.}{}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}dx, x, \sqrt{1-ax}\sqrt{1+ax}\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$
$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-a}\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.69

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}-\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A]
time = 0.10, size = 197, normalized size = 1.70

method	result

default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, x \left(2\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\, a^2 \sqrt{-\frac{c}{a^2}} - 2\sqrt{c}\, \ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\, \sqrt{c} + cx}{\sqrt{c}}\right) a \sqrt{-\frac{c}{a^2}} - \sqrt{-\frac{c}{a^2}}\, \sqrt{\frac{c(a}{}}\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\, a^2 \sqrt{-\frac{c}{a^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)-2*c^(1/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Fricas [A]
time = 0.37, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} + 4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\,\log\left(-\frac{a^2cx^2+2a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right)}{2a}, \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - \sqrt{c}\,\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{c}\,\log\left(2a^2cx^2-2a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x-1)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.867 $\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=112

$$-\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}-\frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\mathrm{ArcSin}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

```
[Out] -(-a*x+1)^2/a^2/x/(c-c/a^2/x^2)^(1/2)-2*(-a*x+1)*(a*x+1)/a^2/x/(c-c/a^2/x^2
)^(1/2)-2*arcsin(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2/x/(c-c/a^2/x^2)^(1/2
)
```

Rubi [A]
time = 0.20, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6294, 6264, 79, 52, 41, 222}

$$-\frac{2\sqrt{ax+1}\,\sqrt{1-ax}\,\mathrm{ArcSin}(ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{(1-ax)^2}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2(ax+1)(1-ax)}{a^2x\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] -((1 - a*x)^2/(a^2*Sqrt[c - c/(a^2*x^2)]*x)) - (2*(1 - a*x)*(1 + a*x))/(a^2
*Sqrt[c - c/(a^2*x^2)]*x) - (2*Sqrt[1 - a*x]*Sqrt[1 + a*x]*ArcSin[a*x])/(a^
2*Sqrt[c - c/(a^2*x^2)]*x)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

$$= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{x\sqrt{1-ax}}{(1+ax)^{3/2}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{\left(2\sqrt{1-ax}\,\sqrt{1+ax}\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

$$= -\frac{(1-ax)^2}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2(1-ax)(1+ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x} - \frac{2\sqrt{1-ax}\,\sqrt{1+ax}\,\sin^{-1}(ax)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.61

$$\frac{-3+2ax+a^2x^2-2\sqrt{-1+a^2x^2}\,\log\left(ax+\sqrt{-1+a^2x^2}\right)}{a^2\sqrt{c-\frac{c}{a^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (-3 + 2*a*x + a^2*x^2 - 2*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])
/(a^2*Sqrt[c - c/(a^2*x^2)]*x)
```

Maple [A]
time = 0.12, size = 179, normalized size = 1.60

method	result
risch	$\frac{a^{2}x^{2}-1}{a^{2}\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}\,x}+\frac{\left(-\frac{2\ln\left(\frac{a^{2}cx}{\sqrt{a^{2}c}}+\sqrt{a^{2}c\,x^{2}-c}\right)}{a\sqrt{a^{2}c}}+\frac{2\sqrt{a^{2}c\left(x+\frac{1}{a}\right)^{2}-2\left(x+\frac{1}{a}\right)ac}}{a^{3}c\left(x+\frac{1}{a}\right)}\right)\sqrt{c\left(a^{2}x^{2}-1\right)}}{\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}\,x}$
default	$-\frac{\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}}}\left(-\sqrt{c}\,\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}}}\,a^{2}x+2\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}}}\right)acx-2a\sqrt{\frac{c(ax+1)(ax-1)}{a^{2}}}\,\sqrt{c}-\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}}}}{\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}\,x\,c^{\frac{3}{2}}a(ax+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(a^2*x^2-1)/a^2)^(1/2)*(-c^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*x+2*ln(c
^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*a*c*x-2*a*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)
*c^(1/2)-(c*(a^2*x^2-1)/a^2)^(1/2)*a*c^(1/2)+2*ln(c^(1/2)*x+(c*(a^2*x^2-1)/
a^2)^(1/2))*c)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/c^(3/2)/a/(a*x+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*sqrt(c - c/(a^2*x^2))), x)
```

Fricas [A]
time = 0.36, size = 212, normalized size = 1.89

$$\left[\frac{(ax+1)\sqrt{c}\,\log\left(2\,a^{2}cx^{2}-2\,a^{2}\sqrt{c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-c\right)+(a^{2}x^{2}+3\,ax)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx+ac},\ \frac{2\,(ax+1)\sqrt{-c}\,\arctan\left(\frac{a^{2}\sqrt{-c}\,x^{2}\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+(a^{2}x^{2}+3\,ax)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx+ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [((a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) - c) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^2
```

```
*c*x + a*c), (2*(a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2
- c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (a^2*x^2 + 3*a*x)*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)))/(a^2*c*x + a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral((a*x - 1)/(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(1/2)*(a*x + 1)), x)
```

3.868 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x}+\frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}+\frac{2(1-ax)^{3/2}(1+ax)^{3/2}\text{ArcSin}(ax)}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

```
[Out] -1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(3/2)/x+2/3*(-a*x+1)^2*(a*x+1)*(2*a*x+5)/
a^4/(c-c/a^2/x^2)^(3/2)/x^3+2*(-a*x+1)^(3/2)*(a*x+1)^(3/2)*arcsin(a*x)/a^4/
(c-c/a^2/x^2)^(3/2)/x^3
```

Rubi [A]
time = 0.29, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {6302, 6294, 6264, 100, 148, 41, 222}

$$-\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(ax+1)^{3/2}(1-ax)^{3/2}\text{ArcSin}(ax)}{a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}+\frac{2(ax+1)(2ax+5)(1-ax)^2}{3a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] -1/3*(1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(3/2)*x) + (2*(1 - a*x)^2*(1 + a*x)
*(5 + 2*a*x))/(3*a^4*(c - c/(a^2*x^2))^(3/2)*x^3) + (2*(1 - a*x)^(3/2)*(1 +
 a*x)^(3/2)*ArcSin[a*x])/(a^4*(c - c/(a^2*x^2))^(3/2)*x^3)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$$

$$= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^3}{(1-ax)^{3/2}(1+ax)^{3/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x^3}{\sqrt{1-ax}\,(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{\left((1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{x(2-4ax)}{\sqrt{1-ax}\,(1+ax)^{3/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{}{\sqrt{}}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/}}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{\left(2(1-ax)^{3/2}(1+ax)^{3/2}\right)\int \frac{}{\sqrt{}}}{a^3\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{3/2}x} + \frac{2(1-ax)^2(1+ax)(5+2ax)}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3} + \frac{2(1-ax)^{3/2}(1+ax)^{3/2}\sin^{-1}(}{a^4\left(c-\frac{c}{a^2x^2}\right)^{3/2}x^3}$$

Mathematica [A]
time = 0.06, size = 95, normalized size = 0.77

$$\frac{-10-4ax+11a^2x^2+3a^3x^3-6(1+ax)\sqrt{-1+a^2x^2}\,\log\left(ax+\sqrt{-1+a^2x^2}\right)}{3a^2c\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (-10 - 4*a*x + 11*a^2*x^2 + 3*a^3*x^3 - 6*(1 + a*x)*Sqrt[-1 + a^2*x^2]*Log[
a*x + Sqrt[-1 + a^2*x^2]])/(3*a^2*c*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(110) = 220.
time = 0.14, size = 326, normalized size = 2.63

method	result

risch	$\frac{a^2x^2-1}{a^2cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^3\sqrt{a^2c}}-\frac{\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{3a^6c\left(x+\frac{1}{a}\right)^2}+\frac{8\sqrt{a^2c\left(x+\frac{1}{a}\right)}}{3a^5c}\right.}{cx\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}$
default	$\frac{\left(3c^{\frac{3}{2}}\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,a^3x^3+15x^2a^2c^{\frac{3}{2}}\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}-4c^{\frac{3}{2}}\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2x^2-6\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(3*c^(3/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*x^3+15*x^2*a^2*c^(3/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*x^2-6*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*c*x-4*c^(3/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a*x-6*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a*c-12*c^(3/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)+2*(c*(a^2*x^2-1)/a^2)^(1/2)*c^(3/2))*(a*x-1)/(c*(a*x+1)*(a*x-1)/a^2)^(1/2)/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(3/2)/a^4/c^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(3/2)), x)

Fricas [A]
time = 0.35, size = 279, normalized size = 2.25

$$\left[\frac{3\left(a^2x^2+2\,ax+1\right)\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\left(3\,a^3x^3+14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)},\ \frac{6\left(a^2x^2+2\,ax+1\right)\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\left(3\,a^3x^3+14\,a^2x^2+10\,ax\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\left(a^3c^2x^2+2\,a^2c^2x+ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas")

[Out] [1/3*(3*(a^2*x^2 + 2*a*x + 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqr

```
t((a^2*c*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2), 1/3*(6*(
a^2*x^2 + 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/
(a^2*x^2))/(a^2*c*x^2 - c)) + (3*a^3*x^3 + 14*a^2*x^2 + 10*a*x)*sqrt((a^2*c
*x^2 - c)/(a^2*x^2)))/(a^3*c^2*x^2 + 2*a^2*c^2*x + a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{3}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(3/2)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{3/2}\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(3/2)*(a*x + 1)), x)
```

3.869 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x}-\frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2}+\frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3}-\frac{2(1-ax)^3(1+ax)^2(28+13ax)}{15a^6\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}-\frac{2(1-$$

```
[Out] -(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(5/2)/x-2/5*(-a*x+1)^3/a^3/(c-c/a^2/x^2)^(5/2
)/x^2+2/15*(-a*x+1)^3*(a*x+1)/a^4/(c-c/a^2/x^2)^(5/2)/x^3-2/15*(-a*x+1)^3*(
a*x+1)^2*(13*a*x+28)/a^6/(c-c/a^2/x^2)^(5/2)/x^5-2*(-a*x+1)^(5/2)*(a*x+1)^(
5/2)*arcsin(a*x)/a^6/(c-c/a^2/x^2)^(5/2)/x^5
```

Rubi [A]
time = 0.31, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6294, 6264, 100, 155, 148, 41, 222}

$$-\frac{(1-ax)^2}{a^2x\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^{5/2}(1-ax)^{5/2}\text{ArcSin}(ax)}{a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(ax+1)^2(13ax+28)(1-ax)^3}{15a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{5/2}}+\frac{2(ax+1)(1-ax)^3}{15a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}}-\frac{2(1-ax)^3}{5a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] -((1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(5/2)*x)) - (2*(1 - a*x)^3)/(5*a^3*(c
- c/(a^2*x^2))^(5/2)*x^2) + (2*(1 - a*x)^3*(1 + a*x))/(15*a^4*(c - c/(a^2*x
^2))^(5/2)*x^3) - (2*(1 - a*x)^3*(1 + a*x)^2*(28 + 13*a*x))/(15*a^6*(c - c/
(a^2*x^2))^(5/2)*x^5) - (2*(1 - a*x)^(5/2)*(1 + a*x)^(5/2)*ArcSin[a*x])/(a^
6*(c - c/(a^2*x^2))^(5/2)*x^5)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^5}{(1-ax)^{5/2}(1+ax)^{5/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^5}{(1-ax)^{3/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^3(4+2ax)}{\sqrt{1-ax}\,(1+ax)^{7/2}}\,dx}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{\left((1-ax)^{5/2}(1+ax)^{5/2}\right)\int \frac{x^2\left(6a+8a^2x\right.}{\sqrt{1-ax}\,(1+}}{5a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^5}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} + \frac{\left((1-ax)^{5/2}(1-\right.}{15}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+}{15a^6\left(c-\right.}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+}{15a^6\left(c-\right.}$$

$$= -\frac{(1-ax)^2}{a^2\left(c-\frac{c}{a^2x^2}\right)^{5/2}x} - \frac{2(1-ax)^3}{5a^3\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^2} + \frac{2(1-ax)^3(1+ax)}{15a^4\left(c-\frac{c}{a^2x^2}\right)^{5/2}x^3} - \frac{2(1-ax)^3(1+}{15a^6\left(c-\right.}$$

Mathematica [A]
time = 0.07, size = 105, normalized size = 0.54

$$\frac{-56-82ax+32a^2x^2+76a^3x^3+15a^4x^4-30(1+ax)^2\sqrt{-1+a^2x^2}\,\log\left(ax+\sqrt{-1+a^2x^2}\right)}{15a^2c^2\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (-56 - 82*a*x + 32*a^2*x^2 + 76*a^3*x^3 + 15*a^4*x^4 - 30*(1 + a*x)^2*Sqrt[
-1 + a^2*x^2]*Log[a*x + Sqrt[-1 + a^2*x^2]])/(15*a^2*c^2*Sqrt[c - c/(a^2*x^
2)]*x*(1 + a*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(175) = 350$.
time = 0.15, size = 462, normalized size = 2.37

method	result
risch	$\frac{a^2x^2-1}{a^2c^2x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^5\sqrt{a^2c}}-\frac{41\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{60a^8c\left(x+\frac{1}{a}\right)^2}+\frac{383\sqrt{a^2c\,(x+}{120}\right.}{}$
default	$\frac{\left(15c^{\frac{5}{2}}\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{3}{2}}a^5x^5+16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^4x^4+45x^4c^{\frac{5}{2}}a^4\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{3}{2}}+16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}c^{\frac{5}{2}}a^3x^3-60c^{\frac{5}{2}}\left(\frac{c(ax+\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(15*c^(5/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^5*x^5+16*(c*(a^2*x^2-1)/a^
2)^(3/2)*c^(5/2)*a^4*x^4+45*x^4*c^(5/2)*a^4*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)+1
6*(c*(a^2*x^2-1)/a^2)^(3/2)*c^(5/2)*a^3*x^3-60*c^(5/2)*(c*(a*x+1)*(a*x-1)/a
^2)^(3/2)*a^3*x^3-30*(c*(a^2*x^2-1)/a^2)^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/
a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^4*c*x-24*(c*(a^2*x^2-1)/a^2)^(3
/2)*c^(5/2)*a^2*x^2-90*c^(5/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a^2*x^2-30*(c*
(a^2*x^2-1)/a^2)^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(
a*x-1)/a^2)^(3/2)*a^3*c-24*(c*(a^2*x^2-1)/a^2)^(3/2)*c^(5/2)*a*x+50*c^(5/2)
*(c*(a*x+1)*(a*x-1)/a^2)^(3/2)*a*x+6*(c*(a^2*x^2-1)/a^2)^(3/2)*c^(5/2)+50*c
^(5/2)*(c*(a*x+1)*(a*x-1)/a^2)^(3/2))*(a*x-1)/(c*(a*x+1)*(a*x-1)/a^2)^(3/2)
/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)/a^6/c^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(5/2)), x)
```

Fricas [A]
time = 0.38, size = 351, normalized size = 1.80

$$\left[\frac{15\,{\left(a^{4} x^{4} + 2\,a^{3} x^{3} - 2\,a x - 1\right)} \sqrt{c} \log\left(2\,a^{2} c x^{2} - 2\,a^{2} \sqrt{c} x^{2} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}} - c\right) + {\left(15\,a^{5} x^{5} + 76\,a^{4} x^{4} + 32\,a^{3} x^{3} - 82\,a^{2} x^{2} - 56\,a x\right)} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}}}{15\,{\left(a^{5} c^{3} x^{4} + 2\,a^{4} c^{3} x^{3} - 2\,a^{2} c^{3} x - a c^{3}\right)}}, \frac{30\,{\left(a^{4} x^{4} + 2\,a^{3} x^{3} - 2\,a x - 1\right)} \sqrt{-c} \arctan\left(\frac{a^{2} \sqrt{-c} x^{2} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}}}{a^{2} c x^{2} - c}\right) + {\left(15\,a^{5} x^{5} + 76\,a^{4} x^{4} + 32\,a^{3} x^{3} - 82\,a^{2} x^{2} - 56\,a x\right)} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}}}{15\,{\left(a^{5} c^{3} x^{4} + 2\,a^{4} c^{3} x^{3} - 2\,a^{2} c^{3} x - a c^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(a^4*x^4 + 2*a^3*x^3 - 2*a*x - 1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2
*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + (15*a^5*x^5 + 76*a^4*x^
4 + 32*a^3*x^3 - 82*a^2*x^2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5
*c^3*x^4 + 2*a^4*c^3*x^3 - 2*a^2*c^3*x - a*c^3), 1/15*(30*(a^4*x^4 + 2*a^3*
x^3 - 2*a*x - 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2
*x^2))/(a^2*c*x^2 - c)) + (15*a^5*x^5 + 76*a^4*x^4 + 32*a^3*x^3 - 82*a^2*x^
2 - 56*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^5*c^3*x^4 + 2*a^4*c^3*x^3 -
 2*a^2*c^3*x - a*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax - 1}{\left(-c\left(-1 + \frac{1}{ax}\right)\left(1 + \frac{1}{ax}\right)\right)^{\frac{5}{2}}(ax + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(5/2)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a\,x - 1}{{\left(c - \frac{c}{a^2\,x^2}\right)}^{5/2}\,\left(a\,x + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)),x)
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(5/2)*(a*x + 1)), x)
```

3.870 $\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x}+\frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2}+\frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3}+\frac{82(1-ax)^4(1+ax)}{105a^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^4}+\frac{2(1-ax)^4(1+a}{35a^6\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

```
[Out] -1/3*(-a*x+1)^2/a^2/(c-c/a^2/x^2)^(7/2)/x+10/3*(-a*x+1)^3/a^3/(c-c/a^2/x^2)
^(7/2)/x^2+12/7*(-a*x+1)^4/a^4/(c-c/a^2/x^2)^(7/2)/x^3+82/105*(-a*x+1)^4*(a
*x+1)/a^5/(c-c/a^2/x^2)^(7/2)/x^4+2/35*(-a*x+1)^4*(a*x+1)^2/a^6/(c-c/a^2/x^
2)^(7/2)/x^5+2/35*(-a*x+1)^4*(a*x+1)^3*(37*a*x+72)/a^8/(c-c/a^2/x^2)^(7/2)/
x^7+2*(-a*x+1)^(7/2)*(a*x+1)^(7/2)*arcsin(a*x)/a^8/(c-c/a^2/x^2)^(7/2)/x^7
```

Rubi [A]
time = 0.33, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6294, 6264, 100, 155, 148, 41, 222}

$$-\frac{(1-ax)^2}{3a^2x\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^{7/2}(1-ax)^{7/2}\text{ArcSin}(ax)}{a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^3(37ax+72)(1-ax)^4}{35a^8x^7\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{2(ax+1)^2(1-ax)^4}{35a^6x^5\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{82(ax+1)(1-ax)^4}{105a^5x^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{12(1-ax)^4}{7a^4x^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}}+\frac{10(1-ax)^3}{3a^3x^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] -1/3*(1 - a*x)^2/(a^2*(c - c/(a^2*x^2))^(7/2)*x) + (10*(1 - a*x)^3)/(3*a^3*
(c - c/(a^2*x^2))^(7/2)*x^2) + (12*(1 - a*x)^4)/(7*a^4*(c - c/(a^2*x^2))^(7
/2)*x^3) + (82*(1 - a*x)^4*(1 + a*x))/(105*a^5*(c - c/(a^2*x^2))^(7/2)*x^4)
 + (2*(1 - a*x)^4*(1 + a*x)^2)/(35*a^6*(c - c/(a^2*x^2))^(7/2)*x^5) + (2*(1
 - a*x)^4*(1 + a*x)^3*(72 + 37*a*x))/(35*a^8*(c - c/(a^2*x^2))^(7/2)*x^7) +
 (2*(1 - a*x)^(7/2)*(1 + a*x)^(7/2)*ArcSin[a*x])/(a^8*(c - c/(a^2*x^2))^(7/
2)*x^7)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{e^{-2\tanh^{-1}(ax)}x^7}{(1-ax)^{7/2}(1+ax)^{7/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^7}{(1-ax)^{5/2}(1+ax)^{9/2}}\,dx}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^5(6+4ax)}{(1-ax)^{3/2}(1+ax)^{9/2}}\,dx}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{\left((1-ax)^{7/2}(1+ax)^{7/2}\right)\int \frac{x^4(-50a-14}{\sqrt{1-ax}\,(1}}{3a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^7}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{\left((1-ax)^{7/2}(1+\right.}{21}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1-}{105a^5\left(c-\frac{c}{a^2x^2}\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1-}{105a^5\left(c-\frac{c}{a^2x^2}\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1-}{105a^5\left(c-\frac{c}{a^2x^2}\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1-}{105a^5\left(c-\frac{c}{a^2x^2}\right.}$$

$$= -\frac{(1-ax)^2}{3a^2\left(c-\frac{c}{a^2x^2}\right)^{7/2}x} + \frac{10(1-ax)^3}{3a^3\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^2} + \frac{12(1-ax)^4}{7a^4\left(c-\frac{c}{a^2x^2}\right)^{7/2}x^3} + \frac{82(1-ax)^4(1-}{105a^5\left(c-\frac{c}{a^2x^2}\right.}$$

Mathematica [A]

time = 0.08, size = 131, normalized size = 0.49

$$\frac{432+654ax-636a^2x^2-1226a^3x^3+74a^4x^4+562a^5x^5+105a^6x^6-210(-1+ax)(1+ax)^3\sqrt{-1+a^2x^2}\log\left(ax+\sqrt{-1+a^2x^2}\right)}{105a^2\sqrt{c-\frac{c}{a^2x^2}}\,x(-1+ax)(c+acx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (432 + 654*a*x - 636*a^2*x^2 - 1226*a^3*x^3 + 74*a^4*x^4 + 562*a^5*x^5 + 10
5*a^6*x^6 - 210*(-1 + a*x)*(1 + a*x)^3*Sqrt[-1 + a^2*x^2]*Log[a*x + Sqrt[-1
 + a^2*x^2]])/(105*a^2*Sqrt[c - c/(a^2*x^2)]*x*(-1 + a*x)*(c + a*c*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(240) = 480$.

time = 0.18, size = 572, normalized size = 2.12

method	result
risch	$\frac{a^2x^2-1}{a^2c^3x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}+\frac{\left(-\frac{2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{a^7\sqrt{a^2c}}-\frac{1753\sqrt{a^2c\left(x+\frac{1}{a}\right)^2-2\left(x+\frac{1}{a}\right)ac}}{1680a^{10}c\left(x+\frac{1}{a}\right)^2}+\frac{3061\sqrt{a^2c\,(x}{8}\right.}{}$
default	$-\frac{\left(-105c^{\frac{7}{2}}\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{5}{2}}a^7x^7+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^6x^6-553x^6c^{\frac{7}{2}}a^6\left(\frac{c(ax+1)(ax-1)}{a^2}\right)^{\frac{5}{2}}+96\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{5}{2}}c^{\frac{7}{2}}a^5x^5+392c^{\frac{7}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(-105*c^(7/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^7*x^7+96*(c*(a^2*x^2-1
)/a^2)^(5/2)*c^(7/2)*a^6*x^6-553*x^6*c^(7/2)*a^6*(c*(a*x+1)*(a*x-1)/a^2)^(5
/2)+96*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a^5*x^5+392*c^(7/2)*(c*(a*x+1)*(a*
x-1)/a^2)^(5/2)*a^5*x^5-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a^4*x^4+1540*
c^(7/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^4*x^4+210*(c*(a^2*x^2-1)/a^2)^(5/2)
*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^6*
c*x-240*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a^3*x^3-350*c^(7/2)*(c*(a*x+1)*(a
*x-1)/a^2)^(5/2)*a^3*x^3+210*(c*(a^2*x^2-1)/a^2)^(5/2)*ln(c^(1/2)*x+(c*(a^2
*x^2-1)/a^2)^(1/2))*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^5*c+180*(c*(a^2*x^2-1)/
a^2)^(5/2)*c^(7/2)*a^2*x^2-1470*c^(7/2)*(c*(a*x+1)*(a*x-1)/a^2)^(5/2)*a^2*x
^2+180*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)*a*x+42*c^(7/2)*(c*(a*x+1)*(a*x-1)/
a^2)^(5/2)*a*x-30*(c*(a^2*x^2-1)/a^2)^(5/2)*c^(7/2)+462*c^(7/2)*(c*(a*x+1)*
(a*x-1)/a^2)^(5/2))*(a*x-1)/(c*(a*x+1)*(a*x-1)/a^2)^(5/2)/x^7/(c*(a^2*x^2-1
)/a^2/x^2)^(7/2)/a^8/c^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)/((a*x + 1)*(c - c/(a^2*x^2))^(7/2)), x)
```

Fricas [A]
time = 0.40, size = 495, normalized size = 1.83

$$\left[\frac{105\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+(105\,a^7x^7+562\,a^6x^6+74\,a^5x^5-1226\,a^4x^4-636\,a^3x^3+654\,a^2x^2+432\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{105\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)},\ \frac{210\,(a^6x^6+2\,a^5x^5-a^4x^4-4\,a^3x^3-a^2x^2+2\,ax+1)\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(105\,a^7x^7+562\,a^6x^6+74\,a^5x^5-1226\,a^4x^4-636\,a^3x^3+654\,a^2x^2+432\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{105\,(a^7c^4x^6+2\,a^6c^4x^5-a^5c^4x^4-4\,a^4c^4x^3-a^3c^4x^2+2\,a^2c^4x+ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/105*(105*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 + 2*a*x +
1)*sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^
2)) - c) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4 - 636*a^3
*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a^7*c^4*x^6
 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*a^2*c^4*x
+ a*c^4), 1/105*(210*(a^6*x^6 + 2*a^5*x^5 - a^4*x^4 - 4*a^3*x^3 - a^2*x^2 +
 2*a*x + 1)*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
)/(a^2*c*x^2 - c)) + (105*a^7*x^7 + 562*a^6*x^6 + 74*a^5*x^5 - 1226*a^4*x^4
 - 636*a^3*x^3 + 654*a^2*x^2 + 432*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/(a
^7*c^4*x^6 + 2*a^6*c^4*x^5 - a^5*c^4*x^4 - 4*a^4*c^4*x^3 - a^3*c^4*x^2 + 2*
a^2*c^4*x + a*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax-1}{\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{\frac{7}{2}}(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Integral((a*x - 1)/((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**(7/2)*(a*x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x-1)/(a*x+1)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a\,x-1}{{\left(c-\frac{c}{a^2\,x^2}\right)}^{7/2}\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)),x)
```

```
[Out] int((a*x - 1)/((c - c/(a^2*x^2))^(7/2)*(a*x + 1)), x)
```

3.871 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx$

Optimal. Leaf size=322

$$-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{4c^4}{a^3\sqrt{}}$$

```
[Out] -1/8*c^4*(c-c/a^2/x^2)^(1/2)/a^9/x^8/(1-1/a^2/x^2)^(1/2)+3/7*c^4*(c-c/a^2/x
^2)^(1/2)/a^8/x^7/(1-1/a^2/x^2)^(1/2)-8/5*c^4*(c-c/a^2/x^2)^(1/2)/a^6/x^5/(
1-1/a^2/x^2)^(1/2)+3/2*c^4*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+
2*c^4*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-4*c^4*(c-c/a^2/x^2)^(
1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)+c^4*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1
/2)-3*c^4*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{c^4x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9x^8\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8x^7\sqrt{1-\frac{1}{a^2x^2}}}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{2a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4c^4\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/8*(c^4*Sqrt[c - c/(a^2*x^2)])/(a^9*Sqrt[1 - 1/(a^2*x^2)]*x^8) + (3*c^4*S
qrt[c - c/(a^2*x^2)])/(7*a^8*Sqrt[1 - 1/(a^2*x^2)]*x^7) - (8*c^4*Sqrt[c - c
/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*c^4*Sqrt[c - c/(a^2*x^2
)])/(2*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (2*c^4*Sqrt[c - c/(a^2*x^2)])/(a^4*
Sqrt[1 - 1/(a^2*x^2)]*x^3) - (4*c^4*Sqrt[c - c/(a^2*x^2)])/(a^3*Sqrt[1 - 1/
(a^2*x^2)]*x^2) + (c^4*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*
c^4*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{9/2}\,dx=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{9/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^6(1+ax)^3}{x^9}\,dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^4\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^9+\frac{1}{x^9}-\frac{3a}{x^8}+\frac{8a^3}{x^6}-\frac{6a^4}{x^5}-\frac{6a^5}{x^4}+\frac{8a^6}{x^3}-\frac{3a^8}{x}\right)dx}{a^9\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{c^4\sqrt{c-\frac{c}{a^2x^2}}}{8a^9\sqrt{1-\frac{1}{a^2x^2}}\,x^8}+\frac{3c^4\sqrt{c-\frac{c}{a^2x^2}}}{7a^8\sqrt{1-\frac{1}{a^2x^2}}\,x^7}-\frac{8c^4\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3c^4\sqrt{}}{2a^5\sqrt{}}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 0.30

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{9/2}\left(-\frac{1}{8x^8}+\frac{3a}{7x^7}-\frac{8a^3}{5x^5}+\frac{3a^4}{2x^4}+\frac{2a^5}{x^3}-\frac{4a^6}{x^2}+a^9x-3a^8\log(x)\right)}{a^9\left(1-\frac{1}{a^2x^2}\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(9/2)/E^(3*ArcCoth[a*x]),x]
```

[Out] ((c - c/(a^2*x^2))^(9/2)*(-1/8*1/x^8 + (3*a)/(7*x^7) - (8*a^3)/(5*x^5) + (3*a^4)/(2*x^4) + (2*a^5)/x^3 - (4*a^6)/x^2 + a^9*x - 3*a^8*Log[x]))/(a^9*(1 - 1/(a^2*x^2))^(9/2))

Maple [A]
time = 0.04, size = 112, normalized size = 0.35

method	result	size
default	$-\frac{\left(-280 a^{9} x^{9}+840 a^{8} \ln \left(x \right) x^{8}+1120 a^{6} x^{6}-560 a^{5} x^{5}-420 a^{4} x^{4}+448 a^{3} x^{3}-120 a x +35\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{9}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{280 \left(a x -1\right)^{3} \left(a^{2} x^{2}-1\right)^{3}}$	112

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/280*(-280*a^9*x^9+840*a^8*ln(x)*x^8+1120*a^6*x^6-560*a^5*x^5-420*a^4*x^4+448*a^3*x^3-120*a*x+35)*x*(c*(a^2*x^2-1)/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a^2*x^2-1)^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")

[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [A]
time = 0.34, size = 96, normalized size = 0.30

$$\frac{\left(280\, a^{9} c^{4} x^{9} - 840\, a^{8} c^{4} x^{8} \log\left(x\right) - 1120\, a^{6} c^{4} x^{6} + 560\, a^{5} c^{4} x^{5} + 420\, a^{4} c^{4} x^{4} - 448\, a^{3} c^{4} x^{3} + 120\, a c^{4} x - 35\, c^{4}\right) \sqrt{a^{2} c}}{280\, a^{10} x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] 1/280*(280*a^9*c^4*x^9 - 840*a^8*c^4*x^8*log(x) - 1120*a^6*c^4*x^6 + 560*a^5*c^4*x^5 + 420*a^4*c^4*x^4 - 448*a^3*c^4*x^3 + 120*a*c^4*x - 35*c^4)*sqrt(a^2*c)/(a^10*x^8)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(9/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(9/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c-\frac{c}{a^2\,x^2}\right)}^{9/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(9/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.872 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}\,dx$

Optimal. Leaf size=324

$$\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{c^3\sqrt{}}{a^2\sqrt{}}$$

```
[Out] 1/6*c^3*(c-c/a^2/x^2)^(1/2)/a^7/x^6/(1-1/a^2/x^2)^(1/2)-3/5*c^3*(c-c/a^2/x^
2)^(1/2)/a^6/x^5/(1-1/a^2/x^2)^(1/2)+1/4*c^3*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1
-1/a^2/x^2)^(1/2)+5/3*c^3*(c-c/a^2/x^2)^(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-5
/2*c^3*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-c^3*(c-c/a^2/x^2)^(1
/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^3*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
-3*c^3*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 90}

$$\frac{c^3x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7x^6\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6x^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{3a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{5c^3\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c^3*Sqrt[c - c/(a^2*x^2)])/(6*a^7*Sqrt[1 - 1/(a^2*x^2)]*x^6) - (3*c^3*Sqrt
[c - c/(a^2*x^2)])/(5*a^6*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (c^3*Sqrt[c - c/(a^2
*x^2)])/(4*a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (5*c^3*Sqrt[c - c/(a^2*x^2)])/(
3*a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (5*c^3*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqr
t[1 - 1/(a^2*x^2)]*x^2) - (c^3*Sqrt[c - c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*
x^2)]*x) + (c^3*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (3*c^3*Sqr
t[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{7/2}dx=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{7/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^5(1+ax)^2}{x^7}dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c^3\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^7-\frac{1}{x^7}+\frac{3a}{x^6}-\frac{a^2}{x^5}-\frac{5a^3}{x^4}+\frac{5a^4}{x^3}+\frac{a^5}{x^2}-\frac{3a^6}{x}\right)dx}{a^7\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{6a^7\sqrt{1-\frac{1}{a^2x^2}}\,x^6}-\frac{3c^3\sqrt{c-\frac{c}{a^2x^2}}}{5a^6\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{c^3\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{5c^3\sqrt{c}}{3a^4\sqrt{1-}}$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 0.29

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{7/2}\left(\frac{10-36ax+15a^2x^2+100a^3x^3-150a^4x^4-60a^5x^5+60a^7x^7}{60x^6}-3a^6\log(x)\right)}{a^7\left(1-\frac{1}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(7/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(7/2)*((10 - 36*a*x + 15*a^2*x^2 + 100*a^3*x^3 - 150*a^4
*x^4 - 60*a^5*x^5 + 60*a^7*x^7)/(60*x^6) - 3*a^6*Log[x]))/(a^7*(1 - 1/(a^2*
x^2))^(7/2))
```

Maple [A]
time = 0.04, size = 112, normalized size = 0.35

method	result	size
default	$-\frac{\left(-60 a^{7} x^{7}+180 a^{6} \ln \left(x \right) x^{6}+60 a^{5} x^{5}+150 a^{4} x^{4}-100 a^{3} x^{3}-15 a^{2} x^{2}+36 a x -10\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{7}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{60 \left(a x -1\right)^{3} \left(a^{2} x^{2}-1\right)^{2}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(-60*a^7*x^7+180*a^6*ln(x)*x^6+60*a^5*x^5+150*a^4*x^4-100*a^3*x^3-15*
a^2*x^2+36*a*x-10)*x*(c*(a^2*x^2-1)/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2)/
(a*x-1)^3/(a^2*x^2-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 96, normalized size = 0.30

$$\frac{\left(60 \, a^{7} c^{3} x^{7} - 180 \, a^{6} c^{3} x^{6} \log\left(x\right) - 60 \, a^{5} c^{3} x^{5} - 150 \, a^{4} c^{3} x^{4} + 100 \, a^{3} c^{3} x^{3} + 15 \, a^{2} c^{3} x^{2} - 36 \, a c^{3} x + 10 \, c^{3}\right)} \sqrt{a^{2} c}}{60 \, a^{8} x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/60*(60*a^7*c^3*x^7 - 180*a^6*c^3*x^6*log(x) - 60*a^5*c^3*x^5 - 150*a^4*c^
3*x^4 + 100*a^3*c^3*x^3 + 15*a^2*c^3*x^2 - 36*a*c^3*x + 10*c^3)*sqrt(a^2*c)
/(a^8*x^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(7/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(7/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c-\frac{c}{a^2\,x^2}\right)^{7/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(7/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.873 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}\,dx$

Optimal. Leaf size=235

$$-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\sqrt{c-\cdots}}{a\sqrt{1\cdots}}$$

```
[Out] -1/4*c^2*(c-c/a^2/x^2)^(1/2)/a^5/x^4/(1-1/a^2/x^2)^(1/2)+c^2*(c-c/a^2/x^2)^
(1/2)/a^4/x^3/(1-1/a^2/x^2)^(1/2)-c^2*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/
x^2)^(1/2)-2*c^2*(c-c/a^2/x^2)^(1/2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c^2*x*(c-c/a
^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-3*c^2*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^
2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 76}

$$\frac{c^2x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] -1/4*(c^2*Sqrt[c - c/(a^2*x^2)])/(a^5*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (c^2*Sqr
t[c - c/(a^2*x^2)])/(a^4*Sqrt[1 - 1/(a^2*x^2)]*x^3) - (c^2*Sqrt[c - c/(a^2*
x^2)])/(a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (2*c^2*Sqrt[c - c/(a^2*x^2)])/(a^2
*Sqrt[1 - 1/(a^2*x^2)]*x) + (c^2*Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x
^2)] - (3*c^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{5/2}dx=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{5/2}dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^4(1+ax)}{x^5}dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\left(c^2\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^5+\frac{1}{x^5}-\frac{3a}{x^4}+\frac{2a^2}{x^3}+\frac{2a^3}{x^2}-\frac{3a^4}{x}\right)dx}{a^5\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^5\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^4\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{c^2\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{2c^2\sqrt{c-}}{a^2\sqrt{1-}}$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 0.34

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{5/2}\left(-\frac{5a^4}{4}-\frac{1}{4x^4}+\frac{a}{x^3}-\frac{a^2}{x^2}-\frac{2a^3}{x}+a^5x-3a^4\log(x)\right)}{a^5\left(1-\frac{1}{a^2x^2}\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - c/(a^2*x^2))^(5/2)/E^(3*ArcCoth[a*x]),x]
[Out] ((c - c/(a^2*x^2))^(5/2)*((-5*a^4)/4 - 1/(4*x^4) + a/x^3 - a^2/x^2 - (2*a^3
)/x + a^5*x - 3*a^4*Log[x]))/(a^5*(1 - 1/(a^2*x^2))^(5/2))
```

Maple [A]
time = 0.04, size = 96, normalized size = 0.41

method	result	size
default	$-\frac{\left(-4 a^{5} x^{5}+12 \ln \left(x \right) a^{4} x^{4}+8 a^{3} x^{3}+4 a^{2} x^{2}-4 a x +1\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{5}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{4 \left(a x -1\right)^{3} \left(a^{2} x^{2}-1\right)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-4*a^5*x^5+12*ln(x)*a^4*x^4+8*a^3*x^3+4*a^2*x^2-4*a*x+1)*x*(c*(a^2*x^
2-1)/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3/(a^2*x^2-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.34, size = 74, normalized size = 0.31

$$\frac{\left(4 \, a^{5} c^{2} x^{5} - 12 \, a^{4} c^{2} x^{4} \log\left(x\right) - 8 \, a^{3} c^{2} x^{3} - 4 \, a^{2} c^{2} x^{2} + 4 \, a c^{2} x - c^{2}\right)} \sqrt{a^{2} c}}{4 \, a^{6} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/4*(4*a^5*c^2*x^5 - 12*a^4*c^2*x^4*log(x) - 8*a^3*c^2*x^3 - 4*a^2*c^2*x^2
 + 4*a*c^2*x - c^2)*sqrt(a^2*c)/(a^6*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(5/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(5/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c-\frac{c}{a^2\,x^2}\right)^{5/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(5/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.874 $\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*c*(c-c/a^2/x^2)^(1/2)/a^3/x^2/(1-1/a^2/x^2)^(1/2)-3*c*(c-c/a^2/x^2)^(1/
2)/a^2/x/(1-1/a^2/x^2)^(1/2)+c*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-3*
c*ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 45}

$$\frac{cx\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3x^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (c*Sqrt[c - c/(a^2*x^2)])/(2*a^3*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (3*c*Sqrt[c -
 c/(a^2*x^2)])/(a^2*Sqrt[1 - 1/(a^2*x^2)]*x) + (c*Sqrt[c - c/(a^2*x^2)]*x)/
Sqrt[1 - 1/(a^2*x^2)] - (3*c*Sqrt[c - c/(a^2*x^2)]*Log[x])/(a*Sqrt[1 - 1/(a
^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^{3/2}\,dx=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int e^{-3\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^{3/2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\frac{(-1+ax)^3}{x^3}\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\left(c\sqrt{c-\frac{c}{a^2x^2}}\right)\int\left(a^3-\frac{1}{x^3}+\frac{3a}{x^2}-\frac{3a^2}{x}\right)\,dx}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{c\sqrt{c-\frac{c}{a^2x^2}}}{2a^3\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{c\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3c\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{}{a^2}}}$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 0.40

$$\frac{\left(c-\frac{c}{a^2x^2}\right)^{3/2}\left(\frac{1}{2x^2}-\frac{3a}{x}+a^3x-3a^2\log(x)\right)}{a^3\left(1-\frac{1}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - c/(a^2*x^2))^(3/2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^(3/2)*(1/(2*x^2) - (3*a)/x + a^3*x - 3*a^2*Log[x]))/(a^3
*(1 - 1/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.03, size = 69, normalized size = 0.47

method	result	size
default	$-\frac{\left(-2 a^{3} x^{3}+6 a^{2} \ln \left(x \right) x^{2}+6 a x -1\right) x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}\right)^{\frac{3}{2}} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 \left(a x -1\right)^{3}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-2*a^3*x^3+6*a^2*ln(x)*x^2+6*a*x-1)*x*(c*(a^2*x^2-1)/a^2/x^2)^(3/2)*(
(a*x-1)/(a*x+1))^(3/2)/(a*x-1)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.34, size = 42, normalized size = 0.28

$$\frac{\left(2 a^{3} c x^{3}-6 a^{2} c x^{2} \log \left(x\right)-6 a c x+c\right) \sqrt{a^{2} c}}{2 a^{4} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*c*x^3 - 6*a^2*c*x^2*log(x) - 6*a*c*x + c)*sqrt(a^2*c)/(a^4*x^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(3/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(3/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c-\frac{c}{a^2\,x^2}\right)^{3/2}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(3/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.875 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6332, 6328, 84}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 0.44

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A]
time = 0.03, size = 67, normalized size = 0.63

method	result	size
default	$-\frac{(-ax+4\ln(ax+1)-\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/
(a*x+1))^(3/2)/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.41, size = 25, normalized size = 0.23

$$\frac{\sqrt{a^2c}\,(ax - 4\log(ax+1) + \log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.876 $\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/x^2)^(1/2)-2*(1-1/a^2/x^2)^(1/2)/a/(a*x+1)/(
c-c/a^2/x^2)^(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 78}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(ax+1)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/Sqrt[c - c/(a^2*x^2)] - (2*Sqrt[1 - 1/(a^2*x^2)])
/(a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x
])/(a*Sqrt[c - c/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx &= \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x(-1+ax)}{(1+ax)^2}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\left(a\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a}+\frac{2}{a(1+ax)^2}-\frac{3}{a(1+ax)}\right)dx}{\sqrt{c-\frac{c}{a^2x^2}}}\\
&= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{\sqrt{c-\frac{c}{a^2x^2}}}-\frac{2\sqrt{1-\frac{1}{a^2x^2}}}{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1+ax)}{a\sqrt{c-\frac{c}{a^2x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.48

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(ax-\frac{2}{1+ax}-3\log(1+ax)\right)}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(a*x - 2/(1 + a*x) - 3*Log[1 + a*x]))/(a*Sqrt[c - c/
(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 87, normalized size = 0.77

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(-a^{2}x^{2}+3\ln(ax+1)ax-ax+3\ln(ax+1)+2\right)}{(ax-1)\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}\,x\,a^{2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x-1)/(a*x+1))^(3/2)/(a*x-1)*(a*x+1)*(-a^2*x^2+3*ln(a*x+1)*a*x-a*x+3*ln
(a*x+1)+2)/(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/sqrt(c - c/(a^2*x^2)), x)
```

Fricas [A]
time = 0.39, size = 47, normalized size = 0.42

$$\frac{\left(a^{2}x^{2}+ax-3\,(ax+1)\log\left(ax+1\right)-2\right)\sqrt{a^{2}c}}{a^{3}cx+a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (a^2*x^2 + a*x - 3*(a*x + 1)*log(a*x + 1) - 2)*sqrt(a^2*c)/(a^3*c*x + a^2*c
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARa*sageVA
Rx+1)]sym
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(1/2), x)
```

3.877 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(1+ax)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c/(c-c/a^2/x^2)^(1/2)+1/2*(1-1/a^2/x^2)^(1/2)/a/c/(a*
x+1)^2/(c-c/a^2/x^2)^(1/2)-3*(1-1/a^2/x^2)^(1/2)/a/c/(a*x+1)/(c-c/a^2/x^2)^
(1/2)-3*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}\,\log(ax+1)}{ac\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2)]
/(2*a*c*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (3*Sqrt[1 - 1/(a^2*x^2)])/(a*c
*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) - (3*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/
(a*c*Sqrt[c - c/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{3/2}}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^3}{(1+ax)^3}\,dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^3\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^3}-\frac{1}{a^3(1+ax)^3}+\frac{3}{a^3(1+ax)^2}-\frac{3}{a^3(1+ax)}\right)dx}{c\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}-\frac{3\sqrt{1-\frac{1}{a^2x^2}}}{ac\sqrt{c-}}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.38

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{3/2}\left(2ax+\frac{-5-6ax}{(1+ax)^2}-6\log(1+ax)\right)}{2a\left(c-\frac{c}{a^2x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(3/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(3/2)*(2*a*x + (-5 - 6*a*x)/(1 + a*x)^2 - 6*Log[1 + a*x]
))/(2*a*(c - c/(a^2*x^2))^(3/2))
```

Maple [A]
time = 0.04, size = 102, normalized size = 0.61

method	result	size
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax+1)\left(-2a^{3}x^{3}+6\ln\left(ax+1\right)a^{2}x^{2}-4a^{2}x^{2}+12\ln\left(ax+1\right)ax+4ax+6\ln\left(ax+1\right)+5\right)}{2a^{4}x^{3}\left(\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}\right)^{\frac{3}{2}}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*x-1)/(a*x+1))^(3/2)*(a*x+1)*(-2*a^3*x^3+6*ln(a*x+1)*a^2*x^2-4*a^2*
x^2+12*ln(a*x+1)*a*x+4*a*x+6*ln(a*x+1)+5)/a^4/x^3/(c*(a^2*x^2-1)/a^2/x^2)^(
3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 81, normalized size = 0.48

$$\frac{\left(2\,a^{3}x^{3}+4\,a^{2}x^{2}-4\,ax-6\,\left(a^{2}x^{2}+2\,ax+1\right)\log\left(ax+1\right)-5\right)\sqrt{a^{2}c}}{2\,\left(a^{4}c^{2}x^{2}+2\,a^{3}c^{2}x+a^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a^3*x^3 + 4*a^2*x^2 - 4*a*x - 6*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1) -
 5)*sqrt(a^2*c)/(a^4*c^2*x^2 + 2*a^3*c^2*x + a^2*c^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(3/2), x)
```

3.878 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx$

Optimal. Leaf size=264

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^3}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}+\frac{\sqrt{1-}}{16a}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^2/(c-c/a^2/x^2)^(1/2)-1/6*(1-1/a^2/x^2)^(1/2)/a/c^2
/(a*x+1)^3/(c-c/a^2/x^2)^(1/2)+9/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)^2/(c-c
/a^2/x^2)^(1/2)-31/8*(1-1/a^2/x^2)^(1/2)/a/c^2/(a*x+1)/(c-c/a^2/x^2)^(1/2)+
1/16*ln(-a*x+1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)-49/16*ln(a*x+
1)*(1-1/a^2/x^2)^(1/2)/a/c^2/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{31\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{49\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{16ac^2\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^2*Sqrt[c - c/(a^2*x^2)]) - Sqrt[1 - 1/(a^2*x^2
)]/(6*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (9*Sqrt[1 - 1/(a^2*x^2)])/
(8*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2) - (31*Sqrt[1 - 1/(a^2*x^2)])/(8
*a*c^2*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)) + (Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)]) - (49*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*
x])/(16*a*c^2*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
```

```
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{5/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{5/2}}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^5}{(-1+ax)(1+ax)^4}\,dx}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^5\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^5}+\frac{1}{16a^5(-1+ax)}+\frac{1}{2a^5(1+ax)^4}-\frac{9}{4a^5(1+ax)^3}+\frac{31}{8a^5(1+ax)^2}-\frac{49}{16a^5(1+ax)}\right.}{c^2\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^2\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\sqrt{1-\frac{1}{a^2x^2}}}{6ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^3} + \frac{9\sqrt{1-\frac{1}{a^2x^2}}}{8ac^2\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2} - \frac{31\sqrt{}}{8ac^2\sqrt{c-}}$$

Mathematica [A]
time = 0.07, size = 85, normalized size = 0.32

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{5/2}\left(48ax-\frac{8}{(1+ax)^3}+\frac{54}{(1+ax)^2}-\frac{186}{1+ax}+3\log(1-ax)-147\log(1+ax)\right)}{48a\left(c-\frac{c}{a^2x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(5/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(5/2)*(48*a*x - 8/(1 + a*x)^3 + 54/(1 + a*x)^2 - 186/(1
+ a*x) + 3*Log[1 - a*x] - 147*Log[1 + a*x]))/(48*a*(c - c/(a^2*x^2))^(5/2))
```

Maple [A]
time = 0.04, size = 175, normalized size = 0.66

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}(ax-1)(ax+1)\left(-48a^4x^4+147\ln(ax+1)a^3x^3-3x^3\ln(ax-1)a^3-144a^3x^3+441\ln(ax+1)a^2x^2-9x^2\ln(ax-1)a^2+42a^2x^2\right.}{48a^6x^5\left(\frac{c\left(a^2x^2-1\right)}{a^2x^2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-48*a^4*x^4+147*ln(a*x+1)*a^
3*x^3-3*x^3*ln(a*x-1)*a^3-144*a^3*x^3+441*ln(a*x+1)*a^2*x^2-9*x^2*ln(a*x-1)
*a^2+42*a^2*x^2+441*ln(a*x+1)*a*x-9*x*ln(a*x-1)*a+270*a*x+147*ln(a*x+1)-3*l
n(a*x-1)+140)/a^6/x^5/(c*(a^2*x^2-1)/a^2/x^2)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

Fricas [A]
time = 0.36, size = 137, normalized size = 0.52

$$\frac{\left(48\,a^4x^4+144\,a^3x^3-42\,a^2x^2-270\,ax-147\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\log\left(ax+1\right)+3\left(a^3x^3+3\,a^2x^2+3\,ax+1\right)\log\left(ax-1\right)-140\right)\sqrt{a^2c}}{48\left(a^5c^3x^3+3\,a^4c^3x^2+3\,a^3c^3x+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="fricas"
)
```

```
[Out] 1/48*(48*a^4*x^4 + 144*a^3*x^3 - 42*a^2*x^2 - 270*a*x - 147*(a^3*x^3 + 3*a^
2*x^2 + 3*a*x + 1)*log(a*x + 1) + 3*(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)*log(a
*x - 1) - 140)*sqrt(a^2*c)/(a^5*c^3*x^3 + 3*a^4*c^3*x^2 + 3*a^3*c^3*x + a^2
*c^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(5/2), x)
```

3.879 $\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx$

Optimal. Leaf size=357

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^4}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^3}+\frac{}{32a}$$

```
[Out] x*(1-1/a^2/x^2)^(1/2)/c^3/(c-c/a^2/x^2)^(1/2)+1/32*(1-1/a^2/x^2)^(1/2)/a/c^
3/(-a*x+1)/(c-c/a^2/x^2)^(1/2)+1/16*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^4/(c-
c/a^2/x^2)^(1/2)-1/2*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^3/(c-c/a^2/x^2)^(1/2
)+59/32*(1-1/a^2/x^2)^(1/2)/a/c^3/(a*x+1)^2/(c-c/a^2/x^2)^(1/2)-75/16*(1-1/
a^2/x^2)^(1/2)/a/c^3/(a*x+1)/(c-c/a^2/x^2)^(1/2)+9/64*ln(-a*x+1)*(1-1/a^2/x
^2)^(1/2)/a/c^3/(c-c/a^2/x^2)^(1/2)-201/64*ln(a*x+1)*(1-1/a^2/x^2)^(1/2)/a/
c^3/(c-c/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 90}

$$\frac{x\sqrt{1-\frac{1}{a^2x^2}}}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}-\frac{75\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{59\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}-\frac{\sqrt{1-\frac{1}{a^2x^2}}}{2ac^3(ax+1)^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3(ax+1)^4\sqrt{c-\frac{c}{a^2x^2}}}+\frac{9\sqrt{1-\frac{1}{a^2x^2}}\log(1-ax)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}-\frac{201\sqrt{1-\frac{1}{a^2x^2}}\log(ax+1)}{64ac^3\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*x)/(c^3*Sqrt[c - c/(a^2*x^2)]) + Sqrt[1 - 1/(a^2*x^2
)]/(32*a*c^3*Sqrt[c - c/(a^2*x^2)]*(1 - a*x)) + Sqrt[1 - 1/(a^2*x^2)]/(16*a
*c^3*Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^4) - Sqrt[1 - 1/(a^2*x^2)]/(2*a*c^3*Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^3) + (59*Sqrt[1 - 1/(a^2*x^2)])/(32*a*c^3*Sqr
t[c - c/(a^2*x^2)]*(1 + a*x)^2) - (75*Sqrt[1 - 1/(a^2*x^2)])/(16*a*c^3*Sqrt
[c - c/(a^2*x^2)]*(1 + a*x)) + (9*Sqrt[1 - 1/(a^2*x^2)]*Log[1 - a*x])/(64*a
*c^3*Sqrt[c - c/(a^2*x^2)]) - (201*Sqrt[1 - 1/(a^2*x^2)]*Log[1 + a*x])/(64*
a*c^3*Sqrt[c - c/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^{7/2}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}}{\left(1-\frac{1}{a^2x^2}\right)^{7/2}}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \frac{x^7}{(-1+ax)^2(1+ax)^5}\,dx}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\left(a^7\sqrt{1-\frac{1}{a^2x^2}}\right)\int \left(\frac{1}{a^7}+\frac{1}{32a^7(-1+ax)^2}+\frac{9}{64a^7(-1+ax)}-\frac{1}{4a^7(1+ax)^5}+\frac{3}{2a^7(1+ax)^4}-\frac{59}{16a^7(1+ \cdots}{c^3\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\,x}{c^3\sqrt{c-\frac{c}{a^2x^2}}}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{32ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}+\frac{\sqrt{1-\frac{1}{a^2x^2}}}{16ac^3\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^4}-\frac{\cdots}{2ac^3\sqrt{c \cdots}}$$

Mathematica [A]
time = 0.12, size = 105, normalized size = 0.29

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{7/2}\left(2\left(32ax+\frac{1}{1-ax}+\frac{2}{(1+ax)^4}-\frac{16}{(1+ax)^3}+\frac{59}{(1+ax)^2}-\frac{150}{1+ax}\right)+9\log(1-ax)-201\log(1+ax)\right)}{64a\left(c-\frac{c}{a^2x^2}\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(3*ArcCoth[a*x])*(c - c/(a^2*x^2))^(7/2)),x]
```

```
[Out] ((1 - 1/(a^2*x^2))^(7/2)*(2*(32*a*x + (1 - a*x)^(-1) + 2/(1 + a*x)^4 - 16/(
1 + a*x)^3 + 59/(1 + a*x)^2 - 150/(1 + a*x)) + 9*Log[1 - a*x] - 201*Log[1 +
 a*x]))/(64*a*(c - c/(a^2*x^2))^(7/2))
```

Maple [A]
time = 0.04, size = 247, normalized size = 0.69

method	result
default	$-\frac{\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}} (ax-1)(ax+1)\left(-64a^6x^6+201\ln(ax+1)a^5x^5-9x^5\ln(ax-1)a^5-192a^5x^5+603\ln(ax+1)a^4x^4-27x^4\ln(ax-1)a^4+174a^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*((a*x-1)/(a*x+1))^(3/2)*(a*x-1)*(a*x+1)*(-64*a^6*x^6+201*ln(a*x+1)*a^
5*x^5-9*x^5*ln(a*x-1)*a^5-192*a^5*x^5+603*ln(a*x+1)*a^4*x^4-27*x^4*ln(a*x-1
)*a^4+174*a^4*x^4+402*ln(a*x+1)*a^3*x^3-18*x^3*ln(a*x-1)*a^3+618*a^3*x^3-40
2*ln(a*x+1)*a^2*x^2+18*x^2*ln(a*x-1)*a^2+118*a^2*x^2-603*ln(a*x+1)*a*x+27*x
*ln(a*x-1)*a-414*a*x-201*ln(a*x+1)+9*ln(a*x-1)-208)/a^8/x^7/(c*(a^2*x^2-1)/
a^2/x^2)^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

Fricas [A]
time = 0.36, size = 208, normalized size = 0.58

$$\frac{\left(64\,a^{6}x^{6}+192\,a^{5}x^{5}-174\,a^{4}x^{4}-618\,a^{3}x^{3}-118\,a^{2}x^{2}+414\,ax-201\,\left(a^{5}x^{5}+3\,a^{4}x^{4}+2\,a^{3}x^{3}-2\,a^{2}x^{2}-3\,ax-1\right)\log\left(ax+1\right)+9\,\left(a^{5}x^{5}+3\,a^{4}x^{4}+2\,a^{3}x^{3}-2\,a^{2}x^{2}-3\,ax-1\right)\log\left(ax-1\right)+208\right)\sqrt{a^{2}c}}{64\,\left(a^{7}c^{4}x^{5}+3\,a^{6}c^{4}x^{4}+2\,a^{5}c^{4}x^{3}-2\,a^{4}c^{4}x^{2}-3\,a^{3}c^{4}x-a^{2}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="fricas"
)
```

```
[Out] 1/64*(64*a^6*x^6 + 192*a^5*x^5 - 174*a^4*x^4 - 618*a^3*x^3 - 118*a^2*x^2 +
414*a*x - 201*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x + 1) + 9*(a^5*x^5 + 3*a^4*x^4 + 2*a^3*x^3 - 2*a^2*x^2 - 3*a*x - 1)*log
(a*x - 1) + 208)*sqrt(a^2*c)/(a^7*c^4*x^5 + 3*a^6*c^4*x^4 + 2*a^5*c^4*x^3 -
 2*a^4*c^4*x^2 - 3*a^3*c^4*x - a^2*c^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))**(3/2)/(c-c/a**2/x**2)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a*x-1)/(a*x+1))^(3/2)/(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}}{\left(c-\frac{c}{a^2\,x^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2),x)
```

```
[Out] int(((a*x - 1)/(a*x + 1))^(3/2)/(c - c/(a^2*x^2))^(7/2), x)
```

3.880 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\, x^m\, dx$

Optimal. Leaf size=80

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\, x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\, x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/2)
/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6332, 6328, 45}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x^{-1+m}+ax^m\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 53, normalized size = 0.66

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{x^m}{m}+\frac{ax^{1+m}}{1+m}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^m,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^m/m + (a*x^(1 + m))/(1 + m)))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [**A**]
time = 0.03, size = 63, normalized size = 0.79

method	result	size
gosper	$\frac{x^{1+m}(axm+m+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(1+m)m(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	63
risch	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, x\sqrt{\frac{c(a^2x^2-1)}{(ax+1)(ax-1)}}\,(ax-1)(axm+m+1)x^m}{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{c}\,(a^2x^2-1)(1+m)m}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] x^(1+m)*(a*m*x+m+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(1+m)/m/(a*x+1)/((a*x-1)/
(a*x+1))^(1/2)
```

Maxima [A]
time = 0.30, size = 44, normalized size = 0.55

$$\frac{\left(a\sqrt{c}\,mx+\sqrt{c}\,(m+1)\right)(ax+1)x^m}{(m^2+m)a^2x+(m^2+m)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] (a*sqrt(c)*m*x + sqrt(c)*(m + 1))*(a*x + 1)*x^m/((m^2 + m)*a^2*x + (m^2 + m
)*a)
```

Fricas [A]
time = 0.35, size = 72, normalized size = 0.90

$$-\frac{(amx^2+(m+1)x)x^m\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-(am^2+am)x+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -(a*m*x^2 + (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**m*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^m*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^m/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((x^m*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.881 $\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/
2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 45}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*
x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
```

```
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.59

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x^2,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 53, normalized size = 0.70

method	result	size

gosper	$\frac{x^{3}(2ax+3)\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53
default	$\frac{x^{3}(2ax+3)\sqrt{\frac{c(a^{2}x^{2}-1)}{a^{2}x^{2}}}}{6(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/6*x^3*(2*a*x+3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.40, size = 24, normalized size = 0.32

$$\frac{(2\,ax^3+3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/6*(2*a*x^3 + 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**B**]
time = 1.44, size = 46, normalized size = 0.61

$$\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x+3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x + 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(a
*x - 1))
```

3.882 $\int e^{\coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2x^2}}\, x \, dx$

Optimal. Leaf size=71

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\, x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\, x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6332, 6328}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)
]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int(1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(x+\frac{ax^2}{2}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)]*x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x + (a*x^2)/2))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 52, normalized size = 0.73

method	result	size
gosper	$\frac{x^2(ax+2)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52
default	$\frac{x^2(ax+2)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*x^2*(a*x+2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.36, size = 21, normalized size = 0.30

$$\frac{\sqrt{a^2c}\,(ax^2+2\,x)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(a^2*c)*(a*x^2 + 2*x)/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B]
time = 1.40, size = 45, normalized size = 0.63

$$\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*x
 - 1))
```

3.883 $\int e^{\coth^{-1}(ax)} \sqrt{c-\frac{c}{a^2x^2}}\, dx$

Optimal. Leaf size=67

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\, x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\, \log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.58

$$
\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 50, normalized size = 0.75

method	result	size

default	$\frac{(ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.33, size = 17, normalized size = 0.25

$$\frac{\sqrt{a^2c}\,(ax+\log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x + log(x))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/sqrt((a*x - 1)/(a*x + 1)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))/sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2),x)
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.884 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1
/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6332, 6328, 45}

$$\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) + (Sqrt[c - c/(a^2*x^2
)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}+\frac{a}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [**A**]
time = 0.02, size = 43, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{x}+a\log(x)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [**A**]
time = 0.03, size = 50, normalized size = 0.71

method	result	size
default	$\frac{(a\ln(x)x-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE
)
```

```
[Out] (a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/(a*x+1)/((a*x-1)/(a*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.36, size = 21, normalized size = 0.30

$$\frac{\sqrt{a^2c}\,(ax\log(x)-1)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) - 1)/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x\sqrt{\frac{ax-1}{ax+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/(x*sqrt((a*x - 1)/(a*x + 1))
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,\sqrt{\frac{a\,x-1}{a\,x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(1/2)), x)
```

3.885 $\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

```
[Out] -1/2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {6332, 6328, 37}

$$-\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.02

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2x^2}-\frac{a}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcCoth[a*x]*Sqrt[c - c/(a^2*x^2)])/x^2,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 - a/x))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 53, normalized size = 1.15

method	result	size
gosper	$-\frac{(2ax+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

default	$-\frac{(2ax+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(ax+1)\sqrt{\frac{ax-1}{ax+1}}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*(2*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x/(a*x+1)/((a*x-1)/(a*x+1))^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Fricas [A]
time = 0.35, size = 21, normalized size = 0.46

$$-\frac{\sqrt{a^2c}\,(2\,ax+1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(a^2*c)*(2*a*x + 1)/(a^2*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(1/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(1/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*sqrt((a*x - 1)/(a*x + 1))), x)
```

Mupad [B]
time = 1.40, size = 63, normalized size = 1.37

$$\frac{\left(x\sqrt{c-\frac{c}{a^2\,x^2}}+\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(1/2)),x)
```

```
[Out] ((x*(c - c/(a^2*x^2))^(1/2) + (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a*
x + 1))^(1/2))/(x/a - x^2)
```

3.886 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=160

$$\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}+\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}}{8a^3\sqrt{1-a}}$$

```
[Out] 7/8*x*(c-c/a^2/x^2)^(1/2)/a^3+7/24*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3+1/6*x*
(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2
-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6302, 6294, 6264, 92, 81, 52, 41, 222}

$$\frac{x^2(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{7x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{8a^3\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}+\frac{7x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}+\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) + (7*Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))
/(24*a^3) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(6*a^3) + (Sqrt[c - c/(a^
2*x^2)]*x^2*(1 + a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/
(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(-1-2ax)(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{4a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right)}{12a^2\sqrt{1-ax}}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1+}{4a^2}$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} +$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} +$$

$$= \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} + \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{24a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{6a^3} +$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.58

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}\,(32+21ax+16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(32 + 21*a*x + 16*a^2*x^2 + 6*
a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A]
time = 0.12, size = 196, normalized size = 1.22

method	result
risch	$\frac{(6a^3x^3+16a^2x^2+21ax+32)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, x}{24a^3}+\frac{7\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{8a^2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(-6x\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^4-16\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-27\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+27c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-\right.}{24\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\,a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4-1
6*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*c*x+27*c^(
3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln(((c*(a*x+1)*(a*x
-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))-48*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a*c)/
(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x - 1), x)
```

Fricas [A]
time = 0.37, size = 222, normalized size = 1.39

$$\left[\frac{2\,(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+21\,\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{48\,a^4},\ \frac{(6\,a^4x^4+16\,a^3x^3+21\,a^2x^2+32\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-21\,\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)}{24\,a^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 + 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
```

```
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 + 16*a^3*x^3 + 21*a^2*x^2 +
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{-c \left(-1+\frac{1}{ax}\right) \left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**3*(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.42, size = 128, normalized size = 0.80

$$\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\operatorname{sgn}(x)}{a^2}+\frac{8\operatorname{sgn}(x)}{a^3}\right)+\frac{21\operatorname{sgn}(x)}{a^4}\right)x+\frac{32\operatorname{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^4|a|}+\frac{(21\,a\sqrt{c}\,\log\left(|c|\right)-64\sqrt{-c}\,|a|)\operatorname{sgn}(x)}{a^5|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 + 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x + 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) - 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.887 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(a*x+
1)^2*(c-c/a^2/x^2)^(1/2)/a^2-x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+1)
^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6302, 6294, 6264, 81, 52, 41, 222}

$$-\frac{x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{x(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x)^2)/(3*a^2) - (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx=-\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$$

$$=-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}x\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$=-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{x(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{3a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int\frac{\sqrt{}}{\sqrt{}}}{a\sqrt{1-ax}\,\sqrt{1+}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(\sqrt{c\right.}}{}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\left(\sqrt{c\right.}}{}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)^2}{3a^2}-\frac{\sqrt{c-}}{a^2\sqrt{}}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.68

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}\,(5+3ax+a^2x^2)+3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 + 3*a*x + a^2*x^2) + 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A]
time = 0.12, size = 174, normalized size = 1.41

method	result
risch	$\frac{(a^2x^2+3ax+5)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{3a^2}+\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{a\sqrt{a^2c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(-\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+3c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-}{a^2}}}{\sqrt{c}}\right.\right.}{3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(
a^2*x^2-1)/a^2)^(1/2)*a^2*c*x+3*c^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1
/2))-6*c^(3/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))-6*(c
*(a*x+1)*(a*x-1)/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x - 1), x)
```

Fricas [A]
time = 0.35, size = 204, normalized size = 1.66

$$\left[\frac{2\,(a^3x^3+3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\ \frac{(a^3x^3+3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)}{3\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*s
qrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c))/a^3, 1/3*((a^3*x^3 + 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)) - 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)))/a^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.41, size = 116, normalized size = 0.94

$$\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{3\,\mathrm{sgn}(x)}{a^3}\right)+\frac{5\,\mathrm{sgn}(x)}{a^4}\right)-\frac{6\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^3|a|}+\frac{(3\,a\sqrt{c}\,\log\left(|c|\right)-10\sqrt{-c}\,|a|)\mathrm{sgn}(x)}{a^4|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 + 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 - 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) + (3*a*sqrt(c)*log(abs(c)) - 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.888 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=98

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] 3/2*x*(c-c/a^2/x^2)^(1/2)/a+1/2*x*(a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*arcsi
n(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {6302, 6294, 6264, 52, 41, 222}

$$-\frac{3x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{x(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2a}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) + (Sqrt[c - c/(a^2*x^2)]*x*(1 + a*x))/(2*
a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a*
x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx &= -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-ax}\,\sqrt{}}}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}}{2\sqrt{1-ax}\,\sqrt{1+ax}}\\
&= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1+ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.79

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left((4+ax)\sqrt{1-a^2x^2}+6\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a
*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a^2*x^2])
```

Maple [A]
time = 0.11, size = 147, normalized size = 1.50

method	result
risch	$\frac{(ax+4)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{2a}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(x\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2-\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)+4\sqrt{c}\,\ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,\sqrt{c}+cx}{\sqrt{c}}\right)+4\sqrt{\cdots}\right.}{2\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c^(1/2
)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))+4*c^(1/2)*ln(((c*(a*x+1)*(a*x-1)/
a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))+4*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a)/(c*(a^2
*x^2-1)/a^2)^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))*x/(a*x - 1), x)
```

Fricas [A]
time = 0.36, size = 188, normalized size = 1.92

$$\left[\frac{2\,(a^2x^2+4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\ \frac{(a^2x^2+4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)}{2\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2
*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1
/2*((a^2*x^2 + 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a
^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [A]
time = 0.41, size = 106, normalized size = 1.08

$$\frac{1}{4}\left(2\,\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}+\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\,\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{\left(3\,a\sqrt{c}\,\log\left(|c|\right)-8\,\sqrt{-c}\,|a|\right)\mathrm{sgn}(x)}{a^3|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 + 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) - 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.889 $\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$\sqrt{c-\frac{c}{a^2x^2}}\,x-\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)-2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$-\frac{2x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x - (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = -\int e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-a-2a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\right.}{}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\sqrt{1+ax}\right)}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\right.}{\sqrt{1-ax}\sqrt{1+ax}}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 0.69

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}-\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 + 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A]
time = 0.10, size = 197, normalized size = 1.70

method	result

default	$\dfrac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, x \left(2\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\, a^2 \sqrt{-\frac{c}{a^2}} + 2\sqrt{c}\, \ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,\sqrt{c}+cx}{\sqrt{c}}\right) a\sqrt{-\frac{c}{a^2}} - \sqrt{-\frac{c}{a^2}}\, \sqrt{\frac{c(a}{}}\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\, a^2\sqrt{-\frac{c}{a^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)+2*c^(1/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/(a*x - 1), x)
```

Fricas [A]
time = 0.37, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - 4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - \sqrt{c}\,\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{ax-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(a*x - 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{a\,x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(a*x - 1), x)
```

3.890 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=117

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x
+1)^(1/2)+2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6302, 6294, 6264, 100, 163, 41, 222, 94, 214}

$$-\frac{ax\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+\sqrt{c-\frac{c}{a^2x^2}}+\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) + (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^2\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-2a-a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} - \frac{\left(a^2\sqrt{c-\right.}{}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-\right.}{\sqrt{1-ax}\,\sqrt{1+a}}$$

Mathematica [**A**]
time = 0.06, size = 82, normalized size = 0.70

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}-2ax\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] - 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(99) = 198.
time = 0.12, size = 306, normalized size = 2.62

method	result
risch	$\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}+\frac{\left(\frac{a^2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}-\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2}}{a^2x^2-1}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3cx^2+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{-\frac{c}{a^2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^3*c*x^2+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+c^(3/2)*ln(c^(1/2)*
x+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*a*x-2*c^(3/2)*(-c/a^2)^(1/2)*ln
(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a*x-2*(-c/a^2)^(1/2)*
(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*c*x+2*(c*(a^2*x^2-1)/a^2)^(1/2)*c*x*a^2*(
-c/a^2)^(1/2)+2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
)*c^2*x)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x), x)
```

Fricas [A]
time = 0.36, size = 252, normalized size = 2.15

$$\left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2+2a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2c}{x^2}\right)+\sqrt{\frac{a^2cx^2-c}{a^2x^2}},\ -2\sqrt{c}\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\frac{1}{2}\sqrt{c}\log\left(2a^2cx^2+2a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), -2*sqrt(c)*arctan(
```

```
a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*
log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) +
sqrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x,x)
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x*(a*x - 1)), x)
```

Giac [A]
time = 0.49, size = 127, normalized size = 1.09

$$\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)}{a}-\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{|a|}+\frac{2c^{\frac{3}{2}}\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="giac")
[Out] (4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
- sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) + 2*
c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs(
a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,(a\,x+1)}{x\,(a\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x*(a*x - 1)), x)
```

3.891 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=111

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] 3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*arcta
nh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)
^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6302, 6294, 6264, 96, 94, 214}

$$\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{2\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(2*x) + (
3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqrt
[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} - \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1+ax}}{x^2\sqrt{1-ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}}}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-a}\right.}{2\sqrt{1-ax}\,\sqrt{}}$$

$$= \frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-a}\right.}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [**A**]
time = 0.05, size = 78, normalized size = 0.70

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((1+4ax)\sqrt{-1+a^2x^2}-3a^2x^2\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((1 + 4*a*x)*Sqrt[-1 + a^2*x^2] - 3*a^2*x^2*ArcTan[1
/Sqrt[-1 + a^2*x^2]]))/(2*x*Sqrt[-1 + a^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(91) = 182$.
time = 0.12, size = 347, normalized size = 3.13

method	result
risch	$\frac{(4a^3x^3+a^2x^2-4ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(a^2x^2-1)} - \frac{3a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{2\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c\,x^3+4\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3x+4c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)
^(1/2)*a^3*c*x^3+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3*x+4*c^(3/2)
*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*a*x^2-4*c^(3/2)*(-c
/a^2)^(1/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a*x^2-4
*(-c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*c*x^2+3*(-c/a^2)^(1/2)*(c
*(a^2*x^2-1)/a^2)^(1/2)*a^2*c*x^2+a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1
/2)+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*c^2*x^2)
/(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^2), x)
```

Fricas [A]
time = 0.36, size = 177, normalized size = 1.59

$$\left[\frac{3\,a\sqrt{-c}\,x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x}, -\frac{3\,a\sqrt{c}\,x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(4\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) + 2*(4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) - (4*a*x + 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^2\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**2*(a*x - 1)),
x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(91) = 182.
time = 0.49, size = 194, normalized size = 1.75

$$\left(3\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)-\frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3ac\operatorname{sgn}(x)-4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2c^{\frac{3}{2}}|a|\operatorname{sgn}(x)-\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)ac^2\operatorname{sgn}(x)-4\,c^{\frac{5}{2}}|a|\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^2a}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) - 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) - 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^2\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^2*(a*x - 1)), x)
```

3.892 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=137

$$a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2}+\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] a^2*(c-c/a^2/x^2)^(1/2)+1/3*a*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(a*x+1)^2*(
c-c/a^2/x^2)^(1/2)/x^2+a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2
/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6294, 6264, 98, 96, 94, 214}

$$a^2\sqrt{c-\frac{c}{a^2x^2}}+\frac{a(ax+1)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(ax+1)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] + (a*Sqrt[c - c/(a^2*x^2)]*(1 + a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 + a*x)^2)/(3*x^2) + (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^4\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^3\sqrt{1-ax}}\,dx}{3\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{}}{x^2}}{\sqrt{1-ax}\sqrt{1+}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} - \frac{\left(a^3\sqrt{}\right.}{}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} + \frac{\left(a^4\sqrt{}\right.}{}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1+ax)^2}{3x^2} + \frac{a^3\sqrt{c}}{}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.63

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(1+3ax+5a^2x^2)-3a^3x^3\operatorname{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 + 3*a*x + 5*a^2*x^2) - 3*a^3*
x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(117) = 234$.
time = 0.12, size = 378, normalized size = 2.76

method	result
risch	$\frac{(5a^4x^4+3a^3x^3-4a^2x^2-3ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{3x^2(a^2x^2-1)} - \frac{a^3 \ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a\left(-6\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3c\,x^4+6\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^3x^2+3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2c\,x^3+6c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/
a^2)^(1/2)*a^3*c*x^4+6*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^3*x^2+3*(
c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2*c*x^3+6*c^(3/2)*ln(c^(1/2)*x+(c
*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*a*x^3-6*c^(3/2)*(-c/a^2)^(1/2)*ln((
(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a*x^3-6*(-c/a^2)^(1/2)*
(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*c*x^3+3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2
)^(1/2)*a^2*x+3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
)*c^2*x^3+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(
1/2)/(-c/a^2)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^3), x)
```

Fricas [A]
time = 0.36, size = 201, normalized size = 1.47

$$\left[\frac{3\,a^2\sqrt{-c}\,x^2\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(5\,a^2x^2+3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2}, -\frac{3\,a^2\sqrt{c}\,x^2\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(5\,a^2x^2+3\,ax+1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, -1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^
2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (5*a^2*x^2 + 3*a*x + 1)*sqrt((a^2*c*x^
2 - c)/(a^2*x^2)))/x^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^3\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**3*(a*x - 1)),
x)
```

Giac [**A**]
time = 0.73, size = 231, normalized size = 1.69

$$\frac{2}{3}\left(3\,a\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5ac\,\mathrm{sgn}(x)-3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^4c^{\frac{3}{2}}|a|\mathrm{sgn}(x)-12\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2c^{\frac{5}{2}}|a|\mathrm{sgn}(x)-3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)ac^3\mathrm{sgn}(x)-5\,c^{\frac{7}{2}}|a|\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^3}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn
(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) - 3*(sqrt(a^2*c
)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) - 12*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c*
x^2 - c))*a*c^3*sgn(x) - 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^2 + c)^3)*abs(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^3\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^3*(a*x - 1)), x)
```

3.893 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{8\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] 4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3+2/3*a*(c-c/a^2/x^2)
^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2)*(
a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{8\sqrt{1-ax}\,\sqrt{ax+1}}+\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) + (2*a*Sqrt
[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*S
qrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a*
x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^5\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-8a-7a^2x}{x^4\sqrt{1-ax}\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2+16a^3x}{x^3\sqrt{1-ax}\sqrt{1+ax}}\,d}{12\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{}{x^2\sqrt{}}}{24\sqrt{1-ax}}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{\left(\sqrt{\right.}}{}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} - \frac{(7a}{}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{(7a}{}$$

$$= \frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{7a^4}{}$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(6+16ax+21a^2x^2+32a^3x^3)-21a^4x^4\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(6 + 16*a*x + 21*a^2*x^2 + 32*a^
3*x^3) - 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(24*x^3*Sqrt[-1 + a^2*x^
2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(128) = 256$.
time = 0.12, size = 410, normalized size = 2.63

method	result
risch	$\frac{(32a^5x^5+21a^4x^4-16a^3x^3-15a^2x^2-16ax-6)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3(a^2x^2-1)} - \frac{7a^4\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c}}{8\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-48\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3cx^5+48\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^3x^3+21\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(c*(a^2*x^2-1)/a^2)^(1/2)*
(-c/a^2)^(1/2)*a^3*c*x^5+48*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^3*x^
3+21*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2*c*x^4+48*(-c/a^2)^(1/2)*c
^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*a*x^4-48*(-c/a^2)^(1/2)*c^(3
/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a*x^4-48*(-c/a^
2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*c*x^4+27*(c*(a^2*x^2-1)/a^2)^(3/
2)*(-c/a^2)^(1/2)*a^2*x^2+21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^2-c)/a^2/x)*c^2*x^4+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+6*(c
*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^
(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^4), x)
```

Fricas [A]
time = 0.37, size = 217, normalized size = 1.39

$$\left[\frac{21\,a^3\sqrt{-c}\,x^3\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{48\,x^3},\ -\frac{21\,a^3\sqrt{c}\,x^3\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(32\,a^3x^3+21\,a^2x^2+16\,ax+6)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{24\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(32*a^3*x^3 + 21*a^2*x^2 + 16*a*x + 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (32*a^3*x^3 + 21*a^
2*x^2 + 16*a*x + 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^4\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**4*(a*x - 1)),
x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(128) = 256.
time = 1.63, size = 316, normalized size = 2.03

$$\frac{1}{12}\left(21\,a^2\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7a^2c\,\mathrm{sgn}(x)+45\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5a^2c^2\mathrm{sgn}(x)-96\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^4ac^{\frac{5}{2}}|a|\mathrm{sgn}(x)-45\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3a^2c^3\mathrm{sgn}(x)-128\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2ac^{\frac{7}{2}}|a|\mathrm{sgn}(x)-21\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)a^2c^4\mathrm{sgn}(x)-32\,ac^{\frac{9}{2}}|a|\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^4}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) - 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) - 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
```

```
4*sgn(x) - 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^4\,\left(a\,x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^4*(a*x - 1)), x)
```

3.894 $\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}+\frac{3a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}}{4\sqrt{1-ax}}$$

```
[Out] 6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4+1/2*a*(c-c/a^2/x^2)
^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2+3/4*a^3*(c-c/a^2/x^2)^(1/2)/x+3/
4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^
(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}+\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}+\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) + (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) + (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) + (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1+ax)^{3/2}}{x^6\sqrt{1-ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{-10a-9a^2x}{x^5\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{36a^2+30a^3x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{20\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\cdots}{x^3\sqrt{\cdots}}}{60\sqrt{1-ax}\,\cdots}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} - \cdots$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\cdots}{\cdots}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\cdots}{\cdots}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\cdots}{\cdots}$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{3a^3\cdots}{\cdots}$$

Mathematica [A]

time = 0.07, size = 102, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(4+10ax+12a^2x^2+15a^3x^3+24a^4x^4)-15a^5x^5\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(2*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 + 10*a*x + 12*a^2*x^2 + 15*a^
3*x^3 + 24*a^4*x^4) - 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(149) = 298$.
time = 0.13, size = 447, normalized size = 2.47

method	result
risch	$\frac{(24a^6x^6+15a^5x^5-12a^4x^4-5a^3x^3-8a^2x^2-10ax-4)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{20x^4(a^2x^2-1)}-\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{4\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-40\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^4cx^6+40\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^4x^4+15\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*
(-c/a^2)^(1/2)*a^4*c*x^6+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^4*x^
4+15*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c*x^5+40*(-c/a^2)^(1/2)*c
^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*a^2*x^5-40*(-c/a^2)^(1/2)*c^
(3/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a^2*x^5-40*(-
c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*c*x^5+25*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2)*a^3*x^3+15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(
1/2)*a^2-c)/a^2/x)*a*c^2*x^5+16*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^
2*x^2+10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+4*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + 1)*sqrt(c - c/(a^2*x^2))/((a*x - 1)*x^5), x)
```

Fricas [A]
time = 0.37, size = 233, normalized size = 1.29

$$\left[\frac{15\,a^4\sqrt{-c}\,x^4\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, -\frac{15\,a^4\sqrt{c}\,x^4\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)-(24\,a^4x^4+15\,a^3x^3+12\,a^2x^2+10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{20\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, -1/20*(15*a^4*sqrt(c)*x^4*a
rctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) - (24*a^
4*x^4 + 15*a^3*x^3 + 12*a^2*x^2 + 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)))/x^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax+1)}{x^5\,(ax-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x + 1)/(x**5*(a*x - 1)),
x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(149) = 298.
time = 1.97, size = 362, normalized size = 2.00

$$\frac{1}{10}\left(15\,a^3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^9a^3c\,\mathrm{sgn}(x)+70\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7a^3c^2\mathrm{sgn}(x)-40\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^6a^2c^{\frac{5}{2}}|a|\mathrm{sgn}(x)-200\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^4a^2c^{\frac{7}{2}}|a|\mathrm{sgn}(x)-70\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3a^3c^4\mathrm{sgn}(x)-120\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2a^2c^{\frac{9}{2}}|a|\mathrm{sgn}(x)-15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)a^3c^5\mathrm{sgn}(x)-24\,a^2c^{\frac{11}{2}}|a|\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^5}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x-1)*(a*x+1)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) - 40*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) - 200*(sqrt(a^2*c)*x - sqrt
(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^3*a^3*c^4*sgn(x) - 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2
*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*c
^5*sgn(x) - 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2
 - c))^2 + c)^5)*abs(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x+1\right)}{x^5\,\left(a\,x-1\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x + 1))/(x^5*(a*x - 1)), x)
```

3.895 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=186

$$\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)/a
^2/(1-1/a^2/x^2)^(1/2)+x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x^
4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/
a^4/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a^
2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*S
qrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2)
]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x^2(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4}{a^2}+\frac{4x}{a}+3x^2+ax^3+\frac{4}{a^2(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.38

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{4x}{a^2}+\frac{2x^2}{a}+x^3+\frac{ax^4}{4}+\frac{4\log(1-ax)}{a^3}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^3,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((4*x)/a^2 + (2*x^2)/a + x^3 + (a*x^4)/4 + (4*Log[1
- a*x])/a^3))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.06, size = 89, normalized size = 0.48

method	result	size
default	$\frac{\left(a^{4} x^{4}+4 a^{3} x^{3}+8 a^{2} x^{2}+16 a x+16 \ln \left(a x-1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x-1\right)}{4 a^{3} \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/4*(a^4*x^4+4*a^3*x^3+8*a^2*x^2+16*a*x+16*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x-1)/a^3/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 48, normalized size = 0.26

$$\frac{\left(a^{4} x^{4} + 4 \, a^{3} x^{3} + 8 \, a^{2} x^{2} + 16 \, a x + 16 \, \log\left(a x - 1\right)\right) \sqrt{a^{2} c}}{4 \, a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/4*(a^4*x^4 + 4*a^3*x^3 + 8*a^2*x^2 + 16*a*x + 16*log(a*x - 1))*sqrt(a^2*c
)/a^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**3*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^3*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.896 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=152

$$\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)+3/2*x^2*(c-c/a^2/x^2)^(1/2)
/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln
(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 78}

$$\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) + (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{x(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(\frac{4}{a}+3x+ax^2+\frac{4}{a(-1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(}{a^3\sqrt{1-\frac{1}{a^2x}}}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.41

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(ax(24+9ax+2a^2x^2)+24\log(1-ax)\right)}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x^2,x]
```

[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 + 9*a*x + 2*a^2*x^2) + 24*Log[1 - a*x]))/(6*a^3*Sqrt[1 - 1/(a^2*x^2)])

Maple [A]
time = 0.03, size = 82, normalized size = 0.54

method	result	size
default	$\frac{\left(2 a^{3} x^{3}+9 a^{2} x^{2}+24 a x +24 \ln \left(a x -1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{6 a^{2} \left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*(2*a^3*x^3+9*a^2*x^2+24*a*x+24*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/a^2/(a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [A]
time = 0.37, size = 41, normalized size = 0.27

$$\frac{\left(2\,a^{3}x^{3}+9\,a^{2}x^{2}+24\,ax+24\,\log\left(ax-1\right)\right)\sqrt{a^{2}c}}{6\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*(2*a^3*x^3 + 9*a^2*x^2 + 24*a*x + 24*log(a*x - 1))*sqrt(a^2*c)/a^4

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x**2*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x^2*(c-c/a^2/x^2)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.897 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=113

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(
1-1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6332, 6328, 45}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
```

```
[Out] (3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^
2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])
/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{-1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(3+ax+\frac{4}{-1+ax}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax(6+ax)+8\log(1-ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)]*x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(6 + a*x) + 8*Log[1 - a*x]))/(2*a^2*Sqrt[1 - 1/
(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 73, normalized size = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}+6 a x+8 \ln \left(a x-1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x-1\right)}{2 a \left(a x+1\right)^{2} \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*(a^2*x^2+6*a*x+8*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/a/(
a*x+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.28

$$\frac{\left(a^{2} x^{2} + 6\, a x + 8\, \log\left(a x - 1\right)\right) \sqrt{a^{2} c}}{2\, a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(a^2*x^2 + 6*a*x + 8*log(a*x - 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*x*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*x*(c-c/a^2/x^2)^(1/2),x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2),x)
[Out] int((x*(c - c/(a^2*x^2))^(1/2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.898 $\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 84}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a-\frac{1}{x}+\frac{4a}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.46

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-\log(x)+4\log(1-ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x] + 4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A]
time = 0.03, size = 65, normalized size = 0.60

method	result	size
default	$-\frac{\left(-ax+\ln\left(x\right)-4\ln\left(ax-1\right)\right)x\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{\left(ax+1\right)^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-a*x+ln(x)-4*ln(a*x-1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2
/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.36, size = 27, normalized size = 0.25

$$\frac{\sqrt{a^2c}\,\left(ax+4\,\log\left(ax-1\right)-\log\left(x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="frica
s")
[Out] sqrt(a^2*c)*(a*x + 4*log(a*x - 1) - log(x))/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.899 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) - (3*Sqrt[c - c/(a^2*x^2)
]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sq
rt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^2(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 0.48

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{x}-3a\log(x)+4a\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^(-1) - 3*a*Log[x] + 4*a*Log[1 - a*x]))/(a*Sqrt[1
- 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 67, normalized size = 0.62

method	result	size
default	$-\frac{\left(3 a \ln \left(x \right) x -4 x \ln \left(a x -1\right) a -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{\left(a x +1\right)^{2} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x,method=_RETURNVERBOSE
)
```

```
[Out] -(3*a*ln(x)*x-4*x*ln(a*x-1)*a-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x
+1)^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.37, size = 32, normalized size = 0.30

$$\frac{\sqrt{a^2c}\,(4\,ax\log(ax-1)-3\,ax\log(x)+1)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] sqrt(a^2*c)*(4*a*x*log(a*x - 1) - 3*a*x*log(x) + 1)/(a^2*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]Warni
ng, choos
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.900 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x/(
1-1/a^2/x^2)^(1/2)-4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a*ln
(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a^2*
x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[
1 - 1/(a^2*x^2)] + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2
*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^3(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^3}-\frac{3a}{x^2}-\frac{4a^2}{x}+\frac{4a^3}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2} + \frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x} - \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}} + \frac{4a\sqrt{c-\frac{c}{a^2x^2}}\ldots}{\sqrt{1-\ldots}}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2x^2}+\frac{3a}{x}-4a^2\log(x)+4a^2\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^2,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*x^2) + (3*a)/x - 4*a^2*Log[x] + 4*a^2*Log[1 -
a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 82, normalized size = 0.56

method	result	size
default	$-\frac{\left(8a^{2}\ln\left(x\right)x^{2}-8x^{2}\ln\left(ax-1\right)a^{2}-6ax-1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{2\left(ax+1\right)^{2}x\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*(8*a^2*ln(x)*x^2-8*x^2*ln(a*x-1)*a^2-6*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x-1)/(a*x+1)^2/x/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 85, normalized size = 0.58

$$\frac{8\,a^{3}\sqrt{c}\,x^{2}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}\,(2\,ax-1)\sqrt{c}\,+ac}{ax^{2}-x}\right)+\sqrt{a^{2}c}\,(6\,ax+1)}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
1)*sqrt(c) + a*c)/(a*x^2 - x)) + sqrt(a^2*c)*(6*a*x + 1))/(a^2*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^2\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^2*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.901 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)/x
^2/(1-1/a^2/x^2)^(1/2)+4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^2*
ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(-a*x+1)*(c-c/a^2/x^2
)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(3*a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a^2*
x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1
- 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^
2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^4(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^4}-\frac{3a}{x^3}-\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.07, size = 76, normalized size = 0.40

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{3x^3}+\frac{3a}{2x^2}+\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^3,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(3*x^3) + (3*a)/(2*x^2) + (4*a^2)/x - 4*a^3*Log[x
] + 4*a^3*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.48

method	result	size
default	$-\frac{\left(24\ln\left(x\right)a^{3}x^{3}-24x^{3}\ln\left(ax-1\right)a^{3}-24a^{2}x^{2}-9ax-2\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax-1\right)}{6\left(ax+1\right)^{2}x^{2}\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/6*(24*ln(x)*a^3*x^3-24*x^3*ln(a*x-1)*a^3-24*a^2*x^2-9*a*x-2)*(c*(a^2*x^2
-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^2/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 93, normalized size = 0.49

$$\frac{24\,a^{4}\sqrt{c}\,x^{3}\log\left(\frac{2\,a^{3}cx^{2}-2\,a^{2}cx-\sqrt{a^{2}c}\,(2\,ax-1)\sqrt{c}+ac}{ax^{2}-x}\right)+\left(24\,a^{2}x^{2}+9\,ax+2\right)\sqrt{a^{2}c}}{6\,a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (24*a^2*x^2 + 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]Warni
ng, choos
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^3\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^3*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.902 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=222

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-}}$$

```
[Out] 1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3/(
1-1/a^2/x^2)^(1/2)+2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2*(c
-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-
1/a^2/x^2)^(1/2)+4*a^3*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(4*a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*x^2
)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) + (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a
^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (
4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^3*Sqrt[c -
 c/(a^2*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^5(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^5}-\frac{3a}{x^4}-\frac{4a^2}{x^3}-\frac{4a^3}{x^2}-\frac{4a^4}{x}+\frac{4a^5}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3} + \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2} + \frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x} -$$

Mathematica [A]
time = 0.08, size = 81, normalized size = 0.36

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{4x^4}+\frac{a}{x^3}+\frac{2a^2}{x^2}+\frac{4a^3}{x}-4a^4\log(x)+4a^4\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^4,x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(4*x^4) + a/x^3 + (2*a^2)/x^2 + (4*a^3)/x - 4*a^4
*Log[x] + 4*a^4*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 98, normalized size = 0.44

method	result	size
default	$-\frac{\left(16\ln \left(x\right) a^{4} x^{4}-16 x^{4} \ln \left(a x -1\right) a^{4}-16 a^{3} x^{3}-8 a^{2} x^{2}-4 a x -1\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{4 \left(a x +1\right)^{2} x^{3} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/4*(16*ln(x)*a^4*x^4-16*x^4*ln(a*x-1)*a^4-16*a^3*x^3-8*a^2*x^2-4*a*x-1)*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^3/((a*x-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 101, normalized size = 0.45

$$\frac{16\, a^{5} \sqrt{c}\, x^{4} \log \left(\frac{2\, a^{3} c x^{2} - 2\, a^{2} c x - \sqrt{a^{2} c}\, {\left(2\, a x - 1\right)} \sqrt{c} + a c}{a x^{2} - x}\right) + {\left(16\, a^{3} x^{3} + 8\, a^{2} x^{2} + 4\, a x + 1\right)} \sqrt{a^{2} c}}{4\, a^{2} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="f
ricas")
```

```
[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x -
 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (16*a^3*x^3 + 8*a^2*x^2 + 4*a*x + 1)*sqrt
(a^2*c))/(a^2*x^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**4,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^4,x, algorithm="g
iac")
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^4\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)),x)
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^4*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.903 $\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=264

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)/x
^4/(1-1/a^2/x^2)^(1/2)+4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+2*
a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^3*(c-c/a^2/x^2)^(1/2)/x
/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*
a^4*ln(-a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\log(1-ax)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(5*a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a^2*
x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) + (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sqrt[
1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)
]*x^2) + (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^4*S
qrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(a^2
*x^2)]*Log[1 - a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(1+ax)^2}{x^6(-1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^6}-\frac{3a}{x^5}-\frac{4a^2}{x^4}-\frac{4a^3}{x^3}-\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{-1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.34

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{12+45ax+80a^2x^2+120a^3x^3+240a^4x^4}{60x^5}-4a^5\log(x)+4a^5\log(1-ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(3*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)])/x^5,x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*((12 + 45*a*x + 80*a^2*x^2 + 120*a^3*x^3 + 240*a^4*x
^4)/(60*x^5) - 4*a^5*Log[x] + 4*a^5*Log[1 - a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)]
)
```

Maple [A]
time = 0.04, size = 106, normalized size = 0.40

method	result	size
default	$-\frac{\left(240 a^{5} \ln \left(x\right) x^{5}-240 x^{5} \ln \left(a x -1\right) a^{5}-240 a^{4} x^{4}-120 a^{3} x^{3}-80 a^{2} x^{2}-45 a x -12\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x -1\right)}{60 \left(a x +1\right)^{2} x^{4} \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x,method=_RETURNVERBO
SE)
```

```
[Out] -1/60*(240*a^5*ln(x)*x^5-240*x^5*ln(a*x-1)*a^5-240*a^4*x^4-120*a^3*x^3-80*a
^2*x^2-45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)^2/x^4/((a*x
-1)/(a*x+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

Fricas [A]
time = 0.36, size = 109, normalized size = 0.41

$$\frac{240\, a^{6} \sqrt{c}\, x^{5} \log\left(\frac{2\, a^{3} c x^{2} - 2\, a^{2} c x - \sqrt{a^{2} c}\, {\left(2\, a x - 1\right)} \sqrt{c} + a c}{a x^{2} - x}\right) + {\left(240\, a^{4} x^{4} + 120\, a^{3} x^{3} + 80\, a^{2} x^{2} + 45\, a x + 12\right)} \sqrt{a^{2} c}}{60\, a^{2} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="f
ricas")
```

```
[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 - 2*a^2*c*x - sqrt(a^2*c)*(2*a*x
 - 1)*sqrt(c) + a*c)/(a*x^2 - x)) + (240*a^4*x^4 + 120*a^3*x^3 + 80*a^2*x^2
 + 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))**(3/2)*(c-c/a**2/x**2)**(1/2)/x**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a*x-1)/(a*x+1))^(3/2)*(c-c/a^2/x^2)^(1/2)/x^5,x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx)]Warni
ng, choos
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{x^5\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)/(x^5*((a*x - 1)/(a*x + 1))^(3/2)), x)
```

3.904 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx$

Optimal. Leaf size=81

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x^m*(c-c/a^2/x^2)^(1/2)/a/m/(1-1/a^2/x^2)^(1/2)+x^(1+m)*(c-c/a^2/x^2)^(1/2
)/(1+m)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 45}

$$\frac{x^{m+1}\sqrt{c-\frac{c}{a^2x^2}}}{(m+1)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^m\sqrt{c-\frac{c}{a^2x^2}}}{am\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x^m)/(a*m*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a
^2*x^2)]*x^(1 + m))/((1 + m)*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^m\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^m\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x^{-1+m}(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-x^{-1+m}+ax^m\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^m}{am\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^{1+m}}{(1+m)\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 0.62

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^m\left(-\frac{1}{m}+\frac{ax}{1+m}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^m)/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x^m*(-m^(-1) + (a*x)/(1 + m)))/(a*Sqrt[1 - 1/(a^2*x^
2)])
```

Maple [A]
time = 0.02, size = 65, normalized size = 0.80

method	result	size
gosper	$\frac{x^{1+m}(axm-m-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{(1+m)m(ax-1)}$	65
risch	$\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\sqrt{\frac{c(a^2x^2-1)}{(ax+1)(ax-1)}}\,(ax+1)(axm-m-1)x^m}{\sqrt{c}\,(a^2x^2-1)(1+m)m}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x^(1+m)*(a*m*x-m-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(
1+m)/m/(a*x-1)
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.57

$$\frac{\left(a\sqrt{c}\,mx-\sqrt{c}\,(m+1)\right)(ax-1)x^m}{(m^2+m)a^2x-(m^2+m)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] (a*sqrt(c)*m*x - sqrt(c)*(m + 1))*(a*x - 1)*x^m/((m^2 + m)*a^2*x - (m^2 + m
)*a)
```

Fricas [A]
time = 0.33, size = 73, normalized size = 0.90

$$-\frac{(amx^2-(m+1)x)x^m\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{m^2-(am^2+am)x+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -(a*m*x^2 - (m + 1)*x)*x^m*sqrt((a*x - 1)/(a*x + 1))*sqrt((a^2*c*x^2 - c)/(
a^2*x^2))/(m^2 - (a*m^2 + a*m)*x + m)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^m*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int(x^m*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.905 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*x^2*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1
/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 45}

$$\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*x^2)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a
^2*x^2)]*x^3)/(3*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int x(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-x+ax^2\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.59

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(-3+2ax)}{6a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x^2*(-3 + 2*a*x))/(6*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 53, normalized size = 0.70

method	result	size

gosper	$\frac{x^3(2ax-3)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53
default	$\frac{x^3(2ax-3)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{6ax-6}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*x^3*(2*a*x-3)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*
x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.33, size = 24, normalized size = 0.32

$$\frac{(2\,ax^3 - 3\,x^2)\sqrt{a^2c}}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*a*x^3 - 3*x^2)*sqrt(a^2*c)/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [B]
time = 1.36, size = 46, normalized size = 0.61

$$\frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(2\,a\,x-3\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{6\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^3*(c - c/(a^2*x^2))^(1/2)*(2*a*x - 3)*((a*x - 1)/(a*x + 1))^(1/2))/(6*(a
*x - 1))
```

3.906 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$, Rules used = {6332, 6328}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
```

```
[Out] -((Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)])) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int(-1+ax)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.58

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(-2+ax)}{2a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(-2 + a*x))/(2*a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 52, normalized size = 0.72

method	result	size
gosper	$\frac{x^2(ax-2)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52
default	$\frac{x^2(ax-2)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2ax-2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*(a*x-2)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [**A**]
time = 0.35, size = 21, normalized size = 0.29

$$\frac{\sqrt{a^2c}\,(ax^2-2\,x)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="frica
s")
[Out] 1/2*sqrt(a^2*c)*(a*x^2 - 2*x)/a^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [**B**]
time = 1.32, size = 45, normalized size = 0.62

$$\frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-2\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{2\,\left(a\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] (x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 2)*((a*x - 1)/(a*x + 1))^(1/2))/(2*(a*x
 - 1))
```

3.907 $\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 45}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] - (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{-1+ax}{x}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(a-\frac{1}{x}\right)\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.60

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax-\log(x))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^ArcCoth[a*x],x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x - Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 52, normalized size = 0.76

method	result	size

default	$-\frac{(-ax+\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-a*x+ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-
1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.28

$$\frac{\sqrt{a^2c}\,(ax-\log(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="fricas"
)
```

```
[Out] sqrt(a^2*c)*(a*x - log(x))/a^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2), x)
```

3.908 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/
a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 45}

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x) + (Sqrt[c - c/(a^2*x^2)]*
Log[x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^2}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{1}{x^2}+\frac{a}{x}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.59

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{x}+a\log(x)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(x^(-1) + a*Log[x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 50, normalized size = 0.72

method	result	size
default	$\frac{(a\ln(x)x+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{\frac{ax-1}{ax+1}}}{ax-1}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (a*ln(x)*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x, x)
```

Fricas [A]
time = 0.34, size = 21, normalized size = 0.30

$$\frac{\sqrt{a^2c}\,(ax\log(x)+1)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="frica
s")
```

```
[Out] sqrt(a^2*c)*(a*x*log(x) + 1)/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ax-1}{ax+1}}\,\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*x - 1)/(a*x + 1))*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))/x,
 x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x,x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(sageVARx),sign(
sageVARa*
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x, x)
```

3.909 $\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

```
[Out] 1/2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 37}

$$\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
```

```
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{-1+ax}{x^3}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{2x^2}-\frac{a}{x}\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^ArcCoth[a*x]*x^2),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(1/(2*x^2) - a/x))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.03, size = 53, normalized size = 1.13

method	result	size
gosper	$-\frac{(2ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2(ax-1)x}$	53
default	$-\frac{(2ax-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\sqrt{\frac{ax-1}{ax+1}}}{2(ax-1)x}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*(2*a*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/(a*x-1
)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.45

$$-\frac{\sqrt{a^2c}\,(2\,ax-1)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(a^2*c)*(2*a*x - 1)/(a^2*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(1/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*sqrt((a*x - 1)/(a*x + 1))/x^2, x)
```

Mupad [B]
time = 1.36, size = 63, normalized size = 1.34

$$\frac{\left(x\,\sqrt{c-\frac{c}{a^2\,x^2}}-\frac{\sqrt{c-\frac{c}{a^2\,x^2}}}{2\,a}\right)\,\sqrt{\frac{a\,x-1}{a\,x+1}}}{\frac{x}{a}-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(1/2))/x^2,x)
```

```
[Out] ((x*(c - c/(a^2*x^2))^(1/2) - (c - c/(a^2*x^2))^(1/2)/(2*a))*((a*x - 1)/(a*
x + 1))^(1/2))/(x/a - x^2)
```

3.910 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=163

$$-\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2}-\frac{7\sqrt{c-\frac{c}{a^2x^2}}}{8a^3\sqrt{1-}}$$

```
[Out] -7/8*x*(c-c/a^2/x^2)^(1/2)/a^3-7/24*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3-1/6*
x*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)/a^3+1/4*x^2*(-a*x+1)^2*(c-c/a^2/x^2)^(1/2)
/a^2-7/8*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^3/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6302, 6294, 6264, 92, 81, 52, 41, 222}

$$\frac{x^2(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{4a^2}-\frac{7x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{8a^3\sqrt{ax+1}\,\sqrt{1-ax}}-\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{6a^3}-\frac{7x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{24a^3}-\frac{7x\sqrt{c-\frac{c}{a^2x^2}}}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-7*Sqrt[c - c/(a^2*x^2)]*x)/(8*a^3) - (7*Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)
)/(24*a^3) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(6*a^3) + (Sqrt[c - c/(a
^2*x^2)]*x^2*(1 - a*x)^2)/(4*a^2) - (7*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])
/(8*a^3*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}x^2\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x^2(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}(-1+2ax)}{\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1-ax)^2}{4a^2} - \frac{\left(7\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{12a^2\sqrt{1-a}}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2(1}{4a^2}$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} +$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} +$$

$$= -\frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x}{8a^3} - \frac{7\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{24a^3} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{6a^3} +$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.57

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}\,(-32+21ax-16a^2x^2+6a^3x^3)+21\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{24a^3\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(-32 + 21*a*x - 16*a^2*x^2 + 6
*a^3*x^3) + 21*Log[a*x + Sqrt[-1 + a^2*x^2]]))/(24*a^3*Sqrt[-1 + a^2*x^2])
```

Maple [A]
time = 0.11, size = 196, normalized size = 1.20

method	result
risch	$\frac{\left(6 a^{3} x^{3}-16 a^{2} x^{2}+21 a x -32\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x}{24 a^{3}}+\frac{7 \ln \left(\frac{a^{2} c x}{\sqrt{a^{2} c}}+\sqrt{a^{2} c \,x^{2}-c}\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \sqrt{c \left(a^{2} x^{2}-1\right)}\, x}{8 a^{2} \sqrt{a^{2} c}\, \left(a^{2} x^{2}-1\right)}$
default	$-\frac{\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, x \left(-6 x \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{4}+16 \left(\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}\right)^{\frac{3}{2}} a^{3}-27 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, a^{2} c x +27 c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\right)\right.}{24 \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2}}}\, c \,a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(-6*x*(c*(a^2*x^2-1)/a^2)^(3/2)*a^4+1
6*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3-27*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2*c*x+27*c^(
3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))-48*c^(3/2)*ln(((c*(a*x+1)*(a*x
-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))+48*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a*c)/
(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^3/(a*x + 1), x)
```

Fricas [A]
time = 0.35, size = 222, normalized size = 1.36

$$\left[\frac{2 \, {\left(6 \, a^{4} x^{4} - 16 \, a^{3} x^{3} + 21 \, a^{2} x^{2} - 32 \, a x\right)} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}} + 21 \, \sqrt{c} \log\left(2 \, a^{2} c x^{2} + 2 \, a^{2} \sqrt{c} x^{2} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}} - c\right)}{48 \, a^{4}}, \frac{{\left(6 \, a^{4} x^{4} - 16 \, a^{3} x^{3} + 21 \, a^{2} x^{2} - 32 \, a x\right)} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}} - 21 \, \sqrt{-c} \arctan\left(\frac{a^{2} \sqrt{-c} x^{2} \sqrt{\frac{a^{2} c x^{2} - c}{a^{2} x^{2}}}}{a^{2} c x^{2} - c}\right)}{24 \, a^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 - 32*a*x)*sqrt((a^2*c*x^2 - c
)/(a^2*x^2)) + 21*sqrt(c)*log(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x
```

```
^2 - c)/(a^2*x^2)) - c))/a^4, 1/24*((6*a^4*x^4 - 16*a^3*x^3 + 21*a^2*x^2 -
32*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 21*sqrt(-c)*arctan(a^2*sqrt(-c)*x
^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(x**3*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A]
time = 0.42, size = 128, normalized size = 0.79

$$\frac{1}{48}\left(2\sqrt{a^2cx^2-c}\left(\left(2x\left(\frac{3x\mathrm{sgn}(x)}{a^2}-\frac{8\,\mathrm{sgn}(x)}{a^3}\right)+\frac{21\,\mathrm{sgn}(x)}{a^4}\right)x-\frac{32\,\mathrm{sgn}(x)}{a^5}\right)-\frac{42\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\mathrm{sgn}(x)}{a^4|a|}+\frac{(21\,a\sqrt{c}\,\log\left(|c|\right)+64\sqrt{-c}\,|a|)\mathrm{sgn}(x)}{a^5|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] 1/48*(2*sqrt(a^2*c*x^2 - c)*((2*x*(3*x*sgn(x)/a^2 - 8*sgn(x)/a^3) + 21*sgn(
x)/a^4)*x - 32*sgn(x)/a^5) - 42*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c
*x^2 - c)))*sgn(x)/(a^4*abs(a)) + (21*a*sqrt(c)*log(abs(c)) + 64*sqrt(-c)*a
bs(a))*sgn(x)/(a^5*abs(a)))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int((x^3*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.911 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\mathrm{ArcSin}(ax)}{a^2\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2+1/3*x*(-a*
x+1)^2*(c-c/a^2/x^2)^(1/2)/a^2+x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/a^2/(-a*x+
1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.259$, Rules used = {6302, 6294, 6264, 81, 52, 41, 222}

$$\frac{x\mathrm{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{ax+1}\,\sqrt{1-ax}}+\frac{x(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3a^2}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/a^2 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(3*a^2)
 + (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x)^2)/(3*a^2) + (Sqrt[c - c/(a^2*x^2)]*x
*ArcSin[a*x])/(a^2*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}x\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{x(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} + \frac{\left(2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{3a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{a\sqrt{1-ax}\,\sqrt{}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} + \frac{(}{}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} + \frac{(}{}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{3a^2} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)^2}{3a^2} + \frac{\sqrt{}}{a}$$

Mathematica [A]
time = 0.05, size = 84, normalized size = 0.68

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}\,(5-3ax+a^2x^2)-3\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{3a^2\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2]*(5 - 3*a*x + a^2*x^2) - 3*Log[
a*x + Sqrt[-1 + a^2*x^2]]))/(3*a^2*Sqrt[-1 + a^2*x^2])
```

Maple [A]
time = 0.11, size = 173, normalized size = 1.40

method	result
risch	$\frac{(a^2x^2-3ax+5)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{3a^2}-\frac{\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{a\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(\left(\frac{c(a^2x^2-1)}{a^2}\right)^{\frac{3}{2}}a^3-3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2cx+3c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)-6c^{\frac{3}{2}}\ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,\sqrt{}}{\sqrt{c}}\right.\right.}{3\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,c\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*((c*(a^2*x^2-1)/a^2)^(3/2)*a^3-3*(c*(a^
2*x^2-1)/a^2)^(1/2)*a^2*c*x+3*c^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2
))-6*c^(3/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))+6*(c*(
a*x+1)*(a*x-1)/a^2)^(1/2)*a*c)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x^2/(a*x + 1), x)
```

Fricas [A]
time = 0.38, size = 204, normalized size = 1.65

$$\left[\frac{2\,(a^3x^3-3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\,\log\left(2\,a^2cx^2-2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{6\,a^3},\ \frac{(a^3x^3-3\,a^2x^2+5\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)}{3\,a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*s
qrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))
- c))/a^3, 1/3*((a^3*x^3 - 3*a^2*x^2 + 5*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2
)) + 3*sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^
2*c*x^2 - c)))/a^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x**2*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A]
time = 0.40, size = 117, normalized size = 0.94

$$\frac{1}{6}\left(2\sqrt{a^2cx^2-c}\left(x\left(\frac{x\operatorname{sgn}(x)}{a^2}-\frac{3\operatorname{sgn}(x)}{a^3}\right)+\frac{5\operatorname{sgn}(x)}{a^4}\right)+\frac{6\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{a^3|a|}-\frac{\left(3a\sqrt{c}\,\log\left(|c|\right)+10\sqrt{-c}\,|a|\right)\operatorname{sgn}(x)}{a^4|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/6*(2*sqrt(a^2*c*x^2 - c)*(x*(x*sgn(x)/a^2 - 3*sgn(x)/a^3) + 5*sgn(x)/a^4)
 + 6*sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^3*abs
(a)) - (3*a*sqrt(c)*log(abs(c)) + 10*sqrt(-c)*abs(a))*sgn(x)/(a^4*abs(a)))*
abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x^2*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.912 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=99

$$-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] -3/2*x*(c-c/a^2/x^2)^(1/2)/a-1/2*x*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/a-3/2*x*arc
sin(a*x)*(c-c/a^2/x^2)^(1/2)/a/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6302, 6294, 6264, 52, 41, 222}

$$-\frac{3x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{x(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2a}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(2*a) - (Sqrt[c - c/(a^2*x^2)]*x*(1 - a*x))/(2
*a) - (3*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(2*a*Sqrt[1 - a*x]*Sqrt[1 + a
*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a} - \frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a} - \frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{}{\sqrt{1-}}}{2\sqrt{1-ax}\,\sqrt{1-}}$$

$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a} - \frac{\left(3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{}{\sqrt{1-}}}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{2a} - \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x(1-ax)}{2a} - \frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{2a\sqrt{1-ax}\,\sqrt{1+ax}}$$

Mathematica [A]
time = 0.04, size = 100, normalized size = 1.01

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{1+ax}\,(4-5ax+a^2x^2)-6\sqrt{1-ax}\,\text{ArcSin}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)\right)}{2a\sqrt{1-ax}\,\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(2*ArcCoth[a*x]),x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[1 + a*x]*(4 - 5*a*x + a^2*x^2) - 6*Sqrt
[1 - a*x]*ArcSin[Sqrt[1 - a*x]/Sqrt[2]]))/(a*Sqrt[1 - a*x]*Sqrt[1 - a^2*x^2
])
```

Maple [A]
time = 0.11, size = 147, normalized size = 1.48

method	result
risch	$\frac{(ax-4)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x}{2a}+\frac{3\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{2\sqrt{a^2c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,x\left(x\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2-\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{\frac{c(a^2x^2-1)}{a^2}}\right)+4\sqrt{c}\,\ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,\sqrt{c}+cx}{\sqrt{c}}\right)-4\sqrt{\frac{c(}{}}\right.}{2\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(x*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c^(1/2
)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))+4*c^(1/2)*ln(((c*(a*x+1)*(a*x-1)/
a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))-4*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a)/(c*(a^2
*x^2-1)/a^2)^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))*x/(a*x + 1), x)
```

Fricas [A]

time = 0.42, size = 188, normalized size = 1.90

$$\left[\frac{2\,(a^2x^2-4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}+3\,\sqrt{c}\,\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)}{4\,a^2},\frac{(a^2x^2-4\,ax)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-3\,\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)}{2\,a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
[Out] [1/4*(2*(a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 3*sqrt(c)*log(2
*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a^2, 1
/2*((a^2*x^2 - 4*a*x)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 3*sqrt(-c)*arctan(a
^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)))/a^2]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
[Out] Integral(x*sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [A]

time = 0.42, size = 106, normalized size = 1.07

$$\frac{1}{4}\left(2\,\sqrt{a^2cx^2-c}\left(\frac{x\mathrm{sgn}(x)}{a^2}-\frac{4\,\mathrm{sgn}(x)}{a^3}\right)-\frac{6\,\sqrt{c}\,\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\,\right|\right)\mathrm{sgn}(x)}{a^2|a|}+\frac{(3\,a\sqrt{c}\,\log\left(|c|\right)+8\,\sqrt{-c}\,|a|)\mathrm{sgn}(x)}{a^3|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
[Out] 1/4*(2*sqrt(a^2*c*x^2 - c)*(x*sgn(x)/a^2 - 4*sgn(x)/a^3) - 6*sqrt(c)*log(ab
s(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/(a^2*abs(a)) + (3*a*sqrt(c)
*log(abs(c)) + 8*sqrt(-c)*abs(a))*sgn(x)/(a^3*abs(a)))*abs(a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
[Out] int((x*(c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.913 $\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=116

$$\sqrt{c-\frac{c}{a^2x^2}}\,x+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)+2*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a
*x+1)^(1/2)+x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a
*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {6302, 6294, 6264, 104, 163, 41, 222, 94, 214}

$$\frac{2x\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+x\sqrt{c-\frac{c}{a^2x^2}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)]*x + (2*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 -
 a*x]*Sqrt[1 + a*x]) + (Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx = -\int e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{a-2a^2x}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\left(2a\sqrt{c-\right.}{}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{\left(a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx, x, \sqrt{1-ax}\,\sqrt{1+a\right.}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}}\,x + \frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-\right.}{\sqrt{1-ax}\,\sqrt{1+a}}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 0.69

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x\left(\sqrt{-1+a^2x^2}-\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)-2\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(2*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x*(Sqrt[-1 + a^2*x^2] - ArcTan[1/Sqrt[-1 + a^2*x^2]]
 - 2*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [A]
time = 0.10, size = 197, normalized size = 1.70

method	result

default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\, x \left(2\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\, a^2 \sqrt{-\frac{c}{a^2}} - 2\sqrt{c}\, \ln\left(\frac{\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\, \sqrt{c} + cx}{\sqrt{c}}\right) a \sqrt{-\frac{c}{a^2}} - \sqrt{-\frac{c}{a^2}}\, \sqrt{\frac{c(a^2 ...}{...}}\right.}{\sqrt{\frac{c(a^2x^2-1)}{a^2}}\, a^2 \sqrt{-\frac{c}{a^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] (c*(a^2*x^2-1)/a^2/x^2)^(1/2)*x*(2*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*(-c/a^
2)^(1/2)-2*c^(1/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*
a*(-c/a^2)^(1/2)-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c*ln(2*((-c/a
^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x))/(c*(a^2*x^2-1)/a^2)^(1/2
)/a^2/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/(a*x + 1), x)
```

Fricas [A]
time = 0.35, size = 267, normalized size = 2.30

$$\left[\frac{2\,ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} + 4\sqrt{-c}\,\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{-c}\,\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)}{2\,a}, \frac{ax\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - \sqrt{c}\,\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right) + \sqrt{c}\,\log\left(2\,a^2cx^2 - 2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}} - c\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) + 4*sqrt(-c)*arctan(a^2*sqrt(-c
)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(-c)*log(-(a^2
*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - 2*c)/x^2))/a, (a*
x*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - sqrt(c)*arctan(a*sqrt(c)*x*sqrt((a^2*c*
x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + sqrt(c)*log(2*a^2*c*x^2 - 2*a^2*sqrt
(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{ax+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(a*x + 1), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{a\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(a*x + 1), x)
```

3.914 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=117

$$\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\text{ArcSin}(ax)}{\sqrt{1-ax}\,\sqrt{1+ax}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] (c-c/a^2/x^2)^(1/2)-a*x*arcsin(a*x)*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x
+1)^(1/2)-2*a*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(
-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6302, 6294, 6264, 100, 163, 41, 222, 94, 214}

$$-\frac{ax\text{ArcSin}(ax)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-ax}\,\sqrt{ax+1}}+\sqrt{c-\frac{c}{a^2x^2}}-\frac{2ax\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*x*ArcSin[a*x])/(Sqrt[1 - a
*x]*Sqrt[1 + a*x]) - (2*a*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqr
t[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^2}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^2\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{2a-a^2x}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(a^2\sqrt{c-\right.}{}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{\left(2a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right.}{}$$

$$= \sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,x\sin^{-1}(ax)}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-\right.}{\sqrt{1-ax}\sqrt{1+a}}$$

Mathematica [**A**]
time = 0.05, size = 82, normalized size = 0.70

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}+2ax\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)+ax\log\left(ax+\sqrt{-1+a^2x^2}\right)\right)}{\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2] + 2*a*x*ArcTan[1/Sqrt[-1 + a^2*x
^2]] + a*x*Log[a*x + Sqrt[-1 + a^2*x^2]]))/Sqrt[-1 + a^2*x^2]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(99) = 198.
time = 0.12, size = 306, normalized size = 2.62

method	result
risch	$\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}+\frac{\left(\frac{a^2\ln\left(\frac{a^2cx}{\sqrt{a^2c}}+\sqrt{a^2c\,x^2-c}\right)}{\sqrt{a^2c}}+\frac{2a\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)}{\sqrt{-c}}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^{2}}}{a^2x^2-1}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c\,x^2+a^3\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}+2\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,a^2cx-2c^{\frac{3}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/a*(-(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)
*a^3*c*x^2+a^3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)+2*(-c/a^2)^(1/2)*(c
*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*c*x-2*c^(3/2)*(-c/a^2)^(1/2)*ln(((c*(a*x+1)
*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a*x+c^(3/2)*ln(c^(1/2)*x+(c*(a^2*
x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*a*x-2*(c*(a^2*x^2-1)/a^2)^(1/2)*c*x*a^2*(
-c/a^2)^(1/2)-2*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
)*c^2*x)/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x), x)
```

Fricas [A]
time = 0.35, size = 252, normalized size = 2.15

$$\left[-\sqrt{-c}\arctan\left(\frac{a^2\sqrt{-c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\sqrt{-c}\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+\sqrt{\frac{a^2cx^2-c}{a^2x^2}},\ 2\,\sqrt{c}\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\frac{1}{2}\,\sqrt{c}\log\left(2\,a^2cx^2+2\,a^2\sqrt{c}\,x^2\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-c\right)+\sqrt{\frac{a^2cx^2-c}{a^2x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="fricas")
```

```
[Out] [-sqrt(-c)*arctan(a^2*sqrt(-c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x
^2 - c)) + sqrt(-c)*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/(
a^2*x^2)) - 2*c)/x^2) + sqrt((a^2*c*x^2 - c)/(a^2*x^2)), 2*sqrt(c)*arctan(a
```

```
*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + 1/2*sqrt(c)*l
og(2*a^2*c*x^2 + 2*a^2*sqrt(c)*x^2*sqrt((a^2*c*x^2 - c)/(a^2*x^2)) - c) + s
qrt((a^2*c*x^2 - c)/(a^2*x^2))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x*(a*x + 1)), x)
```

Giac [**A**]
time = 0.48, size = 127, normalized size = 1.09

$$-\left(\frac{4\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\operatorname{sgn}(x)}{a}+\frac{\sqrt{c}\log\left(\left|-\sqrt{a^2c}\,x+\sqrt{a^2cx^2-c}\right|\right)\operatorname{sgn}(x)}{|a|}-\frac{2c^{\frac{3}{2}}\operatorname{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)|a|}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x,x, algorithm="giac")
```

```
[Out] -(4*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x)/a
 + sqrt(c)*log(abs(-sqrt(a^2*c)*x + sqrt(a^2*c*x^2 - c)))*sgn(x)/abs(a) - 2
*c^(3/2)*sgn(x)/(((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^2 + c)*abs(a)))*abs
(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x*(a*x + 1)), x)
```

3.915 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=112

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{2\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] -3/2*a*(c-c/a^2/x^2)^(1/2)+1/2*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+3/2*a^2*x*arc
tanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+
1)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6302, 6294, 6264, 96, 94, 214}

$$-\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}}+\frac{(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{2x}+\frac{3a^2x\sqrt{c-\frac{c}{a^2x^2}}\,\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{2\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] (-3*a*Sqrt[c - c/(a^2*x^2)])/2 + (Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(2*x) +
(3*a^2*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(2*Sqr
t[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\sqrt{1+ax}}{x^3}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} + \frac{\left(3a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{\sqrt{1-ax}}{x^2\sqrt{1+ax}}\,dx}{2\sqrt{1-ax}\sqrt{1+ax}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} - \frac{\left(3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{}{x\sqrt{1-}}}{2\sqrt{1-ax}\sqrt{1+}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} + \frac{\left(3a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\text{Subst}\Big(\int}{2\sqrt{1-}}$$

$$= -\frac{3}{2}a\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{2x} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\Big(\sqrt{}}{2\sqrt{1-ax}\sqrt{1+}}$$

Mathematica [**A**]
time = 0.05, size = 78, normalized size = 0.70

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left((-1+4ax)\sqrt{-1+a^2x^2}+3a^2x^2\text{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{2x\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^2),x]
```

```
[Out] -1/2*(Sqrt[c - c/(a^2*x^2)]*((-1 + 4*a*x)*Sqrt[-1 + a^2*x^2] + 3*a^2*x^2*Ar
cTan[1/Sqrt[-1 + a^2*x^2]]))/(x*Sqrt[-1 + a^2*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(92) = 184.
time = 0.12, size = 348, normalized size = 3.11

method	result
risch	$-\frac{(4a^3x^3-a^2x^2-4ax+1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{2x(a^2x^2-1)}-\frac{3a^2\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c\,(a^2x^2-1)}\,x}{2\sqrt{-c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\left(-4\sqrt{-\frac{c}{a^2}}\,\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,a^3c\,x^3+4\sqrt{-\frac{c}{a^2}}\,\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}a^3x+4\sqrt{\frac{c(ax+1)(ax-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2c\,x^2-4c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x*(-4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^
(1/2)*a^3*c*x^3+4*(-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(3/2)*a^3*x+4*(c*(a*x+
1)*(a*x-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2*c*x^2-4*c^(3/2)*ln(((c*(a*x+1)*(a*
x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*(-c/a^2)^(1/2)*a*x^2+4*c^(3/2)*ln(c^(
1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*a*x^2-3*(-c/a^2)^(1/2)*(c*
(a^2*x^2-1)/a^2)^(1/2)*a^2*c*x^2-a^2*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/
2)-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*c^2*x^2)/
(-c/a^2)^(1/2)/(c*(a^2*x^2-1)/a^2)^(1/2)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^2), x)
```

Fricas [A]
time = 0.38, size = 176, normalized size = 1.57

$$\left[\frac{3\,a\sqrt{-c}\,x\log\left(-\frac{a^2cx^2+2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)-2\,(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{4\,x},-\frac{3\,a\sqrt{c}\,x\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(4\,ax-1)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a*sqrt(-c)*x*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 - c)/
(a^2*x^2)) - 2*c)/x^2) - 2*(4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x,
-1/2*(3*a*sqrt(c)*x*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2
*c*x^2 - c)) + (4*a*x - 1)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^2\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**2,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**2*(a*x + 1)),
x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(91) = 182.
time = 0.49, size = 194, normalized size = 1.73

$$\left(3\sqrt{c}\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3ac\,\mathrm{sgn}(x)+4\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2c^{\frac{3}{2}}|a|\mathrm{sgn}(x)-\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)ac^2\mathrm{sgn}(x)+4\,c^{\frac{5}{2}}|a|\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^2a}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^2,x, algorithm="giac")
```

```
[Out] (3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sgn(x) -
((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^3*a*c*sgn(x) + 4*(sqrt(a^2*c)*x - sq
rt(a^2*c*x^2 - c))^2*c^(3/2)*abs(a)*sgn(x) - (sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))*a*c^2*sgn(x) + 4*c^(5/2)*abs(a)*sgn(x))/(((sqrt(a^2*c)*x - sqrt(a^2
*c*x^2 - c))^2 + c)^2*a))*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^2\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^2*(a*x + 1)), x)
```

3.916 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=140

$$a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2}-\frac{a^3\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] a^2*(c-c/a^2/x^2)^(1/2)-1/3*a*(-a*x+1)*(c-c/a^2/x^2)^(1/2)/x+1/3*(-a*x+1)^2
*(c-c/a^2/x^2)^(1/2)/x^2-a^3*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a
^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6302, 6294, 6264, 98, 96, 94, 214}

$$a^2\sqrt{c-\frac{c}{a^2x^2}}-\frac{a(1-ax)\sqrt{c-\frac{c}{a^2x^2}}}{3x}+\frac{(1-ax)^2\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}-\frac{a^3x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{\sqrt{1-ax}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] a^2*Sqrt[c - c/(a^2*x^2)] - (a*Sqrt[c - c/(a^2*x^2)]*(1 - a*x))/(3*x) + (Sq
rt[c - c/(a^2*x^2)]*(1 - a*x)^2)/(3*x^2) - (a^3*Sqrt[c - c/(a^2*x^2)]*x*Arc
Tanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^4}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^4\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} + \frac{\left(2a\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^3\sqrt{1+ax}}\,dx}{3\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} - \frac{\left(a^2\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{\sqrt{1-ax}\,\sqrt{1}}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} + \frac{\left(a^3\sqrt{\right.}}{}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} - \frac{\left(a^4\sqrt{\right.}}{}$$

$$= a^2\sqrt{c-\frac{c}{a^2x^2}} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)}{3x} + \frac{\sqrt{c-\frac{c}{a^2x^2}}\,(1-ax)^2}{3x^2} - \frac{a^3\sqrt{c}}{}$$

Mathematica [A]
time = 0.05, size = 86, normalized size = 0.61

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(1-3ax+5a^2x^2)+3a^3x^3\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{3x^2\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(1 - 3*a*x + 5*a^2*x^2) + 3*a^3*
x^3*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(3*x^2*Sqrt[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(120) = 240$.
time = 0.12, size = 378, normalized size = 2.70

method	result
risch	$\frac{\left(5a^4x^4-3a^3x^3-4a^2x^2+3ax-1\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}}{3x^2\left(a^2x^2-1\right)}+\frac{a^3\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,\sqrt{c\left(a^2x^2-1\right)}}{\sqrt{-c}\,\left(a^2x^2-1\right)}$
default	$-\frac{\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2x^2}}\,a\left(-6\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3c\,x^4+6\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^3x^2-3\sqrt{\frac{c\left(a^2x^2-1\right)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2c\,x^3+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^2*a*(-6*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/
a^2)^(1/2)*a^3*c*x^4+6*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^3*x^2-3*(
c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2*c*x^3+6*(c*(a*x+1)*(a*x-1)/a^2)
^(1/2)*(-c/a^2)^(1/2)*a^2*c*x^3-6*c^(3/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)
*c^(1/2)+c*x)/c^(1/2))*(-c/a^2)^(1/2)*a*x^3+6*c^(3/2)*ln(c^(1/2)*x+(c*(a^2*
x^2-1)/a^2)^(1/2))*(-c/a^2)^(1/2)*a*x^3-3*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2
)^(1/2)*a^2*x-3*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x
)*c^2*x^3+a*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(
1/2)/(-c/a^2)^(1/2)/c
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^3), x)
```

Fricas [A]

time = 0.37, size = 200, normalized size = 1.43

$$\left[\frac{3\,a^2\sqrt{-c}\,x^2\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\left(5\,a^2x^2-3\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{6\,x^2},\ \frac{3\,a^2\sqrt{c}\,x^2\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+\left(5\,a^2x^2-3\,ax+1\right)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(-c)*x^2*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2 -
 c)/(a^2*x^2)) - 2*c)/x^2) + 2*(5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2 - c)
/(a^2*x^2)))/x^2, 1/3*(3*a^2*sqrt(c)*x^2*arctan(a*sqrt(c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (5*a^2*x^2 - 3*a*x + 1)*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)))/x^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^3\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**3,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**3*(a*x + 1)),
x)
```

Giac [A]
time = 0.73, size = 231, normalized size = 1.65

$$-\frac{2}{3}\left(3\,a\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^5ac\mathrm{sgn}(x)+3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^4c^{\frac{3}{2}}|a|\mathrm{sgn}(x)+12\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2c^{\frac{5}{2}}|a|\mathrm{sgn}(x)-3\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)ac^3\mathrm{sgn}(x)+5\,c^{\frac{7}{2}}|a|\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^3}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^3,x, algorithm="giac")
```

```
[Out] -2/3*(3*a*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))*sg
n(x) - (3*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a*c*sgn(x) + 3*(sqrt(a^2*
c)*x - sqrt(a^2*c*x^2 - c))^4*c^(3/2)*abs(a)*sgn(x) + 12*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^2*c^(5/2)*abs(a)*sgn(x) - 3*(sqrt(a^2*c)*x - sqrt(a^2*c
*x^2 - c))*a*c^3*sgn(x) + 5*c^(7/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a
^2*c*x^2 - c))^2 + c)^3)*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^3\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^3*(a*x + 1)), x)
```

3.917 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=156

$$-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}+\frac{7a^4\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)}{8\sqrt{1-ax}\,\sqrt{1+ax}}$$

```
[Out] -4/3*a^3*(c-c/a^2/x^2)^(1/2)+1/4*(c-c/a^2/x^2)^(1/2)/x^3-2/3*a*(c-c/a^2/x^2
)^(1/2)/x^2+7/8*a^2*(c-c/a^2/x^2)^(1/2)/x+7/8*a^4*x*arctanh((-a*x+1)^(1/2)*
(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3}+\frac{7a^4x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)}{8\sqrt{1-ax}\,\sqrt{ax+1}}-\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]
```

```
[Out] (-4*a^3*Sqrt[c - c/(a^2*x^2)])/3 + Sqrt[c - c/(a^2*x^2)]/(4*x^3) - (2*a*Sqr
t[c - c/(a^2*x^2)])/(3*x^2) + (7*a^2*Sqrt[c - c/(a^2*x^2)])/(8*x) + (7*a^4*
Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]])/(8*Sqrt[1 - a
*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^5}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^5\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{8a-7a^2x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{4\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{21a^2-16a^3x}{x^3\sqrt{1-ax}\,\sqrt{1+ax}}}{12\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{24\sqrt{1-}}$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} -$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} -$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} +$$

$$= -\frac{4}{3}a^3\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{4x^3} - \frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{3x^2} + \frac{7a^2\sqrt{c-\frac{c}{a^2x^2}}}{8x} +$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.60

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(-6+16ax-21a^2x^2+32a^3x^3)+21a^4x^4\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{24x^3\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^4),x]

[Out] -1/24*(Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(-6 + 16*a*x - 21*a^2*x^2 + 32*a^3*x^3) + 21*a^4*x^4*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(x^3*Sqrt[-1 + a^2*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(128) = 256$.
time = 0.12, size = 410, normalized size = 2.63

method	result
risch	$-\frac{(32a^5x^5-21a^4x^4-16a^3x^3+15a^2x^2-16ax+6)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{24x^3(a^2x^2-1)}-\frac{7a^4\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,\sqrt{c}}{8\sqrt{-c}\,(a^2x^2-1)}$
default	$\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-48\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3c\,x^5+48\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^3x^3-21\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^2c\,x^4+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x,method=_RETURNVERBOSE)

[Out] 1/24*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^3*a^2*(-48*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c*x^5+48*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^3*x^3-21*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^2*c*x^4+48*(-c/a^2)^(1/2)*(c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^2*c*x^4-48*(-c/a^2)^(1/2)*c^(3/2)*ln(((c*(a*x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a*x^4+48*(-c/a^2)^(1/2)*c^(3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*a*x^4-27*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^2*x^2-21*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(1/2)*a^2-c)/a^2/x)*c^2*x^4+16*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)-6*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/(-c/a^2)^(1/2)/c

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="maxima")

[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^4), x)

Fricas [A]
time = 0.35, size = 216, normalized size = 1.38

$$\left[\frac{21\,a^{3}\sqrt{-c}\,x^{3}\log\left(-\frac{a^{2}cx^{2}+2\,a\sqrt{-c}\,x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}-2\,c}{x^{2}}\right)-2\,(32\,a^{3}x^{3}-21\,a^{2}x^{2}+16\,ax-6)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{48\,x^{3}},\ -\frac{21\,a^{3}\sqrt{c}\,x^{3}\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{a^{2}cx^{2}-c}\right)+(32\,a^{3}x^{3}-21\,a^{2}x^{2}+16\,ax-6)\sqrt{\frac{a^{2}cx^{2}-c}{a^{2}x^{2}}}}{24\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(21*a^3*sqrt(-c)*x^3*log(-(a^2*c*x^2 + 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) - 2*(32*a^3*x^3 - 21*a^2*x^2 + 16*a*x - 6)*sqr
t((a^2*c*x^2 - c)/(a^2*x^2)))/x^3, -1/24*(21*a^3*sqrt(c)*x^3*arctan(a*sqrt(
c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (32*a^3*x^3 - 21*a^
2*x^2 + 16*a*x - 6)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^{4}\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**4,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**4*(a*x + 1)),
x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(128) = 256.
time = 1.58, size = 316, normalized size = 2.03

$$\frac{1}{12}\left(21\,a^{2}\sqrt{c}\arctan\left(-\frac{\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{21\left(\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}\right)^{7}a^{2}c\,\mathrm{sgn}(x)+45\left(\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}\right)^{5}a^{2}c^{2}\mathrm{sgn}(x)+96\left(\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}\right)^{4}ac^{\frac{5}{2}}|a|\mathrm{sgn}(x)-45\left(\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}\right)^{3}a^{2}c^{3}\mathrm{sgn}(x)+128\left(\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}\right)^{2}ac^{\frac{7}{2}}|a|\mathrm{sgn}(x)-21\left(\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}\right)a^{2}c^{4}\mathrm{sgn}(x)+32\,ac^{\frac{9}{2}}|a|\mathrm{sgn}(x)}{\left(\left(\sqrt{a^{2}c}\,x-\sqrt{a^{2}cx^{2}-c}\right)^{2}+c\right)^{4}}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(21*a^2*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c))
*sgn(x) - (21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^2*c*sgn(x) + 45*(sq
rt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^5*a^2*c^2*sgn(x) + 96*(sqrt(a^2*c)*x - s
qrt(a^2*c*x^2 - c))^4*a*c^(5/2)*abs(a)*sgn(x) - 45*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^2*c^3*sgn(x) + 128*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a*c^(7/2)*abs(a)*sgn(x) - 21*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^2*c^
```

```
4*sgn(x) + 32*a*c^(9/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c
))^2 + c)^4)*abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^4\,\left(a\,x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)),x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^4*(a*x + 1)), x)
```

3.918 $\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=181

$$\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}+\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}-\frac{3a^5\sqrt{c-\frac{c}{a^2x^2}}\,x\tanh}{4\sqrt{1-a}}$$

```
[Out] 6/5*a^4*(c-c/a^2/x^2)^(1/2)+1/5*(c-c/a^2/x^2)^(1/2)/x^4-1/2*a*(c-c/a^2/x^2)
^(1/2)/x^3+3/5*a^2*(c-c/a^2/x^2)^(1/2)/x^2-3/4*a^3*(c-c/a^2/x^2)^(1/2)/x-3/
4*a^5*x*arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))*(c-c/a^2/x^2)^(1/2)/(-a*x+1)^
(1/2)/(a*x+1)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {6302, 6294, 6264, 100, 156, 12, 94, 214}

$$\frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4}-\frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3}-\frac{3a^5x\sqrt{c-\frac{c}{a^2x^2}}\tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)}{4\sqrt{1-ax}\sqrt{ax+1}}+\frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}}-\frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (6*a^4*Sqrt[c - c/(a^2*x^2)])/5 + Sqrt[c - c/(a^2*x^2)]/(5*x^4) - (a*Sqrt[c
 - c/(a^2*x^2)])/(2*x^3) + (3*a^2*Sqrt[c - c/(a^2*x^2)])/(5*x^2) - (3*a^3*S
qrt[c - c/(a^2*x^2)])/(4*x) - (3*a^5*Sqrt[c - c/(a^2*x^2)]*x*ArcTanh[Sqrt[1
 - a*x]*Sqrt[1 + a*x]])/(4*Sqrt[1 - a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6264

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)*(x_))^(p_.), x_Symbol
] :> Dist[c^p, Int[u*(1 + d*(x/c))^p*((1 + a*x)^(n/2)/(1 - a*x)^(n/2)), x],
 x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[a^2*c^2 - d^2, 0] && (IntegerQ[p] |
| GtQ[c, 0])
```

Rule 6294

```
Int[E^(ArcTanh[(a_.)*(x_)]*(n_))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbol
] :> Dist[x^(2*p)*((c + d/x^2)^p/((1 - a*x)^p*(1 + a*x)^p)), Int[(u/x^(2*p)
)*(1 - a*x)^p*(1 + a*x)^p*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[{a, c, d, n,
p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[p] && IntegerQ[n/2] &&  !GtQ[c, 0
]
```

Rule 6302

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_))*(u_.), x_Symbol] :> Dist[(-1)^(n/2), Int[u
*E^(n*ArcTanh[a*x]), x], x] /; FreeQ[a, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{e^{-2\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = -\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{e^{-2\tanh^{-1}(ax)}\sqrt{1-ax}\,\sqrt{1+ax}}{x^6}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{(1-ax)^{3/2}}{x^6\sqrt{1+ax}}\,dx}{\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{10a-9a^2x}{x^5\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{5\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} - \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int \frac{36a^2-30a^3x}{x^4\sqrt{1-ax}\,\sqrt{1+ax}}}{20\sqrt{1-ax}\,\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} + \frac{\left(\sqrt{c-\frac{c}{a^2x^2}}\,x\right)\int}{60\sqrt{1-ax}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} - \frac{3a^3\sqrt{c-\frac{c}{a^2x^2}}}{4x} -$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} -$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} -$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} -$$

$$= \frac{6}{5}a^4\sqrt{c-\frac{c}{a^2x^2}} + \frac{\sqrt{c-\frac{c}{a^2x^2}}}{5x^4} - \frac{a\sqrt{c-\frac{c}{a^2x^2}}}{2x^3} + \frac{3a^2\sqrt{c-\frac{c}{a^2x^2}}}{5x^2} -$$

Mathematica [A]

time = 0.06, size = 102, normalized size = 0.56

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(\sqrt{-1+a^2x^2}\,(4-10ax+12a^2x^2-15a^3x^3+24a^4x^4)+15a^5x^5\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{-1+a^2x^2}}\right)\right)}{20x^4\sqrt{-1+a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(2*ArcCoth[a*x])*x^5),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(Sqrt[-1 + a^2*x^2]*(4 - 10*a*x + 12*a^2*x^2 - 15*a^
3*x^3 + 24*a^4*x^4) + 15*a^5*x^5*ArcTan[1/Sqrt[-1 + a^2*x^2]]))/(20*x^4*Sqr
t[-1 + a^2*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(149) = 298$.
time = 0.14, size = 447, normalized size = 2.47

method	result
risch	$\frac{(24a^6x^6-15a^5x^5-12a^4x^4+5a^3x^3-8a^2x^2+10ax-4)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{20x^4(a^2x^2-1)}+\frac{3a^5\ln\left(\frac{-2c+2\sqrt{-c}\,\sqrt{a^2c\,x^2-c}}{x}\right)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}}{4\sqrt{-c}\,(a^2x^2-1)}$
default	$-\frac{\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,a^2\left(-40\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^4c\,x^6+40\left(\frac{c\left(a^2x^2-1\right)}{a^2}\right)^{\frac{3}{2}}\sqrt{-\frac{c}{a^2}}\,a^4x^4-15\sqrt{\frac{c(a^2x^2-1)}{a^2}}\,\sqrt{-\frac{c}{a^2}}\,a^3c\,x^5\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)/x^4*a^2*(-40*(c*(a^2*x^2-1)/a^2)^(1/2)*
(-c/a^2)^(1/2)*a^4*c*x^6+40*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^4*x^
4-15*(c*(a^2*x^2-1)/a^2)^(1/2)*(-c/a^2)^(1/2)*a^3*c*x^5+40*(-c/a^2)^(1/2)*(
c*(a*x+1)*(a*x-1)/a^2)^(1/2)*a^3*c*x^5-40*(-c/a^2)^(1/2)*c^(3/2)*ln(((c*(a*
x+1)*(a*x-1)/a^2)^(1/2)*c^(1/2)+c*x)/c^(1/2))*a^2*x^5+40*(-c/a^2)^(1/2)*c^(
3/2)*ln(c^(1/2)*x+(c*(a^2*x^2-1)/a^2)^(1/2))*a^2*x^5-25*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2)*a^3*x^3-15*ln(2*((-c/a^2)^(1/2)*(c*(a^2*x^2-1)/a^2)^(
1/2)*a^2-c)/a^2/x)*a*c^2*x^5+16*(c*(a^2*x^2-1)/a^2)^(3/2)*(-c/a^2)^(1/2)*a^
2*x^2-10*a*(c*(a^2*x^2-1)/a^2)^(3/2)*x*(-c/a^2)^(1/2)+4*(c*(a^2*x^2-1)/a^2)
^(3/2)*(-c/a^2)^(1/2))/(c*(a^2*x^2-1)/a^2)^(1/2)/c/(-c/a^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x - 1)*sqrt(c - c/(a^2*x^2))/((a*x + 1)*x^5), x)
```

Fricas [A]
time = 0.37, size = 232, normalized size = 1.28

$$\left[\frac{15\,a^4\sqrt{-c}\,x^4\log\left(-\frac{a^2cx^2-2\,a\sqrt{-c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}-2\,c}{x^2}\right)+2\,(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{40\,x^4}, \frac{15\,a^4\sqrt{c}\,x^4\arctan\left(\frac{a\sqrt{c}\,x\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{a^2cx^2-c}\right)+(24\,a^4x^4-15\,a^3x^3+12\,a^2x^2-10\,ax+4)\sqrt{\frac{a^2cx^2-c}{a^2x^2}}}{20\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="fricas")
```

```
[Out] [1/40*(15*a^4*sqrt(-c)*x^4*log(-(a^2*c*x^2 - 2*a*sqrt(-c)*x*sqrt((a^2*c*x^2
 - c)/(a^2*x^2)) - 2*c)/x^2) + 2*(24*a^4*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10
*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)))/x^4, 1/20*(15*a^4*sqrt(c)*x^4*ar
ctan(a*sqrt(c)*x*sqrt((a^2*c*x^2 - c)/(a^2*x^2))/(a^2*c*x^2 - c)) + (24*a^4
*x^4 - 15*a^3*x^3 + 12*a^2*x^2 - 10*a*x + 4)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)
))/x^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,(ax-1)}{x^5\,(ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*(a*x-1)/(a*x+1)/x**5,x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*(a*x - 1)/(x**5*(a*x + 1)),
x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(149) = 298.
time = 2.04, size = 362, normalized size = 2.00

$$-\frac{1}{10}\left(15\,a^3\sqrt{c}\,\arctan\left(-\frac{\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}}{\sqrt{c}}\right)\mathrm{sgn}(x)-\frac{15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^9a^3c\,\mathrm{sgn}(x)+70\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^7a^3c^2\mathrm{sgn}(x)+40\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^6a^2c^{\frac{5}{2}}|a|\mathrm{sgn}(x)+200\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^4a^2c^{\frac{7}{2}}|a|\mathrm{sgn}(x)-70\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^3a^3c^4\mathrm{sgn}(x)+120\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2a^2c^{\frac{9}{2}}|a|\mathrm{sgn}(x)-15\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)a^3c^5\mathrm{sgn}(x)+24\,a^2c^{\frac{11}{2}}|a|\mathrm{sgn}(x)}{\left(\left(\sqrt{a^2c}\,x-\sqrt{a^2cx^2-c}\right)^2+c\right)^5}\right)|a|$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*(a*x-1)/(a*x+1)/x^5,x, algorithm="giac")
```

```
[Out] -1/10*(15*a^3*sqrt(c)*arctan(-(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))/sqrt(c)
)*sgn(x) - (15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^9*a^3*c*sgn(x) + 70*(s
qrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^7*a^3*c^2*sgn(x) + 40*(sqrt(a^2*c)*x -
sqrt(a^2*c*x^2 - c))^6*a^2*c^(5/2)*abs(a)*sgn(x) + 200*(sqrt(a^2*c)*x - sqr
t(a^2*c*x^2 - c))^4*a^2*c^(7/2)*abs(a)*sgn(x) - 70*(sqrt(a^2*c)*x - sqrt(a^
2*c*x^2 - c))^3*a^3*c^4*sgn(x) + 120*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))^
2*a^2*c^(9/2)*abs(a)*sgn(x) - 15*(sqrt(a^2*c)*x - sqrt(a^2*c*x^2 - c))*a^3*
c^5*sgn(x) + 24*a^2*c^(11/2)*abs(a)*sgn(x))/((sqrt(a^2*c)*x - sqrt(a^2*c*x^
2 - c))^2 + c)^5)*abs(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(a\,x-1\right)}{x^5\,\left(a\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)),x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*(a*x - 1))/(x^5*(a*x + 1)), x)
```

3.919 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx$

Optimal. Leaf size=186

$$-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -4*x*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)+2*x^2*(c-c/a^2/x^2)^(1/2)/
a^2/(1-1/a^2/x^2)^(1/2)-x^3*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/4*x
^4*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/
a^4/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{2x^2\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^4\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-4*Sqrt[c - c/(a^2*x^2)]*x)/(a^3*Sqrt[1 - 1/(a^2*x^2)]) + (2*Sqrt[c - c/(a
^2*x^2)]*x^2)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (Sqrt[c - c/(a^2*x^2)]*x^3)/(a*
Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^4)/(4*Sqrt[1 - 1/(a^2*x^2
)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^3\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^3\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x^2(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(-\frac{4}{a^2}+\frac{4x}{a}-3x^2+ax^3+\frac{4}{a^2(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= -\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^4}{4\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 69, normalized size = 0.37

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax(-16+8ax-4a^2x^2+a^3x^3)+16\log(1+ax))}{4a^4\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^3)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(-16 + 8*a*x - 4*a^2*x^2 + a^3*x^3) + 16*Log[1
+ a*x]))/(4*a^4*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.06, size = 89, normalized size = 0.48

method	result	size
default	$\frac{\left(a^{4} x^{4}-4 a^{3} x^{3}+8 a^{2} x^{2}-16 a x+16 \ln \left(a x+1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{4 a^{3} \left(a x-1\right)^{2}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/4*(a^4*x^4-4*a^3*x^3+8*a^2*x^2-16*a*x+16*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/
x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^3/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.39, size = 48, normalized size = 0.26

$$\frac{\left(a^{4} x^{4} - 4\, a^{3} x^{3} + 8\, a^{2} x^{2} - 16\, a x + 16\, \log\left(a x + 1\right)\right) \sqrt{a^{2} c}}{4\, a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/4*(a^4*x^4 - 4*a^3*x^3 + 8*a^2*x^2 - 16*a*x + 16*log(a*x + 1))*sqrt(a^2*c
)/a^5
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^3*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{c-\frac{c}{a^2\,x^2}}\,\left(\frac{a\,x-1}{a\,x+1}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^3*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.920 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx$

Optimal. Leaf size=151

$$\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] 4*x*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)-3/2*x^2*(c-c/a^2/x^2)^(1/2)
/a/(1-1/a^2/x^2)^(1/2)+1/3*x^3*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*ln
(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^3/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 78}

$$-\frac{3x^2\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4x\sqrt{c-\frac{c}{a^2x^2}}}{a^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x^3\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (4*Sqrt[c - c/(a^2*x^2)]*x)/(a^2*Sqrt[1 - 1/(a^2*x^2)]) - (3*Sqrt[c - c/(a^
2*x^2)]*x^2)/(2*a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x^2)]*x^3)/(3*S
qrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a^3*Sqrt[1
- 1/(a^2*x^2)])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x^2\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{x(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{4}{a}-3x+ax^2-\frac{4}{a(1+ax)}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{4\sqrt{c-\frac{c}{a^2x^2}}\,x}{a^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^3}{3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(}{a^3\sqrt{1-\frac{1}{a^2x}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.41

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\left(ax(24-9ax+2a^2x^2)-24\log(1+ax)\right)}{6a^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x^2)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(24 - 9*a*x + 2*a^2*x^2) - 24*Log[1 + a*x]))/(6
*a^3*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 82, normalized size = 0.54

method	result	size
default	$-\frac{\left(-2 a^{3} x^{3}+9 a^{2} x^{2}-24 a x +24 \ln \left(a x +1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{6 a^{2} \left(a x -1\right)^{2}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/6*(-2*a^3*x^3+9*a^2*x^2-24*a*x+24*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(
1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/a^2/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 41, normalized size = 0.27

$$\frac{\left(2 a^{3} x^{3} - 9 a^{2} x^{2} + 24 a x - 24 \log\left(a x + 1\right)\right) \sqrt{a^{2} c}}{6 a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*a^3*x^3 - 9*a^2*x^2 + 24*a*x - 24*log(a*x + 1))*sqrt(a^2*c)/a^4
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x^2*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x^2*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.921 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx$

Optimal. Leaf size=112

$$-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -3*x*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)+1/2*x^2*(c-c/a^2/x^2)^(1/2)/
(1-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a^2/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {6332, 6328, 45}

$$\frac{x^2\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3x\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (-3*Sqrt[c - c/(a^2*x^2)]*x)/(a*Sqrt[1 - 1/(a^2*x^2)]) + (Sqrt[c - c/(a^2*x
^2)]*x^2)/(2*Sqrt[1 - 1/(a^2*x^2)]) + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x]
)/(a^2*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,x\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,x\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\frac{(-1+ax)^2}{1+ax}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int\left(-3+ax+\frac{4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$
$$=-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,x}{a\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x^2}{2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [**A**]
time = 0.02, size = 53, normalized size = 0.47

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax(-6+ax)+8\log(1+ax))}{2a^2\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c - c/(a^2*x^2)]*x)/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x*(-6 + a*x) + 8*Log[1 + a*x]))/(2*a^2*Sqrt[1 - 1
/(a^2*x^2)])
```

Maple [**A**]
time = 0.04, size = 73, normalized size = 0.65

method	result	size
default	$\frac{\left(a^{2} x^{2}-6 a x +8 \ln \left(a x +1\right)\right) x \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{2 a \left(a x -1\right)^{2}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2*x^2-6*a*x+8*ln(a*x+1))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a
*x-1)/(a*x+1))^(3/2)/a/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.29

$$\frac{\left(a^{2} x^{2} - 6 \, a x + 8 \, \log\left(a x + 1\right)\right) \sqrt{a^{2} c}}{2 \, a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(a^2*x^2 - 6*a*x + 8*log(a*x + 1))*sqrt(a^2*c)/a^3
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*x*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int(x*(c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.922 $\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=107

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+ln(x)*(c-c/a^2/x^2)^(1/2)/a/(1-1/
a^2/x^2)^(1/2)-4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/a/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6332, 6328, 84}

$$\frac{x\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*x)/Sqrt[1 - 1/(a^2*x^2)] + (Sqrt[c - c/(a^2*x^2)]*Lo
g[x])/(a*Sqrt[1 - 1/(a^2*x^2)]) - (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/(a
*Sqrt[1 - 1/(a^2*x^2)])
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(a+\frac{1}{x}-\frac{4a}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\
&= \frac{\sqrt{c-\frac{c}{a^2x^2}}\,x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{a\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{a\sqrt{1-\frac{1}{a^2x^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 0.44

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\,(ax+\log(x)-4\log(1+ax))}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/E^(3*ArcCoth[a*x]),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(a*x + Log[x] - 4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*
x^2)])
```

Maple [A]
time = 0.03, size = 67, normalized size = 0.63

method	result	size
default	$-\frac{(-ax+4\ln(ax+1)-\ln(x))x\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$	67

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x,method=_RETURNVERBOSE)

[Out] -(-a*x+4*ln(a*x+1)-ln(x))*x*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)

Fricas [**A**]
time = 0.35, size = 25, normalized size = 0.23

$$\frac{\sqrt{a^2c}\,(ax-4\,\log{(ax+1)}+\log{(x)})}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="fricas")

[Out] sqrt(a^2*c)*(a*x - 4*log(a*x + 1) + log(x))/a^2

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2),x)
```

```
[Out] int((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2), x)
```

3.923 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -(c-c/a^2/x^2)^(1/2)/a/x/(1-1/a^2/x^2)^(1/2)-3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)+4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6332, 6328, 90}

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{ax\sqrt{1-\frac{1}{a^2x^2}}}-\frac{3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
```

```
[Out] -(Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x)) - (3*Sqrt[c - c/(a^2*x
^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])
/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x}\,dx &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^2(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^2}-\frac{3a}{x}+\frac{4a^2}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}\\ &= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{a\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{3\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4\sqrt{c-\frac{c}{a^2x^2}}\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.49

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{x}-3a\log(x)+4a\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x),x]
[Out] (Sqrt[c - c/(a^2*x^2)]*(-x^(-1) - 3*a*Log[x] + 4*a*Log[1 + a*x]))/(a*Sqrt[1
 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 66, normalized size = 0.61

method	result	size
default	$\frac{(4\ln(ax+1)ax-3a\ln(x)x-1)\sqrt{\frac{c(a^2x^2-1)}{a^2x^2}}\,(ax+1)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{(ax-1)^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] (4*ln(a*x+1)*a*x-3*a*ln(x)*x-1)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x
-1)/(a*x+1))^(3/2)/(a*x-1)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="maxim
a")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.30

$$\frac{\sqrt{a^2c}\,(4\,ax\log{(ax+1)}-3\,ax\log{(x)}-1)}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="frica
s")
[Out] sqrt(a^2*c)*(4*a*x*log(a*x + 1) - 3*a*x*log(x) - 1)/(a^2*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x,x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x, x)
```

3.924 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/2*(c-c/a^2/x^2)^(1/2)/a/x^2/(1-1/a^2/x^2)^(1/2)+3*(c-c/a^2/x^2)^(1/2)/x/
(1-1/a^2/x^2)^(1/2)+4*a*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)-4*a*l
n(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2ax^2\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] -1/2*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^2) + (3*Sqrt[c - c/(a
^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) + (4*a*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sq
rt[1 - 1/(a^2*x^2)] - (4*a*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(
a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^2}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^2}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^3(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^3}-\frac{3a}{x^2}+\frac{4a^2}{x}-\frac{4a^3}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{2a\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a\sqrt{c-\frac{c}{a^2x^2}}\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\ldots}}{\sqrt{1\ldots}}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.45

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{2x^2}+\frac{3a}{x}+4a^2\log(x)-4a^2\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^2),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/2*1/x^2 + (3*a)/x + 4*a^2*Log[x] - 4*a^2*Log[1 +
 a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 82, normalized size = 0.56

method	result	size
default	$-\frac{\left(8\ln\left(ax+1\right)a^{2}x^{2}-8a^{2}\ln\left(x\right)x^{2}-6ax+1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{2\left(ax-1\right)^{2}x}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/2*(8*ln(a*x+1)*a^2*x^2-8*a^2*ln(x)*x^2-6*a*x+1)*(c*(a^2*x^2-1)/a^2/x^2)^
(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Fricas [A]
time = 0.35, size = 83, normalized size = 0.57

$$\frac{8\,a^{3}\sqrt{c}\,x^{2}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx-\sqrt{a^{2}c}\,(2\,ax+1)\sqrt{c}\,+ac}{ax^{2}+x}\right)+\sqrt{a^{2}c}\,(6\,ax-1)}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="fri
cas")
```

```
[Out] 1/2*(8*a^3*sqrt(c)*x^2*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
1)*sqrt(c) + a*c)/(a*x^2 + x)) + sqrt(a^2*c)*(6*a*x - 1))/(a^2*x^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^2,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^2, x)
```

3.925 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx$

Optimal. Leaf size=187

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,\log(1+ax)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

```
[Out] -1/3*(c-c/a^2/x^2)^(1/2)/a/x^3/(1-1/a^2/x^2)^(1/2)+3/2*(c-c/a^2/x^2)^(1/2)/
x^2/(1-1/a^2/x^2)^(1/2)-4*a*(c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)-4*a^2
*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4*a^2*ln(a*x+1)*(c-c/a^2/x^2
)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^2\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3ax^3\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] -1/3*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^3) + (3*Sqrt[c - c/(a
^2*x^2)])/(2*Sqrt[1 - 1/(a^2*x^2)]*x^2) - (4*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt
[1 - 1/(a^2*x^2)]*x) - (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2
*x^2)] + (4*a^2*Sqrt[c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^3}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^3}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^4(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^4}-\frac{3a}{x^3}+\frac{4a^2}{x^2}-\frac{4a^3}{x}+\frac{4a^4}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{3a\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{2\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 0.40

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{3x^3}+\frac{3a}{2x^2}-\frac{4a^2}{x}-4a^3\log(x)+4a^3\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^3),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/3*1/x^3 + (3*a)/(2*x^2) - (4*a^2)/x - 4*a^3*Log[
x] + 4*a^3*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.48

method	result	size
default	$\frac{\left(24 \ln \left(a x+1\right) a^{3} x^{3}-24 \ln \left(x\right) a^{3} x^{3}-24 a^{2} x^{2}+9 a x-2\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x+1\right) \left(\frac{a x-1}{a x+1}\right)^{\frac{3}{2}}}{6 \left(a x-1\right)^{2} x^{2}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*(24*ln(a*x+1)*a^3*x^3-24*ln(x)*a^3*x^3-24*a^2*x^2+9*a*x-2)*(c*(a^2*x^2-
1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Fricas [A]
time = 0.36, size = 91, normalized size = 0.49

$$\frac{24\, a^{4} \sqrt{c}\, x^{3} \log\left(\frac{2\, a^{3} c x^{2}+2\, a^{2} c x+\sqrt{a^{2} c}\, (2\, a x+1) \sqrt{c}+a c}{a x^{2}+x}\right)-(24\, a^{2} x^{2}-9\, a x+2) \sqrt{a^{2} c}}{6\, a^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(24*a^4*sqrt(c)*x^3*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (24*a^2*x^2 - 9*a*x + 2)*sqrt(a^2*c))/(a^
2*x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^3,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^3, x)
```

3.926 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx$

Optimal. Leaf size=221

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}+\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,\log(x)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\ldots}}{\ldots}$$

```
[Out] -1/4*(c-c/a^2/x^2)^(1/2)/a/x^4/(1-1/a^2/x^2)^(1/2)+(c-c/a^2/x^2)^(1/2)/x^3/
(1-1/a^2/x^2)^(1/2)-2*a*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)+4*a^2*(
c-c/a^2/x^2)^(1/2)/x/(1-1/a^2/x^2)^(1/2)+4*a^3*ln(x)*(c-c/a^2/x^2)^(1/2)/(1
-1/a^2/x^2)^(1/2)-4*a^3*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4ax^4\sqrt{1-\frac{1}{a^2x^2}}}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{x^3\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^3\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] -1/4*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^4) + Sqrt[c - c/(a^2*
x^2)]/(Sqrt[1 - 1/(a^2*x^2)]*x^3) - (2*a*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1
/(a^2*x^2)]*x^2) + (4*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x)
+ (4*a^3*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] - (4*a^3*Sqrt[
c - c/(a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^4}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^4}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^5(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^5}-\frac{3a}{x^4}+\frac{4a^2}{x^3}-\frac{4a^3}{x^2}+\frac{4a^4}{x}-\frac{4a^5}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{4a\sqrt{1-\frac{1}{a^2x^2}}\,x^4}+\frac{\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^3}-\frac{2a\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}+\frac{4a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.36

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{1}{4x^4}+\frac{a}{x^3}-\frac{2a^2}{x^2}+\frac{4a^3}{x}+4a^4\log(x)-4a^4\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^4),x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/4*1/x^4 + a/x^3 - (2*a^2)/x^2 + (4*a^3)/x + 4*a^
4*Log[x] - 4*a^4*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])
```

Maple [A]
time = 0.04, size = 98, normalized size = 0.44

method	result	size
default	$-\frac{\left(16\ln\left(ax+1\right)a^{4}x^{4}-16\ln\left(x\right)a^{4}x^{4}-16a^{3}x^{3}+8a^{2}x^{2}-4ax+1\right)\sqrt{\frac{c\left(a^{2}x^{2}-1\right)}{a^{2}x^{2}}}\,\left(ax+1\right)\left(\frac{ax-1}{ax+1}\right)^{\frac{3}{2}}}{4\left(ax-1\right)^{2}x^{3}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4*(16*ln(a*x+1)*a^4*x^4-16*ln(x)*a^4*x^4-16*a^3*x^3+8*a^2*x^2-4*a*x+1)*(
c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3/2)/(a*x-1)^2/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Fricas [A]
time = 0.36, size = 99, normalized size = 0.45

$$\frac{16\,a^{5}\sqrt{c}\,x^{4}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx-\sqrt{a^{2}c}\,(2\,ax+1)\sqrt{c}+ac}{ax^{2}+x}\right)+\left(16\,a^{3}x^{3}-8\,a^{2}x^{2}+4\,ax-1\right)\sqrt{a^{2}c}}{4\,a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="fri
cas")
```

```
[Out] 1/4*(16*a^5*sqrt(c)*x^4*log((2*a^3*c*x^2 + 2*a^2*c*x - sqrt(a^2*c)*(2*a*x +
 1)*sqrt(c) + a*c)/(a*x^2 + x)) + (16*a^3*x^3 - 8*a^2*x^2 + 4*a*x - 1)*sqrt
(a^2*c))/(a^2*x^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^4,x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4,x)
```

```
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^4, x)
```

3.927 $\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx$

Optimal. Leaf size=263

$$-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x^2}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}-\frac{4a^4\sqrt{c-\frac{c}{a^2x}}}{\sqrt{1-\frac{}{a}}}$$

```
[Out] -1/5*(c-c/a^2/x^2)^(1/2)/a/x^5/(1-1/a^2/x^2)^(1/2)+3/4*(c-c/a^2/x^2)^(1/2)/
x^4/(1-1/a^2/x^2)^(1/2)-4/3*a*(c-c/a^2/x^2)^(1/2)/x^3/(1-1/a^2/x^2)^(1/2)+2
*a^2*(c-c/a^2/x^2)^(1/2)/x^2/(1-1/a^2/x^2)^(1/2)-4*a^3*(c-c/a^2/x^2)^(1/2)/
x/(1-1/a^2/x^2)^(1/2)-4*a^4*ln(x)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)+4
*a^4*ln(a*x+1)*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6332, 6328, 90}

$$\frac{2a^2\sqrt{c-\frac{c}{a^2x^2}}}{x^2\sqrt{1-\frac{1}{a^2x^2}}}-\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5ax^5\sqrt{1-\frac{1}{a^2x^2}}}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4x^4\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3x^3\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^4\log(x)\sqrt{c-\frac{c}{a^2x^2}}}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{4a^4\sqrt{c-\frac{c}{a^2x^2}}\,\log(ax+1)}{\sqrt{1-\frac{1}{a^2x^2}}}-\frac{4a^3\sqrt{c-\frac{c}{a^2x^2}}}{x\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

```
[Out] -1/5*Sqrt[c - c/(a^2*x^2)]/(a*Sqrt[1 - 1/(a^2*x^2)]*x^5) + (3*Sqrt[c - c/(a
^2*x^2)])/(4*Sqrt[1 - 1/(a^2*x^2)]*x^4) - (4*a*Sqrt[c - c/(a^2*x^2)])/(3*Sq
rt[1 - 1/(a^2*x^2)]*x^3) + (2*a^2*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x
^2)]*x^2) - (4*a^3*Sqrt[c - c/(a^2*x^2)])/(Sqrt[1 - 1/(a^2*x^2)]*x) - (4*a^
4*Sqrt[c - c/(a^2*x^2)]*Log[x])/Sqrt[1 - 1/(a^2*x^2)] + (4*a^4*Sqrt[c - c/(
a^2*x^2)]*Log[1 + a*x])/Sqrt[1 - 1/(a^2*x^2)]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 6328

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_.), x_Symb
ol] :> Dist[c^p/a^(2*p), Int[(u/x^(2*p))*(-1 + a*x)^(p - n/2)*(1 + a*x)^(p
+ n/2), x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !Integ
```

```
erQ[n/2] && (IntegerQ[p] || GtQ[c, 0]) && IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}}{x^5}\,dx = \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{e^{-3\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}}{x^5}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \frac{(-1+ax)^2}{x^6(1+ax)}\,dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= \frac{\sqrt{c-\frac{c}{a^2x^2}}\int \left(\frac{1}{x^6}-\frac{3a}{x^5}+\frac{4a^2}{x^4}-\frac{4a^3}{x^3}+\frac{4a^4}{x^2}-\frac{4a^5}{x}+\frac{4a^6}{1+ax}\right)dx}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

$$= -\frac{\sqrt{c-\frac{c}{a^2x^2}}}{5a\sqrt{1-\frac{1}{a^2x^2}}\,x^5}+\frac{3\sqrt{c-\frac{c}{a^2x^2}}}{4\sqrt{1-\frac{1}{a^2x^2}}\,x^4}-\frac{4a\sqrt{c-\frac{c}{a^2x^2}}}{3\sqrt{1-\frac{1}{a^2x^2}}\,x^3}+\frac{2a^2\sqrt{c-\frac{c}{a^2x}}}{\sqrt{1-\frac{1}{a^2x^2}}\,x}$$

Mathematica [A]
time = 0.05, size = 89, normalized size = 0.34

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(-\frac{12-45ax+80a^2x^2-120a^3x^3+240a^4x^4}{60x^5}-4a^5\log(x)+4a^5\log(1+ax)\right)}{a\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - c/(a^2*x^2)]/(E^(3*ArcCoth[a*x])*x^5),x]
```

[Out] (Sqrt[c - c/(a^2*x^2)]*(-1/60*(12 - 45*a*x + 80*a^2*x^2 - 120*a^3*x^3 + 240 *a^4*x^4)/x^5 - 4*a^5*Log[x] + 4*a^5*Log[1 + a*x]))/(a*Sqrt[1 - 1/(a^2*x^2)])

Maple [A]
time = 0.04, size = 106, normalized size = 0.40

method	result	size
default	$\frac{\left(240 \ln \left(a x +1\right) a^{5} x^{5}-240 a^{5} \ln \left(x \right) x^{5}-240 a^{4} x^{4}+120 a^{3} x^{3}-80 a^{2} x^{2}+45 a x -12\right) \sqrt{\frac{c \left(a^{2} x^{2}-1\right)}{a^{2} x^{2}}}\, \left(a x +1\right) \left(\frac{a x -1}{a x +1}\right)^{\frac{3}{2}}}{60 \left(a x -1\right)^{2} x^{4}}$	106

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x,method=_RETURNVERBOSE)

[Out] 1/60*(240*ln(a*x+1)*a^5*x^5-240*a^5*ln(x)*x^5-240*a^4*x^4+120*a^3*x^3-80*a^ 2*x^2+45*a*x-12)*(c*(a^2*x^2-1)/a^2/x^2)^(1/2)*(a*x+1)*((a*x-1)/(a*x+1))^(3 /2)/(a*x-1)^2/x^4

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="max ima")

[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)

Fricas [A]
time = 0.35, size = 107, normalized size = 0.41

$$\frac{240\,a^{6}\sqrt{c}\,x^{5}\log\left(\frac{2\,a^{3}cx^{2}+2\,a^{2}cx+\sqrt{a^{2}c}\,(2\,ax+1)\sqrt{c}+ac}{ax^{2}+x}\right)-\left(240\,a^{4}x^{4}-120\,a^{3}x^{3}+80\,a^{2}x^{2}-45\,ax+12\right)\sqrt{a^{2}c}}{60\,a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="fri cas")

[Out] 1/60*(240*a^6*sqrt(c)*x^5*log((2*a^3*c*x^2 + 2*a^2*c*x + sqrt(a^2*c)*(2*a*x + 1)*sqrt(c) + a*c)/(a*x^2 + x)) - (240*a^4*x^4 - 120*a^3*x^3 + 80*a^2*x^2 - 45*a*x + 12)*sqrt(a^2*c))/(a^2*x^5)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**(1/2)*((a*x-1)/(a*x+1))**(3/2)/x**5,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^(1/2)*((a*x-1)/(a*x+1))^(3/2)/x^5,x, algorithm="gia
c")
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x - 1)/(a*x + 1))^(3/2)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-\frac{c}{a^2\,x^2}}\,{\left(\frac{a\,x-1}{a\,x+1}\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5,x)
[Out] int(((c - c/(a^2*x^2))^(1/2)*((a*x - 1)/(a*x + 1))^(3/2))/x^5, x)
```

3.928 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)\,dx$

Optimal. Leaf size=154

$$\frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}-\frac{2^{1+\frac{n}{2}}c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

```
[Out] 4*c*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)*hypergeom([2, 1-1/2*n],[2-1/2*
n],(a-1/x)/(a+1/x))/a/(2-n)-2^(1+1/2*n)*c*(1-1/a/x)^(1-1/2*n)*hypergeom([-1
/2*n, 1-1/2*n],[2-1/2*n],1/2*(a-1/x)/a)/a/(2-n)
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6329, 130, 71, 133}

$$\frac{4c\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\,{}_2F_1\left(2,1-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(2-n)}-\frac{c2^{\frac{n}{2}+1}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\,{}_2F_1\left(1-\frac{n}{2},-\frac{n}{2};2-\frac{n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (4*c*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2)*Hypergeometric2F1[2
, 1 - n/2, 2 - n/2, (a - x^(-1))/(a + x^(-1))])/(a*(2 - n)) - (2^(1 + n/2)*
c*(1 - 1/(a*x))^(1 - n/2)*Hypergeometric2F1[1 - n/2, -1/2*n, 2 - n/2, (a -
x^(-1))/(2*a)])/(a*(2 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 130

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_Symbol] :> Dist[b*(d/f^2), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)
, x], x] + Dist[(b*e - a*f)*((d*e - c*f)/f^2), Int[(a + b*x)^(m - 1)*((c +
d*x)^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\begin{aligned}\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx &= -\left(c\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{1+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)\right)\\ &= -\frac{2^{2-\frac{n}{2}}c\left(1+\frac{1}{ax}\right)^{\frac{4+n}{2}}F_1\left(\frac{4+n}{2};\frac{1}{2}(-2+n),2;\frac{6+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(4+n)}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 123, normalized size = 0.80

$$\frac{ce^{n\coth^{-1}(ax)}\left(2ax+anx+e^{2\coth^{-1}(ax)}n\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+4e^{2\coth^{-1}(ax)}\,{}_2F_1\left(2,1+\frac{n}{2};2+\frac{n}{2};-e^{2\coth^{-1}(ax)}\right)\right)}{a(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2)),x]
```

```
[Out] (c*E^(n*ArcCoth[a*x])*(2*a*x + a*n*x + E^(2*ArcCoth[a*x])*n*Hypergeometric2
F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])] + 4*E^(2*ArcCoth[a*x])*Hypergeometric2F1[
2, 1 + n/2, 2 + n/2, -E^(2*ArcCoth[a*x])]))/(a*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral((a^2*c*x^2 - c)*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(\int a^{2}e^{n\operatorname{acoth}(ax)}\,dx+\int\left(-\frac{e^{n\operatorname{acoth}(ax)}}{x^{2}}\right)dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2),x)
```

```
[Out] c*(Integral(a**2*exp(n*acoth(a*x)), x) + Integral(-exp(n*acoth(a*x))/x**2,
x))/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate((c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int e^{n \operatorname{acoth}(a x)} \left(c - \frac{c}{a^2 x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)),x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2)), x)
```

3.929 $\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=150

$$-\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn}+\frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c}+\frac{2\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}\ {}_2F_1\left(1,\frac{n}{2};\frac{2+n}{2};\ldots\right.}{ac}$$

```
[Out] -(1+n)*(1+1/a/x)^(1/2*n)/a/c/n/((1-1/a/x)^(1/2*n))+(1+1/a/x)^(1/2*n)*x/c/((
1-1/a/x)^(1/2*n))+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n],(a+1/x
)/(a-1/x))/a/c/((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6329, 105, 160, 12, 133}

$$\frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}\ {}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac}-\frac{(n+1)\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{acn}+\frac{x\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] -(((1 + n)*(1 + 1/(a*x))^(n/2))/(a*c*n*(1 - 1/(a*x))^(n/2))) + ((1 + 1/(a*x
))^(n/2)*x)/(c*(1 - 1/(a*x))^(n/2)) + (2*(1 + 1/(a*x))^(n/2)*Hypergeometric
2F1[1, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a*c*(1 - 1/(a*x))^(n/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{c-\frac{c}{a^2x^2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-1-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-1+\frac{n}{2}}}{x}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} - \frac{a\text{Subst}\left(\int \frac{n^2\left(1-\frac{x}{a}\right)^{-n/}}{a}\right.}{c}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} - \frac{n\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-n/2}(}{x}\right.}{ac}$$

$$= -\frac{(1+n)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}}{acn} + \frac{\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{n/2}x}{c} + \frac{2n\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\right.}{}$$

Mathematica [A]
time = 0.19, size = 94, normalized size = 0.63

$$\frac{e^{n\coth^{-1}(ax)}\left(e^{2\coth^{-1}(ax)}n^2\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+(2+n)\left(-1+anx+n\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)\right)}{acn(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2)),x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(E^(2*ArcCoth[a*x])*n^2*Hypergeometric2F1[1, 1 + n/2, 2
 + n/2, E^(2*ArcCoth[a*x])] + (2 + n)*(-1 + a*n*x + n*Hypergeometric2F1[1,
n/2, 1 + n/2, E^(2*ArcCoth[a*x])])))/(a*c*n*(2 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^2*c*x^2 - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\int\frac{x^2e^{n\,\mathrm{acoth}\,(ax)}}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2),x)
```

```
[Out] a**2*Integral(x**2*exp(n*acoth(a*x))/(a**2*x**2 - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2),x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{c-\frac{c}{a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)),x)
```

```
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2)), x)
```

3.930 $\int \frac{e^{n \coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2} \, dx$

Optimal. Leaf size=289

$$-\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)}+\frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2-n)n(2+n)}-\frac{(6+4n+n^2)\,(}{}$$

```
[Out] -(3+n)*(1-1/a/x)^(-1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/(2+n)+(-n^3-n^2+4*n+
6)*(1-1/a/x)^(1-1/2*n)*(1+1/a/x)^(-1+1/2*n)/a/c^2/n/(-n^2+4)-(n^2+4*n+6)*(1
+1/a/x)^(-1+1/2*n)/a/c^2/n/(2+n)/((1-1/a/x)^(1/2*n))+(1-1/a/x)^(-1-1/2*n)*(
1+1/a/x)^(-1+1/2*n)*x/c^2+2*(1+1/a/x)^(1/2*n)*hypergeom([1, 1/2*n],[1+1/2*n
],(a+1/x)/(a-1/x))/a/c^2/((1-1/a/x)^(1/2*n))
```

Rubi [A]
time = 0.17, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6329, 105, 160, 12, 133}

$$\frac{2\left(\frac{1}{ax}+1\right)^{n/2}\left(1-\frac{1}{ax}\right)^{-n/2}{}_2F_1\left(1,\frac{n}{2};\frac{n+2}{2};\frac{a+\frac{1}{x}}{a-\frac{1}{x}}\right)}{ac^2}-\frac{(n^2+4n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-n/2}}{ac^2n(n+2)}+\frac{(-n^3-n^2+4n+6)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}}{ac^2(2-n)n(n+2)}-\frac{(n+3)\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{ac^2(n+2)}+\frac{x\left(\frac{1}{ax}+1\right)^{\frac{n-2}{2}}\left(1-\frac{1}{ax}\right)^{-\frac{n}{2}-1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] -(((3 + n)*(1 - 1/(a*x))^(-1 - n/2)*(1 + 1/(a*x))^((-2 + n)/2))/(a*c^2*(2 +
 n))) + ((6 + 4*n - n^2 - n^3)*(1 - 1/(a*x))^(1 - n/2)*(1 + 1/(a*x))^((-2 +
 n)/2))/(a*c^2*(2 - n)*n*(2 + n)) - ((6 + 4*n + n^2)*(1 + 1/(a*x))^((-2 + n
)/2))/(a*c^2*n*(2 + n)*(1 - 1/(a*x))^(n/2)) + ((1 - 1/(a*x))^(-1 - n/2)*(1
+ 1/(a*x))^((-2 + n)/2)*x)/c^2 + (2*(1 + 1/(a*x))^(n/2)*Hypergeometric2F1[1
, n/2, (2 + n)/2, (a + x^(-1))/(a - x^(-1))])/(a*c^2*(1 - 1/(a*x))^(n/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x^2}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{n}{a}-\frac{3x}{a^2}\right)\left(1-\frac{x}{a}\right)^{-2-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-2+\frac{n}{2}}}{x}\,dx,x,\frac{1}{x}\right)}{c^2}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}x}{c^2} - \frac{a\text{Subst}\Big(}{}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} - \frac{(6+4n+n^2)\left(1-\frac{1}{ax}\right)^{-n/2}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2n(2+n)}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{ac^2(2-n)n(2+n)}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{ac^2(2-n)n(2+n)}$$

$$= -\frac{(3+n)\left(1-\frac{1}{ax}\right)^{-1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-2+n)}}{ac^2(2+n)} + \frac{(6+4n-n^2-n^3)\left(1-\frac{1}{ax}\right)^{1-\frac{n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-}}{ac^2(2-n)n(2+n)}$$

Mathematica [A]
time = 0.89, size = 180, normalized size = 0.62

$$\frac{e^{n\coth^{-1}(ax)}\left(-6+n^2+6anx-an^3x+6a^2x^2-2a^2n^2x^2-4a^3nx^3+a^3n^3x^3+e^{2\coth^{-1}(ax)}(-2+n)n^2(-1+a^2x^2)\,{}_2F_1\left(1,1+\frac{n}{2};2+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)+n(-4+n^2)(-1+a^2x^2)\,{}_2F_1\left(1,\frac{n}{2};1+\frac{n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{ac^2(-2+n)n(2+n)(-1+a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/(c - c/(a^2*x^2))^2,x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-6 + n^2 + 6*a*n*x - a*n^3*x + 6*a^2*x^2 - 2*a^2*n^2*x
^2 - 4*a^3*n*x^3 + a^3*n^3*x^3 + E^(2*ArcCoth[a*x])*(-2 + n)*n^2*(-1 + a^2*
x^2)*Hypergeometric2F1[1, 1 + n/2, 2 + n/2, E^(2*ArcCoth[a*x])] + n*(-4 + n
^2)*(-1 + a^2*x^2)*Hypergeometric2F1[1, n/2, 1 + n/2, E^(2*ArcCoth[a*x])]))
/(a*c^2*(-2 + n)*n*(2 + n)*(-1 + a^2*x^2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{n\,\mathrm{arccoth}(ax)}}{\left(c-\frac{c}{a^2x^2}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral(a^4*x^4*((a*x + 1)/(a*x - 1))^(1/2*n)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2
 + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4 \int \frac{x^4 e^{n \operatorname{acoth}(ax)}}{a^4x^4-2a^2x^2+1}\, dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**2,x)
```

```
[Out] a**4*Integral(x**4*exp(n*acoth(a*x))/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/(c - c/(a^2*x^2))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e^{n \operatorname{acoth}(a x)}}{\left(c - \frac{c}{a^2 x^2}\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2,x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^2, x)
```

3.931 $\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$

Optimal. Leaf size=295

$$\frac{\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{1-\frac{1}{a^2x^2}}}+\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}-$$

```
[Out] (1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(c-c/a^2/x^2)^(1/2)/(1-1/a^2/
x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1, 1
/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/a^2
/x^2)^(1/2)-2^(1/2+1/2*n)*(1-1/a/x)^(1/2-1/2*n)*hypergeom([1/2-1/2*n, 1/2-1
/2*n],[3/2-1/2*n],1/2*(a-1/x)/a)*(c-c/a^2/x^2)^(1/2)/a/(1-n)/(1-1/a^2/x^2)^
(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6332, 6329, 130, 71, 98, 133}

$$\frac{2n\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}-\frac{2^{\frac{n+1}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{2a}\right)}{a(1-n)\sqrt{1-\frac{1}{a^2x^2}}}+\frac{x\sqrt{c-\frac{c}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{1-\frac{1}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[1 - 1/(a^2*x^2)] + (2*n*Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[1 - 1/(a^2*x^2)]) - (2^((1 + n)
/2)*Sqrt[c - c/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*Hypergeometric2F1[(1 -
n)/2, (1 - n)/2, (3 - n)/2, (a - x^(-1))/(2*a)])/(a*(1 - n)*Sqrt[1 - 1/(a^2
*x^2)])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 130

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_))^2, x_Symbol] :> Dist[b*(d/f^2), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)
, x], x] + Dist[(b*e - a*f)*((d*e - c*f)/f^2), Int[(a + b*x)^(m - 1)*((c +
d*x)^(n - 1)/(e + f*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[m + n, 0] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
 a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx=\frac{\sqrt{c-\frac{c}{a^2x^2}}\int e^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\,dx}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{\sqrt{c-\frac{c}{a^2x^2}}\,\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{\frac{1}{2}+\frac{n}{2}}}{x^2}dx,x,\frac{1}{x}\right)}{\sqrt{1-\frac{1}{a^2x^2}}}$$

$$=-\frac{2^{\frac{3}{2}-\frac{n}{2}}\sqrt{c-\frac{c}{a^2x^2}}\left(1+\frac{1}{ax}\right)^{\frac{3+n}{2}}F_1\left(\frac{3+n}{2};\frac{1}{2}(-1+n),2;\frac{5+n}{2};\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(3+n)\sqrt{1-\frac{1}{a^2x^2}}}$$

Mathematica [A]
time = 0.34, size = 146, normalized size = 0.49

$$\frac{ae^{n\coth^{-1}(ax)}\sqrt{1-\frac{1}{a^2x^2}}\sqrt{c-\frac{c}{a^2x^2}}\,x^2\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}\,x+2e^{\coth^{-1}(ax)}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-e^{2\coth^{-1}(ax)}\right)+2e^{\coth^{-1}(ax)}n\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{(1+n)\left(-1+a^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(n*ArcCoth[a*x])*Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (a*E^(n*ArcCoth[a*x])*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2*(a*(1
 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E^ArcCoth[a*x]*Hypergeometric2F1[1, (1 +
n)/2, (3 + n)/2, -E^(2*ArcCoth[a*x])] + 2*E^ArcCoth[a*x]*n*Hypergeometric2F
1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x])]))/((1 + n)*(-1 + a^2*x^2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int e^{n\,\text{arccoth}(ax)}\sqrt{c-\frac{c}{a^2x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c - c/(a^2*x^2))*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^2)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}\,e^{n\,\mathrm{acoth}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))*exp(n*acoth(a*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\sqrt{c-\frac{c}{a^2\,x^2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2),x)
```

```
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^(1/2), x)
```

3.932 $\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$

Optimal. Leaf size=183

$$\frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}}+\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}$$

```
[Out] (1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(1/2+1/2*n)*x*(1-1/a^2/x^2)^(1/2)/(c-c/a^2/
x^2)^(1/2)+2*n*(1-1/a/x)^(1/2-1/2*n)*(1+1/a/x)^(-1/2+1/2*n)*hypergeom([1, 1
/2-1/2*n],[3/2-1/2*n],(a-1/x)/(a+1/x))*(1-1/a^2/x^2)^(1/2)/a/(1-n)/(c-c/a^2
/x^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6332, 6329, 98, 133}

$$\frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n-1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\frac{a-\frac{1}{x}}{a+\frac{1}{x}}\right)}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}+\frac{x\sqrt{1-\frac{1}{a^2x^2}}\left(\frac{1}{ax}+1\right)^{\frac{n+1}{2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}}{\sqrt{c-\frac{c}{a^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n)/2)*(1 + 1/(a*x))^((1 + n)/2)*
x)/Sqrt[c - c/(a^2*x^2)] + (2*n*Sqrt[1 - 1/(a^2*x^2)]*(1 - 1/(a*x))^((1 - n
)/2)*(1 + 1/(a*x))^((-1 + n)/2)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2,
(a - x^(-1))/(a + x^(-1))])/(a*(1 - n)*Sqrt[c - c/(a^2*x^2)])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 133

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*c - a*d)^n*((a + b*x)^(m + 1)/((m + 1)*(b*e -
```

```
a*f)^(n + 1)*(e + f*x)^(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*
e - c*f))*((a + b*x)/((b*c - a*d)*(e + f*x)))], x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0] && (SumSimplerQ[m, 1]
||  !SumSimplerQ[p, 1]) &&  !ILtQ[m, 0]
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx = \frac{\sqrt{1-\frac{1}{a^2x^2}}\int \frac{e^{n\coth^{-1}(ax)}}{\sqrt{1-\frac{1}{a^2x^2}}}\,dx}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= -\frac{\sqrt{1-\frac{1}{a^2x^2}}\,\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)^{-\frac{1}{2}+\frac{n}{2}}}{x^2}\,dx, x, \frac{1}{x}\right)}{\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} - \frac{\left(n\sqrt{1-\frac{1}{a^2x^2}}\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{-\frac{1}{2}-\frac{n}{2}}\left(1+\frac{x}{a}\right)}{x}\right.}{a\sqrt{c-\frac{c}{a^2x^2}}}$$

$$= \frac{\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1+n}{2}}x}{\sqrt{c-\frac{c}{a^2x^2}}} + \frac{2n\sqrt{1-\frac{1}{a^2x^2}}\left(1-\frac{1}{ax}\right)^{\frac{1-n}{2}}\left(1+\frac{1}{ax}\right)^{\frac{1}{2}(-1+n)}}{a(1-n)\sqrt{c-\frac{c}{a^2x^2}}}$$

Mathematica [A]
time = 0.26, size = 112, normalized size = 0.61

$$\frac{e^{n\coth^{-1}(ax)}(-1+a^2x^2)\left(a(1+n)\sqrt{1-\frac{1}{a^2x^2}}\,x+2e^{\coth^{-1}(ax)}n\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};e^{2\coth^{-1}(ax)}\right)\right)}{a^3(1+n)\sqrt{1-\frac{1}{a^2x^2}}\,\sqrt{c-\frac{c}{a^2x^2}}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*ArcCoth[a*x])/Sqrt[c - c/(a^2*x^2)],x]
```

```
[Out] (E^(n*ArcCoth[a*x])*(-1 + a^2*x^2)*(a*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*x + 2*E
^ArcCoth[a*x]*n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, E^(2*ArcCoth[a*x
])]))/(a^3*(1 + n)*Sqrt[1 - 1/(a^2*x^2)]*Sqrt[c - c/(a^2*x^2)]*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{n\,\mathrm{arccoth}(ax)}}{\sqrt{c-\frac{c}{a^2x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

```
[Out] int(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(a^2*x^2*((a*x + 1)/(a*x - 1))^(1/2*n)*sqrt((a^2*c*x^2 - c)/(a^2*x^
2))/(a^2*c*x^2 - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{n \operatorname{acoth}(ax)}}{\sqrt{-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))/(c-c/a**2/x**2)**(1/2),x)
[Out] Integral(exp(n*acoth(a*x))/sqrt(-c*(-1 + 1/(a*x))*(1 + 1/(a*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))/(c-c/a^2/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(((a*x + 1)/(a*x - 1))^(1/2*n)/sqrt(c - c/(a^2*x^2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{e}^{n\,\mathrm{acoth}(a\,x)}}{\sqrt{c-\frac{c}{a^2\,x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2),x)
[Out] int(exp(n*acoth(a*x))/(c - c/(a^2*x^2))^(1/2), x)
```

3.933 $\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=116

$$-\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}F_1\left(1+\frac{n}{2}+p;\frac{1}{2}(n-2p),2;2+\frac{n}{2}+p;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(2+n+2p)}$$

```
[Out] -2^(1-1/2*n+p)*(c-c/a^2/x^2)^p*(1+1/a/x)^(1+1/2*n+p)*AppellF1(1+1/2*n+p,1/2
*n-p,2,2+1/2*n+p,1/2*(a+1/x)/a,1+1/a/x)/a/(2+n+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6332, 6329, 141}

$$-\frac{2^{-\frac{n}{2}+p+1}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(\frac{1}{ax}+1\right)^{\frac{n}{2}+p+1}F_1\left(\frac{n}{2}+p+1;\frac{1}{2}(n-2p),2;\frac{n}{2}+p+2;\frac{a+\frac{1}{x}}{2a},1+\frac{1}{ax}\right)}{a(n+2p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -((2^(1 - n/2 + p)*(c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + n/2 + p)*AppellF1
[1 + n/2 + p, (n - 2*p)/2, 2, 2 + n/2 + p, (a + x^(-1))/(2*a), 1 + 1/(a*x)]
)/(a*(2 + n + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
```

```
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx=\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{n\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$
$$=-\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int\frac{\left(1-\frac{x}{a}\right)^{-\frac{n}{2}+p}\left(1+\frac{x}{a}\right)^{\frac{n}{2}+p}}{x^2}dx\right.\right.$$
$$=-\frac{2^{1-\frac{n}{2}+p}\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+\frac{n}{2}+p}F_1\left(1+\frac{n}{2}+p;\frac{1}{2}(n-2p),\right.}{a(2+n+2p)}$$

Mathematica [F]
time = 0.56, size = 0, normalized size = 0.00

$$\int e^{n\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] Integrate[E^(n*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int e^{n\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
[Out] int(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^(1/2*n)*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p}e^{n\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*acoth(a*x))*(c-c/a**2/x**2)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(n*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^(1/2*n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{n\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(n*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

3.934 $\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1-\frac{1}{ax}\right)}{a(1+2p)}$$

```
[Out] (c-c/a^2/x^2)^p*(1-1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2+2*p],1-1/a/x)/a/(
1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {6332, 6329, 67}

$$\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(1-\frac{1}{ax}\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(2,2p+1;2(p+1);1-\frac{1}{ax}\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
```

```
[Out] ((c - c/(a^2*x^2))^p*(1 - 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p,
2*(1 + p), 1 - 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) && !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] && !IntegerQ[n/2] && !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{-2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\text{Subst}\left(\int \frac{\left(1-\frac{x}{a}\right)^{2p}}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1-\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1-\frac{1}{ax}\right)}{a(1+2p)}$$

Mathematica [**F**]
time = 0.34, size = 0, normalized size = 0.00

$$\int e^{-2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]),x]
[Out] Integrate[(c - c/(a^2*x^2))^p/E^(2*p*ArcCoth[a*x]), x]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c-\frac{c}{a^2x^2}\right)^p \mathrm{e}^{-2p\,\mathrm{arccoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
[Out] int((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="fricas")
[Out] integral(((a^2*c*x^2 - c)/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p}e^{-2p\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a**2/x**2)**p/exp(2*p*acoth(a*x)),x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(-2*p*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c-c/a^2/x^2)^p/exp(2*p*arccoth(a*x)),x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p/((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{-2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(-2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

3.935 $\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p\,dx$

Optimal. Leaf size=75

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1+\frac{1}{ax}\right)}{a(1+2p)}$$

```
[Out] -(c-c/a^2/x^2)^p*(1+1/a/x)^(1+2*p)*hypergeom([2, 1+2*p],[2+2*p],1+1/a/x)/a/
(1+2*p)/((1-1/a^2/x^2)^p)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6332, 6329, 67}

$$-\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(\frac{1}{ax}+1\right)^{2p+1}\left(c-\frac{c}{a^2x^2}\right)^p{}_2F_1\left(2,2p+1;2(p+1);1+\frac{1}{ax}\right)}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
```

```
[Out] -(((c - c/(a^2*x^2))^p*(1 + 1/(a*x))^(1 + 2*p)*Hypergeometric2F1[2, 1 + 2*p
, 2*(1 + p), 1 + 1/(a*x)])/(a*(1 + 2*p)*(1 - 1/(a^2*x^2))^p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 6329

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*((c_) + (d_.)/(x_)^2)^(p_.), x_Symbol] :>
 Dist[-c^p, Subst[Int[(1 - x/a)^(p - n/2)*((1 + x/a)^(p + n/2)/x^2), x], x,
 1/x], x] /; FreeQ[{a, c, d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n
/2] && (IntegerQ[p] || GtQ[c, 0]) &&  !IntegersQ[2*p, p + n/2]
```

Rule 6332

```
Int[E^(ArcCoth[(a_.)*(x_)]*(n_.))*(u_.)*((c_) + (d_.)/(x_)^2)^(p_), x_Symbo
l] :> Dist[c^IntPart[p]*((c + d/x^2)^FracPart[p]/(1 - 1/(a^2*x^2))^FracPart
[p]), Int[u*(1 - 1/(a^2*x^2))^p*E^(n*ArcCoth[a*x]), x], x] /; FreeQ[{a, c,
d, n, p}, x] && EqQ[c + a^2*d, 0] &&  !IntegerQ[n/2] &&  !(IntegerQ[p] || G
tQ[c, 0])
```

Rubi steps

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx = \left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\int e^{2p\coth^{-1}(ax)}\left(1-\frac{1}{a^2x^2}\right)^p dx$$
$$= -\left(\left(\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\right)\operatorname{Subst}\left(\int\frac{\left(1+\frac{x}{a}\right)^{2p}}{x^2}dx,x,\frac{1}{x}\right)\right)$$
$$= -\frac{\left(1-\frac{1}{a^2x^2}\right)^{-p}\left(c-\frac{c}{a^2x^2}\right)^p\left(1+\frac{1}{ax}\right)^{1+2p}{}_2F_1\left(2,1+2p;2(1+p);1+\frac{1}{ax}\right)}{a(1+2p)}$$

Mathematica [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int e^{2p\coth^{-1}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p,x]
[Out] Integrate[E^(2*p*ArcCoth[a*x])*(c - c/(a^2*x^2))^p, x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int e^{2p\,\mathrm{arccoth}(ax)}\left(c-\frac{c}{a^2x^2}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
[Out] int(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="maxima")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="fricas")
[Out] integral(((a*x + 1)/(a*x - 1))^p*((a^2*c*x^2 - c)/(a^2*x^2))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c\left(-1+\frac{1}{ax}\right)\left(1+\frac{1}{ax}\right)\right)^{p} e^{2p\operatorname{acoth}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*acoth(a*x))*(c-c/a**2/x**2)**p,x)
[Out] Integral((-c*(-1 + 1/(a*x))*(1 + 1/(a*x)))**p*exp(2*p*acoth(a*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*p*arccoth(a*x))*(c-c/a^2/x^2)^p,x, algorithm="giac")
[Out] integrate((c - c/(a^2*x^2))^p*((a*x + 1)/(a*x - 1))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{e}^{2\,p\,\mathrm{acoth}(a\,x)}\,\left(c-\frac{c}{a^2\,x^2}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p,x)
[Out] int(exp(2*p*acoth(a*x))*(c - c/(a^2*x^2))^p, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False



def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```